Sinais e Sistemas

1. a) $x(t) = x(t+T) = \cos\left(\frac{\pi}{3}t + \frac{\pi}{3}T\right) + \sin\left(\frac{\pi}{4}t + \frac{\pi}{4}T\right)$

$= \cos\left(\frac{\pi}{3}t\right)\cdot\underbrace{\cos\left(\frac{\pi}{3}T\right)}_{1} - \operatorname{sen}\left(\frac{\pi}{3}t\right)\underbrace{\operatorname{sen}\left(\frac{\pi}{3}T\right)}_{0} + \operatorname{sen}\left(\frac{\pi}{4}t\right)\underbrace{\cos\left(\frac{\pi}{4}T\right)}_{1} + \operatorname{sen}\left(\frac{\pi}{4}T\right)\cos\left(\frac{\pi}{4}t\right)$

$= \frac{\pi}{3}T = 2\pi k$ e $\frac{\pi}{4}T = 2\pi l \Rightarrow \left.\begin{matrix} T = 6k \\ T = 8l \end{matrix}\right\} k = \frac{8}{6}l = \frac{4}{3}l$ com $l=3$, $T_0 = 24$

b) $x(t+T) = \cos(t+T) + \sin(\sqrt{2}t + \sqrt{2}T)$

$= \cos(t)\cdot\underbrace{\cos(T)}_{=1} - \operatorname{sen}(t)\cdot\underbrace{\operatorname{sen}(T)}_{=0} + \operatorname{sen}(\sqrt{2}t)\cdot\underbrace{\cos(\sqrt{2}T)}_{=1} + \underbrace{\operatorname{sen}(\sqrt{2}T)}_{=0}\cdot\cos(\sqrt{2}t)$

$\left\{\begin{matrix} T = 2\pi k \\ \sqrt{2}T = 2\pi l \end{matrix}\right. \Rightarrow \cancel{2\pi} k = \frac{\cancel{2}\pi l}{\sqrt{2}} \Rightarrow \sqrt{2}k = l$ não existe valor inteiro $\Big\}$ T não existe

c) $x(t+T) = \sin^2(t+T) = \sin^2(t+T) = \left(\sin(t)\cdot\underbrace{\cos(T)}_{=1} + \underbrace{\sin(T)}_{=0}\cdot\cos(t)\right)^2$

periódico $\rightarrow T = k\pi$

2. a) $x(t) = t\,u(t)$

$E = \lim_{T\to\infty}\int_{-T/2}^{T/2} t^2 u^2(t)\,dt = \lim_{T\to\infty}\int_0^{T/2} t^2\,dt = \lim_{T\to\infty}\frac{1}{3}\cdot\frac{T^3}{8}$

$P = \lim_{T\to\infty}\frac{1}{T}\cdot\frac{1}{3}\frac{T^3}{8} = \lim_{T\to\infty}\frac{T^2}{24}$

Não é de Energia ou Potência ?

b) $x(t) = A\cdot\sin(w_0 t + \phi)$

$E = \lim_{T\to\infty}\int_{-T/2}^{T/2} A^2\operatorname{sen}^2(w_0t+\phi)\,dt \xrightarrow[du = w_0 dt]{u = w_0t+\phi} \lim_{T\to\infty}\frac{A^2}{w_0}\int_{-w_0\frac{T}{2}+\phi}^{w_0\frac{T}{2}+\phi}\sin^2(u)\,du$

$= \lim_{T\to\infty}\frac{A^2}{w_0}\left[\frac{u}{2} - \frac{1}{4}\operatorname{sen}(2u)\right]_{-w_0\frac{T}{2}+\phi}^{w_0\frac{T}{2}+\phi} =$

$= \lim_{T\to\infty}\frac{A^2}{w_0}\left[\frac{1}{2}\left(w_0\frac{T}{2}+\cancel{\phi}\right) - \frac{1}{4}\sin(w_0T+2\phi) - \left(\frac{1}{2}\left(-w_0\frac{T}{2}+\cancel{\phi}\right) - \frac{1}{4}\operatorname{sen}(-w_0T+2\phi)\right)\right]$

$P = \lim_{T\to 0}\frac{A^2}{w_0}\cdot\frac{1}{T}\cdot w_0\frac{T}{2} = \frac{A^2}{2}$ W

(c) $x(t) = e^{-at}u(t)$, $a > 0$

$E = \lim_{T\to\infty}\int_{-T/2}^{T/2} u^2(t)\cdot e^{-2\alpha t}\,dt = \lim_{T\to\infty}\int_0^{T/2} e^{-2\alpha t}\,dt = \lim_{T\to\infty}\frac{-1}{2\alpha}\left[e^{-2\alpha t}\right]_0^{T/2}$

$= \lim_{T\to\infty}\frac{-1}{2\alpha}\left[e^{-\alpha T} - 1\right] = \frac{1}{2\alpha}$ J

3. $y(t) = \dfrac{e^{x(t)}}{x(t-t_0)}$; $t_0 > 0$

linear: $= \dfrac{e^{[x_1(t) + x_2(t)]}}{x_1(t-t_0) + x_2(t+t_0)} = \dfrac{e^{x_1(t)} \cdot e^{x_2(t)}}{x_1(t-t_0) + x_2(t-t_0)} \neq \dfrac{e^{x_1(t)}}{x_1(t-t_0)} + \dfrac{e^{x_2(t)}}{x_2(t-t_0)}$ Não Linear

TI $= x(t) \to x(t+t_0) \to \dfrac{e^{x(t+t_0)}}{x(t)}$ Invariante

$y_0 = y_i$

$x(t) \to \dfrac{e^{x(t)}}{x(t-t_0)} \to \dfrac{e^{x(t+t_0)}}{x(t)}$

Estável: $|x(t)| \leq M_x < \infty$

$|y(t)| = \left| \dfrac{e^{x(t)}}{x(t-t_0)} \right| = \dfrac{|e^{x(t)}|}{|x(t-t_0)|}$ → para $x(t-t_0) = 0$

$y(t) \to \infty$

portanto é instável

4. $y_1(t) = x(2t)$

$y_2(t) = x(t/2)$

$x(t) = x(t+T_0)$

1> $y_1(t+T) = x(2(t+T_1))$ → $T_1 = T_0/2$

2> V

3> $y_2(t+T) = x(\frac{1}{2}(t+T_2))$ → $T_2 = 2T_0$

4> V

5.> $g_1(t)$ e $g_2(t)$ são sinais pares $\begin{cases} g_1(t) = g_1(-t) \\ g_2(t) = g_2(-t) \end{cases}$

* $g_1(t) + g_2(t) \to g_1(-t) + g_2(-t) = g_1(t) + g_2(t)$ PAR
* $g_1(t) - g_2(t) \to g_1(-t) - g_2(-t) = g_1(t) - g_2(t)$ PAR
* $g_1(t) \cdot g_2(t) = g_1(-t) \cdot g_2(-t)$ PAR
* $g_1(t) / g_2(t) = g_1(-t) / g_2(-t)$ PAR

> $g_1(t)$ e $g_2(t)$ $\begin{cases} g_1(t) = -g_1(-t) \\ g_2(t) = -g_2(-t) \end{cases}$

* $g_1(t) + g_2(t) = -(g_1(-t) + g_2(-t)) = g_1(t) + g_2(t)$ Impar
* $y(t) = g_1(t) - g_2(t) = -(g_1(-t) - g_2(-t)) = g_1(t) - g_2(t) = y(t)$
* $g_1(t) \cdot g_2(t) = y(t) \overset{PAR}{=} g_1(-t) \cdot g_2(-t) = (-g_1(t))(-g_2(t)) = g_1(t) \cdot g_2(t)$
* $\dfrac{g_1(t)}{g_2(t)} = y(t) \overset{PAR}{=} \dfrac{g_1(-t)}{g_2(-t)} = \dfrac{-g_1(t)}{-g_2(t)} = \dfrac{g_1(t)}{g_2(t)}$

6. $s(t) = e^{-t} u(t)$

$x(t) = u(t-1) - u(t-3)$

$y(t) = T\{x(t)\} = s(t-1) - s(t-3) = e^{-(t-1)} u(t-1) - e^{-(t-3)} u(t-3)$

7.

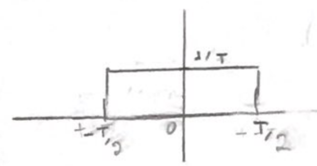
$$y(t) = T\{x(t)\} = \frac{1}{T} \int_{t-T/2}^{t+T/2} x(\tau)\, d\tau$$

a) $h(t)$?

Relação: $x(t) * u(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau$

$$T\{x(t)\} = \frac{1}{T}\Big(x(t) * \big(u(t+T/2) - u(t-T/2) \big) \Big)$$

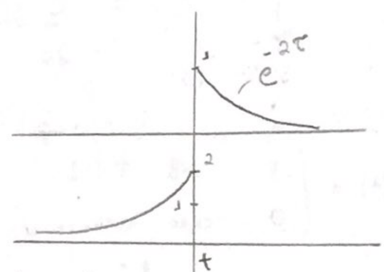
$$h(t) = \frac{1}{T}\big(u(t+T/2) - u(t-T/2) \big)$$



b) para $t<0$, $h(t) \neq 0$

não é causal

8. $x(t) * \big(\underbrace{e^{-2t} u(t) * 2e^{-t} u(t)}_{=h(t)} \big) = y(t)$



$$h(t) = \int_{-\infty}^{\infty} e^{-2\tau} u(\tau) \cdot 2e^{-(t-\tau)} u(t-\tau)\, d\tau$$

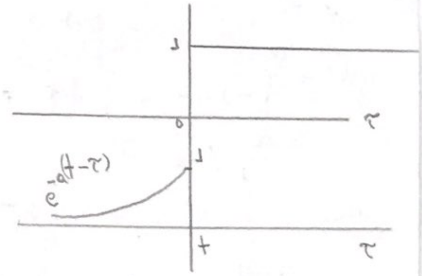
$$2\int_0^{\infty} e^{-\tau} \cdot e^{-t} u(t-\tau)\, d\tau = 2e^{-t}\int_0^t e^{-\tau}\, d\tau = -2e^{-t}\big[\underbrace{e^{-t} - 1}_{<0}\big] \cdot u(t)$$

BIBO?

$$\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty \;\therefore\; \int_{-\infty}^{\infty} \left|(-2e^{-2\tau} + 2e^{-\tau}) \cdot u(\tau)\right| d\tau = 2\int_0^{\infty} \left|-\cancel{2}e^{-2\tau} + \cancel{2}e^{-\tau}\right| d\tau$$

$$= 2\left[\frac{1}{2}e^{-2\tau} - e^{-\tau}\right]_0^{\infty} = -2\left[\frac{1}{2} - 1\right] = 1 < \infty$$

9. $y(t) = u(t) * e^{-at} u(t)$, $a>0$

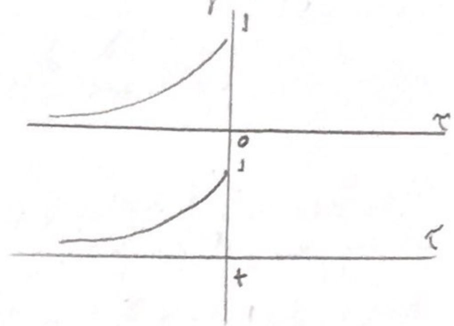


$$y(t) = \int_{-\infty}^{\infty} u(\tau) \cdot e^{-a(t-\tau)} u(t-\tau)\, d\tau = \int_0^t e^{-a(t-\tau)}\, d\tau$$

$$= \frac{e^{-ta}}{a}\left(e^{at} - 1\right)$$

$$y(t) \begin{cases} 0, & t<0 \\ \frac{1}{a}(1 - e^{-ta}), & t>0 \end{cases}$$

10. $x(t) = e^{at} u(-t)$, $a > 0$ $\rightarrow$ $h(t) = e^{-at} u(t)$, $a > 0$ $\rightarrow$ $y(t)$



$$y = \int_{-\infty}^{\infty} e^{a\tau} u(-\tau) \cdot e^{-a(t-\tau)} \cdot u(t-\tau)\, d\tau$$

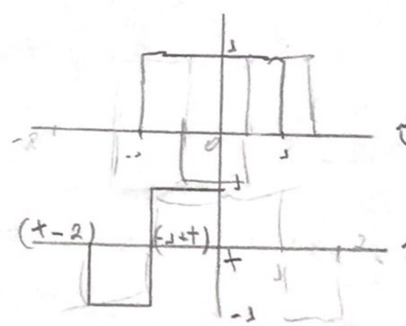
$$= \int_{-\infty}^{t} e^{2a\tau} \cdot e^{-at} \cdot u(-\tau)\, d\tau = e^{-at} \int_{-\infty}^{t} e^{2a\tau} u(-\tau)\, d\tau$$

$$y(t) = e^{-at} \int_{-\infty}^{t} e^{2a\tau}\, d\tau = \frac{e^{-at}}{2a} \left(e^{2at} \right) = \frac{e^{at}}{2a}, \quad t < 0$$

$$y(t) = e^{-at} \int_{-\infty}^{0} e^{2a\tau}\, d\tau = \frac{e^{-at}}{2a}, \quad t \geq 0$$

11.

$$x(t) = \begin{cases} 1 & -1 < t < 1 \\ 0 & \text{caso contrario} \end{cases} \rightarrow h(t) = \begin{cases} 1 & 0 < t < 1 \\ -1 & 1 < t < 2 \\ 0 & \text{caso contrario} \end{cases} \rightarrow y(t)$$



$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(t) = 0, \quad t < -1$$

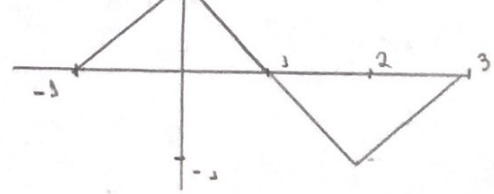
$$y(t) = \int_{-1}^{t} d\tau = (t+1), \quad -1 \leq t < 0$$

$$y(t) = \int_{-1}^{(t-1)} -1\, d\tau + \int_{(t-1)}^{t} d\tau = -[t \cancel{-1} \cancel{+1}] + [\cancel{t} - (\cancel{t}-1)] = (-t+1), \quad 0 \leq t < 1$$

$$y(t) = \int_{(t-2)}^{(t-1)} -1\, d\tau + \int_{(t-1)}^{1} d\tau = -[\cancel{t} - 1 - (\cancel{t} - 2)] + 1 - (t-1) = -(t-1), \quad 1 \leq t < 2$$

$$y(t) = \int_{(t-2)}^{1} -1\, d\tau = -[1 - (t-2)] = -(3-t), \quad 2 \leq t < 3$$

$$y(t) = 0, \quad t \geq 3$$



11. BIBO?

$$\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty \rightarrow \int_{0}^{2} 1 \, d\tau = 2 < \infty$$

$y(t)$ é E ou P?

$$\mathcal{E} = \int_{-\infty}^{\infty} y^2(t)\, dt = \int_{-1}^{0} (t+1)^2\, dt + \int_{0}^{1} (-t+1)^2\, dt + \int_{1}^{2} (-t+1)^2\, dt + \int_{2}^{3} (-3+t)^2\, dt$$

$$= -\left(-\frac{1}{3} + \cancel{1} - \cancel{1}\right) + \left(\cancel{\frac{1}{3}} - \cancel{1} + \cancel{1}\right) + \left(\frac{2^3}{3} - 2^2 + \cancel{2}\right) - \left(\cancel{\frac{1}{3}} - \cancel{1} + \cancel{1}\right) + \left(\frac{3^3}{3} - 3^3 + 9.3\right) -$$

$$- \left(\frac{2^3}{3} - 3.2^2 + 9.2\right) + 10 = 1{,}34 \text{ J}$$

12. $\frac{d}{dt} y(t) + 2y(t) = x(t) + \frac{d}{dt} x(t)$

$$(D+2)\, y(t) = (1+D)\, x(t)$$

a) $H(s)$?

$$L\{Dy(t)\} + 2L\{y(t)\} = L\{x(t)\} + L\{Dx(t)\}$$

$$sY(s) + 2Y(s) = X(s) + sX(s)$$

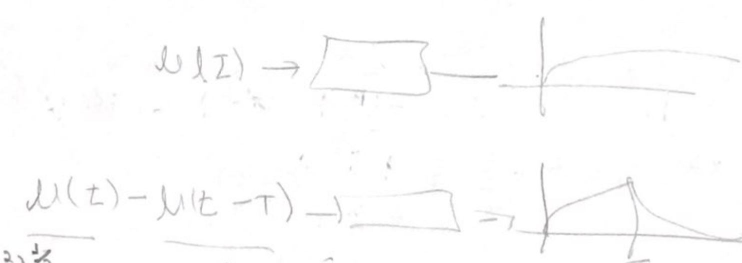
$$(s+2)\, Y(s) = (1+s)\, X(s)$$

$$Y(s) = \frac{1+s}{s+2} X(s) \rightarrow$$

$$H(s) = \frac{1+s}{s+2}$$

$$\begin{array}{r|l} 1+s & s+2 \\ \hline -2-s & 1 \\ -1 & \end{array}$$

b) $h(t) = L^{-1}\{H(s)\} = L^{-1}\left\{1 - \frac{1}{s+2}\right\} = \delta(t) - e^{-2t} u(t)$



c) $H(j\omega) = \frac{1+j\omega}{j\omega+2} = \frac{(1^2+\omega^2)^{1/2}}{(\omega^2+2^2)^{1/2}} \angle tg^{-1}\left(\frac{\omega}{1}\right) - tg^{-1}(\omega/2)$

$$\left(\frac{1+\omega^2}{2^2+\omega^2}\right) \angle tg^{-1}(\omega) - tg^{-1}(\omega/2)$$

13. TBL

$$X(s) = L\{x(t)\} = L\left\{\frac{1}{a^2}(u(t)-u(t-a)) - \frac{1}{a^2}(u(t-a)-u(t-2a))\right\}$$

$$= \frac{1}{a^2} L\{u(t) - 2u(t-a) + u(t-2a)\}$$

$$= \frac{1}{a^2}\left(\frac{1}{s} - 2\frac{1}{s}e^{-as} + \frac{1}{s}e^{-2as}\right) = \frac{1}{a^2 \cdot s}\left(1 - 2e^{-as} + e^{-2as}\right)$$

$$\lim_{a\to 0} X(s) = \infty$$

14. >

$$x_1(t) = \frac{t}{2}(u(t) - u(t-2)) = \frac{t}{2}\cdot u(t) - \frac{t}{2}u(t-2)$$

$$X_1(s) = L\{x_1(t)\} = \frac{1}{2}\left(\frac{1}{s^2} - e^{-2s}\cdot\frac{1}{s^2}\right)$$

$$E = \int_{-\infty}^{\infty}\left[\frac{t}{2}(u(t)-u(t-2))\right]^2 dt = \int_{-\infty}^{\infty}\frac{t^2}{4}(u(t)-u(t-2))^2\, dt = \int_0^2 \frac{t^2}{4}\,dt = \frac{t^3}{12}\Big|_0^2 = \frac{2}{3}$$

>

$$x_2(t) = (u(t+1) - u(t)) + 3(u(t) - u(t-1)) + 2(u(t-1) - u(t-2)) + (u(t-2) - u(t-3))$$

$$= u(t+1) + 2u(t) - u(t-1) - u(t-2) - u(t-3)$$

$$X_2(s) = L\{x_2(t)\} = \frac{1}{s}\left(e^{+s} + 2 - e^{-s} - e^{-2s} - e^{-3s}\right)$$

$$E = \int_{-\infty}^{\infty}\left[u(t+1) + 2u(t) - u(t-1) - u(t-2) - u(t-3)\right]^2 dt$$

15. a) $X(s) = \dfrac{s+5}{s^2(s+2)}$, $Re\{s\} > 0$

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$$= \frac{5/2}{s^2} + \frac{-3/4}{s} + \frac{3/4}{(s+2)} \quad \rightarrow \quad a = \frac{d}{ds}\left[\frac{(s+5)}{(s+2)}\right]\Bigg|_{-a} :$$

$$x(t) = \left(\frac{5}{2}\cdot t - \frac{3}{4} + \frac{3}{4}e^{-2t}\right)u(t)$$

b) $$X(s) = \frac{10s^2}{(s+1)(s+3)} = \left\{\begin{array}{l|l} 10s^2 & s^2+4s+3 \\ -40s-30 & 10 \end{array}\right\} = \frac{-40s-30}{(s+1)(s+3)} + 10$$

$$= \frac{5}{(s+1)} + \frac{-45}{(s+3)} + 10 \rightarrow \left(10\,\delta(t) + (5e^{-t} - 45e^{-3t})u(t)\right.$$

16. A) $X(s) = \dfrac{s}{(s-3)(s^2-4s+5)} = \dfrac{s}{(s-3)((s-2)^2+1)} = \underbrace{\left(\dfrac{1}{(s-3)}\right)}_{X_1}\underbrace{\left(\dfrac{s}{((s-2)^2+1)}\right)}_{X_2}$

$L^{-1}\{X(s)\} = L^{-1}\{X_1 X_2\} = x_1 * x_2 = -t e^{3t} u(+t) * t \cdot \cos(t) \cdot e^{2t} u(+t)$

$\therefore X(s) = \dfrac{s}{(s-3)(s-2-i)(s-2+i)} = \dfrac{3/2}{(s-3)} + \dfrac{\left(-\frac{3}{4}+\frac{i}{4}\right)}{(s-2-i)} + \dfrac{\left(-\frac{3}{4}-\frac{i}{4}\right)}{(s-2+i)}$

$x(t) = \dfrac{3}{2}e^{3t} + \left(-\dfrac{3}{4}+\dfrac{i}{4}\right)\cdot e^{(2+i)t} + \left(-\dfrac{3}{4}-\dfrac{i}{4}\right)e^{(2-i)t}$

$= \dfrac{3}{2}e^{3t} + \dfrac{e^{2t}}{4}\left[(-3+i)(\cos t + i\,\mathrm{sen}\,t) + (-3-i)(\cos(+t) - i\,\mathrm{sen}(+t))\right]$

$-3\cos t \cancel{-3i\,\mathrm{sen}\,t} + \cancel{i\cos t} - \mathrm{sen}\,t - 3\cos t \cancel{+3i\,\mathrm{sen}\,t} \cancel{-i\cos t} - \mathrm{sen}\,t$

$= \dfrac{3}{2}e^{3t} + \dfrac{e^{2t}}{4}(-6\cos t - 2\,\mathrm{sen}\,t) = \left[\dfrac{3}{2}e^{3t} - \dfrac{e^{2t}}{2}(3\cos t + \mathrm{sen}\,t).\right] - u(-t)$

B) $X(s) = \dfrac{5s+13}{s(s^2+4s+13)} = \dfrac{5s+13}{s((s+2)^2+9)} = \dfrac{5s+13}{s(s+2-3i)(s+2+3i)}$

$= \dfrac{1}{s} + \dfrac{\left(-\frac{1}{2}-\frac{i}{2}\right)}{(s+2-3i)} + \dfrac{\left(-\frac{1}{2}+\frac{i}{2}\right)}{(s+2+3i)}$

$x(t) = 1 + \left(-\dfrac{1}{2}-\dfrac{i}{2}\right)e^{-2+3i} + \left(-\dfrac{1}{2}+\dfrac{i}{2}\right)e^{-2-3i}$

$= 1 + \dfrac{e^{-2}}{2}\left[(-1-i)(\cos 3t + i\,\mathrm{sen}\,3t) + (-1+i)(\cos(-3t) + i\,\mathrm{sen}(-3t))\right]$

$-\cos(3t) \cancel{-i\,\mathrm{sen}\,3t} \cancel{-i\cos(3t)} + \mathrm{sen}\,3t - \cos(+3t) + \cancel{i\,\mathrm{sen}(+3t)}$
$\cancel{+i\cos(+3t)} + \mathrm{sen}(+3t)$

$\left[1 + e^{-2}(-\cos(3t) + \mathrm{sen}(3t))\right] u(t)$

17. A) $X(s) = \dfrac{2s+4}{s^2+4s+3} = \dfrac{2s+4}{(s+1)(s+3)} = \dfrac{1}{(s+1)} + \dfrac{(+1)}{(s+3)} \xrightarrow{L^{-1}} x(t)$

$x(t) = (e^{-t} + e^{-3t})\,u(t) \quad , \quad Re\{s\} > -1$

b) $x(t) = (e^{-t} + e^{-3t}) - u(-t) \quad Re\{s\} < -3$

c) $x(t) = e^{-t}(-u(-t)) + e^{-3t}(u(t)) \quad -3 < Re\{s\} < -1$

18. A) $y(t) = h(t) * x(t) = 5e^{-4T} u(t) * u(t)$

$$Y(s) = \frac{5}{(s+4)} \cdot \frac{1}{s} = \frac{-5/4}{(s+4)} + \frac{5/4}{s} \overset{L^{-1}}{\Rightarrow} \frac{5}{4}\left(-e^{-4t} + 1\right) u(t)$$

$$y(t) = -\frac{5}{4} e^{-4t} u(t) + \frac{5}{4} u(t)$$

B) $y(t) = 5e^{-4t} u(t)) * u(-t) = -\frac{5}{4} e^{-4t} u(t) + \frac{5}{4} \cdot -u(-t)$

C) $y(t) = 5e^{4t} u(-t) * u(t) = -\frac{5}{4} e^{4t} u(t) - \frac{5}{4} u(t)$

D) $y(t) = 5e^{4t} u(-t) * u(-t) = -\frac{5}{4} e^{4t} u(-t) + \frac{5}{4} u(-t)$

19. $\frac{d^2}{dt^2} y(t) + \frac{d}{dt} y(t) - 2y(t) = x(t)$ $\qquad H(s) = \frac{1}{(s-1)(s+2)}$

$$s^2 Y(s) + sY(s) - 2Y(s) = X(s)$$

$$(s^2 + s - 2) Y(s) = X(s)$$

$$Y(s) = \frac{1}{(s-1)(s+2)} X(s) = \frac{1/3}{(s-1)} + \frac{-1/3}{(s+2)} X(s)$$

* $h(t) = \frac{1}{3} e^{t} u(t) - \frac{1}{3} e^{-2t} u(t)$

* $h(t) =$

*

20. $\frac{d}{dt} y(t) + 2y(t) = x(t) + \frac{d}{dt} x(t)$

$$sY(s) + 2Y(s) = X(s) + sX(s)$$

$$Y(s) = \frac{(1+s)}{(s+2)} X(s)$$

s

$$H(s) = \frac{(s+1)}{(s+2)} = 1 - \frac{1}{(s+2)} \xrightarrow{L^{-1}} \delta(t) - e^{-2t} u(t) = h(t)$$

21. A) $\frac{d}{dt} y(t) + 10\, y(t) = u(t)$, $y(0^-) = 1$

$$s\,Y(s) - y(0) + 10\, Y(s) = \frac{1}{s}$$

$$(s+10)\, Y(s) = \frac{1}{s} + 1$$

$$Y(s) = \frac{1}{(s+10)} \cdot \left(\frac{1}{s} + 1 \right) = \frac{\frac{1}{10}}{s} + \frac{-\frac{1}{10}}{(s+10)} + \frac{1}{(s+10)}$$

$$y(t) = \frac{1}{10} u(t) - \frac{1}{10} \cdot e^{-10t} u(t) + e^{-10t} u(t)$$

B) $\frac{d^2}{dt^2} y(t) - 2 \frac{d}{dt} y(t) + 4\, y(t) = u(t)$, $y(0^-) = 0$, $\frac{d}{dt} y(0^-) = 4$

$$s^2 Y(s) - s\cancel{y(0)}^{0} - y'(0) - 2s\, Y(s) - 2\cancel{y(0)}^{0} + 4\, Y(s) = \frac{1}{s}$$

$$(s^2 - 2s + 4)\, Y(s) = \frac{1}{s} + 4$$

$$Y(s) = \frac{1}{(s^2 - 2s + 4)} \left(\frac{1}{s} + 4 \right) = \frac{1}{(s - 1 - i\sqrt{3})(s - 1 + i\sqrt{3})\, s} + \frac{4}{(s-1)^2 + 3}$$

$$= \frac{-\frac{1}{8} - \frac{\sqrt{3}}{24} i}{(s - 1 - i\sqrt{3})} + \frac{-\frac{1}{8} + i \frac{\sqrt{3}}{24}}{(s - 1 + i\sqrt{3})} + \frac{\frac{1}{4}}{s} + \frac{4}{((s-1)^2 + 3)}$$

$$y(t) = \left(-\frac{1}{8} - \frac{\sqrt{3}}{24} i \right) e^{(1 + j\sqrt{3})t} + \left(-\frac{1}{8} + i \frac{\sqrt{3}}{24} \right) e^{(1 - j\sqrt{3})t} + \frac{1}{4} + \frac{4}{\sqrt{3}} e^{t} \cdot \text{sen}(\sqrt{3}t)$$

$$= 2e^{t} \left(-\frac{1}{8} \cos(\sqrt{3}t) + \frac{\sqrt{3}}{24} \text{sen}(\sqrt{3}t) \right) + \frac{1}{4} + \frac{4}{\sqrt{3}} e^{t} \text{sen}(\sqrt{3}t)$$

$$= -\frac{e^{t}}{4} \cos(\sqrt{3}t) + e^{t} \left(\frac{\sqrt{3}}{12} + \frac{4}{\sqrt{3}} \right) \cdot \text{sen}(\sqrt{3}t) + \frac{1}{4}$$

$$= \left[-\frac{e^{t}}{4} \cos(\sqrt{3}t) + e^{t} \cdot \frac{17\sqrt{3}}{12} \cdot \text{sen}(\sqrt{3}t) + \frac{1}{4} \right] u(t)$$

c) $\frac{d}{dt} y(t) + 2\, y(t) = \sin(2\pi t)\, u(t)$, $y(0^-) = -4$

$$s\,Y(s) - y(0) + 2\, Y(s) = \mathcal{L}\{ \sin(2\pi t)\, u(t) \} = \frac{2\pi}{s^2 + (2\pi)^2}$$

$$(s+2)\, Y(s) = \frac{2\pi}{s^2 + (2\pi)^2} - 4$$

$$Y(s) = \frac{2\pi}{(s+2)(s^2 + (2\pi)^2)} - \frac{4}{2+s} = \frac{\theta_1}{(s+2)} + \frac{\theta_2 s + \theta_3}{(s^2 + 4\pi^2)} - \frac{4}{(2+s)}$$

$$\begin{cases} \theta_1 = 2\pi / (4 + 4\pi^2) = \pi / 2(1 + \pi^2) \\ \theta_2 = \frac{2\pi}{s+2} = \theta_2 s + \frac{\pi}{(1+\pi^2)} \quad s = \pm 2\pi i \\ \theta_3 = \left(\cancel{2\pi}/\cancel{(2)}(2\pi)^2 - \pi/2(1+\pi^2)2 \right) \cdot 4\pi^2 = \pi - \pi^3/(1+\pi^2) = \pi / (1+\pi^2) \end{cases}$$

$$\Theta_2 = \frac{2\pi}{2+s} \equiv \Theta_2 s + \frac{\pi}{(1+\pi^2)}$$

$$\frac{2\pi}{2 \pm 2\pi i} \equiv \Theta_2(\pm 2\pi i) + \frac{\pi}{1+\pi^2}$$

$$\Theta_2 \equiv \left(\frac{\cancel{\pi}^{1}}{1 \pm \pi i} - \frac{\cancel{\pi}^{1}}{1+\pi^2} \right) \cdot \frac{1}{\pm 2i} \equiv \frac{\cancel{1} + \pi^2 - \cancel{1} \mp \pi i}{(1 \pm \pi i)(1+\pi^2)} \cdot \frac{1}{\pm 2i} = \frac{\pi(\pi \mp i)}{(1 \pm \pi i)(1+\pi^2)(\pm 2i)}$$

$$\equiv \frac{\pi(\pi \mp i)}{2(\pm i - \pi)(1+\pi^2)} = \frac{-\pi}{2(1+\pi^2)}$$

$$y(t) = \frac{\pi}{2(1+\pi^2)} e^{-2t} u(t) + \frac{-\pi}{2(1+\pi^2)} \cdot \cos(2\pi t)\, u(t) + \frac{\pi}{2(1+\pi^2)} \cdot \frac{1}{\pi} \operatorname{sen}(2\pi t)\, u(t) - 4e^{-2t} u(t)$$

$$\left(\frac{\pi}{2(1+\pi^2)} - 4 \right) e^{-2t} u(t) + \frac{1}{2(1+\pi^2)} \left(-\pi \cos(2\pi t) + \operatorname{sen}(2\pi t) \right) u(t)$$

22. EDO

$$\frac{d^2}{dt^2} y(t) + 3 \frac{d}{dt} y(t) + 2 y(t) = 5 x(t) + \frac{d}{dt} x(t) \quad \leftarrow \mathcal{L}\{\ \}$$

$\mathcal{L}\{\}$

$$(s^2 + 3s + 2)\, Y(s) = (5 + s)\, X(s)$$

$$s = \frac{-3 \pm \sqrt{9-8}}{2}$$

$s = -2$ ou -1

$(s+2)(s+1)$

a) $$H(s) = \frac{5+s}{s^2+3s+2} = \rightarrow \quad s = (s + 3/2)^2 - 1/4$$

b) $$H(j\omega) = \frac{5 + j\omega}{(j\omega)^2 + 3j\omega + 2} = \frac{(5^2 + \omega^2)^{1/2} \angle \, tg^{-1}(\omega/5)}{\left((2-\omega^2)^2 + (3\omega)^2\right)^{1/2} \angle \, tg^{-1}\left(\frac{3\omega}{(2-\omega^2)}\right)}$$

$(j\omega)^2 = -\omega^2$

$2^2 - 4\omega^2 + \omega^4 + 9\omega^2$

$$A(\omega) = \left(\frac{5^2 + \omega^2}{\omega^4 + 5\omega^2 + 4} \right)^{1/2} \qquad \phi(\omega) = \operatorname{arctg}\left(\frac{\omega}{5}\right) - \operatorname{arctg}\left(\frac{3\omega}{(2-\omega^2)}\right)$$

c) $$h(t) = \mathcal{L}^{-1}\{H(s)\} = \mathcal{L}^{-1}\left\{ \frac{-3}{s+2} + \frac{4}{s+1} \right\} = \left(-3e^{-2t} + 4e^{-t}\right) u(t) \sim \frac{-3}{(s+2)} + \frac{4}{(s+1)}$$

d) $$y(t) = h(t) * x(t) \rightarrow Y(s) = H(s) \cdot X(s) \quad \Big\} \quad x(t) = 20 \sin(3t + 35°)$$

$$X(s) = 20 \cdot \frac{3}{(s^2+9)} \cdot e^{+35s}$$

22. d) $A(3) = \sqrt{\dfrac{5^2 + 3^2}{3^4 + 5.3^2 + 4}} = 0,5114$

$\phi(3) = \operatorname{arctg}\left(\frac{3}{5}\right) - \operatorname{arctg} \dfrac{3.(3)}{2 - 3^2} = 83,1^\circ$

$\therefore\ y(t) = A(3) . 20 \operatorname{sen}(3t + 35 + \phi(\omega))$

$= 10,228 \operatorname{sen}(3t + 118,1)$

23. $H(s) = \dfrac{s^2 + 2s + 17}{s^2 + 4s + 104}$

a) $H(j\omega) = \dfrac{(j\omega)^2 + 2(j\omega) + 17}{(j\omega)^2 + 4(j\omega) + 104}$; $A(\omega) = \sqrt{\dfrac{(17-\omega^2)^2 + (2\omega)^2}{(104-\omega^2)^2 + (4\omega)^2}}$; $\phi(\omega) = tg^{-1}\left(\dfrac{2\omega}{17-\omega^2}\right) - tg^{-1}\left(\dfrac{4\omega}{104-\omega^2}\right)$

b) $h(t) = \mathcal{L}^{-1}\{H(s)\}$

$s^2 + 2s + 17 \ |\ s^2 + 4s + 104$
$-2s - 87 \quad\quad 1$

; $H(s) = 1 + \dfrac{-2s - 87}{(s^2 + 4s + 104)} = 1 + \dfrac{-2s - 87}{(s+2)^2 + 100}$

$h(t) = \delta(t) - 2e^{-2t}\cos(10t)\,u(t) - 8,7e^{-2t}\operatorname{sen}(10t)\ u(t)$

c) $s(t) = \mathcal{L}^{-1}\{H(s)\,U(s)\}$

$= \dfrac{1}{s} + \dfrac{-2}{(s+2)^2 + 100} + \dfrac{-87}{s(s^2 + 4s + 104)} = \dfrac{1}{s} + \dfrac{-2}{(s+2)^2 + 100} + \dfrac{-87}{s(s+2-j10)(s+2+j10)}$

$= \dfrac{1}{s} + \dfrac{-2}{(s+2)^2 + 100} + \dfrac{-\frac{87}{104}}{s} + \dfrac{\frac{87}{208} - \frac{87}{1040}i}{(s+2-j10)} + \dfrac{\frac{87}{208} + \frac{87}{1040}i}{(s+2+j10)}$

$= \dfrac{17}{104} - \dfrac{2}{10}e^{-2t}\operatorname{sen}(10t) + \left(\dfrac{87}{208} - \dfrac{87}{1040}i\right)e^{(-2+j10)t} + \left(\dfrac{87}{208} + \dfrac{87}{1040}i\right)e^{(-2-j10)t}$

$= \left[\dfrac{17}{104} - \dfrac{2}{10}e^{-2t}\operatorname{sen}(10t) + 2\times 87 e^{-2t}\left(\dfrac{1}{208}\cos(10t) + \dfrac{1}{1040}\operatorname{sen}(10t)\right)\right] u(t)$

$= \left[\dfrac{17}{104} + 2.\,e^{-2t}\left(\dfrac{87}{208}\cos(10t) - \dfrac{17}{1040}\operatorname{sen}(10t)\right)\right] u(t)$

24. $\|H(\omega)\| = \sqrt{\dfrac{1}{1 + \omega^6}}$; $H(s)H(-s) = \left.\dfrac{1}{1 + (j\omega)^6}\right|_{\omega^2 = -s^2} = \dfrac{1}{1 + s^6} =$

$H(s)$? $\quad$ $H(-s)$

?

$H(s)\,H(-s) = \left.\|H(j\omega)\|^2\right|_{\omega^2 = \cancel{s}\ +s^2}$

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