planetmath.org

Math for the people, by the people.

normal family

Canonical name	NormalFamily
Date of creation	2013-03-22 14:17:49
Last modified on	2013-03-22 14:17:49
Owner	jirka (4157)
Last modified by	jirka (4157)
Numerical id	7
Author	jirka (4157)
Entry type	Definition
Classification	msc 30A99

Definition. A set (sometimes called a family) $\mathcal{F}$ of continuous functions $f\colon X \to Y$ for some (`http://planetmath.org/Complete`complete) metric spaces X and Y is called if each sequence of functions in $\mathcal{F}$ contains a subsequence which converges uniformly on compact subsets of X to a continuous function from X to Y.

This definition is often used in complex analysis for spaces of holomorphic functions. It turns out that a sequence of holomorphic functions that converges uniformly on compact sets converges to a holomorphic function. So you can replace X with a region in the complex plane, Y with the complex plane itself and every instance of "continuous" with "holomorphic" and you get a version of the definition most used in complex analysis.

Another space where this is often used is the space of meromorphic functions. This is similar to the holomorphic case, but instead of using the standard metric for convergence we must use the spherical metric. That is if σ is the spherical metric, then want $f_n(z) \to f(z)$ uniformly on compact subsets to mean that $\sigma(f_n(z), f(z))$ goes to 0 uniformly on compact subsets.

Note that this is a classical definition that, while very often used, is not really with modern naming. In more modern , one would give a metric on the space of continuous (holomorphic) functions that corresponds to convergence on compact subsets and then you'd say "precompact set of functions" in such a metric space instead of saying " family of continuous (holomorphic) functions". This added generality however makes it more cumbersome to use since one would need to define the metric mentioned above.

References

[1] John B. Conway. . Springer-Verlag, New York, New York, 1978.