



planetmath.org

Math for the people, by the people.

essential singularity

| | |
|------------------|-------------------------------------|
| Canonical name | EssentialSingularity |
| Date of creation | 2013-03-22 13:32:10 |
| Last modified on | 2013-03-22 13:32:10 |
| Owner | pbruin (1001) |
| Last modified by | pbruin (1001) |
| Numerical id | 7 |
| Author | pbruin (1001) |
| Entry type | Definition |
| Classification | msc 30D30 |
| Related topic | LaurentSeries |
| Related topic | Pole |
| Related topic | RemovableSingularity |
| Related topic | PicardsTheorem |
| Related topic | RiemannsRemovableSingularityTheorem |

Let $U \subset \mathbb{C}$ be a domain, $a \in U$, and let $f : U \setminus \{a\} \rightarrow \mathbb{C}$ be holomorphic. If the Laurent series expansion of $f(z)$ around a contains infinitely many terms with negative powers of $z - a$, then a is said to be an *essential singularity* of f . Any singularity of f is a removable singularity, a pole or an essential singularity.

If a is an essential singularity of f , then the image of any punctured neighborhood of a under f is dense in \mathbb{C} (the Casorati-Weierstrass theorem). In fact, an even stronger statement is true: according to Picard's theorem, the image of any punctured neighborhood of a is \mathbb{C} , with the possible exception of a single point.