



Math for the people, by the people.

polydisc

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Synonym	open polydisc
Defines	bidisc
Defines	distinguished boundary

Definition. We denote the set

$$D^n(z, r) := \{w \in \mathbb{C}^n \mid |z_k - w_k| < r \text{ for all } k = 1, \dots, n\}$$

an *open polydisc*. We can also have *polydiscs* of the form

$$D^1(z_1, r_1) \times \dots \times D^1(z_n, r_n).$$

The set $\partial D^1(z_1, r_1) \times \dots \times \partial D^1(z_n, r_n)$ is called the *distinguished boundary* of the polydisc.

Be careful not to confuse this with the open ball in \mathbb{C}^n as that is defined as

$$B(z, r) := \{w \in \mathbb{C}^n \mid |z - w| < r\}.$$

When $n > 1$ then open balls and open polydiscs are not biholomorphically equivalent (there is no 1-1 biholomorphic mapping between the two).

It is common to write $\bar{D}^n(z, r)$ for the closure of the polydisc. Be careful with this notation however as some texts outside of complex analysis use $D(x, r)$ and the “disc” to represent a closed ball in two real dimensions.

Also note that when $n = 2$ the *bidisc* is sometimes used.

References

- [1] Lars Hörmander. , North-Holland Publishing Company, New York, New York, 1973.
- [2] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.