In Class Practice Problems

1)

$$\begin{aligned} 8x_1 + 3x_2 - 3x_3 &= 14 \\ -2x_1 - 8x_2 + 5x_3 &= 5 \\ 3x_1 + 5x_2 + 10x_3 &= -8 \end{aligned}$$

Solve the system of linear equations using the Gauss-Seidel method, use a pre-defined threshold ϵ=0.01.

Remember to check if the converge condition is satisfied or not.

2) Use numpy.linalg.solve to solve the following equations.

$$\begin{aligned} 4x_1 + 3x_2 - 5x_3 &= 2 \\ -2x_1 - 4x_2 + 5x_3 &= 5 \\ 8x_1 + 8x_2 &= -3 \end{aligned}$$

3) Solve the same system of equations from Problem 2 using the matrix inversion approach.

4) Consider the following equations:

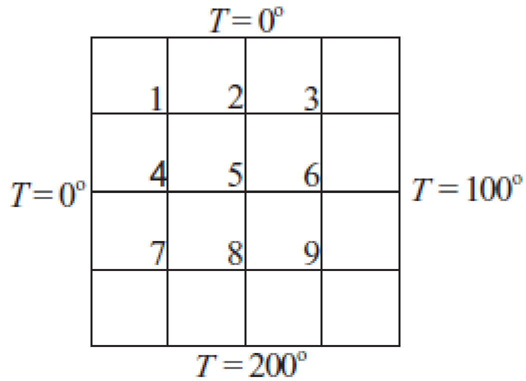
$$\begin{bmatrix} 4 & -1 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 \\ -1 & 4 & -1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & -1 & 4 & -1 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -1 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & -1 & 4 & -1 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-2} \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 100 \end{bmatrix}$$

Solve the system of equations for n = 20. Use the following methods:

a) LU Decomposition (Hint: use the lu function from Scipy)
b) Cramer's Rule
c) Gauss-Seidel
d) Use the solve function in Numpy
e) Use the matrix inversion approach

Now, verify the accuracy of each method. Print the accuracy and the time taken for each method.

5) Consider the following square plate as shown:



The edges of the square plate are kept at the temperatures shown. Assuming steady-state heat conduction, the differential equation governing the temperature T in the interior is as follows

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

If this equation is approximated by finite differences using the mesh shown, we obtain the following algebraic equations for temperatures at the mesh points.

$$\begin{bmatrix} -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -100 \\ 0 \\ 0 \\ -100 \\ -200 \\ -200 \\ -300 \end{bmatrix}$$

Solve these equations and visualize the results.