quasiperiodic function

Canonical name	QuasiperiodicFunction
Date of creation	2013-03-22 14:40:38
Last modified on	2013-03-22 14:40:38
Owner	rspuzio (6075)
Last modified by	rspuzio (6075)
Numerical id	13
Author	rspuzio (6075)
Entry type	Definition
Classification	msc 30A99
Related topic	ComplexTangentAndCotangent
Related topic	CounterperiodicFunction
Defines	quasiperiod
Defines	quasiperiodicity
Defines	period
Defines	periodic function
Defines	periodic
Defines	periodicity

A function f is said to have a *quasiperiod* p if there exists a function g such that

$$f(z+p) = g(z)f(z).$$

In the special case where g is identically equal to 1, we call f a *periodic function*, and we say that p is a *period* of f or that f has *periodicity* p.

Except for the special case of periodicity noted above, the notion of quasiperiodicity is somewhat loose and fuzzy. Strictly speaking, many functions could be regarded as quasiperiodic if one defines $g(z) = f(z+p)/f(z)$. In order for the term "quasiperiodic" not to be trivial, it is customary to reserve its use for the case where the function g is, in some vague, intuitive sense, simpler than the function f. For instance, no one would call the function $f(z) = z^2 + 1$ quasiperiodic even though it meets the criterion of the definition if we set $g(z) = (z^2 + 2z + 2)/(z^2 + 1)$ because the rational function g is "more complicated" than the polynomial f. On the other hand, for the gamma function, one would say that 1 is a quasiperiod because $\Gamma(z+1) = z\Gamma(z)$ and the function $g(z) = z$ is a "much simpler" function than the gamma function.

Note that the every complex number can be said to be a quasiperiod of the exponential function. The term "quasiperiod" is most frequently used in connection with theta functions.

Also note that almost periodic functions are quite a different affair than quasiperiodic functions — there, one is dealing with a precise notion.