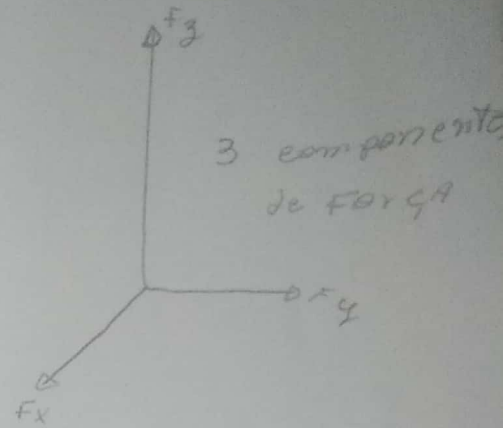
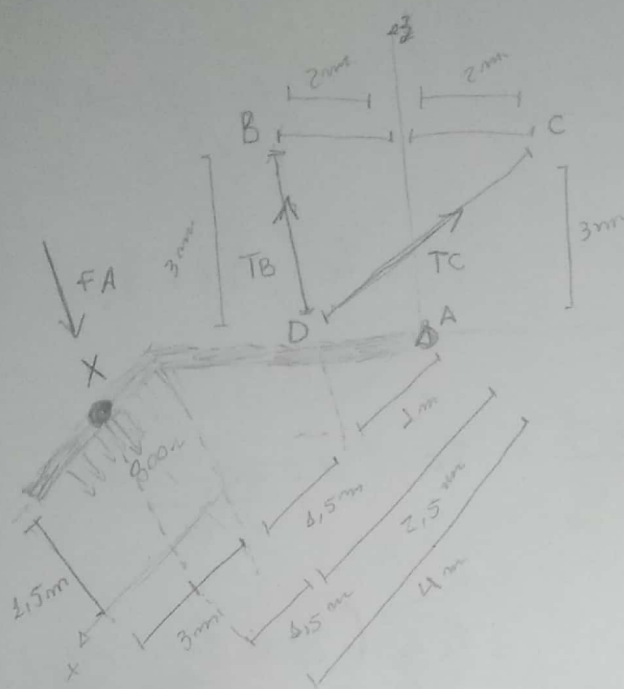


Diagrama pontos Beto

1)



$$AD = D - A$$

Componentes / Posições

$$A = 0\hat{i} + 0\hat{j} + 0\hat{k} \quad (\text{origem})$$

$$B = 0\hat{i} - 2\hat{j} + 3\hat{k}$$

$$C = 0\hat{i} + 1\hat{j} + 3\hat{k}$$

$$D = 1\hat{i} + 0\hat{j} + 1\hat{k}$$

$$X = 4\hat{i} + 0\hat{j} + 1,5\hat{k}$$

distâncias entre pontos

$$AD = (1-0)\hat{i} + (0-0)\hat{j} + (1-0)\hat{k} \\ = 1\hat{i} + 0\hat{j} + 1\hat{k}$$

$$AX = (4-0)\hat{i} + (0-0)\hat{j} + (1,5-0)\hat{k} \\ = 4\hat{i} + 0\hat{j} + 1,5\hat{k}$$

$$DB = (0-1)\hat{i} + (-2-0)\hat{j} + (3-1)\hat{k} \\ = -1\hat{i} - 2\hat{j} + 2\hat{k}$$

$$DC = (0-1)\hat{i} + (1-0)\hat{j} + (3-1)\hat{k} \\ = -1\hat{i} + 1\hat{j} + 2\hat{k}$$

Com a força concentrada no ponto X, podemos calcular a força Resultante no ponto A.

$$F_{AR} = F \cdot d \\ = 800 \cdot 3$$

$$F_{AR} = 2400 \text{ N}$$

calculando as trações e forças

$$\begin{aligned} \textcircled{A} \quad \vec{T}_B &= T_B \cdot \frac{DB}{|DB|} \rightarrow = \frac{T_B (-1\hat{i} - 2\hat{j} + 2\hat{k})}{\sqrt{1^2 + 2^2 + 2^2}} = \frac{T_B (-1\hat{i} - 2\hat{j} + 2\hat{k})}{\sqrt{9}} \\ &= T_B \left(-\frac{1}{3}\hat{i}, -\frac{2}{3}\hat{j}, +\frac{2}{3}\hat{k} \right) \end{aligned}$$

$$\begin{aligned} \textcircled{B} \quad \vec{T}_C &= T_C \cdot \frac{DC}{|DC|} \rightarrow = \frac{T_C (-1\hat{i} + 1\hat{j} + 2\hat{k})}{\sqrt{1^2 + 1^2 + 2^2}} = \frac{T_C (-1\hat{i} + 1\hat{j} + 2\hat{k})}{\sqrt{6}} \\ &= T_C \left(-\frac{1}{\sqrt{6}}\hat{i}, \frac{1}{\sqrt{6}}\hat{j}, +\frac{2}{\sqrt{6}}\hat{k} \right) \end{aligned}$$

$$\textcircled{C} \quad \vec{F}_A = F_{AB} \cdot (0\hat{i}, 0\hat{j}, -1\hat{k})$$

$$\begin{aligned} \textcircled{D} \quad \vec{T}_D &= AD \cdot T_D \\ &= T_D \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & | & \hat{i} & \hat{j} \\ 1 & 0 & 1 & | & 1 & 0 \\ -1 & -2 & 2 & | & -1 & -2 \\ 0 & 0 & 0 & | & 0 & 0 \end{vmatrix} = T_D \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ -1 & -2 & 2 \end{vmatrix} \\ &= T_D \cdot (2\hat{i} - 3\hat{j} - 2\hat{k}) \\ &= \frac{T_D}{3} (2\hat{i} - 3\hat{j} - 2\hat{k}) \end{aligned}$$

$$\begin{aligned} \textcircled{E} \quad \vec{T}_E &= AD \cdot T_E \\ &= T_E \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & | & \hat{i} & \hat{j} \\ 1 & 0 & 1 & | & 1 & 0 \\ -1 & -2 & 2 & | & -1 & -2 \\ 0 & 0 & 0 & | & 0 & 0 \end{vmatrix} = T_E \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ -1 & -2 & 2 \end{vmatrix} \\ &= T_E \cdot (-1\hat{i} - 2\hat{j} - 1\hat{j} + 1\hat{k}) \\ &= T_E \cdot (-1\hat{i} - 3\hat{j} + 1\hat{k}) \\ &= \frac{T_E}{\sqrt{6}} \cdot (-1\hat{i} - 3\hat{j} + 1\hat{k}) \end{aligned}$$

... continuando

$$TC = 5225,58 \quad \text{e} \quad + \frac{12800}{\sqrt{67}}$$

$$TB = \frac{\sqrt{6}}{4} \cdot TC$$

$$= \frac{\sqrt{6}}{4} \cdot \frac{+12800}{\sqrt{67}}$$

$$= \frac{+12800}{4}$$

$$TB = +3200 \text{ N}$$

$$\text{e } TC = +5225,58 \text{ N}$$

SUBSTITUINDO O VALOR DE T_C e T_B NAS EQUAÇÕES "A" e "B" \rightarrow pag 2

$$\vec{T}_B = \left(-\frac{1}{3} \hat{i}, -\frac{2}{3} \hat{j} + \frac{2}{3} \hat{k} \right) \cdot TB$$

$$= \left(-\frac{3200}{3} \hat{i}, -\frac{6400}{3} \hat{j} + \frac{6400}{3} \hat{k} \right) \cdot +3200$$

$$= \left(-1066,67 \hat{i}, -2133,33 \hat{j}, +2133,33 \hat{k} \right)$$

$$\vec{T}_C = \left(-\frac{1}{\sqrt{6}} \hat{i}, \frac{1}{\sqrt{6}} \hat{j}, \frac{2}{\sqrt{6}} \hat{k} \right) \cdot TC$$

$$= \left(-\frac{1}{\sqrt{6}} \hat{i}, \frac{1}{\sqrt{6}} \hat{j}, \frac{2}{\sqrt{6}} \hat{k} \right) \cdot \frac{+12800}{\sqrt{67}}$$

$$= \left(-\frac{+12800}{6} \hat{i}, +\frac{12800}{6} \hat{j}, +\frac{25600}{6} \hat{k} \right)$$

$$= \left(-2133,33 \hat{i}, +2133,33 \hat{j}, +4266,67 \hat{k} \right)$$

1

3

$$T_A = AX \cdot F_A$$

$$= F_A \cdot \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & 0 & -1.5 & 4 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -9 & 0 & 0 \end{vmatrix} = F_A \cdot 4 \hat{j}$$

$$= 1200 \cdot 4 \hat{j}$$

$$= 9600 \cdot (+\hat{j})$$

através das equações 1, 2, 3

$$1) T_B = (2\hat{i}, -3\hat{j}, -2\hat{k}) \frac{T_B}{3}$$

$$= \left(\frac{2}{3}\hat{i}, -\hat{j}, -\frac{2}{3}\hat{k} \right) \cdot T_B$$

$$2) T_C = (-1\hat{i}, -3\hat{j}, +2\hat{k}) \cdot \frac{T_C}{\sqrt{14}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \left(-\frac{1\sqrt{6}}{6}\hat{i}, -\frac{3\sqrt{6}}{6}\hat{j}, +\frac{2\sqrt{6}}{6}\hat{k} \right) T_C$$

$$3) T_{FA} = (0\hat{i}, +9600\hat{j}, 0) N$$

Juntamos as componentes

$$\left(\frac{2T_B}{3} - \frac{\sqrt{6}}{6} T_C + 0 \right) \hat{i} + \left(-T_B - \frac{3\sqrt{6}}{6} T_C + 9600 \right) \hat{j} + \left(-\frac{2T_B}{3} + \frac{\sqrt{6}}{6} T_C + 0 \right) \hat{k}$$

Resolvendo

em i

$$\frac{2T_B}{3} - \frac{\sqrt{6}}{6} T_C = 0$$

$$\frac{12T_B}{3} - \sqrt{6} T_C = 0$$

$$4T_B - \sqrt{6} T_C$$

$$T_B = \frac{\sqrt{6} T_C}{4}$$

em j

$$-T_B - \frac{3\sqrt{6}}{6} T_C + 9600 = 0$$

$$-6T_B - 3\sqrt{6} T_C + 57600 = 0$$

$$-6 \cdot \frac{\sqrt{6} T_C}{4} - 3\sqrt{6} T_C = -57600$$

$$-6\sqrt{6} T_C - 12\sqrt{6} T_C = -230400$$

$$T_C = -\frac{230400}{-18\sqrt{6}} = 5225,58 N$$

$$\begin{aligned}
 \textcircled{c} &= (0\hat{i}, 0\hat{j}, -1\hat{k}) \cdot F_{AR} \\
 &= (0\hat{i}, 0\hat{j}, -1\hat{k}) \cdot 2400 \\
 &= (0\hat{i}, 0\hat{j}, -2400\hat{k}) \text{ N}
 \end{aligned}$$

Encontrando as forças

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$$

Em x

$$\Sigma F_x = 0$$

$$A_x + (TB + TC) = 0$$

$$A_x = -(TB + TC)$$

$$= - \left(-\frac{3200}{3} - \frac{12800}{6} \right)$$

$$\boxed{A_x = +3200 \text{ N}}$$

Em y

$$\Sigma F_y = 0$$

$$A_y + TB + TC = 0$$

$$A_y = -TB - TC$$

$$A_y = - \left(-\frac{6400}{3} + \frac{12800}{6} \right)$$

$$\boxed{A_y = 0 \text{ N}}$$

Em z

$$\Sigma F_z = 0$$

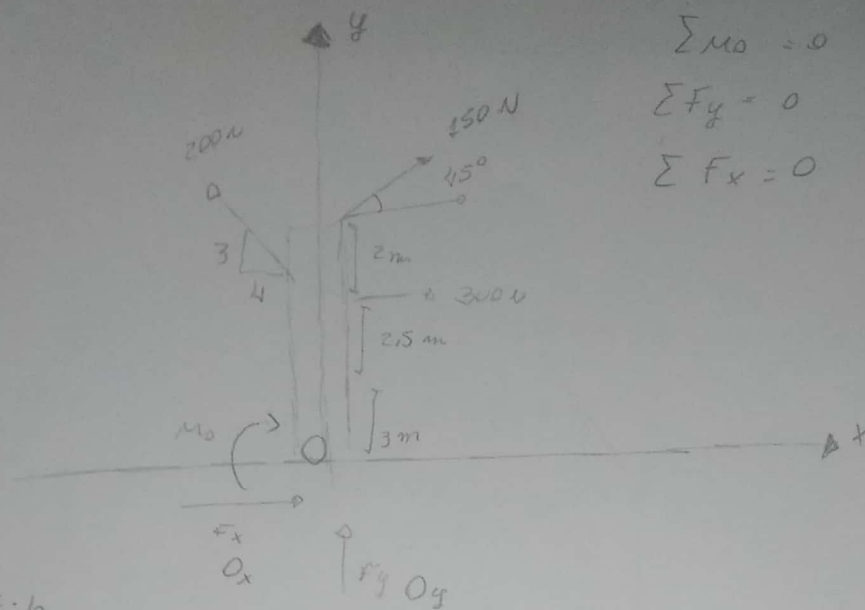
$$A_z + TB + TC = 0$$

$$A_z = -TB - TC$$

$$A_z = - \left(+\frac{6400}{3} + \frac{25600}{6} \right)$$

$$\boxed{A_z = -6400 \text{ N}}$$

2



$$\sum M_O = 0$$

$$\sum F_y = 0$$

$$\sum F_x = 0$$

adotando o sentido

para o momento

em x

$$\sum F_x = 0$$

$$= -300 + (200 \cdot \cos \theta) - (\cos 45^\circ \cdot 150 + O_x) = 0$$

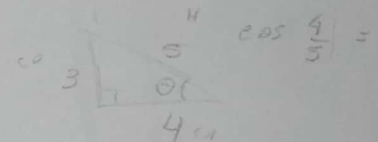
$$= -300 + 200 \cdot 0,8 - \cos 45^\circ \cdot 150 + O_x = 0$$

$$O_x = 300 + \cos 45^\circ \cdot 150 - 200 \cdot 0,8$$

$$O_x = 300 + 106,06 - 160$$

$$O_x = 246,06 \text{ N}$$

$$\sin \left(\frac{3}{5} \right) =$$



$$\theta = \arctg \left(\frac{3}{4} \right) = 36,87^\circ$$

$$\sin \theta = 0,6$$

$$\cos \theta = 0,8$$

em y

$$\sum F_y = +200 \cdot \sin \theta + O_y + 150 \cdot \sin 45^\circ$$

$$O_y = +200 \cdot 0,6 + 106,06$$

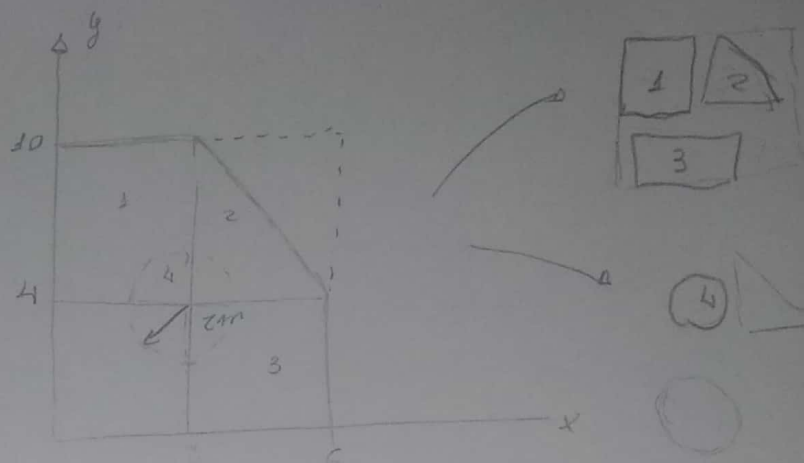
$$O_y = +126,06 \text{ N}$$

$$\sum M_O = -150 \cos 45^\circ \cdot 2,5 - 300 \cdot 3 + 200 \cdot \cos \theta$$

$$= -150 \cos 45^\circ \cdot 2,5 - 900 + 200 \cdot 0,8$$

$$= +815,49 \text{ N}$$

3.



	A (m)	\bar{x}	\bar{y}	$A\bar{x}$	$A\bar{y}$
FIGURA 1	18 m	1,5	7	27	126
FIGURA 2	9	4	6	36	54
FIGURA 3	24	3	2	72	48
FIGURA 4	12,56	3	4	-37,7	-50,27
Σ	38,434			87,30	177,74

Figura 1

$$\bar{x} = \frac{3}{2} = 1,5 \text{ m}$$

$$\bar{y} = 4 + \frac{(10-4)}{2} = 7 \text{ m}$$

$$A = 6 \cdot 3 = 18 \text{ m}^2$$

$$\bar{x} \cdot A = 27 \text{ m}^2$$

$$\bar{y} \cdot A = 126 \text{ m}^2$$

Figura 2

$$\bar{x} = 4$$

$$\bar{y} = 6$$

$$A = \frac{6 \cdot 3}{2} = 9 \text{ m}^2$$

$$\bar{x} \cdot A = 36 \text{ m}^2$$

$$\bar{y} \cdot A = 54 \text{ m}^2$$

Figura 3

Figura 4

$$\bar{x} = 3 \text{ m}$$

$$\bar{y} = 2 \text{ m}$$

$$A = 4 \times 6 = 24 \text{ m}^2$$

$$\bar{x} \cdot A = 72 \text{ m}^2$$

$$\bar{y} \cdot A = 48 \text{ m}^2$$

Figura 4

$$\bar{x} = 3 \text{ m}$$

$$\bar{y} = 4 \text{ m}$$

$$A = \pi \cdot r^2 = \pi \cdot 2^2 = 12,56 \text{ m}^2$$

$$\bar{x} \cdot A = -37,7 \text{ m}^2$$

$$\bar{y} \cdot A = -50,27 \text{ m}^2$$

$$\left. \begin{aligned} I_{x'} &= \frac{\pi r^4}{4} \\ I_{y'} &= \frac{\pi r^4}{4} \end{aligned} \right\} \text{ para círculo}$$

$$I_x = \frac{b h^3}{36} \text{ para triângulo}$$

$$I_y = \frac{b^3 h}{36}$$

$$I_{x'} = \frac{b h^3}{12}$$

$$I_{y'} = \frac{b^3 h}{12}$$

} retângulo

momento de Inercia

$$I_x = I_x' + d_y^2 \cdot A \quad \text{e} \quad I_y = I_y' + d_x^2 \cdot A$$

FIGURA (RETANGULO 1)

$$I_{x1} = \frac{b \cdot h^3}{12} + d_y^2 \cdot A$$

$$= \frac{3 \cdot 6^3}{12} + 7^2 \cdot 18 =$$

$$= 54 + 882$$

$$= 936 \text{ cm}^2$$

FIGURA 3

$$I_{x3} = \frac{b \cdot h^3}{12} + d_y^2 \cdot A$$

$$= \frac{6 \cdot 4^3}{12} + 2^2 \cdot 24$$

$$= 32 + 96$$

$$= 128 \text{ cm}^2$$

FIGURA 2

$$I_{x2} = \frac{b \cdot h^3}{12} + d_y^2 \cdot A$$

$$= \frac{3 \cdot 6^3}{36} + 9 \cdot 6^2$$

$$= 18 + 324$$

$$= 342 \text{ cm}^2$$

FIGURA 4

$$I_{x4} = -\frac{\pi \cdot 16}{4} + 4\pi \cdot 2^2$$

$$= 213,63 \text{ cm}^2$$

é negativo, pois é
necessária tirar o
círculo

somatoria $I_x = I_{x1} + I_{x2} + I_{x3} - I_{x4}$

$$= 936 + 342 + 128 - 213,63$$

$$= 1192,37 \text{ cm}^2$$

... continuando
Em Y

→ FÓRMULA $I = I_{y'} + d^2 \cdot A$

Figura 1

$$\begin{aligned} I_{y1} &= \frac{6 \cdot 3^3}{12} + 1,5^2 \cdot 18 \\ &= 13,5 + 40,5 \\ &= 54 \text{ m}^2 \end{aligned}$$

Figura 2

$$\begin{aligned} I_{y2} &= \frac{6 \cdot 3^3}{36} + 4^2 \cdot 9 \\ &= 4,5 + 114 \\ &= 118,5 \text{ m}^2 \end{aligned}$$

Figura 3

$$\begin{aligned} I_{y3} &= \frac{4 \cdot 6^3}{12} + 3^2 \cdot 24 \\ &= 72 + 216 \\ &= 288 \text{ m}^2 \end{aligned}$$

Figura 4

$$\begin{aligned} I_{y4} &= \frac{\pi \cdot 2^4}{4} + 4\pi \cdot 3^2 \\ &= 125,66 \text{ m}^4 \end{aligned}$$

Somatoria em Y

$$I_y = I_{y1} + I_{y2} + I_{y3} - I_{y4}$$

$$\begin{aligned} &= 54 + 118,5 + 288 - 125,66 \\ &= 364,84 \text{ m}^2 \end{aligned}$$

novamente
retirando o
paralelo de
círculo

4) Questão

ΣA_i

3 = 4 = mesmo Triângulo

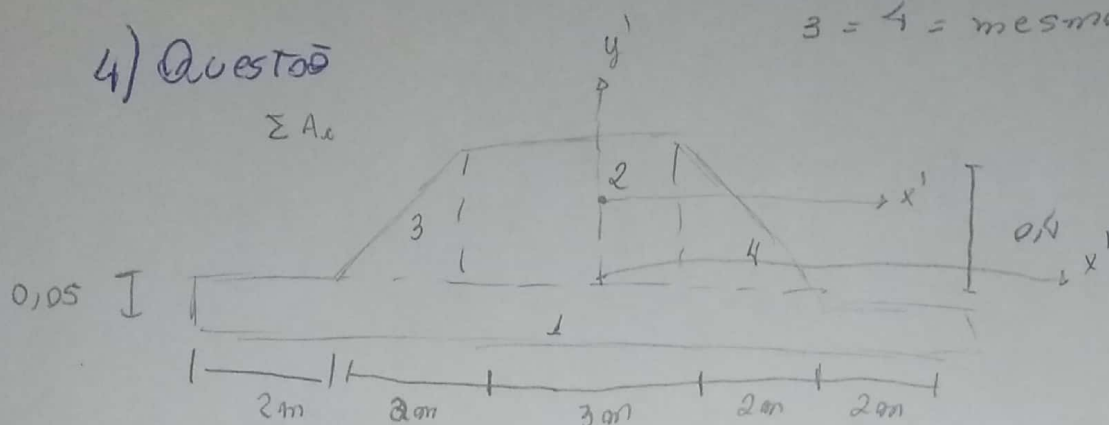


FIGURA	$Y (m)$	$A (m^2)$
1	$\frac{0,055}{2}$ $= 0,025$	$4,1 \cdot 0,05$ $= 0,05$
2	$0,05 + \frac{0,4}{2}$ $= 0,025$	$0,3 \times 0,4$ $= 0,12$
3	$0,05 + \frac{1}{3} \cdot 0,04$ $= 0,1833$	$\frac{0,02 \cdot 0,4}{2}$ $= 0,04$
4	$0,05 + \frac{1}{3} \cdot 0,04$ $= 0,1833$	$\frac{0,2 \cdot 0,4}{2}$ $= 0,04$

não há componente em x, pois apenas varia em y

$$\bar{Y} = \frac{(0,025 \cdot 0,055) + (0,025 \cdot 0,12) + (0,1833 \cdot 0,04) + (0,1833 \cdot 0,04)}{0,055 + 0,12 + 0,04 + 0,04}$$

$$\bar{Y} = 0,1805 \text{ m}$$

calculando momentos de Inercia

P03

$$I_x = I_x' + d_y^2 \cdot A$$

$$I_{x1} = \frac{1,1 \cdot 0,05^3}{12} + (0,1805 - 0,025)^2 \cdot 0,055$$

$$= 1,341 \cdot 10^{-3} \text{ m}^4$$

retângulo

$$I_x = \frac{b \cdot h^3}{12}$$

$$I_{x2} = \frac{0,3 \cdot 0,4^3}{12} + (0,1805 - 0,25)^2 \cdot 0,12$$

$$I_{x2} = 2,179 \cdot 10^{-3} \text{ m}^4$$

Triângulo

$$I_x = \frac{b \cdot h^3}{36}$$

$$I_{x3} = \frac{0,2 \cdot 0,4^3}{36} + (0,1805 - 0,1833)^2 \cdot 0,04$$

$$I_{x3} = 3,558 \cdot 10^{-4} \text{ m}^4$$

como $I_{x3} = I_{x4}$, a calcula ser a mesma

$$I_{x3} = I_{x4} \left. \begin{array}{l} \text{mesma base} \\ \text{mesma altura} \end{array} \right\} = \text{mesma Triângulo}$$

$$I_{x4} = 3,558 \cdot 10^{-4} \text{ m}^4$$

Somatoria

$$I_x = I_{x1} + I_{x2} + I_{x3} + I_{x4}$$

$$= 1,341 \cdot 10^{-3} + 2,179 \cdot 10^{-3} + 3,558 \cdot 10^{-4} + 3,55 \cdot 10^{-4}$$

$$= \underline{4,2317 \cdot 10^{-3} \text{ m}^4}$$

... continuando

encontrando em Y

$$I_y = I_{y'} + d_x^2 \cdot A$$

Figura 1

$$I_{y1} = \frac{(1,1)^3 \cdot 0,005}{12} + 0,055$$

$$= 5,55 \cdot 10^{-3} \text{ m}^4$$

$$I_{y2} = \frac{(0,3)^3 \cdot 0,4}{12} + 0,012$$

$$= 9 \cdot 10^{-4} \text{ m}^4$$

u:

$$I_{y3} = \frac{(0,2)^3 \cdot 0,4}{36} + (0,15 + \frac{1}{3} \cdot 0,02^2) \cdot 0,04$$

$$= 1,97 \cdot 10^{-3} \text{ m}^4$$

$$I_{y4} = \text{como } I_{y4} = I_{y3} \left. \vphantom{\begin{matrix} I_{y4} \\ I_{y3} \end{matrix}} \right\} \begin{array}{l} \text{ambos são} \\ \text{Triângulos de mesma base} \\ \text{mesma altura} \end{array}$$

$$= 1,97 \cdot 10^{-3} \text{ m}^4$$

Somatória

$$I_y = I_{y1} + I_{y2} + I_{y3} + I_{y4}$$

$$= 5,55 \cdot 10^{-3} + 9 \cdot 10^{-4} + 1,97 \cdot 10^{-3} + 1,97 \cdot 10^{-3}$$

$$= 0,06039$$

$$= 10,39 \cdot 10^{-3} \text{ m}^4$$