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## incomplete gamma function recurrence formula

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The incomplete gamma function satisfies the following recurrence formula:

$$\gamma(a+1, x) = a\gamma(a, x) - x^a e^{-x}$$

This can be derived by integration by parts:

$$\begin{aligned} \int_0^x e^{-t} t^a dt &= - \int_0^x t^a de^{-t} \\ &= a \int_0^x t^{a-1} de^{-t} - x^a e^{-x} \end{aligned}$$

In terms of other variants of the incomplete gamma function, the recursion relation appears as follows:

$$\begin{aligned} P(a+1, x) &= P(a, x) - \frac{x^a e^{-x}}{\Gamma(a+1)} \\ \gamma^*(a-1, x) &= x\gamma^*(a, x) + \frac{e^{-x}}{\Gamma(a)} \end{aligned}$$