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a Integral $\int x^n \ln x \, dx$?

(1) Sabendo que $\int_{b-2}^{b} x \, dx = 2$ com relação a b pode ser afirmado?

(a) $b = 0$

(b) $b = -1/2$

(c) $b = -2$

(d) Não existe b

(e) $b = 2$

(2) Com relação ao valor da expressão $\int_a^b \left(\int_a^x (1+z) \, dt \right) dx$ pode ser afirmado que

(a) $(1+z)\left(\frac{b^2}{2} - \frac{a^2}{2}\right)$

(b) $(1-z)(b+a)$

(c) $(b+a)\left(\frac{1}{z} - \frac{z^2}{2}\right)$

(d) $(b+a)\left(\frac{1}{z} - \frac{z^2}{2}\right)$

(e) $(1+z)\frac{(b-a)^2}{2}$

(3) Suponha-se que $\int_a^b f = \frac{1}{2}$ e que $\int_a^b g = -5$. Então, com relação à expressão $\int_a^b \left(5f - \frac{1}{2}g\right)$, pode ser afirmado que seu valor é dado por

(a) $5/2$

(b) $-5/2$

(c) 5

(d) 0

(e) -5

Prova

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4) $\int \left(2x^{1/4} - \cos x + \frac{3}{x^5}\right)$

$2\int x^{1/4} - \int \cos x + 3\int \frac{1}{x^5}$

$2 \cdot \frac{4x^{5/4}}{5} - \operatorname{sen} x - \frac{3}{4} x^{-4}$

5) $\int \frac{\pi^2}{x^2+1} = \pi^2 \int \frac{1}{x^2+1} \quad \pi^2 \cdot \operatorname{arctg} x$

6) $\int \frac{\cos(2x)}{1+\operatorname{sen}^2(2x)}\, dx \qquad u = 2x$

$du = 2dx \Rightarrow \frac{du}{2} = dx$

$\frac{1}{2}\int \frac{\cos(u)}{1+\operatorname{sen}^2(u)}\, du \qquad \operatorname{sen}(u) = y$

$\cos(u) = dy$

$\frac{1}{2}\int \frac{dy}{1+y^2} = \frac{1}{2} \operatorname{arctg}(y) =$

$\frac{1}{2} \cdot \operatorname{arctg}(\operatorname{sen}(2x))$

7) $\int \frac{1}{x \cdot \ln x}\, dx = \qquad u = \log(x)$

10) $\int \operatorname{cosec} dx$ corresponde a ?

(a) $-\frac{1}{2} \ln \left(\frac{\cos x + 1}{\cos x - 1}\right)$

Fórmula de Redução do Seno

$$\int sen^{n} x \, dx = \int \underbrace{sen^{n-1} x}_{f(x)} \cdot \underbrace{sen\, x \, dx}_{g'(x) = -\cos x} =$$

$$= -\cos x \cdot sen^{n-1} x + \int \cos x \cdot (n-1) \cdot sen^{n-2} x \cdot \cos x =$$

$$= -\cos x \cdot sen^{n-1} x + (n-1) \int \underbrace{\cos^{2} x}_{\hookrightarrow (1 - sen^{2} x)} \cdot sen^{n-2} x =$$

$$= -\cos x \cdot sen^{n-1} x + (n-1) \int sen^{n-2} x \, dx - (n-1) \int sen^{n} x \, dx \Rightarrow$$

$$n \int sen^{n} dx = -\frac{\cos x \cdot sen^{n-1} x}{n} + \frac{(n+1)}{n} \int sen^{n-2} x \, dx \quad \forall n$$

$$\int \frac{1}{\operatorname{sen} x}\, dx = \int \frac{\operatorname{sen} x}{\operatorname{sen}^2 x}\, dx = \int \frac{\operatorname{sen} x}{1-\cos^2 x}\, dx =$$

$$= \int \frac{\operatorname{sen} x}{(1+\cos x)\cdot(1-\cos x)} = \frac{1}{2}\left(\int \frac{\operatorname{sen} x}{1+\cos x}\, dx + \int \frac{\operatorname{sen} x}{1-\cos x}\, dx\right)$$

$u = 1+\cos x$

$y = 1-\cos x$

$$\Rightarrow \frac{1}{2}\left(-\int \frac{du}{u} + \int \frac{dy}{y}\right) = \frac{1}{2}\left(-\log u + \log y\right)$$

$$= \frac{1}{2} \log \frac{y}{u} = \frac{1}{2} \log\left(\frac{1-\cos x}{1+\cos x}\right) = \log\left(\frac{1-\cos x}{1+\cos x}\right)^{1/2}$$

$$\int \frac{1}{\cos x}\, dx = \int \frac{\cos x}{\cos^2 x}\, dx$$

$$\int \frac{\cos x}{1-\operatorname{sen}^2 x}\, dx = \int \frac{\cos x}{(1+\operatorname{sen} x)(1-\operatorname{sen} x)}\, dx = \frac{1}{2}\left(\int \frac{\cos x}{1+\operatorname{sen} x}\, dx + \int \frac{\cos x\, dx}{1-\operatorname{sen} x}\right)$$

$u = 1+\operatorname{sen} x$ $\qquad$ $y = 1-\operatorname{sen} x$

$du = \cos x\, dx$ $\qquad$ $dy = -\cos x\, dx$

$$\frac{1}{2}\left(\int \frac{du}{u} + \int \frac{-dy}{y}\right)$$

$$= \frac{1}{2}\left(\log u - \log y\right) = \frac{1}{2} \log\left(\frac{u}{y}\right) = \frac{1}{2} \log\left(\frac{1+\operatorname{sen} x}{1-\operatorname{sen} x}\right) =$$

6.1.10 Tabela de Integrais Imediatas

(1) $\int du = \cancel{u} + c$

(2) $\int \frac{du}{u} = \ln |u| + c$

(3) $\int u^{\alpha}\, du = \frac{u^{\alpha+1}}{\alpha+1} + c$ (α é constante $\neq -1$)

(4) $\int a^u\, du = \frac{a^u}{\ln a} + c$

(5) $\int e^u\, du = e^u + c$

(6) $\int \operatorname{sen} u\, du = -\cos u + c$

(7) $\int \cos u\, du = \operatorname{sen} u + c$

(8) $\int \sec^2 u\, du = \operatorname{tg} u + c$

(9) $\int \operatorname{cosec}^2 u\, du = -\operatorname{cotg} u + c$ $\int \frac{1}{\operatorname{sen}^2 x} = -\operatorname{ctg} x + C$

(10) $\int \underbrace{\sec u \cdot \operatorname{tg} u\, du}_{} = \sec u + c$

$\int \frac{\operatorname{sen} x}{\cos^2 x} = \int \sec x \cdot \operatorname{tg} x\, dx = \sec x + C$

(11) $\int \operatorname{cosec} u \cdot \operatorname{cotg} u\, du = -\operatorname{cosec} u + c$

(12) $\int \frac{du}{\sqrt{1-u^2}} = \operatorname{arc\,sen} u + c$

(13) $\int \frac{du}{1+u^2} = \operatorname{arc\,tg} u + c$

(14) $\displaystyle\int \frac{du}{u\sqrt{u^2-1}} = \text{arc sec } u + c$

*(15) $\displaystyle\int \text{senh } u\, du = \cosh u + c$

(16) $\displaystyle\int \cosh u\, du = \text{senh } u + c$

(17) $\displaystyle\int \text{sech}^2 u\, du = \text{tgh } u + c$

(18) $\displaystyle\int \text{cosech}^2 u\, du = -\text{cotgh } u + c$

(19) $\displaystyle\int \text{sech } u \cdot \text{tgh } u\, du = -\text{sech } u + c$

*(20) $\displaystyle\int \text{cosech } u \cdot \text{cotgh } u\, du = -\text{cosech } u + c$

(21) $\displaystyle\int \frac{du}{\sqrt{1+u^2}} = \text{arg senh } u + c = \ln\left|u + \sqrt{u^2+1}\right| + c$

(22) $\displaystyle\int \frac{du}{\sqrt{u^2-1}} = \text{arg cosh } u + c = \ln\left|u + \sqrt{u^2-1}\right| + c$

(23) $\displaystyle\int \frac{du}{1-u^2} = \begin{cases} \text{arg tgh } u + c\,, & \text{se } |u| < 1 \\ \text{arg cotgh } u + c\,, & \text{se } |u| > 1 \end{cases}$

$$= \frac{1}{2}\ln\left|\frac{1+u}{1-u}\right| + c$$

(24) $\displaystyle\int \frac{du}{u\sqrt{1-u^2}} = -\text{arg sech } |u| + c$

(25) $\displaystyle\int \frac{du}{u\sqrt{1+u^2}} = -\text{arg cosech } |u| + c\,.$

4.14.18 Tabela Geral de derivadas.

Reunindo todas as fórmulas obtidas, formamos a tabela de derivadas que apresentamos a seguir. Nesta tabela u e v são funções deriváveis de x e c, α e a são constantes.

(*1*) $y = c \Rightarrow y' = 0$

(*2*) $y = x \Rightarrow y' = 1$

(*3*) $y = c \cdot u \Rightarrow y' = c \cdot u'$

(*4*) $y = u + v \Rightarrow y' = u' + v'$

(*5*) $y = u \cdot v \Rightarrow y' = u \cdot v' + v \cdot u'$

(*6*) $y = \dfrac{u}{v} \Rightarrow y' = \dfrac{v \cdot u' - u \cdot v'}{v^2}$

(*7*) $y = u^{\alpha}, (\alpha \neq 0) \Rightarrow y' = \alpha \cdot u^{\alpha - 1} \cdot u'$

(*8*) $y = a^u \; (a > 0, a \neq 1) \Rightarrow y' = a^u \cdot \ln a \cdot u'$

(*9*) $y = e^u \Rightarrow y' = e^u \cdot u'$

(*10*) $y = \log_a u \Rightarrow y' = \dfrac{u'}{u} \log_a e$.

(*11*) $y = \ln u \Rightarrow y' = \dfrac{u'}{u}$

(*12*) $y = \underset{(u > 0)}{u^v} \Rightarrow y' = v \cdot u^{v-1} \cdot u' + u^v \cdot \ln u \cdot v'$

(*13*) $y = \text{sen}\, u \Rightarrow y' = \cos u \cdot u'$

(*14*) $y = \cos u \Rightarrow y' = -\text{sen}\, u \cdot u'$

(*15*) $y = \text{tg}\, u \Rightarrow y' = \sec^2 u \cdot u'$

(*16*) $y = \text{cotg}\, u \Rightarrow y' = -\text{cosec}^2 u \cdot u'$

(17) $y = \sec u \Rightarrow y' = \sec u \cdot \operatorname{tg} u \cdot u'$

(18) $y = \operatorname{cosec} u \Rightarrow y' = -\operatorname{cosec} u \cdot \operatorname{cotg} u \cdot u'$

(19) $y = \operatorname{arc\,sen} u \Rightarrow y' = \dfrac{u'}{\sqrt{1-u^2}}$

(20) $y = \operatorname{arc\,cos} u \Rightarrow y' = \dfrac{-u'}{\sqrt{1-u^2}}$

(21) $y = \operatorname{arc\,tg} u \Rightarrow y' = \dfrac{u'}{1+u^2}$

(22) $y = \operatorname{arc\,cotg} u \Rightarrow y' = \dfrac{-u'}{1+u^2}$

(23) $y = \operatorname{arc\,sec} u,\ |u| \geq 1 \Rightarrow y' = \dfrac{u'}{|u|\sqrt{u^2-1}},\ |u| > 1$

(24) $y = \operatorname{arc\,cosec} u,\ |u| \geq 1 \Rightarrow y' = \dfrac{-u'}{|u|\sqrt{u^2-1}},\ |u| > 1$.

(25) $y = \operatorname{senh} u \Rightarrow y' = \cosh u \cdot u'$

(26) $y = \cosh u \Rightarrow y' = \operatorname{senh} u \cdot u'$

(27) $y = \operatorname{tgh} u \Rightarrow y' = \operatorname{sech}^2 u \cdot u'$

(28) $y = \operatorname{cotgh} u \Rightarrow y' = -\operatorname{cosech}^2 u \cdot u'$

(29) $y = \operatorname{sech} u \Rightarrow y' = -\operatorname{sech} u \cdot \operatorname{tgh} u \cdot u'$

(30) $y = \operatorname{cosech} u \Rightarrow y' = -\operatorname{cosech} u \cdot \operatorname{cotgh} u \cdot u'$

X(31) $y = \operatorname{arg\,senh} u \Rightarrow y' = \dfrac{u'}{\sqrt{u^2+1}}$

X(32) $y = \operatorname{arg\,cosh} u \Rightarrow y' = \dfrac{u'}{\sqrt{u^2-1}},\quad u > 1$