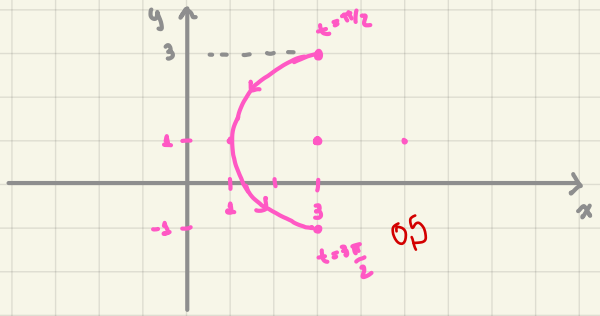
Gabarito P3

1) $\begin{cases} x = 3 + 2\cos t \\ y = 1 + 2\sin t \end{cases} \qquad \frac{\pi}{2} \le t \le \frac{3\pi}{2}$

↳ semicírculo

raio = 2

centro (3, 1) em $\frac{\pi}{2} \le t \le \frac{3\pi}{2}$ } 0,5



0,5

2) $x = \cos t$

$y = \sin t \cdot \cos t \qquad 0 \le t \le 2\pi$

a) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos^2 t - \sin^2 t}{-\sin t}$ 0,5

b) $x = \cos t = 0$

$y = \sin t \cdot \cos t = 0$

$\sin t = 0 \quad$ ou $\quad \cos t = 0$

↓ não

$t = \frac{\pi}{2}, t = \frac{3\pi}{2}$

$t = \pi/2 \Rightarrow \frac{dy}{dx} = 1$

$t = \frac{3\pi}{2} \Rightarrow \frac{dy}{dx} = -1$

0,3

r_1 : $y = x$ 0,3

r_2 : $y = -x$ } retas tangentes

0,2

c) horizontais $\frac{dy}{dt} = 0 = \cos^2 t - \sin^2 t$

$= \cos 2t$

$\Rightarrow 2t = \frac{\pi}{2}, \; 2t = \frac{3\pi}{2}$

$2t = \frac{5\pi}{2}, \; 2t = \frac{7\pi}{2}$

$\Rightarrow t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

0,1

$P_1 = \left(\frac{\sqrt{2}}{2}, \frac{1}{2}\right)$

$P_2 = \left(-\frac{\sqrt{2}}{2}, -\frac{1}{2}\right)$

$P_3 = \left(-\frac{\sqrt{2}}{2}, \frac{1}{2}\right)$

$P_4 = \left(\frac{\sqrt{2}}{2}, -\frac{1}{2}\right)$ } 0,4

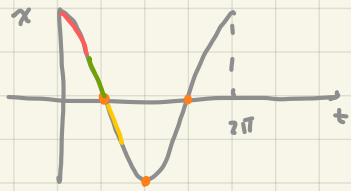
Vertical $\frac{dx}{dt} = -\sin t = 0$

$t = 0, \pi$ 0,1

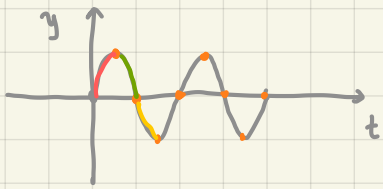
$Q_1 = (1, 0)$

$Q_2 = (-1, 0)$ 0,1

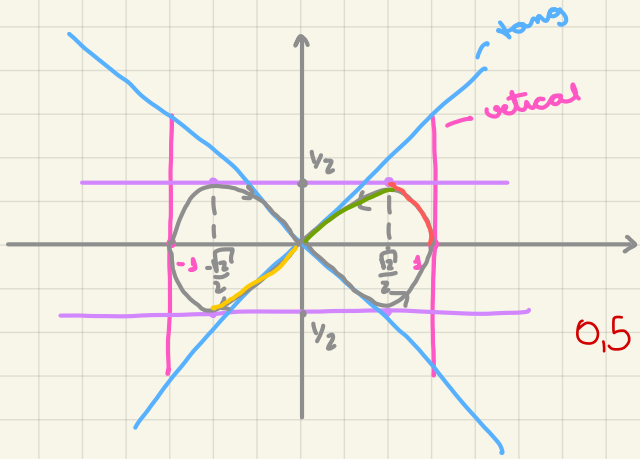
d)



$x = \cos t$

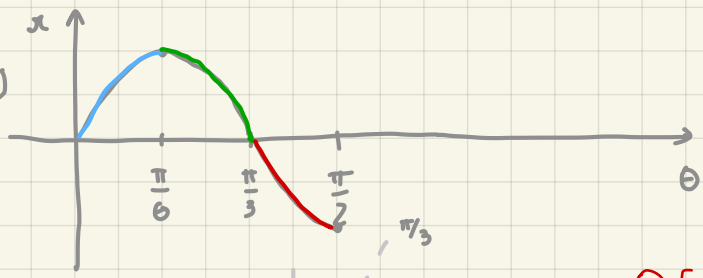


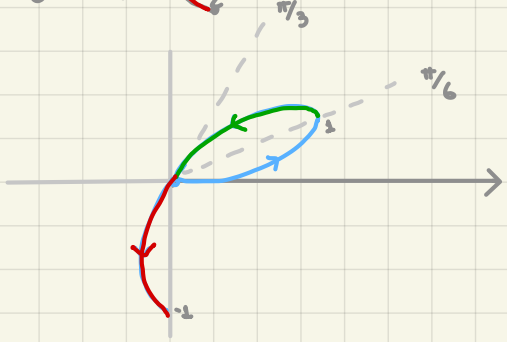
$y = \frac{1}{2}\sin 2t$



0,5

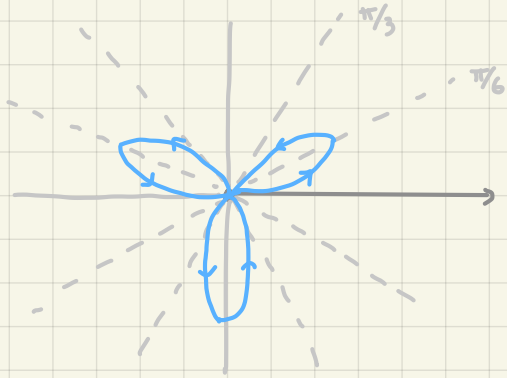
3) a)





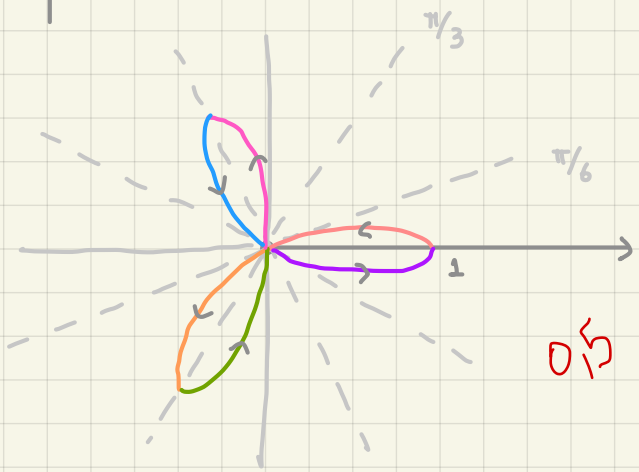
0,5

b)



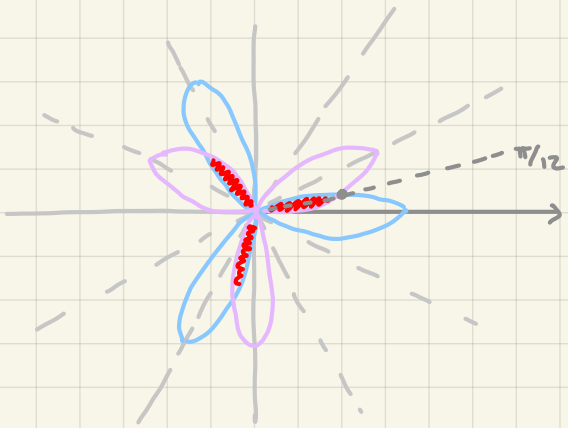
0,5

c)



0,5

d)



0,2

$\cos 3\theta = \operatorname{sen} 3\theta$

$\Rightarrow 3\theta = \pi/4 \Rightarrow \theta = \pi/12$ 0,2

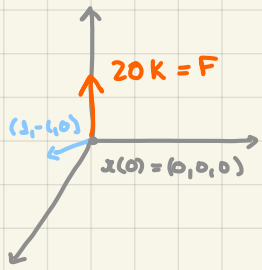
0,4

$$A = 6\int_0^{\pi/12} \frac{(\operatorname{sen} 3\theta)^2}{2}\, d\theta = \frac{3}{2}\int_0^{\pi/12} (1-\cos 6\theta)\, d\theta$$

$$= \frac{3}{2}\left[\theta - \frac{1}{6}\operatorname{sen} 6\theta\right]\Big|_0^{\pi/12}$$

$$= \frac{3}{2}\left[\frac{\pi}{12} - \frac{1}{6}\right] \quad 0,2$$

4)



0,5

$20K = F = m.a = 4a$

$a = 5K$ 0,3

$v(t) = \int 5K\,dt + v_0$

$= 5tK + i - J$ 0,5

$v(t) = i - J + 5tK$

$x(t) = \int (i - J + 5tK)\,dt + x_0$ 0,2

$x(t) = ti - tJ + \frac{5}{2}t^2$ 0,5

5) $x(0) = (0,0,3)$

$$L = \int_0^{t_1} \|x'(t)\| = 5$$ 0,5

$x(t) = 3\operatorname{sen} t\, i + 4tJ + 3\cos K$

$x'(t) = 3\cos t + 4J - 3\operatorname{sen} t K$ 0,5

$$\int_0^{t_1} \sqrt{9+16}\, dt = 5$$ 0,3

$$\int_0^{t_1} 5 \cdot dt = 5$$

$$5t\Big|_0^{t_1} = 5$$

$5t_1 = 5$

$t_1 = 1$ 0,3

$x(1) = (3\operatorname{sen} 1, 4, 3\cos 1)$.

0,4