complex function

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A *complex function* is a function f from a subset A of $\mathbb{C}$ to $\mathbb{C}$.

For every $z = x + iy \in A$ $(x,\, y \in \mathbb{R})$ the complex value $f(z)$ can be split into its real and imaginary parts u and v, respectively, which can be considered as real functions of two real variables:

$$f(z) \;=\; u(x,y) + iv(x,y) \tag{1}$$

The functions u and v are called the *real part* and the *imaginary part* of the complex function f, respectively. Conversely, any two functions $u(x,y)$ and $v(x,y)$ defined in some subset of $\mathbb{R}^2$ determine via (1) a complex function f.

If $f(z)$ especially is defined as a polynomial of z, then both $u(x,y)$ and $v(x,y)$ are polynomials of x and y with real coefficients.

Following are the notations for u and v that are used most commonly (the parentheses around $f(z)$ may be omitted):

$$u(x,y) \;=\; \operatorname{Re}(f(z)) = \Re(f(z))$$
$$v(x,y) \;=\; \operatorname{Im}(f(z)) = \Im(f(z))$$

The of mathematics concerning differentiable complex functions is called *function theory* or *complex analysis*.