# Matemática Discreta

## 1.4    Examples of Proofs

### 1.4.1    A Tautology

**Theorem 9.** *The following proposition is a tautology:*

$$(X \Rightarrow Y) \quad \Leftrightarrow \quad (\neg Y \Rightarrow \neg X)$$

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## 1.4   Examples of Proofs

### 1.4.1   A Tautology

**Theorem 9.** *The following proposition is a tautology:*

$$(X \Rightarrow Y) \quad \Leftrightarrow \quad (\neg Y \Rightarrow \neg X)$$

"If you are wise, then you attend recitation."

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## 1.4   Examples of Proofs

### 1.4.1   A Tautology

**Theorem 9.** *The following proposition is a tautology:*

$$(X \Rightarrow Y) \quad \Leftrightarrow \quad (\neg Y \Rightarrow \neg X)$$

"If you are wise, then you attend recitation."

"If you do not attend recitation, then you are not wise."

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## 1.4   Examples of Proofs

### 1.4.1   A Tautology

**Theorem 9.** *The following proposition is a tautology:*

$$(X \Rightarrow Y) \quad \Leftrightarrow \quad (\neg Y \Rightarrow \neg X)$$

*Proof.* We show that the left side is logically equivalent to the right side for every setting of the variables $X$ and $Y$.

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**Theorem 9.** *The following proposition is a tautology:*

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| $X$ | $Y$ | $X \Rightarrow Y$ | $\neg Y \Rightarrow \neg X$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | F |
| F | T | T | T |
| F | F | T | T |

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**Theorem 9.** *The following proposition is a tautology:*

$$(X \Rightarrow Y) \quad \Leftrightarrow \quad (\neg Y \Rightarrow \neg X)$$

$$\frac{P \Rightarrow Q}{\neg Q \Rightarrow \neg P} \qquad\qquad \frac{\neg Q \Rightarrow \neg P}{P \Rightarrow Q}$$

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## 1.4 Examples of Proofs

### 1.4.2 A Proof by Contradiction

In logical terms, indirect proof relies on the following inference rule:

$$\frac{\neg P \Rightarrow \text{false}}{P}$$

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## 1.4 Examples of Proofs

### 1.4.2 A Proof by Contradiction

In logical terms, indirect proof relies on the following inference rule:

$$\frac{\neg P \Rightarrow \text{false}}{P}$$

tautology

| $P$ | $(\neg P \Rightarrow \text{false}) \Rightarrow P$ |
|-----|------|
| $T$ | $T$ |
| $F$ | $T$ |

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## 1.4   Examples of Proofs

### 1.4.2   A Proof by Contradiction

**Theorem 10.** $\sqrt{2}$ *is an irrational number.*