Puiseux parametrization

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Theorem. *Suppose that $V \subset U \subset \mathbb{C}^2$ is an irreducible complex analytic subset of (complex) dimension 1 where U is a domain. Suppose that $0 \in V$. Then there exists an analytic (holomorphic) map $f \colon \mathbb{D} \to V$, where $\mathbb{D}$ is the unit disc, such that $f(0) = 0$ and $f(\mathbb{D}) = N$ where $N \subset V$ is a neighbourhood of 0 in V, f is one to one, and further $f|_{\mathbb{D}\setminus\{0\}}$ is a biholomorphism onto $N \setminus \{0\}$. In fact there exist suitable local coordinates (z, w) in $\mathbb{C}^2$ such that f is then given by $\xi \mapsto (z, w)$ where $z = \xi^k$, $w = \sum_{n=m}^{\infty} a_n \xi^n$ where $m > k$.*

This is sometimes written as

$$w = \sum_{n=m}^{\infty} a_n z^{n/k}$$

and hence the name *Puiseux series parametrization*. If you do however write it like this, it must be properly interpreted, as the Puiseux series is in general not single valued.

A similar result for arbitrary complex analytic sets with singularities of codimension 1 in higher dimensional spaces under further conditions on the singular set was obtained by Stutz, see Chirka [**?**] page 98.

References

[1] E. M. Chirka. . Kluwer Academic Publishers, Dordrecht, The Netherlands, 1989.

[2] Alexandru Dimca. . Vieweg, Braunschweig, Germany, 1987.