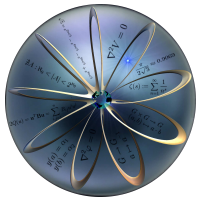

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limit for exp(z)

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For any complex number z, we have

$$\lim_{n\to\infty}\left(1+\frac{z}{n}+o\left(\frac{1}{n}\right)\right)^n=\exp z,$$

where exp denotes the exponential function.
Proof: For $\alpha\to 0$, we have

$$\begin{aligned}\ln(1+\alpha)&=\sum_{k=1}^{\infty}(-1)^{k-1}\cdot\frac{\alpha^k}{k}\\&=\alpha+O(\alpha^2).\end{aligned}$$

Therefore

$$\begin{aligned}\left(1+\frac{z}{n}+o\left(\frac{1}{n}\right)\right)^n&=\exp\left(n\ln\left(1+\frac{z}{n}+o\left(\frac{1}{n}\right)\right)\right)\\&=\exp\left(n\left(\frac{z}{n}+o\left(\frac{1}{n}\right)+O(\frac{1}{n^2})\right)\right)\\&=\exp\left(z+o(1)+O(\frac{1}{n})\right)\to\exp z\quad\text{for}\quad n\to\infty.\quad\square\end{aligned}$$