Θέμα 1ο :

①

$y = \sum_i w_i x_i + b$, $J(y,z) = (y-z)^2$.

1. $\tilde{x}_i = x_i + \varepsilon_i$, $\varepsilon_i \sim \mathcal{N}(0, \sigma_i^2)$, $\sigma_1^2 = \sigma_2^2 = \dots = \sigma_i^2$

Έχουμε $\tilde{y} = \sum_i w_i \tilde{x}_i + b$

$= \sum_i w_i (x_i + \varepsilon_i) + b$

$= \sum_i w_i x_i + \sum_i w_i \varepsilon_i + b$

$= \sum_i w_i x_i + b + \sum_i w_i \varepsilon_i$

$= y + \sum_i w_i \varepsilon_i$.

2. $\tilde{J} = E[J(\tilde{y}, z)] = E[(\tilde{y} - z)^2]$

$= E[\tilde{y}^2 - 2\tilde{y}z + z^2]$

$= E[\tilde{y}^2] - 2z E[\tilde{y}] + z^2$

όμως $\tilde{y}^2 = (y + \sum_i w_i \varepsilon_i)^2 = y^2 + 2y \sum_i w_i \varepsilon_i + (\sum_i w_i \varepsilon_i)^2$

$= y^2 + 2y \sum w_i \varepsilon_i + \sum_i w_i^2 \cdot \varepsilon_i^2 + 2 \sum_{i \neq j} w_i w_j \varepsilon_i \varepsilon_j$

άρα $E[\tilde{y}^2] = E[y^2 + 2\sum w_i \varepsilon_i + \sum_i w_i^2 \varepsilon_i^2 + 2 \sum_{i \neq j} w_i w_j \varepsilon_i \varepsilon_j]$

$= y^2 + 2\sum_i w_i E[\varepsilon_i] + \sum_i w_i^2 E[\varepsilon_i^2] + 2 \sum_{i \neq j} w_i w_j E[\varepsilon_i \varepsilon_j]$

όμως $E[\varepsilon_i] = 0$ και $E[\varepsilon_i \varepsilon_j] = E[\varepsilon_i] \cdot E[\varepsilon_j] = 0 \ \forall i \neq j$.

Άρα $E[\tilde{y}^2] = y^2 + \sum_i w_i^2 \sigma^2$ ②

$= y^2 + \sigma^2 \cdot \sum_i w_i^2$.

Οπότε $\tilde{J} = E[J(\tilde{y}, z)] = y^2 + \sigma^2 \cdot \sum_i w_i^2 - 2zy + z^2$

$= \sigma^2 \sum_i w_i^2 + (y-z)^2$

$= \sigma^2 \cdot \sum_i w_i^2 + J(y, z)$.

3. Από την έκφραση

$\tilde{J} = \sigma^2 \cdot \sum_i w_i^2 + J(y, z)$ είναι ο $\lambda = \sigma^2$