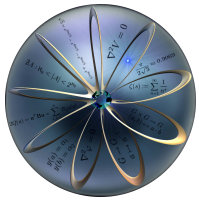

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examples of infinite products

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A classic example is the Riemann zeta function. For $\Re(z) > 1$ we have

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z} = \prod_{p \text{ prime}} \frac{1}{1 - p^{-z}} \, .$$

With the help of a Fourier series, or in other ways, one can prove this infinite product expansion of the sine function:

$$\sin z = z \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2 \pi^2} \right) \tag{1}$$

where z is an arbitrary complex number. Taking the logarithmic derivative (a frequent move in connection with infinite products) we get a decomposition of the cotangent into partial fractions:

$$\pi \cot \pi z = \frac{1}{z} + \sum_{n=1}^{\infty} \left(\frac{1}{z+n} + \frac{1}{z-n} \right) \, . \tag{2}$$

The equation (**??**), in turn, has some interesting uses, e.g. to get the Taylor expansion of an Eisenstein series, or to evaluate $\zeta(2n)$ for positive integers n.