



Math for the people, by the people.

isolated singularity

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Let $\mathbb{C} \cup \{\infty\}$ denote the Riemann sphere, and let $U \subset \mathbb{C}$ be open. Let $f: U \rightarrow \mathbb{C} \cup \{\infty\}$ be a function. We say that z is an *isolated singularity* of f if there exists an open set $V \subset U$ containing z and such that f is analytic on $V \setminus \{z\}$.

In other , if we take the set S of points in U where f is *not* analytic, the isolated singularities are exactly the isolated points of S in the usual topological sense.

There are three kinds of isolated singularities:

- removable singularities $\left(\text{e.g. } z = 0 \text{ for the function } \frac{\sin z}{z} \right)$
- poles $\left(\text{e.g. } z = 0 \text{ for the function } \frac{1}{z^2} \right)$
- essential singularities $\left(\text{e.g. } z = 0 \text{ for the function } \exp \frac{1}{z} \right)$