Nearly optimal pulse control of quantum systems

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Abstract

Quantum technologies is a relatively new field of science in the intersection of physics, engineering and mathematics. It promises to revolutionise many fields such as computation, communication and metrology by exploiting unique properties of small-scale physical systems. It is known that such systems are extremely difficult to manipulate due to their high sensitivity to external noises, making them for many practical purposes quiet short-lived physical objects. For this reason time factor becomes crucial. The goal of this project is to develop a general theory and algorithms for optimal control of quantum systems via pulse controls, which is currently the fastest way of controlling many real-world quantum systems.

Overiview of the project

Quantum systems are very sensitive objects. It is almost impossible to isolate those systems from the environment. For example, it is known that super-conducting qubits are affected by cosmic ray muons [6], which can be a very persistent source of noise. The accumulation of quantum noise in a quantum system is called decoherence. To combat this phenomena one solution is to manipulate quantum systems as quick as possible. Very often this would require a deployment of strong pulse-like interactions. Pulse control, indeed, is a quite common strategy for quantum control [2] and several numerical methods for finding pulse optimal controls currently exist [7, 1, 4], which have their limitations in scalability. The scope of the project is to develop a general theory for optimal pulse control of a certain class of control problems that have quantum applications. The main methodological tool is Filippov's relaxation theorem, that will permit us to apply many theoretical and numerical tools for this kind of problems. The project is essentially separated into three parts: theoretical part, algorithmic part and applications.

Theoretical part

We are interested in the study of control systems of the type:

$$\dot{q} = f_0(q) + \sum_{i=1}^{k} u_i f_i(q), \tag{0.1}$$

where q is a point on the configuration space, $f_0, \dots, f_k$ are vector fields and u_i controls which take values in some set $U \subset \mathbb{R}^k$. We assume that f_i for $i = 1, \dots, k$ are commuting vector fields and all of their trajectories are closed. This happens, for example, for entangled n superconducting qubits. In this case the configuration space is the unit sphere in $\mathbb{C}^{2^n}$, f_i are individual rotations of each qubit in the Bloch sphere and f_0 is a drift that is responsible for the entanglement in the system. If we are interested in a control strategy that combines pulses with zero control, then we can relax the system (0.1) to

$$\dot{q} = \sum_{i=0}^{k} u_i f_i(q), \tag{0.2}$$

where the control u_i will take values in a new appropriate set $\tilde{U}$ constructed from U via convexfication.

The main result of the theoretical part will be understanding of local structure of time-minimal pulse control problems in small dimensions, in the spirit of [3], where a different technique was used. This will give a good benchmark for testing future numerical algorithms.

Algorithmic part

The global analysis of minimal time trajectories of (0.1) is short of impossible for high-dimensional systems. For this reason, it is necessary to develop good numerical tools for finding global and local minima. Thus the second part will be to develop numerical algorithms for nearly optimal control of the system (0.1) using its relaxation (0.2) and latest results from numerical optimal control theory. The algorithms will be tested using finding from theoretical part of the project.

Applications

Finally the tools developed in the previous two parts will be applied to a variety of model systems including: control of entangled super-conducting qubits (both closed and open models), gate generation and ensemble control. We will leverage the use of control-toolbox package (https://control-toolbox.org/) developed by the MCTAO team to perform numerical simulations. The results will be tested on real quantum computers using IBM quantum cloud computing platform and the qiskit package, which allows users to run custom pulse protocols [5]. This will help us to reduce significantly the gap between theory and practice.

Requisites

The candidate should a have strong background in applied mathematics, and especially in optimization and/or control theory, as well as programming (preferably in Python, C++ or Julia).

References

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