Cauchy integral formula in several variables

Canonical name	CauchyIntegralFormulaInSeveralVariables
Date of creation	2013-03-22 15:33:46
Last modified on	2013-03-22 15:33:46
Owner	jirka (4157)
Last modified by	jirka (4157)
Numerical id	7
Author	jirka (4157)
Entry type	Theorem
Classification	msc 32A07
Classification	msc 32A10

Let $D = D_1 \times \ldots \times D_n \subset \mathbb{C}^n$ be a polydisc.

Theorem. *Let f be a function continuous in $\bar{D}$ (the closure of D). Then f is* `http://planetmath.org/HolomorphicFunctionsOfSeveralVariables`*holomorphic in D if and only if for all $z = (z_1, \ldots, z_n) \in D$ we have*

$$f(z_1, \ldots, z_n) = \frac{1}{(2\pi i)^n} \int_{\partial D_1} \cdots \int_{\partial D_n} \frac{f(\zeta_1, \ldots, \zeta_n)}{(\zeta_1 - z_1) \ldots (\zeta_n - z_n)} d\zeta_1 \ldots d\zeta_n.$$

As in the case of one variable this theorem can be in fact used as a definition of holomorphicity. Note that when $n > 1$ then we are no longer integrating over the boundary of the polydisc but over the distinguished boundary, that is over $\partial D_1 \times \ldots \times \partial D_n$.

References

[1] Lars Hörmander. , North-Holland Publishing Company, New York, New York, 1973.

[2] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.