# Iterative Channel Estimation for Block Transmission with Known Symbol Padding—A New Look at TDS-OFDM

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Abstract—This study proposes an iterative channel impulse response (CIR) estimation scheme for multicarrier block transmission with known symbol padding, in the context of the time-domain synchronous OFDM (TDS-OFDM) system. It is shown that the TDS-OFDM still fulfils the circular convolution relationship even if pseudo-noise (PN) sequence is used as guard interval. While conventional methods perform the channel estimation based on the cyclic prefix reconstrction or residual intersymbol interference cancellation in the time domain, by processing the data part and the known symbol padding in one TDS-OFDM block separately, the new proposal uses the onetap frequency-domain equalization to separate the data part and the known symbol padding and the CIR can be estimated in an iterative manner. In a typical example for 3780-point FFT based TDS-OFDM system with 16QAM constellations, the mean square error (MSE) of of the proposed channel estimator can be reduced to about  $10^{-5}/2$  after two iterations when the channel signal-to-noise ratio (SNR) is larger than 8dB, without the initial CIR estimation. The BER performance of a QPSK TDS-OFDM system shows that the signal-to-noise-ratio (SNR) degradation based on the proposed channel estimator is less than 0.3dB compared with the lower bound for both Brazil A and B channels.

### I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) finds a wide utilization in broadband wireless systems, such as the wireless local area network (WLAN) [1], [2], digital audio broadcasting (DAB) [3], digital video broadcasting (DVB) [4], and fixed wireless access [5], etc.

All these systems insert a cyclic-prefix (CP) between adjacent OFDM blocks, which converts the linear convolution of a transmitted signal with a channel impulse response (CIR) to circular convolution and the one-tap frequency-domain equalization can be adopted. An alternative technique called zero-padded OFDM (ZP-OFDM) has been proposed in which the CP is replaced by null symbols [6]. This solution has the capability to recover symbol in presence of channel nulls in absence of noise when the channel is known. In both the CP-OFDM and the ZP-OFDM systems with coherent demodulation, the channel is estimated by the aid of the pilots in the data part or the preambles at the head of the OFDM blocks.

In order to utilize the guard interval between OFDM blocks more efficiently, a technique called time-domain synchronous OFDM (TDS-OFDM) [7] has been proposed in which the CP is replaced by known pseudo-noise (PN) sequence. The PN sequence padding, which may be constant or change from block to block, can be used for synchronization and channel estimation purposes. However, with the known sequence padding, the cyclic property of the symbol block does not hold any more. As a result, neither the conventional channel estimation nor the simple one-tap frequency-domain equalization can be applied to TDS-OFDM directly. One way to solve this problem is to process the PN sequence padding and the data part separately: after substracting the PN sequence padding from the TDS-OFDM signal, the remaining signal can be taken as the ZP-OFDM and all the well-established techniques related to the ZP-OFDM can be applied. Utilizing the padded PN sequence, the channel estimation can be performed in time domain based on correlation, or in the frequency domain based on computation of linear convolution by FFT [8]. Later, constant PN sequence padding, also known as unique-word (UW), has been investigated in the single carrier [9] and multicarrier [10] context, but the exploitation of the known sequence for channel estimation is not detailed and no efficient equalization method is presented. Recently, a semi-blind channel estimation technique has been proposed for the so-called Psedo Random Postfix OFDM sytem [11]. This technique is possible to perform channel estimation for higher order constellations with an initial CIR estimation.

In this paper, we present a different look at TDS-OFDM system with identical PN sequence padding, which has been adopted as one of the optional modes in the recently announced Chinese digital terrestrial television broadcasting (DTTB) standard [7], [12]. We will show that TDS-OFDM still fulfils the circular convolution relationship even if PN sequence is used as guard interval. Based on this new look, an iterative channel estimation scheme is proposed by processing the data part *together* with the known padded PN sequence, instead of dealing with these two parts *separately* as in the conventional method [8]. The proposed channel estimation algorithm has comparable complexity as the conventional one, but does not need the initial CIR for channel estimation.

The remaining paper is organized as follows. The system model of the TDS-OFDM is described in Section II. The new look at TDS-OFDM and the proposed iterative channel estimation algorithm are analyzed in Section III. Simulation

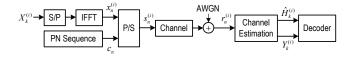


Fig. 1. Block diagram of a typical TDS-OFDM system.

results are presented in section IV to demonstrate the proposed technique, before giving the summary in Section V.

#### II. SIGNAL MODEL OF TDS-OFDM

A typical TDS-OFDM system is shown in Fig. 1. The data stream is divided into blocks of length N and modulated by using an N-point inverse fast Fourier transform (IFFT). For the ith transmitted symbol sequence  $\{X_k^{(i)}\}_{k=0}^{N-1}$ , the time-domain sequence as the output of the IFFT is

$$x_n^{(i)} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k^{(i)} \exp\left\{j\frac{2\pi kn}{N}\right\}, \ 0 \le n < N.$$
 (1)

Here, the elements of the input symbol sequence  $\{X_k^{(i)}\}_{k=0}^{N-1}$  are assumed to be independent identically distributed (i.i.d.) zero-mean random variables of variance  $\sigma_X^2=1$ . An identical PN sequence of length  $\nu$  is inserted between data symbols of length N to form TDS-OFDM signal. The ith transmitted frame,  $\{s_n^{(i)}\}_{n=0}^{N_2-1}$ , is defined as the ith data block with a length- $\nu$  PN sequence appended. Namely,

$$s_n^{(i)} = \begin{cases} x_n^{(i)}, \ 0 \le n < N \\ c_n, \ N \le n < N_2 \end{cases}$$
 (2)

where  $\{x_n^{(i)}\}_{n=0}^{N-1}$  is the ith transmitted data block,  $\{c_n\}_{n=0}^{\nu-1}$  is the PN sequence, and  $N_2=N+\nu$ .

## III. ITERATIVE CHANNEL ESTIMATION

#### A. A New Look at TDS-OFDM

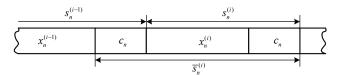
In this subsection, the transmission of TDS-OFDM signal over multipath channel will be investigated. The channel is modeled as an Lth order FIR filter with additive white Gaussian noise (AWGN). Enough length of guard interval is assumed ( $\nu \geq L-1$ ). Let us define an extended transmitted TDS-OFDM block of length  $N_3=N+2\nu$  as

$$\bar{s}_n^{(i)} = \begin{cases} c_n, & -\nu \le n < 0\\ s_n^{(i)}, & 0 \le n < N_2 \end{cases}$$
 (3)

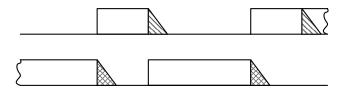
which includes not only the PN sequence appended in the present block  $\{s_n^{(i)}\}_{n=0}^{N_2-1}$  but also the PN sequence appended in the previous block  $\{s_n^{(i-1)}\}_{n=0}^{N_2-1}.$  It is obvious that the PN sequence at the head of the extended signal block  $\{\hat{s}_n^{(i)}\}_{n=0}^{N_3-1}$  serves as the cyclic prefix. As a result, for  $0 \leq n < N_2$ , the linear convolution of  $\{s_n^{(i)}\}_{n=0}^{N_2-1}$  with the CIR

$$\bar{r}_{n}^{(i)} = \bar{s}_{n}^{(i)} \star h_{n} + w_{n}$$
 (4)

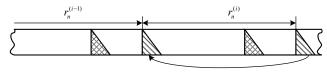
<sup>1</sup>The padded PN sequence may be identical for all data blocks, or different from block to block. We focus on the identical PN sequence padding, which has been adopted as one of the optional modes in the Chinese DTTB standard [13].



(a) Transmitted TDS-OFDM signal.



(b) Decomposition of the received signal.



(c) The received signal.

Fig. 2. Transmission of TDS-OFDM signal over multipath channel.

becomes a circular convolution. Here,  $\star$  denotes linear convolution. Then, for  $0 \le n < N_2$ , the received signal fulfills

$$r_n^{(i)} = \bar{r}_n^{(i)} = s_n^{(i)} \otimes h_n + w_n = \sum_{l=0}^{L-1} h_l s_{(n-l)_{N_2}}^{(i)} + w_n \quad (5)$$

where  $\otimes$  denotes the circular convolution,  $w_n$  represents the AWGN of variance  $\sigma_w^2$  and  $(l)_{N_2}$  is the residue of l modulo  $N_2$ . Accordingly, the frequency-domain representation becomes

$$R_k^{(i)} = S_k^{(i)} H_k + W_k , \quad 0 \le k < N_2$$
 (6)

where  $R_k^{(i)}$ ,  $S_k^{(i)}$ ,  $H_k$  and  $W_k$  denote the discrete Fourier transform of  $r_n^{(i)}$ ,  $s_n^{(i)}$ ,  $h_n$  and  $w_n$ , respectively.

The circular convolution phenomenon is illustrated in Fig. 2, where the shadow areas represent the tails of the PN sequence or data blocks due to the linear convolution of the transmitted signal with the CIR. Since the appended PN sequence in the ith frame is the same as that in the (i-1)th frame, the tail of the ith frame which corrupts the (i+1)th frame is the same as that of the (i-1)th frame. As a result, the linear convolution of the transmitted signal frame with the CIR becomes circular convolution. That is, in a received signal frame, the data block is corrupted by the tail of the PN sequence, and the PN sequence is corrupted by the tail of the data block.

As shown above, the TDS-OFDM signal still fulfils the circular convolution relationship even if the identical PN sequence is padded between OFDM symbols as guard interval. Therefore, one-tap frequency-domain equalization can also be applied in TDS-OFDM system with identical PN sequence padding. This new look at the TDS-OFDM is in accordance

with the different look at the unique-word based single-carrier system [14]. The difference is that the TDS-OFDM signal block is a time-frequency mixture, while the unique-word based single-carrier signal block is a pure time-domain signal.

# B. Channel Estimation

Besides for guard interval, the padded PN sequence can also be used for synchronization and channel estimation. In this subsection, the synchronization at receiver is assumed perfect. The new look at TDS-OFDM described above leads to an iterative channel estimation scheme. The details of the proposed channel estimation algorithm for the *i*th extended received signal block are summarized as follows.

1) Preparation. Define an extended PN sequence as

$$\bar{c}_n = \begin{cases} c_n , & 0 \le n < \nu \\ 0 , & \nu \le n < 2\nu \end{cases}$$
 (7)

and its frequency representation becomes

$$\bar{C}_k = \text{FFT}[\bar{c}_n] , \quad 0 \le k < 2\nu .$$
 (8)

Then, a buffer  $\{b_n\}_{n=0}^{N_3-1}$  is created and the extended received signal block  $\{\bar{r}_n^{(i)}\}_{n=-\nu}^{N_2}$  is copied to it:

$$b_n = \bar{r}_n^{(i)}, \quad -\nu \le n < N_2.$$
 (9)

Finally, set the iteration index I to zero.

2) CIR estimation. Suppose the channel length L is known at the receiver. Since the channel length is assumed smaller than the length of the guard interval  $\nu$ , a new signal  $\{z_n\}_{n=0}^{2\nu-1}$  of length  $2\nu$  is defined as

$$z_n = \begin{cases} b_n , & 0 \le n < \nu + L \\ 0 , & \nu + L \le n < 2\nu \end{cases}$$
 (10)

and the channel frequency response (CFR) is estimated as

$$\hat{H}'_{k} = \frac{\text{FFT}[z_n]}{\bar{C}_k}, \ 0 \le k < 2\nu \ .$$
 (11)

Then, the CIR is calculated as  $\hat{h}_{n}' = \text{IFFT}[\hat{H}_{k}].$ 

3) Filtering and smoothing. Since the length of the CIR is known to be smaller than  $\nu$ , all the CIR values  $\{\hat{h}_n'\}_{n=\nu}^{2\nu-1}$  are set to zeros. To reduce the effect of noise, a threshold  $\triangle$  is used to select the significant taps and the absolute values of  $\{\hat{h}_n'\}_{n=0}^{2\nu-1}$  below the threshold are set to zero. Finally, a new CIR is obtained by weighted averaging the CIR  $\{h_n'\}_{n=0}^{2\nu-1}$  and the CIR  $\{h_n^{I-1}\}_{n=0}^{2\nu-1}$  estimated in the previous iteration, which is expressed as

$$\hat{h}_{n}^{I} = \alpha \cdot h_{n}^{I-1} + (1 - \alpha) \cdot h_{n}^{'}, \ 0 \le n < 2\nu$$
 (12)

where  $\alpha$  is weighting coefficient. When the iteration index  $I=0, \ \alpha$  is set to zero. Otherwise,  $0<\alpha<1.$ 

4) Data part and PN sequence separation. CFR  $\{\hat{H}_k^I\}_{k=0}^{N_2-1}$  of length  $N_2$  is the FFT of the new CIR  $\{\hat{h}_n^I\}_{n=0}^{n_2-1}$  with  $N_2-2\nu$  zeros appended. Based on (6), the received frequency-domain signal  $R_k^{(i)}$  can be equalized by the one-tap frequency-domain equalizer. Two commonly used equalizer types, namely, the zero-forcing (ZF) and

linear minimum mean-square error (LMMSE), can be constructed as follows.

ZF equalizer:

$$\hat{s}_n^{(i)} = \text{IFFT}\left[\frac{R_k^{(i)}}{\hat{H}_k^I}\right],\tag{13}$$

LMMSE equalizer:

$$\hat{s}_{n}^{(i)} = \text{IFFT}\left[\frac{(\hat{H}_{k}^{I})^{*} \odot R_{k}^{(i)} / \hat{H}_{k}^{I}}{\hat{H}_{k}^{I} \odot (\hat{H}_{k}^{I})^{*} + \sigma_{w}^{2} / \sigma_{X}^{2}}\right]$$
(14)

where  $\odot$  and \* denote the elementwise multiplication operation and the conjugate operation, respectively.

5) Remove data part from signal block. In order to select the data part from the equalized signal  $\{\hat{s}_n^{(i)}\}_{n=0}^{N_2-1}$ , a length- $N_2$  sequence is defined as

$$x_n^{(i)} = \begin{cases} \hat{s}_n^{(i)}, & 0 \le n < N \\ 0, & N \le n < N_2. \end{cases}$$
 (15)

Then, the expected received signal can be calculated as

$$y_n^{(i)} = \text{IFFT}[\text{FFT}[x_n^{(i)}] \odot \hat{H}_k^I] \ 0 \le n < N_2.$$
 (16)

The extended signal block after removing the data part can be expressed as

$$b_n = \begin{cases} \bar{r}_n^{(i)}, & -\nu \le n < 0\\ \bar{r}_n^{(i)} - y_n^{(i)}, & 0 \le n < N_2 \end{cases}$$
 (17)

which is used to update the buffer  $\{b_n\}_{n=0}^{N_3-1}$ .

6) Set  $I \longleftarrow I + 1$  and jump to step 2).

After J iterations, the CFR,  $\{H_k^J\}_{k=0}^{N_2-1}$ , can be obtained. For un-coded TDS-OFDM system, the equalized frequency-domain data part can be obtained in the Step 4) described above, by applying FFT to the first N symbols of the equalized signal  $\hat{s}_n^{(i)}$ , i.e.

$$X_k^{eq} = \text{FFT}[\hat{s}_n^{(i)}], \ 0 \le n < N \ . \tag{18}$$

For coded TDS-OFDM system, the frequency-domain data part without equalization is obtained by applying FFT to the first N symbols of  $r_n^{(i)}$ , i.e.

$$Y_k^{(i)} = \text{FFT}[r_n^{(i)}], \ 0 \le n < N \ .$$
 (19)

The CFR  $\{\hat{H}_k^J\}_{k=0}^{N-1}$  of length N is the FFT of the CIR  $\{\hat{h}_n^I\}_{n=0}^{2\nu-1}$  with  $N-2\nu$  zeros appended. The frequency-domain data  $\{Y_k^{(i)}\}_{k=0}^{N-1}$  and the estimated CFR  $\{H_k^J\}_{k=0}^{N-1}$  are applied to the decoder for soft-decision.

Taking the Chinese DTTB system where N=3780 and  $\nu=420$  as an example, the computational complexity comparison in terms of I/FFT resources between the proposed scheme and the conventional method [8] is shown in Table I. As shown in this table, the proposed channel estimation scheme has comparable complexity as the conventional one.

TABLE I

COMPUTATION COMPLEXITY COMPARISON BETWEEN THE PROPOSED AND

CONVENTIONAL SCHEMES.

Complexity	Conventional method	Proposed method	
I/FFT840	0	2(J+1)	
I/FFT2048	4(J+1)	0	
I/FFT3780	2	1	
I/FFT4200	0	5(J+1)	
I/FFT8192	3(J+1)	0	

TABLE II POWER DELAY PROFILES OF THE BRAZIL CHANNELS

	Brazil A		Brazil B	
tap	delay (us)	power (dB)	delay (us)	power (dB)
1	0	0	0	0
2	0.15	-13.8	0.30	-12.0
3	2.22	-16.2	3.50	-4.0
4	3.05	-14.9	4.40	-7.0
5	5.86	-13.6	9.50	-15.0
6	5.93	-16.4	12.7	-22.0

#### IV. SIMULATION RESULTS

Simulations are performed to demonstrate the iterative channel estimation algorithm. The following parameters based on the Chinese DTTB standard [12] are assumed: 1) An N =3780 TDS-OFDM system with each subcarrier modulated in M-QAM constellation. 2) A PN sequence generated by maximal length linear feedback shift register (LFSR) of period 255, and this sequence is cyclic prefixed and BPSK modulated to form the constant PN sequence padding of length  $\nu = 420$ . 3) A baseband symbol rate of 7.56 MHz. 4) Two multipath channel models used in the Brazilian field tests on DTTB. The power delay profiles of these two channel models are shown in Table II. It is assumed that each path is subject to an independent Rayleigh fading, and the average path gains are normalized so that the signal power remains after passing through the multipath channel. The Doppler shift is a fraction of a hertz so that the complex channel gain remains almost constant over one extended signal block. The weighting coefficient  $\alpha$  in (12) is set to 0.125. The threshold  $\triangle$  for selecting the significant taps is 23 dB smaller than the maximum magnitude of the channel taps.

Fig. 3 and Fig. 4 show the mean square error (MSE) of the proposed channel estimator as a function of the signal-to-noise ratio (SNR) when 16-QAM is used, for different number of iteration. The MSE is defined as

$$MSE = \frac{1}{\nu} \sum_{n=0}^{\nu-1} |\hat{h}_n^I - h_n|^2$$
 (20)

where  $h_n$  is the perfect CIR and  $\hat{h}_n^I$  is the estimated CIR in the Ith iteration. The LMMSE equalizer is used in the process of iterative channel estimation. As shown in these figures, for a specific channel SNR, the MSE of the channel estimator

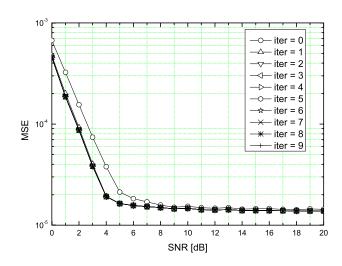


Fig. 3. MSE of channel estimation for TDS-OFDM system over Brazil A channel.

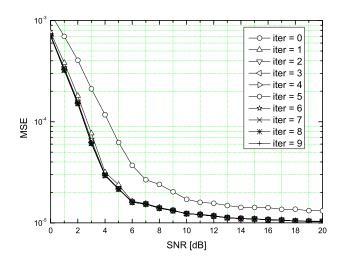


Fig. 4. MSE of channel estimation for TDS-OFDM system over Brazil B channel.

nearly reaches a constant after two or more iterations. While for a specific number of iteration, the MSE maintains when the SNR is larger than 8dB for Brazil A channel, and about 14dB for Brazil B channel. In both figures, the MSE maintains about  $10^{-5}/2$  after two iterations when channel SNR is larger than 8dB.

The bit error rate (BER) as a function of  $E_b/N_o$  defined as the ratio of the energy per bit  $(E_b)$  to the spectral noise density  $(N_o)$  of a QPSK TDS-OFDM system is presented in Fig. 5. The BER performance based on the estimated CIR is compared with the lower bound in which the BER is calculated by perfect CIR. The number of iterations is set to 2 and the LMMSE equalizer is used to do the channel estimation and equalization. For a BER of  $10^{-3}$ , the bit SNR degradataion of the performance based on estimated CIR is less than 3dB for both Brazil A and B channels.

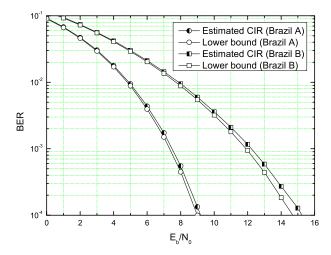


Fig. 5. BER performance of the QPSK TDS-OFDM system over multipath channels.

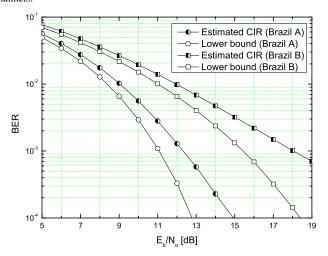


Fig. 6. BER performance of the 16-QAM TDS-OFDM system over multipath channels.

Fig. 6 shows the BER performance of a 16-QAM TDS-OFDM system. For a BER of  $10^{-3}$ , the bit SNR degradataion of the performance based on estimated CIR is about 1.2dB over Brazil A channel, while the degradation is about 2.5dB over Brazil B channel.

# V. CONCLUSION

This contribution presentes an iterative channel estimation algorithm based on the circular convolution theorem, for TDS-OFDM system with identical PN sequence padding. The conventional one-tap frequency-domain equalizer is applied to TDS-OFDM to separate the data part and the guard interval padded with PN sequence. Simulation results show that the MSE of the proposed channel estimator maintains at about  $10^{-5}/2$  after two iterations when channel SNR is larger than 8dB. The BER performance of a QPSK TDS-OFDM system shows that the SNR degradation based on the proposed channel estimator is less than 0.3dB compared with the lower bound

for both Brazil A and B channels. While the SNR degradations of the system based on the proposed estimator are 1.2dB and 2.5dB over Brazil A and B channels, respectively, compared with the lower bound.

The proposed algorithm is expected to give an alternative channel estimation scheme for TDS-OFDM receiver, and this method can also be applied to single-carrier system with known symbol padding such as unique word.

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