D'Angelo finite type

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Let $M \subset \mathbb{C}^n$ be a smooth submanifold of real codimension 1. Let $p \in M$ and let r_p denote the generator of the principal ideal of germs at p of smooth functions vanishing on M. Define the number

$$\Delta_1(M, p) = \sup_z \frac{v(z^* r_p)}{v(z)},$$

where z ranges over all parametrized holomorphic curves $z \colon \mathbb{D} \to \mathbb{C}^n$ (where $\mathbb{D}$ is the unit disc) such that $z(0) = 0$, v is the order of vanishing at the origin, and $z^* r_p$ is the composition of r_p and z. The order of vanishing $v(z)$ is k if k is the smallest integer such that the kth derivative of z is nonzero at the origin and all derivatives of smaller order are zero at the origin. Infinity is allowed for $v(z)$ if all derivatives vanish.

We say M is of (or finite 1-type) at $p \in M$ in the sense of D'Angelo if

$$\Delta_1(M, p) < \infty.$$

If M is real analytic, then M is finite type at p if and only if there does not exist any germ of a complex analytic subvariety at $p \in M$, that is contained in M. If M is only smooth, then it is possible that M is not finite type, but does not contain a germ of a holomorphic curve. However, if M is not finite type, then there exists a holomorphic curve which "touches" M to infinite order.

The Diederich-Fornaess theorem can be then restated to say that every compact real analytic subvariety of $\mathbb{C}^n$ is of D'Angelo finite type at every point.

References

[1] M. Salah Baouendi, Peter Ebenfelt, Linda Preiss Rothschild. , Princeton University Press, Princeton, New Jersey, 1999.

[2] D'Angelo, John P. , CRC Press, 1993.