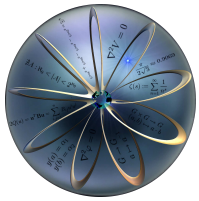

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complex line

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Defines	complex affine space

Definition. Let $a, b \in \mathbb{C}^n$. The set $\ell := \{a + bz \mid z \in \mathbb{C}\}$ is called the *complex line*.

A complex line is a holomorphic complex affine imbedding of $\mathbb{C}$ into $\mathbb{C}^n$ so that if f is holomorphic, then $z \mapsto f(a + bz)$ is also holomorphic. That is the complex structures of ℓ and $\mathbb{C}^n$ are compatible. Hence not every two dimensional real affine space is a complex line.

Definition. Let $a, b_1, \ldots, b_k \in \mathbb{C}^n$ such that $b_1, \ldots, b_k$ are linearly independent over $\mathbb{C}$, then. The set

$$\{a + \sum_{j=1}^{k} b_k z_k \mid z_1, \ldots, z_k \in \mathbb{C}\}$$

is called the *complex affine space*.

References

[1] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.