Picard's theorem

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Let f be an holomorphic function with an essential singularity at $w \in \mathbb{C}$. Then there is a number $z_0 \in \mathbb{C}$ such that the image of any neighborhood of w by f contains $\mathbb{C} - \{z_0\}$. In other words, f assumes every complex value, with the possible exception of z_0, in any neighborhood of w.

Remark. Little Picard theorem follows as a corollary: Given a nonconstant entire function f, if it is a polynomial, it assumes every value in $\mathbb{C}$ as a consequence of the fundamental theorem of algebra. If f is not a polynomial, then $g(z) = f(1/z)$ has an essential singularity at 0; Picard's theorem implies that g (and thus f) assumes every complex value, with one possible exception.