symmetric power

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Let X be a set and let

$$X^m := \underbrace{X \times \cdots \times X}_{m-\text{times}}.$$

Denote an element of X^m by $x = (x_1, \ldots, x_m)$. Define an equivalence relation by $x \sim x'$ if and only if there exists a permutation σ of $(1, \ldots, m)$, such that $x_i = x'_{\sigma i}$.

Definition. The m^{th} symmetric power of X is the set $X^m_{sym} := X^m / \sim$. That is, the set of equivalence classes of X^m under the relation $\sim$.

Let π be the natural projection of X^m onto X^m_{sym}.

Proposition. *$f\colon X^m \to Y$ is a symmetric function if and only if there exists a function $g\colon X^m_{sym} \to Y$ such that $f = g \circ \pi$.*

From now on let R be an integral domain. Let $\tau'\colon X^m \to X^m$ be the map $\tau'(x) := (\tau_1(x), \ldots, \tau_m(x))$, where τ_k is the k^{th} elementary symmetric polynomial. By the above lemma, we have a function $\tau\colon X^m_{sym} \to X^m$, where $\tau' = \tau \circ \pi$.

Proposition. *τ is one to one. If R is algebraically closed, then τ is onto.*

A very useful case is when $R = \mathbb{C}$. In this case, when we put on the natural complex manifold structure onto $\mathbb{C}^m_{sym}$, the map τ is a biholomorphism of $\mathbb{C}^m_{sym}$ and $\mathbb{C}^m$.

References

[1] Hassler Whitney. . Addison-Wesley, Philippines, 1972.