Teichmüller space

Canonical name	TeichmullerSpace
Date of creation	2013-03-22 14:19:48
Last modified on	2013-03-22 14:19:48
Owner	jirka (4157)
Last modified by	jirka (4157)
Numerical id	8
Author	jirka (4157)
Entry type	Definition
Classification	msc 30F60
Defines	Teichmüller metric
Defines	Teichmüller equivalence
Defines	Teichmüller equivalent

Definition. Let S_0 be a Riemann surface. Consider all pairs (S, f) where S is a Riemann surface and f is a sense-preserving quasiconformal mapping of S_0 onto S. We say $(S_1, f_1) \sim (S_2, f_2)$ if $f_2 \circ f_1^{-1}$ is homotopic to a conformal mapping of S_1 onto S_2. In this case we say that (S_1, f_1) and (S_2, f_2) are *Teichmüller equivalent*. The space of equivalence classes under this relation is called the *Teichmüller space* $T(S_0)$ and (S_0, I) is called the initial point of $T(S_0)$. The equivalence relation is called *Teichmüller equivalence*.

Definition. There exists a natural *Teichmüller metric* on $T(S_0)$, where the distance between (S_1, f_1) and (S_2, f_2) is $\log K$ where K is the smallest maximal dilatation of a mapping homotopic to $f_2 \circ f_1^{-1}$.

There is also a natural isometry between $T(S_0)$ and $T(S_1)$ defined by a quasiconformal mapping of S_0 onto S_1. The mapping $(S, f) \mapsto (S, f \circ g)$ induces an isometric mapping of $T(S_1)$ onto $T(S_0)$. So we could think of $T(\cdot)$ as a contravariant functor from the category of Riemann surfaces with quasiconformal maps to the category of Teichmüller spaces (as a subcategory of metric spaces).

References

[1] L. V. Ahlfors. . Van Nostrand-Reinhold, Princeton, New Jersey, 1966