



Math for the people, by the people.

spherical derivative

Canonical name	SphericalDerivative
Date of creation	2013-03-22 14:18:36
Last modified on	2013-03-22 14:18:36
Owner	jirka (4157)
Last modified by	jirka (4157)
Numerical id	7
Author	jirka (4157)
Entry type	Definition
Classification	msc 30D30

Let $G \subset \mathbb{C}$ be a domain.

Definition. Let $f: G \rightarrow \mathbb{C}$ be a meromorphic function, then the *spherical derivative* of f , denoted f^\sharp is defined as

$$f^\sharp(z) := \frac{2|f'(z)|}{1 + |f(z)|^2}$$

for z where $f(z) \neq \infty$ and when $f(z) = \infty$ define

$$f^\sharp(z) = \lim_{\zeta \rightarrow z} f^\sharp(\zeta).$$

The second definition makes sense since a meromorphic functions has only isolated poles, and thus $f^\sharp(\zeta)$ is defined by the first equation when we are close to z . Some basic properties of the spherical derivative are as follows.

Proposition. *If $f: G \rightarrow \mathbb{C}$ is a meromorphic function then*

- f^\sharp is a continuous function,
- $f^\sharp(z) < \infty$ for all $z \in G$.

Note that sometimes the spherical derivative is also denoted as $\mu(f)(z)$ rather than $f^\sharp(z)$.

References

- [1] John B. Conway. . Springer-Verlag, New York, New York, 1978.
- [2] Theodore B. Gamelin. . Springer-Verlag, New York, New York, 2001.