Cálculo 2. 2022.1

Aula 10

Aula passada: plano e Posição

Aula hoje: Exemplo e distâncias

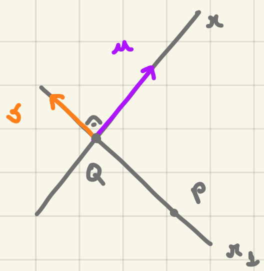
Exemplo Determine uma reta r_1 que passa pelo ponto $P=(-1,3,1)$ e perpendicular a $r: \dfrac{(x-1)}{2} = \dfrac{(y-1)}{3} = z = t$

→ simétrica

Solução

$$\begin{cases} x = 1+2t \\ y = 1+3t \\ z = t \end{cases}$$ paramétricas

vetor diretor $\vec{u} = (2,3,1)$



seja $Q=(1+2t, 1+3t, t)$ interseção de r e r_1

então $\vec{PQ} \perp \vec{u}$

$\vec{PQ} = (2+2t, -2+3t, t-1)$

$$\vec{PQ} \cdot \vec{u} = 2(2+2t)+3(-2+3t)+(t-1) = 0$$
$$4-6-1+4t+9t+t = 0$$
$$14t = +3$$
$$t = 3/14$$

então $\vec{PQ} = \left(2+\dfrac{6}{14}, -2+\dfrac{9}{14}, -1+\dfrac{3}{14}\right)$

$$= \left(\frac{34}{14}, -\frac{19}{14}, -\frac{11}{14}\right)$$

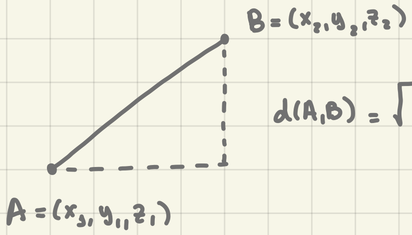
assim $x = -1 + \dfrac{34t}{14}$

$y = 3 - \dfrac{19t}{14}$

$z = 1 - \dfrac{11t}{14}$, $t \in \mathbb{R}$

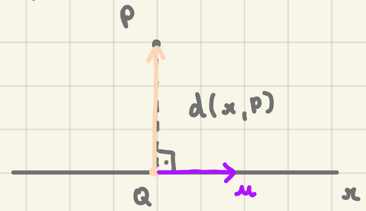
Cap 20 Distâncias

Pontos



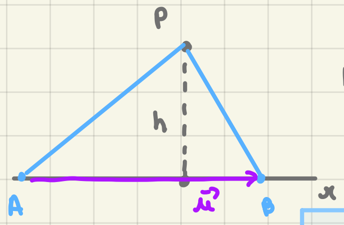
$$d(A,B) = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

reta e ponto



Calcular a distância $d(P,Q)$ onde $\vec{PQ} \cdot \vec{u} = 0$

ou

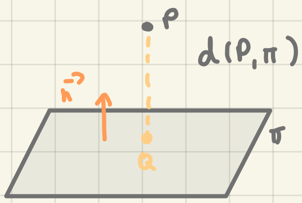


Área = b.h/2

$$\frac{\|\vec{AP} \wedge \vec{AB}\|}{2} = \frac{\|AB\| . d(r,P)}{2}$$

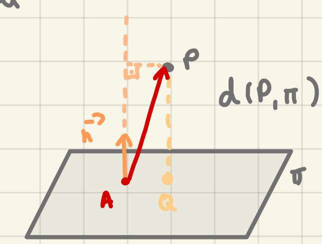
$$\boxed{d(r,P) = \frac{\|\vec{AP} \wedge \vec{u}\|}{\|u\|}}$$

Ponto e plano



Calcular $d(P,Q)$ onde Q é a interseção a reta que passa por P e tem direção $\vec{n} \parallel \vec{PQ}$

ou



$$d(P,\pi) = \left\| \mathrm{proj}_{\vec{n}} \vec{AP} \right\|$$

$$\boxed{d(P,\pi) = \frac{|\vec{AP} \cdot \vec{n}|}{\|\vec{n}\|}} \quad (*)$$

Se $P=(x_0,y_0,z_0)$, $A=(x_1,y_1,z_1)$ $> \vec{PA} = (x_1-x_0, y_1-y_0, z_1-z_0)$

$\vec{n} = (a,b,c)$ em (*)

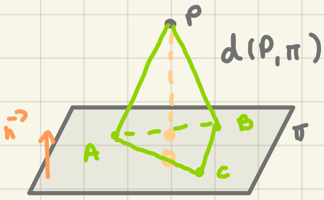
e $\pi: ax+by+cz+d=0$

$A \in \pi \Rightarrow ax_1+by_1+cz_1 = -d$

$$d(P,\pi) = \frac{|a(x_1-x_0)+b(y_1-y_0)+c(z_1-z_0)|}{\sqrt{a^2+b^2+c^2}} \quad = -d$$

$$\boxed{d(P,\pi) = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2+b^2+c^2}}}$$

e por último



$A, B, C \in \pi$

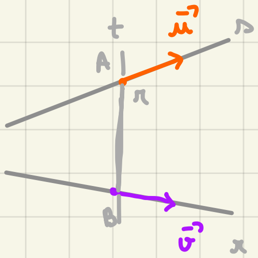
PABC é um tetraedo com $h = d(P,\pi)$

$\text{Área} = \frac{1}{3} h . \text{área da base}$

$$\frac{1}{6}[AP, AB, AC] = \frac{1}{3} d(P,\pi) . \frac{1}{2}\|AB \wedge AC\|$$

$$\Rightarrow \boxed{d(P,\pi) = \frac{|AP . (AB \wedge AC)|}{\|AB \wedge AC\|}}$$

Reta e Reta

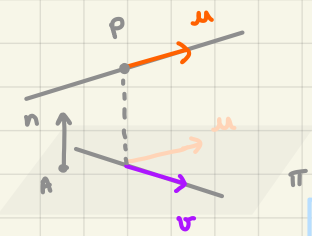


$d(r,s)$

retas reversas

Calcular a reta t perpendicular a r e s
fazer a interseção
e fazer $d(A,B)$

ou

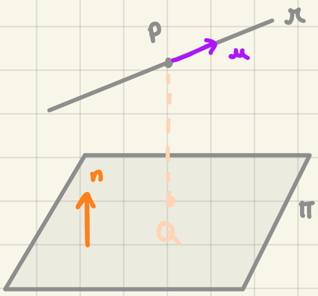


$\vec{n} = \vec{u} \wedge \vec{v}$

$d(r,s) = d(P,\pi)$
pelo item anterior

$$\boxed{d(r,s) = \frac{\|\vec{AP} . \vec{u} \wedge \vec{v}\|}{\|\vec{u} \wedge \vec{v}\|}}$$

Reta e plano

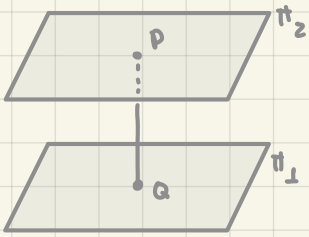


r paralela a π

$d(r,\pi) = d(P,\pi)$

P qualquer ponto de r

Plano e Plano



se $\pi_1 \parallel \pi_2$

$d(\pi_1,\pi_2) = d(P,\pi_1)$
$= d(Q,\pi_2)$

Casos que a distancia é zero:

- $P \in r$ ou $P \in \pi \Rightarrow d(P,r) = 0$ e $d(P,\pi) = 0$
- r_1 e r_2 concorrentes $\Rightarrow d(r_1,r_2) = 0$
- se r é transversal a $\pi \Rightarrow d(r,\pi) = 0$
- se π_1 e π_2 são transversais $\Rightarrow d(\pi_1,\pi_2) = 0$