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meromorphic functions of several variables

Canonical name	MeromorphicFunctionsOfSeveralVariables
Date of creation	2013-03-22 16:01:10
Last modified on	2013-03-22 16:01:10
Owner	jirka (4157)
Last modified by	jirka (4157)
Numerical id	4
Author	jirka (4157)
Entry type	Definition
Classification	msc 32A20
Defines	indeterminacy set

Definition. Let $\Omega \subset \mathbb{C}^n$ be a domain and let $h: \Omega \rightarrow \mathbb{C}$ be a function. h is called meromorphic if for each $p \in \Omega$ there exists a neighbourhood $U \subset \Omega$ ($p \in U$) and two holomorphic functions f, g defined in U where g is not identically zero, such that $h = f/g$ outside the set where $g = 0$.

Note that h is really defined only outside of a complex analytic subvariety. Unlike in one variable, we cannot simply define h to be equal to ∞ at the poles and expect h to be a continuous mapping to some larger space (the Riemann sphere in the case of one variable). The simplest counterexample in \mathbb{C}^2 is $(z, w) \mapsto z/w$, which does not have a unique limit at the origin. The set of points where there is no unique limit, is called the *indeterminacy set*. That is, the set of points where if $h = f/g$, and f and g have no common factors, then the indeterminacy set of h is the set where $f = g = 0$.

References

- [1] Lars Hörmander. , North-Holland Publishing Company, New York, New York, 1973.
- [2] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.