# monodromy theorem


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Let $C(t)$ be a one-parameter family of smooth paths in the complex plane with common endpoints $z_0$ and $z_1$. (For definiteness, one may suppose that the parameter $t$ takes values in the interval $[0, 1]$.) Suppose that an analytic function $f$ is defined in a neighborhood of $z_0$ and that it is possible to analytically continue $f$ along every path in the family. Then the result of analytic continuation does not depend on the choice of path.

Note that it is *crucial* that it be possible to continue $f$ along all paths of the family. As the following example shows, the result will no longer hold if it is impossible to analytically continue $f$ along even a single path. Let the family of paths be the set of circular arcs (for the present purpose, the straight line is to be considered as a degenerate case of a circular arc) with endpoints $+1$ and $-1$ and let $f(z) = \sqrt{z}$. It is possible to analytically continue $f$ along every arc in the family except the line segment passing through 0. The conclusion of the theorem does not hold in this case because continuing along arcs which lie above 0 leads to $f(z_1) = +i$ whilst continuing along arcs which lie below 0 leads to $f(z_1) = -i$.