Stein manifold

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Definition. A complex manifold M of complex dimension n is a *Stein manifold* if it satisfies the following properties

1. M is holomorphically convex,
2. if $z, w \in M$ and $z \neq w$ then $f(z) \neq f(w)$ for some function f holomorphic on M (i.e. M is *holomorphically separable*),
3. for every $z \in M$ there are holomorphic functions $f_1, \ldots, f_n$ which form a coordinate system at z (i.e. M is *holomorphically spreadable*).

Stein manifold is a generalization of the concept of the domain of holomorphy to manifolds. Furthermore, Stein manifolds are the generalizations of Riemann surfaces in higher dimensions. Every noncompact Riemann surface is a Stein manifold by a theorem of Behnke and Stein. Note that every domain of holomorphy in $\mathbb{C}^n$ is a Stein manifold. It is not hard to see that every closed complex submanifold of a Stein manifold is Stein.

Theorem (Remmert, Narasimhan, Bishop)**.** *If M is a Stein manifold of dimension n. There exists a* `http://planetmath.org/ProperMap`*proper holomorphic embedding of M into $\mathbb{C}^{2n+1}$.*

Note that no compact complex manifold can be Stein since compact complex manifolds have no holomorphic functions. On the other hand, every compact complex manifold is holomorphically convex.

References

[1] Lars Hörmander. , North-Holland Publishing Company, New York, New York, 1973.

[2] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.