Course 4 assignment Denys Sikorskyi

Exercise 9.1

Following energy to minimize.

$E(P|\tilde{P}) = \frac{\|P-\tilde{P}\|^2}{2\sigma^2} - \log(P(P))$, where P obeys a GMM, $\tilde{P}$ is a noisy observation of P with Gaussian isotropic noise of variance σ^2. $(\tilde{P} = P + N)$.

$P \sim GMM \Rightarrow P(\tilde{P}|P) = \frac{1}{(2\pi\sigma^2)^{\frac{k^2}{2}}} e^{-\frac{\|P-\tilde{P}\|^2}{2\sigma^2}}$, k - side of the image.

$\min\limits_P E(P|\tilde{P}) = \min\left(\frac{\|P-\tilde{P}\|^2}{2\sigma^2} - \log(P(P))\right) = \max\limits_P\left(\log(P(P)) - \frac{\|P-\tilde{P}\|^2}{2\sigma^2}\right) =$ [illegible] $=$

$= \max\limits_P\left(\log(P(P)) + \log\left(e^{-\frac{\|P-\tilde{P}\|^2}{2\sigma^2}}\right)\right) = \max\limits_P\left(\log\left(P(P)\, e^{-\frac{\|P-\tilde{P}\|^2}{2\sigma^2}}\right)\right) =$

$=$ in order maximize the expression we can omit log because log is increasing function $= \max\limits_P\left(P(P)\, e^{-\frac{\|P-\tilde{P}\|^2}{2\sigma^2}}\right) =$

$\overset{P\sim GMM}{=} \max\limits_P\left(P(P)\, P(\tilde{P}|P)\,(2\pi\sigma^2)^{\frac{k^2}{2}}\right) = \max\limits_P\left(P(\tilde{P})\,(2\pi\sigma^2)^{\frac{k^2}{2}}\, P(P|\tilde{P})\right)$

$=$ ~~[illegible]~~ first part of the expression is constant in terms of P, so $\overset{\text{to max}}{=} \max\limits_P P(P|\tilde{P})$. Therefore, we have verified that minimizing the above energy amounts to compute a maximum a posteriori estimate of P given $\tilde{P}$ where P obeys GMM and $\tilde{P}$ is a noisy observation of P with Gaussian isotropic noise of variance σ^2.

Exercise 9.2

We have $EPLL_p(\mathcal{U}) = \sum_{P \in \mathcal{P}} \log(P(P\mathcal{U}))$ and likelihood d'image $P(\mathcal{U}) = P(\bigcap_{P \in \mathcal{P}} \{P\mathcal{U}\})$, suppose we have independent patches (*) $P(\mathcal{U}) = \prod_{P \in \mathcal{P}} P(P\mathcal{U})$, if we use log on both parts of expression we get ~~log~~ $\log(P(\mathcal{U})) = \log(\prod_{P \in \mathcal{P}} P(P\mathcal{U})) = \sum_{P \in \mathcal{P}} \log P(P\mathcal{U}) = EPLL_p(\mathcal{U})$. But our patches are not independent, so the (*) transition is not correct, so we can't interpret EPLL as log-likelihood of the $\mathcal{U}$.