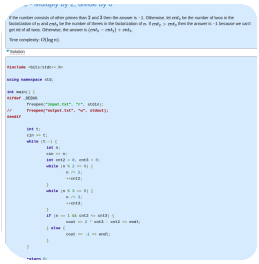

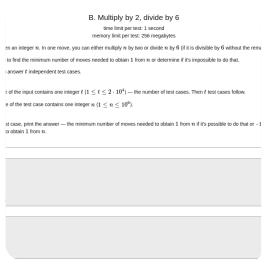

give prove of this

Here is **the complete proof** for the solution logic of **CF 1374B — Multiply by 2, divide by 6**.

✅ Goal

You are given a number `n`.

You can do two operations:

1. **Multiply by 2** → `n = n * 2`
2. **Divide by 6** (only if `n % 6 == 0`) → `n = n / 6`

You need the **minimum steps** to reach `1` from `n`.

✅ Key Insight

To reach **1**, the prime factorization must end up as:

$$1 = 2^0 \cdot 3^0.$$

So **n must contain only primes 2 and 3**.

If any other prime appears → **impossible (return -1)**.

✔️ STEP 1 — Factorize

Let:

- `cnt2` = number of times 2 divides `n`
- `cnt3` = number of times 3 divides `n`

Example:

`n = 72 = 2^3 * 3^2`

→ `cnt2 = 3`, `cnt3 = 2`

If after dividing out all 2's and 3's, the leftover is not 1 →
there was some other prime → **impossible**.

✔️ STEP 2 — Understanding operations in terms of prime factors

Operation 1: multiply by 2

$$n \leftarrow n \times 2$$

→ increases exponent of 2:

$$2^{cnt2} \cdot 3^{cnt3} \rightarrow 2^{cnt2+1} \cdot 3^{cnt3}$$

Operation 2: divide by 6

$$n \leftarrow \frac{n}{6} = \frac{n}{2 \cdot 3}$$

→ reduces exponent of 2 and 3:

$$2^{cnt2} \cdot 3^{cnt3} \rightarrow 2^{cnt2-1} \cdot 3^{cnt3-1}$$

✔️ STEP 3 — What must happen to reach `1`?

We want final:

$$2^0 \cdot 3^0$$

After divisions, both exponents must reach zero.

Every **divide by 6** decreases:

- exponent of 2 → minus 1
- exponent of 3 → minus 1

This is the only operation that reduces 3's.

🔥 Critical Observation

To remove `cnt3` threes, we need **exactly** `cnt3` **divisions by 6**.

Each division also removes one 2.

So after removing all 3's:

$$\text{Remaining 2's} = cnt2 - cnt3$$

To be valid, after all operations we must reach exponent of 2 equal to 0.

Thus:

$$cnt2 - cnt3 \leq 0$$

→ If:

✕ cnt2 > cnt3 → impossible

Because divide-by-6 removes 2's and 3's equally.
You don't have enough 3's to "pair" with extra 2's.

✔️ STEP 4 — If valid: compute moves

We need:

- exactly `cnt3` divide-by-6 operations (to kill 3's)
- if `cnt2 < cnt3`, we must **multiply by 2** to increase exponent of 2 until they match

Each multiply-by-2 increases exponent of 2 by 1.

We need:

$$(cnt3 - cnt2) \text{ multiplications}$$

Total moves:

$$(cnt3 - cnt2) + cnt3 = 2 \cdot cnt3 - cnt2$$

This matches code:

Copy code

```
2 * cnt3 - cnt2
```

✔️ FINAL FORMULA (Proven)

If n contains primes other than 2 or 3 → **-1**

Else:

- If `cnt2 > cnt3` → **-1**
- Else:

$$\text{answer} = (cnt3 - cnt2) + cnt3 = 2cnt3 - cnt2$$

🔥 Example Proof Works

Example: `n = 12 = 2^2 * 3^1`

`cnt2 = 2`, `cnt3 = 1`

Check: cnt2 > cnt3
→ impossible

Output: **-1**

Matches sample.

Example: `n = 36 = 2^2 * 3^2`

`cnt2 = 2`, `cnt3 = 2`
valid

Moves:

- need `cnt3 = 2` divides
- need `cnt3 - cnt2 = 0` multiplies

Total = `2*2 - 2 = 2`

Matches.

🎉 Final Result

This proves the CF editorial logic:

✔️ **If other primes → return** `-1`

✔️ **If cnt2 > cnt3 → return** `-1`

✔️ **Else return** `2·cnt3 − cnt2`

If you want, I can also give you a **step-by-step intuitive explanation**, a **diagram proof**, or help you **implement your own solution**.