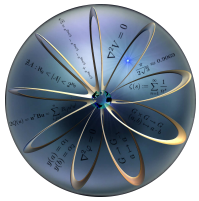

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hyperbolic sine integral

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The function *hyperbolic sine integral* (in Latin *sinus hyperbolicus integralis*) from $\mathbb{R}$ to $\mathbb{R}$ is defined as

$$\operatorname{Shi} x := \int_0^x \frac{\sinh t}{t}\, dt, \tag{1}$$

or alternatively as

$$\operatorname{Shi} x := \int_0^1 \frac{\sinh tx}{t}\, dt.$$

It isn't an elementary function. The equation (1) implies the Taylor series expansion

$$\operatorname{Shi} z = z + \frac{z^3}{3\cdot 3!} + \frac{z^5}{5\cdot 5!} + \frac{z^7}{7\cdot 7!} + \cdots,$$

which converges for all complex values z and thus defines an entire transcendental function. Using the Taylor expansions, it is easily seen that

$$\operatorname{Shi} x \;=\; i \operatorname{Si} ix$$

connects Shi to the sine integral function.

$\operatorname{Shi} x$ satisfies the linear third differential equation

$$xf'''(x) + 2f''(x) - xf'(x) = 0.$$