conformal mapping

Canonical name	ConformalMapping
Date of creation	2013-03-22 13:35:42
Last modified on	2013-03-22 13:35:42
Owner	rspuzio (6075)
Last modified by	rspuzio (6075)
Numerical id	19
Author	rspuzio (6075)
Entry type	Definition
Classification	msc 30E20
Classification	msc 53A30
Related topic	QuasiconformalMapping
Related topic	ConformalMappingTheorem
Related topic	AngleBetweenTwoLines
Related topic	SchwarzChristoffelTransformation
Related topic	CategoryOfRiemannianManifolds
Defines	conformal
Defines	inversely conformal

A mapping $f\colon \mathbb{R}^m \to \mathbb{R}^n$ which preserves the magnitude and orientation of the angles between any two curves which intersect in a given point z_0 is said to be conformal at z_0. A mapping that is conformal at any point in a domain D is said to be conformal in D.

An important special case is when $m = n = 2$. In this case, we may identify $\mathbb{R}^2$ with $\mathbb{C}$ and speak of conformal mappings of the complex plane. It can be shown that a mapping $f\colon \mathbb{C} \to \mathbb{C}$ is conformal if and only if f is a complex analytic function. The complex conjugate $\overline{f}$ of a conformal mapping f is *inversely conformal*, i.e. it preserves the magnitude but reverses the orientation of angles.

If $m = n$, then we can study invertible conformal mappings. It is clear from the definition that the composition of two such maps and the inverse of any such map is again an invertible conformal mapping, so the set of such mappings forms a group. In the case $m = n > 2$, the group of conformal mappings is finite dimensional and is generated by rotations, translations, and spherical inversions.

This notion of conformal mappings can be generalized to any setting in which it makes sense to speak of angles between curves or angles between tangent vectors. In particular, one can consider conformal mappings of Riemannian manifolds. It can be shown that, if (M, g) and (N, h) are Riemannian manifolds, then a map $f\colon M \to N$ is conformal if and only if $f^*h = sg$ for some scalar field s (on M).