pluriharmonic function

Canonical name	PluriharmonicFunction
Date of creation	2013-03-22 14:29:01
Last modified on	2013-03-22 14:29:01
Owner	jirka (4157)
Last modified by	jirka (4157)
Numerical id	7
Author	jirka (4157)
Entry type	Definition
Classification	msc 31C05
Classification	msc 32A50
Classification	msc 31C10
Synonym	pluriharmonic

Definition. Let $f\colon G \subset \mathbb{C}^n \to \mathbb{C}$ be a C^2 (twice continuously differentiable) function. f is called *pluriharmonic* if for every complex line $\{a + bz \mid z \in \mathbb{C}\}$ the function $z \mapsto f(a + bz)$ is harmonic on the set $\{z \in \mathbb{C} \mid a + bz \in G\}$.

Note that every pluriharmonic function is a harmonic function, but not the other way around. Further it can be shown that for holomorphic functions of several complex variables the real (and the imaginary) parts are locally pluriharmonic functions. Do note however that just because a function is harmonic in each variable separately does not imply that it is pluriharmonic.

References

[1] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.