simple boundary point

Canonical name	SimpleBoundaryPoint
Date of creation	2013-03-22 14:23:23
Last modified on	2013-03-22 14:23:23
Owner	jirka (4157)
Last modified by	jirka (4157)
Numerical id	5
Author	jirka (4157)
Entry type	Definition
Classification	msc 30-00
Classification	msc 54-00

Definition. Let $G \subset \mathbb{C}$ be a region and $\omega \in \partial G$ (the boundary of G). Then we call ω a *simple boundary point* if whenever $\{\omega_n\} \subset G$ is a sequence converging to ω there is a path $\gamma \colon [0,1] \to \mathbb{C}$ such that $\gamma(t) \in G$ for $0 \leq t < 1$, $\gamma(1) = \omega$ and there is a sequence $\{t_n\} \in [0,1)$ such that $t_n \to 1$ and $\gamma(t_n) = \omega_n$ for all n.

For example if we let G be the open unit disc, then every boundary point is a simple boundary point. This definition is useful for studying boundary behaviour of Riemann maps (maps arising from the Riemann mapping theorem), and one can prove for example the following theorem.

Theorem. *Suppose that $G \subset \mathbb{C}$ is a bounded simply connected region such that every point in the boundary of G is a simple boundary point, then ∂G is a Jordan curve.*

References

[1] John B. Conway. . Springer-Verlag, New York, New York, 1995.