# Natural Language Processing with Deep Learning
# CS224N/Ling284

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Lecture 2: Word Vectors

# Organization

- PSet 1 is released. Coding Session 1/22: (Monday, PA1 due Thursday)

- Some of the questions from Piazza:
- sharing the choose-your-own final project with another class seems fine--> Yes*
- But how about the default final project? Can that also be used as a final project for a different course?--> Yes*
- Are we allowing students to bring one sheet of notes for the midterm?--> Yes
- Azure computing resources for Projects/PSet4. Part of milestone

# Lecture Plan

1. Word meaning (15 mins)
2. Word2vec introduction (20 mins)
3. Word2vec objective function gradients (25 mins)
4. Optimization refresher (10 mins)

# 1. How do we represent the meaning of a word?

Definition: **meaning** (Webster dictionary)

- the idea that is represented by a word, phrase, etc.
- the idea that a person wants to express by using words, signs, etc.
- the idea that is expressed in a work of writing, art, etc.

Commonest linguistic way of thinking of meaning:

$\boxed{\text{signifier (symbol)} \Leftrightarrow \text{signified (idea or thing)}}$

= denotation

# How do we have usable meaning in a computer?

Common solution: Use e.g. WordNet, a resource containing lists of **synonym sets** and **hypernyms** (“is a” relationships).

*e.g. synonym sets containing “good”:*

```
from nltk.corpus import wordnet as wn
for synset in wn.synsets("good"):
    print "(%s)" % synset.pos(),
    print ", ".join([l.name() for l in synset.lemmas()])
```

```
(adj) full, good
(adj) estimable, good, honorable, respectable
(adj) beneficial, good
(adj) good, just, upright
(adj) adept, expert, good, practiced,
proficient, skillful
(adj) dear, good, near
(adj) good, right, ripe
…
(adv) well, good
(adv) thoroughly, soundly, good
(n) good, goodness
(n) commodity, trade good, good
```

*e.g. hypernyms of “panda”:*

```
from nltk.corpus import wordnet as wn
panda = wn.synset("panda.n.01")
hyper = lambda s: s.hypernyms()
list(panda.closure(hyper))
```

```
[Synset('procyonid.n.01'),
Synset('carnivore.n.01'),
Synset('placental.n.01'),
Synset('mammal.n.01'),
Synset('vertebrate.n.01'),
Synset('chordate.n.01'),
Synset('animal.n.01'),
Synset('organism.n.01'),
Synset('living_thing.n.01'),
Synset('whole.n.02'),
Synset('object.n.01'),
Synset('physical_entity.n.01'),
Synset('entity.n.01')]
```

# Problems with resources like WordNet

- Great as a resource but missing nuance
  - e.g. "proficient" is listed as a synonym for "good". This is only correct in some contexts.
- Missing new meanings of words
  - e.g. wicked, badass, nifty, wizard, genius, ninja, bombest
  - Impossible to keep up-to-date!
- Subjective
- Requires human labor to create and adapt
- Hard to compute accurate word similarity →

# Representing words as discrete symbols

In traditional NLP, we regard words as discrete symbols:
hotel, conference, motel

Means one 1, the rest 0s

Words can be represented by one-hot vectors:

motel = [0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
hotel = [0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]

Vector dimension = number of words in vocabulary (e.g. 500,000)

# Problem with words as discrete symbols

<u>Example:</u> in web search, if user searches for “Seattle motel”, we would like to match documents containing “Seattle hotel”.

But:

motel = [0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]
hotel = [0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]

These two vectors are orthogonal.

There is no natural notion of **similarity** for one-hot vectors!

<u>Solution:</u>

- Could rely on WordNet’s list of synonyms to get similarity?
- **Instead: learn to encode similarity in the vectors themselves**

# Representing words by their context

- <u>Core idea</u>: **A word's meaning is given by the words that frequently appear close-by**
  - *"You shall know a word by the company it keeps"* (J. R. Firth 1957: 11)
  - One of the most successful ideas of modern statistical NLP!
- When a word *w* appears in a text, its **context** is the set of words that appear nearby (within a fixed-size window).
- Use the many contexts of *w* to build up a representation of *w*

| | | |
|---|---|---|
| *…government debt problems turning into* | ***banking*** | *crises as happened in 2009…* |
| *…saying that Europe needs unified* | ***banking*** | *regulation to replace the hodgepodge…* |
| *…India has just given its* | ***banking*** | *system a shot in the arm…* |

These context words will represent ***banking***

# Word vectors

We will build a dense vector for each word, chosen so that it is similar to vectors of words that appear in similar contexts.

$$\textit{linguistics} = \begin{pmatrix} 0.286 \\ 0.792 \\ -0.177 \\ -0.107 \\ 0.109 \\ -0.542 \\ 0.349 \\ 0.271 \end{pmatrix}$$

Note: word vectors are sometimes called word embeddings or word representations.

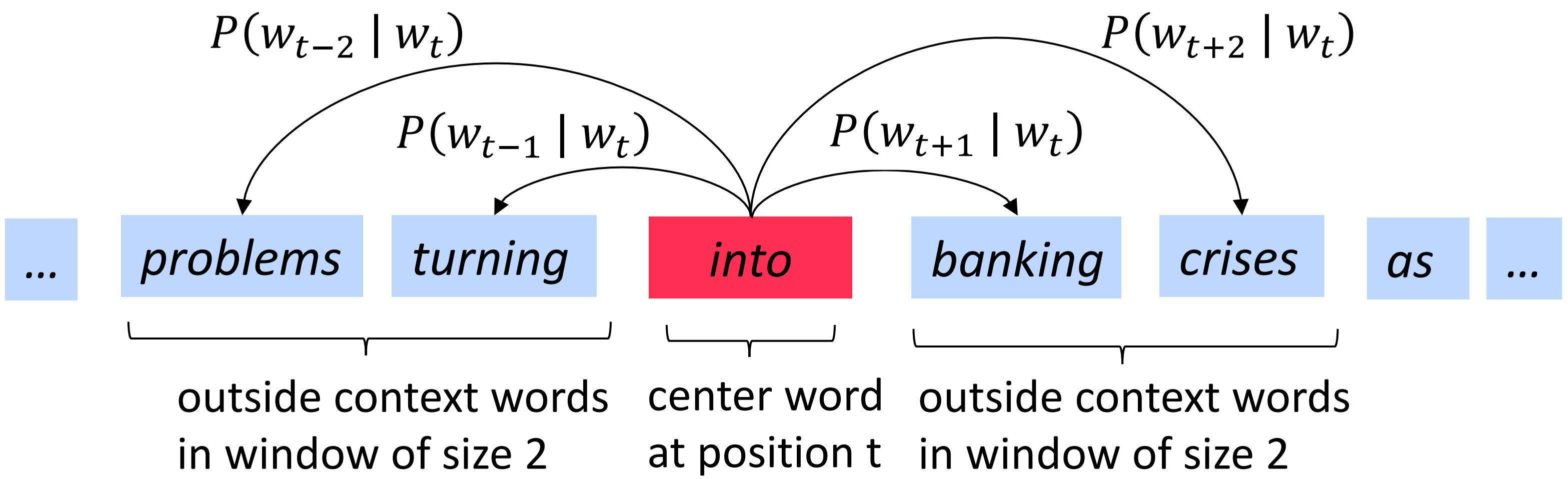
# 2. Word2vec: Overview

Word2vec (Mikolov et al. 2013) is a framework for learning word vectors. Idea:

- We have a large corpus of text
- Every word in a fixed vocabulary is represented by a vector
- Go through each position *t* in the text, which has a center word *c* and context ("outside") words *o*
- Use the similarity of the word vectors for *c* and *o* to calculate the probability of *o* given *c* (or vice versa)
- Keep adjusting the word vectors to maximize this probability

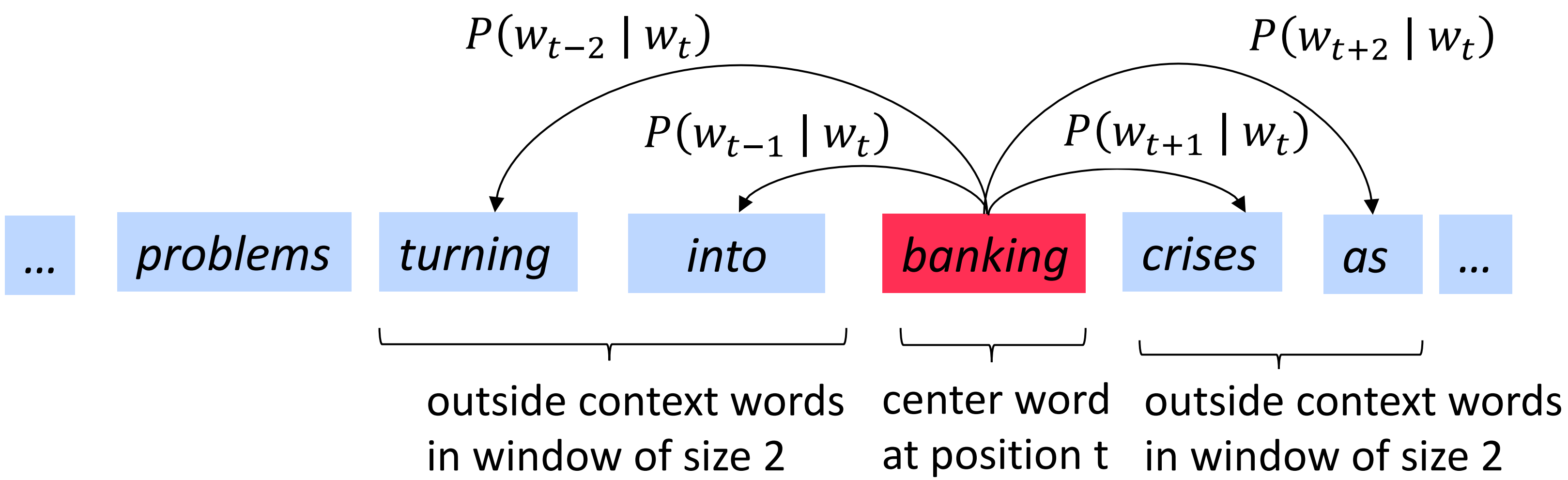
# Word2Vec Overview

- Example windows and process for computing $P\left(w_{t+j} \mid w_t\right)$



# Word2Vec Overview

- Example windows and process for computing $P(w_{t+j} \mid w_t)$



# Word2vec: objective function

For each position $t = 1, \dots, T$, predict context words within a window of fixed size $m$, given center word $w_j$.

Likelihood = $$L(\theta) = \prod_{t=1}^{T} \prod_{\substack{-m \le j \le m \\ j \ne 0}} P\left(w_{t+j} \mid w_t; \theta\right)$$

$\theta$ is all variables to be optimized

sometimes called *cost* or *loss* function

The objective function $J(\theta)$ is the (average) negative log likelihood:

$$J(\theta) = -\frac{1}{T} \log L(\theta) = -\frac{1}{T} \sum_{t=1}^{T} \sum_{\substack{-m \le j \le m \\ j \ne 0}} \log P\left(w_{t+j} \mid w_t; \theta\right)$$

Minimizing objective function $\Leftrightarrow$ Maximizing predictive accuracy

# Word2vec: objective function

- We want to minimize the objective function:

$$J(\theta) = -\frac{1}{T}\sum_{t=1}^{T}\sum_{\substack{-m\le j\le m\\ j\ne 0}} \log P\left(w_{t+j} \mid w_t;\theta\right)$$

- Question: How to calculate $P\left(w_{t+j} \mid w_t;\theta\right)$ ?
- Answer: We will *use two* vectors per word *w*:
  - $v_w$ when *w* is a center word
  - $u_w$ when *w* is a context word

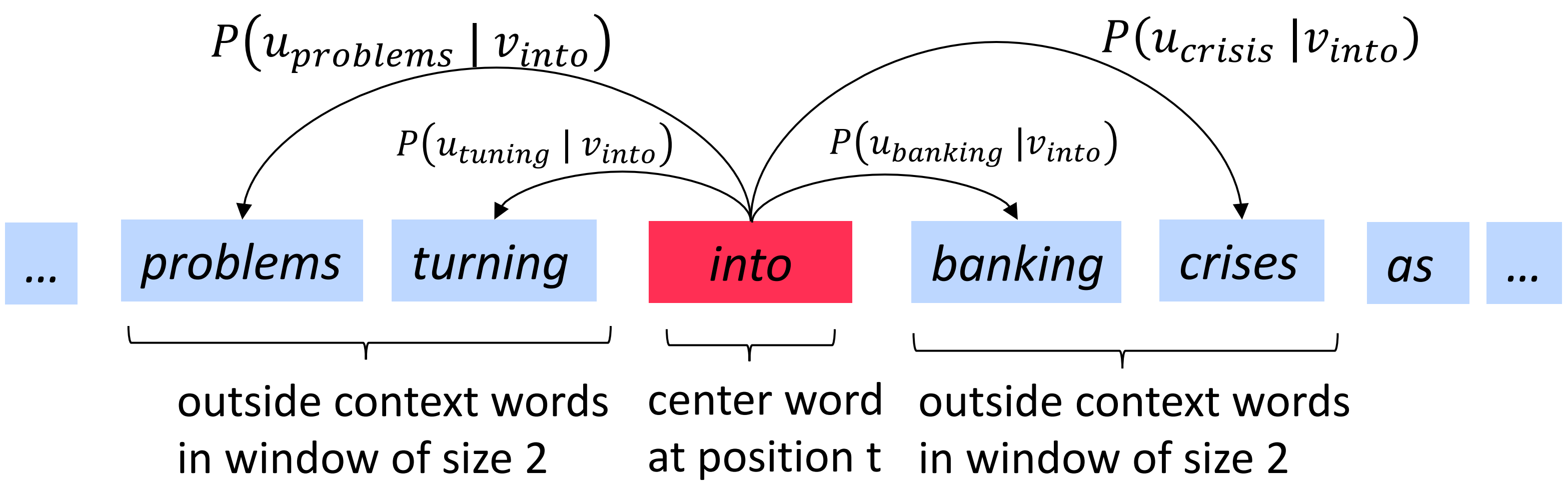
**We can also use one vector per word. However, two vectors per word turns out to perform better and be computationally less expensive than one vector per word.**

- Then for a center word *c* and a context word *o*:

$$P(o|c) = \frac{\exp(u_o^T v_c)}{\sum_{w\in V}\exp(u_w^T v_c)}$$

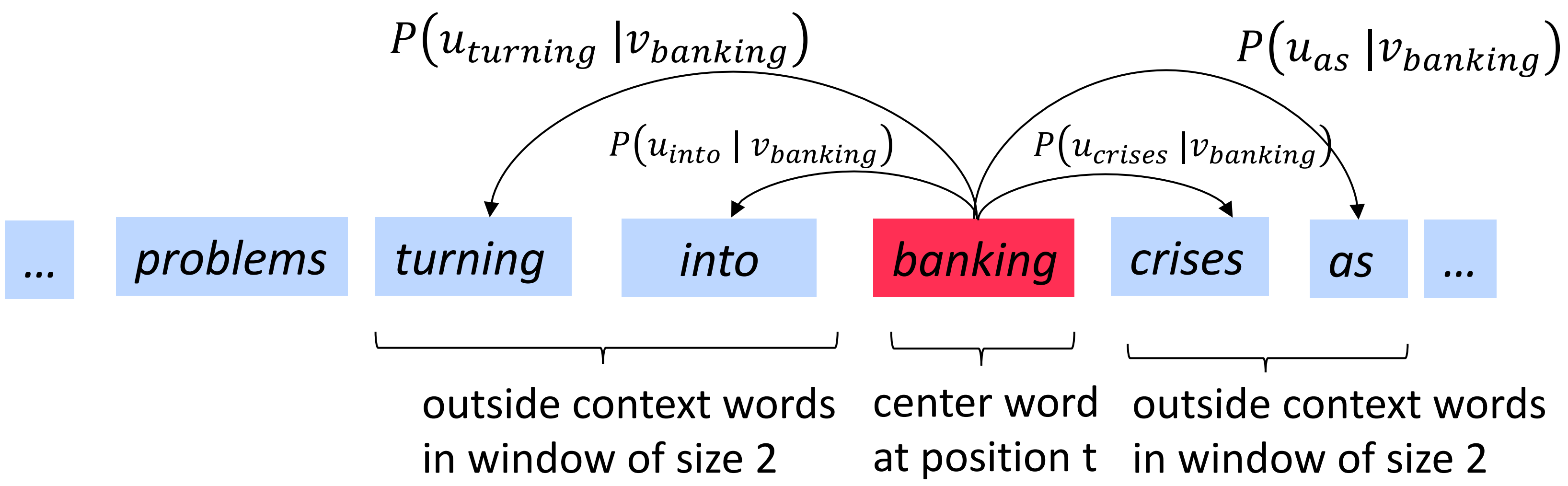
# Word2Vec Overview with Vectors

- Example windows and process for computing $P(w_{t+j} \mid w_t)$
- $P(u_{problems} \mid v_{into})$ short for $\mathrm{P}(problems \mid into\ ; u_{problems}, v_{into}, \theta)$



# Word2Vec Overview with Vectors

- Example windows and process for computing $P\left(w_{t+j} \mid w_t\right)$



# Word2vec: prediction function

$$P(o|c) = \frac{\boxed{\exp(u_o^T v_c)}}{\boxed{\sum_{w \in V} \exp(u_w^T v_c)}}$$

Dot product compares similarity of *o* and *c*.
Larger dot product = larger probability

After taking exponent,
normalize over entire vocabulary

- This is an example of the **softmax function** $\mathbb{R}^n \rightarrow \mathbb{R}^n$

$$\text{softmax}(x_i) = \frac{\exp(x_i)}{\sum_{j=1}^{n} \exp(x_j)} = p_i$$

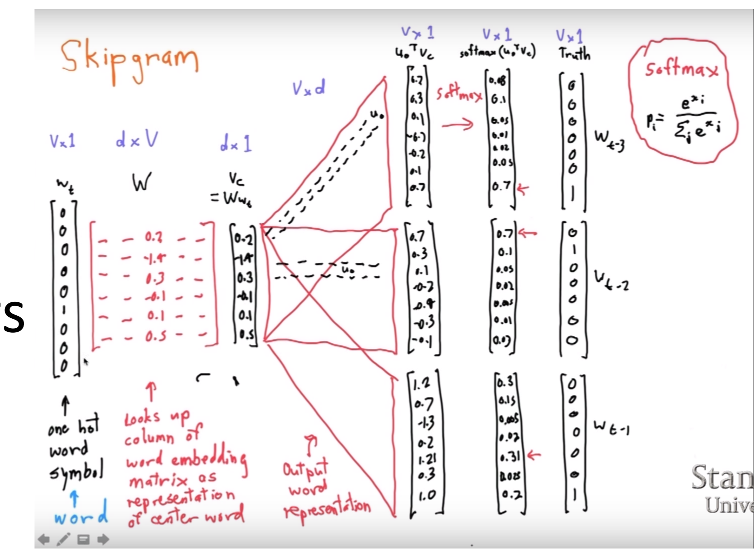
- The softmax function maps arbitrary values $x_i$ to a probability distribution $p_i$
  - "max" because amplifies probability of largest $x_i$
  - "soft" because still assigns some probability to smaller $x_i$
  - Frequently used in Deep Learning

# To train the model: Compute all vector gradients!

- Recall: $\theta$ represents all model parameters, in one long vector
- In our case with $d$-dimensional vectors and $V$-many words:

$$\theta = \begin{bmatrix} v_{aardvark} \\ v_a \\ \vdots \\ v_{zebra} \\ u_{aardvark} \\ u_a \\ \vdots \\ u_{zebra} \end{bmatrix} \in \mathbb{R}^{2dV}$$

- Remember: every word has two vectors
- We then optimize these parameters



# 3. Derivations of gradient

- Whiteboard – see video if you’re not in class ;)
- The basic Lego piece
- Useful basics: $\frac{\partial \mathbf{x}^T \mathbf{a}}{\partial \mathbf{x}} = \frac{\partial \mathbf{a}^T \mathbf{x}}{\partial \mathbf{x}} = \mathbf{a}$
- If in doubt: write out with indices

- Chain rule! If $y = f(u)$ and $u = g(x)$, i.e. $y = f(g(x))$, then:

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

# Chain Rule

- Chain rule! If *y* = *f*(*u*) and *u* = *g*(*x*), i.e. *y* = *f*(*g*(x)), then:

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{df(u)}{du}\frac{dg(x)}{dx}$$

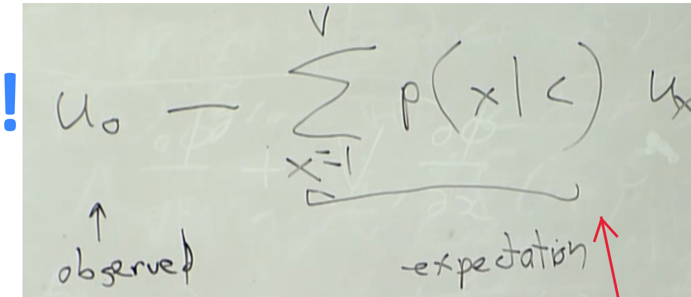
- Simple example: $\frac{dy}{dx} = \frac{d}{dx}5(x^3+7)^4$

$$y = f(u) = 5u^4 \qquad u = g(x) = x^3 + 7$$

$$\frac{dy}{du} = 20u^3 \qquad \frac{du}{dx} = 3x^2$$

$$\frac{dy}{dx} = 20(x^3+7)^3 \cdot 3x^2$$

# Interactive Whiteboard Session!




$$J(\theta) = \frac{1}{T} \sum_{t=1}^{T} \sum_{-m \leq j \leq m,\, j \neq 0} \log p(w_{t+j} | w_t)$$

The sum of all the context vectors weighted by their likelihood of occurrence.

Let's derive gradient for center word together

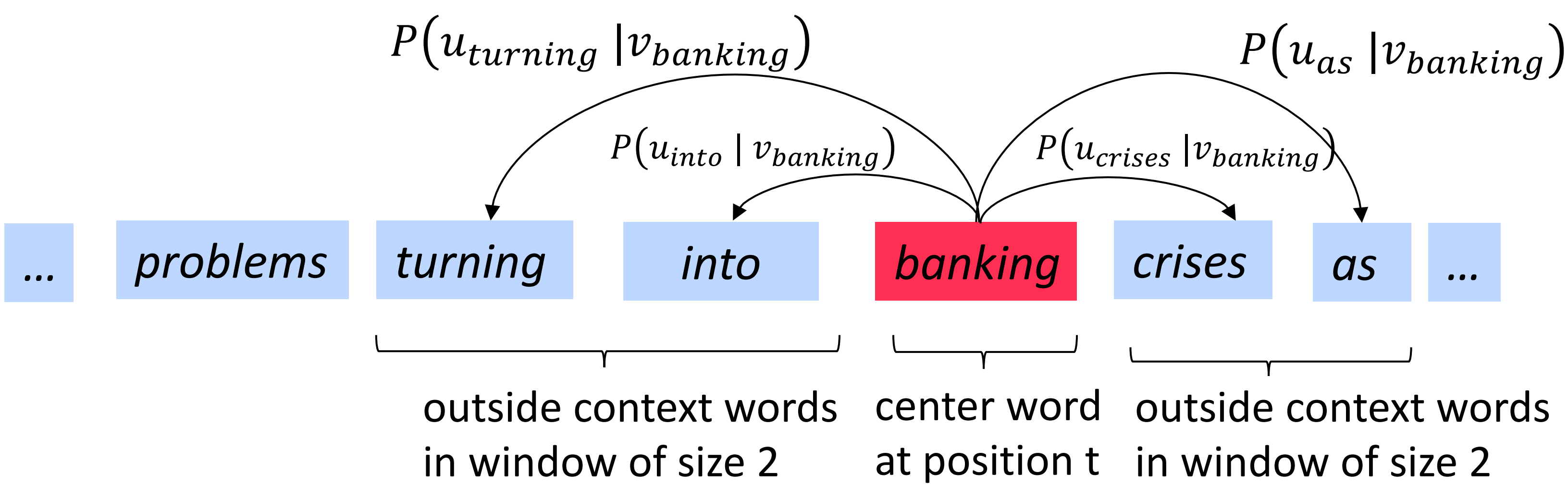
For one example window and one example outside word:

$$\log p(o|c) = \log \frac{\exp(u_o^T v_c)}{\sum_{w=1}^{V} \exp(u_w^T v_c)}$$

You then also need the gradient for context words (it's similar; left for homework). That's all of the parameters $\theta$ here.

# Calculating all gradients!

- We went through gradient for each center vector *v* in a window
- We also need gradients for outside vectors *u*
- Derive at home!
- Generally in each window we will compute updates for all parameters that are being used in that window. For example:



# Word2vec: More details

Why two vectors? → Easier optimization. Average both at the end.

Two model variants:

1. Skip-grams (SG)

   Predict context (”outside”) words (position independent) given center word

2. Continuous Bag of Words (CBOW)

   Predict center word from (bag of) context words

This lecture so far: **Skip-gram model**

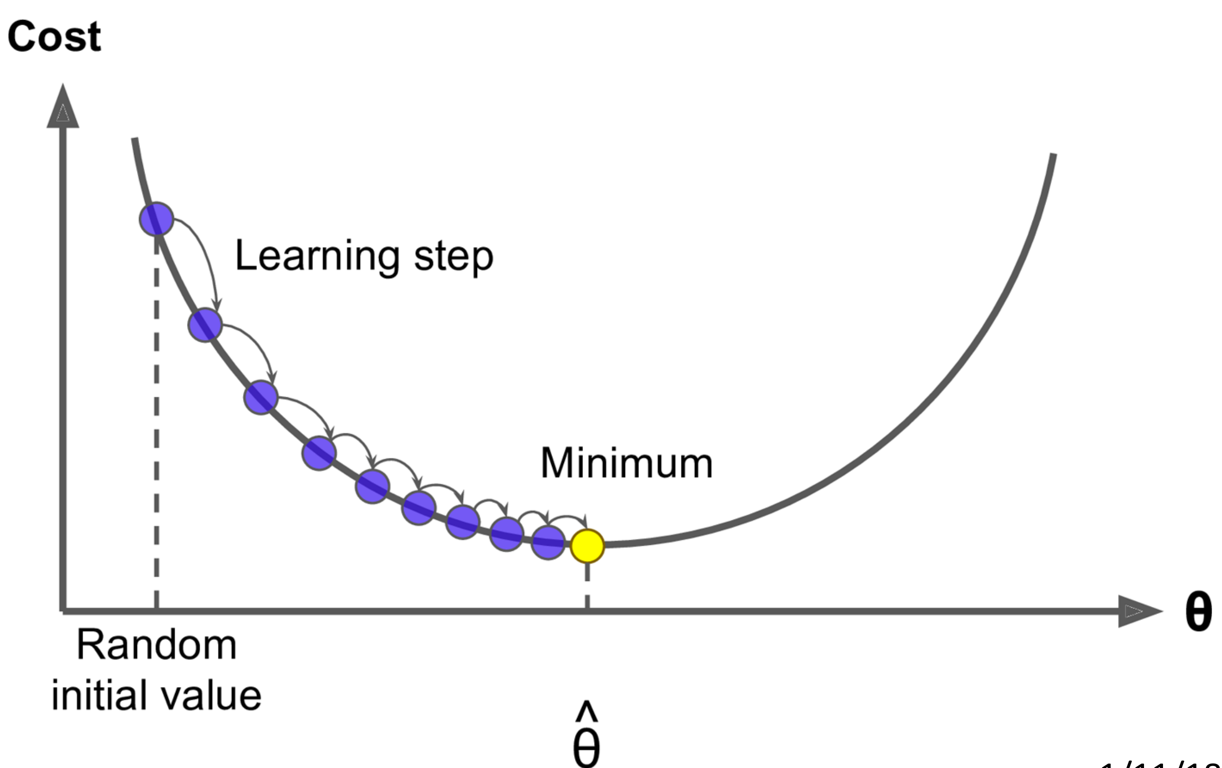
Additional efficiency in training:

1. Negative sampling

So far: Focus on **naïve softmax** (simpler training method)

# Gradient Descent

- We have a cost function $J(\theta)$ we want to minimize
- **Gradient Descent** is an algorithm to minimize $J(\theta)$
- Idea: for current value of $\theta$, calculate gradient of $J(\theta)$, then take small step in direction of negative gradient. Repeat.

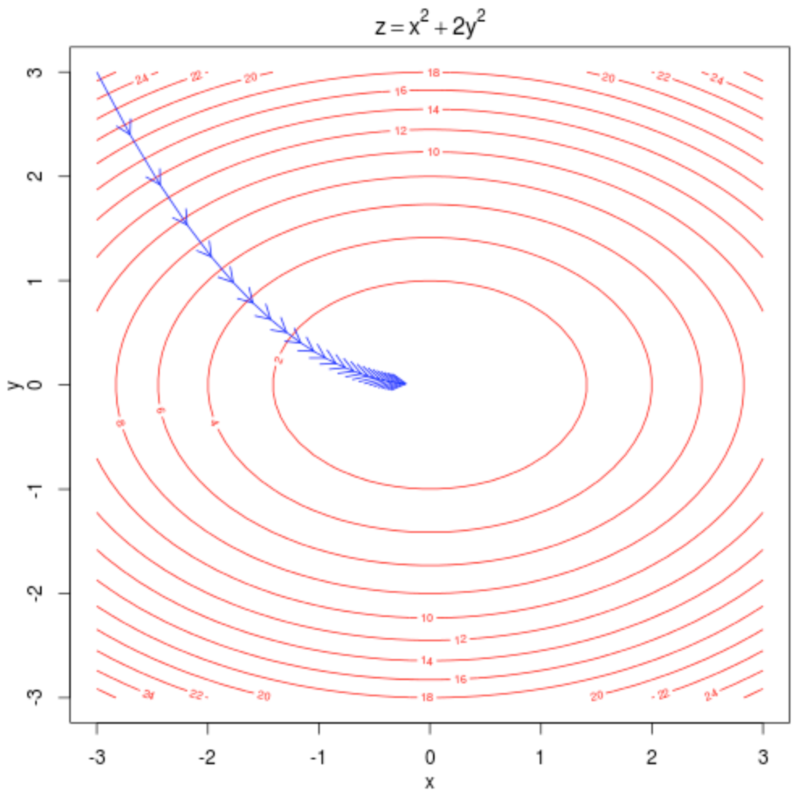



Note: Our objectives are not convex like this :(

# Intuition

For a simple convex function over two parameters.

Contour lines show levels of objective function



# Gradient Descent

- Update equation (in matrix notation):

$$\theta^{new} = \theta^{old} - \alpha \nabla_\theta J(\theta)$$

$\alpha$ = *step size* or *learning rate*

- Update equation (for single parameter):

$$\theta_j^{new} = \theta_j^{old} - \alpha \frac{\partial}{\partial \theta_j^{old}} J(\theta)$$

- Algorithm:

```
while True:
    theta_grad = evaluate_gradient(J,corpus,theta)
    theta = theta - alpha * theta_grad
```

# Stochastic Gradient Descent

- Problem: $J(\theta)$ is a function of **all** windows in the corpus (potentially billions!)
  - So $\nabla_\theta J(\theta)$ is very expensive to compute
- You would wait a very long time before making a single update!

- Very bad idea for pretty much all neural nets!
- Solution: **Stochastic gradient descent (SGD)**
  - Repeatedly sample windows, and update after each one.
- Algorithm:

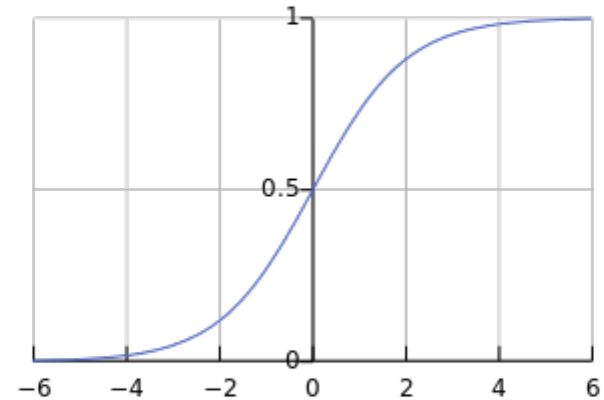
```
while True:
    window = sample_window(corpus)
    theta_grad = evaluate_gradient(J,window,theta)
    theta = theta - alpha * theta_grad
```

# PSet1: The skip-gram model and negative sampling

- From paper: "Distributed Representations of Words and Phrases and their Compositionality" (Mikolov et al. 2013)
- Overall objective function (they maximize): $J(\theta) = \frac{1}{T}\sum_{t=1}^{T} J_t(\theta)$

$$J_t(\theta) = \log \sigma\left(u_o^T v_c\right) + \sum_{i=1}^{k} \mathbb{E}_{j\sim P(w)}\left[\log \sigma\left(-u_j^T v_c\right)\right]$$

- The sigmoid function! $\sigma(x) = \frac{1}{1+e^{-x}}$ (we'll become good friends soon)
- So we maximize the probability of two words co-occurring in first log →



# PSet1: The skip-gram model and negative sampling

- Simpler notation, more similar to class and PSet:

$$J_{neg-sample}(\boldsymbol{o}, \boldsymbol{v}_c, \boldsymbol{U}) = -\log(\sigma(\boldsymbol{u}_o^\top \boldsymbol{v}_c)) - \sum_{k=1}^{K} \log(\sigma(-\boldsymbol{u}_k^\top \boldsymbol{v}_c))$$

- We take k negative samples.
- Maximize probability that real outside word appears, minimize prob. that random words appear around center word

- $P(w)=U(w)^{3/4}/Z$, **Z is the normalization.**
  the unigram distribution U(w) raised to the 3/4 power
  (We provide this function in the starter code).
- The power makes less frequent words be sampled more often

# PSet1: The continuous bag of words model

- Main idea for continuous bag of words (CBOW): Predict center word from sum of surrounding word vectors instead of predicting surrounding single words from center word as in skip-gram model

- To make assignment slightly easier:

  Implementation of the CBOW model is not required (you can do it for a couple of bonus points!), but you do have to do the theory problem on CBOW.