Matemática Discreta

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Matemática Discreta

Ementa da disciplina:

- 1 - O que é uma prova?
  - Proposições
  - Axiomas
  - Deduções Lógicas
  - Exemplos de provas

- 2 - Indução Simples

- 3 – Indução Forte

- Trabalho 1

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Ementa da disciplina:

- 4 – Teoria dos Números I
  - Divisibilidade
  - Aritmetica Modular
- 5 – Teoria dos Números II
  - Alguns teoremas
- 6 – Relações
- Trabalho 2

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Ementa da disciplina:

- 7 – Somatórios, Aproximações e Assintótica
- 8 - Recorrências
- Trabalho 3
- Prova Final

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Avaliação e Bibliografia:

- Trabalhos + Avaliação Final

- Bibliografia

- Mathematics for Computer Science, Eric Lehman and Tom Leighton, 2004

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What is a Proof?

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What is a Proof?

Jury trial. Truth is ascertained by twelve people selected at random.

Word of God. Truth is ascertained by communication with God, perhaps via a third party.

Experimental science. The truth is guessed and the hypothesis is confirmed or refuted by experiments.

Inner conviction. "*My* program is perfect. I know this to be true."

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What is a Proof?

Mathematics its own notion of "proof". In mathematics, a *proof* is a verification of a *proposition* by a chain of *logical deductions* from a base set of *axioms*.

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1.1 Propositions

A *proposition* is a statement that is either true or false.

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Proposition 1. $2 + 3 = 5$

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Proposition 2. $\forall\, n \in \mathbb{N} \quad n^2 + n + 41$ *is a prime number.*

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n	$n^2 + n + 41$	prime or composite?
0	41	prime
1	43	prime
2	47	prime
3	53	prime
...	...	(all prime)
20	461	prime
39	1601	prime

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when $n = 40$, we get $n^2 + n + 41 = 40^2 + 40 + 41 = 41 \cdot 41$, which is not prime.

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1.1 Propositions

Proposition 3. $a^4 + b^4 + c^4 = d^4$ *has no solution when* $a, b, c, d \in \mathbb{N}^+$.

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Here $\mathbb{N}^+$ denotes the *positive* natural numbers, $\{1, 2, 3, \ldots\}$. In 1769, Euler conjectured that this proposition was true. But the it was proven false 218 years later by Noam Elkies at the liberal arts school up Mass Ave. He found the solution $a = 95800, b = 217519, c = 414560, d = 422481$. We could write his assertion symbolically as follows:

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1.1 Propositions

Proposition 3. $a^4 + b^4 + c^4 = d^4$ *has no solution when* $a, b, c, d \in \mathbb{N}^+$.

Here $\mathbb{N}^+$ denotes the *positive* natural numbers, $\{1, 2, 3, \ldots\}$. In 1769, Euler conjectured that this proposition was true. But the it was proven false 218 years later by Noam Elkies at the liberal arts school up Mass Ave. He found the solution $a = 95800, b = 217519, c = 414560, d = 422481$. We could write his assertion symbolically as follows:

$$\exists\, a, b, c, d \in \mathbb{N}^+ \quad a^4 + b^4 + c^4 = d^4$$

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Proposition 4. $313(x^3 + y^3) = z^3$ *has no solution when* $x, y, z \in \mathbb{N}^+$.

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The symbols $\forall$ ("for all") and $\exists$ ("there exists") are called *quantifiers*.

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Proposition 7. $\forall\, n \in \mathbb{Z} \quad (n \geq 2) \Rightarrow (n^2 \geq 4)$

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the implication $P \Rightarrow Q$ is true when P is

false or Q is true.

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P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

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1.1 Propositions

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1.1 Propositions

Proposition 8. $\forall\, n \in \mathbb{Z} \quad (n \geq 2) \Leftrightarrow (n^2 \geq 4)$

A proposition of the form $P \Leftrightarrow Q$ is read "P if and only if Q".

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$P \Leftrightarrow Q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

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1.1 Propositions

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P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$P \Leftrightarrow Q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$n = 3$ $\qquad$ $n = 1$ $\qquad$ $n = -3$