antiperiodic function

Canonical name	AntiperiodicFunction
Date of creation	2015-12-16 15:19:14
Last modified on	2015-12-16 15:19:14
Owner	pahio (2872)
Last modified by	pahio (2872)
Numerical id	12
Author	pahio (2872)
Entry type	Definition
Classification	msc 30A99
Related topic	PeriodicFunctions
Related topic	QuasiperiodicFunction
Defines	antiperiodicity
Defines	antiperiodic
Defines	antiperiod

A special case of the `http://planetmath.org/Period3`quasiperiodicity of functions is the *antiperiodicity.* An *antiperiodic* function f satisfies for a certain constant p the equation

$$f(z+p) \;=\; -f(z)$$

for all values of the variable z. The constant p is the *antiperiod* of f. Then, f has also other antiperiods, e.g. $-p$, and generally $(2n{+}1)p$ with any $n \in \mathbb{Z}$.

The antiperiodic function f is always as well periodic with period $2p$, since

$$f(z+2p) \;=\; f((z+p)+p) \;=\; -f(z+p) \;=\; -(-f(z)) \;=\; f(z).$$

Naturally, then there are all periods $2np$ with $n \in \mathbb{Z}$.

Not all periodic functions are antiperiodic.

For example, the sine and cosine functions are antiperiodic with $p = \pi$, which is their absolutely least antiperiod:

$$\sin(z+\pi) \;=\; -\sin z, \qquad \cos(z+\pi) \;=\; -\cos z$$

The `http://planetmath.org/Trigonometry`tangent and cotangent functions are not antiperiodic although they are periodic (with the prime period π; see complex tangent and cotangent).

The exponential function is antiperiodic with the antiperiod $i\pi$ (see Euler relation):

$$e^{z+i\pi} \;=\; e^z e^{i\pi} \;=\; -e^z$$