CR function

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Definition. Let $M \subset \mathbb{C}^N$ be a CR submanifold and let f be a $C^k(M)$ (k times continuously differentiable) function to $\mathbb{C}$ where $k \geq 1$. Then f is a *CR function* if for every CR vector field L on M we have $Lf \equiv 0$. A `http://planetmath.org/Distribution4`distribution f on M is called a *CR distribution* if similarly every CR vector field annihilates f.

For example restrictions of holomorphic functions in $\mathbb{C}^N$ to M are CR functions. The converse is not always true and is not easy to see. For example the following basic theorem is very useful when you have real analytic submanifolds.

Theorem. *Let $M \subset \mathbb{C}^N$ be a generic submanifold which is real analytic (the defining function is real analytic). And let $f \colon M \to \mathbb{C}$ be a real analytic function. Then f is a CR function if and only if f is a restriction to M of a holomorphic function defined in an open neighbourhood of M in $\mathbb{C}^N$.*

References

[1] M. Salah Baouendi, Peter Ebenfelt, Linda Preiss Rothschild. , Princeton University Press, Princeton, New Jersey, 1999.