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Remmert-Stein theorem

Canonical name	RemmertSteinTheorem
Date of creation	2013-03-22 15:04:55
Last modified on	2013-03-22 15:04:55
Owner	jirka (4157)
Last modified by	jirka (4157)
Numerical id	9
Author	jirka (4157)
Entry type	Theorem
Classification	msc 32A60
Classification	msc 32C25
Synonym	Remmert-Stein extension theorem
Related topic	ChowsTheorem

For a complex analytic subvariety V and $p \in V$ a regular point, let $\dim_p V$ denote the complex dimension of V near the point p.

Theorem (Remmert-Stein)**.** *Let $U \subset \mathbb{C}^n$ be a* `http://planetmath.org/Domain2`*domain and let S be a complex analytic subvariety of U of dimension $m < n$. Let V be a complex analytic subvariety of $U \setminus S$ such that $\dim_p V > m$ for all regular points $p \in V$. Then the closure of V in U is an analytic variety in U.*

The condition that $\dim_p V > m$ for all regular p is the same as saying that all the irreducible components of V are of dimension strictly greater than m. To show that the restriction on the dimension of S is "sharp," consider the following example where the dimension of V equals the dimension of S. Let $(z, w) \in \mathbb{C}^2$ be our coordinates and let V be defined by $w = e^{1/z}$ in $\mathbb{C}^2 \setminus S$, where S is defined by $z = 0$. The closure of V in $\mathbb{C}^2$ cannot possibly be analytic. To see this look for example at $W = \overline{V} \cap \{w = 1\}$. If $\overline{V}$ is analytic then W ought to be a zero dimensional complex analytic set and thus a set of isolated points, but it has a limit point $(0, 1)$ by Picard's theorem.

Finally note that there are various generalizations of this theorem where the set S need not be a variety, as long as it is of small enough dimension. Alternatively, if V is of finite volume, we can weaken the restrictions on S even further.

References

[1] Klaus Fritzsche, Hans Grauert. , Springer-Verlag, New York, New York, 2002.

[2] Hassler Whitney. . Addison-Wesley, Philippines, 1972.