Hartogs functions

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Definition. Let $G \subset \mathbb{C}^n$ be an open set and let $\mathcal{F}_G$ be the smallest class of functions on G to $\mathbb{R} \cup \{-\infty\}$ that contains all of the functions $z \mapsto \log|f(z)|$ where f is holomorphic on G and such that $\mathcal{F}_G$ is closed with respect to the following conditions:

- If $\varphi_1, \varphi_2 \in \mathcal{F}_G$, then $\varphi_1 + \varphi_2 \in \mathcal{F}_G$.
- If $\varphi \in \mathcal{F}_G$ then $a\varphi_1 \in \mathcal{F}_G$ for all $a \geq 0$.
- If $\{\varphi_k\} \in \mathcal{F}_G$ and $\varphi_1 \geq \varphi_2 \geq \ldots$, then $\lim_{k\to\infty} \varphi_k \in \mathcal{F}_G$.
- If $\{\varphi_k\} \in \mathcal{F}_G$ and the sequence is uniformly bounded above on compact sets, then $\sup_k \varphi_k \in \mathcal{F}_G$.
- If $\varphi \in \mathcal{F}_G$ and $\hat{\varphi}(w) := \limsup_{w\to z} \varphi(w)$, then $\hat{\varphi} \in \mathcal{F}_G$
- If $\varphi|_U \in \mathcal{F}_U$ for all $U \subset G$ where U is relatively compact (the closure of U is compact), then $\varphi \in \mathcal{F}_G$.

These functions are called the *Hartogs functions*.

It is known that if $n = 1$ then the upper semi-continuous Hartogs functions are precisely the subharmonic functions on G.

Theorem (H. Bremerman)**.** *All plurisubharmonic functions are Hartogs functions if G is a domain of holomorphy.*

References

[1] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.