Gauss' mean value theorem for harmonic functions

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If the function $u(z) \equiv u(x, y)$ is harmonic in a domain of complex plane which contains the disc $|z - z_0| \leqq r$, then

$$u(z_0) = \frac{1}{2\pi} \int_0^{2\pi} u(z_0 + re^{i\varphi}) \, d\varphi. \tag{1}$$

Conversely, if a real function $u(x, y)$ is continuous in a domain G of $\mathbb{R}^2$ and satisfies on all circles of G the equation (1), then it is harmonic.