Calcular o valor, em função de x, das seguintes integrais, aplicando o método de integração por partes:

(O método tem a seguinte fórmula: $\int u\, dv = u\, v - \int v\, du$)

1) $I = \int x^2 \operatorname{sen}(x)\, dx$;

Solução

considerando: $u = x^2 \Rightarrow du = 2x\, dx$

$dv = \operatorname{sen}(x)dx \Rightarrow v = -\cos(x)$

substituindo em I , temos:

$$I = \int u\, dv = u\, v - \int v\, du = -x^2 \cos(x) - \int -\cos(x)\, 2\, x\, dx =$$

$$-x^2 \cos(x) + 2 \int x \cos(x)\, dx$$

fazendo $J = 2 \int x \cos(x)\, dx$

considerando $w = x \Rightarrow dw = dx$

$dz = \cos(x)\, dx \Rightarrow z = \operatorname{sen}(x)$

substituindo em J , temos:

$$J = 2 \int w\, dz = 2\, w\, z - 2 \int z\, dw = 2\, x \operatorname{sen}(x) - 2 \int \operatorname{sen}(x)\, dx = 2\, x \operatorname{sen}(x) + 2 \cos(x) + K$$

logo, temos: $I = -x^2 \cos(x) + J = -x^2 \cos(x) + 2\, x \operatorname{sen}(x) + 2 \cos(x) + K$

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2) $\quad I = \displaystyle\int \ln(\operatorname{tg}(x))\sec(x)^2\, dx$

Solução

considerando $u = \ln(\operatorname{tg}(x)) \quad => \quad du = \dfrac{\sec(x)^2}{\operatorname{tg}(x)}\, dx$

$dv = \sec(x)^2 dx \quad => \quad v = \operatorname{tg}(x)$

substituindo em I , temos:

$$I = \int u\, dv = u\, v - \int v\, du = \ln(\operatorname{tg}(x))\operatorname{tg}(x) - \int \frac{\operatorname{tg}(x)\sec(x)^2}{\operatorname{tg}(x)}\, dx =$$

$$\ln(\operatorname{tg}(x))\operatorname{tg}(x) - \int \sec(x)^2\, dx =$$

$$= \ln(\operatorname{tg}(x))\operatorname{tg}(x) - \operatorname{tg}(x) + K = \operatorname{tg}(x)\left(\ln(\operatorname{tg}(x)) - 1\right) + K$$

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3) $\quad I = \displaystyle\int \frac{x\, \mathbf{e}^x}{(1+x)^2}\, dx$;

Solução

considerando $u = x\, \mathbf{e}^x \quad => \quad du = (\mathbf{e}^x + x\, \mathbf{e}^x)\, dx = \mathbf{e}^x (1+x)\, dx$

$dv = \dfrac{1}{(1+x)^2}\, dx \quad => \quad v = -\dfrac{1}{1+x}$

sustituindo em I , temos:

$$I = \int u\,dv = u\,v - \int v\,du = -\frac{x\,\mathbf{e}^x}{1+x} - \int -\frac{\mathbf{e}^x\,(1+x)}{1+x}\,dx = -\frac{x\,\mathbf{e}^x}{1+x} + \int \mathbf{e}^x\,dx =$$

$$= -\frac{x\,\mathbf{e}^x}{1+x} + \mathbf{e}^x + K = \mathbf{e}^x\left(1 - \frac{x}{1+x}\right) + K = \mathbf{e}^x\,\frac{1}{1+x} + K = \frac{\mathbf{e}^x}{1+x} + K$$

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4) $I = \int x^2 \ln(x+1)\,dx$;

Solução

considerando u = ln(x + 1) => du = $\frac{1}{x+1}$ dx

dv = x^2 dx => v = $\frac{x^3}{3}$

substituindo em I , temos:

$$I = \int u\,dv = u\,v - \int v\,du = \frac{x^3}{3}\ln(x+1) - \int \frac{x^3}{3\,(x+1)}\,dx = \frac{x^3}{3}\ln(x+1) - \frac{1}{3}$$

$$\int \frac{x^3}{x+1}\,dx =$$

$$= \frac{x^3}{3}\ln(x+1) - \frac{1}{3}\int x^2 - x + 1 - \frac{1}{x+1}\,dx = \frac{x^3}{3}\ln(x+1) - \frac{1}{3}\,($$

$$\int x^2\,dx - \int x\,dx + \int 1\,dx - \int \frac{1}{x+1}\,dx\ ) =$$

$$= \frac{x^3}{3}\ln(x+1) - \frac{1}{3}\left(\frac{x^3}{3} - \frac{x^2}{2} + x - \ln(x+1)\right) + K = \frac{x^3}{3}\ln(x+1) + \frac{1}{3}\ln(x+1) - \frac{x^3}{9} +$$

$$\frac{x^2}{6} - \frac{x}{3} + K$$

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5) $I = \displaystyle\int \frac{\ln(x)}{x}\, dx$;

Solução

considerando $u = \ln(x) \Rightarrow du = \frac{1}{x}\,dx$

$dv = \frac{1}{x}\,dx \Rightarrow v = \ln(x)$

substituindo em I , temos:

$$I = \int u\, dv = u\, v - \int v\, du = \ln(x)\ln(x) - \int \frac{\ln(x)}{x}\, dx = \ln(x)^2 - I \Rightarrow$$

$$I + I = \ln(x)^2 \Rightarrow 2\,I = \ln(x)^2 \Rightarrow I = \frac{1}{2}\ln(x)^2 + K$$

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6) $I = \displaystyle\int x \ln(x)^2\, dx$;

Sulocão

considerando $u = \ln(x)^2 \Rightarrow du = 2\ln(x)\frac{1}{x}\,dx$

$dv = x\,dx \Rightarrow v = \frac{x^2}{2}$

substituindo em I , temos;

$$I = \int u\, dv = u\, v - \int v\, du = \frac{x^2}{2}\ln(x)^2 - \int x \ln(x)\, dx$$

considerando w = ln(x) => dw = $\frac{1}{x}$ dx

dz = x dx => z = $\frac{x^2}{2}$

substituindo nesta última integral de I , temos:

$$I = \frac{x^2}{2}\ln(x)^2 - \int w\,dz = \frac{x^2}{2}\ln(x)^2 - \left(w\,z - \int z\,dw \right) = \frac{x^2}{2}\ln(x)^2 - \frac{x^2}{2}\ln(x) + \int \frac{x^2}{2\,x}\,dx =$$

$$= \frac{x^2}{2}\ln(x)^2 - \frac{x^2}{2}\ln(x) + \frac{1}{2}\int x\,dx = \frac{x^2}{2}\ln(x)^2 - \frac{x^2}{2}\ln(x) + \frac{x^2}{4} + K$$

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7) $I = \int x^3\cos(x^2)\,dx$;

Solução

$$I = \int x^2\cos(x^2)\,x\,dx$$

considerando u = x^2 => du = 2 x dx => $\frac{1}{2}$ du = x dx

substituindo em I , temos:

$$I = \frac{1}{2}\int u\cos(u)\,du$$

considerando w = u => dw = du

dz = cos(u) du z = sen(u)

substituindo nesta última I , temos:

$$I = \frac{1}{2}\int w\,dz = \frac{1}{2}\left(w\,z - \int z\,dw \right) = \frac{1}{2}\left(u\,\mathrm{sen}(u) - \int \mathrm{sen}(u)\,du \right) = \frac{1}{2}(u\,\mathrm{sen}(u) +$$

$\cos(u)$) + k

substituindo u nesta última I , temos:

$$I = \frac{1}{2}\ (\ x^2\ \text{sen}(x^2) + \cos(x^2)\ )\ +\ K\ =\ \frac{1}{2}x^2\ \text{sen}(x^2)\ +\ \frac{1}{2}\cos(x^2)\ +\ K$$

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8) $I = \int \mathbf{e}^{(-x)} \cos(2\,x)\, dx$;

Solução

considerando $u = \mathbf{e}^{(-x)}$ => $du = -\mathbf{e}^{(-x)}\,dx$ => $-du = \mathbf{e}^{(-x)}\,dx$

$dv = \cos(2x)\,dx$ => $v = \frac{1}{2}\,\text{sen}(2x)$

subtituindo em I , temos:

$$I = \int u\, dv = u\, v - \int v\, du = \mathbf{e}^{(-x)}\ \frac{1}{2}\ \text{sen}(2x)\ +\ \frac{1}{2}\int \text{sen}(2\,x)\,\mathbf{e}^{(-x)}\,dx$$

fazendo $J = \frac{1}{2}\int \text{sen}(2\,x)\,\mathbf{e}^{(-x)}\,dx$

considerando $w = \mathbf{e}^{(-x)}$ => $dw = -\mathbf{e}^{(-x)}\,dx$ => $-dw = \mathbf{e}^{(-x)}\,dx$

$dz = \text{sen}(2x)\,dx$ => $z = -\frac{1}{2}\cos(2x)$

substituindo em J , temos:

$$J = \frac{1}{2}\int w\, dz = \frac{1}{2}(\ w\, z - \int z\, dw\ ) = \frac{1}{2}(-\frac{1}{2}\cos(2x)\,\mathbf{e}^{(-x)} - \frac{1}{2}\int \cos(2\,x)\,\mathbf{e}^{(-x)}\,dx\ )$$

=

$$= -\frac{1}{4}\cos(2x)\,\mathbf{e}^{(-x)} - \frac{1}{4}\ I$$

logo, $I = \mathbf{e}^{(-x)}\ \frac{1}{2}\ \text{sen}(2x)\ -\frac{1}{4}\cos(2x)\,\mathbf{e}^{(-x)}\ -\frac{1}{4}\ I\ +\ K$ =>

$$=> \quad I + \frac{1}{4} I = \frac{1}{2} \mathbf{e}^{(-x)} \operatorname{sen}(2x) - \frac{1}{4} \mathbf{e}^{(-x)} \cos(2x) + K \quad =>$$

$$=> \quad \frac{5}{4} I = \frac{1}{2} \mathbf{e}^{(-x)} \operatorname{sen}(2x) - \frac{1}{4} \mathbf{e}^{(-x)} \cos(2x) + K \quad =>$$

$$=> \quad I = \frac{2}{5} \mathbf{e}^{(-x)} \operatorname{sen}(2x) - \frac{1}{5} \mathbf{e}^{(-x)} \cos(2x) + K$$

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9) $I = \int x \operatorname{cossec}(3)^2 \, dx$;

Solução

considerando $u = x \quad => \quad du = dx$

$dv = \operatorname{cossec}(3x)^2 \, dx \quad => \quad v = -\frac{1}{3} \operatorname{cotg}(3x)$

substituindo em I , temos:

$$I = \int u \, dv = u\, v - \int v \, du = -\frac{1}{3} x \operatorname{cotg}(3x) + \frac{1}{3} \int \operatorname{cotg}(3x) \, dx$$

$$\text{fazendo} \quad J = \frac{1}{3} \int \operatorname{cotg}(3x) \, dx = \frac{1}{3} \int \frac{\cos(3x)}{\operatorname{sen}(3x)} \, dx$$

considerando $w = \operatorname{sen}(3x) \quad => \quad dw = 3 \cos(3x)\, dx \quad => \quad \frac{1}{3} dw = \cos(3x)\, dx$

substituindo em J, temos:

$$J = \frac{1}{9} \int \frac{1}{w} \, dw = \frac{1}{9} \ln(|w|) + K = \frac{1}{9} \ln(|\operatorname{sen}(3x)|) + K$$

logo,

$$I = -\frac{1}{3} x \operatorname{cotg}(3x) + \frac{1}{9} \ln(|\operatorname{sen}(3x)|) + K$$

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10) $I = \int \operatorname{arctg}(x)\, dx$;

Solução

considerando u = arctg(x) => $du = \dfrac{1}{1+x^2}\, dx$

dv = dx => v = x

substituindo em I , temos:

$$I = \int u\, dv = u\, v - \int v\, du = x \operatorname{arctg}(x) - \int \frac{x}{1+x^2}\, dx$$

fazendo $J = -\int \dfrac{x}{1+x^2}\, dx$

considerando $w = 1 + x^2$ => dw = 2 x dx => $\dfrac{1}{2}\, dw = x\, dx$

substituindo em J , temos:

$$J = -\frac{1}{2}\int \frac{1}{w}\, dw = -\frac{1}{2}\ln(\,|\,w\,|\,) + K = -\frac{1}{2}\ln(\,1 + x^2\,) + K$$

logo,

$$I = x \operatorname{arctg}(x) - \frac{1}{2}\ln(\,1 + x^2\,) + K$$

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11) $I = \int x \operatorname{arctg}(x)\, dx$;

Solução

considerando u = arctg(x) => du = $\dfrac{1}{1+x^2}$ dx

dv = x dx => v = $\dfrac{1}{2}x^2$

substituindo em I , temos:

$$I = \int u\,dv = u\,v - \int v\,du = \frac{1}{2}x^2\,\mathrm{arctg}(x) - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx =$$

$$= \frac{1}{2}x^2\,\mathrm{arctg}(x) - \frac{1}{2}\int 1 - \frac{1}{1+x^2}\,dx = \frac{1}{2}x^2\,\mathrm{arctg}(x) - \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx =$$

$$= \frac{1}{2}x^2\,\mathrm{arctg}(x) - \frac{1}{2}\mathrm{x} + \frac{1}{2}\mathrm{arctg}(x) + \mathrm{K}$$

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