Bloch's constant

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Bloch's theorem can be stated in the following way:

Bloch's Theorem. *Let $\mathcal{F}$ be the set of all functions f holomorphic on a region containing the closure of the disk $D = \{z \in \mathbb{C} : |z| < 1\}$ and satisfying $f(0) = 0$ and $f'(0) = 1$. For each $f \in \mathcal{F}$ let $\beta(f)$ be the supremum of all numbers r such that there is a disk $S \subset D$ on which f is injective and $f(S)$ contains a disk of radius r. Let B be the infimum of all $\beta(f)$, for $f \in \mathcal{F}$. Then $B \geq 1/72$.*

The constant B is usually referred to as Bloch's constant. Nowadays, better bounds are known and, in fact, it has been conjectured that B has the following tantalizing form

$$B = \frac{\Gamma(1/3) \cdot \Gamma(11/12)}{\left(\sqrt{1+\sqrt{3}}\right) \cdot \Gamma(1/4)}$$

where $\Gamma(x)$ is the gamma function.

References

[1] John B. Conway, *Functions of One Complex Variable I*, Second Edition, 1978, Springer-Verlag, New York.