Möbius transformation cross-ratio preservation theorem

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A Möbius transformation $f : z \mapsto w$ preserves the cross-ratios, i.e.

$$\frac{(w_1 - w_2)(w_3 - w_4)}{(w_1 - w_4)(w_3 - w_2)} = \frac{(z_1 - z_2)(z_3 - z_4)}{(z_1 - z_4)(z_3 - z_2)}$$

Conversely, given two quadruplets which have the same cross-ratio, there exists a Möbius transformation which maps one quadruplet to the other.

A consequence of this result is that the cross-ratio of (a, b, c, d) is the value at a of the Möbius transformation that takes b, c, d, to 1, 0, ∞ respectively.