Cauchy residue theorem

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Let $U \subset \mathbb{C}$ be a simply connected domain, and suppose f is a complex valued function which is defined and analytic on all but finitely many points $a_1, \ldots, a_m$ of U. Let C be a closed curve in U which does not intersect any of the a_i. Then

$$\int_C f(z)\ dz = 2\pi i \sum_{i=1}^{m} \eta(C, a_i)\, \mathrm{Res}(f; a_i),$$

where

$$\eta(C, a_i) := \frac{1}{2\pi i} \int_C \frac{dz}{z - a_i}$$

is the winding number of C about a_i, and $\mathrm{Res}(f; a_i)$ denotes the residue of f at a_i.

The Cauchy residue theorem generalizes both the Cauchy integral theorem (because analytic functions have no poles) and the Cauchy integral formula (because $f(x)/(x-a)^n$ for analytic f has exactly one pole at $x = a$ with residue $\mathrm{Res}(f(x)/(x-a)^n, a) = f^{(n)}(a)/n!$).