A SIMPLE CALCULATOR AND DRAGON CURVES

Lecture notes of the course “*Programming Techniques*”

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1. **Simple calculator**
   - Prefix, infix and postfix expressions
   - Shunting yard algorithm
   - Evaluating postfix expressions

2. **Dragon curves**
   - Dragon curves
   - Drawing dragon curves

3. **Exercises**

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Prefix, infix and postfix expressions

- Stacks and queues allow us to design a simple expression evaluator.
- Prefix, infix and postfix notation: operator before, between and after operands, respectively.
- Infix is more natural to write, postfix is easier to evaluate.

Examples:

Infix	**Prefix**	**Postfix**
$A + B$	$+AB$	$AB+$
$A * B - C$	$- * ABC$	$AB * C-$
$(A + B) * (C - D)$	$* + AB - CD$	$AB + CD - *$

Prefix, infix and postfix expressions

- How can we convert an infix expression to its postfix expression?

$$\underbrace{(A+B)*(C-D)}_{\text{infix}} \Longrightarrow \underbrace{AB+CD-*}_{\text{postfix}}$$

- The "Shunting yard" algorithm of Dijisktra (1961):
  - uses two queues to store input and output
  - use a separate stack for holding operators
- We shall consider the simplest version of this problem where we have *only binary operators*.

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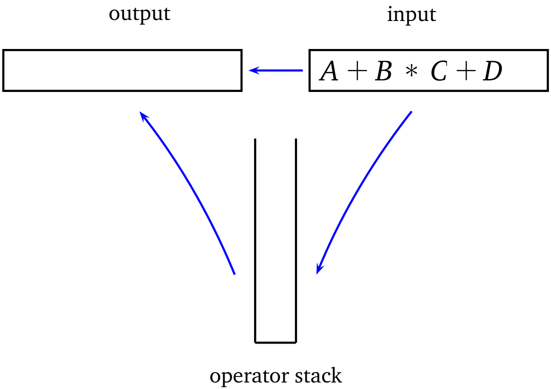
Shunting yard algorithm



Shunting yard algorithm

1. Dequeue token from input;
2. If the token is an operand (number), add it to output queue;
3. If it is an operator then pop operators off stack and add to output queue as long as:
    - top operator on stack has higher precedence, or
    - top operator on stack has same precedence and is *left-associative*

   and push new operator onto stack;
4. Return to step 1 as long as tokens remain in input;
5. Pop remain operators from stack and add to output queue.

Operator associativity

- A property that determines how operators of the same precedence are grouped in the absence of parentheses.
- Operator may be *left-associative* or *right-associative*, meaning that the operators are grouped from the left or from the right.
- Example: consider the expression $a \sim b \sim c$.
  - If the operator $\sim$ is left-associative, the expression would be evaluated as $(a \sim b) \sim c$.
  - If the operator $\sim$ is right-associative, the expression would be evaluated as $a \sim (b \sim c)$.

Normal mathematical usage:

- Addition, subtraction, multiplication and division operators are usually left-associative.
- Exponentiation and assignment operators are typically right-associative.

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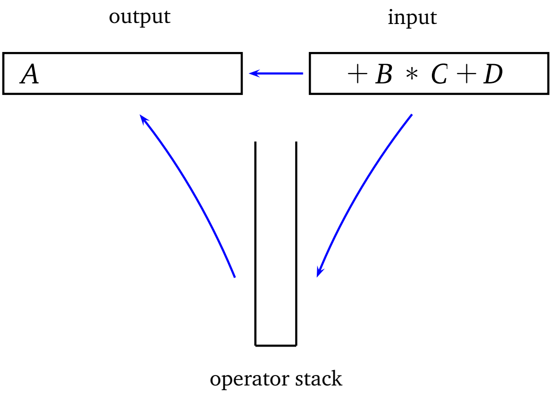

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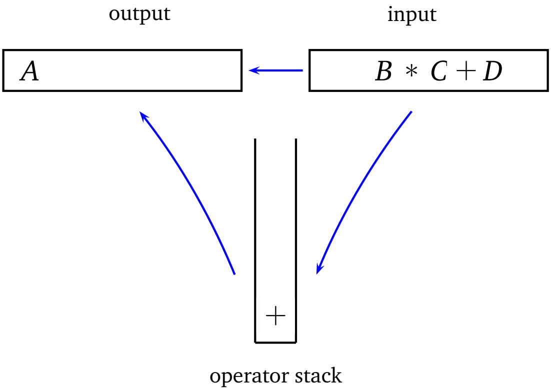

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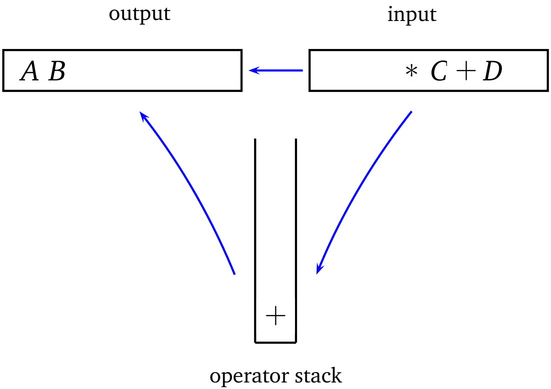

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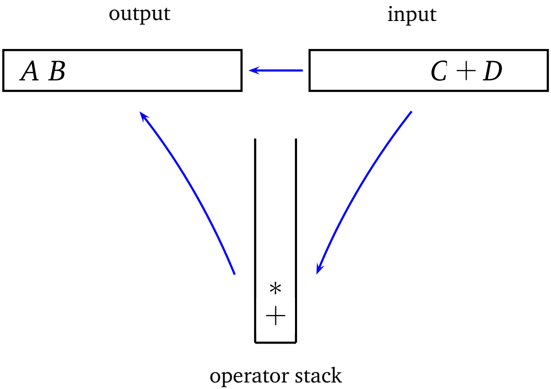

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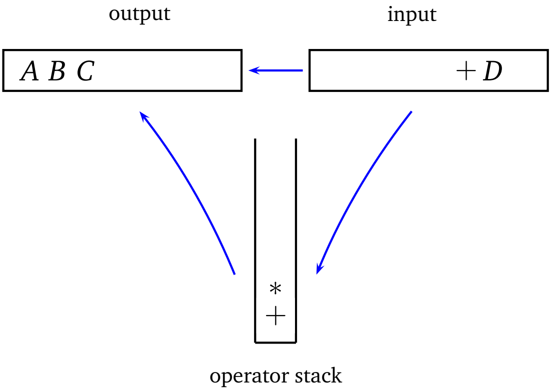

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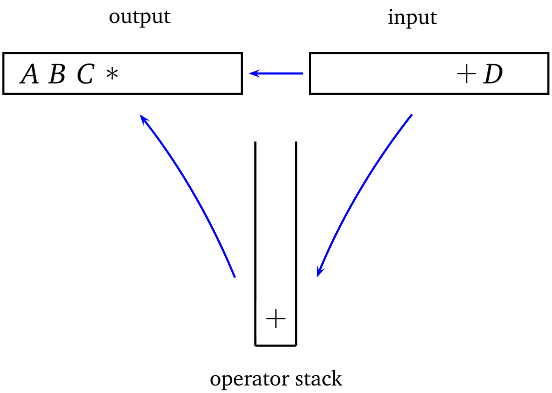

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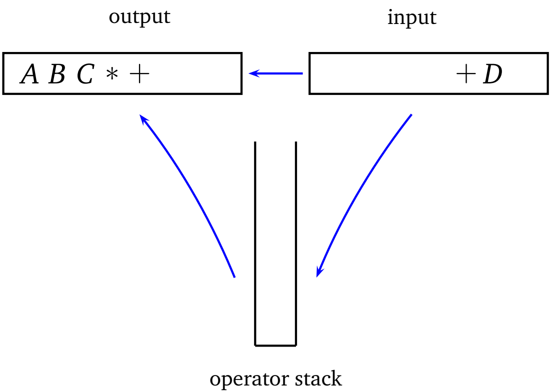

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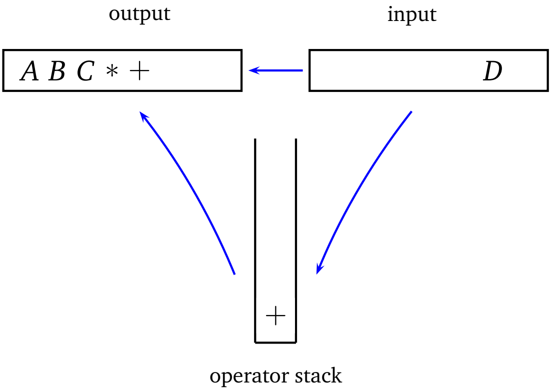

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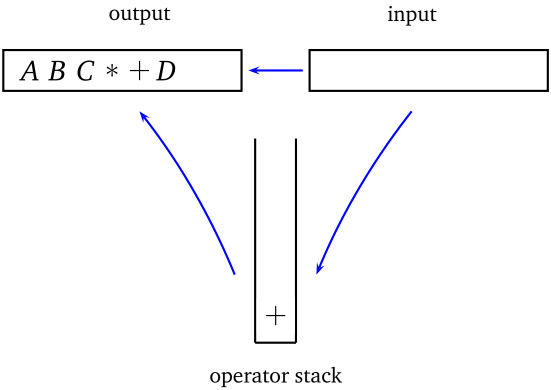
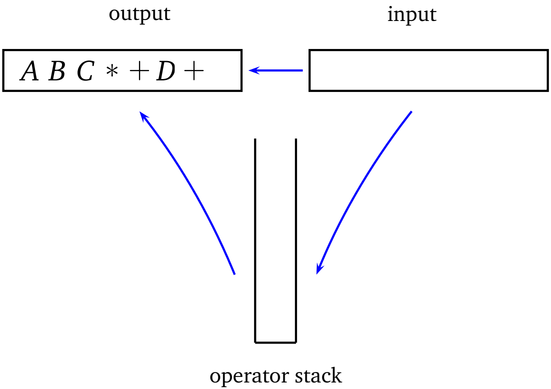


Shunting yard algorithm



Shunting yard algorithm

- What if infix expression includes parentheses?
- Example:

$$(A + B) * (C - D) \Longrightarrow ?$$

- For short, we use a table to track steps of the algorithm.

Shunting yard algorithm

Token	Output	Stack
$($		$($
A	A	$($
$+$	A	$(+$
B	AB	$(+$
$)$	$AB+$	
$*$	$AB+$	$*$
$($	$AB+$	$*($
C	$AB+C$	$*($
$-$	$AB+C$	$*(-$
D	$AB+CD$	$*(-$
$)$	$AB+CD-$	$*$
end	$AB+CD-*$	

The result is $AB+CD-*$.

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Evaluating postfix expressions

Postfix evaluation is easy with a stack. Here is the algorithm:

1. Dequeue a token from the postfix queue;
2. If token is an operand, push it onto stack;
3. If token is an operator:
   - pop operands off stack (2 operands for binary operator);
   - evaluate the expression;
   - push result onto stack;
4. Repeat until postfix queue is empty;
5. Item remaining in stack is the final result.

Evaluating postfix expressions

Example: evaluating the expression $6\,5 + 7\,2 - *$

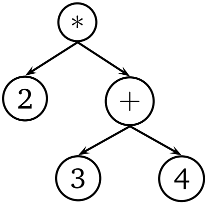
Token	Stack	Evaluation
6	6	
5	$6, 5$	
$+$	11	compute $6 + 5 = 11$
7	$11, 7$	
2	$11, 7, 2$	
$-$	$11, 5$	compute $7 - 2 = 5$
$*$	55	compute $11 * 5 = 55$

The result is 55. Note that the equivalent infix expression is

$$(6 + 5) * (7 - 2).$$

Expression trees

- We can use binary trees to represent expressions: leaf nodes are operands, internal nodes are operators.
- Prefix, infix and postfix expressions are obtained by using pre-order, in-order and post-order traversal on a tree.



- Infix expression: $2 * (3 + 4)$.

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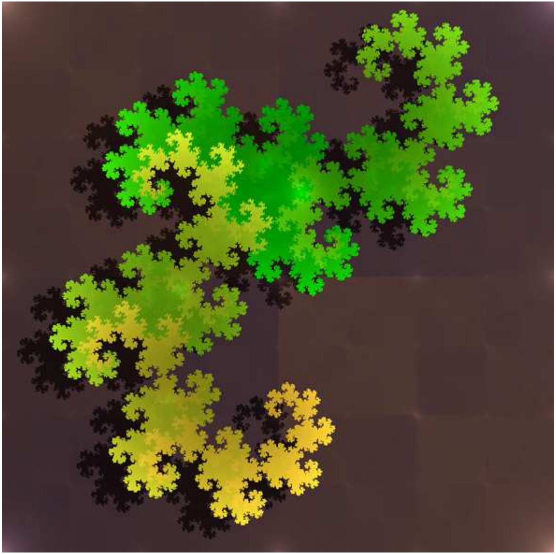
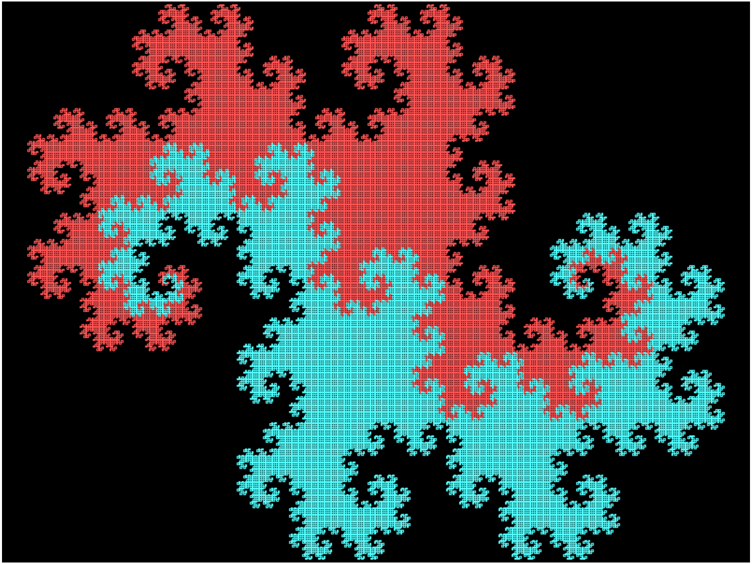
Dragon curves

- A dragon curve is any member of a family of **self-similar fractal curves**, which can be approximated by recursive methods.
- The Jurassic Park dragon was first invented by NASA physicists (John Heighway, Bruce Banks and William Harter).
- It was described in 1967 by Martin Gardner in Mathematical Games column.
- More about this:
  - `http://en.wikipedia.org/wiki/Dragon_curve`
  - `http://mathworld.wolfram.com/DragonCurve.html`

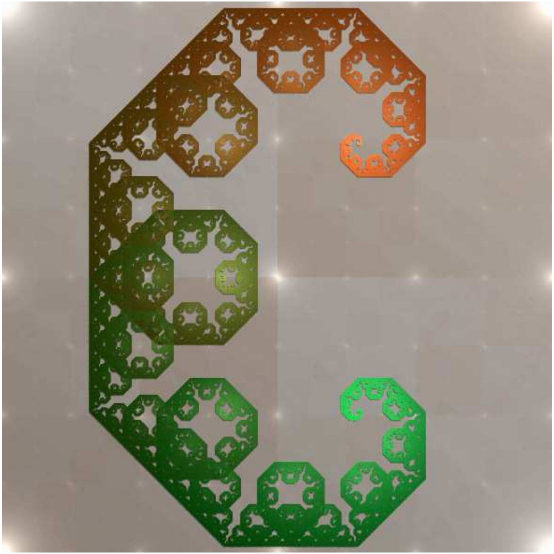
Dragon



Twindragon – Davis–Knuth dragon



Lévy dragon – Lévy C curve



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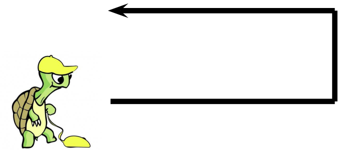
Drawing dragon curves

- How can we draw a dragon programmatically?
- Simple techniques: strait lines and recursive operations
- Simple tools:
  - Postscript (PDF) commands for graphical lines
  - Text file I/O in C

Drawing dragon curves

- Use simple "turtle graphics":
  - `F` (forward): move turtle forward one step (pen down)
  - `L` (left): turn left 90 degrees
  - `R` (right): turn right 90 degrees
- Example: `F L F L F`



Drawing dragon curves

- `dragon(0): F`
- `dragon(1): F L F`
- `dragon(2): F L F L F R F`
- `dragon(3): F L F L F R F L F L F R F R F`
- `dragon(4):`
  - `dragon(3): F L F L F R F L F L F R F R F`
  - `L`
  - `nogard(3): F L F L F R F R F L F R F R F`

Rule?

- The first part of `dragon(n)` is `dragon(n-1)`.
- The second part of `dragon(n)` is the backward of `dragon(n-1)`: reverse string, switch `L` and `R`.

Drawing dragon curves

- `dragon(0): F`
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  - `dragon(3): F L F L F R F L F L F R F R F`
  - `L`
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Simple PostScript commands

```
FILE *f;

void forward() {
  fprintf(f, "5 0 rlineto\n");
}

void left() {
  fprintf(f, "90 rotate\n");
}

void right() {
  fprintf(f, "-90 rotate\n");
}
```

Recursive functions

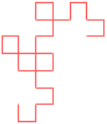
<pre>void dragon(int n) {
  if (n == 0)
    forward();
  else {
    dragon(n-1);
    left();
    nogard(n-1);
  }
}</pre>
	<pre>void nogard(int n) {
	  if (n == 0)
	    forward();
	  else {
	    dragon(n-1);
	    right();
	    nogard(n-1);
	  }
	}</pre>

The main function

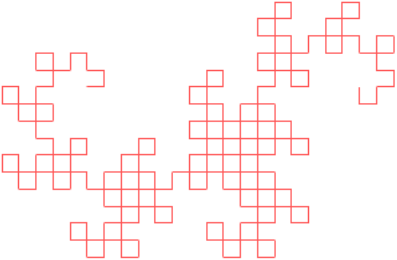
```
int main(int argc, char **argv) {
  f = fopen("dragon.ps", "w");
  fprintf(f, "%%!PS\n");
  fprintf(f,"%% Set the page size to A4\n");
  fprintf(f,"<< /PageSize [595 842] >> setpagedevice\n");
  fprintf(f, "0.1 setlinewidth\n");
  fprintf(f, "400 400 moveto\n");
  dragon(12);
  fprintf(f, "1 0 0 setrgbcolor\n");
  fprintf(f, "stroke\n");
  fprintf(f, "showpage\n");
  fclose(f);
  return 0;
}
```

Examples: $n = 5$, $n = 8$

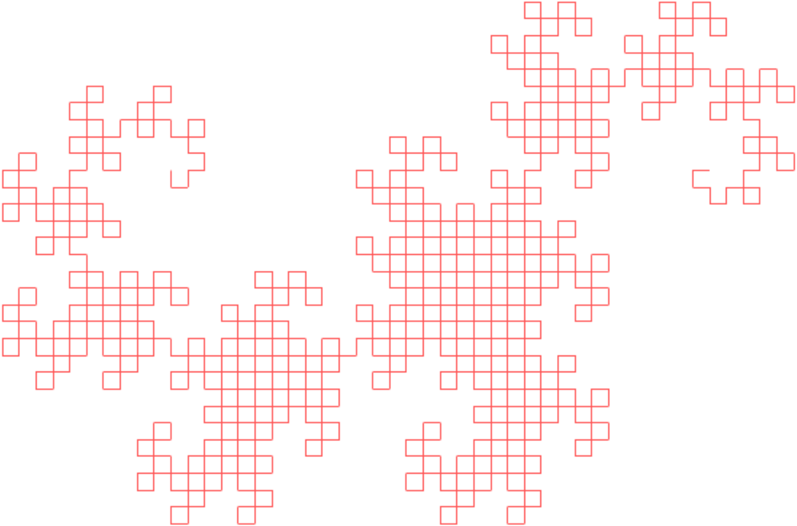
$n = 5$:



$n = 8$:



Examples: $n = 10$



Examples: $n = 12$

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Exercises

Exercise 1. Implement the shunting yard algorithm for converting an infix expression to its postfix expression.

Exercise 2. Implement the algorithm for evaluating a postfix expression.

Exercise 3. Implement a program which draws dragon curves.

- The program accepts an argument which is n;
- Try to use different colors for the dragons.