holomorphic functions of several variables

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Definition. Let $\Omega \subset \mathbb{C}^n$ be a domain and let $f \colon \Omega \to \mathbb{C}$ be a function. f is called if it is http://planetmath.org/Holomorphicholomorphic in each variable separately as a function of one variable.

That means that the function $z_k \mapsto f(z_1, \ldots, z_k, \ldots, z_n)$ is holomorphic as a function of one variable. It is not at all obvious that such a function is even continuous and we must apply the Hartogs's theorem on separate analyticity which is not a trivial result.

Historically and some authors today still continue to do so, the definition of being holomorphic in several variables did include the continuity or at least local boundedness requirement.

Of course we can also characterize holomorphic functions by their power series.

Proposition. *f is holomorphic in Ω if and only if near each point $\zeta \in \Omega$ there is a neighbourhood U and a power series in several variables*

$$\sum_{\alpha} a_\alpha (z - \zeta)^\alpha,$$

where α ranges over all the multi-indices, $a_\alpha \in \mathbb{C}$ and such that the series converges to $f(z)$ for $z \in U$.

Another way to characterize holomorphic functions is by the use of the Cauchy-Riemann equations, which can be given in a very form by the http://planetmath.org/BarpartialOperator$\bar{\partial}$-operator.

Proposition. *f is holomorphic if and only if $\bar{\partial} f = 0$.*

Despite the similarities, one should be careful about carelessly generalizing results about functions of one variable to functions of several variables as the theory is quite different. See the topic entry on several complex variables for more .

References

[1] Lars Hörmander. , North-Holland Publishing Company, New York, New York, 1973.

[2] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.