coherent analytic sheaf

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Entry type	Definition
Classification	msc 32C35
Defines	locally finitely generated sheaf
Defines	sheaf of relations

Let M be a complex manifold and $\mathcal{F}$ be an analytic sheaf. For $z \in M$, denote by $\mathcal{F}_z$ the stalk of $\mathcal{F}$ at z. By $\mathcal{O}$ denote the sheaf of germs of analytic functions. For a section f and a point $z \in M$ denote by f_z the germ of f at z.

$\mathcal{F}$ is said to be *locally finitely generated* if for every $z \in M$, there is a neighbourhood U of z, a finite number of sections $f_1, \ldots, f_k \in \Gamma(U, \mathcal{F})$ such that for each $w \in U$, $\mathcal{F}_w$ is a finitely generated module (as an $\mathcal{O}_w$-module).

Let U be a neighbourhood in M and Suppose that $f_1, \ldots, f_k$ are sections in $\Gamma(U, \mathcal{F})$. Let $\mathcal{R}(f_1, \ldots, f_k)$ be the subsheaf of $\mathcal{O}^k$ over U consisting of the germs

$$\{(g_1, \ldots, g_k) \in \mathcal{O}_z^k \mid \sum_{j=1}^{k} g_j (f_j)_z = 0\}.$$

$\mathcal{R}(f_1, \ldots, f_k)$ is called the *sheaf of relations*.

Definition. $\mathcal{F}$ is called a *coherent analytic sheaf* if $\mathcal{F}$ is locally finitely generated and if for every open subset $U \subset M$, and $f_1, \ldots, f_k \in \Gamma(U, \mathcal{F})$, the sheaf $\mathcal{R}(f_1, \ldots, f_k)$ is locally finitely generated.

References

[1] Lars Hörmander. , North-Holland Publishing Company, New York, New York, 1973.

[2] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.