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analytic space

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A Hausdorff topological space  $X$  is said to be an *analytic space* if:

1. There exists a countable number of open sets  $V_j$  covering  $X$ .
2. For each  $V_j$  there exists a homeomorphism  $\varphi_j: Y_j \rightarrow V_j$ , where  $Y_j$  is a local complex analytic subvariety in some  $\mathbb{C}^n$ .
3. If  $V_j$  and  $V_k$  overlap, then  $\varphi_j^{-1} \circ \varphi_k$  is a biholomorphism.

Usually one attaches to  $X$  a set of coordinate systems  $\mathcal{G}$ , which is a set (now uncountable) of triples  $(V_\iota, \varphi_\iota, Y_\iota)$  as above, such that whenever  $V$  is an open set,  $Y$  a local complex analytic subvariety, and a homeomorphism  $\varphi: Y \rightarrow V$ , such that  $\varphi_\iota^{-1} \circ \varphi$  is a biholomorphism for some  $(V_\iota, \varphi_\iota, Y_\iota) \in \mathcal{G}$  then  $(V, \varphi, Y) \in \mathcal{G}$ . Basically  $\mathcal{G}$  is the set of all possible coordinate systems for  $X$ .

We can also define the singular set of an analytic space. A point  $p$  is if there exists (at least one) a coordinate system  $(V_\iota, \varphi_\iota, Y_\iota) \in \mathcal{G}$  with  $p \in V_\iota$  and  $Y_\iota$  a complex manifold. All other points are the singular points.

Any local complex analytic subvariety is an analytic space, so this is a natural generalization of the concept of a subvariety.

## References

- [1] Hassler Whitney. . Addison-Wesley, Philippines, 1972.