# second form of Cauchy integral theorem

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**Theorem.** Let the complex function $f$ be analytic in a simply connected open domain $U$ of the complex plane, and let $a$ and $b$ be any two points of $U$. Then the contour integral

$$\int_\gamma f(z)\,dz \tag{1}$$

is independent on the path $\gamma$ which in $U$ goes from $a$ to $b$.

**Example.** Let's consider the integral (1) of the real part function defined by

$$f(z) := \mathrm{Re}(z)$$

with the path $\gamma$ going from the point $O = (0,\,0)$ to the point $Q = (1,\,1)$. If $\gamma$ is the line segment $OQ$, we may use the substitution

$$z := (1+i)t, \quad dz = (1+i)\,dt, \quad 0 \leqq t \leqq 1,$$

and (1) equals

$$\int_0^1 t\cdot(1+i)\,dt = \frac{1}{2}+\frac{1}{2}i.$$

Secondly, we choose for $\gamma$ the broken line $OPQ$ where $P = (1,\,0)$. Now (1) is the sum

$$\int_{OP} \mathrm{Re}(z)\,dz + \int_{PQ} \mathrm{Re}(z)\,dz = \int_0^1 x\,dx + \int_0^1 i\,dy = \frac{1}{2}+i.$$

Thus, the integral (1) of the function depends on the path between the two points. This is explained by the fact that the function $f$ is not analytic — its real part $x$ and imaginary part 0 do not satisfy the Cauchy-Riemann equations.