biholomorphically equivalent

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Synonym	biholomorphic equivalence
Defines	biholomorphic mapping

Definition. Let $U, V \subset \mathbb{C}^n$. If there exists a one-to-one and onto holomorphic mapping $\phi\colon U \to V$ such that the inverse ϕ^{-1} exists and is also holomorphic, then we say that U and V are *biholomorphically equivalent* or that they are *biholomorphic*. The mapping ϕ is called a *biholomorphic mapping*.

It is not an obvious fact, but if the source and target dimension are the same then every one-to-one holomorphic mapping is biholomorphic as a one-to-one holomorphic map has a nonvanishing jacobian.

When $n = 1$ biholomorphic equivalence is often called `http://planetmath.org/ConformallyEq` equivalence, since in one complex dimension, the one-to-one holomorphic mappings are conformal mappings.

Further if $n = 1$ then there are plenty of conformal (biholomorhic) equivalences, since for example every simply connected `http://planetmath.org/Domain2`domain other than the whole complex plane is conformally equivalent to the unit disc. On the other hand, when $n > 1$ then the open unit ball and open unit polydisc are not biholomorphically equivalent. In fact there does not exist a `http://planetmath.org/ProperMap`proper holomorphic mapping from one to the other.

References

[1] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.