Bergman space

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Let $G \subset \mathbb{C}^n$ be a domain and let dV denote the Euclidean volume measure on G.

Definition. Let

$$A^2(G) := \left\{ f \text{ holomorpic in } G \;\middle|\; \sqrt{\int_G |f(z)|^2 dV(z)} < \infty \right\}.$$

$A^2(G)$ is called the *Bergman space* on G. The norm on this space is defined as

$$\|f\| := \sqrt{\int_G |f(z)|^2 dV(z)}.$$

Further we define an inner product on $A^2(G)$ as

$$\langle f, g \rangle := \int_G f(z)\overline{g(z)} dV(z).$$

The inner product as defined above really is an inner product and further it can be shown that $A^2(G)$ is complete since convergence in the above norm implies normal convergence (uniform convergence on compact subsets). The space $A^2(G)$ is therefore a Hilbert space. Sometimes this space is also denoted by $L^2_a(G)$.

References

[1] D'Angelo, John P. , CRC Press, 1993.

[2] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.