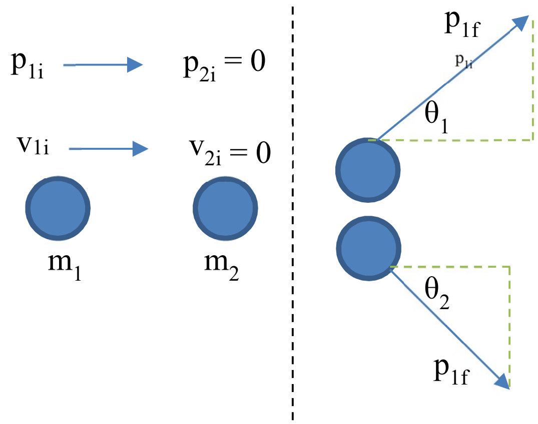
Daniel Abadjiev Section H Extra Credit

Extra Credit 1: Ball 1 and ball 2 collide in a 2 dimensional elastic collision. $v_{2_i} = 0$, and all of the momentum of the first ball is in the x direction



$$\Sigma p_{x_i} = \Sigma p_{x_f}$$
$$EQ1: p_{1i} = p_{1f}\cos\theta_1 + \rho_{2f}\cos\theta_2$$

$$EQ1^2: p_{1i}^2 = \left(p_{1f}\cos\theta_1 + \rho_{2f}\cos\theta_2\right)^2$$
$$p_{1i}^2 = p_{1f}\cos{\theta_1}^2 + 2p_{1f}\cos\theta_1\,\rho_{2f}\cos\theta_2 + \rho_{2f}\cos{\theta_2}^2$$

$$\Sigma p_{y_i} = \Sigma p_{y_f}$$
$$EQ2: 0 = p_{1f}\sin\theta_1 - \rho_{2f}\sin\theta_2$$

$$EQ2^2: 0^2 = \left(p_{1f}\sin\theta_1 - \rho_{2f}\sin\theta_2\right)^2$$
$$0 = p_{1f}\sin{\theta_1}^2 - 2p_{1f}\sin\theta_1\,\rho_{2f}\sin\theta_2 + \rho_{2f}\sin{\theta_2}^2$$

$$EQ1^2 + EQ2^2:\ p_{1i}^2 = (p_{1f}\cos\theta_1)^2 + 2p_{1f}\cos\theta_1\,\rho_{2f}\cos\theta_2 + (\rho_{2f}\cos\theta_2)^2 + (p_{1f}\sin\theta_1)^2 - 2p_{1f}\sin\theta_1\,\rho_{2f}\sin\theta_2 + (\rho_{2f}\sin\theta_2)^2$$
$$p_{1i}^2 = (p_{1f}\cos\theta_1)^2 + (p_{1f}\sin\theta_1)^2 + 2p_{1f}\cos\theta_1\,\rho_{2f}\cos\theta_2 - 2p_{1f}\sin\theta_1\,\rho_{2f}\sin\theta_2 + (\rho_{2f}\cos\theta_2)^2 + (\rho_{2f}\sin\theta_2)^2$$
$$p_{1i}^2 = p_{1f}^2({\cos\theta_1}^2 + {\sin\theta_1}^2) + 2p_{1f}\rho_{2f}(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + \rho_{2f}^2({\cos\theta_2}^2 + {\sin\theta_2}^2)$$
$$p_{1i}^2 = p_{1f}^2(1) + 2p_{1f}\rho_{2f}(\cos(\theta_1+\theta_2)) + \rho_{2f}^2(1)$$
$$EQ3: p_{1i}^2 = p_{1f}^2 + 2p_{1f}\rho_{2f}\cos(\theta_1+\theta_2) + \rho_{2f}^2$$
$$\left(m_1 v_{1_i}\right)^2 = \left(m_1 v_{1_f}\right)^2 + 2m_1 v_{1_f} m_2 v_{2_f}\cos(\theta_1+\theta_2) + \left(m_2 v_{2_f}\right)^2$$
$$m_1^2 v_{1_i}^2 = m_1^2 v_{1_f}^2 + 2m_1 m_2 v_{1_f} v_{2_f}\cos(\theta_1+\theta_2) + m_2^2 v_{2_f}^2$$

$$\Sigma KE_i = \Sigma KE_f$$
$$\frac{1}{2}m_1 v_{1_i}^2 = \frac{1}{2}m_1 v_{1_f}^2 + \frac{1}{2}m_2 v_{2_f}^2$$
$$m_1^2 v_{1_i}^2 = m_1^2 v_{1_f}^2 + m_1 m_2 v_{2_f}^2$$

Substitution:

$$m_1^2 v_{1_f}^2 + 2m_1 m_2 v_{1_f} v_{2_f}\cos(\theta_1+\theta_2) + m_2^2 v_{2_f}^2 = m_1^2 v_{1_f}^2 + m_1 m_2 v_{2_f}^2$$
$$2m_1 m_2 v_{1_f} v_{2_f}\cos(\theta_1+\theta_2) = m_1 m_2 v_{2_f}^2 - m_2^2 v_{2_f}^2$$
$$\cos(\theta_1+\theta_2) = \frac{m_2 v_{2_f}^2 (m_1 - m_2)}{2m_1 m_2 v_{1_f} v_{2_f}}$$
$$\cos(\theta_1+\theta_2) = \frac{v_{2_f}(m_1 - m_2)}{2m_1 v_{1_f}}$$

If we assume m_1=m_2 then m_1-m_2=0 so

$$\cos(\theta_1 + \theta_2) = \frac{v_{2_f}(m_1 - m_2)}{2m_1 v_{1_f}}$$

$$\cos(\theta_1 + \theta_2) = \frac{v_{2_f}(0)}{2m_1 v_{1_f}}$$

$$\cos(\theta_1 + \theta_2) = 0$$

$$\cos(\theta_1 + \theta_2) = \frac{v_{2_f}(0)}{2m_1 v_{1_f}}$$

$$|\theta_1| + |\theta_2| = 90°$$

Q.E.D.