9.2

$E = 73\,GPa \rightarrow 10^9$

$S = 250{,}28 - 250 = 0{,}28$

$L = 250\,mm$

$\varepsilon = \frac{S}{L}$

a) Tensão na barra

$$\sigma = E.\varepsilon = \frac{ES}{L} = \frac{(73.10^9)(0{,}25.10^{-3})}{250.10^{-3}}$$

$$\boxed{\sigma = 81{,}76.10^6\,PA}$$

b) coeficiente de segurança

$$F.S = \frac{\sigma_U}{\sigma} = \frac{140.10^6}{81{,}76.10^6} = 1{,}712 \,//$$

9.3

a) Diametro do fio

tração = 9N

$E = 4{,}8.10^6\,Pa$

$\varepsilon = \frac{S}{L} = \frac{0.114}{\cancel{L}} = 0{,}011$

$S = \frac{PL}{AE} \rightarrow A = \frac{PL}{SE} \rightarrow A = \frac{P}{\varepsilon E}$

$$\frac{9}{(4{,}8.10^6)(0{,}011)} = 1{,}70.10^{-4}$$

$A = \frac{\pi}{4} d^2$

$$\sqrt{\frac{4A}{\pi}} = d$$

$$\sqrt{\frac{4.\ 1{,}70.10^{-4}}{\pi}} = d$$

$d = 0{,}014\,m$

b) $\sigma = \frac{P}{A} = \frac{9}{1{,}70.10^{-4}} = 52\,941{,}17\,N \,//$

9.5

a) menor diametro

$$\delta = \frac{PL}{AE} \rightarrow A = \frac{PL}{\delta E} \rightarrow A = \frac{P}{E\varepsilon}$$

$$\rightarrow \frac{(8{,}5 \cdot 10^{3})(2{,}2)}{(200 \cdot 10^{9})(1{,}2 \cdot 10^{-3})} = 77{,}92 \cdot 10^{-6}\ m$$

$$A = \frac{\pi}{4} d^2$$

$$\rightarrow \sqrt{\frac{4A}{\pi}} = d \rightarrow \sqrt{\frac{(4)(77{,}92 \cdot 10^{-6})}{\pi}} = 9{,}96 \cdot 10^{-3}\ m$$

b) Tensão normal

$$\sigma = \frac{P}{A} = \frac{8{,}5 \cdot 10^{3}}{77{,}92 \cdot 10^{-6}} = 109{,}1 \cdot 10^{6}\ Pa$$

9.8

a) Tensão normal maxima

$E = 69 \cdot 10^{9}\ PA$

$$\varepsilon = \frac{\delta}{L} = \frac{0{,}00025\Delta}{\cancel{x}} = 0{,}00025 = \boxed{17{,}25 \cdot 10^{6}\ Pa}$$

b) espessura minima

$$\tau = \frac{P}{A} \rightarrow A = \frac{P}{\tau} = \frac{7{,}2 \cdot 10^{3}}{17{,}25 \cdot 10^{6}} = 417{,}39 \cdot 10^{-6}\ m$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2)$$

$$\rightarrow d_i^2 = d_o^2 - \frac{4A}{\pi} = 50^2 - \frac{(4 \cdot 417{,}39 \cdot 10^{-6})}{\pi}$$

... continuando

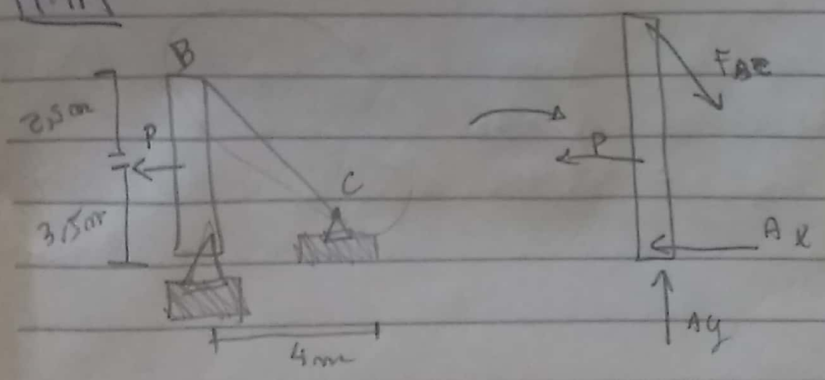
$$50^2 \cdot \frac{(4 \cdot 417,39)}{\pi} = 1968,56\ m^2$$

$$t = \frac{1}{2}(d_0 - d_1)$$

$$t = \frac{1}{2}(50 - 44,368)$$

$$\boxed{t = 2,82\ mm}$$

$\boxed{9.11}$



$$L_{BC} = \sqrt{6^2 + 4^2} = 7,2111\ m$$

$$\curvearrowleft \Sigma M_A = 0$$

$$3,5P - 6 \cdot \left(\frac{4}{7,2111} F_{BC}\right) = 0$$

$$\boxed{P = 0,9509}$$

1 caso

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0,004)^2 = 12,566 \cdot 10^{-6}$$

$$\sigma = \frac{F_{BC}}{A} \Rightarrow F_{BC} = \sigma A = (190 \cdot 10^6 \cdot 12,566 \cdot 10^{-6}) = \boxed{2,388 \cdot 10^3}$$

caso 2

$\delta = \frac{F_{BC} L_{BC}}{AE} \rightarrow F_{BC} = \frac{AE\delta}{L}$

$= \frac{(12{,}566 \cdot 10^{-6})(200 \cdot 10^{9})(6 \cdot 10^{-3})}{7{,}2111}$

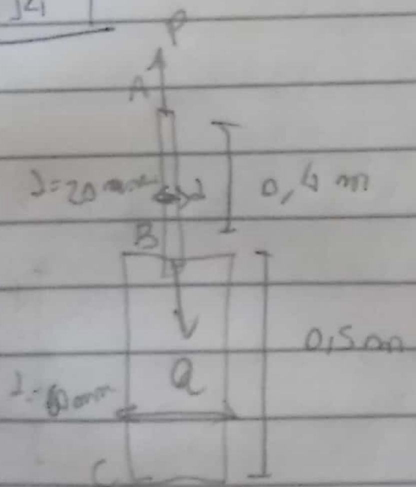
$\boxed{= 2{,}091 \cdot 10^{3}\ N}$

$\therefore P = 0{,}9509 \cdot F_{BC}$

$P = 0{,}9509 \cdot 2{,}091 \cdot 10^{3}$

$\boxed{P = 1{,}988\ kN}$

$\boxed{2.14}$



b) $\delta_{AB} = \delta_{AC} = \delta_B$

$= 72{,}756 \cdot 10^{-6}\ m$

a)

$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0{,}020)^2 = 314{,}16 \cdot 10^{-6}\ m^2$

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0{,}060)^2 = 2{,}8274 \cdot 10^{-3}\ m^2$

$\boxed{AB} \rightarrow P$

$\delta_{AB} = \frac{P L_{AB}}{E A_{AB}} = \frac{4 \cdot 10^{3}(0{,}4)}{(70 \cdot 10^{9})(314{,}16 \cdot 10^{-6})}$

$= \boxed{72{,}756 \cdot 10^{-6}\ m}$

$\boxed{BC} \rightarrow Q - P$

$\delta_{BC} = \frac{(Q-P) L_{BC}}{E A_{BC}} = \frac{(Q-P)(0{,}5)}{(70 \cdot 10^{9})(2{,}8274 \cdot 10^{-3})}$

$= \boxed{2{,}5263 \cdot 10^{-9}(Q-P)}$

9.15

a) Ponto A

... Do exercício 9.14

$A_{AB} = 314,16 \cdot 10^{-6}$

$A_{BC} = 2,8274 \cdot 10^{-3}$

$L_{AB} = 0,4\, m$

$L_{BC} = 0,5\, m$

$P_{AB} = P = 6 \cdot 10^{3}\, N$

$P_{BC} = P - Q = 6 \cdot 10^{3} - 42 \cdot 10^{3} = \boxed{-36 \cdot 10^{3}}$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{A_{AB} E_{AB}} = \frac{(6 \cdot 10^{3})(0,4)}{(314,16 \cdot 10^{-6})(70 \cdot 10^{9})} = 109,135 \cdot 10^{-6}\, m$$

$$\delta_{BC} = \frac{P_{BC} L_{BC}}{A_{BC} E_{BC}} = \frac{(-36 \cdot 10^{3})(0,5)}{(2,8274 \cdot 10^{-3})(70 \cdot 10^{9})} = -90,947 \cdot 10^{-6}\, m$$

$$\delta_A = \delta_{AB} + \delta_{BC}$$
$$= 109,135 \cdot 10^{-6} + (-90,947 \cdot 10^{-6})\, m$$
$$= 18,19 \cdot 10^{-6}\, m$$

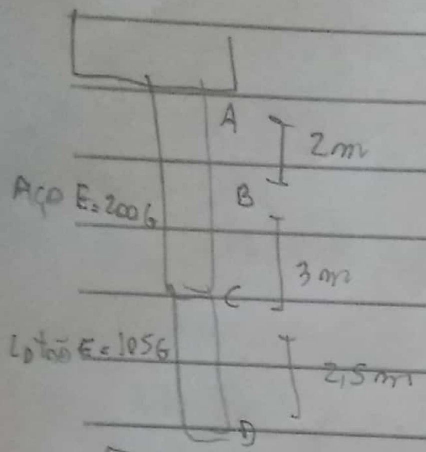
b) Ponto B

$\delta_B = \delta_{BC} \rightarrow = -90,947 \cdot 10^{-6}\, m$

deslocamento

___.___.___

9.18



a) $\delta_C = \delta_{AB} + \delta_{BC}$

$= 1,474 \cdot 10^{-3} + 1,474 \cdot 10^{-3}$

$= 2,948 \cdot 10^{-3}\ m$

b) $\delta_D = \delta_C + \delta_{CD}$

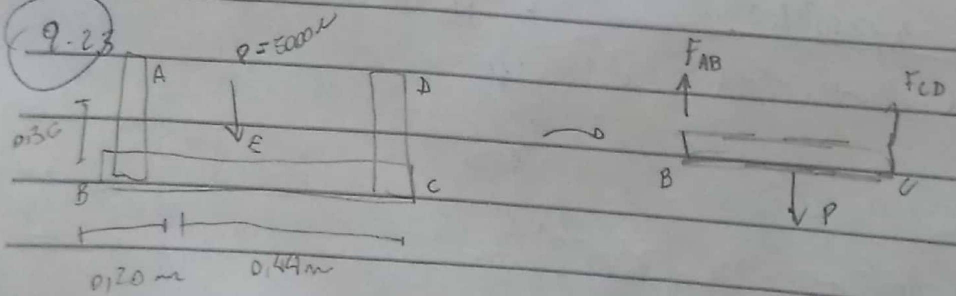
$= 2,948 \cdot 10^{-3} + 2,339 \cdot 10^{-3}$

$= 5,287 \cdot 10^{-3}\ m$

$\boxed{A = \frac{\pi}{4} d^2} \rightarrow \frac{\pi}{4}(0,036)^2 = \boxed{1,01787 \cdot 10^{-3}\ m^2}$

Parte	P_i	L_i	E_i	$P_i \cdot L_i / A E_i$
AB	150 KN	2 m	200 GPa	$1,474 \cdot 10^{-3}$ m
BC	200 KN	3 m	200 GPa	$1,474 \cdot 10^{-3}$ m
CD	100 KN	2,5 m	105 GPa	$2,339 \cdot 10^{-3}$ m

9.23



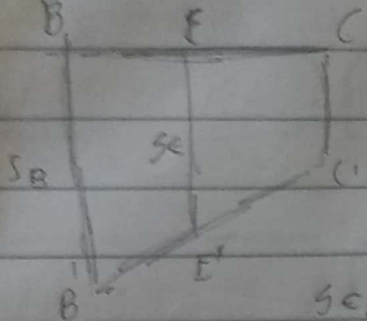
$\circlearrowleft \sum M_C = 0 \rightarrow -(0,64) F_{AB} + (0,44)(5 \cdot 10^3) = 0 \rightarrow F_{AB} = 3,4375 \cdot 10^3 N$

$\circlearrowleft M_B = 0 \quad (0,64) F_{CD} - (0,20)(5 \cdot 10^3) = 0 \rightarrow F_{CD} = 1565 \cdot 10^3 N$

AB e CD $\rightarrow A = 125\ mm^2 = 125 \cdot 10^{-6}$

$\delta_{AB} = \frac{F_{AB} L_{AB}}{EA} = \frac{(3,4375 \cdot 10^3)(0,36)}{(75 \cdot 10^9)(125 \cdot 10^{-6})} = 132 \cdot 10^6 = \delta_B$

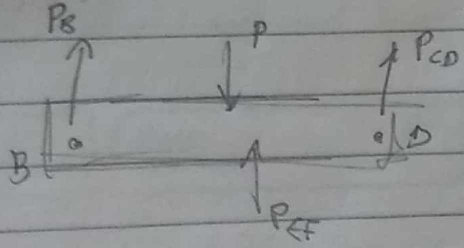
$$\delta_{CD} = \frac{F_{CD} L_{CD}}{E_{CD}} = \frac{15625 \cdot 10^3 \cdot 0,36}{(75 \cdot 10^9)(125 \cdot 10^{-6})} = 60,00 \cdot 10^{-6} = \delta_C$$



$$\theta = \frac{\delta_B - \delta_C}{L_{BC}} = \frac{72 \cdot 10^{-6}}{0,64} = 112,5 \cdot 10^{-6}$$

$$\delta_E = \delta_C + L_{EC}\,\theta$$
$$= 60 \cdot 10^{-6} (0,40)(112,5 \cdot 10^{-6})$$
$$= 109 \cdot 10^{-6}\ m$$

9.24



$P_{CD} = P_{AB}$

$\uparrow \Sigma F_y = 0$

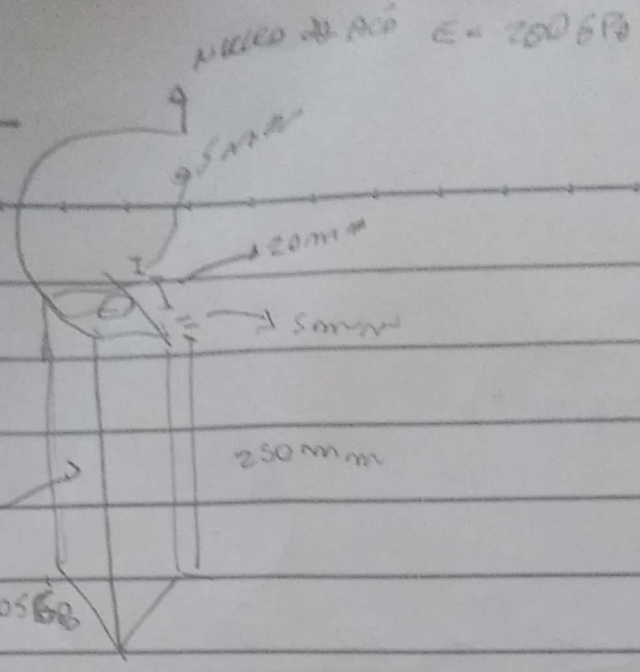
$$-P_{AB} + P_{CD} + P_{EF} + P = 0$$
$$P = 2P_{AB} + P_{EF}$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{E A_{AB}} \qquad \delta_{CD} = \frac{P_{CD} L_{CB}}{E A_{CB}} \qquad \delta = \frac{P_{EF} L_{EF}}{E A_{EF}}$$

a) $$\delta_{EF} = \frac{P_{EF} L_{EF}}{E_{EF} A_{EF}} = \frac{5,6217\,(16)}{29 \cdot 10^3\,(1)} = 0,0031016\ N$$

b) $$\sigma_{AB} = \sigma_{CD} = \frac{P_{AB}}{A_{AB}} = \frac{1,43916}{0,32} = 4,4974$$

$$\sigma_{EF} = \frac{P_{EF}}{A_{EF}} = \frac{5,617}{1} = 5,6217\ kN$$



9.25

$$\delta = \frac{P_b L}{A_b E_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_s L}{A_s E_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$\frac{\delta}{L} = \varepsilon = \frac{P}{E_b A_b + E_s A_s}$$

$$P = P_b + P_s = (E_b A_b + E_s A_s)\frac{\delta}{L}$$

$$A_s = (0,020)(0,020) = 400 \cdot 10^{-6}$$

$$A_b = (0,030)(0,030) - (0,020)(0,020) = 500 \cdot 10^{-6}\ m^2$$

$$\frac{\delta_L}{L} = \varepsilon = \frac{60 \cdot 10^3}{(105 \cdot 10^9)(500 \cdot 10^{-6}) + (200 \cdot 10^9) \cdot (400 \cdot 10^{-6})} = 452{,}83 \cdot 10^{-6}$$

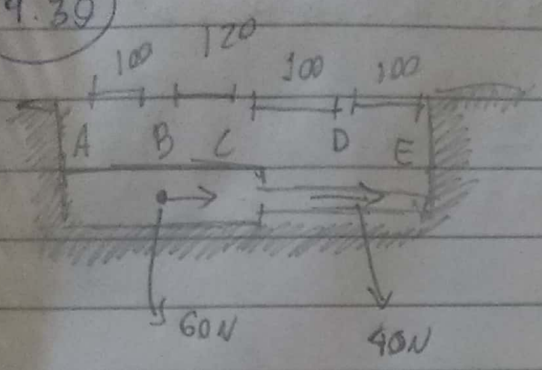
a) $\sigma_L = E_L\,\varepsilon = (105 \cdot 10^9)(452{,}83 \cdot 10^{-6})$

$= 47{,}5 \cdot 10^6\ Pa$

b) $\delta = L\,\varepsilon = (250 \cdot 10^{-3})(452{,}83 \cdot 10^{-6}) = 113 \cdot 10^{-6}\ m$

~~Lista 08~~

9.30



A -> C

$E = 200 \cdot 10^{9}$ Pa

$A = \frac{\pi}{4}(40^2)$

$A = 125664 \cdot 10^{-3}$

$\boxed{EA = 251\,327 \cdot 10^{6} N}$

C -> E

$E = 105 \cdot 10^{9}$ Pa

$A = \frac{\pi}{4}(30)^2$

$A = 706{,}86 \cdot 10^{-6}$

$\boxed{EA = 74\,220 \cdot 10^{6}}$

A -> B

$P = R_A$ e $L = 0{,}180$ m

$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0{,}180)}{251{,}327 \cdot 10^{6}} = 716{,}20^{-12}$ Pa

B -> C

$P = R_A - 60 \cdot 10^{3}$ e $L = 0{,}120$ m

$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \cdot 10^{-3})(0{,}120)}{251\,327 \cdot 10^{6}} = 477{,}47 \cdot 10^{-12} R_A - 26{,}848 \cdot 10^{-6}$

C -> D

$P = R_A - 60 \cdot 10^{3}$ e $L = 0{,}100$ m

$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 100 \cdot 10^{3})(0{,}100)}{74\,220 \cdot 10^{6}} = 1{,}34735 \cdot 10^{-9} R_A - 80{,}841 \cdot 10^{-6}$

$D \to E$

$P = R_A - 100 \cdot 10^3$

$\delta_E = \frac{PL}{EA} = \frac{(R_A - 100 \cdot 10^3)(0,100)}{74220 \cdot 10^6} = 134755 \cdot 10^{-9} - 134,735 \cdot 10^{-6}$

$A \to E$

$\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$= 3,85837 \cdot 10^{-9} R_A - 242,424 \cdot 10^{-6}$

a) $3,85837 R_A - 242,424 \cdot 10^{-6} = 0$

$R_A = 62,831 \cdot 10^3 \, N$

$R_E = R_A - 100 \cdot 10^3$

$= 62831 \cdot 10^3 - 100 \cdot 10^3$

$= -37,2 \cdot 10^3 \, N$

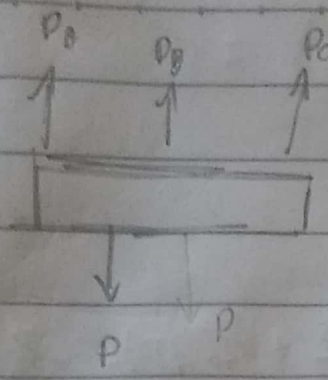
b) $\delta_C = \delta_{AB} + \delta_{BC}$

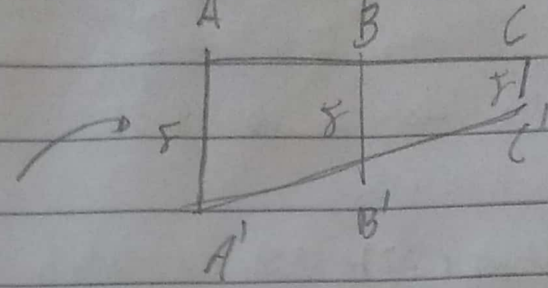
$= 1,16367 \cdot 10^{-9} R_A - 26,848 \cdot 10^{-6}$

$= (1,167 \cdot 10^{-9})(62,831 \cdot 10^3) - 26,848 \cdot 10^{-6}$

$= 46,3 \cdot 10^{-6} \, m$

9.35





$+\circlearrowleft M_A = 0$

$= 2LP_C + LP_B - \frac{3}{4} LP = 0$

$P_C = \frac{3}{8} P - \frac{1}{2} P_B$

$+\circlearrowleft M_C = 0$

$-2LP_A - LP_B + \frac{5}{4} LP = 0$

$P_A = \frac{5}{8} P - \frac{1}{2} P_B$

$\delta_A = \frac{P_A l}{EA} = \frac{l}{EA}\left(\frac{5P}{8} - \frac{P_B}{2}\right)$

$\delta_B = \frac{P_B l}{EA} = \frac{2l}{EA} P_B$

$\delta_C = \frac{P_C l}{EA} = \frac{l}{EA}\left(\frac{3}{8}P - \frac{1}{2}P\right)$

$\delta_A - \delta_B = \delta_B \; \delta_C$

$\delta_B = \frac{1}{2}\left(\delta_A + \delta_C\right)$

$\frac{l}{E \cdot \frac{A}{2}} P_B = \frac{1}{2} \frac{l}{EA}\left(\frac{5P}{8} - \frac{1}{2}P_B + \frac{3}{8}P - \frac{1}{2}P_B\right)$

$\frac{5}{2} P_B = \frac{1}{2} P \;;\; P_B = \frac{1}{5} P$

$P_A = \frac{5}{8} P - \frac{1}{2}\left(\frac{P}{5}\right) = \frac{21}{40} P$

$P_B = 0{,}200P$

$P_A = 0{,}525P$

$P_C = 0{,}275P$

9.37

utilizando o desenho do 9.35

$$\varepsilon_s = \frac{P_s}{E_s A_s} + \alpha_s(\Delta T) \quad e \quad \frac{P_s}{E_s A_s} + \alpha_s(\Delta T) = -\frac{P_s}{E_b A_b} + \alpha(\Delta T)$$

$$\varepsilon_b = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

$$\left(\frac{1}{E_s A_s} + \frac{1}{E_b A_b}\right) P_s = (\alpha_b - \alpha_s)\Delta T$$

$A_s = (0{,}020)(0{,}020) = 400 \cdot 10^{-6} m$

$A_b = (0{,}030)(0{,}030) - (0{,}020)(0{,}020) = 500 \cdot 10^{-6}$

$\alpha_b - \alpha_s = 9{,}2 \cdot 10^{-6}$

$P_s = \sigma_s A_s = (55 \cdot 10^6)(400 \cdot 10^{-6})$

$\boxed{P_s = 22 \cdot 10^3 N}$

$$\frac{1}{E_s A_s} + \frac{1}{E_b A_b} = \frac{1}{(200 \cdot 10^9)(400 \cdot 10^{-6})} + \frac{1}{(105 \cdot 10^9)(500 \cdot 10^{-6})} = 31{,}55 \frac{m}{N}$$

$(31{,}55 \cdot 10^{-9})(22 \cdot 10^3) = (9{,}2 \cdot 10^{-6})\Delta T$

$= 75{,}4°$

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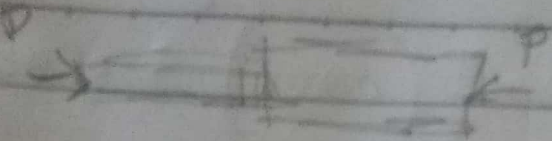
9.44



$\delta T = L_1 \alpha_1 (\Delta T) + L_a \alpha_a (\Delta T)$

$= (0{,}35)(21{,}6 \cdot 10^{-6})(96) + (0{,}45)(23{,}2 \cdot 35\ldots)$

$= 1{,}728 \cdot 10^{-3}\ m$

$\delta = 0{,}5\ mm = 0{,}500 \cdot 10^{-3}\ m$

$\delta P = 1{,}728 \cdot 10^{-3} - 0{,}500 \cdot 10^{-3}$

$= 1{,}228 \cdot 10^{-3}$

$$\delta P = \left(\frac{0{,}35}{(1500 \cdot 10^{-6})(105 \cdot 10^{9})} + \frac{0{,}45}{1800 \cdot 10^{-6} \cdot 73 \cdot 10^{9}} \right) P$$

a) $5{,}6496 \cdot 10^{-9} P = 1{,}228 \cdot 10^{-3}$

$P = 217{,}46 \cdot 10^{3}\ N$

b) $L_b \alpha_b (\Delta T) - \frac{P L_b}{A_b E_b} = (0{,}35)(21{,}6 \cdot 10^{-6}) 96 - \frac{217{,}46 \cdot 10^{3} \cdot 0{,}35}{(1500 \cdot 10^{-6})(105 \cdot 10^{9})}$

$= 725{,}76 \cdot 10^{-6} - 483{,}24 \cdot 10^{-6}$

$= 242{,}5 \cdot 10^{-6}$

9.49

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0,022)^2 = 3,80 \cdot 10^{-4}$

$P = 76,5\,KN = 76500\,N$

$\sigma = \frac{P}{A} = \frac{76500}{3,80 \cdot 10^{-4}} = 201315789,5$

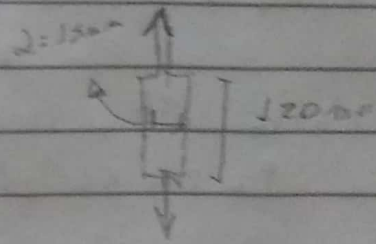
$\varepsilon_x = \frac{\sigma}{E} = \frac{201315789,5}{200 \cdot 10^{9}} = 1,006 \cdot 10^{-3}$

a) $\delta_x = L\varepsilon_x = (0,022)(1,006 \cdot 10^{-3}) = 2,21 \cdot 10^{-5}$

$\varepsilon_y = -\nu\, \varepsilon_x = (-0,3)(1,006 \cdot 10^{-3}) = -3,018 \cdot 10^{-4}$

b) $\delta_y = d\varepsilon_y = (0,022)(-3,018 \cdot 10^{-4}) = 6,64 \cdot 10^{-6}$

9.50



$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (15)^2 = 176,715 \cdot 10^{-6}\,m^2$

$P = 3,5 \cdot 10^3\,N$

$\sigma = \frac{P}{A} = \frac{3,5 \cdot 10^3}{176,715 \cdot 10^{-6}} = 19,806 \cdot 10^6\,Pa$

$\varepsilon_x = \frac{\delta_x}{L} = \frac{11}{120} = 91,667 \cdot 10^{-3}$

$E = \frac{\sigma}{\varepsilon_x} = \frac{19,806 \cdot 10^6}{91,667 \cdot 10^{-3}} = 216 \cdot 10^6\,Pa$

$\delta_y = -0,62\,mm$

$\varepsilon_y = \frac{\delta_y}{d} = \frac{-0,62}{15} = -41,333 \cdot 10^{-3}$

$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = \frac{41,333 \cdot 10^{-3}}{91,667 \cdot 10^{-3}} = 0,4509$

$G = \frac{E}{2(1+\nu)} = \frac{216 \cdot 10^6}{2 \cdot (1+0,4509)} = 74,5 \cdot 10^{6}\,Pa$

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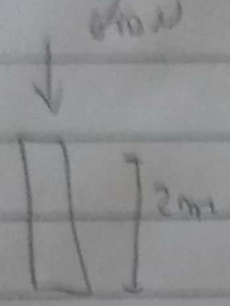
(4.51)



$d_0 = 240$ mm

$t = 10$ mm

$d_i = d_0 - 2t = 220$ mm

$P = 640 \cdot 10^{3}$ N

a) $\delta = -\dfrac{PL}{AE}$

$= -\dfrac{(640 \cdot 10^3)(2)}{(7,2257 \cdot 10^{-3})(73 \cdot 10^9)}$

$= -2,427 \cdot 10^{-3}$

$\varepsilon = \dfrac{\delta}{L} = \dfrac{2,427 \cdot 10^{-3}}{2,00} = -1,2133 \cdot 10^{-3}$

$\varepsilon_{lat} = -v \cdot \varepsilon = (-0,33)(-1,2133 \cdot 10^{-3})$

$= 400,4 \cdot 10^{-6}$

b) $\Delta d_0 = d_0 \cdot \varepsilon_{lat} = (240)(400,4 \cdot 10^{-6}) = 0,0961$ mm

c) $\Delta t = t\,\varepsilon_{lat} = (10)(400,4 \cdot 10^{-6}) = 0,00400$ mm