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Perron family

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Definition. Let $G \subset \mathbb{C}$ be a region, $\partial_\infty G$ the extended boundary of G and $S(G)$ the set of subharmonic functions on G, then if $f\colon \partial_\infty G \to \mathbb{R}$ is a continuous function then the set

$$\mathcal{P}(f,G) := \{\varphi : \varphi \in S(G) \text{ and } \limsup_{z\to a} \varphi(z) \leq f(a) \text{ for all } a \in \partial_\infty G\},$$

is called the *Perron family*.

One thing to note is the $\mathcal{P}(f,G)$ is never empty. This is because f is continuous on $\partial_\infty G$ it attains a maximum, say $|f| < M$, then the function $\varphi(z) := -M$ is in $\mathcal{P}(f,G)$.

Definition. Let $G \subset \mathbb{C}$ be a region and $f\colon \partial_\infty G \to \mathbb{R}$ be a continuous function then the function $u\colon G \to \mathbb{R}$

$$u(z) := \sup\{\phi : \phi \in \mathcal{P}(f,G)\},$$

is called the *Perron function* associated with f.

Here is the reason for all these definitions.

Theorem. *Let $G \subset \mathbb{C}$ be a region and suppose $f\colon \partial_\infty G \to \mathbb{R}$ is a continuous function. If $u\colon G \to \mathbb{R}$ is the Perron function associated with f, then u is a harmonic function.*

Compare this with `http://planetmath.org/RadosTheorem`Rado's theorem which works with harmonic functions with range in $\mathbb{R}^2$, but also gives a much stronger statement.

References

[1] John B. Conway. . Springer-Verlag, New York, New York, 1978.