analytic set

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Defines	regular point
Defines	simple point
Defines	top simple point
Defines	singular point
Defines	locally analytic
Defines	dimension of a variety
Defines	subvariety of a complex analytic variety
Defines	complex analytic subvariety

Let $G \subset \mathbb{C}^N$ be an open set.

Definition. A set $V \subset G$ is said to be *locally analytic* if for every point $p \in V$ there exists a neighbourhood U of p in G and holomorphic functions $f_1, \cdots, f_m$ defined in U such that $U \cap V = \{z : f_k(z) = 0 \text{for all} 1 \leq k \leq m\}$.

This basically says that around each point of V, the set V is analytic. A stronger definition is required.

Definition. A set $V \subset G$ is said to be an *analytic variety* in G (or *analytic set* in G) if for every point $p \in G$ there exists a neighbourhood U of p in G and holomorphic functions $f_1, \cdots, f_m$ defined in U such that $U \cap V = \{z : f_k(z) = 0 \text{ for all } 1 \leq k \leq m\}$.

Note the change, now V is analytic around each point of G. Since the zero sets of holomorphic functions are closed, this for example implies that V is relatively closed in G, while a local variety need not be closed. Sometimes an analytic variety is called an *analytic set*.

At most points an analytic variety V will in fact be a complex analytic manifold. So

Definition. A point $p \in V$ is called a *regular point* if there is a neighbourhood U of p such that $U \cap V$ is a complex analytic manifold. Any other point is called a *singular point*.

The set of regular points of V is denoted by V^- or sometimes V^*.

For any regular point $p \in V$ we can define the dimension as

$$\dim_p(V) = \dim_{\mathbb{C}}(U \cap V)$$

where U is as above and thus $U \cap V$ is a manifold with a well defined dimension. Here we of course take the complex dimension of these manifolds.

Definition. Let V be an analytic variety, we define the dimension of V by

$$\dim(V) = \sup\{\dim_p(V) : p \text{ a regular point of } V\}.$$

Definition. The regular point $p \in V$ such that $\dim_p(V) = \dim(V)$ is called a *top point* of V.

Similarly as for manifolds we can also talk about subvarieties. In this case we modify definition a little bit.

Definition. A set $W \subset V$ where $V \subset G$ is a local variety is said to be a *subvariety* of V if for every point $p \in V$ there exists a neighbourhood U of p in G and holomorphic functions $f_1, \cdots, f_m$ defined in U such that $U \cap W = \{z : f_k(z) = 0 \text{ for all } 1 \leq k \leq m\}$.

That is, a subset W is a subvariety if it is definined by the vanishing of analytic functions near all points of V.

References

[1] E. M. Chirka. . Kluwer Academic Publishers, Dordrecht, The Netherlands, 1989.

[2] Hassler Whitney. . Addison-Wesley, Philippines, 1972.