Course 2 assignment Denys Sikorskyi

Exercise 5.1

Using notation of the exercise: n - initial image dimension, k - image dimension at the lower scale, u_0 - initial image, u_1 - image at the lower scale. Next, we have operators: DCT_n^{iso} and DCT_k^{iso} are the isometric DCTs at scales n and k respectively, ZP_k is the zero-padding operator extracting the first $k \times k$ subimage from an $n \times n$ image. Let's express the image u_1 using mentioned operators. At first, we apply DST_n^{iso} to the initial image $DCT_n^{iso}(u_0)$. Next use $ZP_k \Rightarrow ZP_k(DCT_n^{iso}(u_0))$. ~~The~~ In the end, we scalge by $\sqrt{\frac{Numpix(layer)}{numpix(input)}}$ and use the inverse DCT on the scaled image, so $\Rightarrow$

$$u_1 = IDCT_k^{iso}\left(ZP_k\left(DCT_n^{iso}(u_0)\sqrt{\frac{Numpix(layer)}{Numpix(input)}}\right)\right)$$

We express $u_1 = u_1^* + n_1$, u_1^* - noiseless $\sqrt{\frac{k \times k}{n \times n}}$ part of u_1, n_1 - white noise. ~~For~~ There is a residual of white noise, which has equal components of equal variance, so we can use latter expression.

$$var(u_1) = \cancel{var(\overset{u_1^* \text{ - noiseless and deterministic}}{u_1^* + n_1}) = var(n_1)} = var\left(IDCT_k^{iso}\left(ZP_k\left(DCT_n^{iso}(u_0)\right)\right) \times \sqrt{\frac{k \times k}{n \times n}}\right) = \frac{k \times k}{n \times n}\, var\left(IDCT_k^{iso}\left(ZP_k\left(DCT_n^{iso}(u_0)\right)\right)\right)$$

$\Leftarrow$ this expression is constant due to white noise

So, $\sigma(n_1) = \sqrt{\frac{k \times k}{n \times n}} = \sqrt{\frac{numpix(layer)}{numpix(input)}}$ $(*)$. $\Rightarrow$ noise standard deviation at the lower scale was multiplied by $(*)$.