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## proof of closed curve theorem

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Let

$$f(x + iy) = u(x, y) + iv(x, y).$$

Hence we have

$$\int_C f(z) dz = \int_C \omega + i \int_C \eta$$

where  $\omega$  and  $\eta$  are the differential forms

$$\omega = u(x, y) dx - v(x, y) dy, \quad \eta = v(x, y) dx + u(x, y) dy.$$

Notice that by Cauchy-Riemann equations  $\omega$  and  $\eta$  are closed differential forms. Hence by the lemma on closed differential forms on a simply connected domain we get

$$\int_{C_1} \omega = \int_{C_2} \omega, \quad \int_{C_1} \eta = \int_{C_2} \eta.$$

and hence

$$\int_{C_1} f(z) dz = \int_{C_2} f(z) dz$$