Bergman metric

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Definition. Let $G \subset \mathbb{C}^n$ be a domain and let $K(z,w)$ be the Bergman kernel on G. We define a Hermitian metric on the tangent bundle $T_z\mathbb{C}^n$ by

$$g_{ij}(z) := \frac{\partial^2}{\partial z_i \partial \bar{z}_j} \log K(z,z),$$

for $z \in G$. Then the length of a tangent vector $\xi \in T_z\mathbb{C}^n$ is then given by

$$|\xi|_{B,z} := \sqrt{\sum_{i,j=1}^{n} g_{ij}(z)\xi_i\bar{\xi}_j}.$$

This metric is called the *Bergman metric* on G.

The length of a (piecewise) C^1 curve $\gamma\colon [0,1] \to \mathbb{C}^n$ is then computed as

$$\ell(\gamma) = \int_0^1 \big|\frac{\partial\gamma}{\partial t}(t)\big|_{B,\gamma(t)} dt.$$

The distance $d_G(p,q)$ of two points $p,q \in G$ is then defined as

$$d_G(p,q) := \inf\{\ell(\gamma) \mid \text{ all piecewise } C^1 \text{ curves } \gamma \text{ such that } \gamma(0) = p \text{ and } \gamma(1) = q\}.$$

The distance d_G is called the *Bergman distance*.

The Bergman metric is in fact a positive definite matrix at each point if G is a bounded domain. More importantly, the distance d_G is invariant under biholomorphic mappings of G to another domain G'. That is if f is a biholomorphism of G and G', then $d_G(p,q) = d_{G'}(f(p), f(q))$.

References

[1] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.