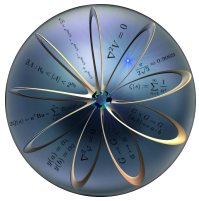

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## failure of Hartogs' theorem in one dimension

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It is instructive to see an example where Hartogs' theorem fails in one dimension. Take $U = \mathbb{C}$ and let $K = \{0\}$. The function $\frac{1}{z}$ is holomorphic in $U \setminus K$, but cannot be extended to $U$.

To understand the example and failure of the theorem it is important to understand `http://planetmath.org/ProofOfHartogsTheorem`the proof. In the proof, the way we construct an extension is that we start with a function holomorphic in $U \setminus K$, modify it in a neighbourhood of $K$ to be zero, hence extending as a smooth function through $K$. Then we solve the `http://planetmath.org/BarPartialOperator`$\bar{\partial}$ operator inhomogeneous equation $\bar{\partial}\psi = g$ to "correct" our extension to be holomorphic. The key point is that $g$ has compact support allowing us to solve the equation and find a $\psi$ with compact support. This fails in dimension 1. While we always get a solution $\psi$, the solution can never have compact support. Hence, if we tried the proof with $\frac{1}{z}$, the new function we obtain in the proof does not agree with $\frac{1}{z}$ on any open set and hence is not an extension.