incomplete gamma function

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The *incomplete gamma function* is defined as the indefinite integral of the integrand of gamma integral. There are several definitions which differ in details of normalization and constant of integration:

$$
\begin{aligned}
\gamma(a,x) &= \int_0^x e^{-t}t^{a-1}\,dt \\
\Gamma(a,x) &= \int_x^\infty e^{-t}t^{a-1}\,dt = \Gamma(a) - \gamma(a,x) \\
P(a,x) &= \frac{1}{\Gamma(a)}\int_0^x e^{-t}t^{a-1}\,dt = \frac{\gamma(a,x)}{\Gamma(a)} \\
\gamma^*(a,x) &= \frac{x^{-a}}{\Gamma(a)}\int_0^x e^{-t}t^{a-1}\,dt = \frac{\gamma(a,x)}{x^a\Gamma(a)} \\
I(a,x) &= \frac{1}{\Gamma(a+1)}\int_0^{x\sqrt{a+1}} e^{-t}t^{a}\,dt = \frac{\gamma(a+1, x\sqrt{a+1})}{\Gamma(a+1)} \\
C(a,x) &= \int_x^\infty t^{a-1}\cos t\,dt \\
S(a,x) &= \int_x^\infty t^{a-1}\sin t\,dt \\
E_n(x) &= \int_1^\infty e^{-xt}t^{-n}\,dt = x^{n-1}\Gamma(1-n) - x^{n-1}\gamma(1-n,x) \\
\alpha_n(x) &= \int_1^\infty e^{-xt}t^{n}\,dt = x^{-n-1}\Gamma(1+n) - x^{-n-1}\gamma(1+n,x)
\end{aligned}
$$

For convenience of translating notations, these variants have been expressed in terms of γ.