



Möbius transformation

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Entry type	Definition
Classification	msc 30D99
Synonym	fractional linear transformation
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Related topic	ProofOfConformalMobiusCircleMapTheorem
Related topic	AutomorphismsOfUnitDisk
Related topic	UnitDiskUpperHalfPlaneConformalEquivalenceTheorem
Related topic	InversionOfPlane

A *Möbius transformation* is a bijection on the extended complex plane $\mathbb{C} \cup \{\infty\}$ given by

$$f(z) = \begin{cases} \frac{a}{c} & \text{if } z = \infty \\ \infty & \text{if } z = -\frac{d}{c} \\ \frac{az+b}{cz+d} & \text{otherwise} \end{cases}$$

where $a, b, c, d \in \mathbb{C}$ and $ad - bc \neq 0$

It can be shown that the inverse, and composition of two Möbius transformations are similarly defined, and so the Möbius transformations form a group under composition.

The geometric interpretation of the Möbius group is that it is the group of automorphisms of the Riemann sphere.

Any Möbius map can be composed from the elementary transformations - dilations, translations and inversions. If we define a line to be a circle passing through ∞ then it can be shown that a Möbius transformation maps circles to circles, by looking at each elementary transformation.