Lista de Exercícios - Derivadas de Funções Trigonométricas Inversas

1) Nos exercícios abaixo, ache a derivada da função.

a) $f(x) = \text{sen}^{-1}\frac{1}{2}x$

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2} = \frac{\frac{1}{2}}{\sqrt{1-\frac{x^2}{4}}} = \frac{\frac{1}{2}}{\sqrt{\frac{4-x^2}{4}}} = \frac{\frac{1}{2}}{\frac{\sqrt{4-x^2}}{2}} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{4-x^2}}$$

$$f'(x) = \frac{1}{\sqrt{4-x^2}}$$

b) $g(x) = \text{tg}^{-1}2x$

$$g'(x) = \frac{1}{1+(2x)^2} \cdot 2 = \frac{2}{1+4x^2}$$

c) $F(x) = 2\cos^{-1}\sqrt{x}$

$$F'(x) = 2 \cdot \left(-\frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}}\right) \cdot \left(\frac{1}{2}x^{-1/2}\right)$$

$$F'(x) = \left(-\frac{\cancel{2}}{\sqrt{1-x}}\right) \cdot \left(\frac{1}{\cancel{2}\sqrt{x}}\right)$$

$$F'(x) = -\frac{1}{\sqrt{x-x^2}}$$

d) $g(t) = \sec^{-1}5t + \cos\sec^{-1}5t$

$$g'(t) = \frac{1}{5t \cdot \sqrt{(5t)^2 - 1}} \cdot 5 - \frac{1}{5t \cdot \sqrt{(5t)^2 - 1}} \cdot 5$$

$$g'(t) = 0$$

e) $f(x)=\operatorname{sen}^{-1}\sqrt{1-x^2}$

$$f'(x)=\frac{1}{\sqrt{1-\left(\sqrt{1-x^2}\right)^2}}\cdot\frac{1}{\cancel{2}}\left(1-x^2\right)^{-1/2}\cdot(-\cancel{2}x)$$

$$f'(x)=\frac{1}{\sqrt{1-1+x^2}}\cdot\left(1-x^2\right)^{-1/2}\cdot(-x)$$

$$f'(x)=-\frac{x}{\sqrt{x^2}\cdot\sqrt{1-x^2}}$$

$$f'(x)=-\frac{x}{|x|\cdot\sqrt{1-x^2}}$$

f) $F(x)=\operatorname{cotg}^{-1}\frac{2}{x}+\operatorname{tg}^{-1}\frac{x}{2}$

$$F'(x)=-\frac{1}{1+\left(\frac{2}{x}\right)^2}\cdot\left(-2x^{-2}\right)+\frac{1}{1+\left(\frac{x}{2}\right)^2}\cdot\frac{1}{2}$$

$$F'(x)=\frac{2x^{-2}}{1+\frac{4}{x^2}}+\frac{1/2}{1+\frac{x^2}{4}}$$

$$F'(x)=\frac{2x^{-2}}{\frac{x^2+4}{x^2}}+\frac{1/2}{\frac{4+x^2}{4}}$$

$$F'(x)=2x^{-2}\cdot\frac{x^2}{x^2+4}+\frac{1}{\cancel{2}}\cdot\frac{\cancel{4}}{x^2+4}$$

$$F'(x)=\frac{2}{x^2+4}+\frac{2}{x^2+4}$$

$$F'(x)=\frac{4}{x^2+4}$$

g) $h(y)=y\operatorname{sen}^{-1}2y$

$$h'(y)=y\cdot\frac{d}{dy}\left[\operatorname{sen}^{-1}2y\right]+\operatorname{sen}^{-1}2y\cdot\frac{d}{dy}[y]$$

$$h'(y)=y\cdot\frac{1}{\sqrt{1-(2y)^2}}\cdot 2+\operatorname{sen}^{-1}2y$$

$$h'(y) = \frac{2y}{\sqrt{1-4y^2}} + \text{sen}^{-1} 2y$$

h) $g(x) = x^2 \sec^{-1} \frac{1}{x}$

$$g'(x) = x^2 \cdot \frac{d}{dx}\left[\sec^{-1} \frac{1}{x}\right] + \sec^{-1} \frac{1}{x} \cdot \frac{d}{dx}\left[x^2\right]$$

$$g'(x) = x^2 \cdot \frac{1}{\frac{1}{x} \cdot \sqrt{\left(\frac{1}{x}\right)^2 - 1}} \cdot \left(-x^{-2}\right) + \sec^{-1} \frac{1}{x} \cdot (2x)$$

$$g'(x) = -\frac{1}{\frac{1}{x} \cdot \sqrt{\frac{1}{x^2} - 1}} + 2x \cdot \sec^{-1} \frac{1}{x}$$

$$g'(x) = -\frac{1}{\frac{1}{x} \cdot \sqrt{\frac{1-x^2}{x^2}}} + 2x \cdot \sec^{-1} \frac{1}{x}$$

$$g'(x) = -\frac{1}{\frac{1}{x} \cdot \frac{1}{|x|}\sqrt{1-x^2}} + 2x \cdot \sec^{-1} \frac{1}{x}$$

$$g'(x) = -\frac{x \cdot |x|}{\sqrt{1-x^2}} + 2x \cdot \sec^{-1} \frac{1}{x}$$

$$g'(x) = 2x \cdot \sec^{-1} \frac{1}{x} - \frac{x \cdot |x|}{\sqrt{1-x^2}}$$

i) $f(x) = \cos^{-1}(\text{sen}\, x)$

$$f'(x) = -\frac{1}{\sqrt{1-\text{sen}^2 x}} \cdot \frac{d}{dx}[\text{sen}\, x]$$

$$f'(x) = -\frac{1}{\sqrt{\cos^2 x}} \cdot \cos x$$

$$f'(x) = -\frac{\cos x}{|\cos x|}$$

j) $F(x) = \ln(\text{tg}^{-1} x^2)$

$$F'(x) = \frac{1}{\text{tg}^{-1} x^2} \cdot \frac{d}{dx}\left[\text{tg}^{-1} x^2\right]$$

$$F'(x) = \frac{1}{tg^{-1}x^2} \cdot \frac{1}{1+\left(x^2\right)^2} \cdot 2x$$

$$F'(x) = \frac{2x}{\left(1+x^4\right) \cdot tg^{-1}x^2}$$

k) $f(x) = 4sen^{-1}\frac{1}{2}x + x\sqrt{4-x^2}$

$$f'(x) = \cancel{4} \cdot \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \cdot \frac{1}{\cancel{2}} + x \cdot \frac{d}{dx}\left[\sqrt{4-x^2}\right] + \sqrt{4-x^2} \cdot \frac{d}{dx}[x]$$

$$f'(x) = \frac{2}{\sqrt{1-\frac{x^2}{4}}} + x \cdot \frac{1}{\cancel{2}}\left(4-x^2\right)^{-1/2} \cdot \left(-\cancel{2}x\right) + \sqrt{4-x^2}$$

$$f'(x) = \frac{2}{\sqrt{\frac{4-x^2}{4}}} - \frac{x^2}{\left(4-x^2\right)^{1/2}} + \sqrt{4-x^2}$$

$$f'(x) = \frac{2}{\frac{\sqrt{4-x^2}}{2}} - \frac{x^2}{\sqrt{4-x^2}} + \sqrt{4-x^2}$$

$$f'(x) = \frac{4}{\sqrt{4-x^2}} - \frac{x^2}{\sqrt{4-x^2}} + \sqrt{4-x^2}$$

$$f'(x) = \frac{4-x^2}{\sqrt{4-x^2}} + \sqrt{4-x^2}$$

$$f'(x) = \frac{4-x^2+4-x^2}{\sqrt{4-x^2}}$$

$$f'(x) = \frac{8-2x^2}{\sqrt{4-x^2}}$$

$$f'(x) = \frac{2\cdot\left(4-x^2\right)}{\sqrt{4-x^2}}$$

$$f'(x) = \frac{2\cdot\left(4-x^2\right)}{\sqrt{4-x^2}} \cdot \frac{\sqrt{4-x^2}}{\sqrt{4-x^2}}$$

$$f'(x) = \frac{2\cdot\cancel{\left(4-x^2\right)}\cdot\sqrt{4-x^2}}{\cancel{4-x^2}}$$

$$f'(x) = 2\sqrt{4-x^2}$$

l) $h(x) = \text{cossec}^{-1}(2e^{3x})$

$$h^{'}(x) = -\frac{1}{(2e^{3x}) \cdot \sqrt{(2e^{3x})^2 - 1}} \cdot \frac{d}{dx}\left[2e^{3x}\right]$$

$$h^{'}(x) = -\frac{1}{\cancel{(2e^{3x})} \cdot \sqrt{4e^{6x} - 1}} \cdot \cancel{2e^{3x}} \cdot 3$$

$$h^{'}(x) = -\frac{3}{\sqrt{4e^{6x} - 1}}$$

m) $G(x) = x\cot g^{-1}x + \ln\sqrt{1+x^2}$

$$G^{'}(x) = x \cdot \frac{d}{dx}\left[\cot g^{-1}x\right] + \cot g^{-1}x \cdot \frac{d}{dx}\left[x\right] + \frac{1}{\sqrt{1+x^2}} \cdot \frac{d}{dx}\left[\sqrt{1+x^2}\right]$$

$$G^{'}(x) = x \cdot \left(-\frac{1}{1+x^2}\right) + \cot g^{-1}x + \frac{1}{\sqrt{1+x^2}} \cdot \frac{1}{\cancel{2}} \cdot \left(1+x^2\right)^{-1/2} \cdot \cancel{2}x$$

$$G^{'}(x) = -\frac{x}{1+x^2} + \cot g^{-1}x + \frac{1}{\sqrt{1+x^2}} \cdot \left(1+x^2\right)^{-1/2} \cdot x$$

$$G^{'}(x) = -\frac{x}{1+x^2} + \cot g^{-1}x + \frac{x}{\sqrt{1+x^2} \cdot \sqrt{1+x^2}}$$

$$G^{'}(x) = \cancel{-\frac{x}{1+x^2}} + \cot g^{-1}x + \cancel{\frac{x}{1+x^2}}$$

$$G^{'}(x) = \cot g^{-1}x$$