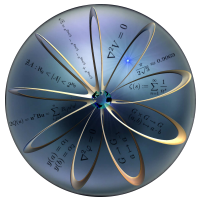

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example of harmonic functions on graphs

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1. Let $G = (V, E)$ be a connected finite graph, and let $a, z \in V$ be two of its vertices. The function

$$f(v) = \mathbb{P}\{\text{simple random walk from } v \text{ hits } a \text{ before } z\}$$

is a harmonic function except on $\{a, z\}$.

Finiteness of G is required only to ensure f is well-defined. So we may replace "G finite" with "simple random walk on G is recurrent".

2. Let $G = (V, E)$ be a graph, and let $V' \subseteq V$. Let $\alpha : V' \to \mathbb{R}$ be some boundary condition. For $u \in V$, define a random variable X_u to be the first vertex of V' that simple random walk from u hits. The function

$$f(v) = \mathbb{E}\,\alpha(X_v)$$

is a harmonic function except on V'.

The first example is a special case of this one, taking $V' = \{a, z\}$ and $\alpha(a) = 1, \alpha(z) = 0$.