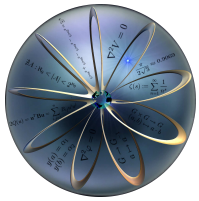

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proof of Gauss' mean value theorem

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We can parameterize the circle by letting $z = z_0 + re^{i\phi}$. Then $dz = ire^{i\phi}d\phi$. Using the Cauchy integral formula we can express $f(z_0)$ in the following way:

$$\begin{aligned} f(z_0) &= \frac{1}{2\pi i}\oint_C \frac{f(z)}{z - z_0}dz \\ &= \frac{1}{2\pi i}\int_0^{2\pi} \frac{f(z_0 + re^{i\phi})}{re^{i\phi}} ire^{i\phi}d\phi \\ &= \frac{1}{2\pi}\int_0^{2\pi} f(z_0 + re^{i\phi})d\phi. \end{aligned}$$