COMP4106 - Reasoning with Fuzzy Logic Lecture 2

September 30th, 2009

These slides are based on the book
Artificial Intelligence, A Guide to Intelligent Systems by Michael Negnevitsky
and lecture slides by Franz Oppacher

Building a Fuzzy Expert System

- Get the specification of the problem
  - determine the ranges of problem inputs and outputs
- Determine the fuzzy sets
  - simple shapes (triagle, trapezoids) are usually "good enough"
  - complex problems may require more complex membership functions
- Construct the fuzzy rules
  - use knowlege provided by experts in the problem domain
- Program the fuzzy sets, rules and procedures to perform inference
  - build your own or use use a library/toolkit
- Test and tune the system
  - provide sufficient overlap between the rules
  - add or remove rules
  - adjust rule execution weights
  - revise the shapes of the sets
  - use alternative techniques to devise the rules

Fuzzy Rules

Example

IF a is A THEN b is B

- a and b are linguistic variables
- A,B are linquistic values from the universes of discourse M,N

Classic Rules

IF speed is $>$ 100 THEN stopping_distance is long
IF speed is $<$ 40 THEN stopping_distance is short

Fuzzy Rules

IF speed is high THEN stopping_distance is long
IF speed is low THEN stopping_distance is short

Using Fuzzy Rules

- Each rule includes two distinct parts
  - *antecedent* - the IF part of the rule
  - *concequent* - the THEN part of the rule
- Each rule "fires" to some extent
- Rules can have multiple antecedents
- Rules can hvae multiple concequents

Rule with multiple antecedents

IF project_duration is long AND project_staffing is large AND project_funding is inadequate THEN risk is high

Rule with multiple consequents

IF temperature is high THEN hot_water is reduced; cold_water is increased

Fuzzy Inference

- The process of mapping from a given input to some output, using fuzzy set theory
- Two styles of reasoning:
  - Mandani style (most commonly used)
  - Sugeno style
- Four steps:
  - Fuzzification - determining to what degree do the crisp inputs belong to each of the appropriate fuzzy sets
  - Rule evaluation - given the fuzzy inputs apply the antecedents are applied
  - Aggregation - unify the outputs of all the rules into a single fuzzy set
  - Defuzzification - given a fuzzy output return a crisp value

Project Staffing Example - Rules

Rule 1

IF project_funding is adequate OR project_staffing is small THEN risk is low

Rule 2

If project_funding is marginal AND project_staffing is large THEN risk is normal

Rule 3

IF project_funding is inadequate THEN risk is high

- {project_funding, project_staffing, risk} are linguistic variables
- {adequate, marginal, inadequate} are linguistic values in the fuzzy sets of project_funding
- {large, small} are linguistic values from the fuzzy sets of project_staffing
- {low, normal, high} are linguistic values from the fuzzy sets of risk

Project Staffing Example - Inputs

- Crisp values, provided by an expert

$$project\_funding = 35\%$$
$$project\_staffing = 60\%$$

- Each input is fuzzified over all the membership functions used by fuzzy rules
- Fuzzy values

$$\mu_{F=inadequate}(35) = 0.5$$
$$\mu_{F=marginal}(35) = 0.2$$
$$\mu_{S=small}(60) = 0.1$$
$$\mu_{S=large}(60) = 0.7$$

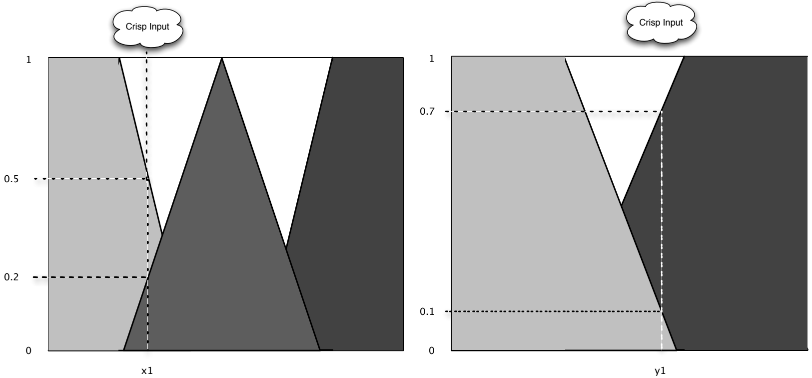

Figure: Fuzzifying

Project Staffing Example - Rule Evaluation

- Given the fuzzy inputs apply them in the antecedents of the rules
- Evaluate the disjunction of the antecedents using the union operation

Rule 1

IF project_funding is adequate (0.0) OR project_staffing is small (0.1) THEN risk is low

$$\mu_{A \cup B}(\chi) = max[\mu_A(\chi), \mu_B(\chi)]$$
$$\mu_{R=low}(z) = max[0.0, 0.1] = 0.1$$
$$\mu_{A \cup B}(\chi) = \mu_A(\chi) + \mu_B(\chi) - \mu_A(\chi) * \mu_B(\chi)$$
$$\mu_{R=low}(z) = 0.0 + 0.1 - 0.0 * 0.1 = 0.1$$

Project Staffing Example - Rule Evaluation

- Evaluate the conjuction of the antecedents using the AND operation

Rule 2

If project_funding is marginal (0.2) AND project_staffing is large (0.7) THEN risk is normal

$$\mu_{A\cap B}(\chi) = min[\mu_A(\chi), \mu_B(\chi)]$$
$$\mu_{R=normal}(z) = min[0.2, 0.7] = 0.2$$
$$\mu_{A\cap B}(\chi) = \mu_A(\chi) * \mu_B(\chi)$$
$$\mu_{R=normal}(z) = 0.2 * 0.7 = 0.14$$

Project Staffing Example - Rule Evaluation

- Simple rules assign the value to the consquent

Rule 3

IF project_funding is inadequate (0.5) THEN risk is high

$$\mu_{R=high}(z) = \mu_{F=inadequate}(\chi) = 0.5$$

- Finally

$$\begin{aligned}\mu_{R=low}(z) &= 0.1\\ \mu_{R=normal}(z) &= 0.2\\ \mu_{R=high}(z) &= 0.5\end{aligned}$$

Project Staffing Example - Rule Evaluation

- Use the obtained values to *clip* or *scale* the consequent membership function
- Clipping (correlation minimum)
  - Cuts the consequent membership function at the level of the antecedent truth
  - Tends to lose some information
  - Requires less calculations
  - Generates output suface that is easier to defuzzify
- Scaling (correlation product)
  - Multiply the consequent membership function by the level of antecedent truth
  - Loses less information
  - More difficult to deal with in later stages

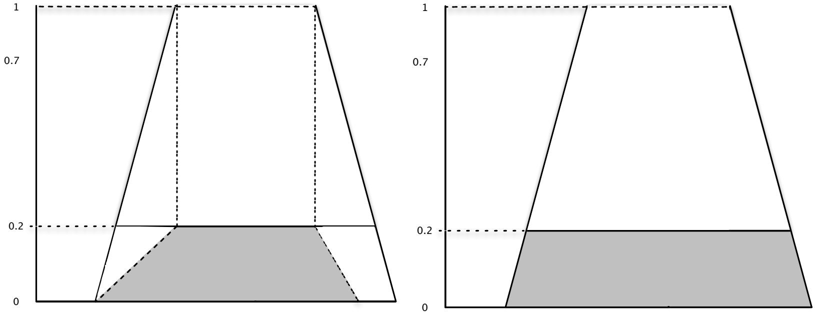

Figure: Scaling vs Clipping

Project Staffing Example - Rule Output Aggregation

- The process of unifying the fuzzy outputs of each fuzzy rules
- The clipped/scaled membership functions are combined into a single fuzzy set

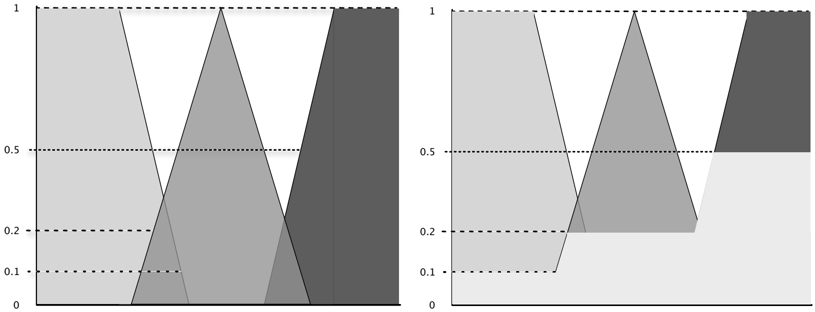

Figure: Output Set

Project Staffing Example - Defuzzification

- Given the aggregate output fuzzy set returns a single crisp value
- There are multiple methods to perform defuzzification:
  - Centroid method (most popular)

$$COG = \frac{\int_a^b \mu_A(\chi) x dx}{\int_a^b \mu_A(\chi) dx} \quad COG = \frac{\sum_{x=a}^b \mu_A(\chi) x}{\sum_{x=a}^b \mu_A(\chi)}$$

  - Bisector
  - Mean of maximum
  - Smallest value of maximum
  - Largest value of maximum

Project Staffing Example - Defuzzification

$$COG = \frac{\sum_{x=a}^{b} \mu_A(\chi)x}{\sum_{x=a}^{b} \mu_A(\chi)}$$

$$\begin{aligned} COG &= \frac{(0+10+20)*0.1+(30+40+50+60)*0.2+(70+80+90+100)*0.5}{0.1*3+0.2*4+0.5*3} \\ &= 67.4 \end{aligned}$$

- The final crisp output value is 67.4.
- The risk involved in the project is $\approx 67\%$.

Sugeno-style Inference

- Mamdani-style inference uses integration, which is computationally intensive
- The output values can be instead represented as *singletons*
- *Fuzzy singleton* - membership function that is unity at a single point and zero everywhere else.
- Sugeno-style inference uses singletons to speed up processing

Sugeno-style Inference

- Given that each rule consequent is a singleton, weighted average can be used instead of center of gravity

$$\begin{aligned} WA &= \frac{\mu(k_1) * k_1 + \mu(k_2) * k_2 + \mu(k_3) * k_3}{\mu(k_1) + \mu(k_2) + \mu(k_3)} \\ &= \frac{0.1 * 20 + 0.2 * 50 + 0.5 * 80}{0.1 + 0.2 + 0.5} \\ &= 65 \end{aligned}$$

- The final output value is different but is close to the previous value

Crane Example - Rules

Rule 1

IF distance is far and angle is zero small THEN apply positive medium power

Rule 2

IF distance is far and angle is small negative THEN apply big positive power

Rule 3

IF distance is far and angle is big negative THEN apply positive medium power

Crane Example - Rules

Rule 4

IF distance is medium AND angle is small negative THEN apply negative medium power

Rule 5

IF distance is small AND angle is positive small THEN apply positive medium power

Rule 6

IF distance is 0 AND angle is 0 THEN power is 0

- Suspose:

$$\mu_{D=far}(12yards) = 0.1 \qquad \mu_{A=zero}(+4\,\text{deg}) = 0.2$$
$$\mu_{D=medium}(12yards) = 0.9 \qquad \mu_{A=pos\_small}(+4\,\text{deg}) = 0.8$$

- Aggregate the antecedents:
  - Rule 1
    IF distance = medium & angle = pos_small THEN power = pos_medium

    $$\mu_{P=pos\_power}(z) = min(0.9, 0.8) = 0.8$$

  - Rule 2
    IF distance = medium & angle = zero THEN power = zero

    $$\mu_{P=pos\_power}(z) = min(0.9, 0.2) = 0.2$$

  - Rule 3
    IF distance = far & angle = zero THEN power = pos_medium

    $$\mu_{P=pos\_power}(z) = min(0.1, 0.2) = 0.1$$

Crane Example - Aggregation & Defuzzification

- Rule 1 result
  power = pos_medium to degree 0.8
- Rule 2 result
  power = zero to degree 0.2
- Rule 3 result
  power = pos_medium to degree 0.1
- Thefore

$$\mu_{P=pos\_power}(z) = max(0.8, 0.1) = 0.8 \qquad \mu_{P=zero}(z) = 0.2$$

- Defuzzify using the any of the previously mentioned methods

Final Notes on Fuzzy Logic

- Why use fuzzy logic
  - Simple and flexible mathematics
  - Based on natural language
  - Easy to model non-linear functions of arbitrary complexity
  - Can be easily mixed with conventional control techniques
  - Good performace
- When not to use fuzzy logic
  - Working the problem into the fuzzy framework is too troublesome
  - Large scale problems can be very difficult to implement