MATÉRIA DATA

Exercícios para o Capítulo 1 15/03/11

1.2.1: Prove as seguintes identidades em F:

(a) $a^2 - b^2 = (a-b)(a+b)$

$(a-b)(a+b) \overset{P_2}{=} (a-b+0)(a+b) \overset{P_9}{=} a^2 - ab + a.0 + ab - b^2 + b.0 \overset{P_3}{=} a^2 + a.0 - b^2 + b.0 \overset{lema (c)}{=} a^2 - b^2$

(b) $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$(a-b)(a^2+ab+b^2) \overset{P_2}{=} (a-b+0)(a^2+ab+b^2) \overset{P_9}{=} a^3 - a^2b + a^2.0 + a^2.b - a.b^2 + b^2.0 + a.b^2 - b^3 + b^2.0 \overset{lema(c)}{=} a^3 - a^2b + a^2b - ab^2 + a.b^2 - b^3 \overset{P_3}{=} a^3 - b^3$

(c) $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

$(a+b)(a^2-ab+b^2) \overset{P_2}{=} (a+b+0)(a^2-ab+b^2) \overset{P_9}{=} a^3 + a^2b + a^2.0 - a^2.b - ab^2 - ab.0 + ab^2 + b^3 + b^2.0 \overset{lema(c)}{=} a^3 + a^2b - a^2.b - ab^2 + ab^2 + b^3 \overset{P_3}{=} a^3 + b^3$

1.2.2

$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b \ldots + ab^{n-2} + b^{n-1})$

$(a-b+0)(a^{n-1} + a^{n-2}.b \ldots ab^{n-2} + b^{n-1}) \overset{P_9}{=}$
$a^n + (a^{n-1}).(-b) + a^{n-1}.0 + a^{n-1}.b + (-b).(a^{n-2}.b) + a^{n-2}.b.0 \ldots a^2.b^{n-2} + (-b)(a.b^{n-2}) + ab^{n-2}.0 + ab^{n-1} + (-b)(b^{n-1}) + b^{n-1}.0 \overset{lema (c)}{=}$

$a^n + (a^{n-1})(-b) + a^{n-1}.b + (-b).(a^{n-2}.b) \ldots a^2.b^{n-2} + (-b)(ab^{n-2}) + ab^{n-1} + (-b)(b^{n-1}) \overset{lema\ e,\ lema\ d}{=}$

$a^n - a^{n-1}.b + a^{n-1}.b - a^{n-2}.b^2 \ldots + a^2.b^{n-2} - ab^{n-1} + ab^{n-1} + b^n \overset{P_3}{=} a^n - a^{n-2}.b^2 \ldots a^2.b^{n-2} + b^n$

MATÉRIA

DATA

Desigualdades

2.4.1

(a) Se $a < b$ então $a + c < b + c$, para todo $c \in F$.

$a - b < c - c$ (se prova isso, prova que é verdade)

(P6) $a - b < 0$

$a < b$

(b) Se $a < b$ e $c < d$, então $a + c < b + d$

$a + c < b + d$ (a e c são os menores valores, b e d são os maiores, logo...)

$a - b < d - c$ ($a - b$ somente pode ser um número negativo e $d - c$, somente pode ser um número positivo)

Naturais

3.5.1

(a) Demonstrar

$$1 + r + r^2 + \dots r^n = \frac{1 - r^{n+1}}{1 - r} \quad (r \neq 1)$$

para $n = 1$

$$\sum_{k=0}^{1} r^k = r^0 + r^1 = \frac{1 - r^2}{1 - r}$$

$$= 1 + r = \frac{(1+r)\cancel{(1-r)}}{\cancel{(1-r)}}$$

$$= 1 + r = 1 + r$$

3.5.1

(a) Demonstrar por indução que: $(1-r) \neq 1$

$$1 + r + r^2 + \dots r^n = \frac{1 - r^{n+1}}{1-r}$$

$$= \sum_{K=0}^{n} r^K = \frac{1 - r^{n+1}}{1 - r}$$

• 1 pertence ao corpo:

$$\sum_{K=0}^{1} r^1 = 1 + r = \frac{1 - r^2}{1 - r}$$

$$= 1 + r = \frac{(1 + r)\cancel{(1-r)}}{\cancel{1-r}}$$

• $n+1$ pertence ao corpo:

$$\sum_{K=0}^{n+1} r^K = \frac{1 - r^{n+2}}{1-r}$$

$$\sum_{K=0}^{n+1} r^K = \sum_{K=0}^{n} r^K + r^{n+1}$$

$$= \frac{1 - r^{n+1}}{1 - r} + r^{n+1}\frac{(1-r)}{(1-r)}$$

$$\frac{1 + r^{n+1}(\cancel{-1} + \cancel{1} - r)}{1-r}$$

$$\frac{1 - r^{n+2}}{1 - r}$$

Exercício

3.6.1

(a) $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

• 1 pertence ao corpo

$\sum_{k=1}^{1} k = \frac{1(2)}{2} \Rightarrow 1 = 1$

• n+1 pertence ao corpo

isso é o q precisa provar

$\sum_{k=1}^{n+1} k = \frac{(n+1)(n+1+1)}{2} \Rightarrow \sum_{k=1}^{n+1} k = \frac{(n+1)(n+2)}{2}$

$\sum_{k=1}^{n+1} k = \sum_{k=1}^{n} + n+1 \Rightarrow \frac{n(n+1)}{2} + \frac{2(n+1)}{2} = \frac{(n+1)(n+2)}{2}$ //

(B)

• 1 pertence ao corpo:

$\sum_{k=1}^{1} k^2 = 1^2 = \frac{1(2)(2+1)}{6}$

$1 = 1$

(b) $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

isso q precisa provar

• n+1 pertence ao corpo

$\sum_{k=1}^{n+1} k^2 = \frac{(n+1)(n+1+1)(2(n+1)+1)}{6}$

$= (n+1)(n+2)(2n+3)/6$

$\sum^{n+1} k^2 = \sum_{k=1}^{n} k^2 + (n+1)^2 \Rightarrow \frac{n(n+1)(2n+1)}{6} + \frac{6(n+1)^2}{6}$

$\Rightarrow (n+1)\frac{[6(n+1) + n(2n+1)]}{6}$ //

(c)
$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4} =$$

- 1 pertence ao corpo

$$\sum_{k=1}^{1} k^3 = 1^3 = \frac{1^2(2^2)}{4} \Leftrightarrow 1 = 1$$

- (n+1) pertence ao corpo

$$\sum_{k=1}^{n+1} k^3 = \frac{(n+1)^2(n+1+1)^2}{4} = \frac{(n+1)^2(n+2)^2}{4}$$

Isso → precisa provar

$$\sum_{k=1}^{n+1} k^3 = \sum_{k=1}^{n} k^3 + (n+1)^3 \Leftrightarrow \frac{n^2(n+1)^2}{4} + \frac{4(n+1)^3}{4}$$

$$\Leftrightarrow \frac{(n+1)^2(n^2+4(n+1))}{4} \Leftrightarrow \frac{(n+1)^2(n+2)^2}{4}$$

Exercício

15.5 Domínio e Imagens

15.5.1 - Determine o domínio

(a) $f(x) = \dfrac{2}{|x - 1/3|}$

$x - 1/3 \neq 0$

$x \neq 1/3$

$S = \{x \in \mathbb{R} / x \neq 1/3\}$

(b) $g(x) = 3 - \sqrt{2(x-1)}$

$2(x-1) \geq 0$

$2x - 2 \geq 0$

$2x \geq 2$

$x \geq 1$

$S = \{x \in \mathbb{R} / x \geq 1\}$

$y = 3 - \sqrt{2(x-1)}$

$(y-3)^2 = (-\sqrt{2(x-1)})^2$

$y^2 - 6y + 9 = 2(x-1)$

$\dfrac{y^2 - 6y + 9}{2} + \dfrac{2}{2} = x$

(c) $h(x) = \dfrac{x+2}{2x^2+7x+6}$

$2x^2 + 7x + 6 \neq 0$

$\Delta = 49 - 4 \cdot 2 \cdot 6$

$\Delta = 1$

$x = \dfrac{-7 \pm 1}{4}$

$\rightarrow \dfrac{-6}{4} = -\dfrac{3}{2}$

$\rightarrow \dfrac{-8}{4} = -\dfrac{4}{2} = -2$

$S = \{x \in \mathbb{R} / x \neq -3/2 \wedge x \neq -2\}$

(d) $k(x) = \dfrac{1}{x-1} + \dfrac{1}{x-2}$

$S = \{x \in \mathbb{R} / x \neq 1 \wedge x \neq 2\}$

(e) $l(x) = 2x^2 + 4x + 3.$

$S = \mathbb{R}$

(f) $m(x) = \sqrt{3x^2 - 1}$

$3x^2 - 1 \geq 0$

$3x^2 \geq 1$

$x^2 \geq 1/3$

$x \geq \pm\sqrt{\dfrac{1}{3}}$

$S = \{x \in \mathbb{R} / x \geq 1/\sqrt{3} \vee x \leq -1/\sqrt{3}\}$

2

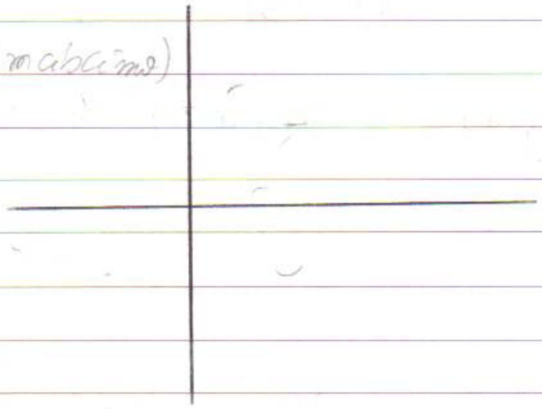
15.5.2 - Imagem

(a) $f(x) = \dfrac{2}{|x - 1/3|}$

quando $x - 1/3 = 2$ y (máximo)

$x = 2 + 1/3$

$x = 7/3$ $\qquad y <$



(b) $g(x) = 3 - \sqrt{2(x-1)}$

$\left(\sqrt{2(x-1)}\right)^2 \geq 0^2$

$2(x-1) \geq 0$

$2x - 2 \geq 0$

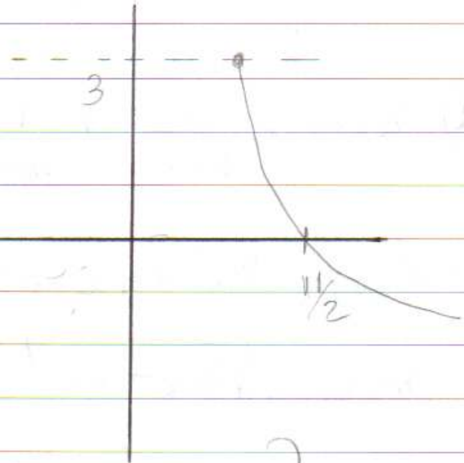
$2x \geq 2$

$x \geq 1$

quando $x = 1$, y será máximo

$y = 3$.

$\therefore y \leq 3$



$3 - \sqrt{2(x-1)} = y$

$3 - y = \sqrt{2(x-1)}$

$\dfrac{(3-y)^2}{2} + 1 = x$

em princípio, y está em $\mathbb{R}$, mas $3 - y \geq 0$

$\left.\begin{array}{l} 3 - y \geq 0 \\ \dfrac{(3-y)^2}{2} + 1 = x \end{array}\right\}$

15.5.3

(a) $f(1) = \frac{2}{|1 - 1/3|} = \frac{2}{|3/3 - 1/3|} = \frac{2}{1} \cdot \frac{3}{2} = 3$

$S = \{3\}.$

(b) $g(2) = 3 - \sqrt{2(2-1)} = 3 - \sqrt{2} \quad S = \{3 - \sqrt{2}\}$

(c) $h(x+1) = \frac{x+1+2}{2(x+1)^2 + 7(x+1)\ 6} = \frac{x+3}{2x^2+11x+15}$

$= \frac{\cancel{(x+3)}}{2\cancel{(x+3)}(x+5/2)} = \frac{1}{2x+5}$

(d) $l(\sqrt{2}) = 2(\sqrt{2})^2 + 4(\sqrt{2}) + 3 = 4 + 4\sqrt{2} + 3$
$= 7 + 4\sqrt{2}$

(e) $m(-1) = \sqrt{3(-1)^2 - 1} = \sqrt{2}$

~~(f) $m(x) = \sqrt{3x^2 - 1}$~~

(f) $\frac{l(1+x) - l(1)}{x} = \frac{2(1+x)^2 + 4(1+x) + 3 - 9}{x} =$

$\frac{2(1+2x+x^2) + 4 + 4x - 6}{x} = \frac{2 + 4x + 2x^2 + 4 + 4x - 6}{x}$

$= \frac{+2x^2 + 8x}{x} = \frac{2\cancel{x}(x+4)}{\cancel{x}} = 2(x+4)$

15.5.3

(a) $f(1) = \dfrac{2}{|1-1/3|} = \dfrac{2}{|3/3-1/3|} = \dfrac{2}{1} \cdot \dfrac{3}{2} = 3$

$S = \{3\}.$

(b) $g(2) = 3-\sqrt{2(2-1)} = 3-\sqrt{2}$ $\quad S = \{3-\sqrt{2}\}$

(c) $h(x+1) = \dfrac{x+1+2}{2(x+1)^2+7(x+1)+6} = \dfrac{x+3}{2x^2+11x+15}$

$= \dfrac{\cancel{(x+3)}}{2\cancel{(x+3)}(x+5/2)} = \dfrac{1}{2x+5}$

(d) $\ell(\sqrt{2}) = 2(\sqrt{2})^2+4(\sqrt{2})+3 = 4+4\sqrt{2}+3$
$= 7+4\sqrt{2}$

(e) $m(-1) = \sqrt{3(-1)^2-1} = \sqrt{2}$

~~(f) $m(x) = \sqrt{3x^2-1}$~~

(f) $\dfrac{\ell(1+x)-\ell(1)}{x} = \dfrac{2(1+x)^2+4(1+x)+3-9}{x} =$

$\dfrac{2(1+2x+x^2)+4+4x-6}{x} = \dfrac{2+4x+2x^2+4+4x-6}{x}$

$= \dfrac{+2x^2+8x}{x} = \dfrac{2\cancel{x}(x+4)}{\cancel{x}} = 2(x+4)$

15.5.5 - Seja $f(x) = \frac{1}{1+x}$; $x \neq -1$

(a) $f(f(x)) = \frac{1}{1+\left(\frac{1}{1+x}\right)} = \frac{1}{\frac{1+x+1}{1+x}} = \frac{1}{1} \cdot \frac{1+x}{x+2} =$

$\frac{1+x}{x+2}$, com $x \neq -2$

(b) $f\left(\frac{1}{x}\right) = \frac{1}{1+\frac{1}{x}} = \frac{1}{\frac{x+1}{x}} = \frac{x}{x+1}$

(c) $f(cx) = \frac{1}{1+cx}$

(d) $f(x+y) = \frac{1}{1+x+y} =$

(e) $f(x)+f(y) = \frac{1}{1+x} + \frac{1}{1+y} = \frac{1+y+1+x}{(1+x)(1+y)} = \frac{2+x+y}{(1+x)(1+y)}$

(f) $f(cx) = f(x)$

$\frac{1}{1+c \cdot x} = \frac{1}{1+x} \Leftrightarrow \cancel{1}+\cancel{x} = \cancel{1}+c\cancel{x}$

$c = 1$

↳ para qualquer caso, pois $c = 1$.

$\mathbb{R}$

15.6 Composição:

15.6.1

(a) $l \circ m = l(m(x)) = 2\left(\sqrt{3x^2-1}\right)^2 + 4\left(\sqrt{3x^2-1}\right) + 3$

$2(3x^2-1) + 4\sqrt{3x^2-1} + 3$

$6x^2 - 2 + 4\sqrt{3x^2-1} + 3$

$l \circ m = 6x^2 + 1 + 4\sqrt{3x^2-1}$

$3x^2 - 1 \geq 0$ $\quad D\, l \circ m =$

$3x^2 \geq 1$

$x^2 \geq \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$-\frac{\sqrt{3}}{2}$ $\quad \frac{\sqrt{3}}{3}$

$x \geq \pm \frac{\sqrt{3}}{3}$

Imagem $= x_v = -\frac{b}{2a} \Rightarrow x_v = 0$

Imagem $[3, +\infty[$

(b) $m \circ l$ $\quad m(l(x)) = \sqrt{3(2x^2+4x+3)^2 - 1}$

15.6.2 $\quad S(x) = x^2$, $P(x) = 2^x$ e $s(x) = \operatorname{sen} x$

(a) $(S \circ P)(y)$ $\quad S(P(y)) = (2^y)^2 = 2^{2y}$

(b) $(S \circ s)(y)$ $\quad S(s(y)) = (\operatorname{sen} y)^2 = \operatorname{sen}^2(y)$

(c) $(S \circ P \circ s)(t) + (s \circ P)(t)$

$S(P(s(t))) + s(P(t))$

$\left(2^{\operatorname{sen} t}\right)^2 + \operatorname{sen} 2^t = 2^{2\operatorname{sen} t} + \operatorname{sen} 2^t$

(d) $s(t^3)$ $\quad \operatorname{sen} t^3$

Exercício

15.6.3

(a) $f(x) = 2^{\operatorname{sen} x}$ $P \circ s$

(b) $f(x) = \operatorname{sen}(2^{x})$ $s \circ P$

(c) $f(x) = \operatorname{sen} x^2$ $s \circ S$

(d) $f(x) = \operatorname{sen}^2 x$ $(\operatorname{sen} x)^2$ $S \circ s$

(e) $f(x) = 2^{2^{x}} = 2^{2x}$ $P \circ P$

(f) $f(u) = (\operatorname{sen} 2^{u} + 2^{u^2})$ $s \circ P(u) + S \circ P(u)$

(g) $f(y) = \operatorname{sen}(\operatorname{sen}(\operatorname{sen} 2^{2^{2\operatorname{sen} y}})))$. $s \circ s \circ s \circ P \circ P \circ P \circ s$

(h) $f(a) = 2^{\operatorname{sen}^2 a} + \operatorname{sen}(a^2) + 2^{\operatorname{sen}(a^2 + \operatorname{sen} a)}$

$P \circ S \circ s + s \circ S + P \circ (s \circ (S + s))$

15.6.4

$f(f(y)) = y$

a) $\underbrace{f(f(f(\ldots(f(f(y))\ldots)))}_{80} = \underbrace{f(f(f(\ldots y \ldots)))}_{78} = {}_{76} = y$

b) $\underbrace{f(f(f(\ldots f(f(y))\ldots)))}_{80} = \underset{81}{f(y)}$

c) $\underbrace{f(f(f(\ldots f(y))\ldots)))}_{80} = \underbrace{f(f(f(\ldots f(y))\ldots)))}_{78} = f(y)$

15.10 Funções Racionais

Para que valores de a, b, c e d $f(x) = \dfrac{ax+b}{cx+d}$ satisfaz

$f(f(x)) = x$

$$\frac{a\left(\dfrac{ax+b}{cx+d}\right)+b}{c\left(\dfrac{ax+b}{cx+d}\right)+d} = \frac{\left(\dfrac{a^2x+ab}{cx+d}\right)+b}{\left(\dfrac{cax+cb}{cx+d}\right)+d} =$$

$$\frac{a^2x+ab+bcx+bd}{\cancel{cx+d}} \cdot \frac{\cancel{cx+d}}{cax+cb+dcx+d^2}$$

$$\frac{a^2x+ab+bcx+bd}{cax+cb+dcx+d^2}$$

$$\frac{a^2x+b(a+cx+d)}{d^2+c(ax+b+dx)}$$

Resumo

7.3.2 Exercícios: Sejam $\{s_n\}_{n \in \mathbb{N}}$ e $\{t_n\}_{n \in \mathbb{N}}$ sequências convergente. Ou seja, tais que existem os limites $\lim_{n \to \infty} s_n =: s$ e $\lim_{n \to \infty} t_n =: t$. Então:

(a) $\lim_{n \to \infty} (s_n + t_n) = s + t$. limite da soma, é a soma dos limites

(b) $\lim_{n \to \infty} s_n \cdot t_n = s \cdot t$.

(c) Se $t \neq 0$, então $\lim_{n \to \infty} \frac{1}{t_n} = \frac{1}{t}$

(d) Se $t \neq 0$, então $\lim_{n \to \infty} \frac{s_n}{t_n} = \frac{s}{t}$

7.3.4 Exercícios. les Suites Ménagers

Sejam $\{r_n\}_{n \in \mathbb{N}}$, $\{s_n\}_{n \in \mathbb{N}}$, $\{t_n\}_{n \in \mathbb{N}}$ sequências tais que $r_n \leq s_n \leq t_n$, para todo $n \in \mathbb{N}$ (ou para todo $n \geq N$, para algum $N \in \mathbb{N}$).
Suponha-se que $\lim_{n \to \infty} r_n = l = \lim_{n \to \infty} t_n$. Então $\{s_n\}_{n \in \mathbb{N}}$ é convergente com $\lim_{n \to \infty} s_n = C$

7.5 Convergência de algumas sequências. Seja F corpo arquimediano. então:

(a) $\lim_{n - \infty} \frac{1}{n} = 0$

(b) $\lim_{n - \infty} 2^{-n} = 0$

18.14 - Funções Racionais

(a) $\lim_{x \to +\infty} \dfrac{2x^5 - 11}{3x^5 + 10x^3 - 7}$ $\dfrac{\cancel{x^5}\left(2 - \frac{11}{x^5}\right)}{\cancel{x^5}\left(3 + \frac{10}{x^2} - \frac{7}{x^5}\right)} = 2/3$

(b) $\lim_{x \to -\infty} \dfrac{x^3 + 1}{1 - 9x^3}$ $\dfrac{\cancel{x^3}\left(1 + \frac{1}{x^3}\right)}{\cancel{x^3}\left(\frac{1}{x^3} - 9\right)} = -\dfrac{1}{9}$

(c) $\lim_{x \to -\infty} \dfrac{x^5}{1 + x + x^2 + x^3 + x^4 + x^5} = \dfrac{\cancel{x^5}}{\cancel{x^5}\left(\frac{1}{x^5} + \frac{1}{x^4} + \dots 1\right)}$

$= 1$

(d) $\lim_{x \to -\infty} \dfrac{7x^2 + 5}{1 - x - x^2}$ $\dfrac{\cancel{x^2}\left(7 + \frac{5}{x^2}\right)}{\cancel{x^2}\left(\frac{1}{x^2} - \frac{1}{x} - 1\right)} = \dfrac{7}{-1}$

(e) $\lim_{x \to +\infty} \dfrac{x^3}{x^2 + 1}$ $\dfrac{\cancel{x^3}}{\cancel{x^2}\left(\frac{1}{x^2} + 1\right)} = \dfrac{x}{1} = \infty +$

(f) $\lim_{x \to -\infty} \dfrac{x^3}{x^2 + 1} = -\infty$

18.15 - Limites ~~ao~~ Impróprios valores

(a) $\lim\limits_{x \to +\infty} \sqrt{x^2+x} - x \quad \frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x} = \frac{\cancel{x^2}+x-\cancel{x^2}}{x+\sqrt{x^2+x}}$

$= \frac{x}{x+\sqrt{x^2(1+1/x)}} = \frac{\cancel{x}}{\cancel{x}+\cancel{x}\sqrt{1+1/x}} = \frac{1}{\sqrt{1+1/x}+1}$

$= 1/2$

Exercício

funções racionais

18.10.1 - Calcular os limites

(a) $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} \quad 1 + 1 = 2$

(b) $\lim_{x \to 2} \frac{x^3 - 8}{x - 2} = (x-y)(x^2 + xy + y^2)$

$\frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{x-2}} = 12$

(c) $\lim_{x \to y} \frac{x^n - y^n}{x - y} = \lim \frac{\cancel{(x-y)}(x^{n-1} + x^{n-2} \cdot y + \dots + y^{n-2} \cdot x + y^{n-1})}{\cancel{(x-y)}} =$

$y^{n-1} + y^{n-2} \cdot y + \dots + y^{n-2} \cdot y + y^{n-1} = n \cdot y^{n-1}$

(d) $\lim_{y \to x} \frac{x^n - y^n}{x - y} = \lim \frac{\cancel{x-y}(x^{n-1} + x^{n-2} \cdot y + \dots + y^{n-2} \cdot x + y^{n-1})}{\cancel{x-y}}$

$= x^{n-1} + x^{n-2} \cdot x + \dots x^{n-2} \cdot x + x^{n-1} = n x^{n-1}$

18.11.1 limites com raízes

(a) $\lim_{x \to 1} \frac{1 - \sqrt{x}}{1 - x} \cdot \frac{1 + \sqrt{x}}{1 + \sqrt{x}} = \frac{\cancel{1 - x}}{\cancel{1 - x}(1 + \sqrt{x})} = \frac{1}{1 + \sqrt{x}}$

$= \frac{1}{1 + 1} = 2$

(b) $\lim_{x \to 0} \frac{1 - \sqrt{1 - x^2}}{x} \cdot \frac{1 + \sqrt{1 - x^2}}{1 + \sqrt{1 - x^2}} = \frac{\cancel{1} - \cancel{1} - x^{\cancel{2}}}{\cancel{x}(1 + \sqrt{1 - x^2})} =$

$\frac{-x}{1 + \sqrt{1 - x^2}} = 0$

(c) $\lim_{x \to 0} \frac{1-\sqrt{1-x^2}}{x^2} \cdot \frac{(1+\sqrt{1-x^2})}{(1+\sqrt{1-x^2})} = \frac{\cancel{1}-\cancel{1}-x^2}{x^2(1+\sqrt{1-x^2})} =$

$\frac{1}{1+\sqrt{1-x^2}} = \frac{1}{2}$

(d) $\lim_{h \to 0} \frac{\sqrt{a+h}-\sqrt{a}}{h} \cdot \frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}} = \frac{a+h-a}{h(\sqrt{a+h}+\sqrt{a})} =$

$= \frac{\cancel{h}}{\cancel{h}(\sqrt{a+h}+\sqrt{a})} = \frac{1}{\sqrt{a+h}+\sqrt{a}} = \frac{1}{2\sqrt{a}}$

18.13 - Funções Trigonométricas

(a) $\lim_{x \to 0} \frac{sen(2x)}{x} = 2\,\frac{sen(2x)}{2x} = 2 \cdot 1 = 2$

(b) $\lim_{x \to 0} \frac{sen^2(2x)}{x^2} = 2\,\underbrace{\frac{sen(2x)}{2x}}_{1} \cdot 2\,\underbrace{\frac{sen(2x)}{2x}}_{1} = 2 \cdot 2 = 4$

(c) $\lim_{x \to 0} \frac{sen(ax)}{bx} \quad a\,\frac{sen(ax)}{b \cdot ax} = \frac{a}{b}$

(d) $\lim_{x \to 1} \frac{sen(x^2-1)}{x-1} \cdot \frac{x+1}{x+1} = x+1\,\frac{sen(x^2-1)}{(x^2-1)}$

(e) $\lim_{x \to 0} \frac{1-\cos x}{x^2} \cdot \frac{1+\cos x}{1+\cos x} = \frac{1-\cos^2 x}{x^2(1+\cos x)} =$

$\frac{\overset{1}{\cancel{sen^2 x}}}{\cancel{x^2}(1+\cos x)} = \frac{1}{1+1} = \frac{1}{2}$

(f) $\lim_{x \to 0} \frac{1-\cos x}{x} \cdot \frac{1+\cos x}{1+\cos x} = \frac{1-\cos^2 x}{x(1+\cos x)} =$

$\frac{\overset{1}{x \cdot sen^2 x}}{x \cdot x(1+\cos x)} = \frac{0}{2} = 0$

(g) $\lim_{x \to 0} \frac{x \cdot sen\, x}{1-\cos x} \cdot \frac{sen\, x}{sen\, x} = \frac{x \cdot sen^2 x}{(1-\cos x)\, sen\, x} =$

$\frac{x(1-\cos^2 x)}{(1-\cos x) \cdot sen\, x} = \frac{\overset{\to 1}{x}(1+\cos x)}{sen\, x} = 2$

2

(h) $\lim_{x \to 0} \frac{x^2(3+sen\, x)}{(x+sen\, x)^2} = \frac{\cancel{x^2}(3+sen\, x)}{\frac{\cancel{x^2}(x+sen\, x)^2}{x^2}}$

(i) $\lim_{x \to 0} \frac{tg\, x}{x} = \frac{sen\, x}{\cos x \cdot x} = \frac{1}{\cos 0} = 1$

(j) $\lim_{x \to 0} \frac{tg^2 x + 2x}{x+x^2} = \frac{tg^2 x + 2x}{x(x+1)} = \frac{\overset{0}{x} \overset{1}{tg^2 x}}{x \cdot x(x+1)} + \frac{2\cancel{x}}{x(x+1)}$

$0 + \frac{2\cancel{x}}{\cancel{x} \cdot (x+1)} = \frac{2}{x+1} = 2$

(k) $\lim_{h \to 0} \dfrac{sen(x+h) - sen\,x}{h} =$

$$\frac{sen\,x \cdot cos\,h + sen\,h \cdot cos\,x - sen\,x}{h}$$

$$\frac{sen\,x(cos\,h - 1) + sen\,h \cdot cos\,x}{h}$$

$$sen\,x \frac{(cos\,h - 1)}{h} + \frac{sen\,h}{h} \cdot cos\,x$$

$\to$ $\to 1$

$$sen\,x \frac{(cos\,h - 1)}{h} \frac{(cos\,h + 1)}{(cos\,h + 1)}$$

$$sen\,x \frac{(cos^2 h - 1)}{h(cos\,h + 1)} + cos\,x$$

$$sen\,x \frac{(- sen^2 h)\,h}{h(cos\,h + 1)\,h} + cos\,x$$

$$- \frac{sen\,x \cdot h \cdot sen^2 h}{(cos\,h + 1) \cdot h^2} + cos\,x$$

$\to 0$

18.15

a) $\lim_{x \to +\infty} \sqrt{x^2 + x} - x$

$= \sqrt{x^2 \left(1 + \frac{1}{x}\right)} - x$

tende a zero

$= x\left(\sqrt{1 + \frac{1}{x}} - 1\right)$ → vai para zero

não pode multiplicar por infinito

$\frac{1}{x}$ → tende a zero

outro = multiplica

$\left(\frac{\sqrt{x^2 + x} + x}{\sqrt{x^2 + x} + x}\right) \cdot \lim_{x \to \infty} \sqrt{x^2 + x} - x =$

$\lim_{x \to \infty} \frac{\cancel{x^2} + \cancel{x} - \cancel{x^2}}{\cancel{x}\left(\sqrt{1 + \frac{1}{x}} + 1\right)}$

18.15

e)

até 1

chute

$\lim_{x \to +\infty} \frac{x \operatorname{sen} x}{x^2 + 5} = \frac{1}{x}$

quando x é muito grande, pode ignorar o 5. $\frac{\cancel{x}}{x^{\cancel{2}}} = \frac{1}{x}$ →

vai para zero

tendendo a zero $\left(\frac{\operatorname{sen} x}{x}\right)^2$

$\frac{x \cdot \operatorname{sen} x}{x^2+5}$ ⊳ se estiver indo para $+\infty$

$-1 \leq \operatorname{sen} x \leq 1$

$-\frac{x}{x^2+5} \leq \frac{x}{x^2+5} \cdot \operatorname{sen} x \leq \frac{x}{x^2+5}$

vai para zero — vai para zero

$\frac{x}{x^2(1+5/x)} = \frac{1}{x} \cdot \frac{1}{(1+\frac{5}{x})} = 0 \cdot 1 = 0$

18.14

e)

$\lim_{x \to +\infty} \frac{1-\cos x}{x^2} \cdot \frac{1+\cos x}{1+\cos x}$

$\lim_{x \to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)} = \frac{\overset{1}{\operatorname{sen}^2 x}}{x^2(1+\cos x)} \Rightarrow \frac{1}{1+\cos x}$

$= \frac{1}{1+1} = \frac{1}{2}$

sequência convergente, tem limite

7.3

(b)

$\forall \varepsilon > 0 \quad \exists N \in \mathbb{N} / n \geq N \Rightarrow |s_n - \ell| < \varepsilon$

Se $m, n \geq N \Rightarrow |s_n - s_m| = |s_n \underbrace{- \ell + \ell}_{\text{zero}} - s_m|$

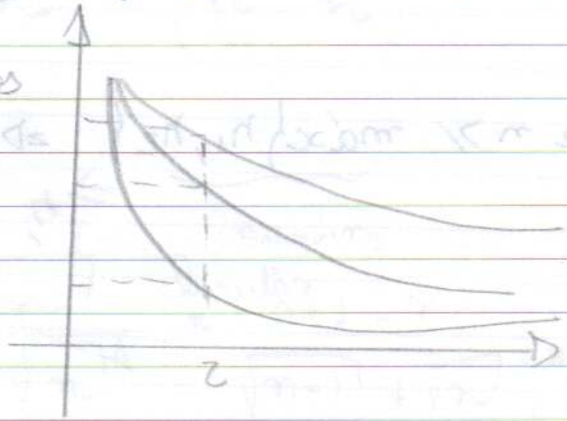
$\leq \underbrace{|s_n - \ell|}_{<\varepsilon} + \underbrace{|\ell - s_m|}_{<\varepsilon}$

$< \varepsilon + \varepsilon = 2\varepsilon$

7.3.4

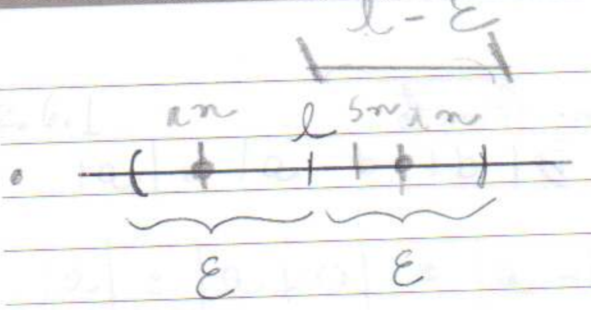
três sequências: $r_n \leq s_n \leq t_n$

supondo que as sequências dos extremos tem o mesmo limite



(i) Se $r_n \leq s_n \leq t_n$ e $\lim\limits_{n \to \infty} r_n = \ell = \lim\limits_{n \to \infty} t_n$

$\Rightarrow \lim\limits_{n \to \infty} s_n = \ell$



$|a| < b$ equivale $-b < a < b$

- $|s_n - l| < \varepsilon$

$-\varepsilon < s_n - l < \varepsilon$

$\boxed{l - \varepsilon < s_n < l + \varepsilon}$

$\forall \varepsilon > 0 \quad \exists N_1 \in \mathbb{N} / n \geq N_1 \Rightarrow |r_n - l| < \varepsilon \quad l - \varepsilon < r_n < l + \varepsilon$

$\forall \varepsilon > 0 \quad \exists N_2 \in \mathbb{N} / n \geq n_2 \Rightarrow |t_n - l| < \varepsilon \quad l - \varepsilon < t_n < l + \varepsilon$

$\therefore$ se $n \geq \underbrace{\max\{n_1, n_2\}}_{\text{número natural}} \geq n_1 \Rightarrow \quad l - \varepsilon < r_n \leq s_n \leq t_n$

por hipótese

$l - \varepsilon < r_n \leq s_n \leq t_n < l + \varepsilon$

isso prova que $l - \varepsilon < s_n < l + \varepsilon$

$\lim_{n \to \infty} s_n = l$

em sequências, n vai para infinito
n tende a zero

ex: $\lim_{n \to \infty} \frac{4n+1}{n} = \lim_{n \to \infty} 4 + \frac{1}{n} = 4 + 0 = 4$

↳ exemplo do professor

pois $\lim (s_n + t_n) = \lim s_n + \lim t_n$

$\lim_{n \to \infty} \frac{4n+1}{n^{10000}}$

ex 2: $\lim_{n \to \infty} 4 \cdot \frac{1}{n^{9999}} + \frac{1}{n^{10000}} = 4 \cdot 0 + 0 = 0$

37.4

$\lim_{n \to \infty} \frac{n+3}{n^3+4} = \lim_{n \to \infty} \frac{n\left(1 + \frac{3}{n}\right)}{n^3\left(1 + \frac{4}{n^3}\right)} = \frac{(1+0)}{(1+0)} \cdot 0$

d) $\frac{n!}{n^n} = \frac{1 \cdot 2 \cdot 3 \cdots n}{n \cdot n \cdot n \cdots n} = \frac{1}{n} \cdot \frac{2}{n} \cdots 1$ $\left(\frac{1}{n} \to 0\right)$

37.4.3

$\sqrt{n+1} - \sqrt{n} = \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} = \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} =$

$= \frac{1}{\sqrt{n+1} + \sqrt{n}}$

2.6.1

$|a| = |a-b+b| \leq |a-b| + |b|$

$|a| = |a+0| = |a-b+b| = |(a-b)+b| \leq |a-b| + |b|$

$|a| \leq |a-b| + |b|$

15.4.2

$$\frac{x+2}{2x^2+7x+6} = \frac{\cancel{x+2}}{2\cancel{(x+2)}(x+3/2)} = \frac{1}{2(x+3/2)}$$

$ax^2+bx+c = a(x-r_1)(x-r_2)$

• $\lim_{n \to \infty} \sqrt[n]{n^2+n} = 1$

$n < n+1 < n^2$

$n^2 < n(n+1) < n.n^2$

$\sqrt[n]{n}.\sqrt[n]{n} < \sqrt[n]{n(n+1)} < \sqrt[n]{n}.\sqrt[n]{n}.\sqrt[n]{n}$

$\downarrow \qquad\qquad\qquad\qquad \downarrow$

$1 \quad < \qquad\qquad < 1$

o que vai para os extremos é 1, o que está dentro é 1.

• $\lim_{n \to \infty} \sqrt[n]{n} = 1$

$s_n \to l$

$z_n = \frac{s_1+s_2+\dots+s_n}{n} \to l$ } equivalente monitoria Cruel

15/4/11

$$\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1$$

$$\lim_{x \to a} \frac{\operatorname{sen}(f(x))}{f(x)} = 1 \; *$$

se $\lim_{x \to a} f(x) = 0$

$$\lim_{x \to 0} \frac{\operatorname{sen}(x^2-1)}{x-1} = \frac{\operatorname{sen} -1}{-1} = \operatorname{sen} 1$$

$$\lim_{x \to 1} \frac{\operatorname{sen}(x^2-1)}{(x-1)}$$ pode ~~dar~~ não existir

$$\frac{\operatorname{sen}(x^2-1)}{(x-1)} \cdot \frac{x+1}{x+1} = (x+1) \cdot \frac{\operatorname{sen}(x^2-1)}{x^2-1} =$$

$$\lim_{\to 1} (x+1) \cdot \lim \frac{\operatorname{sen}(x^2-1)}{x^2-1} = 2 \cdot 1 = 2$$

$$\lim_{h \to 0} \frac{\operatorname{sen}(x+h) - \operatorname{sen} x}{h}$$

$$= \frac{\operatorname{sen} x \cdot \cos h}{h} + \frac{\operatorname{sen} h \cdot \cos x}{h} - \frac{\operatorname{sen} x}{h}$$

$$= \operatorname{sen} x \underbrace{\left(\frac{\cos h - 1}{h}\right)}_{0 \text{ (exercício 1)}} + \underbrace{\frac{\operatorname{sen} h}{h}}_{\to 1} \cdot \cos x = \operatorname{sen} x \cdot 0 + \cos x$$

$$= \cos x$$

n Ímpar

$x^n = y^n \Rightarrow x = y$

ou $x = 0$. $\Rightarrow 0 = y^n \Rightarrow y = 0$

$y = 0$

$x < 0 \Rightarrow x = -\alpha$

$x^n = (-\alpha)^n = (-1)^n \alpha^n = -\alpha^n$

$\left.\begin{matrix} x > 0 \\ y > 0 \end{matrix}\right\} \Rightarrow x < y$

$x \cdot x < y \cdot x < y \cdot y \Rightarrow x^2 \cdot x < y^2 \cdot x < y^2 \cdot y$

$\Rightarrow x^3 < y^3 \Rightarrow x^n < y^n$ (contradição)

$\therefore$ se o maior não pode ser, nem o menor, só pode ser o igual.

3.6.2

(a) $\sum_{k=1}^{n} (2k-1) = n^2$

• hipótese indutiva

$$\sum_{k=1}^{n+1} (2k-1) = \sum_{k=1}^{n} + (2(n+1)-1)$$

$$\sum_{k=1}^{n+1} (2k-1) = n^2 + (2n+2-1)$$

$$= n^2 + 2n + 1$$

$$= (n+1)^2$$

• provar que {1} ∃ ao conjunto indutivo

$1 = 1^2$

(b) $\sum_{k=1}^{n} (2k-1)^2 = \frac{4n^3+5n^2+n}{2}$

prova da hipótese indutiva

$$\sum_{k=1}^{n+1} (2k-1)^2 = \sum_{k=1}^{n} (2k-1)^2 + [2(n+1)-1]^2$$

$(2n-1)^2$

$$= \frac{4n^3+5n^2+n}{2} + 4n^2+4n+1 \left(\frac{2}{2}\right)$$

$$= \frac{4n^3+5n^2+n + 8n^2+8n+2}{2}$$

$$= \frac{4n^3+13n^2+9n+2}{2}$$

$$\frac{\left(1 + (2n-1)^2\right) \cdot n}{2}$$

$$\frac{(\ldots + 4n^2 + 5n + 1)\, n}{2}$$

$$\frac{+4n^3 + 5n^2 + n}{2}$$

$$\frac{n(4n^2 + 5n + 1)}{2}$$