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antiderivative of complex function

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By the of a complex function  $f$  in a domain  $D$  of  $\mathbb{C}$ , we every complex function  $F$  which in  $D$  satisfies the condition

$$\frac{d}{dz}F(z) = f(z).$$

- If  $f$  is a continuous complex function in a domain  $D$  and if the integral

$$F(z) := \int_{\gamma_z} f(t) dt \tag{1}$$

where the path  $\gamma_z$  begins at a fixed point  $z_0$  of  $D$  and ends at the point  $z$  of  $D$ , is independent of the path  $\gamma_z$  for each value of  $z$ , then (1) defines an analytic function  $F$  with domain  $D$ . This function is an antiderivative of  $f$  in  $D$ , <http://planetmath.org/Iei>.e. at all points of  $D$ , the condition

$$\frac{d}{dz} \int_{\gamma_z} f(t) dt = f(z)$$

is true.

- If  $f$  is an analytic function in a simply connected open domain  $U$ , then  $f$  has an antiderivative in  $U$ , <http://planetmath.org/Ege>.e.g. the function  $F$  defined by (1) where the path  $\gamma_z$  is within  $U$ . If  $\gamma$  lies within  $U$  and connects the points  $z_0$  and  $z_1$ , then

$$\int_{\gamma} f(z) dz = F(z_1) - F(z_0),$$

where  $F$  is an arbitrary antiderivative of  $f$  in  $U$ .