PathSmooth

November 7, 2024

This code uses QP and convex optimization to smooth the path found by the A* algorithm. I've completed the initial model definition and code implementation. However, there are three issues that need to be addressed and improved:

1. The A* algorithm seems to have some bugs and could be improved.
2. We could use real maps from https://movingai.com/benchmarks/grids.html to simulate more complex maps and paths.
3. It would be beneficial to add collision volumes to the moving points to simulate real robot movement in the scene.

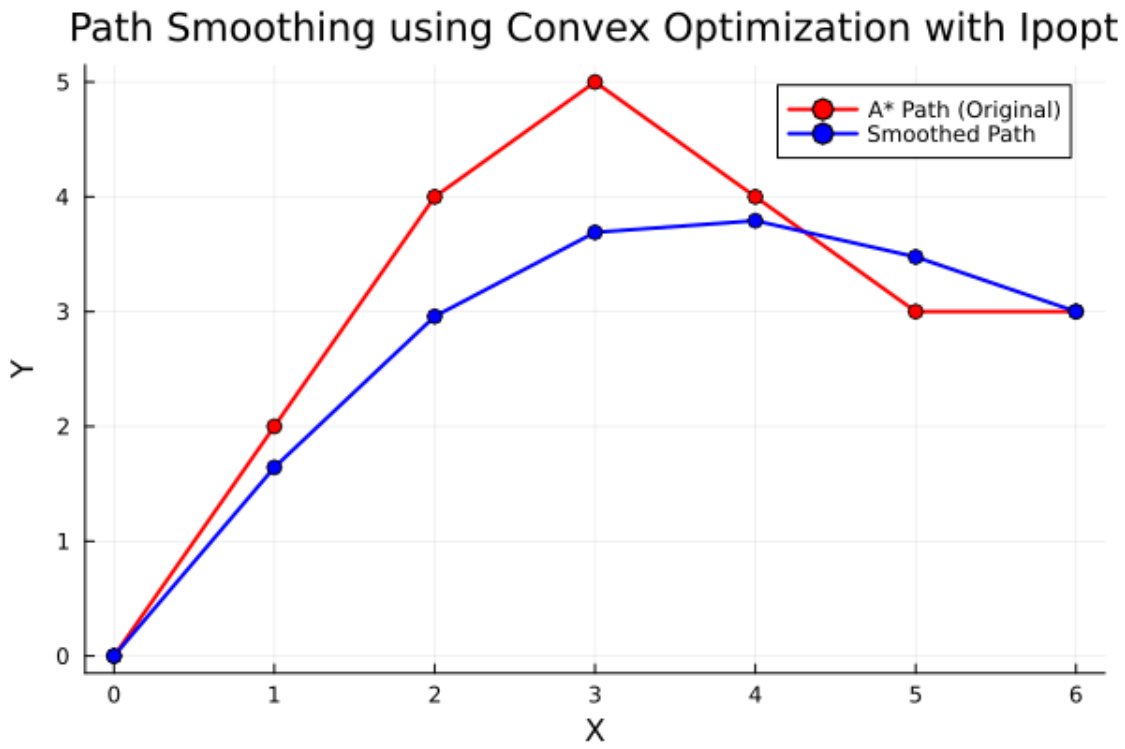
[1]:
```
using LinearAlgebra
using Ipopt
using JuMP
using Plots

function generate_astar_path()
    return [0.0 0.0; 1.0 2.0; 2.0 4.0; 3.0 5.0; 4.0 4.0; 5.0 3.0; 6.0 3.0]
end

function smooth_path(path, alpha=0.5, beta=0.1)

    N = size(path, 1)
    x0, y0 = path[:, 1], path[:, 2]

    model = Model(Ipopt.Optimizer)

    @variable(model, x[1:N])
    @variable(model, y[1:N])

    @constraint(model, x[1] == x0[1])
    @constraint(model, y[1] == y0[1])
    @constraint(model, x[N] == x0[N])
    @constraint(model, y[N] == y0[N])

    smoothness_loss = sum((x[i-1] - 2 * x[i] + x[i+1])^2 + (y[i-1] - 2 * y[i] +␣
↪y[i+1])^2 for i in 2:N-1)
    proximity_loss = sum((x[i] - x0[i])^2 + (y[i] - y0[i])^2 for i in 1:N)
```

```
    @objective(model, Min, alpha * smoothness_loss + beta * proximity_loss)

    optimize!(model)

    optimized_x = value.(x)
    optimized_y = value.(y)

    return hcat(optimized_x, optimized_y)
end

path = generate_astar_path()
smoothed_path = smooth_path(path)

plot(path[:, 1], path[:, 2], label="A* Path (Original)", lw=2, marker=:o,␣
 ↪color=:red)
plot!(smoothed_path[:, 1], smoothed_path[:, 2], label="Smoothed Path", lw=2,␣
 ↪marker=:o, color=:blue)
xlabel!("X")
ylabel!("Y")
title!("Path Smoothing using Convex Optimization with Ipopt")
```

```
******************************************************************************
This program contains Ipopt, a library for large-scale nonlinear optimization.
 Ipopt is released as open source code under the Eclipse Public License (EPL).
         For more information visit https://github.com/coin-or/Ipopt
******************************************************************************

This is Ipopt version 3.14.16, running with linear solver MUMPS 5.7.3.

Number of nonzeros in equality constraint Jacobian…:        4
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian…:       36

Total number of variables…:       14
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints…:        4
Total number of inequality constraints…:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  1.7000000e+01 6.00e+00 1.00e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  8.4226190e-01 1.97e-31 1.78e-15  -1.0 6.00e+00    -  1.00e+00 1.00e+00f  1
```

```
Number of Iterations…: 1

                                   (scaled)                 (unscaled)
Objective…:   8.4226190476193086e-01    8.4226190476193086e-01
Dual infeasibility…:   1.7763568394002505e-15    1.7763568394002505e-15
Constraint violation…:   1.9721522630525295e-31    1.9721522630525295e-31
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity…:   0.0000000000000000e+00    0.0000000000000000e+00
Overall NLP error…:   1.7763568394002505e-15    1.7763568394002505e-15


Number of objective function evaluations             = 2
Number of objective gradient evaluations             = 2
Number of equality constraint evaluations            = 2
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 1
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 1
Total seconds in IPOPT                               = 0.003

EXIT: Optimal Solution Found.
```

[1]:



[ ]:
```
using Random
using LinearAlgebra
using JuMP
using Ipopt
using Plots

function generate_environment(grid_size, obstacle_ratio=0.3)
    env = rand(grid_size, grid_size) .< obstacle_ratio
    return env
end

function print_environment(env, start, goal)
    grid_size = size(env, 1)
    println("Environment:")
    for i in 1:grid_size
        for j in 1:grid_size
            if env[i, j]
                print(" X ")
            elseif (i, j) == start
                print(" S ")
            elseif (i, j) == goal
                print(" G ")
            else
                print(" . ")
            end
        end
        println()
    end
end

using DataStructures

function manhattan_distance(a, b)
    return abs(a[1] - b[1]) + abs(a[2] - b[2])
end

function astar(env, start, goal)
    grid_size = size(env, 1)
    open_set = PriorityQueue{Tuple{Int,Int}, Float64}()
    open_set[start] = 0.0
    came_from = Dict{Tuple{Int,Int}, Tuple{Int,Int}}()
    g_score = Dict{Tuple{Int,Int}, Float64}(start => 0.0)
    f_score = Dict{Tuple{Int,Int}, Float64}(start => manhattan_distance(start,␣
 ↪goal))

    neighbors = [(1, 0), (-1, 0), (0, 1), (0, -1)]
```

```
    while !isempty(open_set)
        current = dequeue!(open_set)

        if current == goal
            path = []
            while haskey(came_from, current)
                push!(path, current)
                current = came_from[current]
            end
            push!(path, start)
            return reverse(path)
        end

        for (dx, dy) in neighbors
            neighbor = (current[1] + dx, current[2] + dy)

            if 1 <= neighbor[1] <= grid_size && 1 <= neighbor[2] <= grid_size␣
 ↪&& !env[neighbor...]
                tentative_g_score = g_score[current] + 1
                if tentative_g_score < get(g_score, neighbor, Inf)
                    came_from[neighbor] = current
                    g_score[neighbor] = tentative_g_score
                    f_score[neighbor] = tentative_g_score +␣
 ↪manhattan_distance(neighbor, goal)
                    open_set[neighbor] = f_score[neighbor]
                end
            end
        end
    end
    return []  # No path found
end


function smooth_segment(path_segment, alpha=0.5, beta=0.1)
    N = size(path_segment, 1)
    x0, y0 = path_segment[:, 1], path_segment[:, 2]

    model = Model(Ipopt.Optimizer)

    @variable(model, x[1:N])
    @variable(model, y[1:N])

    @constraint(model, x[1] == x0[1])
    @constraint(model, y[1] == y0[1])
    @constraint(model, x[N] == x0[N])
    @constraint(model, y[N] == y0[N])
```

```
    # Smoothness loss  use the second derivative of the path to penalize sharp␣
 ↪turns
    smoothness_loss = sum((x[i-1] - 2 * x[i] + x[i+1])^2 + (y[i-1] - 2 * y[i] +␣
 ↪y[i+1])^2 for i in 2:N-1)
    # Proximity loss: penalize the deviation of the smoothed path from the␣
 ↪original path
    proximity_loss = sum((x[i] - x0[i])^2 + (y[i] - y0[i])^2 for i in 1:N)

    @objective(model, Min, alpha * smoothness_loss + beta * proximity_loss)

    optimize!(model)

    optimized_x = value.(x)
    optimized_y = value.(y)

    return hcat(optimized_x, optimized_y)
end

function smooth_path_with_segments(rough_path, segment_size=5)
    smoothed_path = Matrix{Float64}(undef, 0, 2)

    for i in 1:segment_size:size(rough_path, 1) - segment_size + 1
        segment = rough_path[i:i+segment_size-1, :]
        smoothed_segment = smooth_segment(segment)

        if i == 1
            smoothed_path = vcat(smoothed_path, smoothed_segment)
        else
            smoothed_path = vcat(smoothed_path, smoothed_segment[2:end, :])
        end
    end

    if smoothed_path[end, :] != rough_path[end, :]
        smoothed_path = vcat(smoothed_path, reshape(rough_path[end, :], 1, 2))
    end

    return smoothed_path
end

grid_size = 20
obstacle_ratio = 0.3
env = generate_environment(grid_size, obstacle_ratio)
start = (1, 1)
goal = (grid_size, grid_size)

# print_environment(env, start, goal)
```

```
rough_path_coords = astar(env, start, goal)
if isempty(rough_path_coords)
    println("No path found!")
else

    rough_path = hcat([x[1] for x in rough_path_coords], [x[2] for x in␣
 ↪rough_path_coords])
    smoothed_path = smooth_path_with_segments(rough_path)


    obs_coords = collect(Tuple.(findall(env .== 1)))
    obs_x = [coord[2] for coord in obs_coords]
    obs_y = [coord[1] for coord in obs_coords]


    scatter(obs_x, obs_y, color=:black, label="Obstacles", markersize=5,␣
 ↪yflip=true)


    plot!(rough_path[:, 1], rough_path[:, 2], label="A* Rough Path", color=:
 ↪red, lw=2, marker=:o)
    plot!(smoothed_path[:, 1], smoothed_path[:, 2], label="Smoothed Path",␣
 ↪color=:blue, lw=2)


    scatter!([start[2]], [start[1]], label="Start", color=:green, markersize=8)
    scatter!([goal[2]], [goal[1]], label="Goal", color=:orange, markersize=8)


    xlabel!("X")
    ylabel!("Y")
    title!("A* Path and Smoothed Path with Obstacle Avoidance")
    plot!(legend=:outertopright)
end
```

```
This is Ipopt version 3.14.16, running with linear solver MUMPS 5.7.3.

Number of nonzeros in equality constraint Jacobian…:        4
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian…:       24

Total number of variables…:       10
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints…:        4
Total number of inequality constraints…:        0
```

```
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  6.0000000e+00 5.00e+00 8.00e-01  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  0.0000000e+00 0.00e+00 1.78e-15  -1.0 5.00e+00    -  1.00e+00 1.00e+00f  1

Number of Iterations…: 1

                                   (scaled)                 (unscaled)
Objective…:   0.0000000000000000e+00    0.0000000000000000e+00
Dual infeasibility…:   1.7763568394002505e-15    1.7763568394002505e-15
Constraint violation…:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity…:   0.0000000000000000e+00    0.0000000000000000e+00
Overall NLP error…:   1.7763568394002505e-15    1.7763568394002505e-15


Number of objective function evaluations             = 2
Number of objective gradient evaluations             = 2
Number of equality constraint evaluations            = 2
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 1
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 1
Total seconds in IPOPT                               = 0.001

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.16, running with linear solver MUMPS 5.7.3.

Number of nonzeros in equality constraint Jacobian…:        4
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian…:       24

Total number of variables…:       10
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints…:        4
Total number of inequality constraints…:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  2.9300000e+01 8.00e+00 1.60e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  1.9254658e-01 0.00e+00 7.11e-15  -1.0 8.00e+00    -  1.00e+00 1.00e+00f  1
```

```
Number of Iterations…: 1

                                   (scaled)                 (unscaled)
Objective…:   1.9254658385093393e-01    1.9254658385093393e-01
Dual infeasibility…:   7.1054273576010019e-15    7.1054273576010019e-15
Constraint violation…:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity…:   0.0000000000000000e+00    0.0000000000000000e+00
Overall NLP error…:   7.1054273576010019e-15    7.1054273576010019e-15


Number of objective function evaluations             = 2
Number of objective gradient evaluations             = 2
Number of equality constraint evaluations            = 2
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 1
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 1
Total seconds in IPOPT                               = 0.000

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.16, running with linear solver MUMPS 5.7.3.

Number of nonzeros in equality constraint Jacobian…:        4
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian…:       24

Total number of variables…:       10
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints…:        4
Total number of inequality constraints…:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  5.1600000e+01 9.00e+00 1.80e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  6.4182195e-02 0.00e+00 1.07e-14  -1.0 9.00e+00    -  1.00e+00 1.00e+00f  1

Number of Iterations…: 1

                                   (scaled)                 (unscaled)
Objective…:   6.4182194616989818e-02    6.4182194616989818e-02
Dual infeasibility…:   1.0658141036401503e-14    1.0658141036401503e-14
Constraint violation…:   0.0000000000000000e+00    0.0000000000000000e+00
```

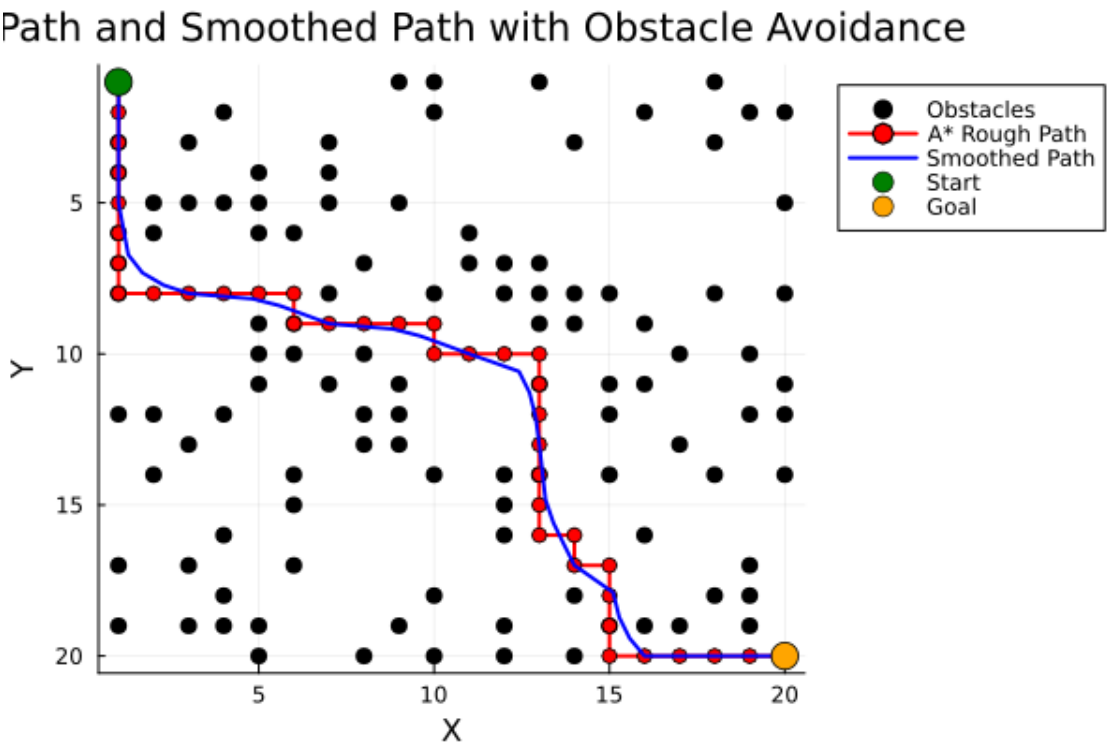
```
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity…:   0.0000000000000000e+00    0.0000000000000000e+00
Overall NLP error…:   1.0658141036401503e-14    1.0658141036401503e-14


Number of objective function evaluations             = 2
Number of objective gradient evaluations             = 2
Number of equality constraint evaluations            = 2
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 1
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 1
Total seconds in IPOPT                               = 0.000

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.16, running with linear solver MUMPS 5.7.3.

Number of nonzeros in equality constraint Jacobian…:        4
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian…:       24

Total number of variables…:       10
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints…:        4
Total number of inequality constraints…:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  9.0900000e+01 1.10e+01 2.00e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  6.4182195e-02 0.00e+00 1.07e-14  -1.0 1.10e+01    -  1.00e+00 1.00e+00f  1

Number of Iterations…: 1

                                   (scaled)                 (unscaled)
Objective…:   6.4182194617018240e-02    6.4182194617018240e-02
Dual infeasibility…:   1.0658141036401503e-14    1.0658141036401503e-14
Constraint violation…:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity…:   0.0000000000000000e+00    0.0000000000000000e+00
Overall NLP error…:   1.0658141036401503e-14    1.0658141036401503e-14


Number of objective function evaluations             = 2
Number of objective gradient evaluations             = 2
```

```
Number of equality constraint evaluations            = 2
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 1
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 1
Total seconds in IPOPT                               = 0.000

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.16, running with linear solver MUMPS 5.7.3.

Number of nonzeros in equality constraint Jacobian…:        4
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian…:       24

Total number of variables…:       10
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints…:        4
Total number of inequality constraints…:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  1.4540000e+02 1.30e+01 2.60e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  1.2008282e-01 0.00e+00 7.11e-15  -1.0 1.30e+01    -  1.00e+00 1.00e+00f  1

Number of Iterations…: 1

                                   (scaled)                 (unscaled)
Objective…:   1.2008281573515944e-01    1.2008281573515944e-01
Dual infeasibility…:   7.1054273576010019e-15    7.1054273576010019e-15
Constraint violation…:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity…:   0.0000000000000000e+00    0.0000000000000000e+00
Overall NLP error…:   7.1054273576010019e-15    7.1054273576010019e-15


Number of objective function evaluations             = 2
Number of objective gradient evaluations             = 2
Number of equality constraint evaluations            = 2
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 1
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 1
Total seconds in IPOPT                               = 0.000
```

```
EXIT: Optimal Solution Found.
This is Ipopt version 3.14.16, running with linear solver MUMPS 5.7.3.

Number of nonzeros in equality constraint Jacobian…:        4
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian…:       24

Total number of variables…:       10
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints…:        4
Total number of inequality constraints…:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  2.1210000e+02 1.70e+01 3.20e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  6.4182195e-02 0.00e+00 7.11e-15  -1.0 1.70e+01    -  1.00e+00 1.00e+00f  1

Number of Iterations…: 1

                                   (scaled)                 (unscaled)
Objective…:   6.4182194617160349e-02    6.4182194617160349e-02
Dual infeasibility…:   7.1054273576010019e-15    7.1054273576010019e-15
Constraint violation…:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity…:   0.0000000000000000e+00    0.0000000000000000e+00
Overall NLP error…:   7.1054273576010019e-15    7.1054273576010019e-15


Number of objective function evaluations             = 2
Number of objective gradient evaluations             = 2
Number of equality constraint evaluations            = 2
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 1
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 1
Total seconds in IPOPT                               = 0.000

EXIT: Optimal Solution Found.
This is Ipopt version 3.14.16, running with linear solver MUMPS 5.7.3.

Number of nonzeros in equality constraint Jacobian…:        4
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian…:       24
```

```
Total number of variables…:       10
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints…:        4
Total number of inequality constraints…:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  2.9300000e+02 2.00e+01 4.00e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  1.2008282e-01 0.00e+00 2.84e-14  -1.0 2.00e+01    -  1.00e+00 1.00e+00f  1

Number of Iterations…: 1

                                   (scaled)                 (unscaled)
Objective…:   1.2008281573525892e-01    1.2008281573525892e-01
Dual infeasibility…:   2.8421709430404007e-14    2.8421709430404007e-14
Constraint violation…:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity…:   0.0000000000000000e+00    0.0000000000000000e+00
Overall NLP error…:   2.8421709430404007e-14    2.8421709430404007e-14


Number of objective function evaluations             = 2
Number of objective gradient evaluations             = 2
Number of equality constraint evaluations            = 2
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 1
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 1
Total seconds in IPOPT                               = 0.000

EXIT: Optimal Solution Found.
```

[ ]:


Path and Smoothed Path with Obstacle Avoidance
Obstacles
A* Rough Path
Smoothed Path
Start
Goal
Y
X
5
10
15
20
5
10
15
20