Line Assignment

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Problem Statement:

The straight line 5x +4y = 0 passes through the point of intersection of the straight lines x + 2y - 10 =0 and 2x+y+5=0. Find the point of intersection of 2 and 3 lines . and then passes 1st eqaution in 3rd equation.

Solution:

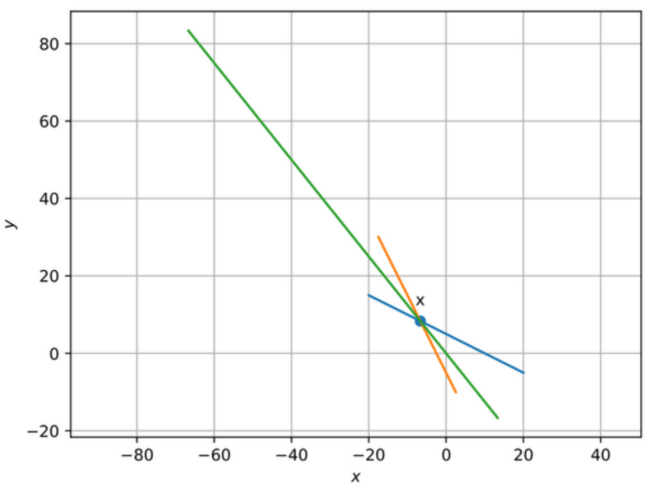

Figure 1: Diagram generated using python

0.1 Theory:

They given three lines one line i.e) 5x + 4y =0 passes through the point of intersection of the straight lines x + 2y - 10 =0 and 02x+y+5=0.

0.2 Mathematical Calculation:

$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Given line equations are

$$x + 2y = 10 \tag{1}$$

$$2x + y = -5 \tag{2}$$

The parametric equation of a line is given by

$$\mathbf{n_1}^\top \mathbf{x} = \mathbf{c_1} \tag{3}$$

$$\mathbf{n_2}^\top \mathbf{x} = \mathbf{c_2} \tag{4}$$

$$\mathbf{n_3}^\top \mathbf{x} = \mathbf{c_3} \tag{5}$$

$$\begin{pmatrix} \mathbf{n_1} \\ \mathbf{n_2} \\ \mathbf{n_3} \end{pmatrix}^\top \mathbf{x} = \begin{pmatrix} \mathbf{c_1} \\ \mathbf{c_2} \\ \mathbf{c_3} \end{pmatrix} \tag{6}$$

The vector equation of the line x+2y-10=0 is

$$\mathbf{n_1}^\top \mathbf{x} = \mathbf{c_1} \tag{7}$$

$$(1\ 2)\left(\mathbf{X}\right) = 10 \tag{8}$$

The vector equation of the line 2x+y+5=0 is

$$\mathbf{n_2}^\top \mathbf{x} = \mathbf{c_2} \tag{9}$$

$$(2\ 1)\left(\mathbf{X}\right) = -5 \tag{10}$$

The vector equation of the line 5x+4y=0 is

$$\mathbf{n_3}^\top \mathbf{x} = \mathbf{c_3} \tag{11}$$

$$(5\ 4)\left(\mathbf{X}\right) = 0 \tag{12}$$

The above equations can be written in matrix form as,

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} \mathbf{X} \end{pmatrix} = \begin{pmatrix} 10 \\ -5 \\ 0 \end{pmatrix}$$

The augmented matrix can be expressed as,

$$\begin{pmatrix} 1 & 2 & 10 \\ 2 & 1 & -5 \\ 5 & 4 & 0 \end{pmatrix}$$

Through pivoting, the augmented matrix will become as,

$$\left(\begin{array}{cc|c} 1 & 2 & 10 \\ 2 & 1 & -5 \\ 5 & 4 & 0 \end{array}\right)$$

$$[]R_2 \leftarrow \; R_2 - 2R_1 \left(\begin{array}{cc|c} 1 & 2 & 15 \\ 0 & -3 & 20 \\ 5 & 4 & 0 \end{array}\right) \tag{13}$$

$$[]R_1 \leftarrow \; 2R_1 - R_3 \left(\begin{array}{cc|c} 1 & 2 & 10 \\ 0 & -3 & -25 \\ 5 & 4 & 0 \end{array}\right) \tag{14}$$

$$[]R_3 \leftarrow \; 3R_3 + 5R_1 \left(\begin{array}{cc|c} -3 & 0 & 20 \\ 0 & -3 & -25 \\ 0 & 12 & 0 \end{array}\right) \tag{15}$$

$$[]R_2 \leftarrow \; R_2 - 2R_1 \left(\begin{array}{cc|c} 1 & 2 & 15 \\ 0 & -3 & 20 \\ 5 & 4 & 0 \end{array}\right) \tag{16}$$

$$[]R_2 \leftarrow \; 4R_2 + R_3 \left(\begin{array}{cc|c} -3 & 0 & 15 \\ 0 & -3 & 20 \\ 0 & 0 & 0 \end{array}\right) \tag{17}$$

On solving above equation the crossing point of the given equations will be,

$$\text{X=} \begin{pmatrix} -20/3 \\ 25/3 \\ 0 \end{pmatrix}$$

$$\text{X=} \begin{pmatrix} -6.8 \\ 8.3 \\ 0 \end{pmatrix}$$

hence the straight are passing through the concentric

1 Construction:

The construction of given lines can be done only the x and y of given equations

variable	**point**	**Description**
X	(x/y)	point of intersection
line1	$\mathrm{n}_1 = (1,2)c_1 = (-10)$	-6,8
line2	$\mathrm{n}_2(2,1)c_2 = (5)$	-6,8
line3	$\mathrm{n}_3 = (5,4)c_3 = (0)$	-6,8