

Estatística

3 - Probabilidades

Exemplo 1 – Espaço Amostral

ESPAÇO AMOSTRAL: S

Conjunto de todos os resultados possíveis de uma variável do fenômeno em observação

EVENTO : A

Sub-conjunto de resultados possíveis

Experimento:

Dois dados equilibrados são lançados e observa-se o número da face superior.

Seja:

x_1 = número da face superior do 1º dado

x_2 = número da face superior do 2º dado.

Espaço Amostral “S”:

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4, 5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Exemplo 1 – Operações

a) Intersecção de eventos: $(A \cap B)$

Consideremos os eventos:

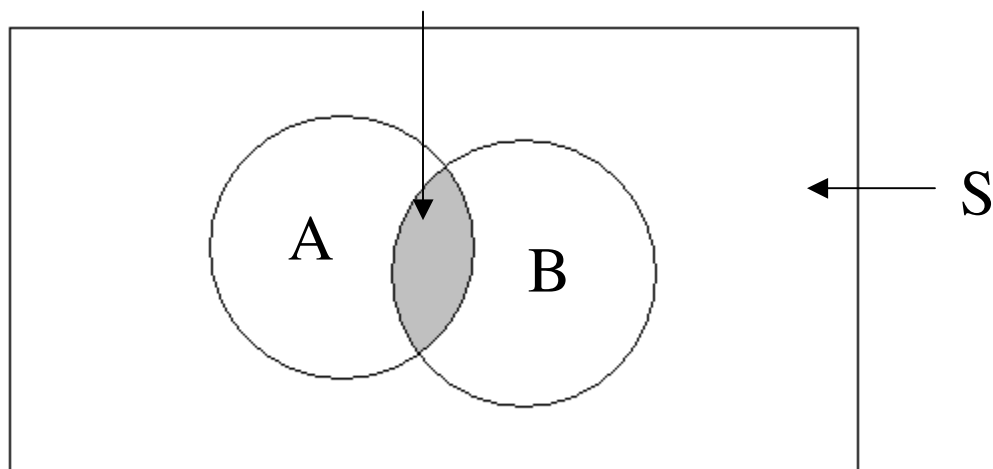
$$A = x_1 + x_2 = 10 = \{(4,6), (5,5), (6,4)\}$$

$$B = x_1 > x_2 = \{(2,1), (3,1), (3,2), \dots, (6,4), (6,5)\}$$

ESPAÇO AMOSTRAL S

	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	
	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	
<div>B</div> →	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)	
	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)	← <div>A</div>
	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)	
	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)	

$(A \cap B)$



Exemplo 1 – Operações

a) União de eventos: $(A \cup B)$

Consideremos os eventos:

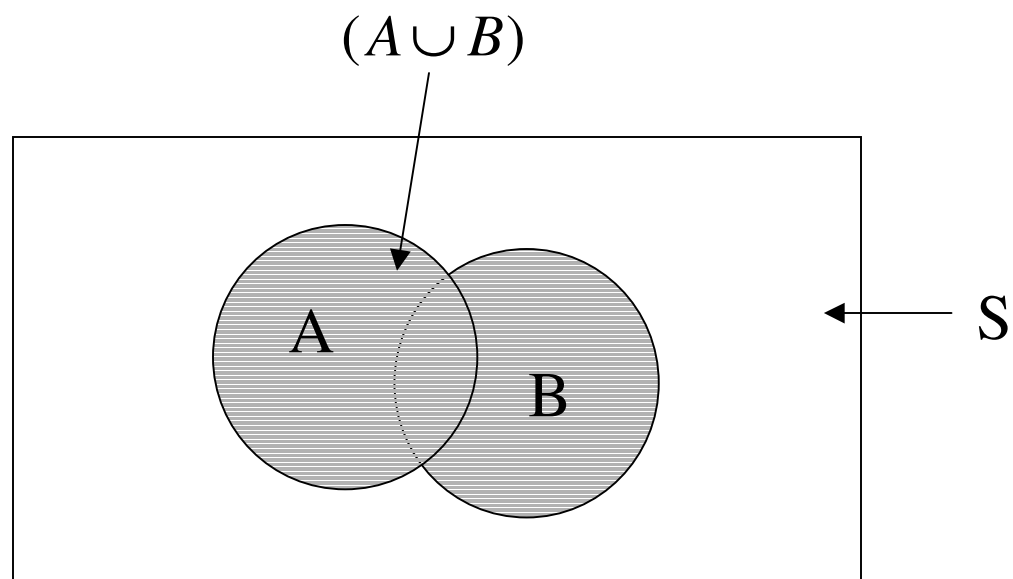
$$A = x_1 + x_2 = 10 = \{(4,6), (5,5), (6,4)\}$$

$$B = x_1 > x_2 = \{(2,1), (3,1), (3,2), \dots, (6,4), (6,5)\}$$

ESPAÇO AMOSTRAL S

	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
B →	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

← **A**



Exemplo 1 – Operações

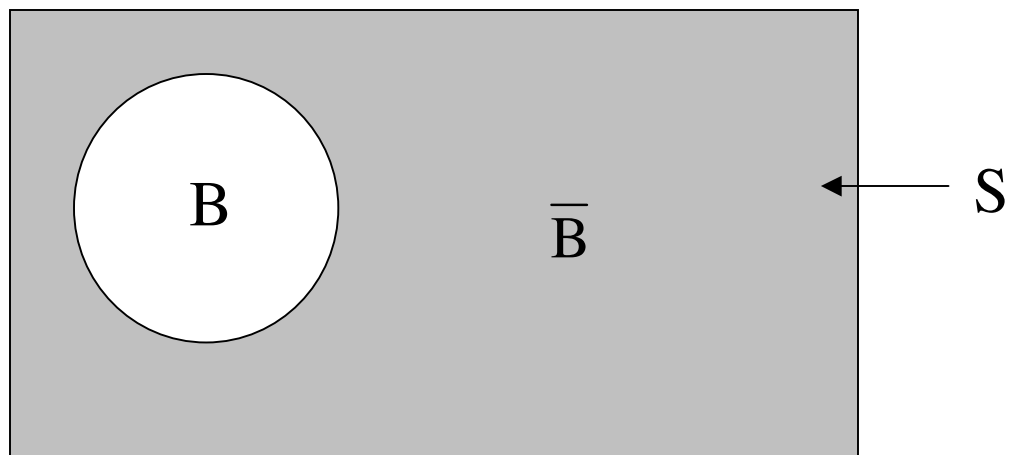
a) Evento complementar: (\bar{B})

Consideremos o evento:

$$B = x_1 > x_2 = \{(2,1), (3,1), (3,2), \dots, (6,4), (6,5)\}$$

ESPAÇO AMOSTRAL S

\bar{B}	→	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
		(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
B	→	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
		(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
		(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
		(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)



Exemplo 1 – Operações

a) Eventos excludentes: $se(E \cap F) = \emptyset$

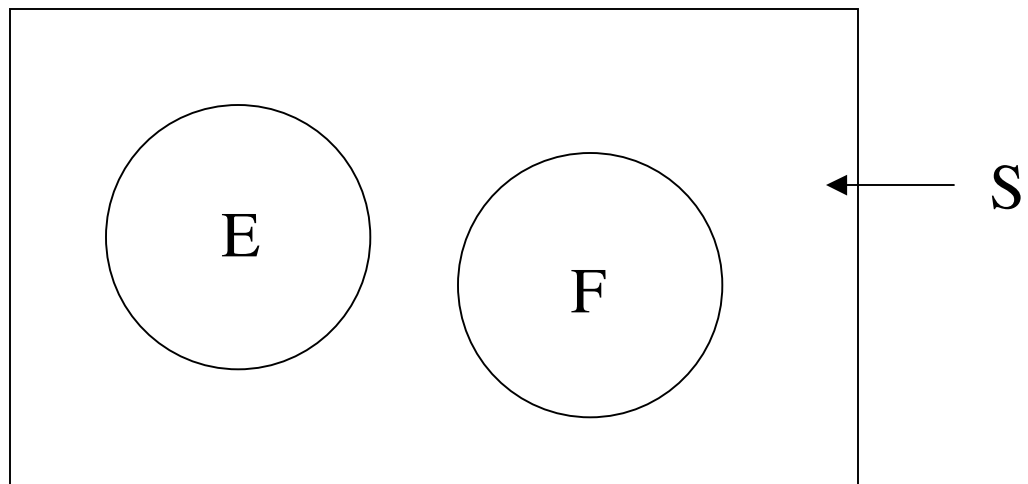
Consideremos os eventos:

$$E = x_1 = x_2 = 1 = \{(1,1)\}$$

$$F = x_1 + x_2 = 5 = \{(1,4), (2,3), (3,2)\}$$

ESPAÇO AMOSTRAL S

E	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
F	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)



Probabilidades

ESPAÇO AMOSTRAL: S

Conjunto de todos os resultados possíveis de uma variável do fenômeno em observação

EVENTO : A

Sub-conjunto de resultados possíveis

FUNÇÃO PROBABILIDADE: P

$$P : S \longrightarrow [0, 1]$$

PROBABILIDADE:

$$P(A) = \frac{m}{n}$$

onde

m = número de resultados favoráveis ao evento A

n = número de resultados possíveis

$$P(S) = 1$$

Exemplo 1 – Propriedades

a) Intersecção de eventos: $(A \cap B)$

Consideremos os eventos:

$$A = x_1 + x_2 = 10 = \{(4,6), (5,5), (6,4)\}$$

$$B = x_1 > x_2 = \{(2,1), (3,1), (3,2), \dots, (6,4), (6,5)\}$$

ESPAÇO AMOSTRAL S

	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	
	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	
	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)	
B →	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)	← A
	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)	
	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)	

$$P(A \cap B) = ?$$

$$P(A \cap B) = \frac{1}{36}$$

Exemplo 1 – Propriedades

a) União de eventos: $(A \cup B)$

Consideremos os eventos:

$$A = x_1 + x_2 = 10 = \{(4,6), (5,5), (6,4)\}$$

$$B = x_1 > x_2 = \{(2,1), (3,1), (3,2), \dots, (6,4), (6,5)\}$$

ESPAÇO AMOSTRAL S

	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	
	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	
	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)	
B →	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)	← A
	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)	
	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)	

Propriedades:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{3}{36} + \frac{15}{36} - \frac{1}{36} = \frac{17}{36}$$

$$P(A \cup B) = 1 - P(\bar{A} \cap \bar{B})$$

$$P(A \cup B) = 1 - \frac{19}{36} = \frac{17}{36}$$

Exemplo 1 – Propriedades

a) União de eventos: $(A \cup B)$

Consideremos os eventos:

$$A = x_1 + x_2 = 10 = \{(4,6), (5,5), (6,4)\}$$

$$B = x_1 > x_2 = \{(2,1), (3,1), (3,2), \dots, (6,4), (6,5)\}$$

ESPAÇO AMOSTRAL S

\bar{B}	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
\bar{A}	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Propriedades:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{3}{36} + \frac{15}{36} - \frac{1}{36} = \frac{17}{36}$$

$$P(A \cup B) = 1 - P(\bar{A} \cap \bar{B})$$

$$P(A \cup B) = 1 - \frac{19}{36} = \frac{17}{36}$$

Exemplo 1 – Propriedades

a) Evento complementar: (\bar{B})

Consideremos o evento:

$$B = x_1 > x_2 = \{(2,1), (3,1), (3,2), \dots, (6,4), (6,5)\}$$

ESPAÇO AMOSTRAL S

\bar{B} →	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$$P(\bar{B}) = \frac{21}{36}$$

Propriedade:

$$P(\bar{B}) = 1 - P(B)$$

$$P(\bar{B}) = 1 - \frac{15}{36} = \frac{21}{36}$$

Exemplo 1 – Operações

a) Eventos excludentes: $se(E \cap F) = \emptyset$

Consideremos os eventos:

$$E = x_1 = x_2 = 1 = \{(1,1)\}$$

$$F = x_1 + x_2 = 5 = \{(1,4), (2,3), (3,2)\}$$

ESPAÇO AMOSTRAL S

E	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
F	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Propriedades:

$$P(\emptyset) = 0$$

$$P(E \cup F) = P(E) + P(F)$$

$$P(E \cup F) = \frac{1}{36} + \frac{3}{36} = \frac{4}{36}$$