monodromy group

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Consider an ordinary linear differential equation

$$\sum_{k=0}^{n} c_k(x) \frac{d^n y}{dx^n} = 0$$

in which the coefficients are polynomials. If c_n is not constant, then it is possible that the solutions of this equation will have branch points at the zeros of c_n. (To see if this actually happens, we need to examine the indicial equation.)

By the persistence of differential equations, the analytic continuation of a solution of this equation will be another solution. Pick a neighborhood which does not contain any zeros of c_n. Since the differential equation is of order n, there will be n independent solutions $y = f_1(x), \ldots, y = f_n(x)$. (For example, one may exhibit these solutions as power series about some point in the neighborhood.)

Upon analytic continuation back to the original neighborhood via a chain of neghborhoods, suppose that the solution $y = f_i(x)$ is taken to a solution $y = g_i(x)$. Because the solutions are linearly independent, there will exist a matrix $\{m_{ij}\}_{i,j=1}^{n}$ such that

$$g_i = \sum_{j=1}^{n} m_{ij} f_j.$$

Now consider the totality of all such matrices corresponding to all possible ways of making analytic continuations along chains which begin and end wit the original neighborhood. They form a group known as the *monodromy group* of the differential equation. The reason this set is a group is some basic facts about analytic continuation. First, there is the trivial analytic continuation which simply takes a function element to itself. This will correspond to the identity matrix. Second, we can reverse a process of analytic continuation. This will correspond to the inverse matrix. Third, we can follow continuation along one chain of neighborhoods by continuation along another chain. This will correspond to multiplying the matrices corresponding to the two chains.