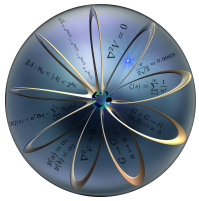

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complex logarithm

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The z is defined as every complex number w which satisfies the equation

$$e^w = z. \tag{1}$$

This is is denoted by

$$\log z := w.$$

The solution of (1) is obtained by using the form $e^w = re^{i\varphi}$, where $r = |z|$ and $\varphi = \arg z$; the result is

$$w = \log z = \ln|z| + i\arg z.$$

Here, the $\ln|z|$ means the usual Napierian or `http://planetmath.org/NaturalLogarithm2`natural logarithm ('*logarithmus naturalis*') of the real number $|z|$. If we fix the phase angle φ of $|z|$ so that $0 \leqq \varphi < 2\pi$, we can write

$$\log z = \ln r + i\varphi + n \cdot 2\pi i \quad (n = 0, \pm 1, \pm 2, ...).$$

The complex logarithm $\log z$ is defined for all $z \neq 0$ and it is infinitely multivalued − e.g. $\log(-1) = (2n+1)\pi i$ where n is an arbitrary integer. The values with $n = 0$ are called the of the ; if z is real, the value of $\log z$ coincides with $\ln z$.