Marty's theorem

Canonical name	MartysTheorem
Date of creation	2013-03-22 14:18:39
Last modified on	2013-03-22 14:18:39
Owner	jirka (4157)
Last modified by	jirka (4157)
Numerical id	7
Author	jirka (4157)
Entry type	Theorem
Classification	msc 30D30

Theorem (Marty). *A set $\mathcal{F}$ of meromorphic functions is a normal family on a domain $G \subset \mathbb{C}$ if and only if the spherical derivatives are uniformly bounded (uniformly over $\mathcal{F}$) on each compact subset of G.*

Here normal convergence (convergence on compact subsets) is given using the spherical metric and not the standard metric of the complex plane. That is, if σ is the spherical metric then we will say $f_n \to f$ normally if $\sigma(f_n(z), f(z))$ converges to 0 uniformly on compact subsets.

A related theorem can be stated.

Theorem. *If $f_n(z) \to f(z)$ uniformly in the spherical metric on compact subsets of $G \subset \mathbb{C}$ then $f_n^\sharp(z) \to f^\sharp(z)$ uniformly on compact subsets of G.*

Here $f^\sharp$ denotes the spherical derivative of f.

References

[1] Theodore B. Gamelin. . Springer-Verlag, New York, New York, 2001.