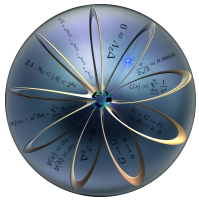

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table of partial fraction expansions

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The purpose of this entry is to collect a table of Mittag-Leffler type partial fraction expansions for various functions.

1 Elementary Functions

$$\pi \cot(\pi z) = \frac{1}{z} + \sum_{n=1}^{\infty} \left(\frac{1}{z-n} + \frac{1}{z+n} \right) \tag{1}$$

$$\pi \sec(\pi z) = \frac{2}{1-2z} + \sum_{k=1}^{\infty} (-1)^{k+1} \left(\frac{2}{2k-1-2z} - \frac{2}{2k+1-2z} \right) \tag{2}$$

$$\tag{3}$$

2 Hypergeometric Functions

$${}_2F_1(z, 1; z+1; w) = \sum_{k=0}^{\infty} \frac{w^k}{z+k} \tag{4}$$

$$\tag{5}$$

3 Gamma Functions

$$\psi(z) = \frac{\Gamma'(z)}{\Gamma(z)} + \gamma = \frac{1}{z} + \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{z+k} \right) \tag{6}$$

$$(-1)^n \frac{\psi^{(n)}(z)}{n!} = \sum_{k=0}^{\infty} \frac{1}{(z+k)^n} \tag{7}$$

$$\frac{\Gamma(x)\Gamma(\frac{1}{2})}{\Gamma(x+\frac{1}{2})} = \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2} \frac{1}{x+n} \tag{8}$$

$$\tag{9}$$

Here γ is Mascheroni's constant.

4 Elliptic Functions

$$\wp\left(z\left|\frac{1}{2}\omega,\frac{1}{2}\omega'\right.\right) = \frac{1}{z^2}+\sum_{|k|+|k'|\neq 0}\left(\frac{1}{(z-k\omega-k'\omega')^2}-\frac{1}{(k\omega+k'\omega')^2}\right) \quad (10)$$

$$(11)$$