Schwarz-Christoffel transformation

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Let

$$w = f(z) = c\int\frac{dz}{(z-a_1)^{k_1}(z-a_2)^{k_2}\ldots(z-a_n)^{k_n}}+C,$$

where the a_j's are real numbers satisfying $a_1 < a_2 < \ldots < a_n$, the k_j's are real numbers satisfying $|k_j| \leqq 1$; the integral expression means a complex antiderivative, c and C are complex constants.

The transformation $z \mapsto w$ maps the real axis and the upper half-plane `http://planetmath.org/ConformalMapping`conformally onto the closed area bounded by a broken line. Some vertices of this line may be in the infinity (the corresponding angles are $= 0$). When z moves on the real axis from $-\infty$ to ∞, w moves along the broken line so that the direction turns the amount $k_j\pi$ anticlockwise every z passes a point a_j. If the broken line closes to a polygon, then $k_1+k_2+\ldots+k_n = 2$.

This transformation is used in solving two-dimensional potential problems. The parameters a_j and k_j are chosen such that the given polygonal domain in the complex w-plane can be obtained.

A half-trivial example of the transformation is

$$w = \frac{1}{2}\int\frac{dz}{(z-0)^{\frac{1}{2}}} = \sqrt{z},$$

which maps the upper half-plane onto the first quadrant of the complex plane.