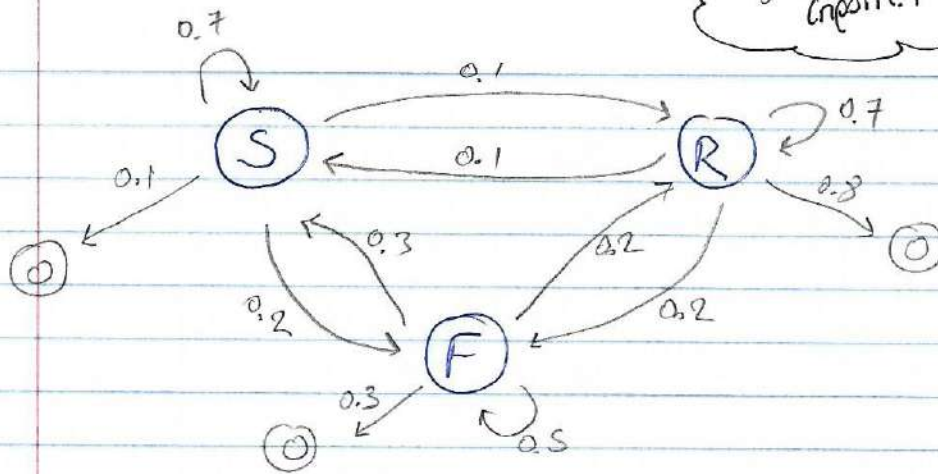


HMM
2021-2022
Exercice 1



$$b_S(0) = 0.1 \quad b_R(0) = 0.8 \quad b_F(0) = 0.3$$

$$b_S(10) = 0.9 \quad b_R(10) = 0.2 \quad b_F(10) = 0.7$$

$$(a) p(q_1 = S, q_2 = R | q_0 = S) = \frac{\pi_S \cdot \alpha_{SS} \alpha_{SR}}{\pi_S} = 0.7 \cdot 0.1 = 0.07$$

$$(b) p(q_2 = R | q_0 = F) = \sum_{q_1} p(q_2 = R, q_1 | q_0 = F) =$$

$$= \frac{\pi_F \cdot (\alpha_{FR} \alpha_{RR} + \alpha_{FS} \alpha_{SR} + \alpha_{FF} \alpha_{FR})}{\pi_F} =$$

$$= 0.2 \cdot 0.7 + 0.3 \cdot 0.1 + 0.5 \cdot 0.2 = 0.14 + 0.03 + 0.1 = 0.27$$

$$(c) \pi_0 = 0.5 \quad (p(0) = 0.5)$$

$$p(q_1 = R | q_0 = S, o_1 = 0) =$$

$$= \frac{p(o_1 = 0 | q_0 = S, q_1 = R) \cdot p(q_1 = R | q_0 = S)}{p(o_1 = 0 | q_0 = S)} =$$

$$= \frac{p(o_1 = 0 | q_1 = R) \cdot p(q_1 = R | q_0 = S)}{p(o_1 = 0 | q_0 = S)} = \frac{b_R(0) \cdot \alpha_{SR}}{p(o_1 = 0 | q_0 = S)}$$

Οπως $p(O_1=0|q_0=S) =$

$$\sum_{q_1} p(O_1=0, q_1|q_0=S) = \sum_{q_1} p(O_1=0|q_1, q_0=S) \cdot p(q_1|q_0=S)$$

$$= \sum_{q_1} p(O_1=0|q_1) \cdot p(q_1|q_0=S) =$$

$$b_S(0) \cdot \kappa_{SS} + b_M(0) \cdot \kappa_{SM} + b_F(0) \cdot \kappa_{SF} =$$

$$= 0.1 \cdot 0.7 + 0.8 \cdot 0.1 + 0.3 \cdot 0.2 = 0.07 + 0.08 + 0.06 = 0.21$$

Αρα $Prob = \frac{0.8 \cdot 0.1}{0.21} \approx 0.381$

(δ) Δεδομένα: $q_1=S, O_2=0, O_3=NO$

$$p(q_3=F|q_1=S, O_2=0, O_3=NO) =$$

$$\frac{p(O_2=0, O_3=NO|q_3=F, q_1=S) \cdot p(q_3=F|q_1=S)}{p(O_2=0, O_3=NO|q_1=S)}$$

• $p(O_2=0, O_3=NO|q_3=F, q_1=S) =$

$$\sum_{q_2} p(O_2=0, O_3=NO|q_3=F, q_2, q_1=S) \cdot p(q_2|q_3=F, q_1=S)$$

$$= \sum_{q_2} p(O_2=0|q_3=F, q_2, q_1=S) p(O_3=NO|q_3=F, q_2, q_1=S) \cdot$$

$$\frac{p(q_3=F|q_2, q_1=S) \cdot p(q_2|q_1=S)}{p(q_3=F|q_1=S)}$$

$$= \sum_{q_2} p(O_2=0|q_2) p(O_3=NO|q_3=F) \cdot p(q_3=F|q_2) \cdot p(q_2|q_1=S)$$

$$= \sum_{q_2} \overbrace{b_{q_2}(0) \cdot b_F(NO) \cdot \alpha_{q_2 F} \cdot \alpha_{S q_2}}^{(1)} / p(q_3 = F | q_1 = S)$$

$$\bullet p(O_2 = 0, O_3 = NO | q_1 = S) =$$

$$\sum_{q_2, q_3} p(O_2 = 0, O_3 = NO | q_3, q_2, q_1 = S) \cdot p(q_3 | q_2, q_1 = S) \cdot p(q_2 | q_1 = S)$$

$$= \sum_{q_2, q_3} p(O_2 = 0 | q_2) \cdot p(O_3 = NO | q_3) \cdot p(q_3 | q_2) \cdot p(q_2 | q_1 = S)$$

$$= \sum_{q_2, q_3} b_{q_2}(0) b_{q_3}(NO) \cdot \alpha_{q_2 q_3} \cdot \alpha_{S q_2} \quad (2)$$

$$(1) = b_F(0) b_F(NO) \alpha_{FF} \alpha_{SF} + b_S(0) b_F(NO) \alpha_{SF} \alpha_{SS}$$

$$+ b_R(0) b_F(NO) \cdot \alpha_{RF} \cdot \alpha_{SR} =$$

$$= 0.7 (0.3 \cdot 0.5 \cdot 0.2 + 0.1 \cdot 0.2 \cdot 0.7 + 0.8 \cdot 0.2 \cdot 0.1)$$

$$= 0.7 \cdot (0.030 + 0.014 + 0.016) = 0.7 \cdot 0.06 = 0.042$$

(2):	q_2	q_3	
	F	F	$0.3 \cdot 0.7 \cdot 0.5 \cdot 0.2 = 210 \cdot 10^{-4}$
	F	R	$0.3 \cdot 0.2 \cdot 0.2 \cdot 0.2 = 24 \cdot 10^{-4}$
	F	S	$0.3 \cdot 0.9 \cdot 0.3 \cdot 0.2 = 192 \cdot 10^{-4}$
	R	F	$0.8 \cdot 0.7 \cdot 0.2 \cdot 0.1 = 112 \cdot 10^{-4}$
	R	R	$0.8 \cdot 0.2 \cdot 0.7 \cdot 0.1 = 112 \cdot 10^{-4}$
	R	S	$0.8 \cdot 0.9 \cdot 0.1 \cdot 0.1 = 56 \cdot 10^{-4}$
	S	F	$0.1 \cdot 0.7 \cdot 0.2 \cdot 0.7 = 98 \cdot 10^{-4}$
	S	R	$0.1 \cdot 0.2 \cdot 0.1 \cdot 0.7 = 14 \cdot 10^{-4}$
	S	S	$0.1 \cdot 0.9 \cdot 0.7 \cdot 0.7 = 441 \cdot 10^{-4}$

(+)

$$(2) = 1259 \cdot 10^{-4}, \text{ onore } \text{prob} = \frac{0.042}{0.1259} \approx 0.3336$$