2019 / Ζητημα Γ

exponential family: $f_x(x,\theta) = \exp\left[\eta(\theta)\cdot T(x) - A(\theta) + B(x)\right]$ (y, μ του Poisson)

(Γ_1) $f(y) = \dfrac{e^{-\mu}\cdot\mu^y}{y!}$ $\quad y = 0,1,2,\dots$

(i) $f(y) = \dfrac{e^{-\mu}\cdot\mu^y}{y!} = \dfrac{1}{y!}\exp\left(\ln(\mu)\cdot y - \mu\right) = \exp\left(\ln\tfrac{1}{y!}\right)\cdot\exp\left(\ln(\mu)\cdot y - \mu\right) =$

$= \exp\left(-\ln y!\right)\cdot\exp\left(\ln(\mu)\cdot y - \mu\right) = \exp\left(\underbrace{y}_{T(x)}\cdot\underbrace{\ln\mu}_{\eta(\theta)} - \underbrace{\ln(y!)}_{B(x)} - \underbrace{\mu}_{A(\theta)}\right).$

άρα ανήκει στην exponential family.

Το link function είναι το $\ln\mu$ (παράμετρος του y):

$g(x) = \ln x \rightarrow g(\mu x) = \ln\mu x$

Μοντέλο Poisson: $g(E(y)) = g(\mu x) = \beta' X$. γραμμικό μεν.

$\ln\mu x$

(ii) $\ln\mu x = \beta' X \rightarrow \mu x = e^{\beta' x} = e^{\beta_0+\beta_1 x_1+\beta_2 x_2+\beta_3 x_3}$.