National Technical University of Athens
School of Civil Engineering

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Course: Data-driven models in engineering applications

2nd Assignment

Consider a rectangular plate of dimensions $1\ m\ \times 1\ m$. There is a candle located at position (0.55,0.45) below the plate which heats it. The steady state equation that describes the temperature field $T(x, y)$ along the plate at thermal equilibrium is:

$$-\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = f(x, y)$$

where $f(x, y)$ is the external heat source due the candle, given by:

$f(x, y) = 100\ \cdot exp\left(-\frac{(x-0.55)^2+(y-0.45)^2}{r}\right)$ with $r \sim N(0.05, 0.005)$ being a normal random variable.

At all edges of the plate we assign $T = 0$ (Dirichlet Boundary conditions).

1) Discretize the plate using a 40×40 grid, with $h = \Delta x = \Delta y = 1/40$. Using the following second-order central difference approximation to 2nd derivatives

   - $\frac{\partial^2 T}{\partial x^2} \approx \frac{T(x+h,y)-2T(x,y)+T(x-h,y)}{h^2}$
   - $\frac{\partial^2 T}{\partial y^2} \approx \frac{T(x,y+h)-2T(x,y)+T(x,y-h)}{h^2}$

   you can get the finite difference scheme:

   $$-T(x+h, y) - T(x, y+h) + 4T(x, y) - T(x-h, y) - T(x, y-h) \approx h^2 f(x, y)$$

Derive the linear system of equations $\boldsymbol{Kx} = \boldsymbol{b}$ for the problem. (<u>Comment</u>: Instead of this scheme, you can use quadrilateral finite elements to derive a linear system for this problem, or any other scheme of your choice.)

2) Perform Monte Carlo simulation to obtain the probability density function of the temperature at the midpoint of the plate (0.5, 0.5).
3) Perform a small number of deterministic simulations for different values of r, and use these solutions as your initial data set. Implement the PCA/POD method to reduce the dimensionality of the linear system that describes the problem and perform the Monte Carlo simulation on the reduced system. Compare the pdf of T at point $(0.5, 0.5)$ to the one from the previous question.