QUIZ: TA session

FINAL EXAM: Dec 11th $12^{30} - 3^{pm}$

10 problems

$\left.\begin{array}{l} 2.2 \\ 2.6 \end{array}\right\}$ Limits

3.2 – 3.7 } Differential Calculus

implicit diff.

3.10 Related rates

4.1 – 4.6 Min/Max

applied optimization

$\underbrace{4.8,\ 5.4,\ 5.5,\ 5.6}_{\text{Integrals}}$, 8.2

Int. by parts

TRIG INTEGRALS

$$\boxed{\int (\sin x)^m (\cos x)^n \, dx}$$

m, n one of them odd.

- split off one

sin x	cos x

- substitute $u = \cos x$ | $u = \sin x$

- use $\sin x = \sqrt{1-\cos^2 x}$

$\cos x = \sqrt{1-\sin^2 x}$

<u>Ex:</u> $\int (\sin x)^4 (\cos x)^3 \, dx$

$$= \int (\sin x)^4 (\cos x)^2 (\cos x) \, dx$$

$$= \int (\sin x)^4 (\cos x)^2 \cancel{(\cos x)} \frac{du}{\cancel{\cos x}}$$

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x \, dx \\ dx &= \frac{du}{\cos x} \end{aligned}}$$

$$= \int u^4 (\cos x)^2 \, du$$

$$= \int u^4 (1 - \sin^2 x) \, du$$

$$= \int u^4 (1 - u^2) \, du$$

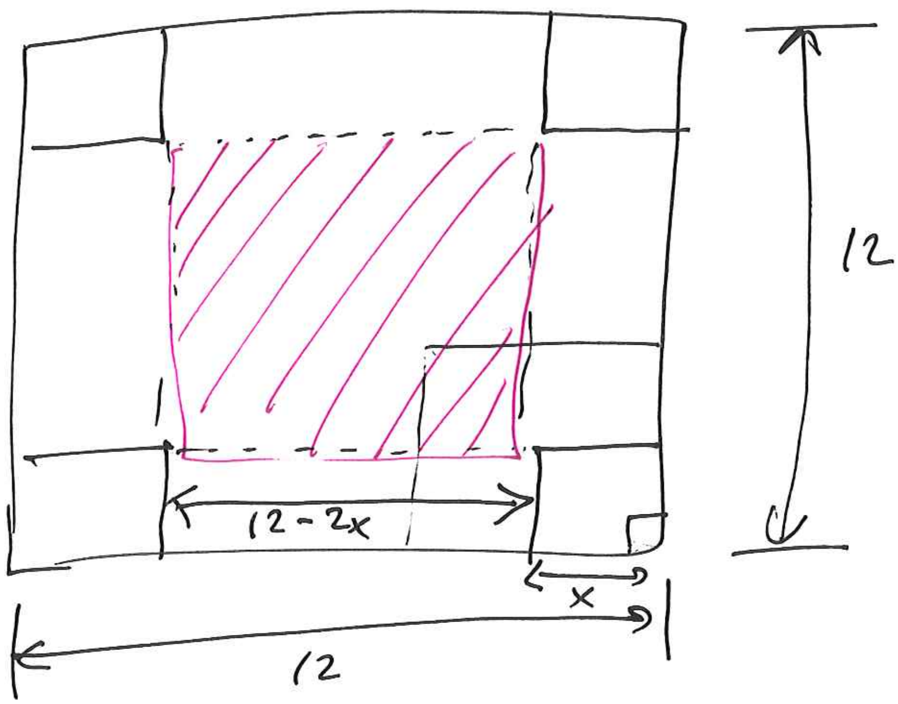
$$= \int (u^4 - u^6) \, du$$

$$= \frac{1}{5} u^5 - \frac{1}{7} u^7 + C = \frac{1}{5} (\sin x)^5 - \frac{1}{7} (\sin x)^7 + C$$

APPLIED OPTIMIZATION (4.6)

- "Real world" problem
- Draw a picture
- What are the variables?
- What is $f(x)$? ← need to optimize
- Range for x?

Ex:



Make a box, cut off at corners. Find x such that the volume is maximized.

① $V = (\text{area of base}) \times (\text{height})$

$$= (12-2x)^2 \cdot x$$

② Range of x: $0 \le x \le 6$

③ $V(x) = (12-2x)^2 \cdot x$

$$V'(x) = 2\underbrace{(12-2x)}(-2)x + (12-2x)^2$$

$$= (12-2x)\left(-4x + 12 - 2x\right)$$

$$= (12-2x)(-6x+12)$$

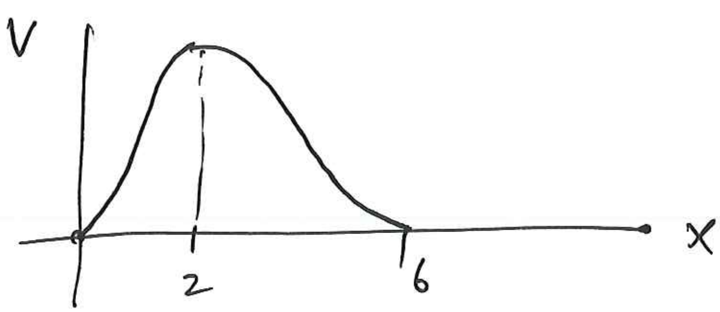
$$V'(x) = 0 \iff \begin{cases} 12-2x=0 & \text{or} \\ -6x+12=0 \end{cases}$$

$$\iff x = 6, \quad x = 2.$$

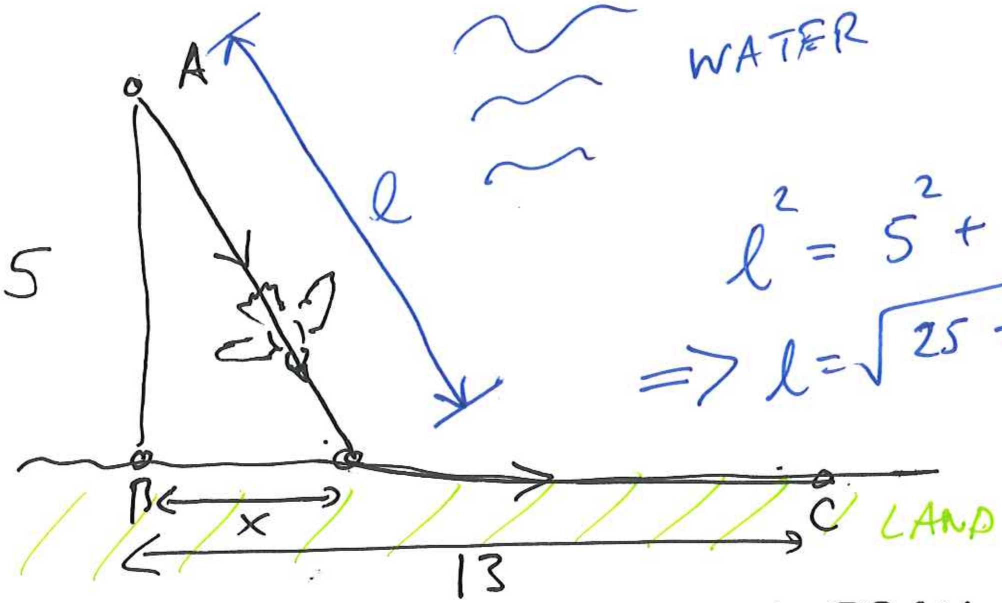
CP: $\underline{x=0}$, $x=2$, $\underline{x=6}$ endpoints

$V(x=0)=0$, $V(x=2)=128$, $V(x=6)=0$

$x=2$ is the optimal value.



Ex:



$\ell^2 = 5^2 + x^2$

$\Rightarrow \ell = \sqrt{25 + x^2}$

FLYING OVER LAND ~ ENERGY IS PROP. TO LENGTH OF PATH

FLYING OVER WATER ~ ENERGY IS PROP TO 1.4 × LENGTH.

Minimize the energy the bird needs.

$$E(x) = \underbrace{(13 - x)}_{\text{LAND}} + 1.4 \cdot \underbrace{\sqrt{25 + x^2}}_{\text{WATER}}$$

$0 \leq x \leq 13$

$$\frac{dE}{dx} = -1 + 1.4 \cdot \frac{1}{\cancel{2}} \cdot \frac{1}{\sqrt{25 + x^2}} \cdot \cancel{2}x$$

$$= -1 + \frac{1.4\,x}{\sqrt{25 + x^2}} \overset{!}{=} 0$$

$$\frac{dE}{dx} = 0 \iff -1 + \frac{1.4\,x}{\sqrt{25 + x^2}} = 0 \quad \Big| \cdot \sqrt{\ldots}$$

$$\Leftrightarrow \quad -\sqrt{25+x^2} + 1.4x = 0$$

$$\Leftrightarrow \quad 1.4x = \sqrt{25+x^2} \qquad \Big|\ (\ )^2$$

$$\Leftrightarrow \quad (1.4)^2 x^2 = 25 + x^2$$

$$\Leftrightarrow \quad 1.96\,x^2 - x^2 = 25$$

$$\Leftrightarrow \quad 0.96\,x^2 = 25$$

$$\Leftrightarrow \quad x^2 = \frac{25}{0.96}$$

$$\Rightarrow \quad x = \sqrt{\frac{25}{0.96}} = \frac{5}{\sqrt{0.96}} \approx 5.1 \qquad E(5.1) \cong \cancel{19.5}\ 17.9$$

endpoints: $x = 0 \Rightarrow E(0) = 20$

$x = 13 \Rightarrow E(13) = 19.5$

$\Rightarrow$ $x = 5.1$ is the optimum.