MAT1856/APM466: Mathematical Finance **Winter 2020**

Assignment #1: Yield Curves

Professor: Luis Seco, TA: Jonathan Mostovoy

Note: Please bring any questions about this assignment to your TA's, Jonathan's, weekly office hour.

1.1 Introduction

Due: Monday, February 3rd, 2020 at 11AM to be submitted online via Crowdmark.

For each weekday from Jan 2nd, 2020, until Jan 15th, 2020 (inclusive, 2 weeks worth, 10 days), collect all historical close prices for the 32 Canadian Government Bonds which have a maturity less than 10 years from January 15th, 2020 on the "Frankfurt" Exchange; I.e., all bonds listed via the following two links:

1. `https://markets.businessinsider.com/bonds/finder?borrower=71&maturity=shortterm&yield=&bondtype=2%2c3%2c4%2c16&coupon=&currency=184&rating=&country=19`
2. `https://markets.businessinsider.com/bonds/finder?borrower=71&maturity=midterm&yield=&bondtype=2%2c3%2c4%2c16&coupon=&currency=184&rating=&country=19`

The data surrounding the 32 bonds found on the above two links will be used for calculating the yield curve (ytm curve), spot curve, and forward curve.

To collect the close prices, after markets close on Jan 15th, you must click-through to each of the 32 bonds' unique page and under "Go In-Depth" click "Historical". The close prices will be the rightmost column of prices after selecting "Frankfurt" as the exchange.

For each bond, you will also need to collect the following information: coupon, ISIN, issue date, maturity date. All this data is available on the "Snapshot" page before clicking through to the "Historical" page.

This assignment is split into 2 parts, the first asks questions about some fundamentals of fixed income and mathematical finance. The second will be an empirical exercise in generating yield curves, and in particular the 1, 2, 3, 4 and 5 year rates, and analyzing these rates through PCA.

1.2 Expectations

1. You may use R, Python, or any programming language (no Excel without approval from TA beforehand) of your choice to answer the "empirical questions".
2. Please have your final report typeset using LaTeXand the following template: template. The website: www.overleaf.com is particularly useful.
3. **Important/New:** Your report must be no longer than 3 pages long in total.
4. Each of the "fundamental questions" must be answered in clear and coherent sentences - no math.
5. At the end of your report you must cite all references and include a link to a GitHub repository with all your code used for the project.
6. You may, and are encouraged, to discuss how to do these questions with your peers. However, your write-up must be done individually, and the sharing of your write-up or code before the deadline is prohibited.
7. A 5% penalty per day past the deadline up until 1 week (35%) will apply for late submissions.

Additional Notes: Marks will be awarded for each question as either full-, half-, or zero-marks according to if the question was answered with a few small mistakes, substantial mistakes but fundamental idea still correct, or fundamental idea wrong / no answer respectively. -10 marks (each) if expectations 2, 3, or 4 not adhered to.

2 Questions

2.1 Fundamental Questions - 25 points

1. *(5 points total)* (One sentence each.)
   (a) *(1 point)* Why does a government issue bonds?
   (b) *(2 points)* From the government's perspective, why does the yield curve matter?
   (c) *(2 points)* How can a government reduce the money supply through bonds?

2. *(10 points)* We asked you to pull data for 32 bonds, but if you'd like to construct a yield a "0-5 year" yield & spot curves, as the government of Canada issues all of its bonds with a semi-annual coupon, when bootstrapping you'll only need 10 bonds to perform this task. Ideally, the bonds in any yield curve should be consistent in some way with one another. Select (list) 10 bonds that you will use to construct the aforementioned curves with an explanation of why you selected those 10 bonds based on the characteristics we asked you to collect for each bond (coupon, issue date, maturity date).
   (*Note: 1) There is a unique ideal answer, 2) To easily refer to a bond, please use the following convention: "CAN 2.5 Jun 24" refers to the Canadian Government bond with a maturity in June 24 and a coupon of 2.5*).

3. *(10 points)* In a few plain English sentences, in general, if we have several stochastic processes for which each process represents a unique point along a stochastic curve (assume points/processes are evenly distributed along the curve), what do the eigenvalues and eigenvectors associated with the covariance matrix of those stochastic processes tell us?
   (*Hint: This is called Principal Component Analysis*)

2.2 Empirical Questions - 75 points

4. *(40 points total)*
   (a) *(10 points)* First, calculate each of your 10 selected bonds' yield (ytm). Then provide a well-labeled plot with a 5-year yield curve (ytm curve) corresponding to each day of data (Jan 2 to Jan 15) superimposed on-top of each other. You may use any interpolation technique you deem appropriate provided you include a reasonable explanation for the technique used.
   (b) *(15 points)* Write a pseudo-code (a simple explanation of an algorithm) for how you would derive the spot curve with terms ranging from 1-5 years from your chosen bonds in part 2. (Please also recall the day convention simplifications provided in part 2 as well.) Then provide a well-labeled plot with a 5-year spot curve corresponding to each day of data superimposed on-top of each other.
   (c) *(15 points)* Write a pseudo-code for how you would derive the 1-year forward curve with terms ranging from 2-5 years from your chosen bonds in part 2 (I.e., a curve with the first point being the 1yr-1yr forward rate and the last point being the 1yr-4yr rate). Then provide a well-labeled plot with a forward curve corresponding to each day of data superimposed on-top of each other.

5. *(20 points)* Calculate two covariance matrices for the time series of daily log-returns of yield, and forward rates (no spot rates). In other words, first calculate the covariance matrix of the random variables X_i, for $i = 1, \ldots, 5$, where each random variable X_i has a time series $X_{i,j}$ given by:
$$X_{i,j} = \log(r_{i,j+1}/r_{i,j}), \quad j = 1, \ldots, 9$$
   then do the same for the following forward rates - the 1yr-1yr, 1yr-2yr, 1yr-3yr, 1yr-4yr.

6. *(15 points)* Calculate the eigenvalues and eigenvectors of both covariance matrices, and in one sentence, explain what the first (in terms of size) eigenvalue and its associated eigenvector imply.