

which is greater
 $\text{ceil}(a/x) + \text{ceil}(b/x)$
or
 $\text{ceil}(a+b/x)$
explain in cp/dsa style how its is used in coding
with examples and code
in large and bold text



Below is the **CP/DSA-style explanation**, written exactly the way you expect when solving problems.

📌 WHICH IS GREATER?

$\text{ceil}(a/x) + \text{ceil}(b/x)$ VS $\text{ceil}((a+b)/x)$

🔥 FINAL VERDICT (Remember This ALWAYS)

$\text{ceil}(a/x) + \text{ceil}(b/x) \geq \text{ceil}((a+b)/x)$

Equality holds only when

a % x == 0 AND b % x == 0

Otherwise:

$\text{ceil}(a/x) + \text{ceil}(b/x) > \text{ceil}((a+b)/x)$

⚔️ CP / DSA STYLE EXPLANATION

Why does this happen?

Because

bash

Copy code

$\text{ceil}(a/x) = a/x$ rounded UP
 $\text{ceil}(b/x) = b/x$ rounded UP
 $\text{ceil}((a+b)/x) = (a+b)/x$ rounded UP

When you round separately, you **over-count**.

This is exactly like **splitting work into two batches**, each batch creates its own rounding-up waste.



Analogy (CP Intuition)

You have a bucket size x .

- You want to store **a items** → may need 1 extra bucket if remainder exists.
- You want to store **b items** → may need 1 extra bucket if remainder exists.

But

- If you store **a+b** together, you share leftover space.

So using two buckets individually always requires **equal or more buckets** than combining.



Small Examples

Example 1: $a = 4, b = 5, x = 3$

$$\text{ceil}(4/3) = 2$$

$$\text{ceil}(5/3) = 2$$

$$\text{So LHS} = 2 + 2 = 4$$

Right side:

$$\text{ceil}((4+5)/3) = \text{ceil}(9/3) = 3$$

Result:

$$4 > 3$$

Example 2: $a = 6, b = 9, x = 3$

$$\text{ceil}(6/3) = 2$$

$$\text{ceil}(9/3) = 3$$

$$\text{LHS} = 2 + 3 = 5$$

$$\text{RHS} = \text{ceil}(15/3) = 5$$

Result:

Equal only because both divisible by x

Example 3: a = 7, b = 8, x = 4

$$\begin{aligned} \text{LHS} &= \text{ceil}(7/4) + \text{ceil}(8/4) \\ &= 2 + 2 = 4 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \text{ceil}((7+8)/4) \\ &= \text{ceil}(15/4) = 4 \end{aligned}$$

Result:

Sometimes equal even if only sum fits perfectly.

 **General Rule You MUST Remember in CP**

If $(a \% x \neq 0) + (b \% x \neq 0) \geq 1 \rightarrow \text{LHS} > \text{RHS}$

Meaning:

 If **any one** has remainder \rightarrow rounding twice \rightarrow more cost.

 If **BOTH** divisible \rightarrow equal.

 **Why This Is Used In CP?**

This appears in problems involving:

-  **Distributing work in buckets**
-  **Splitting people into groups**
-  **Counting number of operations**
-  **Minimizing number of segments**
-  **Greedy partition problems**

In these problems you compare:

- Doing operations **separately**

- Doing operations **combined**

This inequality decides if merging is beneficial.

C++ / Python Code

C++

cpp

 Copy code

```
#include <bits/stdc++.h>
using namespace std;

int main() {
    long long a, b, x;
    cin >> a >> b >> x;

    long long lhs = (a + x - 1) / x + (b + x - 1) / x; // ceil division
    long long rhs = (a + b + x - 1) / x;

    if (lhs > rhs) cout << "LHS > RHS\n";
    else if (lhs == rhs) cout << "LHS = RHS\n";
    else cout << "LHS < RHS\n"; // Never happens except weird negatives
}
```

Python

python

 Copy code

```
import math

a, b, x = map(int, input().split())

lhs = math.ceil(a/x) + math.ceil(b/x)
rhs = math.ceil((a+b)/x)

if lhs > rhs:
    print("LHS > RHS")
elif lhs == rhs:
    print("LHS = RHS")
```

```
else:  
    print("LHS < RHS")
```

📌 KEY TAKEAWAY (MEMORIZE THIS)

$$\text{ceil}(a/x) + \text{ceil}(b/x) \geq \text{ceil}((a+b)/x)$$

Equality when $a \% x == 0$ AND $b \% x == 0$

Otherwise strictly greater

If you want, I can show prove using number line, graph, or CF problem examples where this directly appears.

