harmonic function

Canonical name	HarmonicFunction
Date of creation	2013-03-22 12:43:46
Last modified on	2013-03-22 12:43:46
Owner	mathcam (2727)
Last modified by	mathcam (2727)
Numerical id	9
Author	mathcam (2727)
Entry type	Definition
Classification	msc 31C05
Classification	msc 31B05
Classification	msc 31A05
Classification	msc 30F15
Related topic	RadosTheorem
Related topic	SubharmonicAndSuperharmonicFunctions
Related topic	DirichletProblem
Related topic	NeumannProblem

A twice-differentiable real or complex-valued function $f\colon U \to \mathbb{R}$ or $f\colon U \to \mathbb{C}$, where $U \subseteq \mathbb{R}^n$ is some , is called *harmonic* if its Laplacian vanishes on U, i.e. if

$$\Delta f \equiv 0.$$

Any harmonic function $f\colon \mathbb{R}^n \to \mathbb{R}$ or $f\colon \mathbb{R}^n \to \mathbb{C}$ satisfies Liouville's theorem. Indeed, a holomorphic function *is* harmonic, and a real harmonic function $f\colon U \to \mathbb{R}$, where $U \subseteq \mathbb{R}^2$, is locally the real part of a holomorphic function. In fact, it is enough that a harmonic function f be below (or above) to conclude that it is .