convergence condition of infinite product

Canonical name	ConvergenceConditionOfInfiniteProduct
Date of creation	2013-03-22 14:37:22
Last modified on	2013-03-22 14:37:22
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	16
Author	pahio (2872)
Entry type	Theorem
Classification	msc 30E20
Related topic	OrderOfFactorsInInfiniteProduct
Related topic	NecessaryConditionOfConvergence
Defines	infinite product
Defines	value of infinite product

Let us think the sequence $u_1, u_1u_2, u_1u_2u_3, \dots$ In the complex analysis, one often uses the definition of the convergence of an *infinite product* $\prod_{k=1}^{\infty} u_k$ where the case $\lim_{k\to\infty} u_1u_2\dots u_k = 0$ is excluded. Then one has the

Theorem. The infinite product $\prod_{k=1}^{\infty} u_k$ of the non-zero complex numbers u_1, u_2, ... is convergent iff for every positive number ε there exists a positive number n_ε such that the condition

$$|u_{n+1}u_{n+2}\dots u_{n+p} - 1| < \varepsilon \quad \forall\, p \in \mathbb{Z}_+$$

is true as soon as $n \geqq n_\varepsilon$.

Corollary. If the infinite product converges, then we necessarily have $\lim_{k\to\infty} u_k = 1$. (Cf. the necessary condition of convergence of series.)

When the infinite product converges, we say that the *value of the infinite product* is equal to $\lim_{k\to\infty} u_1u_2\dots u_k$.