THE KALMAN FILTER APPLICATION TO THE PURCHASING POWER PARITY

by

Demet Tunali

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Abstract

The purchasing power parity (PPP) theory links the relative price of domestic and foreign goods and services goods to the spot exchange rate. One stream of the literature reports that the PPP has a poor empirical value due to the random walk in the real exchange rate. The other stream in contrast, demonstrates with more powerful tests that the real exchange rate displays a slow mean revesion thereby validating the PPP relation. In this study we examine the weaker version; namely the relative PPP (RPPP); which states that the percentage change in the exchange rate between two countries is equal to the inflation differentials. We ask whether RPPP holds in the long run. To answer this question we formulate a state-space model with stochastically varying coefficients. The estimation using the Kalman Filter technique is conducted both under the classical and the Bayesian approaches to provide a comparison. We analyze the Japanese ¥ to the U.S.$ exchange rate using monthly data since the collapse of the Bretton Woods system. We find that, under both approaches, the real exchange rate displays stationarity around zero. However, the slope parameter of the RPPP relation, though stationary, is far from unity.

Keywords: Purchasing power parity, state-space models, time-varying paramaters, Bayesian inference, classical inference.

1 Introduction

The purchasing power parity is a view of the long-run exchange rate behavior depicting a link between a country's overall price level and its exchange rate. Its strongest form; known as the absolute PPP (APPP); is a generalization of the law of one price. The law of one price states that the price of any particular traded good will be the same in every country after controlling for either trade barriers, transportation costs, or non-traded inputs. Thus the APPP implies that the nominal exchange rate between two currencies should be equal to the relative price levels of the two countries. This means that the real exchange rate equals unity. This relation is hard to hold though even if the law of one price holds far all commodities. This is because foreign and domestic price indices are not identically constructed in terms of either their composition or their weighting scheme. So far this problem has been addressed through the use of the Penn World Table; which computes price indices on a common market basket; or even easier, with the comparison of the "Big Mac" sandwich prices in various countries around the world as published by *The Economist*. Yet substantial deviations are observed under both price measures as reported by Pakko and Pollard (1996). Authors show that for various industrialized countries PPP relative to the U.S. dollar strictly holds neither for a given year nor in a ten-year period where persistent deviations are observed for most currencies. Furthermore, it is found by Isaard (1977) that the law of one price fails in products of different countries that are not close substitutes. Isaard provides evidence from the prices of U.S. manufactured goods that have foreign counterparts from the most disaggregated list of commodities to show that exchange rate fluctuations substantially and persistently alter the relative dollar-equivalent prices.

Given these evidences against the APPP, another view of this relation is suggested. It is then formulated that the PPP holds upto to a constant. In other words, this view expresses that the purchasing power ratio between the domestic country and the foreign country is constant or likewise the real exchange rate is fixed at some level. Two opposite arguments then emerged from this formulation. One of these arguments suggested that the real exchange rate followed a random walk. Thus accordingly, given common large fluctuations in the real exchange rate, it is rather put forward that the change in the real exchange rate must be expected at any horizon to be zero. Support for this argument came from Cumby and Obstfeld (1984) and Roll (1979) among others. Cumby and Obstfeld (1984) analyze the real exchange rate of the currencies of Germany, Switzerland, Canada and Japan against U.S. dollar with monthly data from 1960 to 1980. Authors find that the real exchange rate changes are serially uncorrelated for this time span under both one-month and three-month forecasting horizons. Adler and Lehmann (1983) study the monthly and annual data for the period from 1915 to 1972 for forty three industrial and developing countries' currencies against the U.S.$ and conclude that the real exchange rate follows a martingale process instead of a stable autoregression as suggested by the long-run PPP hypothesis.

The other argument posits that the real exchange rate is mean reverting; in other words that the PPP holds in the long run. At this front, Abuaf and Jorion (1990) and Diebold, Husted and Rush (1991) consider a larger sample and more sophisticated estimation methods. As such, these studies aimed to solve the power problem of the stationarity tests in earlier work, that they blamed for the findings of random walk in the real exchange rate. Abuaf and Jorion utilize a system of univariate autoregressions instead of country-by country regressions; to exploit the cross-equation correlations; for the period between 1900 and 1972 for ten industrial countries. Diebold, Husted and Rush work on sixteen real exchange rates in the gold standard era through autoregressive fractionally integrated moving average models. The intention in such method relies on better consideration of the important low-frequency components in the real exchange rate. Both studies find that the PPP holds in the long run albeit substantial and prolonged short-term deviations with an approximately three years of a half-life of a shock.

In this study we consider a weaker version of the theory, namely the relative PPP (RPPP). This version states that the percentage change in the exchange rate between two countries is the same with the inflation differentials. We work with the monthly data on the Japanese Yen to U.S. dollar exchange rate for the post-Bretton Woods era until 2006. We examine whether this relation holds in the long run for this currency. We estimate a state-space model using Kalman filter and applying both classical and Bayesian inference methods for a comparison purpose. The observation equation represents the relative PPP. The state equation models the stochastic process of the observation equation's intercept and slope parameters. In this setup, the intercept correspond to the change in the real exchange rate and hence is trivially expected to follow a stationary process around zero. Of more interest is the path of the slope parameter which should come stationary around one for the relative PPP relation to hold. We find that under both approaches, the real exchange rate displays stationarity around zero. However, the slope parameter of the RPPP relation, though stationary, is far from unity. The paper proceeds as follows. Section 2 presents the data, Section 3 describes the model, Sections 4 and 5 briefly explains the classical and the Bayesian estimation of the model respectively. Section 6 reports the results and Section 7 concludes. Tables and graphs are included to the appendix in section 8.

2 Data

The data on which the state space model is based consist of the monthly exchange rate between the Japanese ¥ and the U.S.$ and monthly CPI since 1971. The RPPP equation is constructed for the month on month inflation and exchange rate change. To get an initial idea about the RPPP relation between these currencies, to be detailed in the next section, we present a cursory look at the real exchange rate and at a simple OLS estimation.

The OLS estimates of the regression of the month-on-month change in the nominal exchange rate on monthly inflaion differential show the following. Both parameters are close to zero and have wrong signs. Additionally none of these parameters is statistically different from zero under 5% significance level. While this is encouraging for the intercept parameter which corresponds to the change in the real exchange rate, it is disappointing to find that slope parameter is not equal to one.

The plot of the real exchange rate in Figure 1 reveals that this sereis had exerienced several breaks in its trend since 1971; e.g. among many others an upward trend from 1971 to 1978 followed by a reversal from 1978 to 1985 and recently a downward trend from 1995 to 1998. Through these periods the series seem to have followed a trend stationary process. We conduct a unit root test for the whole series on the real exchange rate. We selected the lag order using the SIC and assumed no constant and no trend in the Augmented Dicky Fuller equation based on the overall behavior of the series. Table 1 shows that the null hypothesis of a unit root in the real exchange rate is not rejected confirming the findings of one stream of earlier studies. Yet, it is well known that given these breaks a standard unit root test would have low power and tests of the type suggested by Perron (1989) should rather be aplied. Hence this is a week finding in favor of the pessimistic literature. We drop such refinement for the sake of brievety and due to its irrelavance to the the main analysis.

In this study, we are rather concerned with the change in the real exchange rate and the path of the slope parameter. The plot of the change in the real exchange rate in Figure 2 displays a stationary behavior around zero; as is also confirmed by the unit root test. This indeed stands as a positive sign towards the validity of the RPPP. Nevertheless, it is the slope parameter's pattern that will ultimately determine our conclusion.

3 The Model

To capture this long-run relationship between the nominal exchange rate and the relative price levels we express this doctrine in terms of relative changes in these variables.

The time varying parameter model we consider for this weaker form of the purchasing power parity takes the following form.

$$\widehat{s_t} = \beta_{0,t} + \beta_{1,t}(\widehat{p_t^*} - \widehat{p_t}) + w_t \tag{1}$$

$$\beta_{0,t+1} = \mu_0 + \rho_0\beta_{0,t} + v_{0,t+1} \tag{2}$$

$$\beta_{1,t+1} = \mu_1 + \rho_1\beta_{1,t} + v_{1,t+1} \tag{3}$$

where $\widehat{s_t}$ is the change in log of the ¥ per \$ exchange rate, $\widehat{p_t^*}$and $\widehat{p_t}$ are the Japanese and U.S. month-on-month inflation rates respectively. Notice from the definition of the real exchange rate that, $\beta_{0,t}$ stands for the change in the log real exchange rate. It is assumed that, given $(\widehat{p_t^*} - \widehat{p_t})$ and Υ_t denoting the data observed through date $t-1$, the vector of random variables $(v'_{0,t}, v'_{1,t}, w_t)'$ has a white noise Gaussian distribution

$$\begin{bmatrix} v_{0,t+1} \\ v_{1,t+1} & \mid (\widehat{p_t^*} - \widehat{p_t}), \Upsilon_t \\ w_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{v_0} & 0 & 0 \\ 0 & \sigma^2_{v_1} & 0 \\ 0 & 0 & \sigma^2_w \end{bmatrix} \right) \tag{4}$$

The state space representation of this model then is,

$$\begin{aligned} y_t &= x_t \xi_t + w_t \\ \xi_{t+1} &= F \xi_t + v_{t+1} \end{aligned} \tag{5}$$

Here matrix x_t has dimension $1X3$ so that each row has the structure $\left[1 \mid (\widehat{p_t^*} - \widehat{p_t}) \mid 0 \right]$. The state vector ξ_t is $3X1$ and represents the parameters of the observation equation with the specification as $\xi_t = (\beta_{0,t}, \beta_{1,t}, 1)$ 'and $v_t = [v'_{0,t}, v'_{1,t}, 0]'$. The martices of unknown parameters F, Q and R are specified as follows.

$$F = \begin{bmatrix} \rho_0 & 0 & \mu_0 \\ 0 & \rho_1 & \mu_1 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$Q = \begin{bmatrix} \sigma^2_{v_0} & 0 & 0 \\ 0 & \sigma^2_{v_1} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{7}$$

$$R = \sigma^2_w \tag{8}$$

Notice that we assume no correlation between the state variables; yielding a diagonal variance covariance matrix of Q. Likewise the shick to the observation equation is also assumed to be uncorrelated with those to the state variables as represented in R. These assumptions will later prove highly useful in the application of both the classical and the Bayesian estimation methods.

For this form of the PPP relation to hold, the real exchange rate growth should be zero and the slope parameter has to be one. Under this framework of time varying model estimation with AR(1) processes, we thus have to expect these parameters to be stationary processes around these respective long-run means of zero and one. A priori, given the unit root test reported above this estimation will unsurprisingly yield the desired stationary path for $\beta_{0,t}$. Thus, the result for $\beta_{1,t}$ stands more interesting. In this context, our problem is to estimate the unknown parameter matrices Q and F and to calculate the path of the state vector for given parameters. To do so we will apply both the classical and the Bayesian inference methods. For the former we first maximize the likelihood function for the observation series $\{y_t\}_{t=1}^{T}$. Then forecasted series and the smoothed estimate of the state vector are constructed using the Kalman filter for given maximum likelihood estimates. As for the later, the state vector and the model's parameters are simulated applying Gibs-sampling approach using the appropriate respective natural conjugate priors. These steps are briefly described below.

4 The Classical Inference

4.1 The Maximum Likelihood Function

The likelihood function for the series of exchange rate changes $\{y_t\}_{t=1}^{T}$ that will be maximized to estimate unknown parameters can be constructed given the assumption about the distribution of the initial state ξ_t. Accordingly, we assume $\xi_1\,\tilde{}\,N(\widehat{\xi}_{1|0}, P_{1|0})$, where $\widehat{\xi}_{1|0}$ is the forecast with the information in period zero; i.e. no observation on y_t and x_t, about the state in period one and $P_{1|0}$ represents the forecast mean squared error. Combined with the multivariate Gaussian distribution of the vector of errors, we can obtain the conditional distribution of y_t for given information up to period t, Υ_t , from the observation and state equations above. As such y_t is normally distributed with mean $(x_t\widehat{\xi}_{t|t-1})$ and variance Ω_t where

$$
\begin{aligned}
\Omega_t &= E\left[\left(y_t - x_t\widehat{\xi}_{t|t-1}\right)\left(y_t - x_t\widehat{\xi}_{t|t-1}\right)'\right] \qquad (9a)\\
&= E\left[(x_t\left(\xi_t - \widehat{\xi}_{t|t-1}\right) + w_t)(x_t\left(\xi_t - \widehat{\xi}_{t|t-1}\right) + w_t)'\right]\\
&= E\left[(x_t\left(\xi_t - \widehat{\xi}_{t|t-1}\right)\left(\xi_t - \widehat{\xi}_{t|t-1}\right)' x_t' + w_t w_t')\right]\\
&= x_t' P_{t|t-1} x_t' + R
\end{aligned}
$$

with the expectation of cross term vanishing as w_t is uncorrelated with all realization of ξ_t $t = 1, ..., T$. The sample log likelihood function is then

$$
\sum_{t=1}^{T} \log f(y_t|x_t, \Upsilon_t\ ) = -(T/2)\log(2\pi) - (1/2)\sum_{t=1}^{T}\log(\Omega_t) - (1/2)\sum_{t=1}^{T}(y_t - x_t\widehat{\xi}_{t|t-1})^2/\Omega_t \qquad (10)
$$

where $\{\Omega_t\}_{t=1}^{T}$and $\left\{{}_t\widehat{\xi}_{t|t-1}\right\}_{t=1}^{T}$ are obtained from the Kalman Filter equations.

4.2 Filter Equations

As mentioned above, the evaluation of the likelihood function is made possible by the use of the Kalman Filter, that would recursively produce the optimal estimate and projections of the state vector at time t by using observations up to time t and $t-1$ respectively denoted as $\widehat{\xi_{t|t}}$ and $\widehat{\xi_{t|t-1}}$. To generate such state vector, the filter has to be initialized with the forecast of the state at period 0. Let $\widehat{\xi_{t|t-1}}$ denote this forecast which is just the unconditional expectation of the state vector;

$$
\widehat{\xi_{1|0}} = \widehat{E}(\xi_1) \qquad (11)
$$

while the expected squared error associated with this forecast; which thus summarizes the researcher's belief in its initial forecast is denoted as

$$P_{1|0} = E\left\{\left(\xi_1 - \widehat{\xi_{1|0}}\right)\left(\xi_1 - \widehat{\xi_{1|0}}\right)'\right\} \tag{12}$$

Once the vector and the matrix $\widehat{\xi_{t|t-1}}$ and $P_{t|t-1}$ are in hand the following steps are iterated.

$$\widehat{\xi_{t|t}} = \widehat{\xi_{t|t-1}} + \left\{P_{t|t-1}x_t'\Omega_t^{-1}\left[y_t - x_t\widehat{\xi_{t|t-1}}\right]\right\} \tag{13}$$

$$\widehat{\xi_{t+1|t}} = F\widehat{\xi_{t|t}} \tag{14}$$

$$P_{t|t} = P_{t|t-1} - \left\{P_{t|t-1}x_t'\Omega_t^{-1}x_tP_{t|t-1}\right\} \tag{15}$$

$$P_{t+1|t} = FP_{t|t}F' + Q \tag{16}$$

where$x_t\widehat{\xi_{t|t-1}} = E\left(y_t|\Upsilon_{t-1}\right)$ and Ω_t is the variance of y_t as mentioned above. Equations 13 to 16 are the Kalman Filter equations. In constructing our forecasts $\widehat{\xi_{t|t-1}}$, we let $\widehat{\xi_{1|0}} = (0,1,1)'$ thus assuming that the state vector was at the long run average that would make the PPP relationship hold. As for $P_{1|0}$ we considered three different cases. In one case we formulated $P_{1|0}$ such that the variance of the states are expressed by the unconditional variance of their corresponding AR(1) processes as $\sigma_v^2/\left(1-\rho^2\right)$. In the other two cases we set the variances to be 1% and 1 for both states respectively. Finally, each time, $P_{1|0}$ is a diagonal matrix with the variance of the third state unambiguously set to zero.

4.3 Smoothing

We are also interested in the inference about the va;ues of the state vector ξ_t based on the full data set. This sequence of smoothed estimates $\left\{\widehat{\xi_{t|T}} \equiv \widehat{E}(\xi_t|\Upsilon_T)\right\}_{t=1}^{T}$ is obtained after the Kalman filter describes above is calculated. This iteration starts with $\widehat{\xi_{T|T}}$ which is simply the last value of the sequence $\left\{\widehat{\xi_{t|t}}\right\}_{t=1}^{T}$. The estimate of $\widehat{\xi_{t|T}}$ for $t = T-1, T-2, ..., 1$ proceeds backwards according to the following formula:

$$\widehat{\xi_{t|T}} = \widehat{\xi_{t|t}} + P_{t|t}F'P_{t+1|t}^{-1}\left(\widehat{\xi}_{t+1|T} - \widehat{\xi}_{t+1|t}\right) \tag{17}$$

where $\widehat{\xi}_{t+1|T} - \widehat{\xi}_{t+1|t}$ are computed through the from the Kalman filter equations. Finally and analogously, the error associated with this estimate has the squared mean

$$E\left\{\left(\xi_1-\widehat{\xi_{1|0}}\right)\left(\xi_1-\widehat{\xi_{1|0}}\right)'\right\} = P_{t|T} \tag{18}$$
$$= P_{t|t}+\left(P_{t|t}F'P_{t+1|t}^{-1}\right)\left(P_{t+1|T}-P_{t+1|t}\right)\left(P_{t|t}F'P_{t+1|t}^{-1}\right)'$$

[1]

5 Bayesian Inference

As opposed to the classical inference, under the Bayesian approach the model's parameters in the matrices F, Q, and R together with the state vector ξ_t are treated as random. To apply Gibbs sampling based on the set of conditional posterior densities, two steps are to be iterated large enough times for a good approximation of the true distributions. These steps are:

1) $\{\xi_t\}_{t=1}^{T}$ is generated conditional on $\{y_t\}_{t=1}^{T}$ and on the model's parameters in F, Q, and R; which is initialized using a draw from the prior distribution of these parameters. for arbitrary.

2) F, Q, and R are generated conditional on $\{\xi_t\}_{t=1}^{T}$ and on $\{y_t\}_{t=1}^{T}$.

While the first step requires the use of the Kalman filter, the second simply involves a standard Bayesian estimation of a linear regression model given in the latter equality in equation 5; due to the independence assumption between w_t and v_t.

5.1 Generating the Sate Vector

Let $\widetilde{\xi}_T = [\xi_1, ...\xi_T]$ and $\widetilde{y}_T = [y_1, ...y_T]'$. We generate $\widetilde{\xi}_T$ from the joint distribution $p(\widetilde{\xi}_T|\widetilde{y}_T)$ which can be expressed as follows using the Markov property of the state-space model; implying that information beyond $t-1$ does not affect the current state t.

$$p(\widetilde{\xi}_T|\widetilde{y}_T) = p(\xi_T|\widetilde{y_T})\Pi_{t=1}^{t=T} p(\xi_t|\xi_{t+1}, \widetilde{y}_t) \tag{19}$$

where $\widetilde{y}_t = [y_1, ...y_t]$. Thus the vector $\widetilde{\xi}_T$ can be generated from the individual pieces $p(\xi_T|\widetilde{y_T})$, and $p(\xi_t|\xi_{t+1}, \widetilde{y}_t)$ for $t = 1, ..., T$. The linearity and the Gaussian assumptions applied to the state-space model imply that the conditional distributions of ξ_T and ξ_t are also Gaussian with

$$\xi_T|\widetilde{y}_T \sim N(\xi_{T|T}, P_{T|T}) \tag{20}$$

and

$$\xi_t|\widetilde{y}_t, \xi_{t+1} \sim N(\xi_{t|t,\xi_{t|t+1}}, P_{t|t,\xi_{t|t+1}}) \tag{21}$$

[1] Hamilton (1994) Ch.13 provides the details of the Kalman filter equations and the estimation of the smoothed state vector.

Notice that $\xi_{T|T}$ is the updated estimate of the state in period T using the new information in y_T while $\xi_{t|t,\xi_{t|t+1}}$ represents an update additionally combining information in y_{t+1} and thus in ξ_{t+1}. The accompanying covariance thus represent the error associated with these forecasts. These can be obtained from the updating procedure of the Kalman filter given in equation 13 above. The former is simply the last iteration of this procedure. Then $\xi_{t|t,\xi_{t|t+1}}$ and $P_{T|T,\xi_{t|t+1}}$ are derived backwards from

$$\xi_{t|t,\xi_{t|t+1}} = \xi_{t|t} + P_{t|t}F'(FP_{t|t}F' + Q)^{-1}(\xi_{t+1} - F\xi_{t|t}) \tag{22}$$

$$P_{t|t,\xi_{t|t+1}} = P_{t|t} - P_{t|t}F'(FP_{t|t}F' + Q)^{-1}FP_{t|t} \tag{23}$$

for $t = T-1, ..., 1$. Here $(\xi_{t+1} - F\xi_{t|t})$ is the forecast error for ξ_{t+1} as suggested by equation 5 thereby representing the additional information contained in ξ_{t+1} with the variance of this error given by $FP_{t|t}F' + Q$. The path of the state vector is based on 2000 drawings from the posterior distribution. The mean of the last 1500 draws across iterations are computed to generate Figure 4.

5.2 Generating the Model's Parameters

Given $\widetilde{\xi}_T$, the second equality in equation 5 can be treated as two independent linear regression due to the independence between w_t and v_t. To estimate the parameters of the state equation, we first specify the conditional prior distributions for Q and F. Accordingly we assume an inverted Gamma distribution for the elements in the former and a normal distribution for those in the latter.

Let $\phi_0 = [\mu_0\ \rho_0]'$ and $\phi_1 = [\mu_1\ \rho_1]'$, thus conditional prior distributions of ϕ_0 and ϕ_1 given $\sigma^2_{v_0}$ and σ^2_{v1} respectively are

$$\phi_0|\sigma^2_{v_0} \sim N(\phi_{00}, \Sigma_{00}) \tag{24}$$

$$\phi_1|\sigma^2_{v1} \sim N(\phi_{10}, \Sigma_{10}) \tag{25}$$

where we set $\phi_{00} = [0\ 0.8]'$, $\phi_{10} = [0.5\ 0.5]'$ to result in long-run mean of $\beta_{0,t}$ and $\beta_{1,t}$as zero and one respectively. The covariances are determined as $\Sigma_{00} = \Sigma_{00} = I_2$. Combining with the likelihood function $L(\phi_i|\sigma^2_{v_i}, \beta_{i,T}) = \left(2\pi\sigma^2_{v_i}\right)^{-T/2} \exp(\beta_{i,T} - \beta_{i,T-1}\phi_i)'(\beta_{i,T} - \beta_{i,T-1}\phi_i)$ for $i = 0, 1$ and $\beta_{i,T} = [\beta_{i,2}, ..., \beta_{i,T}]'$, the conditional posterior density is given by

$$\phi_i|\sigma^2_{vi} \sim N(\phi_{i1}, \Sigma_{i1}) \tag{26}$$

where

$$\phi_{i1} = \left(\Sigma^{-1}_{i0} + \sigma^{-2}_{vi}\beta'_{i,T-1}\beta_{i,T-1}\right)^{-1}\left(\Sigma^{-1}_{i0}\phi_{i0} + \sigma^{-2}_{vi}\beta'_{i,T-1}\beta_{i,T}\right) \tag{27}$$

$$\Sigma_{i1} = \left(\Sigma^{-1}_{i0} + \sigma^{-2}_{vi}\beta'_{i,T-1}\beta_{i,T-1}\right)^{-1} \tag{28}$$

The prior distribution of the σ^2_{vi} $i = 0, 1$ for given respective ϕ_i takes the form

$$\sigma^2_{vi}|\phi_i \sim IG\left(\frac{v_0}{2}, \frac{\delta_0}{2}\right) \tag{29}$$

where we specified v_0 and δ_0 arbitrarily as zero. Multiplied with the likelihood function $L(\sigma^2_{v_i}|\phi_i, \beta_{i,T}) = \left(2\pi\sigma^2_{v_i}\right)^{-T/2} \exp(\beta_{i,T} - \beta_{i,T-1}\phi_i)'(\beta_{i,T} - \beta_{i,T-1}\phi_i)$, the conditional posterior distribution is again given by the inverted gamma distribution as

$$\sigma^2_{vi}|\phi_i \sim IG\left(\frac{v_1}{2}, \frac{\delta_1}{2}\right) \tag{30}$$

with

$$\begin{aligned} v_1 &= v_0 + T & (31) \\ \delta_1 &= \delta_0 + (\beta_{i,T} - \beta_{i,T-1}\phi_i)'(\beta_{i,T} - \beta_{i,T-1}\phi_i) & (32) \end{aligned}$$

The conditional posterior distribution of σ^2_w given $\widetilde{\xi}_T$ is derived analogously using the likelihood function $L(\sigma^2_w|\widetilde{\xi}_T, Y\,\{y_t\}^T_{t=1}) = \left(2\pi\sigma^2_w\right)^{-T/2} + (1/2)\sum^T_{t=1} \exp((y_t - x_t\widehat{\xi}_t)^2/\sigma^2_w)$; which differs from that in equation 10 as the likelihood here is derived for fixed ξ_t. Then

$$\sigma^2_w|\widetilde{\xi}_T \sim IG\left(\frac{v_1}{2}, \frac{\delta_1}{2}\right) \tag{33}$$

with

$$\begin{aligned} v_1 &= v_0 + T & (34) \\ \delta_1 &= \delta_0 + (Y - X\widetilde{\xi}_T)'(Y - X\widetilde{\xi}_T) & (35) \end{aligned}$$

for arbitrarily pre-specified v_0 and δ_0 as zero.

Posterior means for these parameters in Q, R and F are computed at each iteration for the state vector. Each posterior mean and the corresponding standard deviation is obtained from 2000 draws from the respective posterior distributions; where the first 500 are ignores. Finally, the values in table 4 are computed as the average of these posterior means and standard deviations across iterations of the state vector.[2]

[2] Kim and Nelson (1999) provide the details of the Bayesian inference of the state-space models using the Gibbs sampling.

6 Results

The purchasing power parity is a view of the long-run exchange rate behavior depicting a link between a country's overall price level and its exchange rate. Its strongest form; known as the absolute PPP (APPP) which implies a real exchange rate of unity is hard to hold though even if the law of one price holds far all commodities. This is mainly due to differing composition and weighting scheme in the construction of price indices by individual countries together with market imperfections. The research on the validity of PPP then rather focused on examining the stability of the real exchange rate. In this respect, the recent literature is divided into two competing views. One stream of research reports that the PPP has a poor empirical value for the determination of the exchange rate; due to the findings of random walk behavior in the real exchange rate. The other stream in contrast, demonstrate with more powerful tests that the PPP in fact hold in the long run as suggested by the findings of though slow mean-reversion.

In this study we considered the relative RPPP which states that the percentage change in the exchange rate between two countries equals the inflation differentials. The examination of the monthly data on the Japanese Yen to U.S. dollar exchange rate for the post-Bretton Woods era until 2006 revealed the following. The results from the classical inference approach are shown in tables 3 and 4 and figures 3.to 6. Unsurprisingly; both the forecasts and the smoothed estimates of the intercept parameter; i.e. the change in the real exchange rate; is fairly stable around zero as can be seen on figures 3 and 5. These figures are produced for the case where the variances of the initial state estimates are 1; for the sake of brevity and due to the observation that the choice of $P_{1|0}$ is irrelevant for the results. This stable autoregressive path can be also traced from the parameter estimates of the respective state equation. Accordingly, irrespective of the covariance of the forecast error for our belief of this state's initial value, ρ_0 comes at around 0.5 and μ_0 very close to zero yielding a zero long-run mean as reported on table 3. However, the behavior of the intercept; the other state variable; appears to violate the RPPP. While a stable behavior around unity is required for this relation to hold in the long run, the smooth estimate of this state wanders around zero and its forecast stands very close to zero which can also be confirmed on table 3.

As for the results from the Bayesian inference, while the implied long-run means for the state variables are somewhat maintained at near zero, individual parameter estimates differ noticeably. All parameters but ρ_1 are on average lot smaller with mostly a higher standard deviation than their counterpart value under the classical approach. Yet all parameters are again near zero. Additionally the posterior mean of the respective σ^2 are also noticeably smaller. Nevertheless, despite these discrepancies; the main message remains unchanged across these methods. As the graphs of the Bayesian estimate for the state variable's path display on figures 7 and 8, while the real exchange rate moves around zero, the behavior of the slope parameter violates the RPPP.

7 Conclusion

In this study we considered the weak version of the PPP theory, namely the relative PPP (RPPP);.that states that the percentage change in the exchange rate between two countries is equal to the these countries' inflation differentials. We worked with monthly data on the Japanese ¥ to U.S.$ exchange rate for the floating rate period from 1971 to the end of 2006. We examined the long-run validity of this equality by formulating a state-space model with time varying parameters. Using the Kalman filter and applying both classical and Bayesian inference methods we reached to the conclusion that on a month-on-month basis, the RPPP fails to explain the long-run behavior of the change in the ¥/$ nominal exchange rate. Despite differences in the point estimates of the state-space model's parameters accross the inference methods, both approaches yielded a stationary change in the real exchange rate around zero. However, the series for the slope parameter jeopardized the validity of this relation due to values oscillating far from unity.

The following extensions can be suggested for this analysis. The first one concerns the forecast horizon in the RPPP relation. The monthly horizon used here might be blamed to be too restrictive. In this respect, annual data might be more promising as mentioned in the studies of Abuaf and Jorion (1990) and Diebold, Husted and Rush (1991). Secondly, disaggregation of the price indices to trim the prices of non-traded good would also potentially improve the results. Finally other currencies should also be subjected to this method to reach a final conclusion about the validity of the RPPP.

8 Appendix

TABLES

Table 1: OLS Estimates

Dependent Variable: $\widehat{s}_t$	β_0	β_1	Adjusted R^2
Yen / U.S. Dollar	-0.003	-0.099	-0.002
	(0.002)	(0.240)	

* Notes: Standard errors in parantheses.

Table 2: The Unit Root Test

Augmented Dicky-Fuller Unit Root Test				
	Level		First Difference	
	Test Statistic	P-value	Test Statistic	P-value
Real Exchange Rate	-0.055	0.664	-18.816	0.000

Table 3: Parameter Estimates - Matrix F and R - Classical Approach

ρ_0	ρ_1	μ_0	μ_1	$\mu_0/(1-\rho_0)$	$\mu_1/(1-\rho_1)$
PANEL A - unconditional variance					
0.59**	1.50*10^{-5}	2.93*10^{-11}	2.38*10^{-6}	7.13*10^{-11}	2.38*10^{-6}
(0.08)	(0.01)	(1.38*10^{-6})	(0.217)	n.a.	n.a.
PANEL B - 1%					
0.55**	2.80*10^{-5}	1.32*10^{-9}	5.43*10^{-6}	2.93*10^{-9}	5.43*10^{-6}
(0.08)	(0.01)	(1.47*10^{-6})	(0.217)	n.a.	n.a.
PANEL C - 1					
0.55**	3.28*10^{-5}	3.01*10^{-9}	6.42*10^{-6}	6.68*10^{-9}	6.42*10^{-6}
(0.08)	(0.01)	(1.48*10^{-6})	(0.218)	n.a.	n.a.

* Notes: Standard errors in parantheses. The last two colums report the unconditional mean of the state variables. Across panels A to C, the variance-covariance matrix of the initial state is set equal to the unconditional variance of the AR(1) state variable processess, 1

Table 4: Parameter Estimates - Matrices Q and R - Classical Approach

σ_{v_0}	σ_{v1}	σ_w
PANEL A - unconditional variance		
0.01**	0.66**	0.03**
(4.19*10^{-5})	(0.049)	(7.09*10^{-6})
PANEL B - 1%		
0.01**	0.663**	0.03**
(4.35*10^{-5})	(0.043)	(8.20*10^{-6})
PANEL C - 1		
0.01**	0.664**	0.03**
(4.36*10^{-5})	(0.051)	(7.01*10^{-5})

* Notes: Standard errors in parantheses. Across panels A to C, the variance-covariance matrix of the initial state is set equal to the unconditional variance of the AR(1) state variable processess, 1

Table 5: Estimates of the State Equations - Bayesian Approach

	Prior		Posterior	
	Mean	Standard Deviation	Mean	Standard Deviation
ρ_0	0.8	1	$0.44*10^{-3}$	0.470
μ_0	0.0	1	$-0.30*10^{-5}$	0.031
ρ_1	0.5	1	$0.20*10^{-3}$	0.404
μ_1	0.5	1	$0.36*10^{-6}$	0.032
$\sigma^2_{v_0}$	0.5	0.01	$0.79*10^{-5}$	0.001
$\sigma^2_{v_1}$	0.5	0.01	$0.08*10^{-4}$	0.002
σ^2_w	0.5	0.01	$0.50*10^{-5}$	$0.01*10^{-2}$

* Notes: Posterior means and standard deviation in this table are computed as the average of the posterior means and standard deviations obtained at each iterations for the state vector.

GRAPHS

Figure 1: The Real Exchange Rate (¥/$)

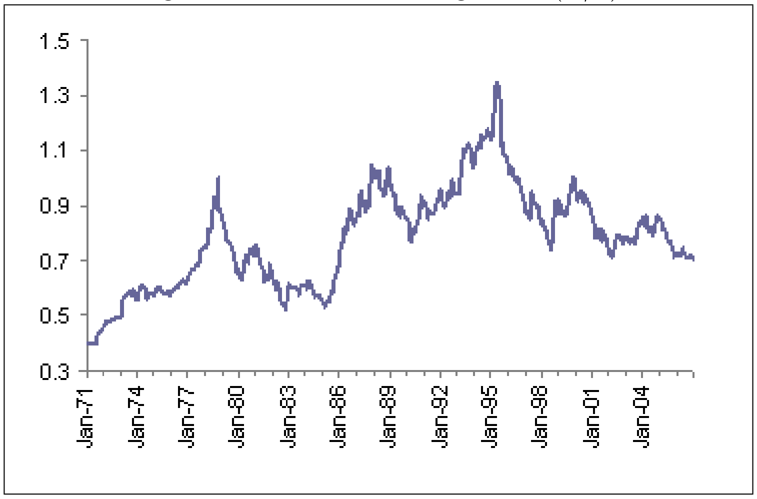


Figure 2: The Real Exchange Rate (¥/$) - First Difference

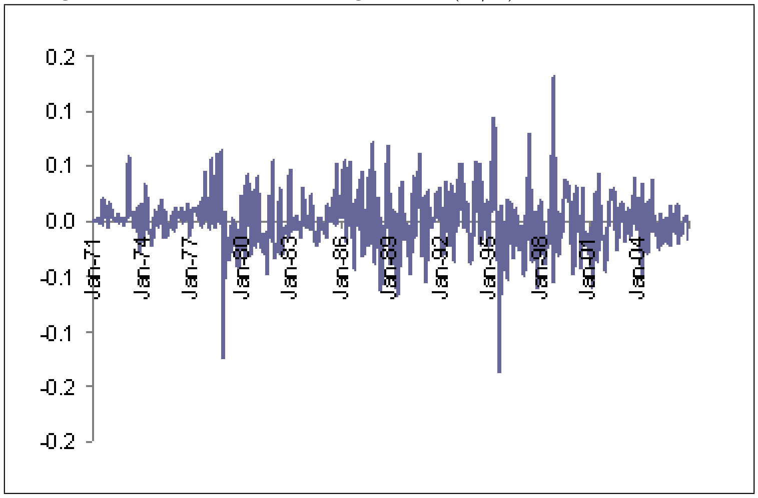


Figure 3: Forecasts of the Interecept Parameter $\beta_{0,t}$ - Classical Approach

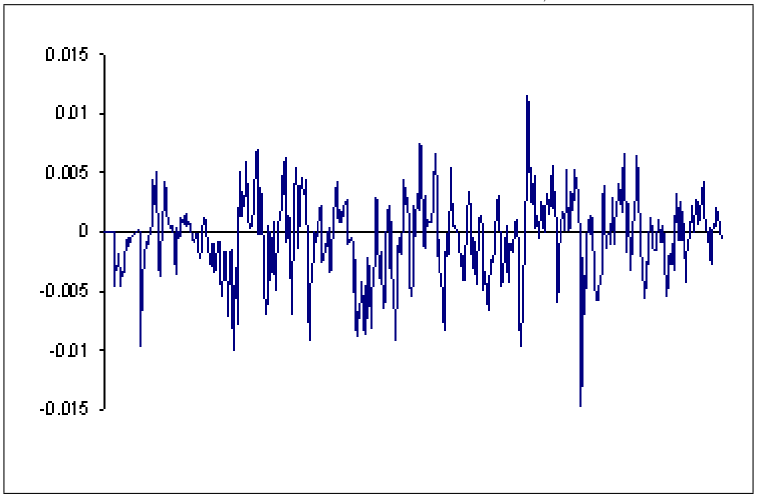


Figure 4: Forecasts of the Slope Parameter $\beta_{1,t}$- Classical Approach

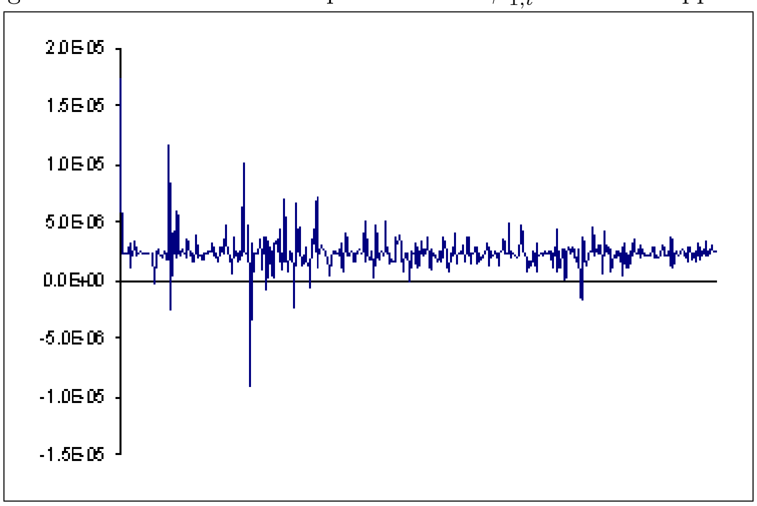


Figure 5: Smooth Estimate of the Interecept Parameter $\beta_{0,t}$ - Classical Approach

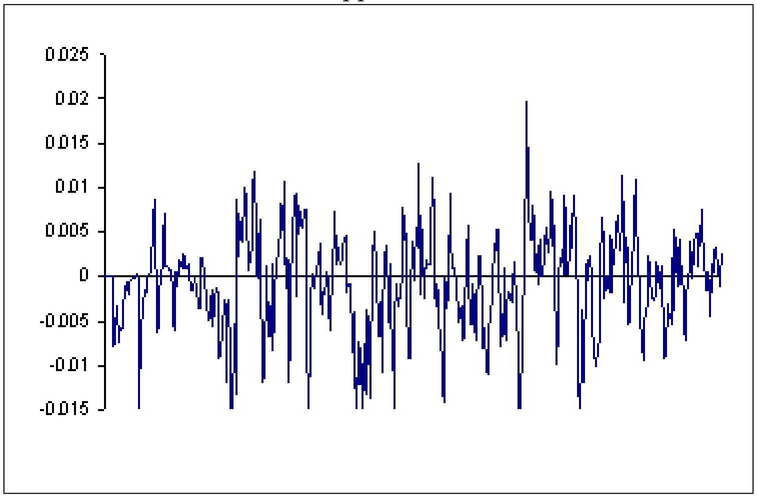


Figure 6: Smooth Estimate of the Slope Parameter $\beta_{1,t}$ - Classical Approach

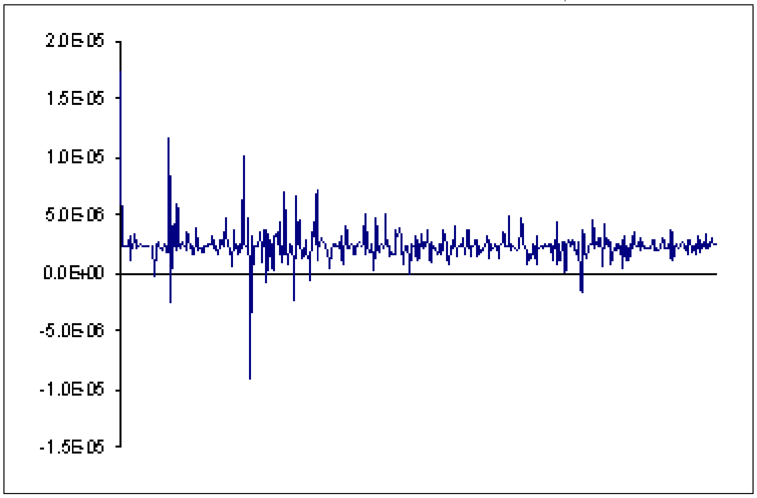


Figure 7: Smooth Estimate of the Interecept Parameter $\beta_{0,t}$ - Bayesian Approach

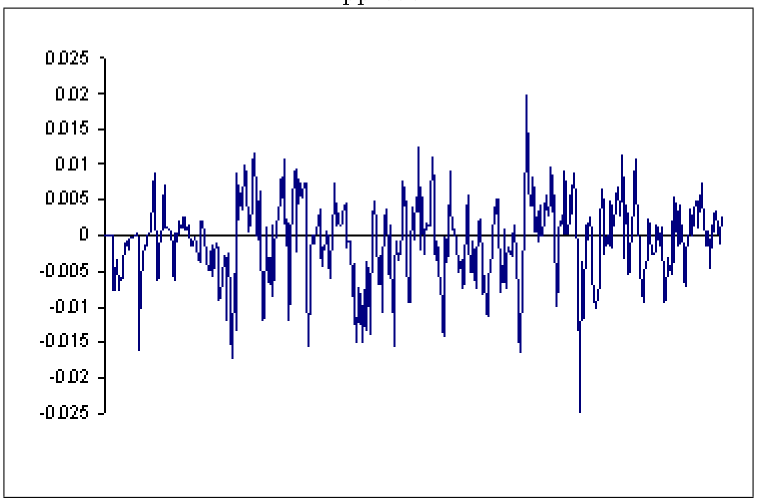
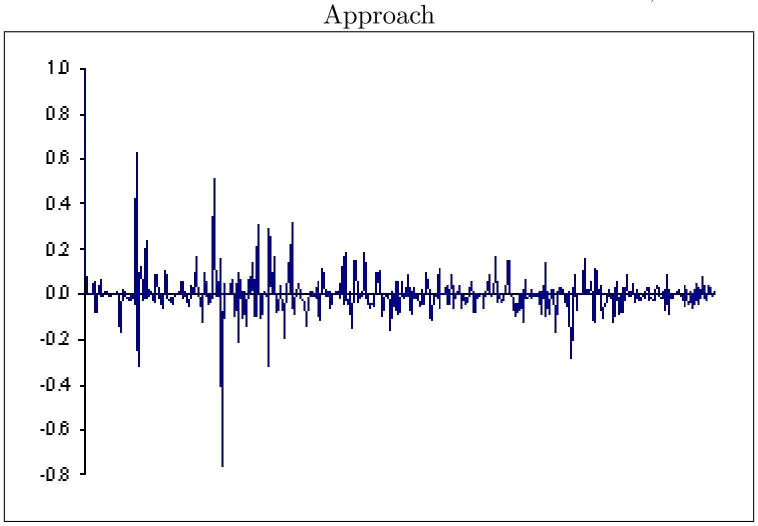


Figure 8: Smooth Estimate of the Interecept Parameter $\beta_{1,t}$ - Bayesian Approach



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