analytic curve

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There are several somewhat different definitions of the word *analytic curve* depending on context. In the context of a real analytic manifold (for example $\mathbb{R}^n$), the most generic definition is perhaps the following.

Definition. Suppose X is a real analytic manifold. A curve $\gamma \subset X$ is an *analytic curve* if it is a real analytic submanifold of dimension 1. Equivalently if near each point $p \in \gamma$, there exists a real analytic mapping $f\colon (-1, 1) \to X$, such that f has nonvanishing differential and maps onto a neighbourhood of p in γ.

It is sometimes common to equate the mapping f and the curve γ. If the curve is as above but instead in the complex plane, we can instead make the following equivalent definition.

Definition. A curve $\gamma \subset \mathbb{C}$ is said to be an *analytic curve* (or *analytic arc*) if every point of γ has an open neighbourhood Δ for which there is an onto conformal map $f\colon \mathbb{D} \to \Delta$ (where $\mathbb{D} \subset \mathbb{C}$ is the unit disc) such that $\mathbb{D} \cap \mathbb{R}$ is mapped onto $\Delta \cap \gamma$ by f.

Other words for this concept are *smooth analytic curve*, in which case the word *analytic curve* would be reserved for curves with singularities. That is, for real analytic subvarieties of X. Some authors will emphasize the fact that this is a real curve and say *real analytic curve*.

In the context of subvarieties the following definition may be used.

Definition. An *analytic curve* is a complex analytic subvariety of dimension 1 of a complex manifold.

Note that locally all complex analytic subvarieties of dimension 1 in $\mathbb{C}^2$ can be parametrized by a the Puiseux parametrization theorem. Perhaps that is why there is the confusion in using the term.

References

[1] Theodore B. Gamelin. . Springer-Verlag, New York, New York, 2001.

[2] Hassler Whitney. . Addison-Wesley, Philippines, 1972.