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exhaustion function

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Author	jirka (4157)
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Defines	bounded exhaustion function
Defines	hyperconvex

Definition. Let $G \subset \mathbb{C}^n$ be a domain and let $f: G \rightarrow \mathbb{R}$ is called an *exhaustion function* whenever

$$\{z \in G \mid f(z) < r\}$$

is relatively compact in G for all $r \in \mathbb{R}$.

For example G is pseudoconvex if and only if G has a continuous plurisubharmonic exhaustion function.

We can also define a bounded version.

Definition. Let $G \subset \mathbb{C}^n$ be a domain and let $f: G \rightarrow (-\infty, c]$ for some $c \in \mathbb{R}$, is called a *bounded exhaustion function* whenever

$$\{z \in G \mid f(z) < r\}$$

is relatively compact in G for all $r < c$.

A domain which has a bounded plurisubharmonic exhaustion function is usually referred to as a *hyperconvex* domain. Note that not all pseudoconvex domains have a bounded plurisubharmonic exhaustion function. For example the Hartogs's triangle does not, though it does have an unbounded one.

References

- [1] Steven G. Krantz. , AMS Chelsea Publishing, Providence, Rhode Island, 1992.