



Hadamard three-circle theorem

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Let $f(z)$ be a complex analytic function on the annulus $r_1 \leq |z| \leq r_3$. Let $M(r)$ be the maximum of $|f(z)|$ on the circle $|z| = r$. Then $\log M(r)$ is a convex function of $\log r$. Moreover, if $f(z)$ is not of the form cz^λ for some λ , then $\log M(r)$ is a <http://planetmath.org/ConvexFunction> strictly convex as a function of $\log r$.

The conclusion of the theorem can be restated as

$$\log \frac{r_3}{r_1} \log M(r_2) \leq \log \frac{r_3}{r_2} \log M(r_1) + \log \frac{r_2}{r_1} \log M(r_3)$$

for any three concentric circles of radii $r_1 < r_2 < r_3$.