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annulus

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Related topic	Annulus
Defines	closed annulus

An *annulus* is the region bounded between two (usually concentric) circles.

An *open annulus* is a domain in the complex plane of the form

$$A = A_w(r, R) = \{z \in \mathbb{C} : r < |z - w| < R\},$$

where w is an arbitrary complex number, and r and R are real numbers with $0 < r < R$. Such a set is often called an *annular region*.

It should be noted that the annulus usually refers to an open annulus.

More generally, one can allow $r = 0$ or $R = \infty$. (This makes sense for the purposes of the bound on $|z - w|$ above.) This would make an annulus include the cases of a punctured disc, and some unbounded domains.

Analogously, a *closed annulus* is a set of the form

$$\overline{A} = \overline{A}_w(r, R) = \{z \in \mathbb{C} : r \leq |z - w| \leq R\},$$

where $w \in \mathbb{C}$, and r and R are real numbers with $0 < r < R$.

One can show that two annuli $D_w(r, R)$ and $D_{w'}(r', R')$ are conformally equivalent if and only if $R/r = R'/r'$. More generally, the complement of any closed disk in an open disk is conformally equivalent to precisely one annulus of the form $D_0(r, 1)$.