

Acad. Year: 2020-2021

## **Course: Data-driven models in engineering applications**

## 2<sup>nd</sup> Assignment

Consider a rectangular plate of dimensions 1  $m \times 1 m$ . There is a candle located at position (0.55,0.45) below the plate which heats it. The steady state equation that describes the temperature field T(x,y) along the plate at thermal equilibrium is:

$$-\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = f(x, y)$$

where f(x, y) is the external heat source due the candle, given by:

 $f(x,y) = 100 \cdot exp\left(-\frac{(x-0.55)^2 + (y-0.45)^2}{r}\right)$  with  $r \sim N(0.05, 0.005)$  being a normal random variable.

At all edges of the plate we assign T = 0 (Dirichlet Boundary conditions).

1) Discretize the plate using a  $40 \times 40$  grid, with  $h = \Delta x = \Delta y = 1/40$ . Using the following second-order central difference approximation to  $2^{\text{nd}}$  derivatives

• 
$$\frac{\partial^2 T}{\partial x^2} \approx \frac{T(x+h,y)-2T(x,y)+T(x-h,y)}{h^2}$$

• 
$$\frac{\partial^2 T}{\partial y^2} \approx \frac{T(x,y+h)-2T(x,y)+T(x,y-h)}{h^2}$$

you can get the finite difference scheme:

$$-T(x+h,y) - T(x,y+h) + 4T(x,y) - T(x-h,y) - T(x,y-h) \approx h^2 f(x,y)$$

Derive the linear system of equations Kx = b for the problem. (Comment: Instead of this scheme, you can use quadrilateral finite elements to derive a linear system for this problem, or any other scheme of your choice.)

- 2) Perform Monte Carlo simulation to obtain the probability density function of the temperature at the midpoint of the plate (0.5, 0.5).
- 3) Perform a small number of deterministic simulations for different values of r, and use these solutions as your initial data set. Implement the PCA/POD method to reduce the dimensionality of the linear system that describes the problem and perform the Monte Carlo simulation on the reduced system. Compare the pdf of T at point (0.5, 0.5) to the one from the previous question.