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# Regime Switching

## How Regimes Affect Asset Allocation

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# Introduction



## Table of Contents

- Regime-Switching Beta Model
- Regime-Switching Market Timing model
- Out-of-Sample Test of the Model
- Conclusion



## 1. Regime-Switching Beta Model

International CAPM model with world market return:

$$y_t^w = \mu^w(s_t) + \sigma^w(s_t)\epsilon_t^w,$$

Individual excess return for security  $j$ :

$$y_{t+1}^j = (1 - \beta^j)\mu^z + \beta^j\mu^w(s_{t+1}) + \beta^j\sigma^w(s_{t+1})\epsilon_{t+1}^w + \bar{\sigma}^j\epsilon_{t+1}^j.$$

Where:

$\mu^w(s_t)$  denotes the world market expected excess return

$\sigma^w(s_t)$  denotes the conditional volatility

$\epsilon_t^w$  is the unexpected shock or return drawn from a standard normal distribution

$\mu^z$  denotes zero beta excess return

$\bar{\sigma}^j$  denotes idiosyncratic term, volatility



## Regime Dynamics

- Assume that the portfolio manager knows the realized regime but does not know which regime will be realized next time period.
- The regime variable follows a Markov process with constant transition probabilities  $P$  and  $Q$ :

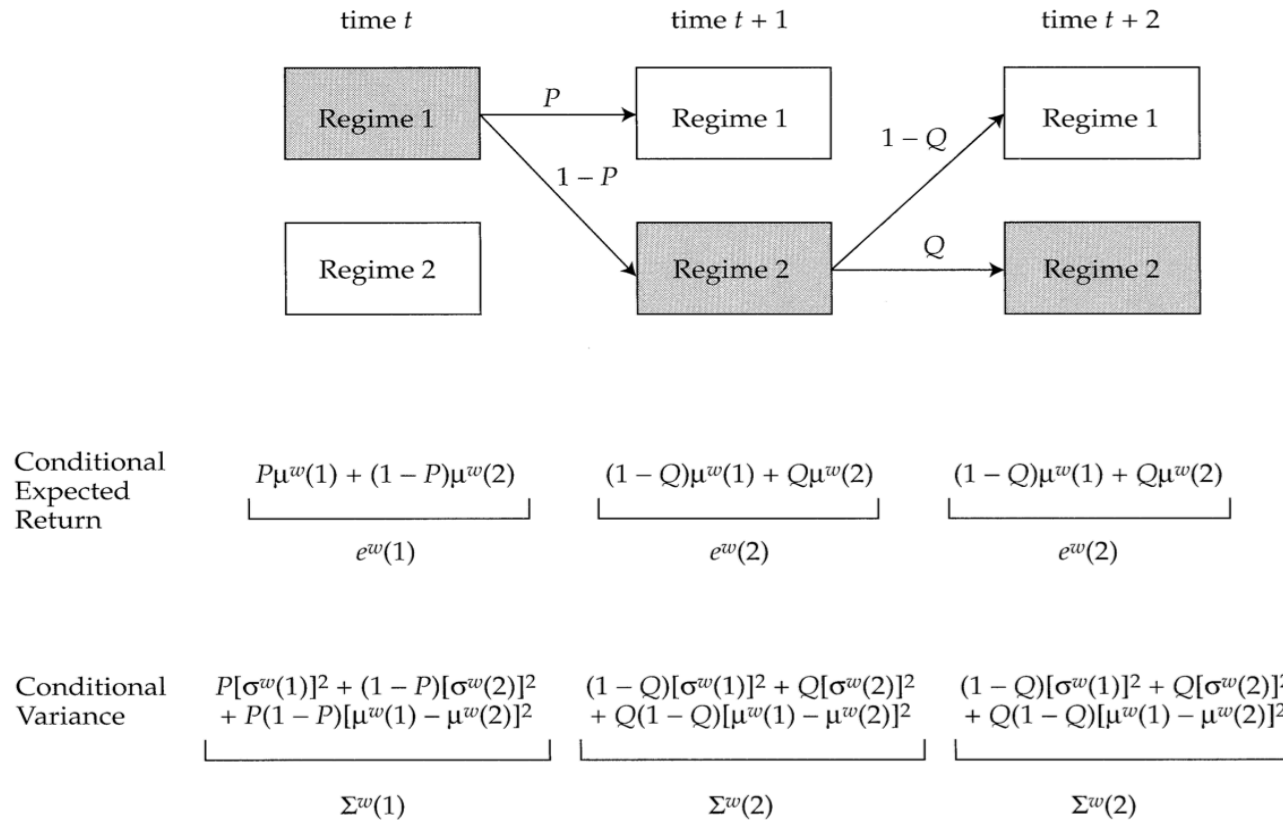
$$P = p(s_t = 1 | s_{t-1} = 1)$$

$$Q = p(s_t = 2 | s_{t-1} = 2).$$

- The situation  $P = 1 - Q$  usually does not happen. Empirical studies (e.g., Gray(1996)) find  $P$  and  $Q$  are well over 50%, indicating persistent states.



## A Regime-Switching Model for the world





## Data

The table lists the country composition of the geographic returns. Within each geographic region, they construct monthly returns, value-weighted in US dollars. The sample period is February 1975 through the end of 2000.

North America	UK	Japan	Europe large	Europe small	Pacific ex-Japan
Canada			France	Austria	Australia
US			Germany	Belgium	New Zealand
			Italy	Denmark	Singapore
				Finland	
				Ireland	
				Netherlands	
				Norway	
				Spain	
				Sweden	
				Switzerland	



## Regime-Switching Beta Model Parameter Estimates

### A. Transition probabilities and $\mu_z$

Measure	P	Q	$\mu^z$
Estimate	0.8917	0.8692	0.74
Standard error	0.0741	0.1330	0.68

### B. World market

Measure	$\mu(1)$	$\mu(2)$	$\sigma(1)$	$\sigma(2)$
Estimate	0.90	0.13	2.81	5.04
Standard error	0.32	0.62	0.44	0.55

### C. Country betas, $\beta$

Measure	North America	United Kingdom	Japan	Europe: Large	Europe: Small	Pacific ex Japan
Estimate	0.88	1.03	1.21	0.90	0.89	0.92
Standard error	0.03	0.06	0.07	0.05	0.04	0.07

### D. Idiosyncratic volatilities, $\bar{\sigma}$

Measure	North America	United Kingdom	Japan	Europe: Large	Europe: Small	Pacific ex Japan
Estimate	2.40	4.50	4.62	3.87	2.72	4.99
Standard error	0.09	0.18	0.19	0.16	0.11	0.20

Notes: All parameters are monthly; the mean,  $\mu$ , and standard deviation,  $\sigma$ , parameters are expressed in percentages.





## RS Equity Model Estimation Results, 1975-2000

Regime	North America	United Kingdom	Japan	Europe: Large	Europe: Small	Pacific ex Japan
<i>A. Regime-dependent excess returns</i>						
Regime 1	9.64	9.76	9.90	9.65	9.65	9.67
Regime 2	3.47	2.54	1.42	3.36	3.39	3.22
<i>B. Regime-dependent covariances and correlations</i>						
Regime 1						
North America	1.35	0.44	0.48	0.45	0.54	0.38
United Kingdom	0.90	3.08	0.37	0.35	0.42	0.29
Japan	1.06	1.25	3.60	0.38	0.46	0.32
Europe: Large	0.79	0.92	1.08	2.30	0.43	0.30
Europe: Small	0.78	0.91	1.07	0.80	1.53	0.36
Pacific ex Japan	0.81	0.94	1.11	0.82	0.82	3.33
Regime 2						
North America	2.37	0.64	0.68	0.65	0.73	0.58
United Kingdom	2.10	4.49	0.58	0.55	0.63	0.49
Japan	2.47	2.89	5.53	0.58	0.66	0.52
Europe: Large	1.83	2.14	2.52	3.36	0.63	0.49
Europe: Small	1.82	2.13	2.50	1.85	2.58	0.56
Pacific ex Japan	1.88	2.20	2.58	1.91	1.90	4.45
<i>C. Tangency portfolio weights</i>						
Regime 1	0.42	0.06	-0.01	0.15	0.31	0.08
Regime 2	0.79	-0.14	-0.55	0.25	0.54	0.10
Unconditional	0.52	0.04	-0.16	0.18	0.37	0.09
Average market cap	0.50	0.09	0.22	0.08	0.08	0.02



## Asset Allocation

- Expected excess return of country  $j$  is given by

$$e_{j,i} = (1-\beta^j) \mu^z + \beta^j e_i^\omega$$

Where  $i$  is the prevailing regime and  $e_i^\omega$  is the world's expected excess return,  $i = 1, 2$

- Expected returns differ across individual equity indices only through their different betas w.r.t the world market
- Let

$$\mathbf{e}_1 = (e_{j,1})^T \quad \text{and} \quad \mathbf{e}_2 = (e_{j,2})^T$$



## The Covariance Matrix

- Idiosyncratic part

$$V = \text{diag} [\bar{\sigma}_j^2]$$

- Regime- dependent systematic part

$$\Omega_i = (\beta\beta')(\sigma^w(s_{t+1} = i))^2 + V, \quad i = 1, 2$$

- Then we can get different covariance matrices

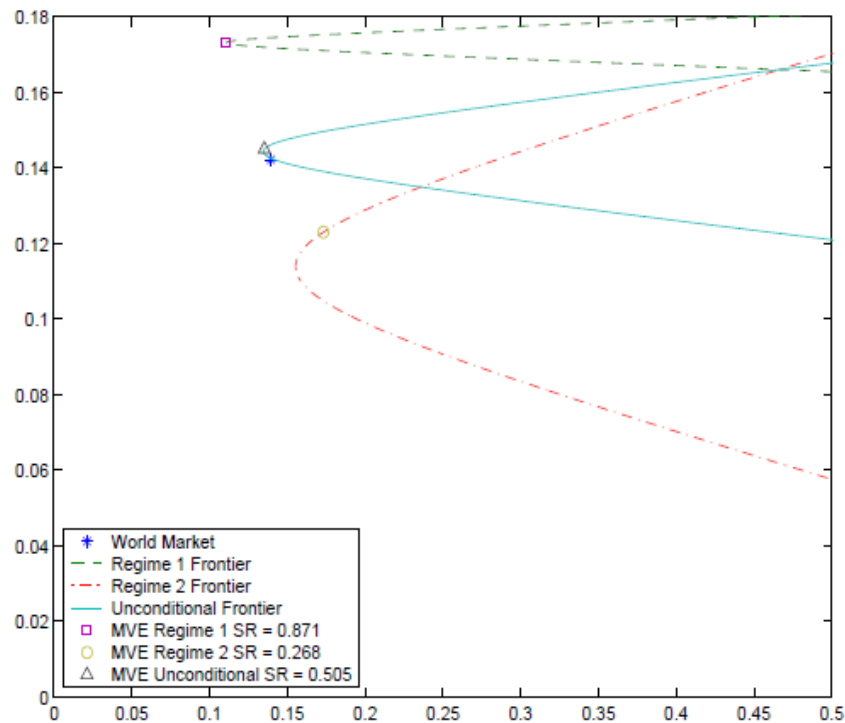
$$\Sigma_1 = P\Omega_1 + (1 - P)\Omega_2 + P(1 - P)(e_1 - e_2)(e_1 - e_2)'$$

$$\Sigma_2 = (1 - Q)\Omega_1 + Q\Omega_2 + Q(1 - Q)(e_1 - e_2)(e_1 - e_2)'$$



## RS Portfolio Strategy Results

Mean-Standard Deviation Frontiers of the Regime-Switching Beta Model



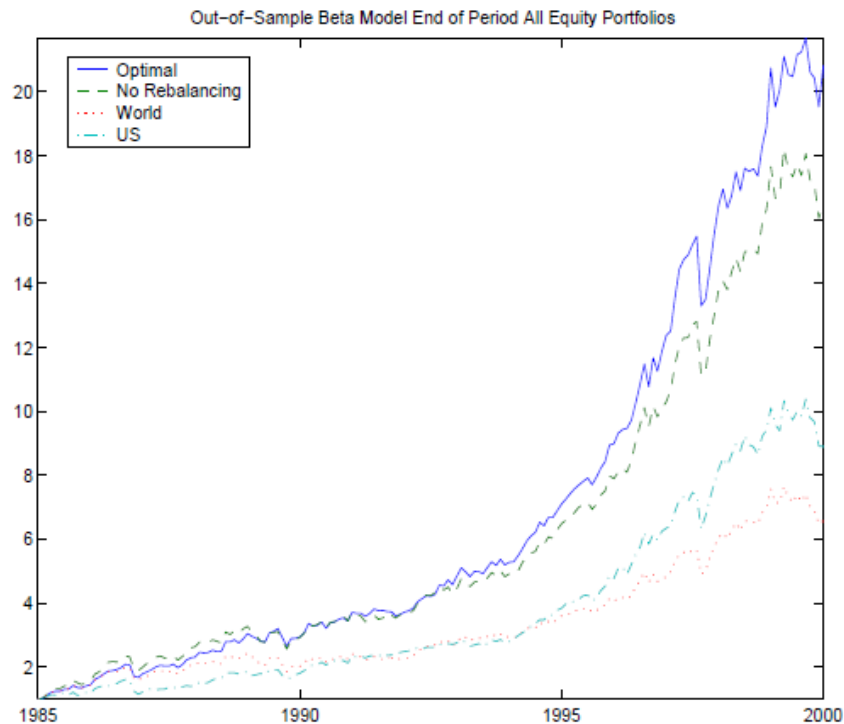
- The standard optimal mean-variance portfolio vector is given by:

$$\omega_i = \frac{1}{\gamma} \sum_i^{-1} e_i$$

- In regime 1, the unconditional (conditional) tangency portfolio yields a sharp ratio of 0.62(0.87).
- In regime 2, the unconditional (conditional) tangency portfolio yields a sharp ratio of 0.13(0.27).



## RS Portfolio Strategy Results



- The model was rebalanced every month.
- The outperformance is particularly striking for the last five years. It is also over the last five years that the RS strategy outperformed the non-regime-dependent strategy particularly successfully.



## 2. Regime Switching Market Timing Model

When a volatile bear market is expected, the optimal asset allocation may be to switch to a safe asset or a bond.

The market-timing model is

$$\begin{aligned}r_t &= \mu^r(i) + \rho(i)r_{t-1} + \varepsilon_t^1; \\r_t^b &= \mu^b + \varepsilon_t^2; \\r_t^e &= \mu^e + \varepsilon_t^3.\end{aligned}$$

$r_t$  : the risk-free rate (the nominal T-bill rate)

$r_t^b$  : the excess bond return

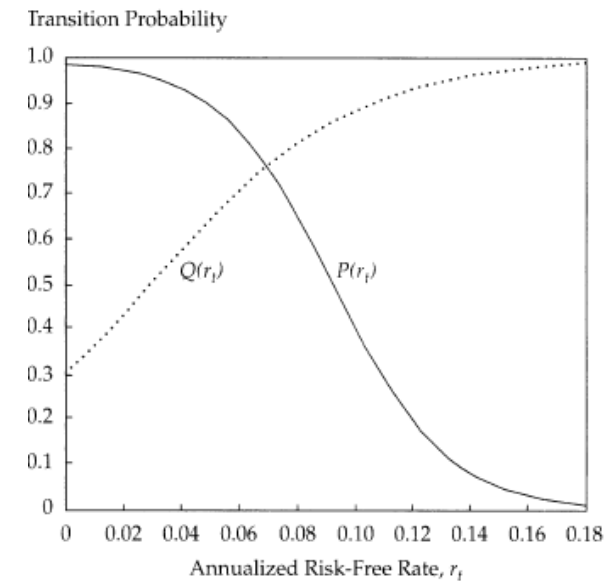
$r_t^e$  : the excess return on U.S. equity



## Transition probabilities of the Market Timing Model

Specify the transition probabilities to be a function of the short rate:

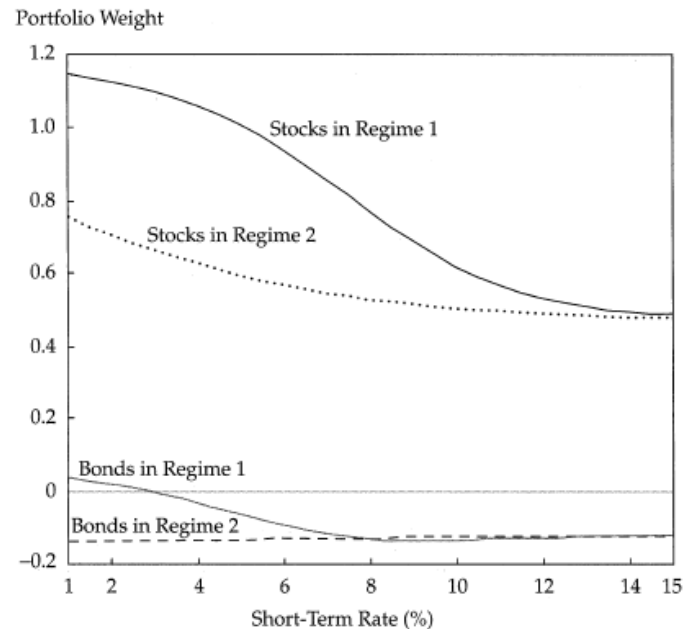
$$P_{t-1} \equiv p(s_t = 1 | s_{t-1} = 1, I_{t-1}) = \frac{\exp(a_1 + b_1 r_{t-1})}{1 + \exp(a_1 + b_1 r_{t-1})}$$
$$Q_{t-1} \equiv p(s_t = 2 | s_{t-1} = 2, I_{t-1}) = \frac{\exp(a_2 + b_2 r_{t-1})}{1 + \exp(a_2 + b_2 r_{t-1})}$$





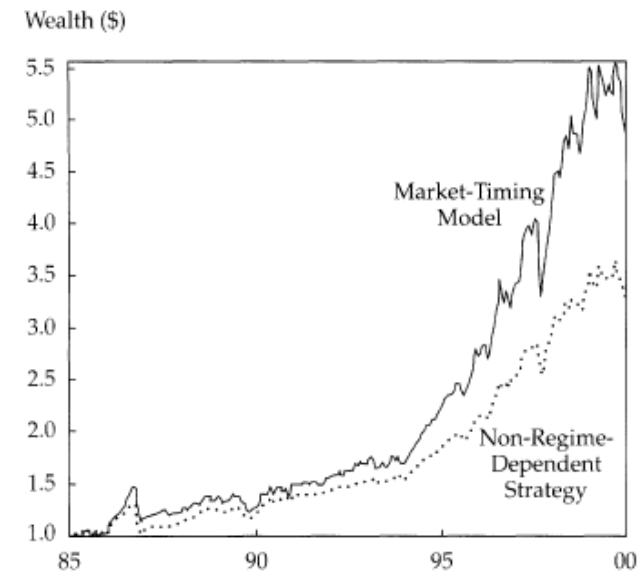
## Mean-Variance Market Timing

Asset Allocation of the Market-Timing Model  
as a Function of the Short Rate



Note: Risk aversion = 5.

Out-of-Sample Wealth for the Market-Timing Model, 1985-2000



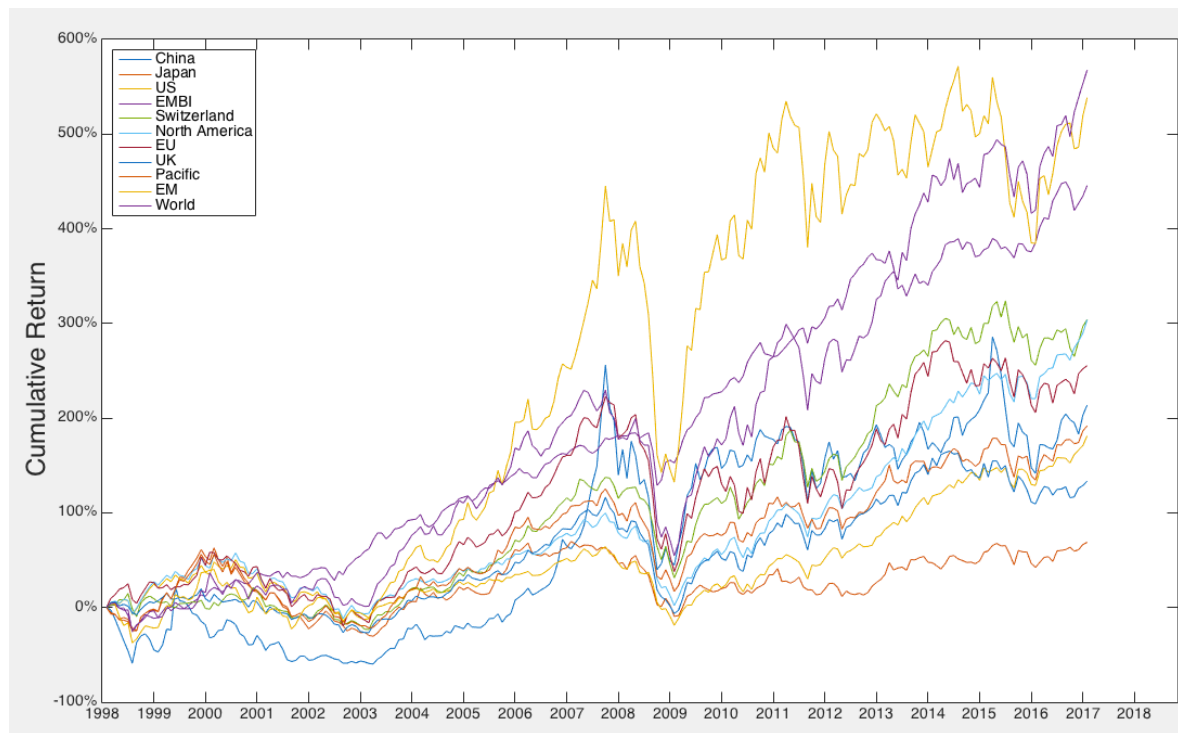
Notes: Accumulation of \$1.00 invested on 1 January 1985. Risk aversion = 5.





### 3. Out-of-Sample Test of the Model

Historical Data of 12 Indices Were Used

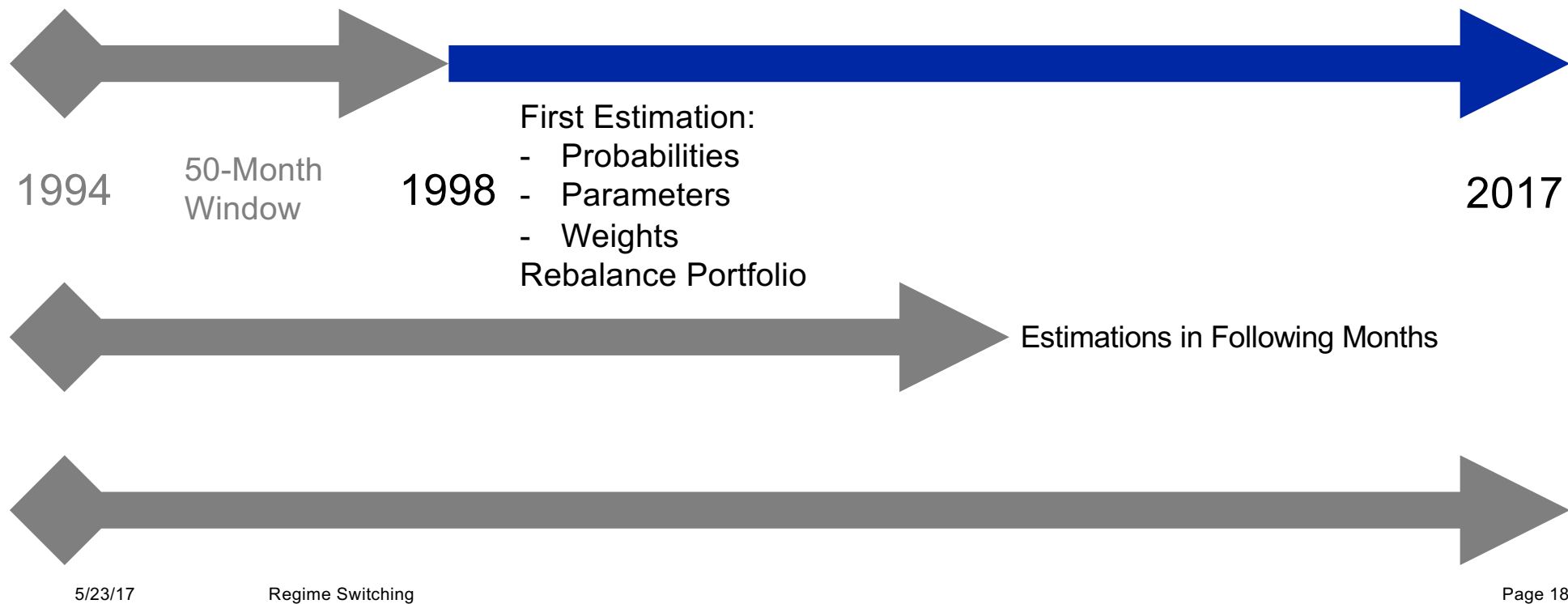


- JPM US CASH 3MO TOT RET IDX
- MSCI China Index
- MSCI Japan Index
- S&P500 Index
- JPM EMBI GLOBAL COMPOSITE
- Swiss Performance Index
- MSCI North America Index
- MSCI EMU Index
- MSCI UK Index
- MSCI Pacific Index
- MSCI EM Index
- MSCI World Index

Source: *BenchmarkReturns\_v2.xlsx*, Bloomberg



## Out-of-Sample Test - The Process





## Estimation Results: 1994 - 2017

### A. Transition probabilities and world market (monthly, in percentages)

	P	Q	$\mu_1^\omega$	$\mu_2^\omega$	$\sigma_1^\omega$	$\sigma_2^\omega$
Est.	0.98	0.98	1.11	0.07	3.51	6.80
Std. err.	0.06	0.17	0.31	0.66	0.02	0.07

### B. Country Betas

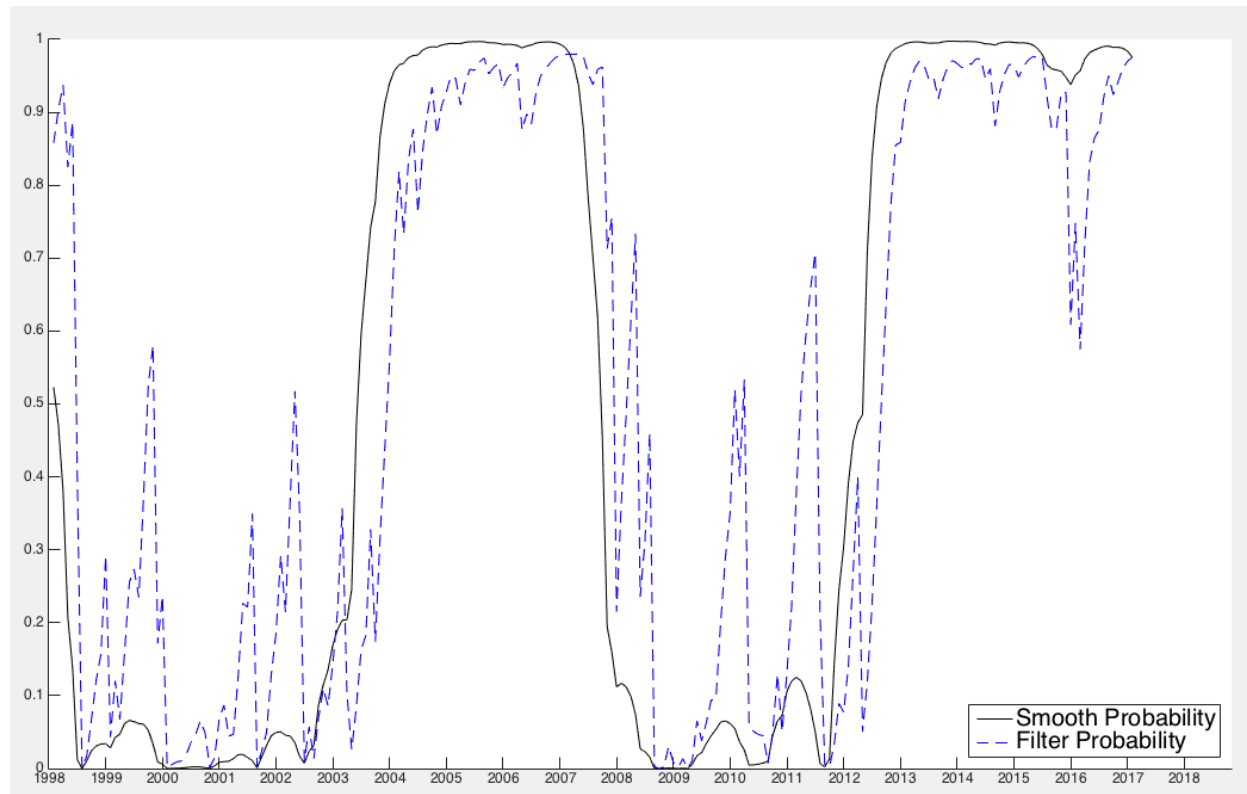
	China	Japan	US	EMBI	Swiss	North America	EMU	UK	Pacific	Emerging Market
Est.	0.95	0.56	0.66	0.37	0.58	0.68	0.89	0.65	0.66	0.99
Std. err.	0.11	0.06	0.05	0.04	0.05	0.05	0.07	0.05	0.06	0.08

### C. Idiosyncratic Volatilities (monthly, in percentages)

	China	Japan	US	EMBI	Swiss	North America	EMU	UK	Pacific	Emerging Market
Est.	0.08	0.04	0.02	0.03	0.04	0.02	0.04	0.03	0.08	0.04



## Ex-Ante and Smoothed Probabilities: Being in Regime 1



$$p \geq 0.5 \rightarrow \mu_1^\omega = 1.17, \sigma_1^\omega = 0.31$$
$$p < 0.5 \rightarrow \mu_2^\omega = 0.07, \sigma_2^\omega = 0.66$$

*We calculate the  $E(R)$  and  $\sigma$  of each indices using the estimated  $\mu_i^\omega$  and  $\mu_i^\omega$  of regime  $i$ , provided that the current state is more likely to be in regime  $i$  ( $i=1$  if  $p \geq 0.5$ , else  $i=0$ )*



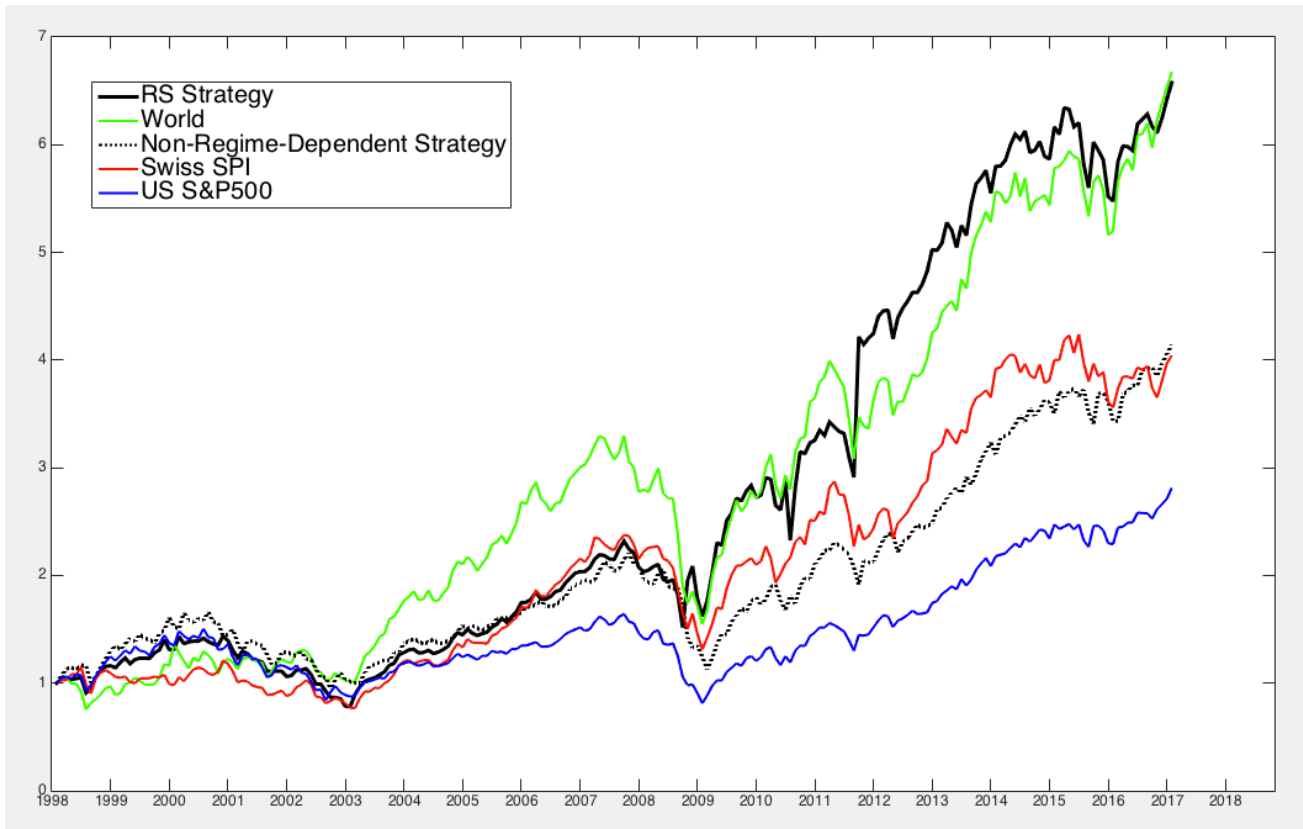
## Covariance and Correlation Matrices & Portfolio Weights

MSCI Indices	China	Japan	US (SPX)	EMB JPM	CH (SPI)	N. Am	EU	UK	Pacific	EM
<b>Regime-dependent Covariance and Correlation Matrices Under Different Regimes</b>										
Regime 1	1.05	0.12	0.16	0.13	0.13	0.16	0.16	0.15	0.14	0.16
	0.07	0.31	0.19	0.14	0.14	0.18	0.16	0.18	0.16	0.17
	0.08	0.05	0.23	0.16	0.20	0.26	0.25	0.25	0.22	0.22
	0.05	0.03	0.03	0.15	0.15	0.16	0.16	0.15	0.14	0.17
	0.07	0.04	0.05	0.03	0.26	0.20	0.21	0.19	0.18	0.18
	0.08	0.05	0.06	0.03	0.05	0.24	0.25	0.24	0.22	0.25
	0.11	0.06	0.08	0.04	0.07	0.08	0.44	0.21	0.22	0.22
	0.08	0.05	0.06	0.03	0.05	0.06	0.07	0.26	0.21	0.21
	0.08	0.05	0.06	0.03	0.05	0.06	0.08	0.06	0.31	0.19
	0.12	0.07	0.08	0.05	0.07	0.09	0.11	0.08	0.08	0.56
avg. corr. = 0.2660										
Regime 2	1.34	0.42	0.57	0.40	0.48	0.58	0.56	0.54	0.51	0.56
	0.24	0.41	0.64	0.42	0.53	0.62	0.62	0.60	0.55	0.60
	0.28	0.17	0.38	0.59	0.70	0.85	0.82	0.78	0.75	0.84
	0.16	0.09	0.11	0.19	0.51	0.58	0.58	0.56	0.51	0.59
	0.25	0.15	0.17	0.10	0.37	0.72	0.68	0.65	0.60	0.68
	0.29	0.17	0.20	0.11	0.18	0.39	0.83	0.80	0.73	0.82
	0.38	0.23	0.26	0.15	0.23	0.27	0.70	0.77	0.73	0.81
	0.28	0.17	0.19	0.11	0.17	0.20	0.26	0.40	0.70	0.76
	0.29	0.17	0.20	0.11	0.17	0.20	0.27	0.20	0.45	0.72
	0.43	0.25	0.30	0.17	0.26	0.30	0.40	0.29	0.30	0.88
avg. corr. = 0.6772										
<b>Tangency Portfolio Weights</b>										
Regime 1	0.0315	0.0837	0.1356	0.1448	0.1045	0.133	0.0835	0.1165	0.0978	0.069
Regime 2	-0.0437	0.1385	0.1022	0.5899	0.1595	0.0825	-0.0831	0.0984	0.0681	-0.1123
Unconditional	0	0	0.0004	0.5718	0.1958	0.2313	0.0002	0.0002	0.0002	0

Regime Switching



## Out-of-Sample Wealth for Various Markets or Strategies, 1998-2017



	World	RS Strategy	Non-RS Strategy*	Swiss SPI	US SP500
Return	10.34%	9.91%	7.48%	7.80%	5.79%
Volatility	18.98%	19.22%	15.31%	16.61%	15.19%
Max Draw-down	53.12%	45.61%	48.85%	44.70%	50.48%
Sharpe Ratio	0.51	0.44	0.40	0.38	0.29

Annual Turnover: 84.26%	Commission	Return
	1%	9.06%
	0.50%	9.48%
	0.10%	9.82%
	0.05%	9.86%
	0.01%	9.90%

\*Non-Regime-Dependent Strategy: Out-of-sample, monthly rebalancing, with short-sell constraint



## Conclusion

- Equity returns perform badly and were highly correlated during high-volatility period.
- Changes between low- and high-volatility regimes can help to add value to portfolios.
- RS models are very valuable in tactical asset allocation programs that allow switching to a risk-free asset.
- The results in RS strategy are highly linked to historical period
- Further studies
  - build a Black-Litterman (1992) type equilibrium into the regime-switching model and into the estimation of the parameters
  - test whether there are country specific regimes



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