INSTITUTO FEDERAL DE
EDUCAÇÃO, CIÊNCIA E TECNOLOGIA
BAHIA

Lista de Exercício

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Disciplina: Análise de sinais e Sistemas

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Problemas

2.2 Avalie as somas de convolução de tempo discreto dadas abaixo:

a) $y[n]=u[n]*u[n-3]$

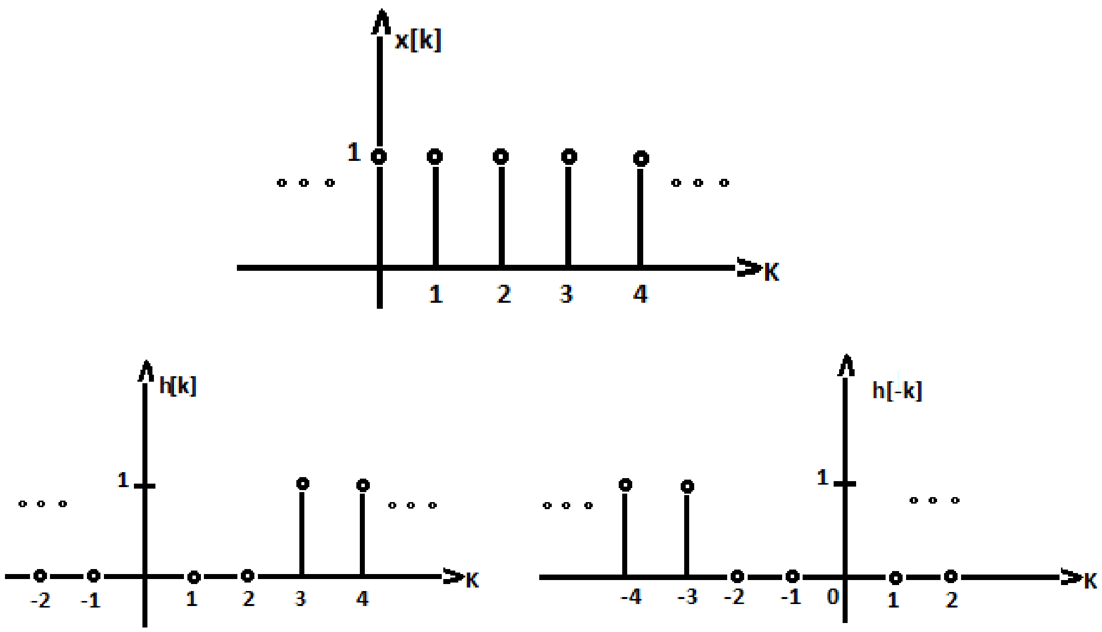
b) $y[n]=2^{n}.u[-n+2]*u[n-3]$

Letra a:

$y[n]=u[n]*u[n-3]$

$x[k]=u[n]$
$h[k]=u[n-3]$

$$y[n]=\sum_{k=-\infty}^{\infty}x[k].h[n-k]$$



$n-3<0$
$n<3,$
$y[n]=0$

$n=3;\ y[3]=1.1=1$
$n=4;\ y[4]=1.+1.1=2$
$n=5;\ y[5]=1+1+1.1=3$

Logo:
$$\boxed{\begin{array}{l} n=x \\ y[x]=x-2 \\ y[n]=(n-2).u[n-2] \end{array}}$$

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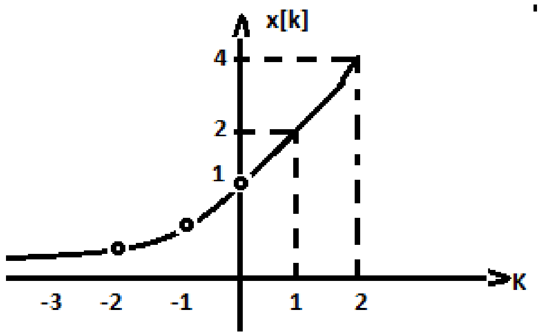
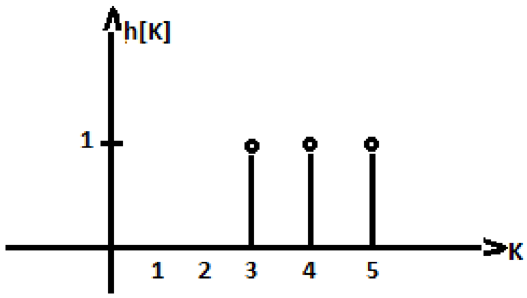
Letra b :

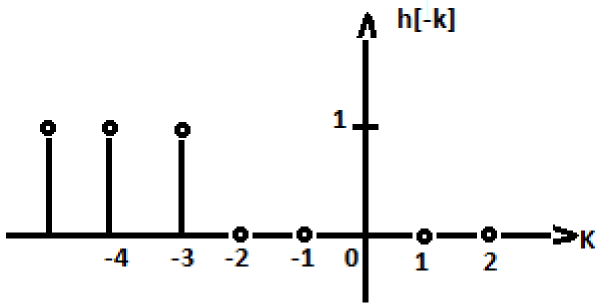
$$y[n] = 2^{n}.u[-n+2]*u[n-3]$$

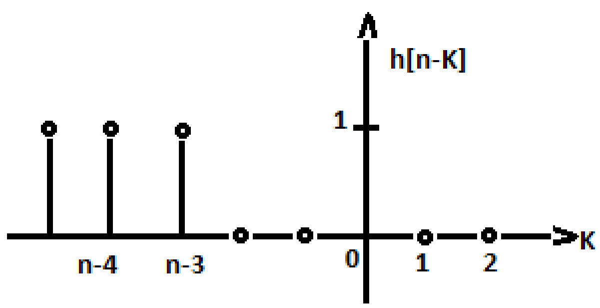
$$x[k] = 2^{n}u[-n+2]$$
$$h[k] = [n-3]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k].h[n-k]$$









$n-3<2$
$n<5$

$para$
$n=0; k=-3$
$n=1; k=-2$
$n=2; k=-1$

$n\leq 5$

$$y[n]=\sum_{k=-\infty}^{\infty} x[k].h[n-k]$$

$$y[n]=\sum_{k=-\infty}^{n-3} 2^{k}$$

$$\boxed{\begin{array}{l} l=-k \\ k=-\infty \\ l=\infty \\ k=n-3 \\ l=3-n \\ a=l \\ b=3-n \end{array}}$$

$$\sum_{a=b}^{\infty} \beta^{a}=\frac{\beta^{a}}{1-\beta}$$
$|\beta|<1$

$$y[n]=\sum_{l=\infty}^{3-n} 2^{-l}=\sum_{3-n}^{\infty}\left(\frac{1}{2}\right)^{l}$$

$$y[n]=\frac{\left(\frac{1}{2}\right)^{3-n}}{\frac{1}{2}}$$

Logo: $\boxed{y[n]=\left(\frac{1}{8}\right)\left(\frac{1}{2}\right)^{-n}\left(\frac{1}{2}\right)^{-1}=\left(\frac{1}{4}\right)^{2^{n}}}$

2.5) Avalie as de integrais de convolução de tempo contínuo apresentadas abaixo:

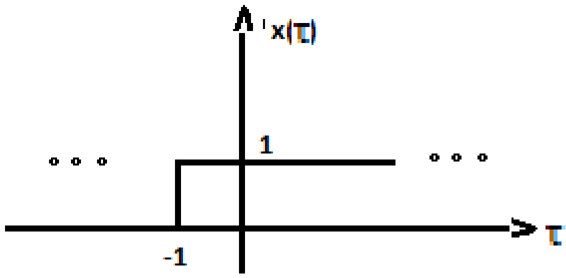
a) $y(t)=u(t+1)*u(t-2)$

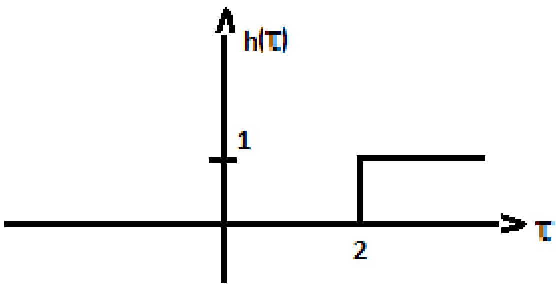
b) $y(t)=e^{-2t}u(t)*u(t+2)$

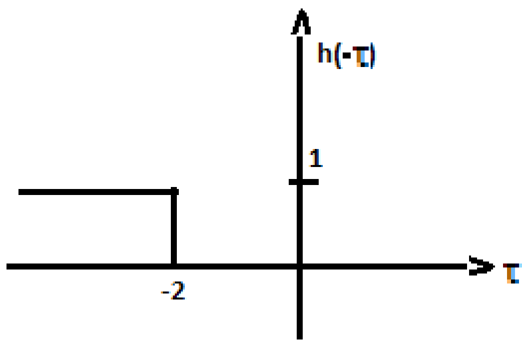
Letra a:

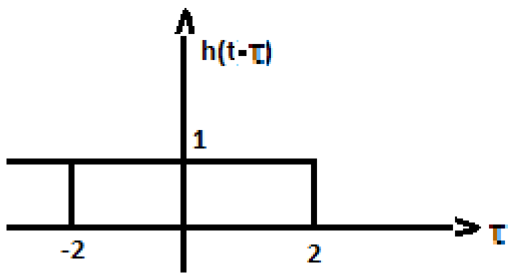
$$y(t) = u(t+1) * u(t-2)$$
$$x(t) = u(t+1)$$
$$h(t) = u(t-2)$$

$$y(t) = h(t) * x(t)$$
$$y(t) = \int_{-\infty}^{\infty} x(\tau).h(t-\tau)d\tau$$
$$y(t) = \int_{-\infty}^{\infty} u(\tau+1).u(t-2-\tau)d\tau$$









$$y(t) = \int_{-\infty}^{\infty} u(\tau+1).u(\tau-2)d\tau$$

$$\boxed{\begin{array}{l} \tau = t-2 \\ t-2 < -1 \\ t < 1 \end{array}}$$

$$y(0) = \int_{-\infty}^{\infty} 0 d\tau = 0, t < 1$$
$$y(t) = \int_{-1}^{t-2} 1 d\tau = t-2+1$$

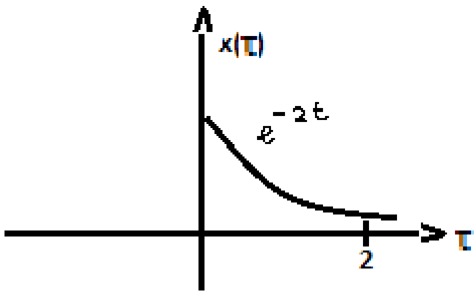
Logo: $\boxed{y(t) = t-1, t \geq 1}$

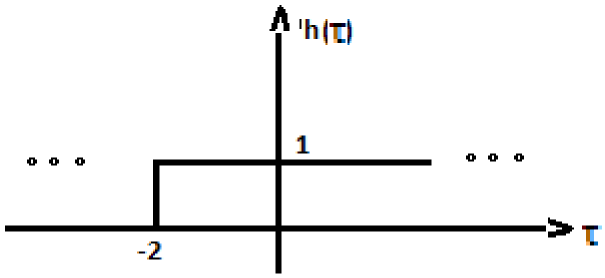
Letra b:

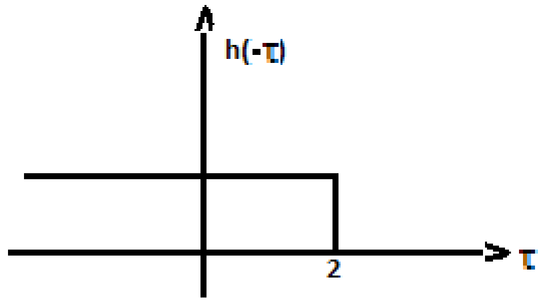
$$y(t) = e^{-2t}u(t) * u(t+2)$$

$$x(t) = e^{-2t}$$
$$h(t) = u(t+2)$$

$$y(t) = x(t) * h(t)$$
$$y(t) = \int_{-\infty}^{\infty} x(\tau).h(t-\tau)d\tau = \int_{-\infty}^{\infty} x(\tau)u(t+2-\tau)d\tau$$

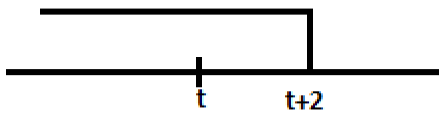






$t+2>0$

t



$$\tau = t+2$$
$$t+2<0$$
$$t<-2$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau).h(t-\tau)d\tau$$
$$y(0) = \int_{-\infty}^{\infty} 0 d\tau = 0, t<-2$$

$$y(t) = \int_{0}^{t+2} e^{-2t}.1d\tau$$
$$y(t) = -\frac{1}{2}e^{-2t} /_{0}^{t+2} = -\frac{1}{2}\left(e^{-2(t+2)} - 1\right)$$

Logo: $\boxed{y(t) = \frac{1}{2}\left(1 - e^{-2(t+2)}\right), t \geq -2}$

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2.11 Uma interconexão de sistemas LTI é descrita na figura P2.11. As respostas ao impulso são $h_1 = \left(\frac{1}{2}\right)^n (u[n+2]-u[n-3]), h_2[n] = \delta[n], h_3[n] = u[n-1]$. Admitamos que a reposta ao impulso do sistema global de $x[n]$ até $y[n]$ seja denotada como $h[n]$.

a) Expresse $h[n]$ em termos de $h_1[n]$, $h_2[n]$, $h_3[n]$.

b) Avalie $h[n]$ usando os resultados de (a).

Nas parteas (c)-(e) determine se o sistema corresponde a cada resposta ao impulso é (i) estável, (ii) causal, e (iii)sem memória.

c) $h_1[n]$

d) $h_2[n]$

e) $h_3[n]$

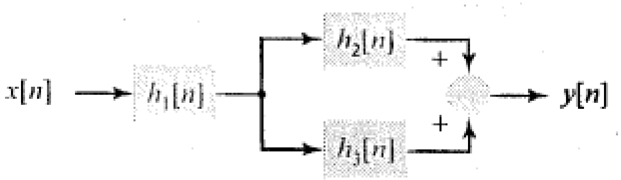


FIGURA P2.11

Letra a:

$$y[n] = x[n] * h[n]$$

Logo: $\boxed{h[n] = h_1[n]*(h_1[n]+h_3[n])}$

Letra b:

$$h_4[n] = (h_2[n]*h_3[3]) = \delta[n] + u[n-1] = u[n]$$

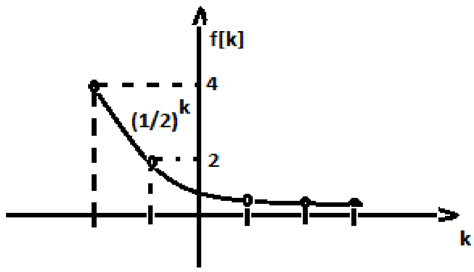
$$h[n] = h_1[n]*h_4[n] = \left(\frac{1}{2}\right)^n \left(u[n+2]-u[n-3]\right)*u[n]$$

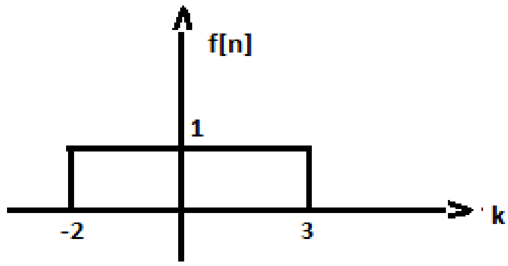
$$f[n] = \left(\frac{1}{2}\right)^n \left(u[n+2]-u[n-3]\right)$$

$$g[n] = u[n]$$

$$z[n] = f[n]*g[n]$$

$$z[n] = \sum_{k=-\infty}^{\infty} f[n].g[n-k]$$





$n < -2,$
$h[n] = 0$
$-2 \le n \le 3,$

Logo: $\boxed{h[n] = \sum_{k=-2}^{n} \left(\frac{1}{2}\right)^{k}}$

Letra c:

$i)$

$$\sum_{k=-\infty}^{\infty} |h_1[n]| = \left(\frac{1}{2}\right)^{n} .((u[n+2] - u[n-3])),$$

estável

$ii)$

$h_1[n] \neq 0, n < 0$

não _ causal

$iii)$

$h_1[n] \neq k\delta[n]$

dinâmico

Letra d:

$h_2[n] = \delta[n]$

$i)$

$$\sum_{k=-\infty}^{\infty} |h_2[n]| = 1$$

estável

$ii)$

$h_2[n] = 0, n < 0$

causal

$iii)$

$h_2[n] = k\delta[n]$

instâneo

Letra e:

$$h_3[n] = u[n-1]$$
$i)$
$$\sum_{k=-\infty}^{\infty} |h_3[n]| \to \infty,$$
$não\_estável$

$ii)$
$$h_3[n] = 0, n < 0$$
$causal$

$iii)$
$$h_3[n] \neq k\delta[n]$$
$dinâmico$

2.12 Para cada resposta ao impulso listada abaixo, determine se o sistema correspondente é (i) sem memória, (ii) causal, (iii) estável.

a) $h(t) = e^{-2|t|}$

b) $h(t) = e^{2t}u(t-1)$

c) $h(t) = u(t+1) - 2u(t-1)$

d) $h(t) = 3\delta(t)$

e) $h(t) = \cos(\pi.t)u(t)$

f) $h[n] = 2^n u[-n]$

g) $h[n] = e^{2n}u[n-1]$

h) $\cos(\frac{1}{8}\pi.n)\{u[n] - u[n-10]\}$

i) $h[n] = 2u[n] - 2u[n-1]$

j) $h[n] = sen(\frac{1}{2}\pi.n)$

k) $h[n] = \delta[n] + sen(\pi.n)$

Letra a:

$$h(t) = e^{-2|t|}$$

$i) h(t) = k\delta(t)$
$dinâmico$
$ii)$
$$h(t) \neq 0, t < 0$$
$não\_causal$
$iii)$
$$h(t) = \int_{-\infty}^{\infty} |h(t)| dt < \infty$$
$estável$

Letra b:

$h(t) = e^{2t}u(t-1)$

$i)$
$dinâmico$
$ii)$
$\int_{-\infty}^{\infty} |h(t)| dt \to \infty$
$não\_estável$
$iii)$
$h[t] = 0, t < 0$
$causal$

Letra c:

$h(t) = u(t+1) - 2u(t-1)$

$i)$
$dinâmico$
$ii)$
$\int_{-\infty}^{\infty} |h(t)| dt \to \infty$
$não\_estável$
$iii)$
$h[t] \neq 0$
$não\_causal$

Letra d:

$h(t) = 3\delta(t)$

$i)$
$h(t) = 0, t \neq 0$
$instân\tan io$
$ii)$
$h(t) = 0, t < 0$
$causal$
$iii)$
$\int_{-\infty}^{\infty} h(t) dt = 1(< \infty)$
$estável$

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Letra f:

$h[n] = 2^n u[-n]$

$i)$
$h[n] \neq 0, n < 0$
$não\_causal$
$ii)$
$\sum_{-\infty}^{\infty} |h[n]| = \sum_{k=-\infty}^{\infty} 2^k = 2 < \infty$
$iii)$
$h[n] \neq 0, n < 0$
$dinâmico$

Letra h:

$h[n] = \cos(\frac{1}{8}\pi.n)\{u[n] - u[n-10]\}$
$i)$
$h[n] \neq 0,$
$0 < n < 10$
$dinãmico$
$ii)$
$\sum_{-\infty}^{\infty} |h[n]| < \infty$
$estável$
$iii)$
$h[n] = 0, n < 0$
$causal$

Letra i:

$h[n] = 2u[n] - 2u[n-1]$

$i)$
$h[n] = 0, n < 0$
$causal$
$ii)$
$\sum_{-\infty}^{\infty} |h[n]| < \infty$
$estável$
$iii)$
$instâ\tan io$

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Letra j:

$$h[n] = sen(\tfrac{1}{2}\pi.n)$$

$$i)h[n] \neq 0, n < 0$$
$$não\_causal$$
$$ii)$$
$$\sum_{-\infty}^{\infty} |h[n]| \to \infty$$
$$não\_estável$$
$$iii)$$
$$h[n] \neq 0, n < 0$$
$$dinâmico$$

Letra k:

$$h[n] = \delta[n] + sen(\pi.n)$$

$$i)$$
$$h[n] = 0, n < 0$$
$$causal$$
$$ii)$$
$$\sum_{-\infty}^{\infty} |h[n]| < \infty$$
$$estável$$
$$iii)$$
$$instân\tan io$$

2.21) Determine a saída do sistma descrita pelas seguintes equações diferenciais com entrada e condições iniciais conforme especificado:

a)

$$\frac{d}{dt}y(t) + 10y(t) = 2x(t), \quad y(0) = 1, \quad x(t) = u(t)$$

b)

$$\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 4y(t) = \frac{d}{dt}x(t), \quad y(0) = 0, \quad \frac{d}{dt}y(t)/_{t=0} = 1, \quad x(t) = e^{-2t}u(t)$$

c)

$$\frac{d^2}{dt^2}y(t)+3\frac{d}{dt}y(t)+2y(t)=2x(t), \quad y(0)=-1, \quad \frac{d}{dt}y(t)/_{t=0}=1, \quad x(t)=\cos(t)u(t)$$

d)

$$\frac{d^2}{dt^2}y(t)+y(t)=2\frac{d}{dt}x(t), \quad y(0)=-1, \quad \frac{d}{dt}y(t)/_{t=0}=1, \quad x(t)=2e^{-t}u(t)$$

Letra a:

$$\frac{d}{dt}y(t)+10y(t)=2x(t),$$

Solução homogênea:

$$\frac{d}{dt}y(t)+10y(t)=0$$

$$r+10=0$$
$$r=-10$$
$$y_h(t)=A.e^{-10t}$$

Solução particular:

$$x(t)=2u(t)$$

$$y_p(t)=k.u(t)$$
$$y_p^{'}=0$$

Substituindo na equação:

$$0+10[k.u(t)]=2u(t)$$

$$k=\frac{2u(t)}{10u(t)}=\frac{1}{5}$$

Solução geral:

$$y(t)=A.e^{-10t}+{}^1\!/_5$$

$$y(0)=1=A+{}^1\!/_5$$

$$A={}^4\!/_5$$

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Logo: $$\boxed{y(t) = \frac{4}{5}e^{-10t} + \frac{1}{5}, t \geq 0}$$

Letra b:

$$\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 4y(t) = \frac{d}{dt}x(t)$$

Solução homogênea:

$$\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 4y(t) = 0$$

$$r^2 + 5r + 4 = 0$$

$$\Delta = 25 - 16 = 9$$

$$r_1 = \frac{-5+3}{2} = -1$$

$$r_2 = \frac{-5-3}{2} = -4$$

$$y_h(t) = c_1 e^{-4t} + c_2 e^{-t}$$

Solução Particular:

$$x(t) = -2^{-2t}$$

$$y_p(t) = ke^{-2t}$$

$$y'_p(t) = -2ke^{-2t}$$

$$y''_p(t) = 4ke^{-2t}$$

$$4ke^{-2t} + 5.(-2ke^{-2t}) + 4ke^{-2t} = -2ke^{-2t}$$

$$4ke^{-2t} - 10ke^{-2t} + 4ke^{-2t} = -2ke^{-2t}$$

$$k.e^{-2t}(4 - 10 + 4) = -2ke^{-2t}$$

$$k = \frac{-2ke^{-2t}}{-2ke^{-2t}} = 1$$

Solução geral:

$$y(t) = y_p(t) + y_h(t)$$

$$y(t) = e^{-2t} + c_1 e^{-4t} + c_2 e^{-t}$$
$$y(0) = 0$$
$$0 = 1 + c_1 + c_2$$
$$c_1 = -c_2 - 1$$

$$y'(t) = -2e^{-2t} - 4c_1 e^{-4t} - c_2 e^{-t}$$
$$y'(0) = 1$$
$$1 = -2 - 4c_1 - c_2$$

$$c_2 = -\frac{1}{3}$$
$$c_1 = -\frac{2}{3}$$

Logo: $\boxed{y(t) = e^{-2t} - \frac{2}{3}e^{-4t} - \frac{1}{3}^{-t}, t \geq 0}$

Letra c:

$$\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = 2x(t)$$

Solução homogênea:

$$\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = 0$$

$$r^2 + 3r + 2 = 0$$
$$\Delta = 9 - 8 = 1$$

$$r_1 = \frac{-3+1}{2} = -1$$
$$r_2 = \frac{-3-1}{2} = -2$$

$$y_h(t) = c_1 e^{-t} + c_2 e^{-2t}$$

Solução Particular (I):

$$x(t) = 2\cos(t).u(t)$$
$$y_p(t) = A\cos(t) + Bsen(t)$$

$$y'_p(t) = -Asen(t) + B\cos(t)$$
$$y''_p(t) = -A\cos(t) - Bsen(t)$$

Solução particular:

$$-A\cos(t) - Bsen(t) - 3Asen(t) + 3B\cot(t) + A\cos(t) + 2Bsen(t) = 2\cos(t)$$
$$(-A + 3B + 2A).\cos(t) + (-B - 3A + 2B)sen(t) = 2\cos(t)$$

$$A + 3B = 2$$
$$-3A + B = 0$$
$$B = 3A$$
$$A + 9A = 2$$
$$A = {}^{1}\!/_{5}$$
$$B = {}^{3}\!/_{5}$$

Solução geral:

$$y(t) = {}^{1}\!/_{5}\cos(t) + {}^{3}\!/_{5}\,sen(t) + c_1 e^{-t} + c_2 e^{-2t}$$

$$y(0) = -1$$
$$-1 = {}^{1}\!/_{5} + c_1 + c_2$$

$$c_1 = \frac{-6}{5} - c_2$$

$$y'(0) = 1$$
$$y'(t) = -\frac{1}{5}sent(t) + \frac{3}{5}\cos(t) - c_1 e^{-t} - 2c_2 e^{-2t}$$
$$y'(0) = \frac{3}{5} - c_1 - 2c_2 = 1$$
$$c_1 = -\frac{2}{5} - 2c_2$$
$$-\frac{2}{5} - 2c_2 = -\frac{6}{5} - c_2$$
$$c_2 = \frac{4}{5}$$
$$c_1 = -2$$

Logo: $\boxed{y(t) = {}^{1}\!/_{5}\cos(t) + {}^{3}\!/_{5}\,sen(t) - 2e^{-t} + {}^{4}\!/_{5}\,e^{-2t}, t > 0}$

Letra d:

$$\frac{d^2}{dt^2}y(t)+y(t)=2\frac{d}{dt}x(t)=0$$

Solução homogênea:

$$r^2+1=0$$
$$r=\pm 1$$

$$y_h(t)=c_1\cos(t)+c_2 sen(t)$$

Solução particular:

$$x(t)=-6e^{-t}$$
$$y_p(t)=ke^{-t}$$
$$y'(t)=-ke^{-t}$$
$$y''_p(t)=ke^{-t}$$

$$ke^{-t}+ke^{-t}=-6e^{-t}$$
$$2(ke^{-t})=-6e^{-t}$$
$$k=\frac{-6e^{-t}}{2ke^{-t}}=-3$$

$$y_p(t)=-3e^{-t}$$

Solução geral:

$$y(t)=y_h(t)+y_p(t)$$

$$y(t)=c_1\cos(t)+c_2 sen(t)-3e^{-t}$$
$$y'(t)=-c_1 sen(t)+c_2\cos(t)+3e^{-t}$$

$$y(0)=-1=c_1-3$$
$$c_1=2$$
$$y'0)=1=c_2+3$$
$$c_2=-2$$

Logo: $\boxed{2\cos(t)-2sen(t)-3e^{-t}, t\geq 0}$

2.22 Determine a saída dos sistemas descritos pelas seguintes equações de diferenças com entrada e condições iniciais conforme especificado:

a)

$$y[n]-\frac{1}{2}y[n-1]=2x[n],\quad y[1]=3,\quad x[n]=2\left(-\frac{1}{2}\right)^n u[n]$$

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b)

$$y[n]-\frac{1}{9}y[n-2]=x[n-1],\quad y[-1]=1,\quad y[-2]=0,\quad x[n]=u[n]$$

c)

$$y[n]-\frac{1}{4}y[n-1]-\frac{1}{8}y[n-2]=x[n]+x[n-1],\quad y[-1]=2,\quad y[-2]=-1,\quad x[n]=2^{n}u[n]$$

Letra a:

$$y[n]-\frac{1}{2}y[n-1]=2x[n]$$

Solução homogênea:

$$y[n]-\frac{1}{2}y[n-1]=0$$

$$r-\tfrac{1}{2}=0$$

$$r=\tfrac{1}{2}$$

$$y_h[n]=A\left(\frac{1}{2}\right)^{n},\forall n$$

Solução particular:

$$x[n]=4.\left(-\frac{1}{2}\right)^{n}$$

$$y_p[n]=k\left(-\frac{1}{2}\right)^{n}$$

$$k\left(-\frac{1}{2}\right)^{n}-\frac{1}{2}k\left(-\frac{1}{2}\right)^{n-1}=4.\left(-\frac{1}{2}\right)^{n}$$

$$k\left(-\frac{1}{2}\right)^{n}-\frac{1}{2}k\left(-\frac{1}{2}\right)^{n}.\left(-\frac{1}{2}\right)^{-1}=4.\left(-\frac{1}{2}\right)^{n}$$

$$k+k=4$$

$$2k=4$$

$$k=2$$

$$y_p[n]=2.\left(-\frac{1}{2}\right)^{n},n\geq 0$$

Solução geral:

$$y[n] = y_h[n] + y_p[n]$$

$$y[n] = A\left(\frac{1}{2}\right)^n + 2\left(-\frac{1}{2}\right)^n, n \geq 0$$

$$y[-1] = 3$$

$$\det er \min ar$$

$$y[o] = ?$$

$$y[n] - \frac{1}{2}y[n-1] = 2x[n]$$

$$y[0] - \frac{1}{2}y[-1] = 2x[0]$$

$$y[o] = -\frac{1}{2}.3 = 4$$

$$y[o] = 4 + \frac{3}{2} = \frac{11}{2}$$

Substitui na solução completa:

$$y[o] = A + 2 = \frac{11}{2}$$

$$A = \frac{7}{2}$$

Logo : $\boxed{y[n] = \frac{7}{2}\left(\frac{1}{2}\right)^n + 2\left(-\frac{1}{2}\right)^n, n \geq 0}$

Letra b:

$$y[n]-\frac{1}{9}y[n-2]=x[n-1]$$

Solução homogênea:

$$y[n]-\frac{1}{9}y[n-2]=0$$

$$r^2-\frac{1}{9}=0$$

$$r=\pm\left(\frac{1}{3}\right)$$

$$y_h[n]=A\left(\frac{1}{3}\right)^n+B\left(-\frac{1}{3}\right)^n,\forall n$$

Solução particular:

$$x[n]=1$$
$$y_p[n]=k$$

$$k-\frac{1}{9}k=1$$

$$\frac{8}{9}k=1$$

$$k=9/8$$

$$y_p[n]=9/8$$

Solução geral:

$$y[n]=y_h[n]+y_p[n]$$

$$y[n]=\frac{9}{8}+A\left(\frac{1}{3}\right)^n+B\left(-\frac{1}{3}\right)^n,n\geq 0$$

$\det er \min ar$

$$y[0]=?$$

$$y[n]-\frac{1}{9}y[n-2]=x[n-1]$$

$$y[0]-\frac{1}{9}y[-2]=x[-1]$$

$$y[0]-1/9.(0)=0$$

$\det er \min ar$

$$n=1$$

$$y[1]-1/9\,y[-1]=x[0]$$

$$y[1]=1/9+1$$

$$y[1]=\frac{10}{9}$$

Substitui na equação completa:

$$y[0]=\frac{9}{8}+A+B=0$$
$$B=-A-\frac{9}{8}$$

$$y[1]=\frac{9}{8}+A\left(\tfrac{1}{3}\right)+B\left(-\tfrac{1}{3}\right)=\tfrac{10}{9}$$
$$\tfrac{10}{9}-\tfrac{9}{8}=A\left(\tfrac{1}{3}\right)+B\left(-\tfrac{1}{3}\right)$$
$$A=-\frac{3}{72}+B$$

$$B=\frac{3}{72}-B-\frac{9}{8}$$
$$B=-\frac{13}{24}$$
$$A=-\frac{7}{12}$$

Logo: $\boxed{y[n]=\frac{9}{8}-\frac{7}{12}\left(\frac{1}{3}\right)^{n}-\frac{13}{24}\left(-\frac{1}{3}\right)^{n},n\geq 0}$

Letra c:

$$y[n]-\frac{1}{4}y[n-1]-\frac{1}{8}y[n-2]=x[n]+x[n-1],$$

Solução homogênea:

$$y[n]-\frac{1}{4}y[n-1]-\frac{1}{8}y[n-2]=0$$

$$r^{2}-\frac{1}{4}r-\frac{1}{8}=0$$
$$\Delta=\frac{1}{16}+\frac{1}{2}=\frac{9}{16}$$
$$r_{1}=\frac{\tfrac{1}{4}+\tfrac{3}{4}}{2}=\frac{1}{2}$$
$$r_{2}=\frac{\tfrac{1}{4}-\tfrac{3}{4}}{2}=-\frac{1}{4}$$

Solução particular:

$$y_h[n] = A\left(\frac{1}{2}\right)^n + B\left(-\frac{1}{4}\right)^n$$

$$x[n] = 2^n$$

$$y_p[n] = k.2^n, n \geq 2$$

$$k.2^n - \frac{1}{4}k.2^{n-1} - \frac{1}{8}k.2^{n-2} = 2^n + 2^{n-1}$$

$$k.2^n - \frac{1}{4}k.2^n.\frac{1}{2} - \frac{1}{8}k.2^n.\frac{1}{4} = 2^n + 2^n.\frac{1}{2}$$

$$k = \frac{16}{9}$$

$$y_p = \frac{16}{9}2^n$$

Solução geral:

$$y[n] = y_h[n] + y_p[n]$$

$$y[n] = A\left(\frac{1}{2}\right)^n + B\left(-\frac{1}{4}\right)^n + \frac{16}{9}2^n$$

$$\det er \min ar$$

$$y[0] = ?$$

$$y[1] = ?$$

$$y[0] - \frac{1}{4}y[-1] - \frac{1}{8}y[-2] = x[0] + x[-1]$$

$$y[0] - \frac{1}{4}(2) - \frac{1}{8}(-1) = 1 + 0$$

$$y[0] = \frac{1}{2} - \frac{1}{8}(-1) = \frac{11}{8}$$

$$y[1] - \frac{1}{4}y[0] - \frac{1}{8}y[-1] = x[1] + x[0]$$

$$y[1] = \frac{1}{4}.\frac{11}{8} + \frac{1}{8}.(2) = 2 + 1$$

$$y[1] = \frac{115}{32}$$

Substituindo na solução geral:

$$y[0] = A + B + \frac{16}{9} = \frac{11}{8}$$

$$A = -\frac{29}{72} - B$$

$$-\frac{29}{72} - B = \frac{11}{144} + \frac{B}{2}$$

$$B = -\frac{23}{72}$$

$$y[1] = A\left(\frac{1}{2}\right) \quad + B\left(-\frac{1}{4}\right) + \frac{16}{9}2 = \frac{115}{32}$$

$$y[1] = A\left(\frac{1}{2}\right) \quad + \frac{23}{72}\left(-\frac{1}{4}\right) + \frac{16}{9}2 = \frac{115}{32}$$

$$A = -\frac{1}{12}$$

Logo: $\boxed{y[n] = -\frac{1}{12}\left(\frac{1}{2}\right) \quad - \frac{23}{72}\left(-\frac{1}{4}\right) + \frac{16}{9}2^n, n \geq 0}$