pseudoconvex

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Definition. Let $G \subset \mathbb{C}^n$ be a domain (open connected subset). We say G is *pseudoconvex* (or *Hartogs pseudoconvex*) if there exists a continuous plurisubharmonic function φ on G such that the sets $\{z \in G \mid \varphi(z) < x\}$ are relatively compact subsets of G for all $x \in \mathbb{R}$. That is we say that G has a continuous plurisubharmonic exhaustion function.

When G has a C^2 (twice continuously differentiable) boundary then this notion is the same as `http://planetmath.org/LeviPseudoconvex`Levi pseudoconvexity, which is easier to work with if you have such nice boundaries. If you don't have nice boundaries then the following approximation result can come in useful.

Proposition. *If $G \subset \mathbb{C}^n$ is pseudoconvex then there exist bounded, strongly Levi pseudoconvex domains $G_k \subset G$ with C^∞ (smooth) boundary which are relatively compact in G, such that $G = \bigcup_{k=1}^{\infty} G_k$.*

This is because once we have a φ as in the definition we can actually find a C^∞ exhaustion function.

The reason for the definition of pseudoconvexity is that it classifies domains of holomorphy. One thing to note then is that every open domain in one complex dimension (in the complex plane $\mathbb{C}$) is then pseudoconvex.

References

[1] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.