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generic manifold

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Definition. Let $M \subset \mathbb{C}^N$ be a real submanifold of real dimension n. We say that M is a *generic manifold* if for every $x \in M$ we have

$$T_x(M) + JT_x(M) = T_x(\mathbb{C}^N),$$

where J denotes the operator of multiplication by the imaginary unit in $T_x(\mathbb{C}^N)$. That is every vector in $T_x(\mathbb{C}^N)$ can be written as $X + JY$ where $X, Y \in T_x(M)$.

For more details about the tangent spaces and the J operator see the entry on `http://planetmath.org/CRSubmanifold`CR manifolds. In fact every generic manifold is also CR manifold (the converse is not true however). A basic important result about generic submanifolds is.

Theorem. *Let $M \subset \mathbb{C}^N$ be a generic submanifold and let $f: U \subset \mathbb{C}^N \to \mathbb{C}$ be a holomorphic function where U is a connected open set such that $M \cap U \neq \emptyset$, and further suppose that $f(M \cap U) = \{0\}$, that is f is zero when restricted to M. Then in fact $f \equiv 0$ on U.*

For example in $\mathbb{C}^1$ the real line is a generic submanifold, and any holomorphic function which is zero on the real line is zero everywhere (if the domain of the function is connected and intersects the real line of course). There are of course much stronger uniqueness results for the complex plane so the above is mostly useful for higher dimensions.

References

[1] M. Salah Baouendi, Peter Ebenfelt, Linda Preiss Rothschild. , Princeton University Press, Princeton, New Jersey, 1999.