several complex variables

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Several complex variables is a study of http://planetmath.org/HolomorphicFunctionsOfSev[...] functions on the space $\mathbb{C}^n$ or on abstract complex manifolds. It is also the study of objects and spaces under biholomorphic mappings. While the http://planetmath.org/FundamentalTheoremsInComplexAnalysistheory of one complex variable is really a subset of the study of several complex variables, the study of several complex variables is substantially different. As an example, in one variable theory, the Riemann mapping theorem tells us that all simply connected domains (with the exception of the entire plane) are biholomorphically equivalent to each other, in just $\mathbb{C}^2$ the open unit ball and the open polydisc are not biholomorphic.

A large part of the study of several complex variables is the study of domains of holomorphic functions, namely, the domains of holomorphy. Topics related to this study include

- Bergman space,

- pseudoconvex and strongly pseudoconvex domains,

- Reinhardt domains,

- http://planetmath.org/BarpartialOperator$\bar{\partial}$-operator and the solution to the inhomogeneous equation $\bar{\partial}u = f$,

- Stein manifolds.

Important theorems in the theory of several complex variables include,

- Cauchy integral formula in several variables,

- Oka coherence theorem,

- Cartan theorem A and http://planetmath.org/CartanTheoremBtheorem B,

- Hartogs' theorem on separate analyticity,

- Hartogs' phenomenon,

- Remmert-Stein theorem,

- Weierstrass preparation theorem.

Several complex variables also includes the study of the zero sets of complex analytic functions and these are called complex analytic varieties. Study of such objects in one complex dimension is invariably boring as zero sets of complex analytic functions of one variable are just isolated points. However, the zero set of a holomorphic function of n complex variables is at most points an $n-1$ dimensional complex manifold.

The study of real manifolds in complex spaces (such as the boundary of a domain) and their behaviour under biholomorphic mappings is called CR geometry. See the entries on CR submanifold and CR function for more information.

The topics in several complex variables have many connections to other parts of modern mathematics. There are many connections to both real and complex algebraic geometry on one hand, and partial differential equations on the other.

References

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