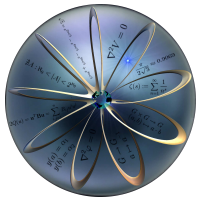

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zero as contour integral

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Suppose that f is a complex function which is defined in some open set $D \subseteq \mathbb{C}$ which has a simple zero at some point $p \in D$. Then we have

$$p = \frac{1}{2\pi i} \oint_C \frac{z f'(z)}{f(z)} \, dz$$

where C is a closed path in D which encloses p but does not enclose or pass through any other zeros of f.

This follows from the Cauchy residue theorem. We have that the poles of f'/f occur at the zeros of f and that the residue of a pole of f'/f is 1 at a simple zero of f. Hence, the residue of $zf'(z)/f(z)$ at p is p, so the above follows from the residue theorem.