



Course: Data-driven models in engineering applications

1st Assignment

A) Consider the following stochastic field:

$$E(x) = 10(1 + f(x))$$

where $f(x)$ is a zero-mean stationary Gaussian field with unit variance and $x \in [0, 5](m)$. The autocorrelation function for f is $R_f(\tau) = \exp(-|\tau|/2)$.

1. Use the Karhunen-Loeve series expansion method to generate $N=5000$ realizations of the field $E(x)$.
2. Justify the number of terms you retained in the KL-expansion.
3. Calculate the ensemble average and the ensemble variance from these realizations. To which values would they converge as we increase the number N of realizations?

B) Consider the zero-mean Gaussian process $X(t)$, $t \in [0, 10]$ (sec), which has the following one-sided power-spectrum

$$G(\omega) = \begin{cases} \omega - 1, & 1 \leq \omega \leq 2 \\ 3 - \omega, & 2 < \omega \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

1. Use the Spectral Representation method to generate $N=5000$ time-histories (realizations) of the process $X(t)$.
2. Calculate the ensemble average and the ensemble variance from these time-histories. To which values would they converge as we increased the number N of realizations.
3. Calculate the temporal average and temporal variance from a single realization. What do you observe?