Riemannian manifolds category R_M

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Defines	conformal Riemannian manifold
Defines	conformal mapping
Defines	c_R

Definition 0.1. A category $\mathcal{R}_M$ whose objects are all Riemannian manifolds R and whose morphisms are mappings between Riemannian manifolds m_R is defined as the *category of Riemannian manifolds.*

0.1 Applications of Riemannian manifolds in mathematical physics

1. The *conformal Riemannian subcategory* $\mathcal{R}_C$ of $\mathcal{R}_M$, whose objects are Riemannian manifolds R, and whose morphisms are conformal mappings of Riemannian manifolds c_R, is an important category for mathematical physics, in conformal theories.

2. It can be shown that, if (R_1, g) and (R_2, h) are Riemannian manifolds, then a map $f\colon R_1 \to R_2$ is `http://planetmath.org/ConformalMapping`conformal iff $f^*h = s.g$ for some scalar field s (on R_1), where f^* is the complex conjugate of f.

0.1.1 Category of pseudo-Riemannian manifolds

The category of `http://planetmath.org/PseudoRiemannianManifold`pseudo-Riemannian manifolds $\mathcal{R}_P$ that generalize Minkowski spaces M_k is similarly defined by replacing the Riemanian manifolds R in the above definition with pseudo-Riemannian manifolds R_P. Pseudo-Riemannian manifolds R_Ps were claimed to have applications in Einstein's theory of general relativity (GR), whereas the subcategory **Mink** of four-dimensional Minkowski spaces in $\mathcal{R}_P$ plays the central role in special relativity (SR) theories.