Lecture 9:

Fuzzy Logic

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OVERVIEW

1. Historical background.
2. Fuzzy variables and sets.
3. Reasoning with fuzzy facts and rules.
4. Defuzzyfication.
5. Strengths and weaknesses.
6. Application of fuzzy logic.

HISTORICAL BACKGROUND

400 B.C. Classical logic of Aristotle: every proposition is either `True` or `False` (no middle). Plato objected.

1900 Jan Lukasiewicz proposed three–valued logic: `True`, `False` and `Possible`.

1965 Lotfi A. Zadeh published "*Fuzzy Sets*" article.

1979 S. Haack "*Do we need fuzzy logic?*"

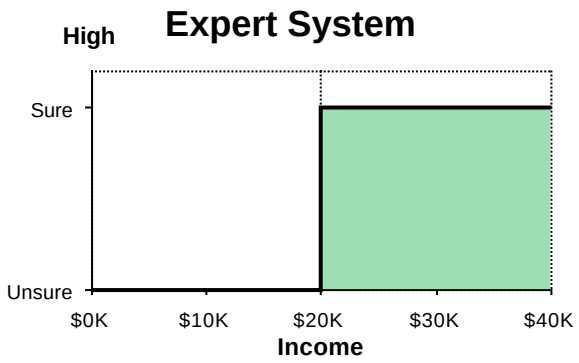
CRISP vs FUZZY

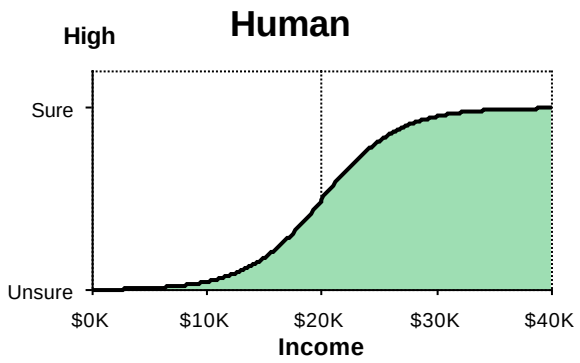
Consider two facts describing an income, High and Low, and two rules concluding about the degree of risk:

IF *income is high* THEN *risk is low*

IF *income is low* THEN *risk is high*

- How much income is high?
- If $£20,000$ p.a. is high, is $£19,500$ high too?





TRADITIONAL ES 'SOLUTION'

Traditional Rule–Based system would have to use many rules:

IF	*income* $\pounds 20,000$	THEN	*risk is very low*
IF	*income* $\pounds 19,500$	THEN	*risk is almost very low*
IF	*income* $\pounds 19,000$	THEN	*risk is almost low*
IF	*income* $\pounds 18,500$	THEN	*risk is low*
IF	*income* $\pounds 18,000$	THEN	*risk is quite low*
IF	*income* $\pounds 17,500$	THEN	*risk is nearly low*
IF	*income* $\pounds 17,000$	THEN	*risk is somewhat low*

How many rules would we need to account for all cases?

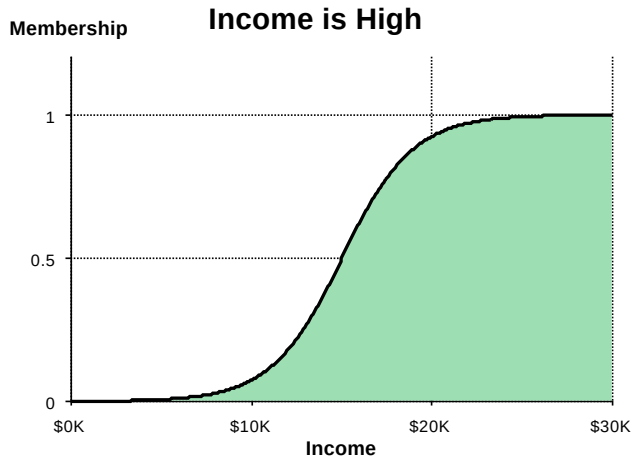
FUZZY MEMBERSHIP

Membership function $M(x)$ represents the degree of belief (confidence factor) that x belongs to some category.

$$0 \leq M(x) \leq 1 \quad \text{or} \quad 0\% \leq M(x) \leq 100\%$$

$M(x) = 0$ does not belong, $M(x) = 1$ does belong. For example, the fact "Income" belongs to category `High`:

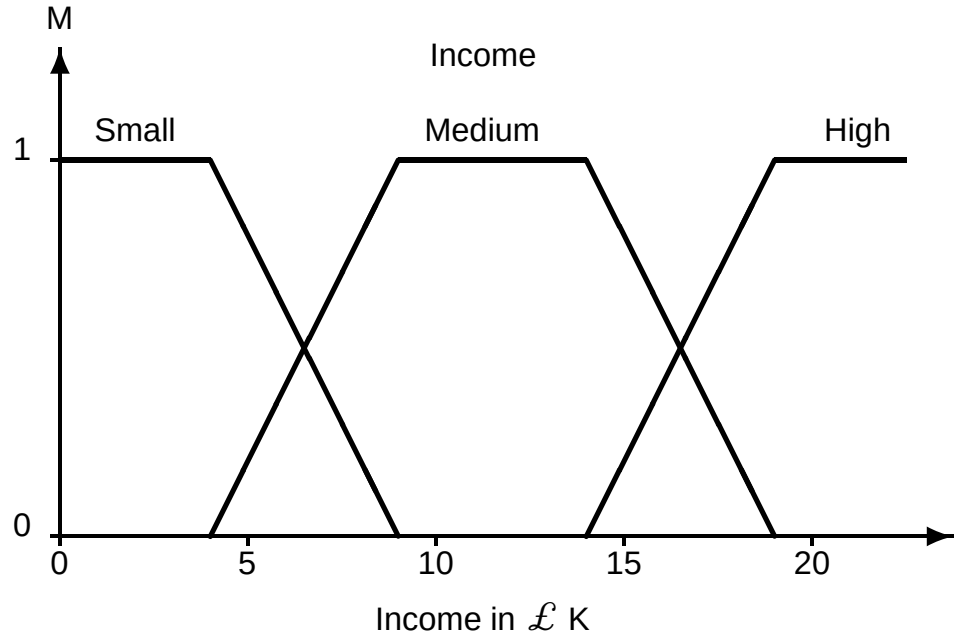
Income	belongs to `High`
$£20,000$	$M = 0.90$
$£19,500$	$M = 0.85$
$£19,000$	$M = 0.80$



FUZZY SETS

A set of several fuzzy variables (categories) is called a **fuzzy set**. For example, an income can be $\{\texttt{Small}, \texttt{Medium}, \texttt{High}\}$

Each category in the set has its membership function:



Income $\pounds 15$K

High	$M = 0.2$
Medium	$M = 0.8$
Small	$M = 0$

WHAT IS THE ADVANTAGE?

£20K				
£19K		High	IF *High*	THEN
⋮	⟶	Medium	⟶ IF *Medium*	THEN
⋮		Small	IF *Small*	THEN
£5K				

- Many real values are converted into few fuzzy variables with different memberships.
- Then just a few rules can be used for reasoning.

REASONING WITH FUZZY RULES

Reasoning with fuzzy facts with different degrees of membership (confidence) is similar to Stanford algebra:

$$\begin{aligned} M(\text{NOT } x) &= 1 - M(x) \\ M(x_1 \text{ AND } x_2) &= \min[M(x_1), M(x_2)] \\ M(x_1 \text{ OR } x_2) &= \max[M(x_1), M(x_2)] \end{aligned}$$

Rules in fuzzy systems also have degrees of confidence:

IF *condition* THEN *action*

$$M(\textit{action}) = M(\textit{condition}) \times M(\textit{rule})$$

EXAMPLE

Consider a rule with confidence $M = 0.8$:

IF *income is high* OR *income is stable* THEN *risk is low*

What is the confidence that *risk is low* if $M(\textit{income is high}) = 0.6$ $M(\textit{income is stable}) = 0.2$?

Answer: because condition (left–hand–side) of the rule uses OR, the confidence of the condition is the largest of the two:

$$M(\textit{income is high} \text{ OR } \textit{income is stable}) = \max[0.6, 0.2] = 0.6$$

and the confidence of action (right–hand–side) is

$$M(\textit{risk is low}) = 0.6 \times 0.8 = 0.48$$

MULTIPLE RULES

In a fuzzy logic system **ALL** rules are firing in parallel. This means that we can have multiple conclusions about the same fact or even contradictory conclusions.

- If two rules arrive to the same conclusion x with different $M_1(x)$ and $M_2(x)$, then we use a formula to combine the beliefs

$$M(x) = M_1(x) + M_2(x) - M_1(x) \times M_2(x)$$

  For example, let $M_1 = 0.2$, $M_2 = 0.6$ that the *risk is low*, then $M = 0.2 + 0.6 - 0.2 \cdot 0.6 = 0.68$.

- Different and even contradictory conclusions (e.g. *risk is low* and *risk is high*) are all allowed to stay in the system.

CRISP CONCLUSIONS

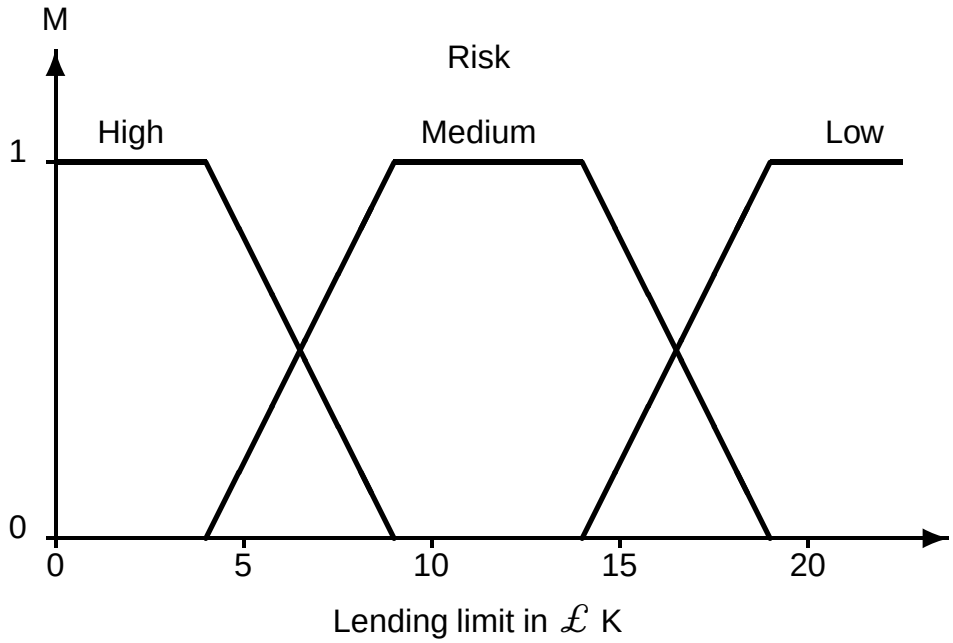
- The conclusions that a fuzzy reasoning system will arrive to are fuzzy facts (facts with degrees of membership). For example, *risk is low* with $M = 0.5$.
- The outcome, however, must be a precise decision (e.g. how much money exactly can be let).
- The process of converting a fuzzy fact into a crisp value is called **defuzzyfication**.

DEFUZZYFICATION

Suppose the system has arrived to three conclusions:

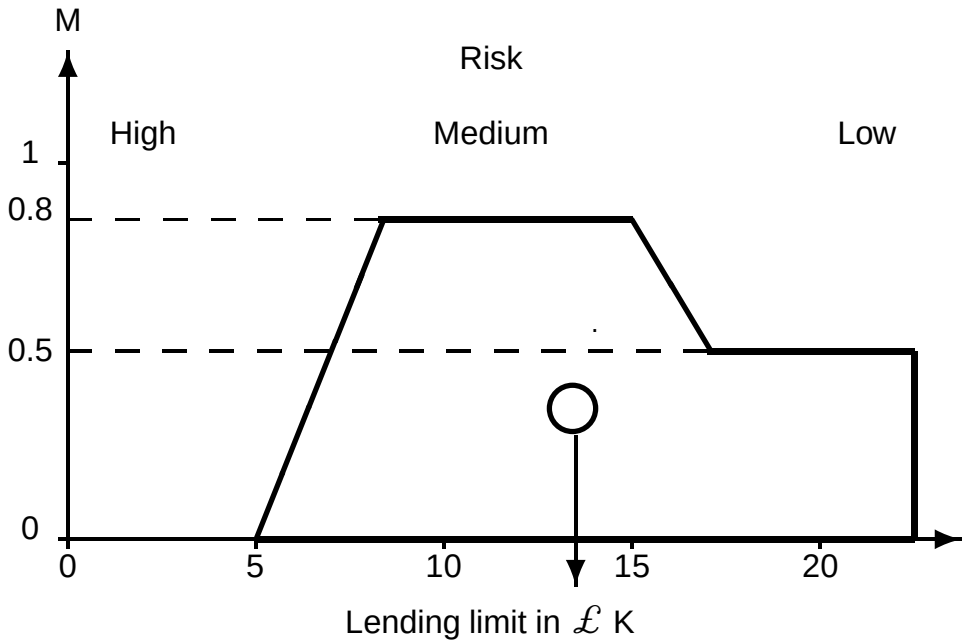
Risk

Low	$M = 0.5$
Medium	$M = 0.8$
High	$M = 0$



DEFUZZYFICATION (CONT.)

One of the way to of defuzzyfication is called **centroid** method (finding the ‘centre of gravity’):



Unite and select the areas under the belief values M. The centre of the selected area provides the crisp output value.

FUZZY LOGIC EXPERT SYSTEMS

£20K								£80K
£19K		High		IF THEN		Low		£70K
⋮	→	Medium	→	IF THEN	→	Medium	→	⋮
⋮		Small		IF THEN		High		⋮
£5K								£1K

- From crisp values (Salary) to fuzzy variables (Income).
- Reasoning leads to conclusions with other fuzzy variables (Risk).
- Defuzzyfication provides crisp output outcomes (Lending limit).

STRENGTHS AND WEAKNESSES

Strengths are

- Use fewer rules as there is no need to cover all the cases.
- Thus, it is easier to understand the system.
- Fine tuning by changing the membership functions.

Weaknesses are

- Limited explanatory system.
- Still necessary to consult experts.
- Saturation problem (if memberships are not defined carefully).

APPLICATION OF FUZZY LOGIC

- Risk assessment.
- Loan assessment.
- Investment advice.
- Strategic planning.
- Control systems (appliances such as washing machines, etc)
- Robotics.