Regime Switching

How Regimes Affect Asset Allocation

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Introduction

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1. Regime-Switching Beta Model

International CAPM model with world market return:

$$y_t^w = \mu^w(s_t) + \sigma^w(s_t)\epsilon_t^w,$$

Individual excess return for security *j*:

$$y_{t+1}^j = (1-\beta^j)\mu^z + \beta^j\mu^w(s_{t+1}) + \beta^j\sigma^w(s_{t+1})\epsilon_{t+1}^w + \bar{\sigma}^j\epsilon_{t+1}^j.$$

Where:

$\mu^\omega(S_t) denotes\ the\ world\ market\ expected\ excess\ return$

$\sigma^\omega(S_t) denotes\ the\ conditional\ volatility$

$\varepsilon_t^\omega is\ the\ unexpected\ shock\ or\ return\ drawn\ from\ a\ standard\ normal\ distribution$

$\mu^z\ denotes\ zero\ beta\ excess\ return$

$\bar{\sigma}^j denotes\ idiosyncratic\ term, volatility$

Regime Dynamics

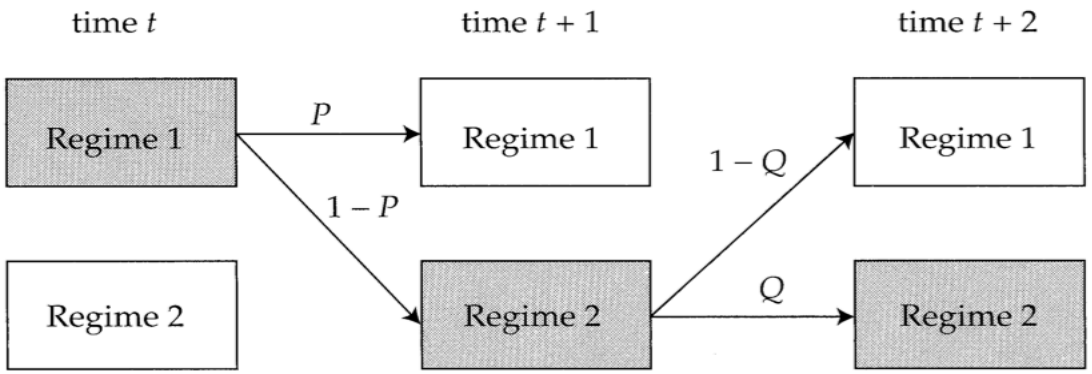
- Assume that the portfolio manager knows the realized regime but does not know which regime will be realized next time period.

- The regime variable follows a Markov process with constant transition probabilities P and Q:

$$
\begin{aligned}
P &= p(s_t = 1 | s_{t-1} = 1) \\
Q &= p(s_t = 2 | s_{t-1} = 2).
\end{aligned}
$$

- The situation P =1-Q usually does not happen. Empirical studies (e.g., Gray(1996)) find P and Q are well over 50%, indicating persistent states.

A Regime-Switching Model for the world



Conditional Expected Return

$$\underbrace{P\mu^w(1) + (1-P)\mu^w(2)}_{e^w(1)} \qquad \underbrace{(1-Q)\mu^w(1) + Q\mu^w(2)}_{e^w(2)} \qquad \underbrace{(1-Q)\mu^w(1) + Q\mu^w(2)}_{e^w(2)}$$

Conditional Variance

$$\underbrace{\begin{gathered}P[\sigma^w(1)]^2 + (1-P)[\sigma^w(2)]^2 \\ + P(1-P)[\mu^w(1) - \mu^w(2)]^2\end{gathered}}_{\Sigma^w(1)} \qquad \underbrace{\begin{gathered}(1-Q)[\sigma^w(1)]^2 + Q[\sigma^w(2)]^2 \\ + Q(1-Q)[\mu^w(1) - \mu^w(2)]^2\end{gathered}}_{\Sigma^w(2)} \qquad \underbrace{\begin{gathered}(1-Q)[\sigma^w(1)]^2 + Q[\sigma^w(2)]^2 \\ + Q(1-Q)[\mu^w(1) - \mu^w(2)]^2\end{gathered}}_{\Sigma^w(2)}$$

Data

The table lists the country composition of the geographic returns. Within each geographic region, they construct monthly returns, value-weighted in US dollars. The sample period is February 1975 through the end of 2000.

North America	UK	Japan	Europe large	Europe small	Pacific ex-Japan
Canada			France	Austria	Australia
US			Germany	Belgium	New Zealand
			Italy	Denmark	Singapore
				Finland	
				Ireland	
				Netherlands	
				Norway	
				Spain	
				Sweden	
				Switzerland	

Regime-Switching Beta Model Parameter Estimates

A. Transition probabilities and μ_z

Measure	P	Q	μ^z
Estimate	0.8917	0.8692	0.74
Standard error	0.0741	0.1330	0.68

B. World market

Measure	$\mu(1)$	$\mu(2)$	$\sigma(1)$	$\sigma(2)$
Estimate	0.90	0.13	2.81	5.04
Standard error	0.32	0.62	0.44	0.55

C. Country betas, β

Measure	North America	United Kingdom	Japan	Europe: Large	Europe: Small	Pacific ex Japan
Estimate	0.88	1.03	1.21	0.90	0.89	0.92
Standard error	0.03	0.06	0.07	0.05	0.04	0.07

D. Idiosyncratic volatilities, $\bar{\sigma}$

Measure	North America	United Kingdom	Japan	Europe: Large	Europe: Small	Pacific ex Japan
Estimate	2.40	4.50	4.62	3.87	2.72	4.99
Standard error	0.09	0.18	0.19	0.16	0.11	0.20

Notes: All parameters are monthly; the mean, μ, and standard deviation, σ, parameters are expressed in percentages.

RS Equity Model Estimation Results, 1975-2000

Regime	North America	United Kingdom	Japan	Europe: Large	Europe: Small	Pacific ex Japan
A. Regime-dependent excess returns						
Regime 1	9.64	9.76	9.90	9.65	9.65	9.67
Regime 2	3.47	2.54	1.42	3.36	3.39	3.22
B. Regime-dependent covariances and correlations						
Regime 1						
North America	1.35	0.44	0.48	0.45	0.54	0.38
United Kingdom	0.90	3.08	0.37	0.35	0.42	0.29
Japan	1.06	1.25	3.60	0.38	0.46	0.32
Europe: Large	0.79	0.92	1.08	2.30	0.43	0.30
Europe: Small	0.78	0.91	1.07	0.80	1.53	0.36
Pacific ex Japan	0.81	0.94	1.11	0.82	0.82	3.33
Regime 2						
North America	2.37	0.64	0.68	0.65	0.73	0.58
United Kingdom	2.10	4.49	0.58	0.55	0.63	0.49
Japan	2.47	2.89	5.53	0.58	0.66	0.52
Europe: Large	1.83	2.14	2.52	3.36	0.63	0.49
Europe: Small	1.82	2.13	2.50	1.85	2.58	0.56
Pacific ex Japan	1.88	2.20	2.58	1.91	1.90	4.45
C. Tangency portfolio weights						
Regime 1	0.42	0.06	–0.01	0.15	0.31	0.08
Regime 2	0.79	–0.14	–0.55	0.25	0.54	0.10
Unconditional	0.52	0.04	–0.16	0.18	0.37	0.09
Average market cap	0.50	0.09	0.22	0.08	0.08	0.02

Asset Allocation

- Expected excess return of country *j* is given by

$$e_{j,i} = (1\text{-}\beta^{j})\,\mu^{z} + \beta^{j}\,e_{i}^{\omega}$$

Where *i* is the prevailing regime and e_{i}^{ω} is the world's expected excess return, *i* = 1,2

- Expected returns differ across individual equity indices only through their different betas w.r.t the world market

- Let

$$\boldsymbol{e_1} = (e_{j,1})^{T} \quad \text{and} \quad \boldsymbol{e_2} = (e_{j,2})^{T}$$

The Covariance Matrix

- Idiosyncratic part

$$V=\mathrm{diag}\left[\bar{\sigma}^2{}_j\right]$$

- Regime- dependent systematic part

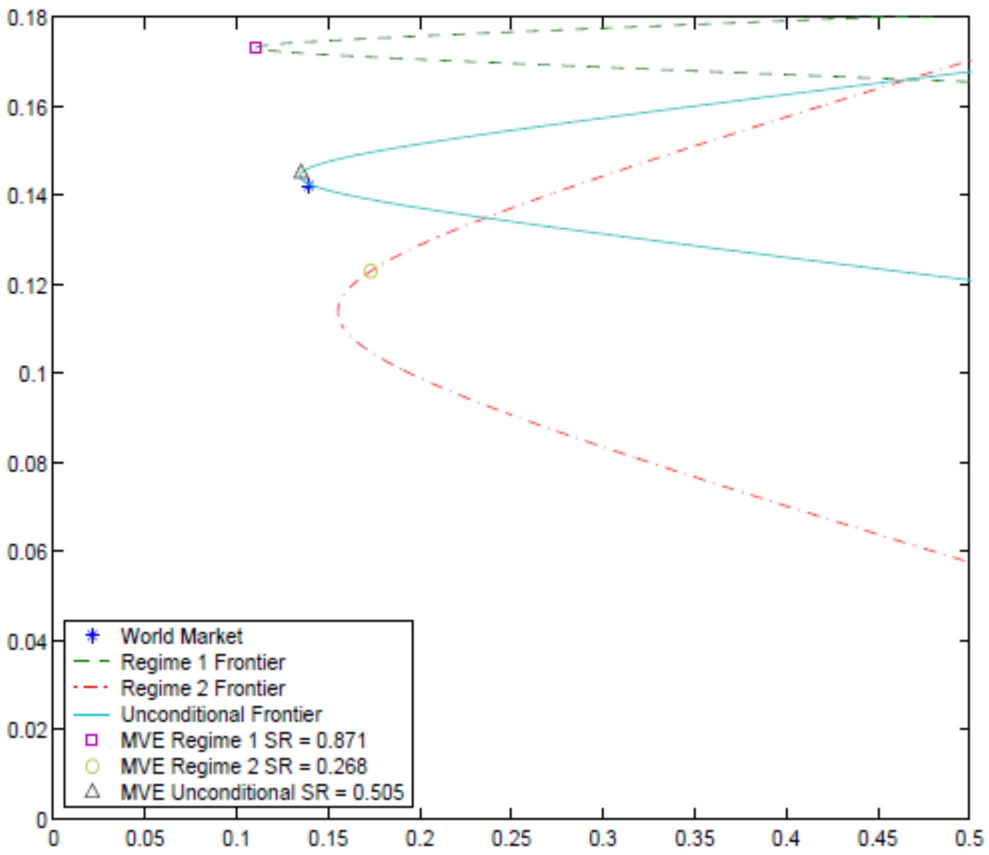
$$\Omega_i = (\beta\beta')(\sigma^w(s_{t+1}=i))^2 + V, \qquad i=1,2$$

- Then we can get different covariance matrices

$$\begin{aligned}\Sigma_1 &= P\Omega_1 + (1-P)\Omega_2 + P(1-P)(e_1-e_2)(e_1-e_2)' \\ \Sigma_2 &= (1-Q)\Omega_1 + Q\Omega_2 + Q(1-Q)(e_1-e_2)(e_1-e_2)'\end{aligned}$$

RS Portfolio Strategy Results

Mean-Standard Deviation Frontiers of the Regime-Switching Beta Model

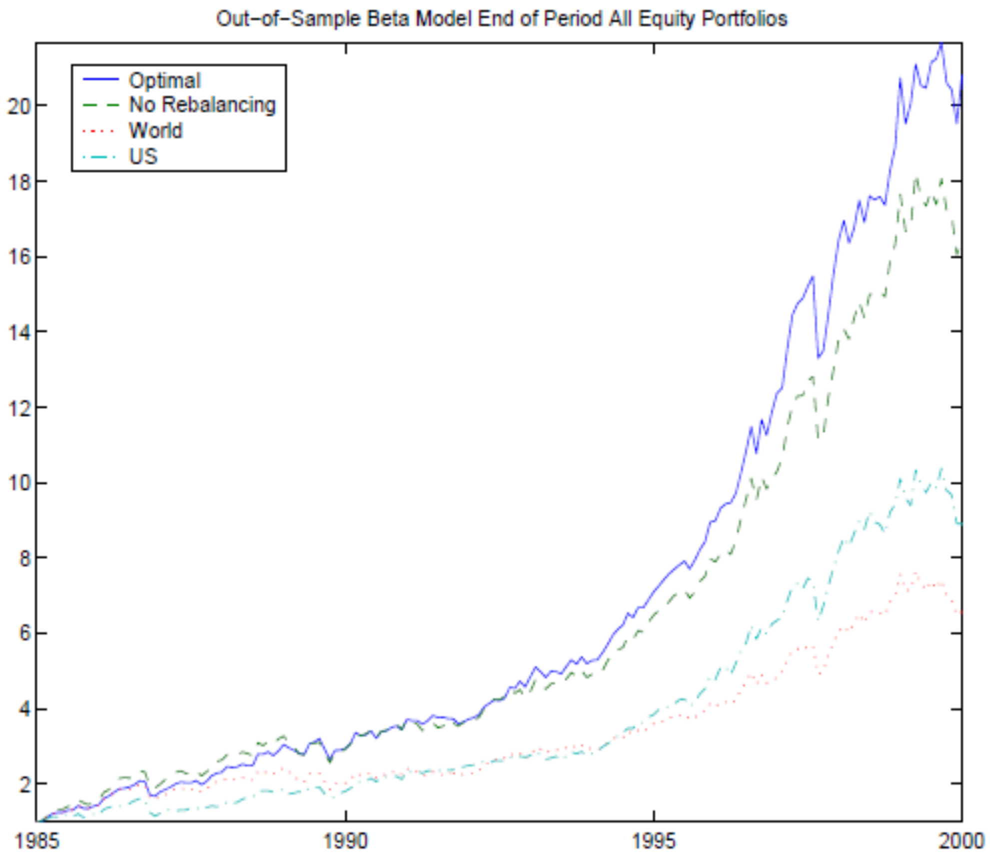


- The standard optimal mean-variance portfolio vector is given by:

$$\omega_i = \frac{1}{\gamma} \sum\nolimits_i^{-1} e_i$$

- In regime 1, the unconditional (conditional) tangency portfolio yields a sharp ratio of 0.62(0.87).
- In regime 2, , the unconditional (conditional) tangency portfolio yields a sharp ratio of 0.13(0.27).

RS Portfolio Strategy Results



- The model was rebalanced every month.
- The outperformance is particularly striking for the last five years. It is also over the last five years that the RS strategy outperformed the non-regime-dependent strategy particularly successfully.

2. Regime Switching Market Timing Model

When a volatile bear market is expected, the optimal asset allocation may be to switch to a safe asset or a bond.

The market-timing model is

$$
\begin{aligned}
r_t &= \mu^r(i) + \rho(i) r_{t-1} + \varepsilon_t^1; \\
r_t^b &= \mu^b + \varepsilon_t^2; \\
r_t^e &= \mu^e + \varepsilon_t^3.
\end{aligned}
$$

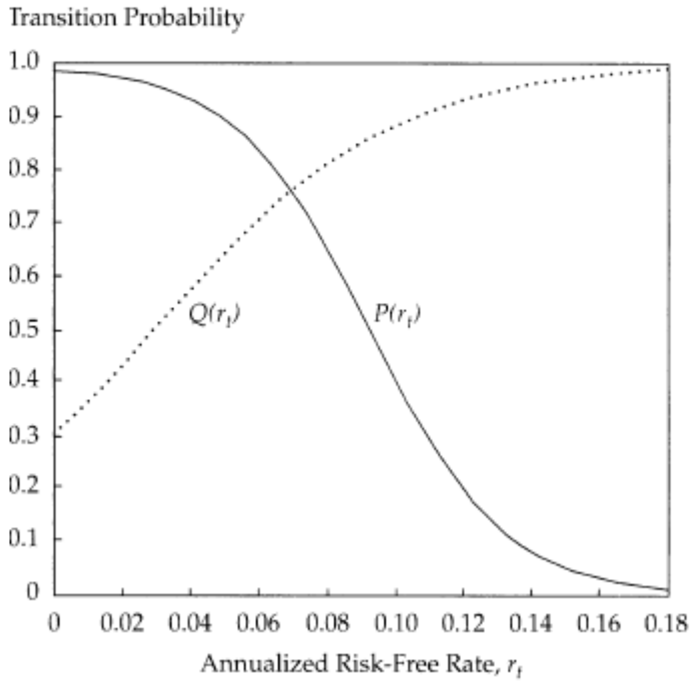
r_t : the risk-free rate (the nominal T-bill rate)
r_t^b: the excess bond return
r_t^e : the excess return on U.S. equity

Transition probabilities of the Market Timing Model

Specify the transition probabilities to be a function of the short rate:

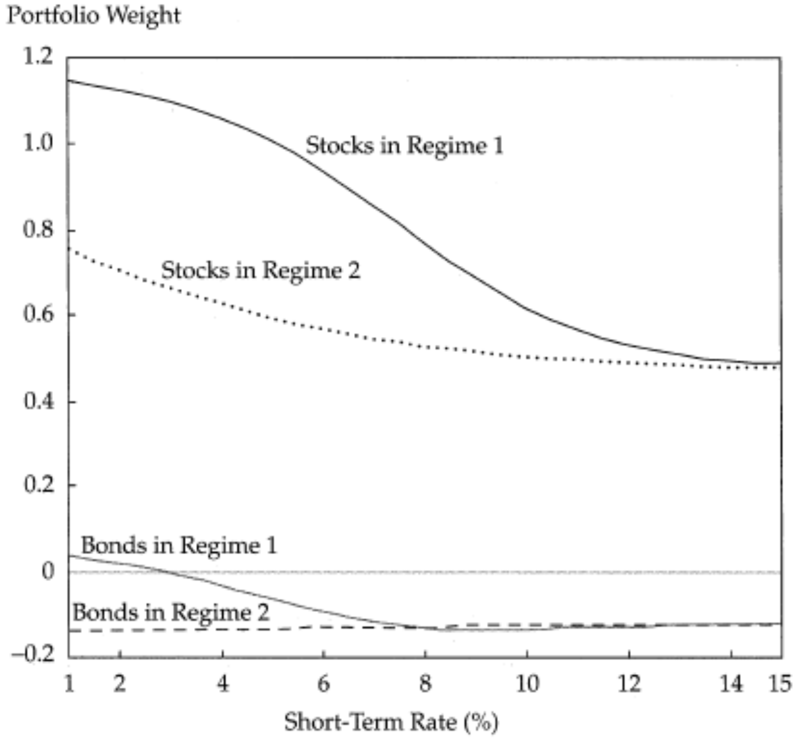
$$P_{t-1} \equiv p(s_t = 1 | s_{t-1} = 1, I_{t-1}) = \frac{\exp(a_1 + b_1 r_{t-1})}{1 + \exp(a_1 + b_1 r_{t-1})}$$

$$Q_{t-1} \equiv p(s_t = 2 | s_{t-1} = 2, I_{t-1}) = \frac{\exp(a_2 + b_2 r_{t-1})}{1 + \exp(a_2 + b_2 r_{t-1})}$$



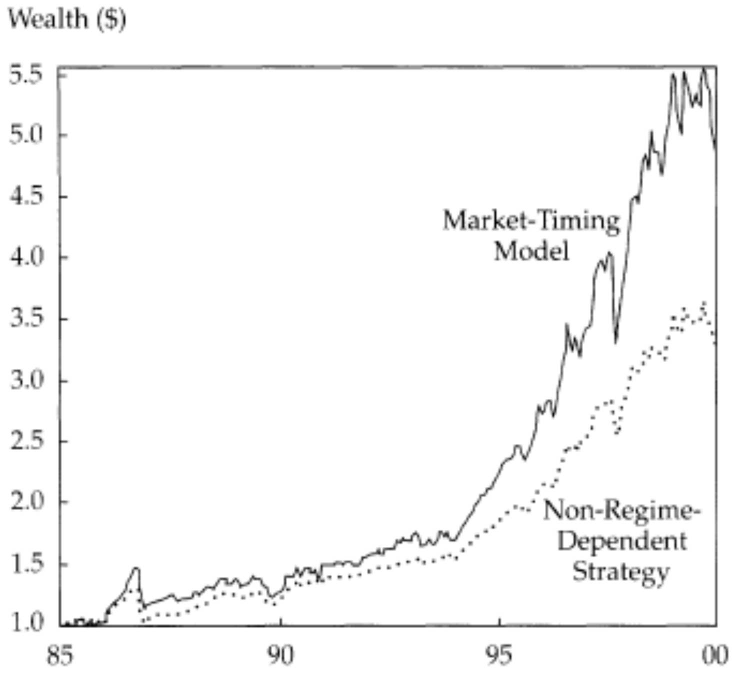
Mean-Variance Market Timing

Asset Allocation of the Market-Timing Model as a Function of the Short Rate



Note: Risk aversion = 5.

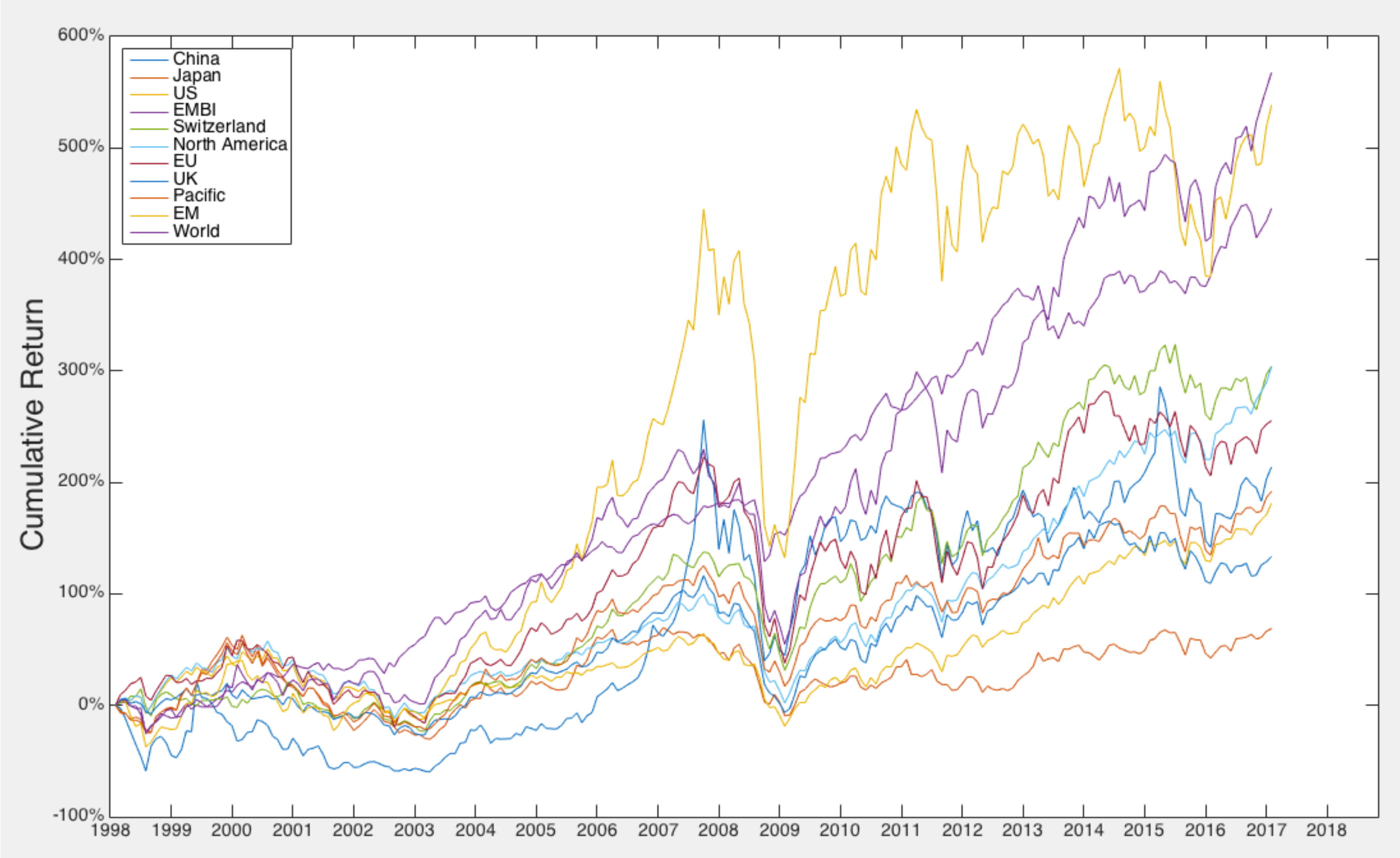
Out-of-Sample Wealth for the Market-Timing Model, 1985-2000



Notes: Accumulation of $1.00 invested on 1 January 1985. Risk aversion = 5.

3. Out-of-Sample Test of the Model

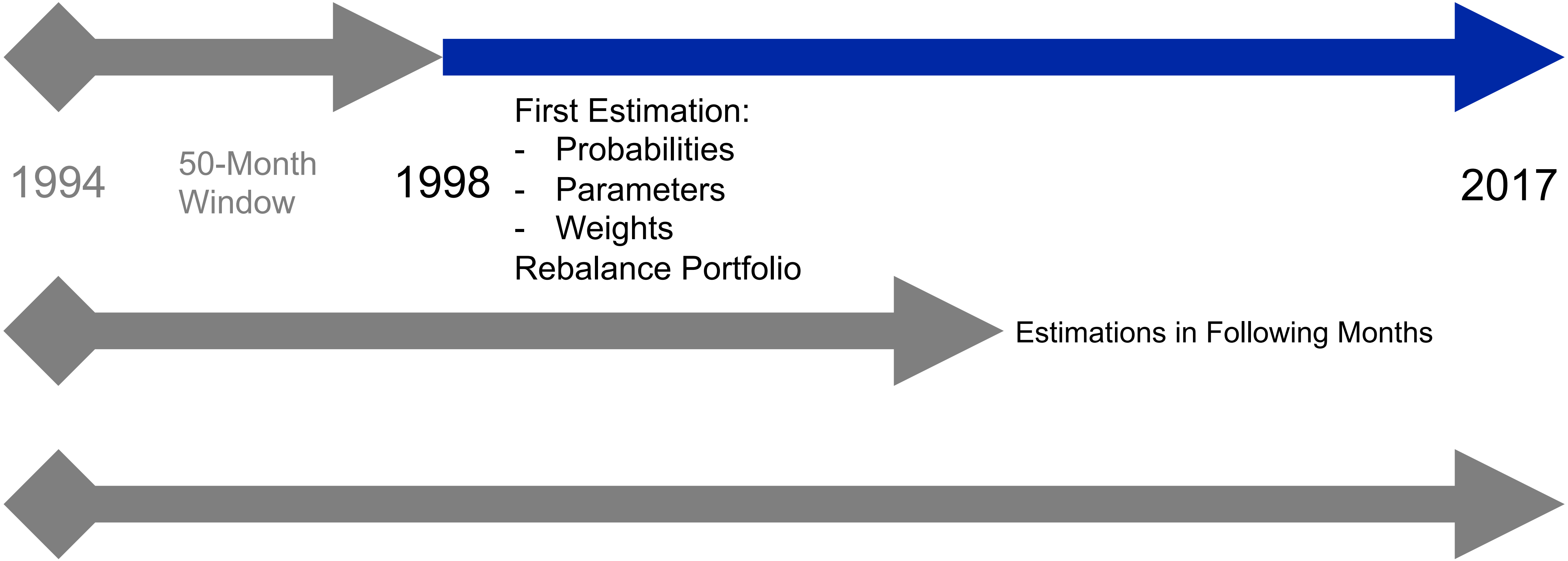
Historical Data of 12 Indices Were Used



- JPM US CASH 3MO TOT RET IDX
- MSCI China Index
- MSCI Japan Index
- S&P500 Index
- JPM EMBI GLOBAL COMPOSITE
- Swiss Performance Index
- MSCI North America Index
- MSCI EMU Index
- MSCI UK Index
- MSCI Paficic Index
- MSCI EM Index
- MSCI World Index

Source: BenchmarkReturns_v2.xlsx, Bloomberg

Out-of-Sample Test - The Process



Estimation Results: 1994 - 2017

A. Transition probabilities and world market (monthly, in percentages)

	P	Q	μ_1^ω	μ_2^ω	σ_1^ω	σ_2^ω
Est.	0.98	0.98	1.11	0.07	3.51	6.80
Std. err.	0.06	0.17	0.31	0.66	0.02	0.07

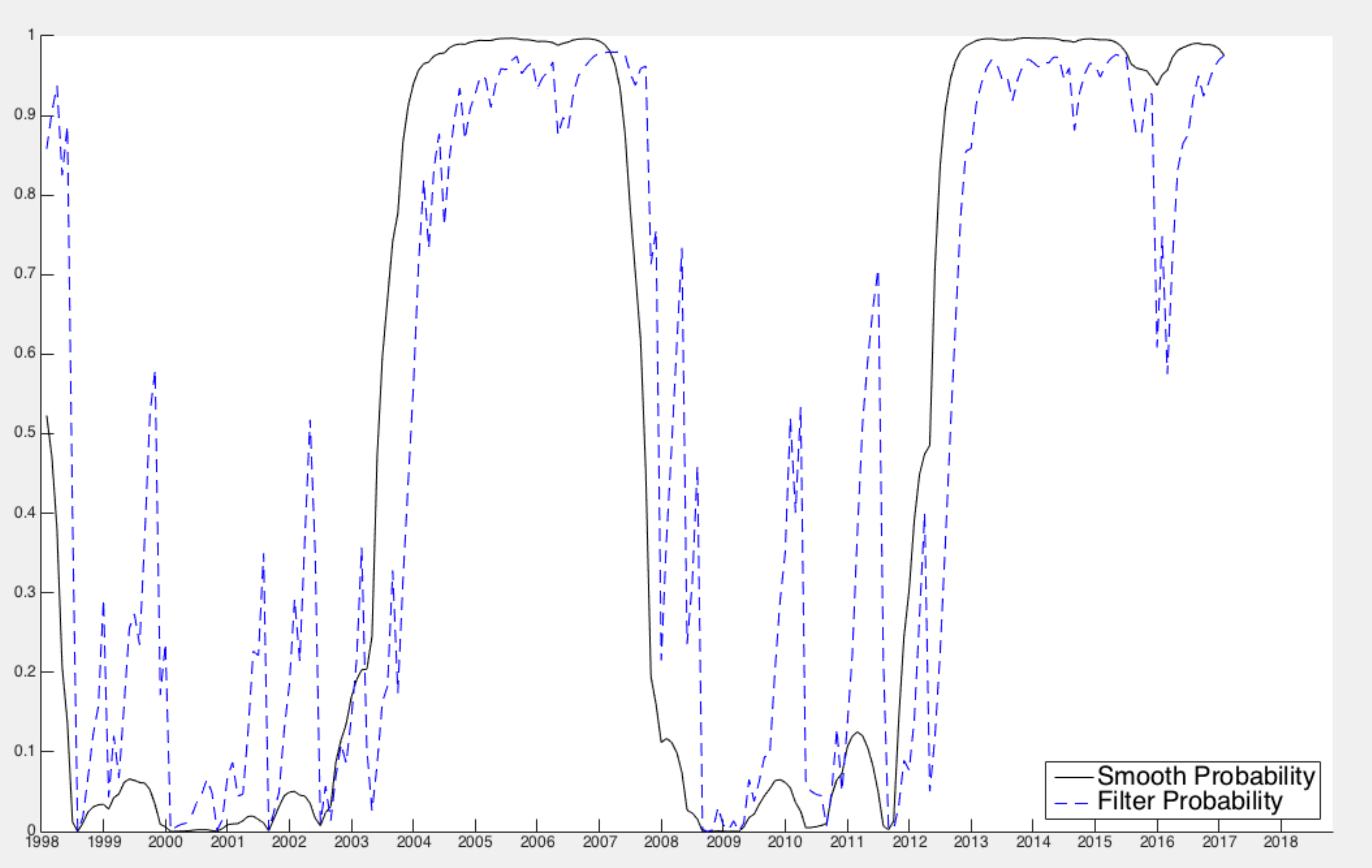
B. Country Betas

	China	Japan	US	EMBI	Swiss	North America	EMU	UK	Pacific	Emerging Market
Est.	0.95	0.56	0.66	0.37	0.58	0.68	0.89	0.65	0.66	0.99
Std. err.	0.11	0.06	0.05	0.04	0.05	0.05	0.07	0.05	0.06	0.08

C. Idiosyncratic Volatilities (monthly, in percentages)

	China	Japan	US	EMBI	Swiss	North America	EMU	UK	Pacific	Emerging Market
Est.	0.08	0.04	0.02	0.03	0.04	0.02	0.04	0.03	0.08	0.04

Ex-Ante and Smoothed Probabilities: Being in Regime 1



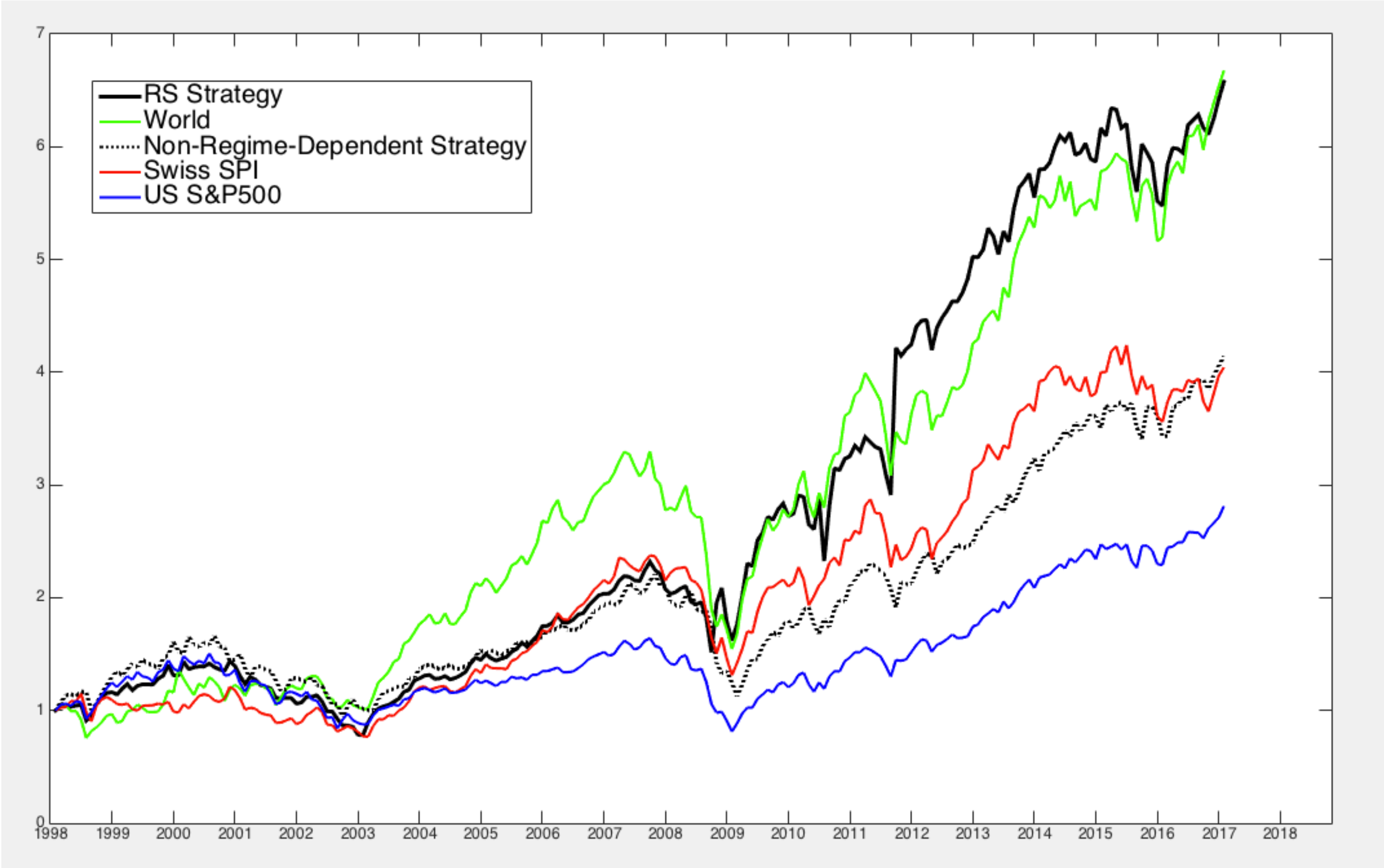
$p >= 0.5 \rightarrow \mu_1^{\omega} = 1.17,\ \sigma_1^{\omega} = 0.31$
$p < 0.5 \rightarrow \mu_2^{\omega} = 0.07,\ \sigma_2^{\omega} = 0.66$

We calculate the $E(R)$ and σ of each indices using the estimated μ_i^{ω} and μ_i^{ω} of regime i, provided that the current state is more likely to be in regime i (i=1 if p>=0.5, else i=0)

Covariance and Correlation Matrices & Portfolio Weights

MSCI Indices	China	Japan	US (SPX)	EMB JPM	CH (SPI)	N. Am	EU	UK	Pacific	EM
Regime-dependent Covariance and Correlation Matrices Under Different Regimes										
Regime 1	1.05	0.12	0.16	0.13	0.13	0.16	0.16	0.15	0.14	0.16
	0.07	0.31	0.19	0.14	0.14	0.18	0.16	0.18	0.16	0.17
avg. corr. =	0.08	0.05	0.23	0.16	0.20	0.26	0.25	0.25	0.22	0.22
0.2660	0.05	0.03	0.03	0.15	0.15	0.16	0.16	0.15	0.14	0.17
	0.07	0.04	0.05	0.03	0.26	0.20	0.21	0.19	0.18	0.18
	0.08	0.05	0.06	0.03	0.05	0.24	0.25	0.24	0.22	0.25
	0.11	0.06	0.08	0.04	0.07	0.08	0.44	0.21	0.22	0.22
	0.08	0.05	0.06	0.03	0.05	0.06	0.07	0.26	0.21	0.21
	0.08	0.05	0.06	0.03	0.05	0.06	0.08	0.06	0.31	0.19
	0.12	0.07	0.08	0.05	0.07	0.09	0.11	0.08	0.08	0.56
Regime 2	1.34	0.42	0.57	0.40	0.48	0.58	0.56	0.54	0.51	0.56
	0.24	0.41	0.64	0.42	0.53	0.62	0.62	0.60	0.55	0.60
avg. corr. =	0.28	0.17	0.38	0.59	0.70	0.85	0.82	0.78	0.75	0.84
0.6772	0.16	0.09	0.11	0.19	0.51	0.58	0.58	0.56	0.51	0.59
	0.25	0.15	0.17	0.10	0.37	0.72	0.68	0.65	0.60	0.68
	0.29	0.17	0.20	0.11	0.18	0.39	0.83	0.80	0.73	0.82
	0.38	0.23	0.26	0.15	0.23	0.27	0.70	0.77	0.73	0.81
	0.28	0.17	0.19	0.11	0.17	0.20	0.26	0.40	0.70	0.76
	0.29	0.17	0.20	0.11	0.17	0.20	0.27	0.20	0.45	0.72
	0.43	0.25	0.30	0.17	0.26	0.30	0.40	0.29	0.30	0.88
Tangency Portfolio Weights										
Regime 1	0.0315	0.0837	0.1356	0.1448	0.1045	0.133	0.0835	0.1165	0.0978	0.069
Regime 2	-0.0437	0.1385	0.1022	0.5899	0.1595	0.0825	-0.0831	0.0984	0.0681	-0.1123
Unconditional	0	0	0.0004	0.5718	0.1958	0.2313	0.0002	0.0002	0.0002	0

Out-of-Sample Wealth for Various Markets or Strategies, 1998-2017



	World	RS Strategy	Non-RS Strategy*	Swiss SPI	US SP500
Return	10.34%	9.91%	7.48%	7.80%	5.79%
Volatility	18.98%	19.22%	15.31%	16.61%	15.19%
Max Draw-down	53.12%	45.61%	48.85%	44.70%	50.48%
Sharpe Ratio	0.51	0.44	0.40	0.38	0.29

**Non-Regime-Dependent Strategy: Out-of-sample, monthly rebalancing, with short-sell constraint*

Conclusion

- Equity returns perform badly and were highly correlated during high-volatility period.
- Changes between low- and high-volatility regimes can help to add value to portfolios.
- RS models are very valuable in tactical asset allocation programs that allow switching to a risk-free asset.
- The results in RS strategy are highly linked to historical period
- Further studies
  - build a Black-Litterman (1992) type equilibrium into the regime-switching model and into the estimation of the parameters
  - test whether there are country specific regimes

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