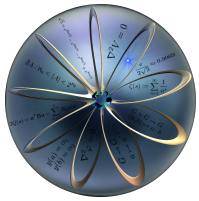

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Proof of Möbius transformation cross-ratio preservation theorem

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From the definition of Möbius transform we get that the image w_k of a point z_k is

$$w_k = \frac{az_k + b}{cz_k + d}$$

From this we get

$$w_i - w_j = \frac{az_i + b}{cz_i + d} - \frac{az_j + b}{cz_j + d} = \frac{(ad - bc)(z_i - z_j)}{(cz_i + d)(cz_j + d)}$$

and by inserting this into the cross-ratios

$$\frac{(w_1 - w_2)(w_3 - w_4)}{(w_1 - w_4)(w_3 - w_2)} = \frac{\frac{(ad-bc)(z_1-z_2)}{(cz_1+d)(cz_2+d)} \frac{(ad-bc)(z_3-z_4)}{(cz_3+d)(cz_4+d)}}{\frac{(ad-bc)(z_1-z_4)}{(cz_1+d)(cz_4+d)} \frac{(ad-bc)(z_3-z_2)}{(cz_3+d)(cz_2+d)}} = \frac{(z_1 - z_2)(z_3 - z_4)}{(z_1 - z_4)(z_3 - z_2)}$$