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Measuring the Drag Force on a Falling Ball

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The effect of the aerodynamic drag force on an object in flight is well known and has been described in this and other journals many times. At speeds less than about 1 m/s, the drag force on a sphere is proportional to the speed and is given by Stokes' law. At higher speeds, the drag force is proportional to the velocity squared and is usually small compared with the gravitational force if the object mass is large and its speed is low. In order to observe a significant effect, or to measure the terminal velocity, experiments are often conducted with very light objects such as a balloon[1,2] or coffee filter[3] or muffin cup,[4] or are conducted in a liquid rather than in air. The effect of the drag force can also be increased by increasing the surface area of the object.

Suppose that the object of the experiment is to measure the drag force on a tennis ball or a baseball or some other ball such as a basketball or a football. Can that be done in a high school environment? The answer is yes, provided that the ball is traveling fast enough. A simple technique is to film the flight and to record the change in speed or to record the fall time from a given height. Such an experiment is not easy in practice, especially if the ball is following a parabolic trajectory, since accurate measures of speed and position are required to distinguish between the drag and gravitational forces. A simplification arises if the ball is allowed to fall vertically from rest. The problem is then one dimensional and can be described theoretically by analytical solutions.

Qualitatively, it is easy to see visually the difference in fall time between, say, a baseball and a Ping-Pong ball, even if the drop height is only about 2 m. The difference is even more obvious if the drop height is increased to, say, 6 m, by dropping the balls from an upstairs window or balcony. It is not immediately obvious that the baseball should land first. The gravitational force on the baseball is larger, but so is the drag force since the surface area is larger. In addition, the drag coefficient is not necessarily the same for both balls given that one ball has a smooth surface and one has a rougher surface. Surface roughness can play a major role in determining the drag coefficient, which is why golf balls have dimples.

To investigate and analyze differences in fall times, the authors dropped eight different balls from heights of about 2 m and 6 m and filmed each fall with a video camera. The properties of each ball are listed in Table I. The quantity S, defined below, is a measure of the effect of air resistance on each ball. In Table I, the drag coefficient was assumed to be 0.5 for each ball in order to calculate S. The balls were dropped by hand at zero initial speed and allowed to fall next to a vertical length of string marked at 1.0-m intervals to calibrate the distance scale on the film. To obtain clear images of the ball, the experiment was conducted in sunlight against a dark background, using a shutter speed of 1/4000 s.

The equation of motion for a ball of mass m falling vertically at speed v is given by

$$m\frac{dv}{dt} = mg - F_D, \tag{1}$$

where $F_D = 0.5C_D\,\rho A v^2$ is the drag force, C_D is the drag coefficient, ρ is the density of air, and $A = \pi D^2/4$ is the cross-sectional area of the ball. Equation (1) can be written as

$$\frac{dv}{dt} = g\left(1 - \frac{S^2v^2}{4g^2}\right), \tag{2}$$

where $S^2 = 2C_D\,\rho Ag/m$. Equation (2) can be integrated by parts to show that

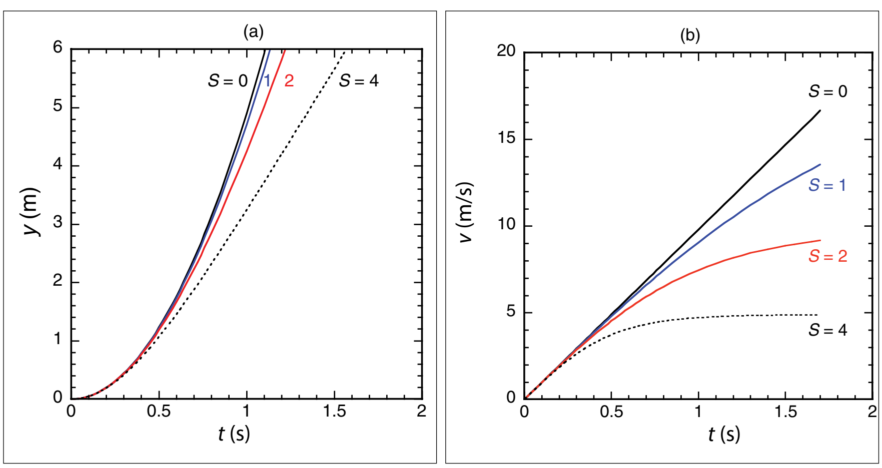

Fig. 1. Theoretical values of (a) y vs t and (b) v vs t when a ball is dropped vertically in air from rest. The quantity S increases with the area of the ball and decreases when the ball mass increases.

$$v = \frac{dy}{dt} = \frac{2g(e^{St}-1)}{S(e^{St}+1)}, \tag{3}$$

assuming that $v = 0$ at $t = 0$. Equation (3) can also be integrated by parts to show that

$$y = \left(\frac{2g}{S}\right)\left[\frac{2}{S}\ln_e\left(\frac{e^{St}+1}{2}\right) - t\right], \tag{4}$$

assuming that $y = 0$ at $t = 0$. In the limit where $C_D = 0$, we recover the usual results that $v = gt$ and $y = gt^2/2$ [using the relations $e^x = 1 + x$ and $\ln_e(1 + x) = x$ when x is small]. The usual results are also valid when t or v is small since the drag force is then much smaller than mg. Solutions of Eqs. (3) and (4) are shown in Fig. 1 for values of S relevant to the present experiment. As

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Table I. Ball properties. M = mass, D = diameter.

Ball	M (g)	D (mm)	S (s^{-1})
Baseball	149.8	73.0	0.58
Tennis	58.0	66.0	0.84
Rubber	46.3	59.5	0.84
Basketball	90.65	175	1.77
Ping-Pong 1	6.16	54.7	2.13
Ping-Pong 2	2.59	43.7	2.62
Polystyrene 1	10.19	97.0	2.93
Polystyrene 2	4.76	79.0	3.50

Table II. Experimental results for 2.0-m and 6.0-m drop heights.

Ball	t (2)	S (2)	C_D(2)	t (6)	S (6)	C_D(6)
Baseball	0.640	0.55	0.46	1.116	0.64	0.62
Tennis	0.641	0.85	0.52	1.130	0.91	0.59
Rubber	0.640	0.70	0.34	1.130	0.91	0.58
Basketball	0.660	2.20	0.77	1.201	1.85	0.54
Ping-Pong 1	0.657	2.06	0.47	1.195	1.95	0.42
Ping-Pong 2	0.667	2.50	0.45	1.275	2.44	0.43
Polystyrene 1	0.690	3.20	0.59	1.442	2.95	0.50
Polystyrene 2	0.720	3.65	0.54	1.476	3.36	0.46

shown in Fig. 1, there is very little difference in y versus t for the first 0.4 s of the drop, regardless of the value of S. Consequently, a reliable measurement of the drag coefficient using this technique needs to focus on the behavior of the ball after it has been falling for at least 0.4 s. Eventually, the ball will approach a terminal velocity $v = 2g/S$, but the drop height would need to be greater than 6 m if S is less than about 3.

For a 2.0-m drop height, an accurate measurement of S can be obtained, in theory, by plotting y versus t over the time interval 0.4 s < t < 0.7 s while v increases from about 3 m/s to about 6 m/s. A simple method of analysis is to fit a quadratic to the y-versus-t data, to measure the average acceleration. That method yields an average value of the drag force, from Eq. (1). However, the variation of v gives a corresponding uncertainty in C_D. Nevertheless, such a result shows clearly that the average acceleration is less than g, especially for light balls such as Ping-Pong and polystyrene balls. The technique adopted in this paper was to fit Eq. (4) to the experimental data to find a best fit value of S. The same technique was used for the 6-m drop height results, ignoring the first 0.5 s of the drop. Results for both drop heights are shown in Table II, where t(2) and t(6) are the measured times to fall through heights of 2 m and 6 m, respectively. The drop time in a vacuum is 0.639 s for a 2.00-m drop and 1.106 s for a 6.00-m drop.

In order to fit the y-versus-t data using Eq. (4), it is necessary to estimate the start time, $t = 0$, as accurately as possible. An error in the start time of only 0.01 s can lead to an error in S of up to about 0.5, especially when S is less than 1.0. When filming at 30 fps, there is a potential error of up to 0.033 s in estimating the start of the fall. We filmed at 300 fps to reduce this problem, but even then the error in the start time may have been as large as 0.01 s, given the manual method of dropping the ball. It was easy to see the fingers opening to release the ball, but the precise start of the drop was uncertain, given that it takes 0.01 s for a ball to fall by 0.5 mm when released from rest. A solution to this problem was found by treating the starting value of y and the starting value of t as unknowns, together with the unknown value of S, as part of the curve fit procedure. We used KaleidaGraph software, which allows the user to define a curve with several unknown constants. Each constant is then determined separately to obtain a best fit curve. For example, t in Eq. (4) was replaced by $t + t_0$, t_0 being an unknown constant, and the curve fit procedure then returned best fit values of both t_0 and S. The best fit value of t_0 was typically about 0.01 s, representing the experimental error in estimating the actual start time.

The results in Table II represent the average of three drops for each ball. The maximum error in C_D was typically about ±0.05. The 2-m drop results for the rubber ball and the basketball appear to have a larger error than the other results. One of the problems with a small drop height in this experiment is that a small error in the measured height of the ball will generate a relatively large error in the calculated value of C_D. The sensitivity to small errors in y was calculated by assuming that all y values are overestimated by 1%. Such a result could be obtained if there was a 1% error in the calibration scale or if the ball falls away from the vertical as it falls or if it was dropped from a point slightly in front of the vertical calibration string. For a drop height of 2 m, a 1% overestimate of y results in a 32% reduction in C_D when the data are curve fit using Eq. (4). However, for a drop height of 6 m, the error in C_D is only 8%. Consequently, the results in Table II for the 2-m drop height are less reliable than those for the 6-m drop height. Nevertheless, they do show that reasonable estimates of the drag coefficient for a sports ball can be obtained in a student laboratory with a 2-m drop, and more accurate results can be expected for greater drop heights and if special care is taken to drop the ball vertically in an accurately calibrated vertical plane.

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