sum of values of holomorphic function

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Let $w(z)$ be a holomorphic function on a simple closed curve C and inside it. If a_1, a_2, $\ldots$, a_m are inside C the simple zeros of a function $f(z)$ holomorphic on C and inside, then

$$w(a_1)+w(a_2)+\ldots+w(a_m) \;=\; \frac{1}{2i\pi}\oint_C w(z)\frac{f'(z)}{f(z)}\,dz \qquad (1)$$

where the contour integral is taken anticlockwise.

The if some of the zeros are multiple and are counted with http://planetmath.org/Polemultip

If the zeros a_j of $f(z)$ have the multiplicities α_j and the function has inside C also the poles b_1, b_2, $\ldots$, b_n with the multiplicities β_1, β_2, $\ldots$, β_n, then (1) must be written

$$\sum_j \alpha_j w(a_j) - \sum_k \beta_k w(b_k) \;=\; \frac{1}{2i\pi}\oint_C w(z)\frac{f'(z)}{f(z)}\,dz. \qquad (2)$$

The special case $w(z) \equiv 1$ gives from (2) the argument principle.

References

[1] Ernst Lindelöf: *Le calcul des résidus et ses applications à la théorie des fonctions.* Gauthier-Villars, Paris (1905).