Heaviside step function

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The *Heaviside step function* is the function $H : \mathbb{R} \to \mathbb{R}$ defined as

$$H(x) = \begin{cases} 0 & \text{when } x < 0, \\ 1/2 & \text{when } x = 0, \\ 1 & \text{when } x > 0. \end{cases}$$

Here, there are many conventions for the value at $x = 0$. The motivation for setting $H(0) = 1/2$ is that we can then write H as a function of the signum function (see `http://planetmath.org/SignumFunction`this page). In applications, such as the Laplace transform, where the Heaviside function is used extensively, the value of $H(0)$ is irrelevant. The Fourier transform of heaviside function is

$$\mathcal{F}_0 H(t) = \frac{1}{2}\left(\delta(t) - \frac{i}{\pi t}\right)$$

where δ denotes the Dirac delta centered at 0. The function is named after Oliver Heaviside (1850-1925) [**?**]. However, the function was already used by Cauchy[**?**], who defined the function as

$$u(t) = \frac{1}{2}\big(1 + t/\sqrt{t^2}\big)$$

and called it a *coefficient limitateur* [**?**].

References

[1] The MacTutor History of Mathematics archive, `http://www-gap.dcs.st-and.ac.uk/ history/Mathematicians/Heaviside.html`Oliver Heaviside.

[2] The MacTutor History of Mathematics archive, `http://www-gap.dcs.st-and.ac.uk/ history/Mathematicians/Cauchy.html`Augustin Louis Cauchy.

[3] R.F. Hoskins, *Generalised functions*, Ellis Horwood Series: Mathematics and its applications, John Wiley & Sons, 1979.