antiholomorphic

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A complex function $f : D \to \mathbb{C}$, where D is a domain of the complex plane, having the derivative

$$\frac{df}{d\overline{z}}$$

in each point z of D, is said to be *antiholomorphic* in D.

The following conditions are http://planetmath.org/Equivalent3equivalent:

- $f(z)$ is antiholomorphic in D.
- $\overline{f(z)}$ is holomorphic in D.
- $f(\overline{z})$ is holomorphic in $\overline{D} := \{\overline{z} \vdots\ z \in D\}$.
- $f(z)$ may be to a power series $\sum_{n=0}^{\infty} a_n(\overline{z}-u)^n$ at each $u \in D$.
- The real part $u(x,\,y)$ and the imaginary part $v(x,\,y)$ of the function f satisfy the equations

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}, \qquad \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}.$$

  N.B. the of minus; cf. the http://planetmath.org/CauchyRiemannEquationsCauchy–Riemann equations.

Example. The function $z \mapsto \dfrac{1}{\overline{z}}$ is antiholomorphic in $\mathbb{C}\smallsetminus\{0\}$. One has

$$f(z) = \frac{z}{|z|^2} = \underbrace{\frac{x}{x^2+y^2}}_{u} + i\,\underbrace{\frac{y}{x^2+y^2}}_{v}$$

and thus

$$\frac{\partial u}{\partial x} = \frac{y^2-x^2}{(x^2+y^2)^2}, \qquad \frac{\partial v}{\partial y} = \frac{x^2-y^2}{(x^2+y^2)^2}, \qquad \frac{\partial u}{\partial y} = -\frac{2xy}{(x^2+y^2)^2}, \qquad \frac{\partial v}{\partial x} = -\frac{2xy}{(x^2+y^2)^2}.$$