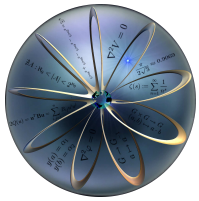

planetmath.org

Math for the people, by the people.

limit of $\frac{a^x - 1}{x}$ as x approaches 0

Canonical name	LimitOfdisplaystylefracax1xAsXApproaches0
Date of creation	2013-03-22 17:40:21
Last modified on	2013-03-22 17:40:21
Owner	Wkbj79 (1863)
Last modified by	Wkbj79 (1863)
Numerical id	5
Author	Wkbj79 (1863)
Entry type	Corollary
Classification	msc 32A05

Corollary. *For $a > 0$, we have*

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a.$$

Proof. Recall that $a^x = e^{x \ln a}$. Thus,

$$\begin{aligned}
\lim_{x\to 0} \frac{a^x - 1}{x} &= \lim_{x\to 0} \frac{e^{x\ln a} - 1}{x} \\
&= \lim_{x\to 0} \frac{(e^{x\ln a} - 1)\ln a}{x \ln a} \\
&= (\ln a) \lim_{x\to 0} \frac{e^{x\ln a} - 1}{x \ln a}.
\end{aligned}$$

Let $t = x \ln a$. Then $t \to 0$ as $x \to 0$. Therefore,

$$\begin{aligned}
\lim_{x\to 0} \frac{a^x - 1}{x} &= (\ln a) \lim_{t\to 0} \frac{e^t - 1}{t} \\
&= (\ln a) 1 \\
&= \ln a.
\end{aligned}$$

□

The formula from the corollary is useful for proving that $\dfrac{d}{dx} a^x = a^x \ln a$. On the other hand, once this fact is known, the corollary is easily proven via `http://planetmath.org/LHpitalsRule`l'Hôpital's rule:

$$\begin{aligned}
\lim_{x\to 0} \frac{a^x - 1}{x} &= \lim_{x\to 0} \frac{a^x \ln a}{1} \\
&= a^0 \ln a \\
&= \ln a.
\end{aligned}$$