complex analytic manifold

Canonical name	ComplexAnalyticManifold
Date of creation	2013-03-22 15:04:40
Last modified on	2013-03-22 15:04:40
Owner	jirka (4157)
Last modified by	jirka (4157)
Numerical id	6
Author	jirka (4157)
Entry type	Definition
Classification	msc 32Q99
Synonym	complex manifold
Defines	complex analytic submanifold
Defines	complex submanifold
Defines	analytic structure
Defines	holomorphic structure

Definition. A manifold M is called a *complex analytic manifold* (or sometimes just a *complex manifold*) if the transition functions are holomorphic.

Definition. A subset $N \subset M$ is called a *complex analytic submanifold* of M if N is closed in M and if for every point $z \in N$ there is a coordinate neighbourhood U in M with coordinates $z_1, \ldots, z_n$ such that $U \cap N = \{p \in U \mid z_{d+1}(p) = \ldots = z_n(p)\}$ for some integer $d \leq n$.

Obviously N is now also a complex analytic manifold itself.

For a complex analytic manifold, dimension always means the complex dimension, not the real dimension. That is M is of dimension n when there are neighbourhoods of every point homeomorphic to $\mathbb{C}^n$. Such a manifold is of real dimension $2n$ if we identify $\mathbb{C}^n$ with $\mathbb{R}^{2n}$. Of course the tangent bundle is now also a complex vector space.

A function f is said to be holomorphic on M if it is a holomorphic function when considered as a function of the local coordinates.

Examples of complex analytic manifolds are for example the Stein manifolds or the Riemann surfaces. Of course also any open set in $\mathbb{C}^n$ is also a complex analytic manifold. Another example may be the set of regular points of an analytic set.

Complex analytic manifolds can also be considered as a special case of CR manifolds where the CR dimension is maximal.

Complex manifolds are sometimes described as manifolds carrying an or . This refers to the atlas and transition functions defined on the manifold.

References

[1] Lars Hörmander. , North-Holland Publishing Company, New York, New York, 1973.

[2] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.