Bergman kernel

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Let $G \subset \mathbb{C}^n$ be a `http://planetmath.org/Domain2`domain. And let $A^2(G)$ be the Bergman space. For a fixed $z \in G$, the functional $f \mapsto f(z)$ is a bounded linear functional. By the Riesz representation theorem (as $A^2(G)$ is a Hilbert space) there exists an element of $A^2(G)$ that represents it, and let's call that element $k_z \in A^2(G)$. That is we have that $f(z) = \langle f, k_z \rangle$. So we can define the *Bergman kernel*.

Definition. The function

$$K(z, w) := \overline{k_z(w)}$$

is called the *Bergman kernel*.

By definition of the inner product in $A^2(G)$ we then have that for $f \in A^2(G)$

$$f(z) = \int_G f(w) K(z, w) dV(w),$$

where dV is the volume measure.

As the $A^2(G)$ space is a subspace of $L^2(G, dV)$ which is a separable Hilbert space then $A^2(G)$ also has a countable orthonormal basis, say $\{\varphi_j\}_{j=1}^{\infty}$.

Theorem. *We can compute the Bergman kernel as*

$$K(z, w) = \sum_{j=1}^{\infty} \varphi_j(z) \overline{\varphi_j(w)},$$

where the sum converges uniformly on compact subsets of $G \times G$.

Note that integration against the Bergman kernel is just the orthogonal projection from $L^2(G, dV)$ to $A^2(G)$. So not only is this kernel reproducing for holomorphic functions, but it will produce a holomorphic function when we just feed in any $L^2(G, dV)$ function.

References

[1] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.