1 → a)

$$P_{\beta\leftarrow\alpha} = [\text{BASE NOVA} \mid \text{BASE VELHA}] \longrightarrow [I \mid P_{\beta\leftarrow\alpha}]$$

$$P_{\beta\leftarrow\alpha} = \left[\begin{array}{ccc|ccc} -6 & -2 & -2 & -3 & -3 & 1 \\ -6 & -6 & -3 & 0 & 2 & 6 \\ 0 & 4 & 7 & -3 & -1 & -1 \end{array}\right]$$

$L_1 \times -1/6$

$$\left[\begin{array}{ccc|ccc} 1 & 1/3 & 1/3 & 1/2 & 1/2 & -1/6 \\ -6 & -6 & -3 & 0 & 2 & 6 \\ 0 & 4 & 7 & -3 & -1 & -1 \end{array}\right]$$

$L_2 + 6L_1$

$$\left[\begin{array}{ccc|ccc} 1 & 1/3 & 1/3 & 1/2 & 1/2 & -1/6 \\ 0 & -4 & -1 & 3 & 5 & 5 \\ 0 & 4 & 7 & -3 & -1 & -1 \end{array}\right]$$

$L_2 \times -1/4$

$$\left[\begin{array}{ccc|ccc} 1 & 1/3 & 1/3 & 1/2 & 1/2 & -1/6 \\ 0 & 1 & 1/4 & -3/4 & -5/4 & -5/4 \\ 0 & 4 & 7 & -3 & -1 & -1 \end{array}\right]$$

$L_3 - 4L_2$

$$\left[\begin{array}{ccc|ccc} 1 & 1/3 & 1/3 & 1/2 & 1/2 & -1/6 \\ 0 & 1 & 1/4 & -3/4 & -5/4 & -5/4 \\ 0 & 0 & 6 & 0 & 4 & 4 \end{array}\right]$$

$L_3 \times 1/6$

$$\left[\begin{array}{ccc|ccc} 1 & 1/3 & 1/3 & 1/2 & 1/2 & -1/6 \\ 0 & 1 & 1/4 & -3/4 & -5/4 & -5/4 \\ 0 & 0 & 1 & 0 & 2/3 & 2/3 \end{array}\right]$$

$L_1 - \frac{1}{3}L_3 \;\|\; L_2 - \frac{1}{4}L_3$

$$\left[\begin{array}{ccc|ccc} 1 & 1/3 & 0 & 1/2 & 5/18 & -7/18 \\ 0 & 1 & 0 & -3/4 & -17/12 & -17/12 \\ 0 & 0 & 1 & 0 & 2/3 & 2/3 \end{array}\right]$$

$L_1 - \frac{1}{3}L_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/4 & 3/4 & 1/12 \\ 0 & 1 & 0 & -3/4 & -17/12 & -17/12 \\ 0 & 0 & 1 & 0 & 2/3 & 2/3 \end{array}\right]$$

$$P_{\beta\leftarrow\alpha} = \frac{1}{12}\begin{bmatrix} 9 & 9 & 1 \\ -9 & -17 & -17 \\ 0 & -8 & -8 \end{bmatrix}$$

b) $P_{\alpha \leftarrow \beta} = (P_{\beta \leftarrow \alpha})^{-1}$

$$P_{\alpha \leftarrow \beta} = \begin{bmatrix} 3/4 & 3/4 & 1/12 \\ -3/4 & -17/12 & -17/12 \\ 0 & 2/3 & 2/3 \end{bmatrix}^{-1}$$

A matriz de mudança de base de $\beta \rightarrow \alpha$ é a inversa da matriz de $\alpha \rightarrow \beta$.

$$P_{\alpha \leftarrow \beta} = \begin{bmatrix} 0 & 4/3 & -17/6 \\ 3/2 & 3/2 & 3 \\ -3/2 & -3/2 & -3/2 \end{bmatrix} \text{ ou } P_{\alpha \rightarrow \beta} = \frac{1}{6}\begin{bmatrix} 0 & 8 & -17 \\ 9 & 9 & 18 \\ -9 & -9 & -9 \end{bmatrix}$$

c) Para encontrar $[W]_\alpha$ devemos encontrar a matriz $P_{\alpha \leftarrow E}$, onde E é a matriz cujas colunas são os vetores canônicos de R^3 (base canônica).

$$\alpha = \begin{bmatrix} -3 & -3 & 1 \\ 0 & 2 & 6 \\ -3 & -1 & -1 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} -3 & -3 & 1 & 1 & 0 & 0 \\ 0 & 2 & 6 & 0 & 1 & 0 \\ -3 & -1 & -1 & 0 & 0 & 1 \end{array}\right]$$

Escalonando...

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/12 & -1/12 & -5/12 \\ 0 & 1 & 0 & -3/8 & 1/8 & 3/8 \\ 0 & 0 & 1 & 1/8 & 1/8 & -1/8 \end{array}\right]$$

$$P_{\alpha \leftarrow E} = \begin{bmatrix} 1/12 & -1/12 & -5/12 \\ -3/8 & 1/8 & 3/8 \\ 1/8 & 1/8 & -1/8 \end{bmatrix} \text{ ou } P_{\alpha \rightarrow E} = \frac{1}{24}\begin{bmatrix} 2 & -2 & -10 \\ -9 & 3 & 9 \\ 3 & 3 & -3 \end{bmatrix}$$

Como a matriz $P_{\alpha \leftarrow E}$ faz a transformação da base canônica onde encontram-se $w = (-5, 8, -5)$ para a base α, podemos calcular $[W]_\alpha^*$ como se segue:

$[W]_\alpha = P_{\alpha \to E} \cdot [W]_E$

$$[W]_\alpha = \begin{bmatrix} 1/12 & -1/12 & -5/12 \\ -3/8 & 1/8 & 3/8 \\ 1/8 & 1/8 & -1/8 \end{bmatrix} \begin{bmatrix} -5 \\ 8 \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$[W]_\alpha = (1,1,1)$

Por último calcula-se $[W]_\beta$ utilizando a matriz de transformação de α para β.

$[W]_\beta = P_{\beta \leftarrow \alpha} \cdot [W]_\alpha$

$$[W]_\beta = \begin{bmatrix} 3/4 & 3/4 & 0 \\ -3/4 & -17/12 & -3/2 \\ 0 & 2/3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3/2 \\ -11/3 \\ 5/3 \end{bmatrix}$$

$[W]_\beta = \left(\frac{3}{2}, -\frac{11}{3}, \frac{5}{3}\right)$

* Isso segue do fato que qualquer vetor em R^3 é gerado pela base canônica.

2 - $\alpha = \{(0,1,0), (1,0,1), (0,1,1)\}$
$\beta = \{(1,0,0), (1,1,0), (0,0,1)\}$

a) $\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$

$$P_{\beta\leftarrow\alpha} = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

b) $P_{\alpha\leftarrow\beta} = (P_{\beta\leftarrow\alpha})^{-1}$

$$P_{\alpha\leftarrow\beta} = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$$

c) $[p(x)]_E = (4, -2, -1)$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$P_{\alpha\leftarrow E} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$[p(x)]_\alpha = P_{\alpha\leftarrow E} \cdot [p(x)]_E$

$$[p(x)]_\alpha = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix} ; [p(x)]_\alpha = 3 + 4x - 5x^2$$

$$[p(x)]_\beta = P_{\beta \leftarrow \alpha} \cdot [p(x)]_\alpha$$

$$[p(x)]_\beta = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ -1 \end{bmatrix}$$

$$[p(x)]_\beta = 6 - 2x - x^2$$

3 → $\alpha = \{(0,1,2)\ (1,0,1)\ (0,1,0)\}$

$\beta = \{v_1, v_2, v_3\}$

$v_1 = (0+1-0,\ 1+0-1,\ 2+1-0) = (1,0,3)$

$v_2 = (0-1-0,\ 1-0-1,\ 2-1-0) = (-1,0,1)$

$v_3 = (0-1+0,\ 1-0+1,\ 2-1+0) = (-1,2,1)$

a) $\left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 & 1 \\ 3 & 1 & 1 & 2 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & -1/2 \\ 0 & 0 & 1 & 1/2 & 0 & 1/2 \end{array}\right]$

$$P_{\beta \leftarrow \alpha} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & -1/2 & -1/2 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

b) $P_{\alpha \leftarrow \beta} = (P_{\beta \leftarrow \alpha})^{-1}$

$$P_{\alpha \leftarrow \beta} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & -1/2 & -1/2 \\ 1/2 & 0 & 1/2 \end{bmatrix}^{-1}$$

$$P_{\alpha \leftarrow \beta} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

4)
$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

$$\det(A-\lambda I) = 0$$

$$\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 3-\lambda & 0 \\ 8 & -1-\lambda \end{bmatrix}$$

$$\det(A-\lambda I) = 0$$

$$\begin{vmatrix} 3-\lambda & 0 \\ 8 & -1-\lambda \end{vmatrix} = (3-\lambda)(-1-\lambda) = 0$$

Os autovalores são as raízes da equação $(3-\lambda)(-1-\lambda) = 0$, logo, $\lambda_1 = 3$ e $\lambda_2 = -1$ são autovalores de A.

$$B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 4 & -17 & 8-\lambda \end{vmatrix} = 0$$

$$-\lambda \begin{vmatrix} -\lambda & 1 \\ -17 & 8-\lambda \end{vmatrix} - 0 \begin{vmatrix} 1 & 0 \\ -17 & 8-\lambda \end{vmatrix} + 4 \begin{vmatrix} 1 & 0 \\ -\lambda & 1 \end{vmatrix} = 0$$

$$-\lambda \cdot [-\lambda(8-\lambda) + 17] + 4(1-0) = 0$$

$$-\lambda(-8\lambda + \lambda^2 + 17) + 4 = 0$$

$$-\lambda^3 + 8\lambda^2 - 17\lambda + 4 = 0$$

$$\left.\begin{array}{l} \lambda_1 = 4 \\ \lambda_2 = 2-\sqrt{3} \approx 0,268 \\ \lambda_3 = 2+\sqrt{3} \approx 3,732 \end{array}\right\} \text{Autovalores de A}$$

3

5) Por definição, $X = [X_1\ X_2]^T$ é um autovetor de A associado ao autovalor λ, se e somente, X é solução não trivial de $(\lambda I - A)X = 0$.

Primeiro encontra-se os autovalores de A.

$$\begin{vmatrix} -\lambda & 0 & -2 \\ 1 & 2-\lambda & 1 \\ 1 & 0 & 3-\lambda \end{vmatrix} = 0 \qquad -(2-\lambda)\begin{vmatrix} -\lambda & -2 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)[-\lambda(3-\lambda)+2] = 0$$

$$-\lambda^3 + 5\lambda^2 - 8\lambda + 4 = 0$$

Os autovalores são, $\lambda_1 = \lambda_2 = 2$, $\lambda_3 = 1$

$$\begin{bmatrix} -\lambda & 0 & -2 \\ 1 & 2-\lambda & 1 \\ 1 & 0 & 3-\lambda \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Para $\lambda = 2$

$$\begin{bmatrix} -2 & 0 & -2 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Escalonando

$L_1 \times -1/2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$L_2 - L_1 \;||\; L_3 - L_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$X_1 + X_3 = 0 \to X_1 = -X_3$

$X_2 = \forall \mathbb{R}$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -X_3 \\ X_2 \\ X_3 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = t\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \forall, t\ e\ s \in \mathbb{R}$$

Logo, os autovetores associados ao autovalor $\lambda = 2$ são

$$\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

Para $\lambda = 1$

$$\begin{bmatrix} -1 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$L_2 + L_1$ || $L_3 + L_1$

$$\begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_1 - 2x_3 = 0 \rightarrow x_1 = -2x_3$

$x_2 - x_3 = 0 \rightarrow x_2 = x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_3 \\ x_3 \\ x_3 \end{bmatrix}, \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \quad \forall\, t \in \mathbb{R}$$

Logo o autovetor associado ao autor valor $\lambda = 1$ é

$$\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$$

6→ a) $\begin{vmatrix} 1-\lambda & 3 \\ 2 & 2-\lambda \end{vmatrix} = 0$ $\quad det(A-\lambda I)=0$

$(1-\lambda)(2-\lambda)-6=0$

$\lambda^2-3\lambda-4=0$

$(\lambda-4)(\lambda+1)=0 \quad \lambda_1=-1, \ \lambda_2=4$

Para $\lambda=4$

$\begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$L_{1\,x} = 1/3$

$\begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$

$L_2 - 2L_1$

$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$

$x_1 - x_2 = 0 \rightarrow x_1 = x_2$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t\begin{bmatrix} 1 \\ 1 \end{bmatrix} \ \forall t \in \mathbb{R}$

$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ é o autovetor associado ao autovalor $\lambda=4$

Para $\lambda=-1$

$\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$

$L_2 - L_1$

$\begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}$

$2x_1+3x_2=0 \rightarrow x_1 = -\frac{3}{2}x_2$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t\begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$ ou $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

$\begin{bmatrix} -3 \\ 2 \end{bmatrix}$ é o autovetor associado ao autovalor $\lambda=-1$

Já que a multiplicidade algébrica (número de vezes que um fator λi apareceu como um fator do polinômio característico $\lambda-\lambda i$) foi igual a multiplicidade geométrica de A (dimensão do autoespaço associado à λi).

b) $P = \begin{bmatrix} 1 & -3 \\ 1 & 2 \end{bmatrix}$; $D = \begin{array}{cc} \lambda_1 & \lambda_2 \\ \end{array}\begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$

Pode-se verificar que $AP = PD$

7- a) $\begin{vmatrix} -1-\lambda & 0 & 1 \\ -3 & -\lambda & -3 \\ 1 & 0 & -1-\lambda \end{vmatrix} = 0$

$-\lambda \begin{vmatrix} -1-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} = 0$

$-\lambda[(-1-\lambda)(-1-\lambda) - 1] = 0$

$-\lambda^3 - 2\lambda^2 = 0$ ou $-\lambda^2(\lambda+2)$, $\lambda_1 = \lambda_2 = 0$, $\lambda_3 = -2$

Para $\lambda = 0$

$\begin{bmatrix} -1 & 0 & 1 \\ -3 & 0 & -3 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$L_2 - 3L_1$;; $L_3 + L_1$

$\begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & -6 \\ 0 & 0 & 0 \end{bmatrix}$

$L_2 \times -1/6$

$\begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$L_1 - L_2$

$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$-x_1 + 0x_2 + 0x_3 = 0 \rightarrow x_1 = 0$

$x_2 = x_2$

$x_3 = 0$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Como o multiplicidade algébrica do $\lambda = 0$ é dois, mas a multiplicidade geométrica é 1 (pois gerou somente um vetor) A não é diagonalizável.

Para $\lambda = 2$

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$L_3 + L_1$

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

$L_3 + L_2$

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$-x_1 + x_3 = 0 \rightarrow x_1 = x_3$

$-x_2 + x_3 = 0 \rightarrow x_2 = x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \forall t \in \mathbb{R}$$

b) A matriz P é obtida utilizando as bases para o autoespaço (autovetores)

$$P = \begin{bmatrix} -1 & -1 & 1 \\ 1 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix} \qquad D = \begin{array}{c} \begin{array}{ccc} \lambda_1 & \lambda_2 & \lambda_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \end{array}$$

A matriz P acima diagonaliza A.

Para $\lambda = 1$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$L_1 \leftrightarrow L_3$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$L_3 - L_2$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$L_1 + L_2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 + x_2 = 0 \rightarrow x_1 = -x_2$

$x_3 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_2 \\ x_2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad \forall\, t \in \mathbb{R}$$

Para $\lambda = -1$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$L_1 \times 1/2$

$$\begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$L_3 - L_1$

$$\begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 2 & 1 \\ 0 & 1 & 1/2 \end{bmatrix}$$

$L_3 - \frac{1}{2} L_2$

$$\begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 + \frac{1}{2} x_3 = 0 \rightarrow x_1 = -\frac{1}{2} x_3$

$2x_2 + x_3 = 0 \rightarrow x_2 = -\frac{1}{2} x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1/2\, x_3 \\ -1/2\, x_3 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s \begin{bmatrix} -1/2 \\ -1/2 \\ 1 \end{bmatrix} \quad \text{ou} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

$\forall\, s \in \mathbb{R}$

8→ a) $\begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$

$(1-\lambda)\begin{vmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 1-\lambda & 1 \end{vmatrix} = 0$

$(1-\lambda)[(1-\lambda)(-\lambda) - 1\times 1] + 1[0\times 1 - (1-\lambda)\times 1] = 0$

$(1-\lambda)(-\lambda+\lambda^2-1) + (\lambda-1) = 0$

$-\lambda^3 + 2\lambda^2 + \lambda - 2 = 0$

ou $(\lambda-2)(\lambda+1)(\lambda-1) = 0$

$\lambda_1 = 2,\ \lambda_2 = -1,\ \lambda_3 = 1$

Como existe 3 autovalores distintos, A é diagonalizável