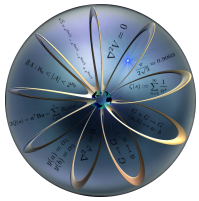

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expressible in closed form

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Defines	closed form

An expression is , if it can be converted (simplified) into an expression containing only elementary functions, combined by a finite amount of rational operations and compositions. Thus, such a closed form must not e.g. limit signs, integral signs, sum signs and "...".

For example,

$$\int \frac{dx}{x^4+1},$$

may be expressed in the closed form

$$\frac{1}{4\sqrt{2}} \ln \frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1} + \frac{1}{2\sqrt{2}} \arctan \frac{x\sqrt{2}}{1-x^2} + C$$

but for

$$\int \frac{dx}{\sqrt{x^4+1}}\, dx,$$

there exists no closed form.

In certain contexts, the of the "elementary functions" may be enlarged by allowing in it some other functions, e.g. the error function.