analytic polyhedron

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Definition. Suppose $G \subset \mathbb{C}^n$ is a domain and let $W \subset G$ be an open set. Let $f_1, \ldots, f_k \colon W \to \mathbb{C}$ be holomorphic functions. Then if the set

$$\Omega := \{z \in W \mid |f_j(z)| < 1, j = 1, \ldots, k\}$$

is relatively compact in W, we say that Ω is an *analytic polyhedron* in G. Sometimes it is denoted $\Omega(f_1, \ldots, f_k)$. Further $(W, f_1, \ldots, f_k)$ is called the *of the analytic polyhedron.*

An analytic polyhedron is automatically a domain of holomorphy by using the functions that define it as $g(z) := \frac{1}{e^{i\theta} - f_j(z)}$ to show that g cannot be extended beyond a point where $f_j(z) = e^{i\theta}$. Every boundary point of Ω is of that form for some f_j.

Furthermore every domain of holomorphy can be exhausted by analytic polyhedra (that is, every compact subset is contained in an analytic polyhedron) and in fact only domains of holomorphy can be exhausted by analytic polyhedra, see the Behnke-Stein theorem.

Note that sometimes W is required to be homeomorphic to the unit ball.

References

[1] Lars Hörmander. , North-Holland Publishing Company, New York, New York, 1973.

[2] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.