Quant 2, Lab 3

DAGs, Sensitivity Analysis

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2025-02-13

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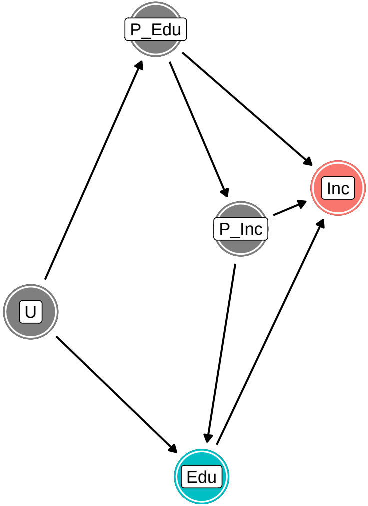
- Representation of the data generating process (DGP)
  - Nodes (X,D,Y etc.) are random variables
  - Edges ($X \to Y$) denote a direct causal effect of X on Y
- Tools to help understand whether a research design can identify a causal relationship

Example: Becker, 1994



- Main relationship of interest: Education effect on Income

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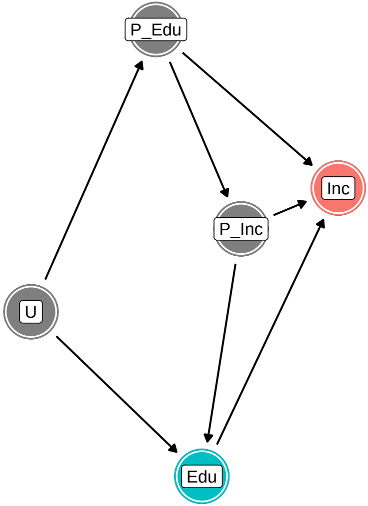

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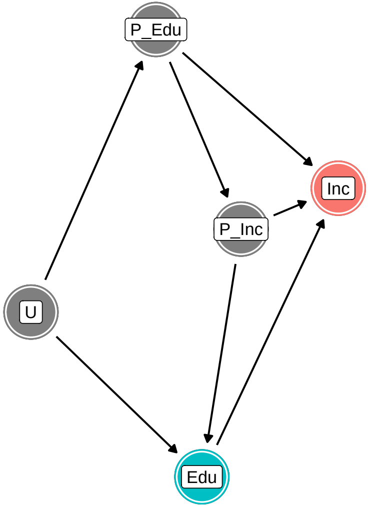
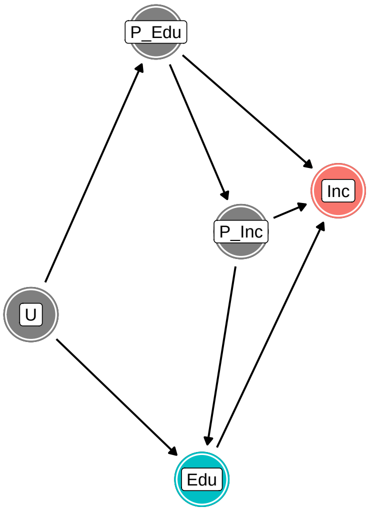


- Main relationship of interest: Education effect on Income
- Parental effects (income, education) affect both child income and education
- Unobserved **family specific** factors (ie, genetics) affect parent and child education

DAGs and Identification



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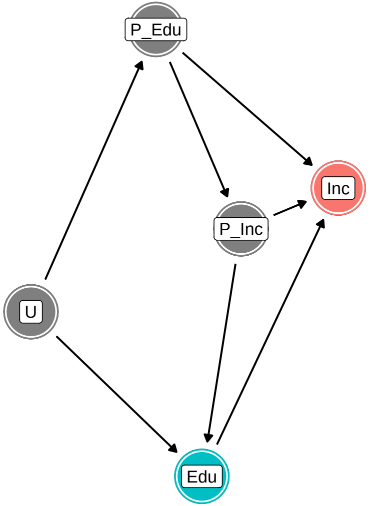
DAGs and Identification



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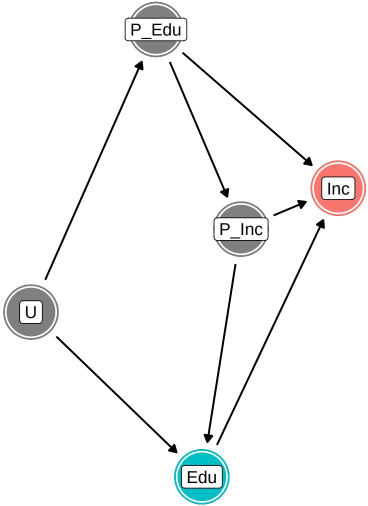

- To identify the effect of some D on Y
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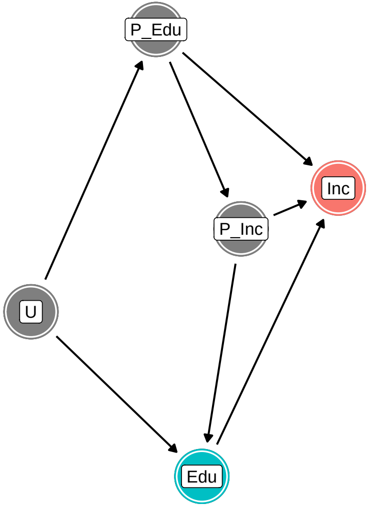
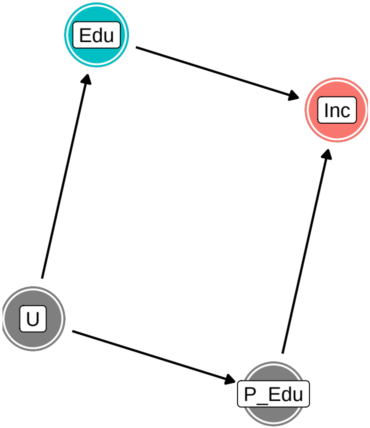
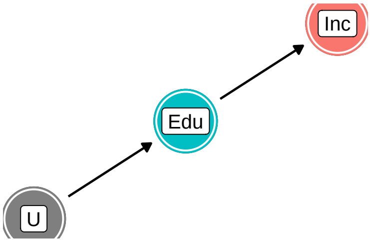


- To identify the effect of some D on Y
- DAG **must** satisfy the **backdoor criterion** (no backdoor paths)
  - A **backdoor path** is an alternate path between D and Y that does not go through a collider (more on these later)
- Eg, we cannot identify the effect of `Edu` on `Inc` because there is a **backdoor path**, eg through `P_Inc`

Controlling for a variable

- If we *control for a variable* in a DAG, we **remove its node and corresponding edges**





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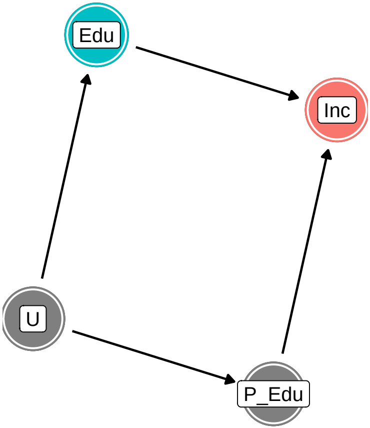
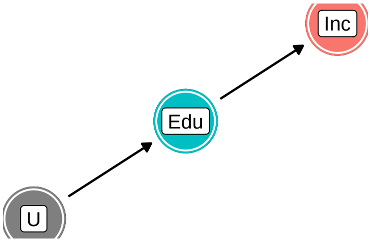
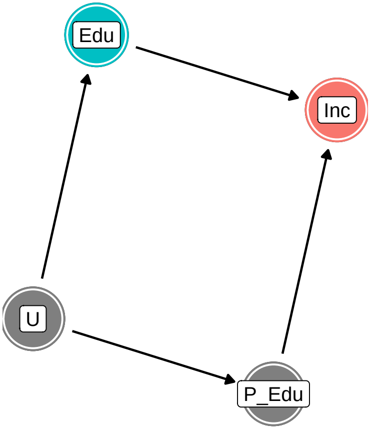
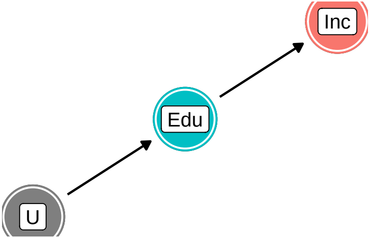




Controlling for a variable

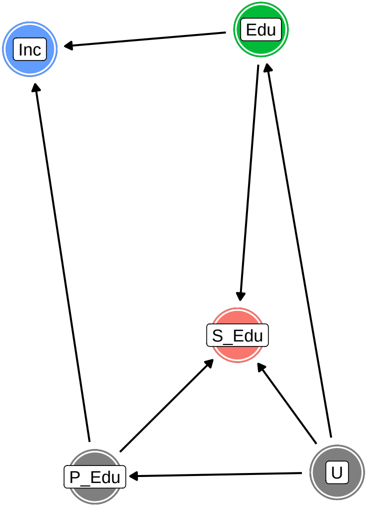
- If we *control for a variable* in a DAG, we **remove its node and corresponding edges**
  - Unless it's a collider
- Ex, if we control for `P_Inc` and `P_Edu`, we get the following DAGs:





Colliders

- A **collider** is a node that has multiple arrows leading into it



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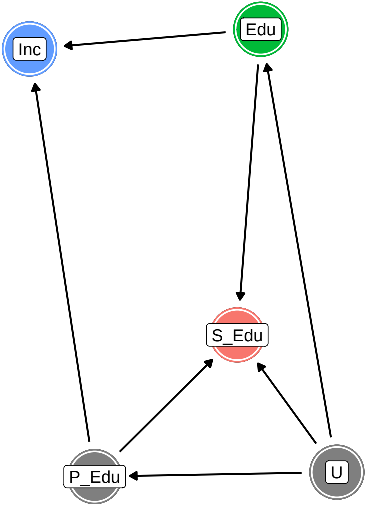

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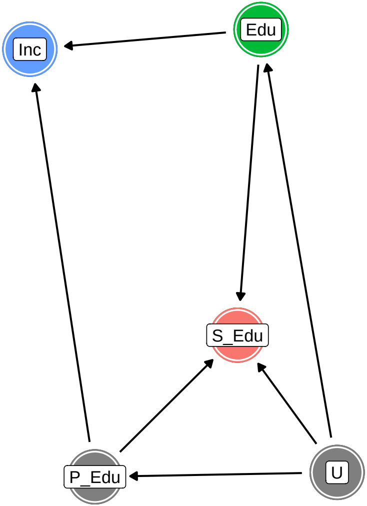

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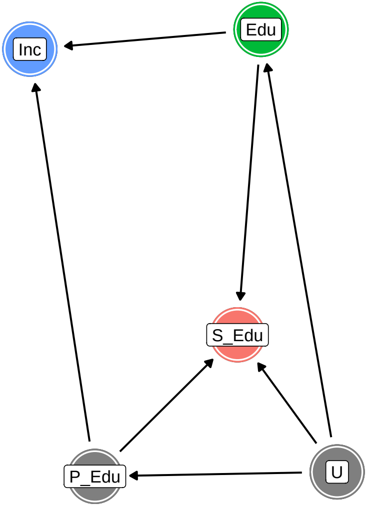

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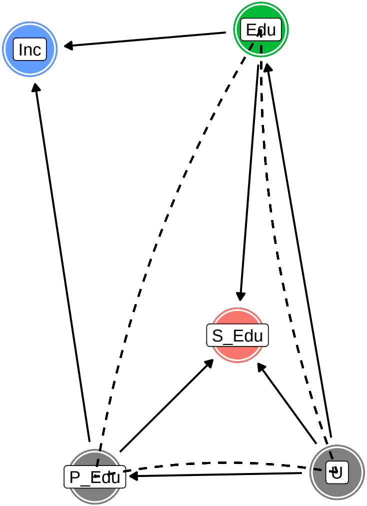
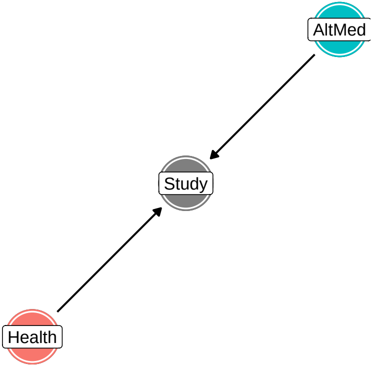
- Consider the following DAG that includes a sibling's education `S_Edu`.
- Suppose we are interested in understanding the relationship between sibling education (`Edu -> S_Edu`)
- Should we control for `Inc`?
  - No. Because `Inc` is a collider, the backdoor path is closed.



Colliders | Sample Selection

- Collider bias often discussed in the context of sample selection



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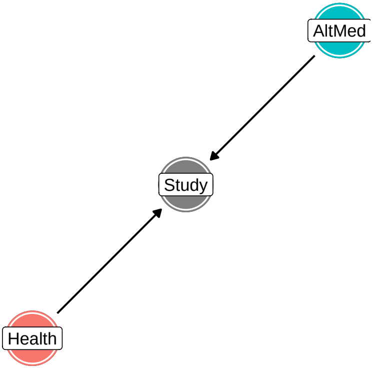

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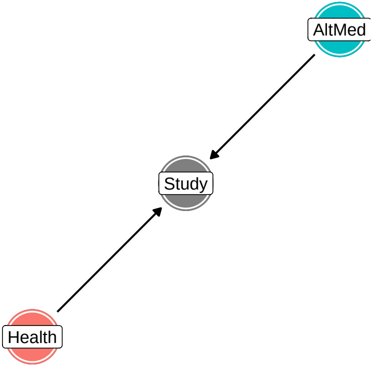

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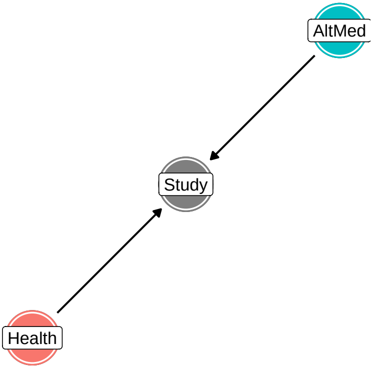

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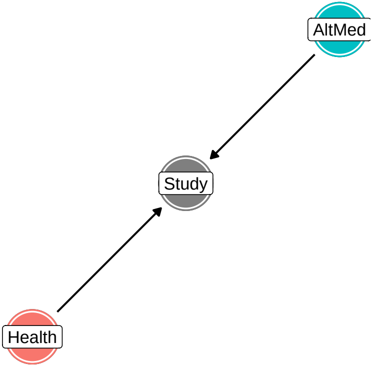

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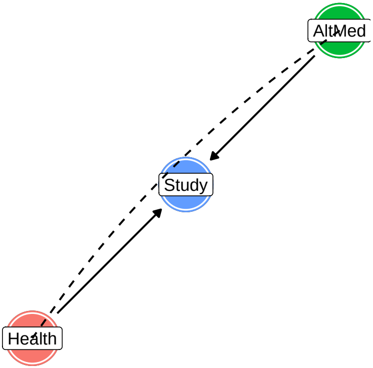

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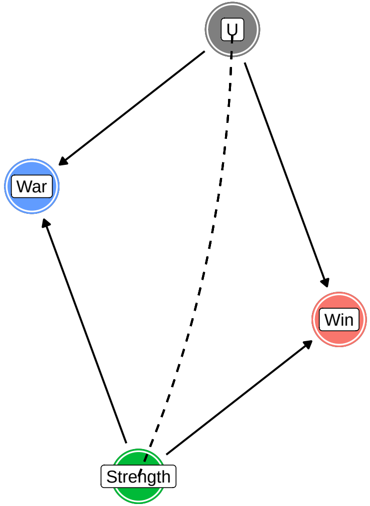

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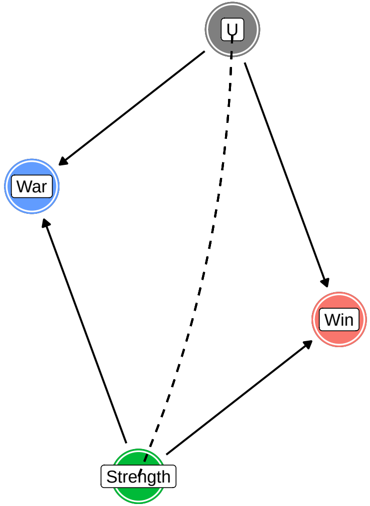
- Country military strength appears to be uncorrelated with winning a war



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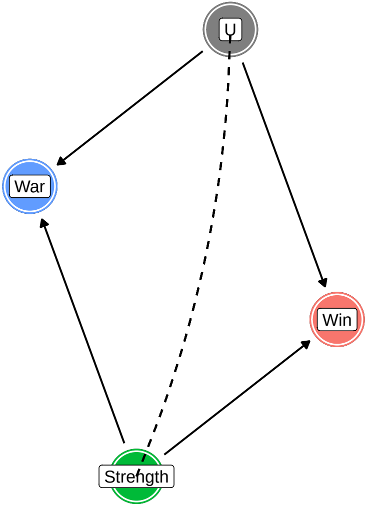

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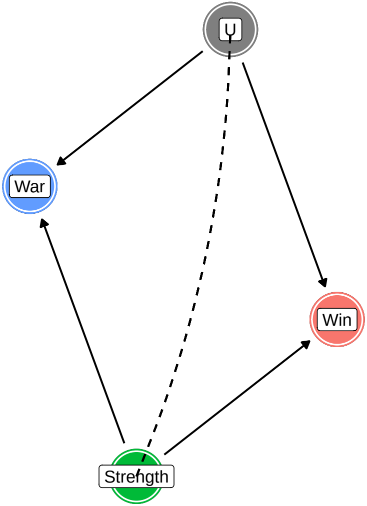
  - Conditioning on `War` opens a backdoor path through `U`



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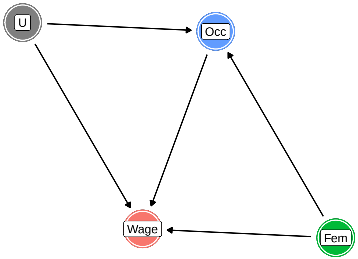

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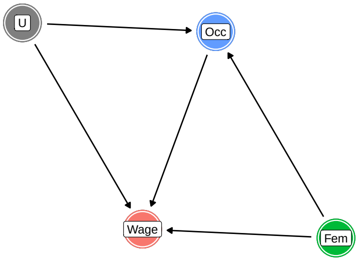

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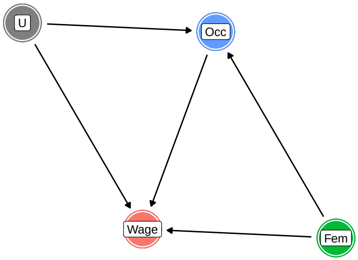

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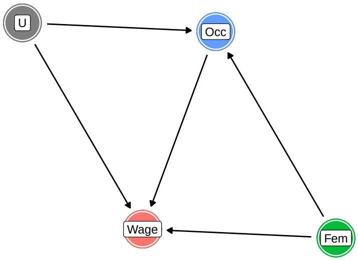

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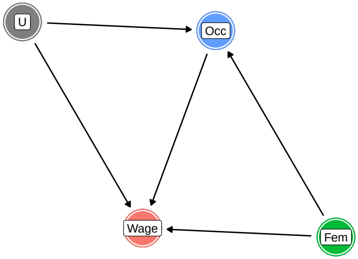

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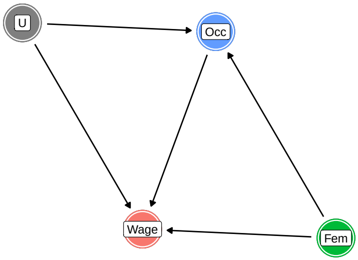

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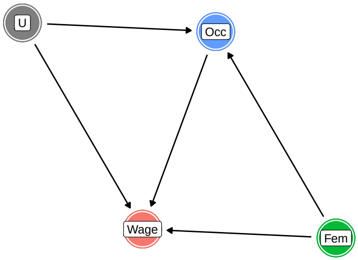

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  - $\epsilon_1, \epsilon_2, u \;\; N(0, 1)$

Colliders | Gender Wage Gap

Simulation Setup

```
N <- 10000
tb <- tibble(
    # Gender is exogenous
    female = sample(c(0, 1), N, replace = T),
    # U is exogenous
    u = rnorm(N),
    # Occupation choice a function of u and gender
    occupation = u - 0.1 * female + rnorm(N),
    # Wage is a function of u and occupation
    #  AND very slightly directly affected by gender
    wage = -0.1 * female + occupation + 2 * u + rnorm(N)
)
```

Colliders | Gender Wage Gap

Simulation Results

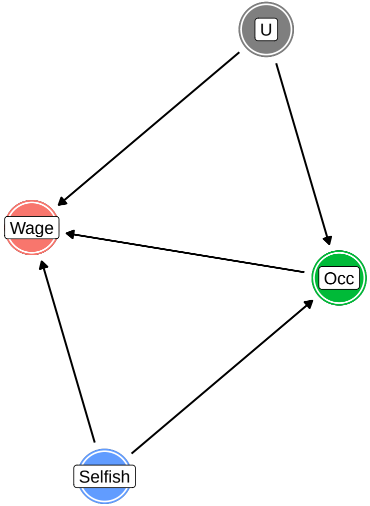
Dependent Variable:		wage	
Model:	(1)	(2)	(3)
Variables			
Constant	-0.0563	-0.0285	-0.0012
	(0.0461)	(0.0244)	(0.0142)
female	-0.1045	0.0197	-0.0986***
	(0.0653)	(0.0346)	(0.0202)
occupation		1.969***	0.9759***
		(0.0123)	(0.0101)
u			2.007***
			(0.0144)
Fit statistics			
Observations	10,000	10,000	10,000

IID standard-errors in parentheses

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Bias Amplification

- Same DAG as before, but let's say we are now interested in `Occ->Wage`



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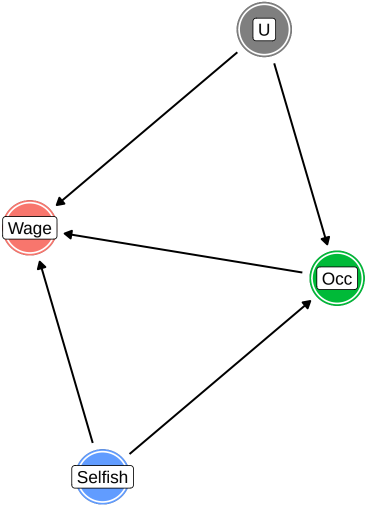

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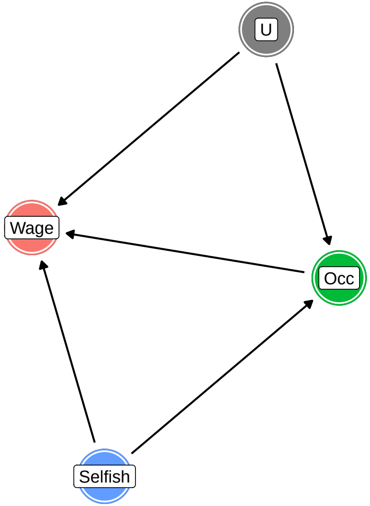

Bias Amplification

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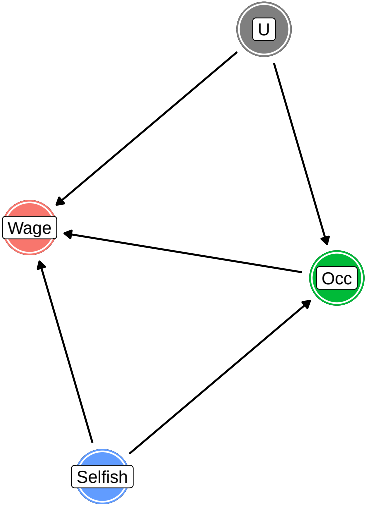
- Suppose `Selfish` increases `Occ` but decreases `Wages`
- Suppose we observe `Selfish`
- Should we control for `Selfish`?



Bias Amplification

Simulation Setup

```r
tb <- tibble(
    # U and Selfish exogenous
    u = rnorm(N),
    selfish = rnorm(N),
    # Selfish positively affects occupation
    occupation = u + selfish + rnorm(N),
    # Selfish negatively affects wages
    wage = occupation + 2 * u - 0.5 * selfish + rnorm(N)
)
```

Bias Amplification

Simulation Results

Dependent Variable:	wage	
Model:	(1)	(2)
Variables		
Constant	-0.0374*	-0.0313*
	(0.0213)	(0.0175)
occupation	1.495***	1.985***
	(0.0123)	(0.0124)
selfish		-1.485***
		(0.0215)
Fit statistics		
Observations	10,000	10,000

IID standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Post-treatment bias

Homola, Pereira, and Tavits (2020, APSR) argue that living closer to a Nazi era concentration camp increases modern day far right support. - Pepinsky, Goodman, Ziller (2023, APSR) argue that "state-level differences confound the relationship between distance to camps and out-group intolerance" - They add state level fixed effects and show that the original effect disappears. - "Länder cannot be posttreatment variables unless we assume that the creation of Länder was caused by their distance from concentration camps." HPT (2024, APSR) rebuttal. "contemporary state fixed effects induce post-treatment bias if any factor (observable or not) that varies across German Länder is a direct or indirect descendant of proximity to concentration camps." "

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- Sensitivity analysis and Double ML can also help
  - (Double ML next week)

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- General idea - quantify **how large** an omitted variable would have to be to mess up your results

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- General idea - quantify **how large** an omitted variable would have to be to mess up your results
  - Roman and D'Urso show a correlation between anti LGBTQ attitudes and dislike for "Latinx" group label, controlling for several factors
  - Sensitivity analysis: Omitted variable would have to have as large an effect on "Latinx" favorability as partisanship

Sensitivity analysis: Attitudes in Darfur (Hazlett, 2019)

- 2003-2004 government violence against civilians

```
library(sensemakr)
data("darfur")
darfur.model <- feols(
    peacefactor ~ directlyharmed + female +
        age + farmer_dar + herder_dar + pastvoted + hhsize_
    data = darfur
)
```

Sensitivity analysis: Attitudes in Darfur (Hazlett, 2019)

- 2003-2004 government violence against civilians
- Outcome (Y): attitudes toward peace

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Sensitivity analysis: Attitudes in Darfur (Hazlett, 2019)

- 2003-2004 government violence against civilians
- Outcome (Y): attitudes toward peace
- Treatment (D): exposure to violence

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Sensitivity analysis: Attitudes in Darfur (Hazlett, 2019)

- Specification with lots of controls shows a positive relationship

Dependent Variable:	peacefactor
Model:	(1)
Variables	
directlyharmed	0.0973***
	(0.0238)
female	-0.2321***
	(0.0244)
age	-0.0021***
	(0.0007)
farmer_dar	-0.0404
	(0.0296)
herder_dar	0.0143
	(0.0365)
pastvoted	-0.0480*
	(0.0269)
hhsize_darfur	0.0012
	(0.0022)
Fixed-effects	

Sensitivity analysis: Attitudes in Darfur (Hazlett, 2019)

- `sensemakr` package lets us conduct sensitivity analysis **relative to a covariate of choice**

```
darfur.sensitivity <- sensemakr(
    model = darfur.model,
    treatment = "directlyharmed",
    benchmark_covariates = "female",
    kd = 1:3,
    ky = 1:3,
)
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- Ex, gender played an important role in exposure to violence: women were specifically targeted
- `kd` and `ky` arguments: we investigate a hypothetical confounder **1-3x** as strong as `female`

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)
```

Sensitivity Analysis

```
ovb_minimal_reporting(darfur.sensitivity)
```

Outcome: *peacefactor*

Treatment:	$R^2_{Y\sim D\|\mathbf{X}}$	$RV_{q=1}$	$RV_{q=1,\alpha=0.05}$
directlyharmed	2.2%	13.9%	7.6%
Bound (1x female): $R^2_{Y\sim Z\|\mathbf{X},D} = 12.5\%$, $R^2_{D\sim Z\|\mathbf{X}} = 0.9\%$			

Sensitivity Analysis

```
summary(darfur.sensitivity)
```

- Partial R2 of the treatment with the outcome: an extreme confounder (orthogonal to the covariates) that explains 100% of the residual variance of the outcome, would need to explain at least 2.19% of the residual variance of the treatment to fully account for the observed estimated effect.

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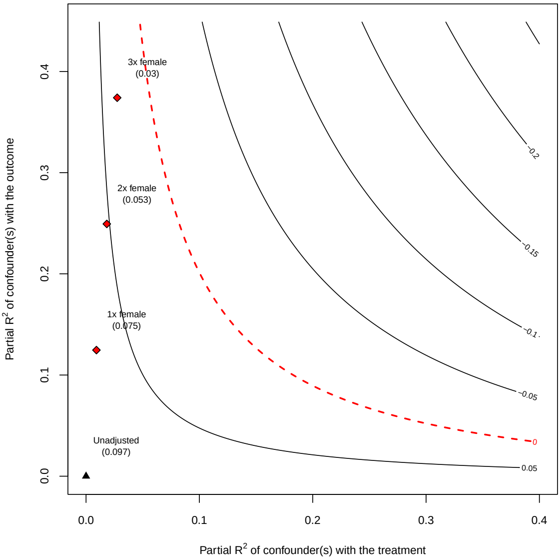
- Robustness Value, q = 1, alpha = 0.05: unobserved confounders (orthogonal to the covariates) that explain more than 7.63% of the residual variance of both the treatment and the outcome are strong enough to bring the estimate to a range where it is no longer 'statistically different' from 0 at the significance level of alpha = 0.05.

Sensitivity: Plots

```r
plot(darfur.sensitivity)
```



HW1

- Grades coming soon

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  - $\frac{1}{n}\sum y_i^2$ approaches a finite value
  - So $V(\bar{Y_N}) \to 0$ and now we can say unbiased $=$ consistency for $\bar{Y}_n$

HW1 | Q8 (Simulation question)

N	$\text{Var}\left[\overline{Y}_N\right]$	$\text{Var}[\hat{\mu}]$	$\text{Var}\left[\overline{Y}_N\right] - \text{Var}[\hat{\mu}]$	$(\frac{1}{N} - \frac{1}{n})\mu^2$	μ
20	0.2097426	0.07680564	0.132937	0.132937	1.647119
50	0.08132877	0.02978178	0.05154699	0.05154699	1.647119
100	0.03852416	0.01410716	0.024417	0.024417	1.647119

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for (N in sample_sizes) {
    do stuff
    for (j in 1:nsims) {
        do more stuff
    }
}
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