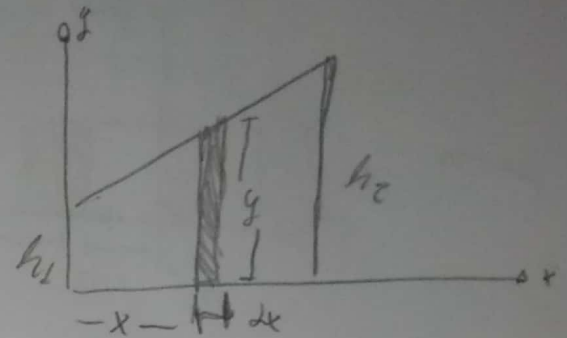
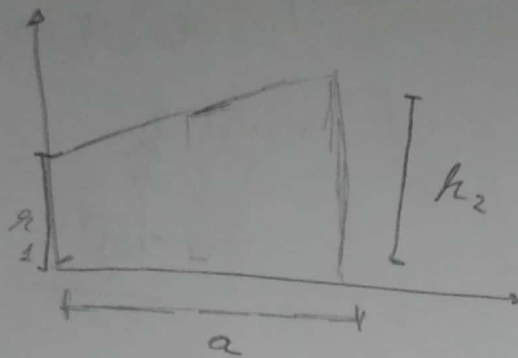


7.1



$$y = h_1 + (h_2 - h_1) \cdot \frac{x}{a} \quad \text{and} \quad dA = y dx$$

$$dI_g = x^2 dA$$

$$= x^2 \left[h_1 + (h_2 - h_1) \frac{x}{a} \right] dx$$

$$= \int_0^a \left[h_1 \cdot x^2 + \frac{h_2 - h_1}{a} x^3 \right] dx$$

$$= \left[h_1 \frac{a^3}{3} + \frac{h_2 - h_1}{a} \cdot \frac{a^4}{4} \right]$$

$$= \frac{h_1 a^3}{12} + \frac{h_2 a^3}{4}$$

$$I_g = \frac{a^3}{12} (h_1 + 3h_2)$$

7. 5

$$y = h_1 + (h_2 - h_1) \cdot \frac{x}{a}$$

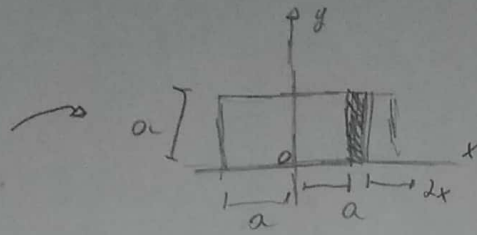
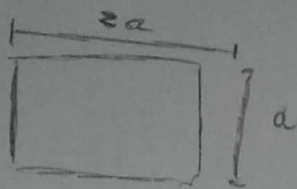
$$dI_x = \frac{1}{3} y^3 dx$$

$$I_x = \int dI_x = \frac{1}{3} \int_0^a \left[h_1 + (h_2 - h_1) \frac{x}{a} \right]^3 dx$$

$$= \left[\frac{1}{12} \left[h_1 + (h_2 - h_1) \frac{x}{a} \right]^4 \cdot \frac{a}{h_2 - h_1} \right]_0^a$$

$$\begin{aligned} &= \frac{a}{12(h_2 - h_1)} \cdot (h_2^4 - h_1^4) \\ &= \frac{a}{12} \cdot \frac{(h_2^2 + h_1^2)(h_2 + h_1)(h_2 - h_1)}{h_2 - h_1} \\ I_x &= \frac{a}{12} (h_1^2 + h_2^2) (h_1 + h_2) \end{aligned}$$

7.12



$$a) dI_x = \frac{1}{3} a^3 dx$$

$$I_x = \frac{1}{3} a^3 \int_{-a}^a dx$$

$$= \frac{a^3}{3} [a - (-a)]$$

$$= \frac{a^3}{3} + 2a$$

$$= \frac{2a^4}{3}$$

$$dI_y = x^2 dA = x^2 (a dx)$$

$$I_y = a \int_{-a}^a x^2 dx$$

$$I_y = a \left[\frac{x^3}{3} \right]_{-a}^a$$

$$= a \left[\frac{a^3}{3} - \left(-\frac{a^3}{3} \right) \right]$$

$$= a \left[\frac{2a^3}{3} \right]$$

$$= \frac{2a^4}{3}$$

$$I_0 = r_0^2 \cdot A \quad 0,816a$$

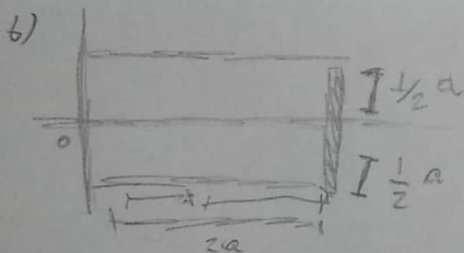
$$r_0^2 = \frac{I_0}{A}$$

$$= \frac{4 \frac{a^4}{3}}{2a^2} = \frac{2a^2}{3}$$

$$I_0 = I_x + I_y$$

$$= \frac{2a^4}{3} + \frac{2a^4}{3}$$

$$= \frac{4a^4}{3}$$



$$dx = 2 \left[\frac{1}{3} \left(\frac{a}{2} \right)^3 dx \right] = \frac{1}{12} a^3 dy$$

$$I_x = \int_0^{2a} \frac{1}{12} a^3 dy = \frac{a^3}{12} \Big|_0^{2a}$$

$$I_x = \frac{a^3}{12} (2a)$$

$$I = \frac{1}{6} a^4$$

$$dI_y = x^2 dA$$

$$= x^2 (a dy)$$

$$I_y = a \int_0^{2a} x^2 dx$$

$$I_y = a \left[\frac{x^3}{3} \right]_0^{2a}$$

$$I_y = a \left(\frac{(2a)^3}{3} \right)$$

$$I_y = a^4 \cdot \frac{8}{3}$$

$$I_0 = I_x + I_y$$

$$= \frac{1}{6} a^4 + \frac{8a^4}{3}$$

$$= \frac{a^4 + 16a^4}{6}$$

$$= \frac{17a^4}{6}$$

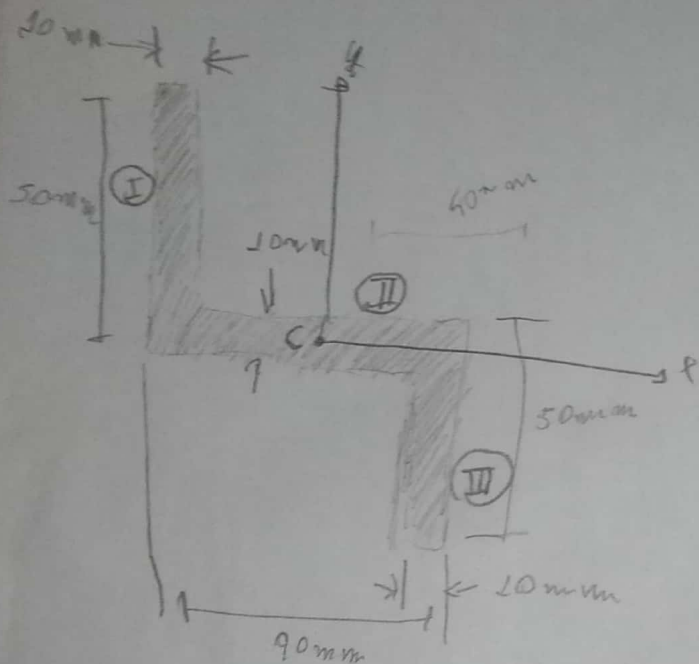
$$I_0 = r_0^2 \cdot A$$

$$r_0^2 = \frac{I_0}{A}$$

$$r_0^2 = \frac{17a^4/6}{2a^2}$$

$$r_0^2 = \frac{17}{12} a^2$$

7.25



$$A = \text{Area} : 2(10 \cdot 40) + (10 \cdot 90) = 1700 \text{ mm}^2$$

$$r^2 = \frac{I_x}{A} = \frac{614166,66}{1700} = 361,27$$

$$r = \sqrt{361,27}$$

$$r = 19,01 \text{ mm}$$

Ponto I

$$\begin{aligned} I_x &= \bar{I}_x + A d^2 \\ &= \frac{1}{12} (10)(40^3) + (10)(40)(25^2) \\ &= 303333,33 \text{ mm}^4 \end{aligned}$$

Ponto II

$$\begin{aligned} I_x &= \frac{1}{12} \cdot 90 \cdot 10^3 \\ &= 7500 \text{ mm}^4 \end{aligned}$$

Ponto III

$$I = \bar{I}$$

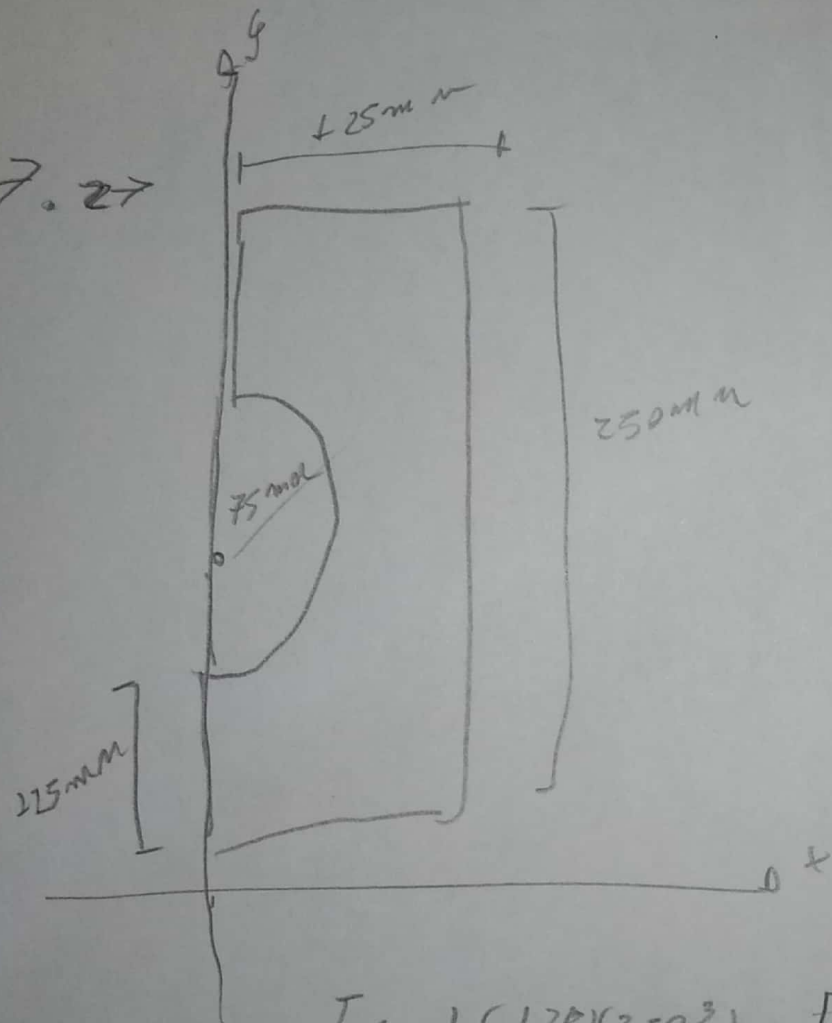
$$\text{Logo} : I_x = 303333,33 \text{ mm}^4$$

$$I_x = (303333,33 + 7500 + 303333,33)$$

$$I_x = 614166,66 \text{ mm}^4$$

entre
áreas

7.27

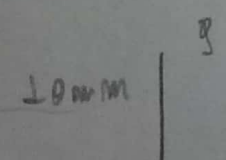


$$\begin{aligned} \text{Area} &= (125)(250) - \frac{\pi (75)^2}{2} \\ &= 22414 \text{ mm}^2 \end{aligned}$$

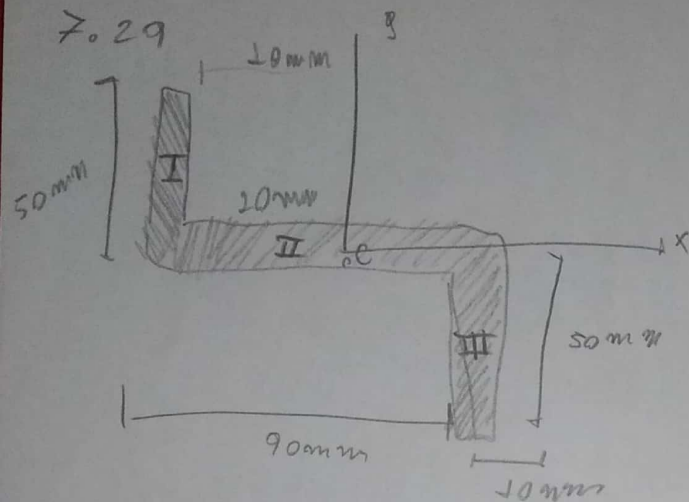
$$\begin{aligned} r_x^2 &= \frac{I_x}{\text{Area}} = \frac{500558151,8}{22414} = \\ r &= \sqrt{22332338,38} \\ r &= 149,44 \text{ mm} \end{aligned}$$

$$\begin{aligned} I_x &= \frac{1}{3} (125)(250^3) - \left[\frac{\pi}{8} (75^4) + \frac{\pi}{2} (75^2)(125^2) \right] \\ &= 500558151,4 \text{ mm}^4 \end{aligned}$$

7.29



$$\text{Area} = 2(10 \cdot 40) + 10 \cdot 90$$



$$\text{Area} = 2(10 \cdot 90) + 10 \cdot 90$$

$$= 1700 \text{ mm}^2$$

$$r_g^2 = \frac{2894100}{1700}$$

$$r_g = \sqrt{1702,41}$$

$$r_g = 41,26 \text{ mm}$$

Ponto I

$$I_g = I_g + Ad^2$$

$$= \frac{1}{12} (10)(90^3) + (10)(90)(45^2)$$

$$= 643300 \text{ mm}^4$$

Ponto III

I = III

Iguál a ponto I

$$I_g = 643300 \text{ mm}^4$$

Ponto II

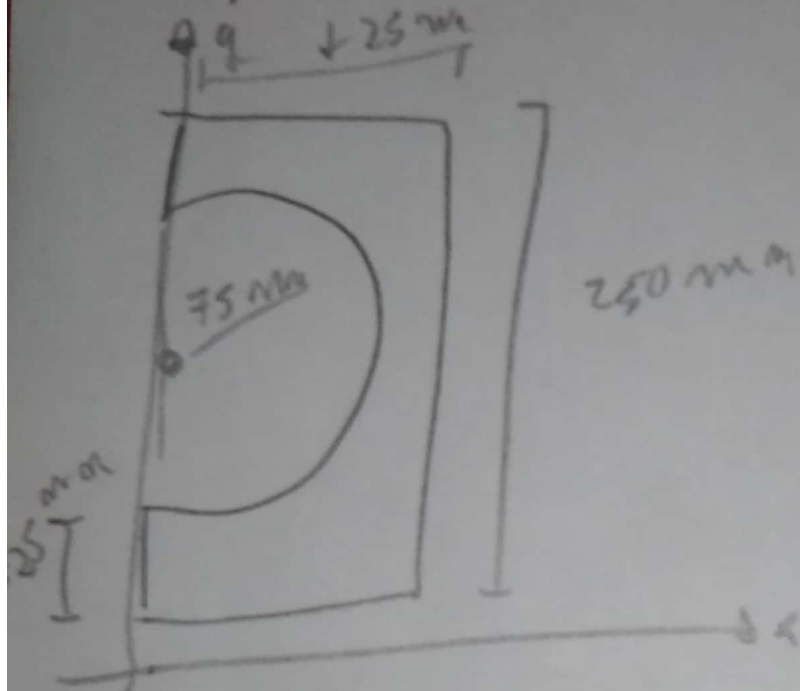
$$I_g = \frac{1}{12} (10)(90^3)$$

$$I_g = 607500 \text{ mm}^4$$

$$I_g = (643300 + 607500 + 643300)$$

$$= 1894100$$

7.31



$$A = (125)(250) - \frac{\pi (75)^2}{2}$$

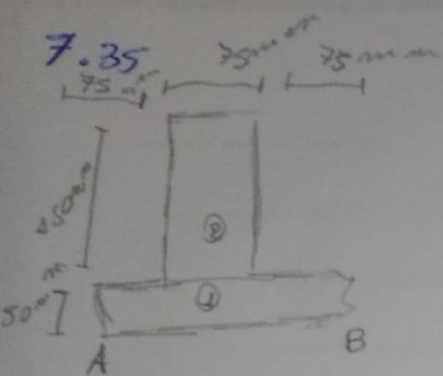
$$A = 22\,414 \text{ mm}^2$$

$$I_y = \frac{1}{3} (250) (125^3) - \frac{\pi}{8} (75^4)$$

$$I_y = 150\,300\,000 \text{ mm}^4$$

$$r_y^2 = \frac{I_y}{A} = \sqrt{\frac{150\,300\,000}{22\,414}} = 67$$

$$r = 81,89 \text{ mm}$$



	$A \text{ mm}^2$	$\bar{y} \text{ mm}$	\bar{y}_{mm}
②	11250	25	281250
①	11250	125	1406250
Σ	22500		1687500

$$\textcircled{1} - I_x = I_x + A d^2$$

$$= \frac{1}{12} \cdot 225 \cdot 50^3$$

$$= 46875 \text{ mm}^4$$

② $A \text{ mm}^2$

$$A = 150 \cdot 75$$

$$A = 11250 \text{ mm}^2$$

①

$$A = 50 \cdot 225$$

$$A = 11250$$

$$\textcircled{2} - I_x = \frac{1}{12} \cdot 75 \cdot 150^3$$

$$= 140625 \text{ mm}^4$$

③

$$I_x = I_{x1} + I_{x2}$$

$$= 46875 + 140625$$

$$= 187500 \text{ mm}^4$$

$$= 140,672 \text{ mm}$$

$$\textcircled{1} - I_y = \frac{1}{12} \cdot 50 \cdot 225^3$$

$$= 4760937,5 \text{ mm}^4$$

$$I_y = I_{y1} + I_{y2}$$

$$= 4760937,5 + 5273437,5$$

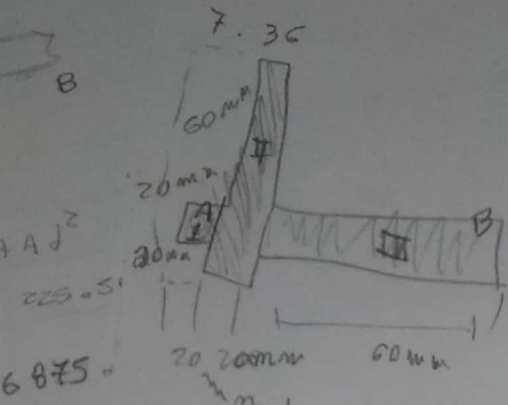
$$= 10034375 \text{ mm}^4$$

$$\textcircled{2} - I_y = \frac{1}{12} \cdot 150 \cdot 75^3$$

$$= 5273437,5 \text{ mm}^4$$

$$= 10034,4 \text{ mm}$$

mm
A mm
Y mm
X mm
= 81250



Areas

① $20 \cdot 20 = 400$

② $100 \cdot 20 = 2000$

③ $20 \cdot 60 = 1200$

	A (mm ²)	\bar{x} (mm)	\bar{y} (mm)	$\bar{x}A$	$\bar{y}A$
I	400	10	30	4000	12000
II	2000	30	50	60000	100000
III	1200	70	30	84000	36000
	3600			148000	148000

$\sum \bar{x} = \frac{148000}{3600} = 41,11$

$I_x = (62715 + 1224700 + 188145)$
 $= 2075600 \text{ mm}^4$

no segs as 0

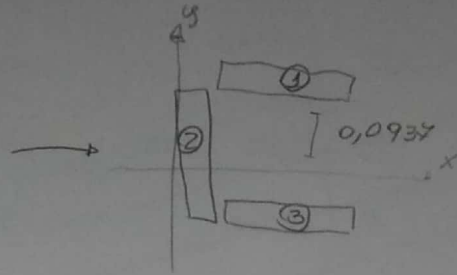
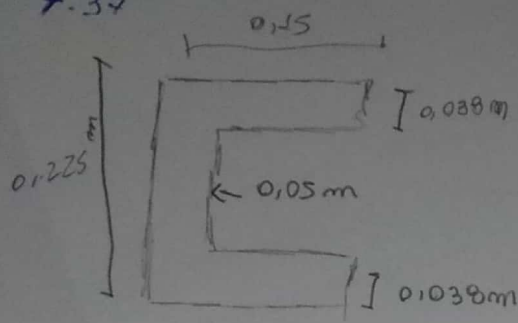
① $I_x = \bar{I}_x + Ad^2$
 $= \frac{1}{12} (20 \times 20^3) + 400 (41,11 - 30)^2$
 $= 62715 \text{ mm}^4$

② $I_x = \bar{I}_x + Ad^2$
 $= \frac{1}{12} (20 \times 100^3) + (2000) (50 - 41,11)^2$
 $= 1224700$

③ $I_x = \bar{I}_x + Ad^2$
 $= \frac{1}{12} (60 \times 20^3) + (1200) (41,11 - 30)^2$
 $= 188145$

$I_y = I_x = 2075600 \text{ mm}^4$

7.37



I

①

$$I_x = I_v + A d^2$$

$$= \frac{1}{12} \cdot (0,038)^3 \cdot 0,15 + (0,038 \cdot 0,15) (0,9375)^2$$

$$= 5 \cdot 10^{-3}$$

$$② \quad I_y = \frac{1}{3} \cdot (0,15)^3 \cdot 0,038$$

$$= 4,27 \cdot 10^{-5}$$

③

$$I_x = 5 \cdot 10^{-5} + 0,8 + 5 \cdot 10^{-5}$$

$$= 0,8$$

$$I_y = 4,27 \cdot 10^{-5} + 0,4 + 4,27 \cdot 10^{-5}$$

$$= 0,4$$

II

$$x \Sigma A = \Sigma x A \rightarrow x (0,225 + 0,3 + 0,225) = 3 \cdot 0,225 + 0,3 + 3 \cdot 0,225$$

$$\bar{x} = 2,25 \quad \bar{y} = 0$$

III

$$\bar{I}_x - I_x = 0,8$$

$$\bar{I}_y = I_y - A \bar{x}^2$$

$$= 0,4 - 0,75 \cdot (2,2)^2$$

$$= -3,23$$

