Compressive Sensing Based Channel Estimation for OFDM Systems Under Long Delay Channels

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***Abstract*—Time-domain synchronous orthogonal frequency division multiplexing (TDS-OFDM) has advantages in spectral efficiency and synchronization. However, its iterative interference cancellation algorithm will suffer from performance loss especially under severely fading channels with long delays and has difficulty supporting high-order modulations like 256 QAM, which may not accommodate the emerging ultra-high definition television service. To solve this problem, a channel estimation method for OFDM under the framework of compressive sensing (CS) is proposed in this paper. Firstly, by exploiting the signal structure of recently proposed time-frequency training OFDM scheme, the auxiliary channel information is obtained. Secondly, we propose the auxiliary information based subspace pursuit (A-SP) algorithm to utilize a very small amount of frequency-domain pilots embedded in the OFDM block for the exact channel estimation. Moreover, the obtained auxiliary channel information is adopted to reduce the complexity of the classical SP algorithm. Simulation results demonstrate that the CS-based OFDM outperforms the conventional dual pseudo noise padded OFDM and CS-based TDS-OFDM schemes in both static and mobile environments, especially when the channel length is close to or even larger than the guard interval length, where the conventional schemes fail to work completely.**

***Index Terms*—Channel estimation (CE), compressive sensing (CS), long delays, orthogonal frequency division multiplexing (OFDM), ultra-high definition television (UHDTV).**

I. Introduction

WITH the official approval of the digital television/terrestrial multimedia broadcasting (DTMB) as the fourth international digital television terrestrial broadcasting (DTTB) standard by International Telecommunications Union (ITU) in Dec. 2011 [1], DTMB has attracted lots of interests from both academia and industry. As the core technique of DTMB, time-domain synchronous orthogonal frequency division multiplexing (TDS-OFDM) distinguishes the standard cyclic prefix OFDM (CP-OFDM) by replacing CP with the prior known pseudo noise (PN) sequence as the guard interval (GI) [2]. The PN sequence can also work as the training sequence (TS) for both synchronization and channel estimation (CE) at the receiver side, which saves a large amount of frequency-domain pilots commonly used in CP-OFDM. Consequently, TDS-OFDM usually has higher spectral efficiency under the same condition [3]. Additionally, faster and reliable synchronization could be also achieved by TDS-OFDM [4], [5].

However, the main drawback of TDS-OFDM is that the TS and the OFDM block will cause mutual inter-block interferences (IBI) to each other. Thus, iterative interference cancellation algorithm with high complexity has to be adopted for CE and channel equalization in TDS-OFDM systems [6]. Unfortunately, this will result in an open problem of TDS-OFDM: Under the severely fading channels, it is difficult for the iterative algorithm to perfectly remove the IBI when the maximum channel delay spread is large, which is common in the single frequency network (SFN) environment [7]. This will cause the degradation of the whole system performance and the difficulty to support the high-order modulations like 256 QAM [2] to accommodate the emerging ultra-high definition television (UHDTV) service requirement (DVB-T2 has claimed to support the 256QAM for UHDTV services [8]).

Some alternative solutions have been proposed to solve this problem [9]–[12]. One exciting solution is the dual-PN padded OFDM (DPN-OFDM) scheme, whereby the PN sequence is duplicated twice to make the second PN sequence immune from the IBI caused by the preceding OFDM block [9]. Thus, the second received PN sequence can be directly used for CE, which avoids the iterative interference cancellation algorithm with high complexity, and improves the performance over severely fading channels as well. However, the spectral efficiency of the DPN-OFDM scheme is significantly decreased by the doubled length of the PN sequence. For example, when the length of the single PN sequence is $1/8$ that of the OFDM block, the spectral efficiency of TDS-OFDM is 88.89%, while it is reduced to 80% in DPN-OFDM. To prevent the spectral efficiency loss, the compressive sensing (CS) theory is exploited to solve the problem of TDS-OFDM, whereby the

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IBI-free region of small size within the received PN sequence is utilized to reconstruct the propagation channel with high dimension [10]. However, such CS-based TDS-OFDM scheme cannot work once the maximum channel delay spread is close to the GI length due to the reduced size of the required observations. Furthermore, to the best of the authors' knowledge, there is no effective solution to improve the CE accuracy for channels with long delays larger than the GI length in TDS-OFDM up to now [11], [12].

To enhance the CE performance under severely fading channels with long delays, we propose the CE method under the framework of CS. The work is inspired by the recently proposed time-frequency training OFDM (TFT-OFDM) scheme which is modified from TDS-OFDM [13], [14]. The specific contributions of this paper can be summarized as follows: 1) Compared to the conventional TDS-OFDM or CP-OFDM which merely relies on either the time-domain TS or frequency-domain pilots for channel recovery [5], the proposed CE method uses the PN sequence to acquire the coarse channel path delay estimation, while the exact channel impulse response (CIR) estimation depends on a very small amount of frequency-domain pilots embedded in the OFDM block based on CS. 2) Unlike the conventional scheme of TDS-OFDM where the IBI caused by the preceding OFDM block to the current TS has to be cancelled completely to achieve good performance, it is not necessary to remove such interference in the proposed scheme since the received PN sequence is only used for the coarse channel path delay estimation. The main CE task is transferred to the pilots in the OFDM block, which could combat the maximum channel delay spread close to or even larger than the GI length. 3) With the use of CS and sparse channel nature, the number of pilots embedded in the OFDM block could be significantly reduced (about 1% of the total sub-carrier number), and hence high spectral efficiency can still be maintained. 4) Moreover, the coarse channel path delay estimation is used as the auxiliary information to reduce the complexity of the classical CS algorithm, making it applicable for practical systems.

The remainder of this paper is organized as follows. The system model is introduced in Section II. The proposed CS-based CE method is addressed in Section III. Section IV presents the performance analysis of the proposed method. The simulation results are demonstrated in Section V. Finally, the conclusions are provided in Section VI.

Notation: We use boldface letters to denote matrices and column vectors; $\mathbf{F}_{N\times N}$ denotes the N-point fast Fourier transform (FFT) matrix with the $(n+1, k+1)$th entry being $\exp(-j2\pi nk/N)/\sqrt{N}$; $\mathbf{0}$ denotes the zero vector; $\otimes$ represents the circular correlation; $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$, $(\cdot)^\dagger$, $\mathrm{diag}(\cdot)$, and $\|\cdot\|_p$ denote the transpose, conjugate transpose, matrix inversion, Moore-Penrose matrix inversion, diagonal matrix, and l_p norm operations, respectively; $\mathbf{x}_S$ is generated by restricting the vector $\mathbf{x}$ to its S largest components, and $\sup(\mathbf{x}_S)$ denotes the location set of the S largest components of $\mathbf{x}$; $\mathbf{x}|_\Gamma$ denotes the entries of the vector $\mathbf{x}$ in the set of Γ; $\mathbf{\Phi}_\Gamma$ represents the sub-matrix comprising the Γ columns of $\mathbf{\Phi}$, and $\mathbf{\Phi}^\Omega$ represents the sub-matrix comprising the Ω rows of $\mathbf{\Phi}$; Finally, $\mathrm{E}(\cdot)$ stands for the expectation operator.

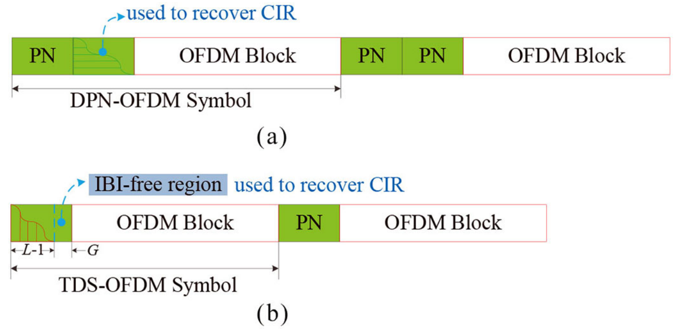

Fig. 1. System models of the two conventional schemes. (a) DPN-OFDM. (b) CS-based TDS-OFDM.

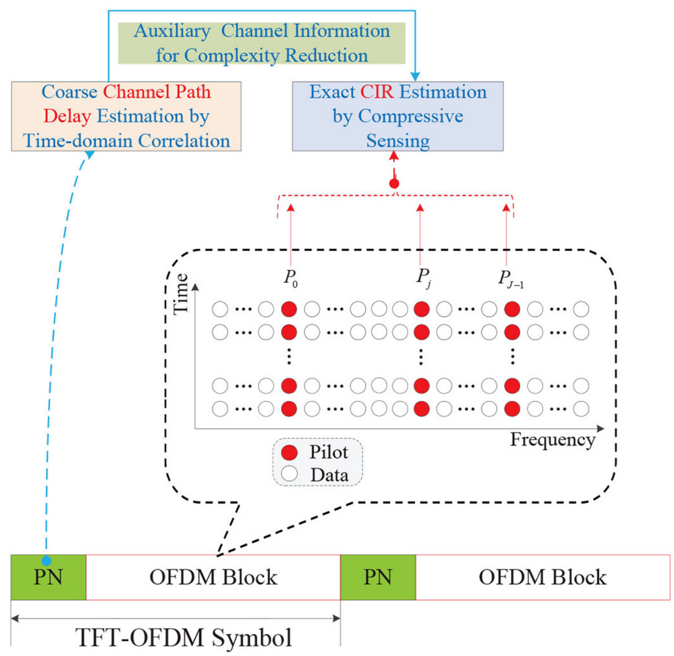

Fig. 2. Proposed frame structure and the corresponding CS-based CE for the TFT-OFDM scheme.

II. System Model

Fig. 1 presents the system model of two conventional schemes. As shown in Fig. 1(a), the DPN-OFDM scheme uses the second received PN sequence immune from IBI for CE. Thus, the iterative IBI removal can be avoided, but significant loss in spectral efficiency will be introduced [9]. In Fig. 1(b), the CS-based TDS-OFDM scheme utilizes the IBI-free region of size G within the received PN sequence for CE, but the maximum estimated channel length is limited to $L = M - G + 1$, where M is the length of the PN sequence [10]. When the channel length L becomes larger, the size of the required IBI-free region becomes smaller, which leads to severe performance deterioration due to the reduced number of observations in CS.

Unlike the conventional TDS-OFDM or CP-OFDM where the training information only exists in either the time or frequency domain [15], [16], Fig. 2 shows that TFT-OFDM has training information in both the time and frequency domains for every signal symbol, i.e., the time-domain TS and the frequency-domain pilots scattered over the signal bandwidth are jointly used in TFT-OFDM. The ith TFT-OFDM signal symbol $\mathbf{s}^i = \left[s_0^i, s_1^i, \cdots, s_{M+N-1}^i\right]^T$ is composed of the known PN sequence $\mathbf{c} = \left[c_0, c_1, \cdots, c_{M-1}\right]^T$ of length M and the OFDM block $\mathbf{x}^i = \left[x_0^i, x_1^i, \cdots, x_{N-1}^i\right]^T$ of length N, i.e.,

$$\mathbf{s}^i = \begin{bmatrix} \mathbf{c} \\ \mathbf{x}^i \end{bmatrix}_{(M+N)\times 1} = \begin{bmatrix} \mathbf{c} \\ \mathbf{F}_{N\times N}^H \tilde{\mathbf{x}}^i \end{bmatrix}_{(M+N)\times 1}, \tag{1}$$

where $\tilde{\mathbf{x}}^i = \left[\tilde{x}_0^i, \tilde{x}_1^i, \cdots, \tilde{x}_{N-1}^i\right]^T$ is the ith OFDM block in the frequency domain. In contrast to the conventional TDS-OFDM, the OFDM block in TFT-OFDM contains not only the traffic data, but also a small number J of pilots denoted as $\tilde{\mathbf{x}}^i|_\Gamma$, where Γ is the pilot location set and represented as

$$\Gamma = \{P_0, P_1, \cdots, P_{J-1}\}, \tag{2}$$

where $0 \le P_0 < P_1 < \cdots < P_{J-1} \le N-1$ can be assumed without loss of generality. By the use of CS and sparse channel nature, the pilot number J could be reduced significantly (far less than L, around 1% of N) and hence the spectral efficiency loss is negligible. The power of the pilots could be boosted for better CE performance, which is similar to that in the traditional pilot-aided CE method [17]. The difference is that the pilots are equally spaced in conventional schemes, while the pilots are randomly located in the proposed scheme to ensure good CE performance based on CS. The reason will be discussed later in Section IV.

In wireless communications, the discrete-time CIR $\mathbf{h}^i = [h_0^i, h_1^i, \ldots, h_{L-1}^i]^T$ of length L comprising S resolvable propagation paths can be modeled as [18]

$$h_l^i = \sum_{s=0}^{S-1} \alpha_s \delta\left[l - \tau_s\right], \; 0 \le l \le L-1, \tag{3}$$

where α_s and τ_s denote the path gain and the normalized path delay of the sth path, respectively. The path delay set T is defined as

$$\mathrm{T} = \{\tau_0, \tau_1, \cdots, \tau_{S-1}\}, \tag{4}$$

where $0 \le \tau_0 < \tau_1 < \cdots < \tau_{S-1} \le L-1$ can be assumed without loss of generality.

If the channel is exactly known at the receiver, the IBI of the PN sequence on the OFDM block can be completely removed. Then, by using the idea of the classical overlap and add (OLA) algorithm [16], the cyclicity reconstruction of the OFDM block can be obtained and hence the CP effect can be restored [6]. This process will be described in Section III-B in detail.

The time-domain OFDM block after cyclicity reconstruction $\mathbf{y}^i = \left[y_0^i, y_1^i, \cdots, y_{N-1}^i\right]^T$ can be represented as

$$\mathbf{y}^i = \mathbf{\Psi}\mathbf{h}_N^i + \mathbf{w}^i, \tag{5}$$

$$\mathbf{\Psi} = \begin{bmatrix} x_0^i & x_{N-1}^i & \cdots & x_1^i \\ x_1^i & x_0^i & \cdots & x_2^i \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-1}^i & x_{N-2}^i & \cdots & x_0^i \end{bmatrix}_{N\times N}, \tag{6}$$

$$\mathbf{h}_N^i = \begin{bmatrix} \mathbf{h}^i \\ \mathbf{0}_{N-L} \end{bmatrix}, \tag{7}$$

where $\mathbf{\Psi}$ is the Toeplitz matrix of size $N \times N$ determined by the transmitted OFDM block $\mathbf{x}^i$, $\mathbf{h}_N^i$ is the N-length CIR vector extended from the original L-length CIR vector $\mathbf{h}^i$ with $N-L$ zeros, and $\mathbf{w}^i = [\omega_0^i, \omega_1^i, \ldots, \omega_{N-1}^i]^T$ denotes the additive white Gaussian noise (AWGN) with zero mean and the variance σ^2.

The frequency-domain OFDM block $\tilde{\mathbf{y}}^i = \left[\tilde{y}_0^i, \tilde{y}_1^i, \cdots, \tilde{y}_{N-1}^i\right]^T$ after cyclicity reconstruction can be represented as

$$\begin{aligned} \tilde{\mathbf{y}}^i = \mathbf{F}_{N\times N}\mathbf{y}^i &= \mathbf{F}_{N\times N}\mathbf{\Psi}\mathbf{F}_{N\times N}^H\mathbf{F}_{N\times N}\mathbf{h}_N^i + \tilde{\mathbf{w}}^i \\ &= \mathrm{diag}(\tilde{\mathbf{x}}^i)\mathbf{F}_{\mathrm{N}\times\mathrm{N}} \begin{bmatrix} \mathbf{h}^i \\ \mathbf{0}_{N-L} \end{bmatrix} + \tilde{\mathbf{w}}^i, \\ &= \mathrm{diag}(\tilde{\mathbf{x}}^i)\mathbf{F}_{(\mathrm{L})}\mathbf{h}^i + \tilde{\mathbf{w}}^i \end{aligned} \tag{8}$$

where $\mathbf{F}_{N\times N}\mathbf{\Psi}\mathbf{F}_{N\times N}^H = \mathrm{diag}(\tilde{\mathbf{x}}^i)$, $\mathbf{F}_{(L)}$ is the matrix comprising the first L columns of the $\mathbf{F}_{N\times N}$, and $\tilde{\mathbf{w}}^i = \mathbf{F}_{N\times N}\mathbf{w}^i$ denotes the AWGN in the frequency domain. Then the received frequency-domain pilots $\tilde{\mathbf{y}}^i|_\Gamma$ within $\mathbf{y}^i$ can be represented by

$$\tilde{\mathbf{y}}^i|_\Gamma = \mathrm{diag}(\tilde{\mathbf{x}}^i|_\Gamma)\mathbf{F}_{(\mathrm{L})}^\Gamma\mathbf{h}^i + \tilde{\mathbf{w}}^i|_\Gamma, \tag{9}$$

where $\mathbf{F}_{(L)}^\Gamma$ represents the sub-matrix comprising the rows of the $\mathbf{F}_{(L)}$ on the set Γ.

In the proposed scheme, the time-domain PN sequence and the frequency-domain pilots are jointly exploited to perform the CE, which will be addressed in detail in the next section.

III. Compressive Sensing Based Channel Estimation

For (9), CS theory has proved that the target signal $\mathbf{h}^i$ of large dimension L can be exactly recovered by a very small number of observations J if $\mathbf{h}^i$ is sparse, i.e., the number of nonzero entries of the target signal is much smaller than its dimension [19], [20]. Fortunately, numerous theoretical analysis and experimental results have verified that wireless channels are sparse in nature, i.e., in the CIR model (3), the dimension of the CIR L may be large, but the number of the active paths S with significant gains is usually small, i.e., $S \ll L$, especially in the wireless wideband communications [1], [21]. This indicates that the sparse channel can be recovered by a very small amount of frequency-domain pilots based on CS.

Many efficient signal recovery algorithms have been developed for CS. Among them, the subspace pursuit (SP) algorithm is a widely used CS algorithm due to its robustness to noise, where the most significant S components of the original sparse signal are identified in an iterative manner [22]. However, it requires the priori knowledge on the sparsity level of the signal, and has relatively high complexity.

In this section, by fully exploiting the joint time-frequency signal feature of TFT-OFDM, we propose a CS-based CE method with the auxiliary information based SP (A-SP) algorithm. Unlike the conventional TS-based or pilot-aided CE schemes which depend on either time- or frequency-domain information [6], [17], the proposed CS-based CE method firstly utilizes the PN-based correlation in the time domain to acquire the auxiliary channel information, and then the

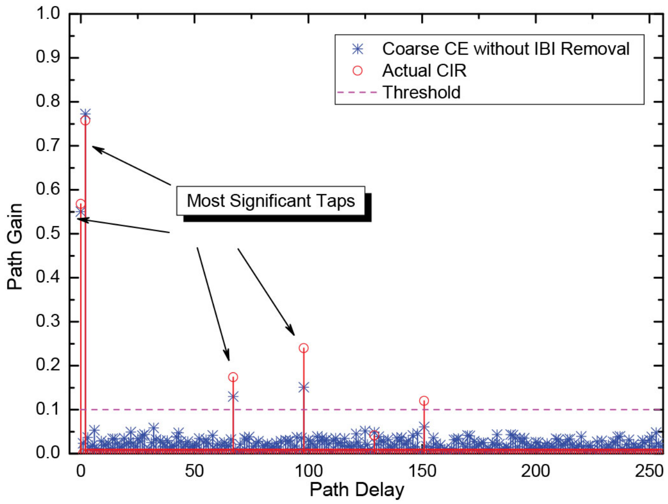

Fig. 3. Coarse path delay estimation for auxiliary channel information acquisition under ITU-VB channel with SNR of 10 dB.

frequency-domain pilots are used for the final exact CIR estimation based on CS. The specific procedure of this method can be divided into three steps: 1) PN-based coarse path delay estimation; 2) Cyclicity reconstruction of the OFDM block; 3) Exact CIR estimation using A-SP.

A. Step 1: PN-Based Coarse Path Delay Estimation

Based on the good auto-correlation property of the PN sequence [23], the received PN sequence $\mathbf{d}^i$ is directly correlated with the locally known PN sequence $\mathbf{c}$ to acquire the coarse channel estimate $\bar{\mathbf{h}}^i$ as

$$\bar{\mathbf{h}}^i = \frac{1}{M}\mathbf{c} \otimes \mathbf{d}^i = \mathbf{h}^i + \mathbf{v}, \tag{10}$$

where $\mathbf{v}$ denotes the AWGN as well as the effect of interference caused by the preceding OFDM block. As shown in Fig. 3, where the ITU Vehicular B (ITU-VB) channel model [24] with the signal-to-noise ratio (SNR) of 10 dB is considered, although the coarse path delay estimation is not accurate due to the existence of IBI, the good auto-correlation property of the PN sequence ensures that the auxiliary channel information necessary for the following A-SP algorithm could be obtained. Such information includes the locations of the most significant taps and the approximate channel sparsity level.

The path gains in $\bar{\mathbf{h}}^i$ are discarded directly, and only the path delays of the most significant taps (partial path delays) are retained in the initial coarse path delay set

$$\mathrm{T}_0 = \left\{l : \left\|\bar{h}_l^i\right\|_2 \geq p_{th}\right\}_{l=0}^{L-1}, \tag{11}$$

where p_{th} is the power threshold configured according to [25].

Then, the channel sparsity level S can be approximated by

$$S = S_0 + a = \|\mathrm{T}_0\|_0 + a, \tag{12}$$

where $S_0 = \|\mathrm{T}_0\|_0$ denotes the initial channel sparsity level, and a is a number used to combat the interference effect, since some low-power active paths may be treated as noise in $\bar{\mathbf{h}}^i$. This auxiliary channel information is essential for practical CS-based signal recovery algorithm with improved performance as well as reduced complexity in *Step 3*.

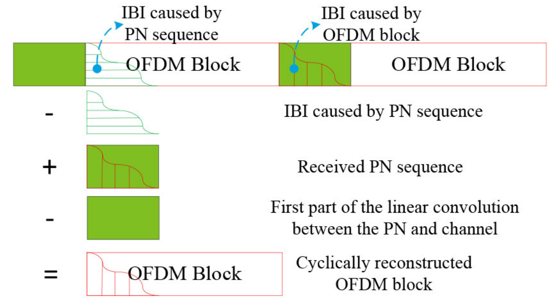

Fig. 4. Illustration for the OLA-based OFDM block cyclicity reconstruction.

B. Step 2: Cyclicity Reconstruction of the OFDM Block

The cyclicity reconstruction of the OFDM block is achieved by firstly subtracting the IBI caused by the PN sequence from the received OFDM block, then adding the received PN sequence and finally subtracting the first part of linear convolution outputs between the PN sequence and channel [6]. This process is based on the idea of OLA algorithm [16] and is briefly illustrated in Fig. 4.

In this step, the IBI caused by the PN sequence is obtained by computing the linear convolution between the local PN sequence and the estimated CIR obtained in the preceding symbol. Under the slow time-varying channels, which can be assumed in many wireless broadcasting systems [26], the estimated CIR obtained in the preceding symbol can be used for the IBI removal in the current symbol. In fact, the received PN sequence contains not only the useful part which is the IBI caused by the OFDM block, but also the useless part that is the linear convolution between the PN sequence and channel. Hence, the useless part should be removed after the received PN sequence is added to achieve the cyclicity reconstruction.

C. Step 3: Exact CIR Estimation Using A-SP

After *Step 2*, the pilots can be extracted from the OFDM block after cyclicity reconstruction for the final accurate CE. Based on the basic idea of classical SP algorithm [22], we propose the A-SP algorithm, whereby the auxiliary channel information obtained in *Step 1* is exploited to improve the CE performance and lower the computational complexity. The proposed A-SP algorithm is described by pseudo code as shown in **Algorithm 1**.

Compared to the classical SP algorithm, the proposed A-SP algorithm has quite similar procedure, but differs from SP in the following three aspects:

1) ***Initial Configuration***. In the SP algorithm, the initial approximation of the support T^0 is set to the S indices corresponding to the largest magnitude entries in the vector $\mathbf{\Phi}^H\mathbf{r}$ because no prior knowledge of the signal is available. However, in the proposed A-SP scheme, by exploiting the obtained partial path delays T_0 in *Step 1*, the initial approximation can be directly configured as $\mathrm{T}^0 \leftarrow \mathrm{T}_0$.

2) ***Significant Entry Identification***. Unlike SP where the S most significant entries of the vector $\mathbf{\Phi}^H\mathbf{r}$ are identified in each

Algorithm 1 Auxiliary information based SP (A-SP)

Inputs:

1) Initial coarse path delay set T_0, channel sparsity level S, initial channel sparsity level S_0;
2) Noisy measurements $\mathbf{m} \overset{\Delta}{=} \tilde{\mathbf{y}}^i|_\Gamma$;
3) Observation matrix $\mathbf{\Phi} \overset{\Delta}{=} \mathrm{diag}(\tilde{\mathbf{x}}^i|_\Gamma)\mathbf{F}^\Gamma_{(L)}$.

Output: The S-sparse CIR estimation $\hat{\mathbf{h}}^i$.

$\mathrm{T}^0 \leftarrow \mathrm{T}_0$;
$\mathbf{r} \leftarrow \mathbf{m} - \mathbf{\Phi}_{\mathrm{T}^0}\mathbf{\Phi}^\dagger_{\mathrm{T}^0}\mathbf{m}$;
$k \leftarrow 0$;
while $k < S - S_0$ **do**
  $k \leftarrow k + 1$;
  $\mathbf{p} \leftarrow \mathbf{\Phi}^H\mathbf{r}$;
  $\tilde{\mathrm{T}}^k \leftarrow \tilde{\mathrm{T}}^{k-1} \cup \mathrm{sup}(\mathbf{p}_{S-S_0})$;
  $\mathbf{q} \leftarrow \mathbf{\Phi}^\dagger_{\tilde{\mathrm{T}}^k}\mathbf{m}$;
  $\mathrm{T}^k \leftarrow \mathrm{sup}(\mathbf{q}_S)$;
  $\mathbf{r} \leftarrow \mathbf{y} - \mathbf{\Phi}_{\mathrm{T}^k}\mathbf{\Phi}^\dagger_{\mathrm{T}^k}\mathbf{m}$;
end while
Actual path delay acquisition: $\mathrm{T} \leftarrow \mathrm{T}^k$;
Then the signal model (9) can be simplified as

$$\tilde{\mathbf{y}}^i|_\Gamma = \mathbf{\Phi}_\mathrm{T}\mathbf{h}^i|_\mathrm{T} + \mathbf{w}^i|_\Gamma. \tag{13}$$

Final least squares (LS):

$$\hat{\mathbf{h}}^i|_\mathrm{T} = \mathbf{\Phi}^\dagger_\mathrm{T}\tilde{\mathbf{y}}^i|_\Gamma = \left(\mathbf{\Phi}^H_\mathrm{T}\mathbf{\Phi}_\mathrm{T}\right)^{-1}\mathbf{\Phi}^H_\mathrm{T}\tilde{\mathbf{y}}^i|_\Gamma. \tag{14}$$

iteration, we remain the S_0 most significant entries unchanged, and identify the next $S - S_0$ most significant ones instead.

3) ***Iteration Number***. The required number of iterations is reduced from S in SP to $S - S_0$ in A-SP, so the computational complexity can be reduced.

IV. Performance Analysis of CS-Based TFT-OFDM

This section addresses the performance analysis of the proposed scheme in terms of the spectral efficiency, the Cramér-Rao lower bound (CRLB) of the proposed CE method, and the computational complexity as well.

A. Spectral Efficiency

Table I compares the spectral efficiency of different TDS-OFDM schemes with the ideal OFDM system without any overhead [27]. The length of the PN sequence is $M = 256$ and the pilot number embedded in TFT-OFDM is $J = 36$.

The conventional TDS-OFDM scheme has the highest spectral efficiency, but the iterative interference cancellation is required, which results in high complexity and performance loss. In the DPN-OFDM scheme, the PN sequence is duplicated to avoid the interference from the preceding OFDM block to the second PN sequence, but the spectral efficiency is significantly reduced. In the proposed scheme, the embedded pilots would somehow reduce the spectral efficiency, but the penalty is very small, since the proposed A-SP signal recovery

TABLE I
Spectral Efficiency Comparison

OFDM Block Length	TDS-OFDM	DPN-OFDM	TFT-OFDM
1024	80.00%	66.67%	77.19%
2048	88.89%	80.00%	87.33%
4096	94.12%	88.89%	93.29%
8192	96.97%	94.12%	96.54%

algorithm only requires a very small number of observations $\mathcal{O}\left(S\log_2\left(L/S\right)\right)$ [22]. In fact, for the most common broadcasting channels with six active paths [1], the number $J = 36$ of the embedded pilots is enough for good channel recovery performance (note that $S\log_2\left(L/S\right) = 6 \times \log_2\left(256/6\right) \approx 32$). In the typical system configuration of $M = 256$ and $N = 4096$ [10], the spectral efficiency of the proposed CS-based TFT-OFDM scheme is 93.29%, which is only 0.83% less than that of the conventional TDS-OFDM system.

Moreover, the constant 36-symbol transmission parameter signaling (TPS) used in the practical DTMB systems can be regarded as the pilots once after being successfully detected [28], which indicates that even the negligible spectral efficiency loss can be avoided.

B. CRLB of the Channel Estimator

According to the signal model (13) in which the AWGN vector $\tilde{\mathbf{w}}|_\Gamma$ is subject to the distribution $\mathcal{CN}\left(\mathbf{0}, \sigma^2\mathbf{I}_J\right)$, where $\mathbf{I}_J$ is the identity matrix of size $J \times J$, the conditional probability density function (PDF) of $\tilde{\mathbf{y}}^i|_\Gamma$ with the given $\mathbf{h}^i|_\mathrm{T}$ is

$$p\left(\tilde{\mathbf{y}}^i|_\Gamma \middle| \mathbf{h}^i|_\mathrm{T}\right) = \frac{1}{\left(2\pi\sigma^2/A\right)^{J/2}} \exp\left(-\frac{1}{2\sigma^2/A}\left\|\tilde{\mathbf{y}}^i|_\Gamma - \mathbf{\Phi}_\mathrm{T}\mathbf{h}^i|_\mathrm{T}\right\|_2\right), \tag{15}$$

where A denotes the average boosted power of the pilots.

Then, we can derive the CRLB of the unbiased estimator (14) by using the vector estimation theory [29], which is given as

$$\mathrm{CRLB} = \mathrm{E}\left(\left\|\hat{\mathbf{h}}^i|_\mathrm{T} - \mathbf{h}^i|_\mathrm{T}\right\|_2\right) = \frac{S\sigma^2}{JA}. \tag{16}$$

Compared with the channel estimator in DPN-OFDM systems, where the best mean square error (MSE) performance is σ^2, the channel estimator based on A-SP can achieve much better MSE performance, since S is much smaller than J, and the boosted power A is usually larger than 1.

Note that if the observation matrix $\mathbf{\Phi}_\mathrm{T}$ does not have orthogonal columns, the CRLB (16) cannot be achieved by the practical channel estimator. However, due to the random positions of the pilots used in TFT-OFDM and the random locations of active paths of wireless channels, the matrix $\mathbf{\Phi}_\mathrm{T}$ has imperfect but approximate orthogonal columns (the requirement of near orthogonality is equivalently similar to the restricted isometry property (RIP) of the observation matrix in the CS theory [30]). Thus, the CRLB can be approximately approached, which will be validated by the simulation results in Section V.

TABLE II
MULTIPATH CHANNEL PARAMETERS FOR SIMULATION

Path Index	ITU-VB		SARFT-8		Mod ITU-VB	
	Delay (μs)	Gain (dB)	Delay (μs)	Gain (dB)	Delay (μs)	Gain (dB)
1	0.00	-2.5	0.00	-18.0	0.00	-2.5
2	0.30	0.0	1.80	0.00	0.30	0.0
3	8.90	-12.8	1.95	-20.00	8.90	-12.8
4	12.90	-10.0	3.60	-20.00	12.90	-10.0
5	17.10	-25.2	7.50	-10.00	17.10	-25.2
6	20.00	-16.0	31.80	0.00	20.00	-16.0
7	/	/	/	/	40.00	-10.0

C. Computational Complexity

In the proposed CS-based CE method, the M-point circular correlation in *Step 1* could be efficiently implemented by M-point FFT, so the corresponding complexity is $\mathcal{O}\left(M\log_2(M)/2\right)$. In *Step 2*, it requires the complexity of $\mathcal{O}\left(M\log_2(M)/4+3M\right)$ for the cyclicity reconstruction operation.

In fact, the main computational burden of the proposed method is the A-SP algorithm used to acquire the actual path delays in *Step 3*. Each iteration has the complexity of $\mathcal{O}\left(J\left(L+S^2\right)\right)$, and the overall complexity of SP comprising S iterations is $\mathcal{O}\left(JS\left(L+S^2\right)\right)$ [22]. However, as has been discussed in Section III-C, only $S-S_0$ iterations are required by A-SP, since some of the locations of the significant taps have been detected already, the complexity of the proposed A-SP is reduced to $\mathcal{O}\left(J\left(S-S_0\right)\left(L+S^2\right)\right)$. So the complexity of A-SP is lower than that of SP.

V. SIMULATION RESULTS

This section investigates the performance of the CS-based CE for TFT-OFDM under typical broadcasting channels. The signal bandwidth is 7.56 MHz locating at a central frequency of 760 MHz. The OFDM block length N is 4096, and the GI length M is 256. The low-density parity-check (LDPC) code with code rate of 0.6 and code length of 7488 in DTMB is adopted [1]. The well-known iterative decoding algorithm called belief propagation (BP) is used with the maximum iteration number of 30 [31]. The modulation schemes 256 QAM for the static channel and 16 QAM with a receiver velocity of 60 km/h are both considered to evaluate the support for UHDTV and mobile services, respectively.

The multipath channel parameters used for simulations are listed in Table II. The typical six-tap ITU-VB channel model ($S=6$, $L=152$) is adopted for performance evaluation. Furthermore, the State Administration of Radio, Film, and Television 8 (SARFT-8) channel model ($S=6$, $L=241$) with a very strong echo path close to the GI length [1] and the modified ITU-VB channel ($S=7$, $L=303$) with an extremely long path delay exceeding the GI length are also adopted to evaluate the system performance under severely fading channel with long delays which may exist for broadcasting systems, especially in SFN. In the CS-based TDS-OFDM scheme, the last $G=36$ samples of the IBI-free region are used for CE, while the number of frequency-domain pilots $J=36$ (around 1% of the total sub-carrier number $N=4096$) with the boosted power $A=2$ is utilized in the proposed scheme.

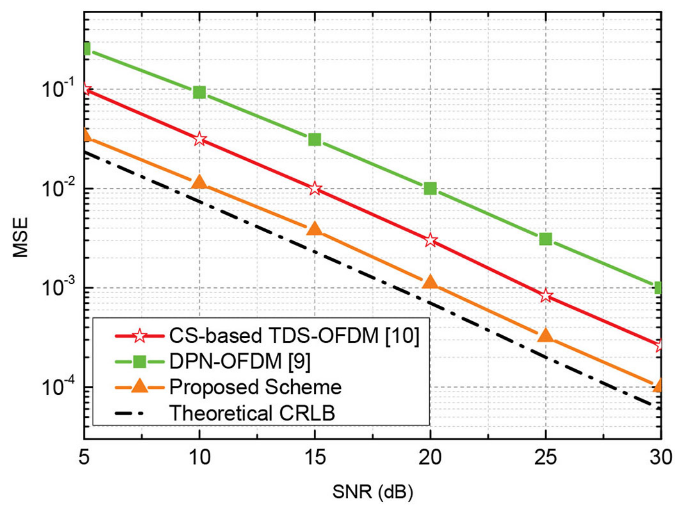

Fig. 5. MSE performance comparison under the ITU-VB channel with the channel length smaller than the GI length.

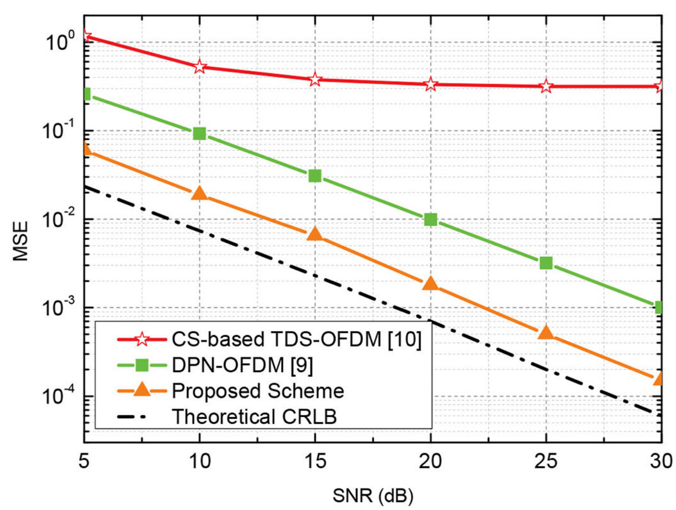

Fig. 6. MSE performance comparison under the SARFT-8 channel with the channel length close to the GI length.

Figs. 5–7 present the MSE performance comparison of the proposed scheme with the conventional DPN-OFDM and CS-based TDS-OFDM schemes under three different channels with different channel lengths. It can be seen from Fig. 5 that under the ITU-VB channel, the MSE performance of the proposed scheme enjoys a significant SNR gain of 4 dB and 10 dB compared to those of CS-based TDS-OFDM and DPN-OFDM, respectively, when the target MSE of 10^{-3} is considered. If the channel length L is fairly close to the GI length M, see the SARFT-8 channel considered in Fig. 6, the MSE performance of the proposed scheme is 7.5 dB better than that of DPN-OFDM, while the recent CS-based TDS-OFDM cannot work due to the reduced size of the IBI-free region.

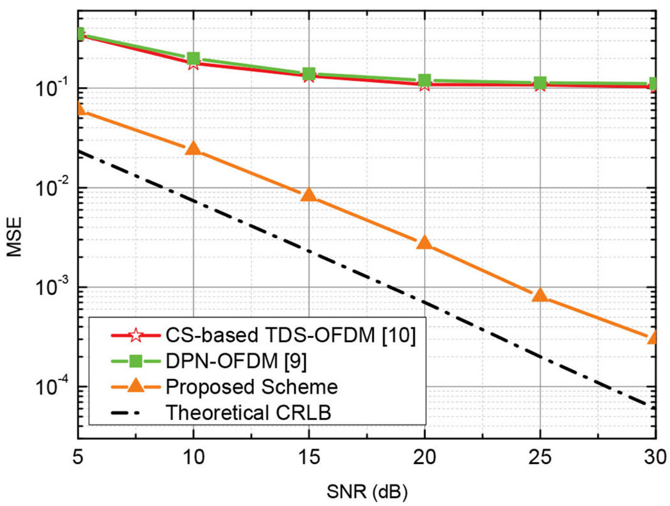

Fig. 7. MSE performance comparison under the modified ITU-VB channel with the channel length larger than the GI length.

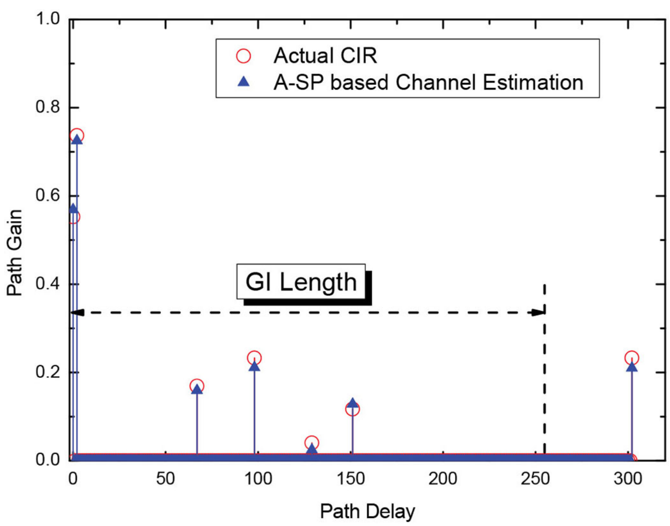

Fig. 8. Snapshot of the channel estimate of the proposed CS-based CE method under the modified ITU-VB channel with a long path delay $L = 303$ when SNR is 10 dB.

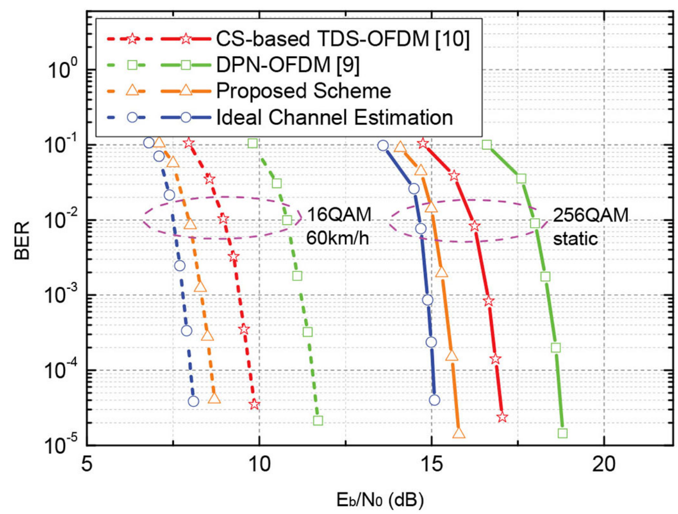

Fig. 9. BER performance comparison under the ITU-VB channel with the channel length smaller than the GI length.

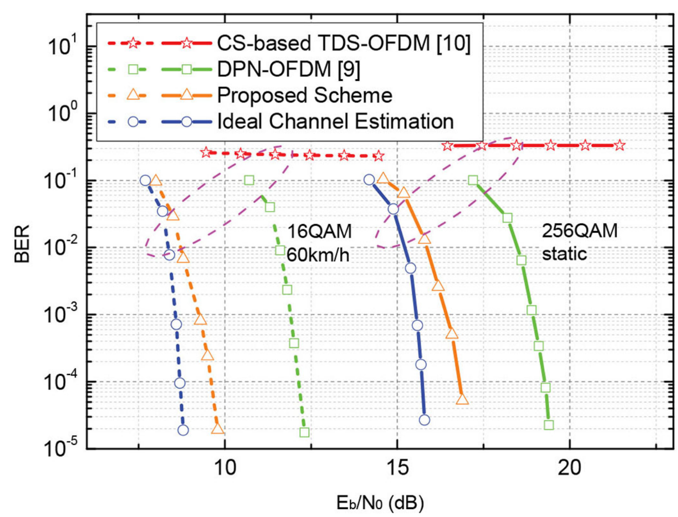

Fig. 10. BER performance comparison under the SARFT-8 channel with the channel length close to the GI length.

In the extreme case that the actual channel length L is larger than the GI length M (i.e., $L > M$), the conventional DPN-OFDM and CS-based TDS-OFDM schemes will fail to work completely due to the limitation of the PN length, while the proposed scheme can still have good MSE performance, as shown in Fig. 7. This can be explained by Eq. (9) that the maximum CE length of the proposed scheme is only limited by the column number of $\mathbf{F}^{\Gamma}_{(L)}$, which could be N at most, i.e., $L \leq N$. In this way, the maximum channel delay spread of the proposed CE method is extended from M to N, as long as the cyclicity reconstruction of the OFDM block is near-perfect. Fig. 8 shows a snapshot of the channel estimate of the proposed CS-based CE method under the modified ITU-VB channel with a long path delay $L = 303$ (larger than the GI length $M = 256$) when SNR is 10 dB. It is clear that the proposed scheme can acquire very exact CIR estimation compared with the actual channel, including both the path delays and path gains.

Figs. 9–11 compare the LDPC-coded bit error rate (BER) performance of the proposed scheme with the conventional DPN-OFDM and CS-based TDS-OFDM schemes under the three considered channels with different channel lengths. The BER performance with the ideal CE is also presented as the benchmark for comparison. Since the schemes have different spectral efficiencies, for fair comparison, the energy per bit to noise power spectral density ratio (E_b/N_0) is more suitable for the performance evaluation than the traditional SNR. As shown in Fig. 9, under the ITU-VB channel, the BER performance of the proposed scheme enjoys a significant E_b/N_0 gain of about 1 dB and 2.5 dB compared with CS-based TDS-OFDM and DPN-OFDM, respectively, when the target BER of 10^{-4} is considered. It can be observed from Fig. 10 that under the SARFT-8 channel, the BER performance of the proposed scheme is about 2 dB better than that of the DPN-OFDM scheme, while the CS-based TDS-OFDM scheme cannot work due to the IBI-free region is severely contaminated by the long

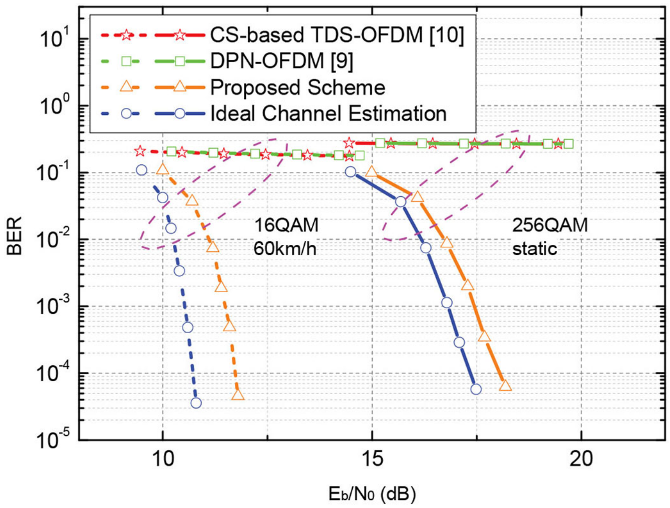

Fig. 11. BER performance comparison under the modified ITU-VB channel with the channel length larger than the GI length.

channel. Fig. 11 shows that when the actual channel length is larger than the GI length, the two conventional schemes fail to work, while the proposed scheme still performs well. In addition, the actual BER curve is very close to that of the ideal case, which indicates the good performance of the proposed scheme.

VI. Conclusion

We have proposed a CS-based CE method for TFT-OFDM in this paper. The MSE performance of this method outperforms the conventional schemes and is close to the CRLB by simultaneously exploiting the time-domain PN sequence and frequency-domain pilots. Simulation results show that the proposed scheme has a good BER performance in both static and mobile scenarios and can well support the 256 QAM, especially when the maximum channel delay spread is fairly close to or even larger than the GI length. Besides, by using the auxiliary channel information, the proposed A-SP algorithm has lower complexity than the conventional SP algorithm. Thus, this scheme is expected to extend TDS-OFDM in the emerging UHDTV applications under SFN with long channel delay spread. Furthermore, for CP-OFDM system with time-domain preamble or TS, this scheme can also be applied.

References

[1] *Error-Correction, Data Framing, Modulation and Emission Methods for Digital Terrestrial Television Broadcasting*, ITU-R BT. 1306-6 Standard, Dec. 2011.

[2] J. Song *et al.,* "Technical review on Chinese digital terrestrial television broadcasting standard and measurements on some working modes," *IEEE Trans. Broadcast.*, vol. 53, no. 1, pp. 1–7, Feb. 2007.

[3] L. Dai, Z. Wang, and Z. Yang, "Next-generation digital television terrestrial broadcasting systems: Key technologies and research trends," *IEEE Commun. Mag.*, vol. 50, no. 6, pp. 150–158, Jun. 2012.

[4] B. Ai *et al.,* "On the synchronization techniques for wireless OFDM systems," *IEEE Trans. Broadcast.*, vol. 52, no. 2, pp. 236–244, May 2006.

[5] L. He, F. Yang, C. Zhang, and Z. Wang, "Synchronization for TDS-OFDM over multipath fading channels," *IEEE Trans. Consum. Electron.*, vol. 56, no. 4, pp. 2141–2147, Nov. 2010.

[6] J. Wang, Z. Yang, C. Pan, and J. Song, "Iterative padding subtraction of the PN sequence for the TDS-OFDM over broadcast channels," *IEEE Trans. Consum. Electron.*, vol. 51, no. 11, pp. 1148–1152, Nov. 2005.

[7] K. Yan, F. Yang, C. Pan, and J. Song, "Reception quality prediction in a single frequency network for the DTMB standard," *IEEE Trans. Broadcast.*, vol. 58, no. 4, pp. 629–636, Dec. 2012.

[8] L. Vangelista *et al.,* "Key technologies for next-generation terrestrial digital television standard DVB-T2," *IEEE Commun. Mag.*, vol. 47, no. 10, pp. 146–153, Oct. 2009.

[9] J. Fu, J. Wang, J. Song, C. Pan, and Z. Yang, "A simplified equalization method for dual PN-sequence padding TDS-OFDM systems," *IEEE Trans. Broadcast.*, vol. 54, no. 4, pp. 825–830, Dec. 2008.

[10] L. Dai, Z. Wang, and Z. Yang, "Compressive sensing based time domain synchronous OFDM transmission for vehicular communications," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 9, pp. 460–469, Sep. 2013.

[11] S. Li, J. Xiong, L. Gui, and Y. Xu, "A generalized analytical solution to channel estimation with intersymbol interference cancelation and co-channel interference cancelation for single input single output/multiple input single output digital terrestrial multimedia broadcasting systems," *IEEE Trans. Broadcast.*, vol. 59, no. 1, pp. 116–128, Mar. 2013.

[12] J. Xiong, L. Gui, H. Liu, and P. Cheng, "On channel estimation and equalization in 2x1 MISO TDS-OFDM based terrestrial DTV systems," *IEEE Trans. Broadcast.*, vol. 58, no. 1, pp. 130–138, Mar. 2012.

[13] L. Dai, Z. Wang, and Z. Yang, "Time-frequency training OFDM with high spectral efficiency and reliable performance in high speed environments," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 4, pp. 695–707, May 2012.

[14] C. Pan, L. Dai, and Z. Yang, "Unified time-frequency OFDM transmission with self interference cancellation," *IEICE Trans. Fund.*, vol. E96-A, no. 4, pp. 807–813, Apr. 2013.

[15] J. Wang *et al.,* "A general SFN structure with transmit diversity for TDS-OFDM system," *IEEE Trans. Broadcast.*, vol. 52, no. 2, pp. 245–251, Jun. 2006.

[16] B. Muquet, Z. Wang, G. Giannakis, M. D. Courville, and P. Duhamel, "Cyclic prefixing or zero padding for wireless multicarrier transmissions?" *IEEE Trans. Commun.*, vol. 50, no. 12, pp. 2136–2148, Dec. 2002.

[17] Z. Tang, R. Cannizzaro, G. Leus, and P. Banelli, "Pilot-assisted time-varying channel estimation for OFDM systems," *IEEE Trans. Signal Process.*, vol. 55, no. 5, pp. 2226–2238, May 2007.

[18] B. Yang, K. Letaief, R. Cheng, and Z. Cao, "Channel estimation for OFDM transmission in multipath fading channels based on parametric channel modeling," *IEEE Trans. Commun.*, vol. 49, no. 3, pp. 467–479, Mar. 2001.

[19] E. Candès and M. Wakin, "An introduction to compressive sampling," *IEEE Signal Process. Mag.*, vol. 25, no. 2, pp. 21–30, Feb. 2008.

[20] W. Bajwa, J. Haupt, A. Sayeed, and R. Nowak, "Compressed channel sensing: A new approach to estimating sparse multipath channels," *Proc. IEEE*, vol. 98, no. 6, pp. 1058–1076, Jun. 2010.

[21] C. Berger, Z. Wang, J. Huang, and S. Zhou, "Application of compressive sensing to sparse channel estimation," *IEEE Commun. Mag.*, vol. 48, no. 11, pp. 164–174, Nov. 2010.

[22] W. Dai and O. Milenkovic, "Subspace pursuit for compressive sensing signal reconstruction," *IEEE Trans. Inf. Theory*, vol. 55, no. 5, pp. 2230–2249, May 2009.

[23] B. Ai, J. Ge, and Y. Wang, "Symbol synchronization technique in COFDM systems," *IEEE Trans. Broadcast.*, vol. 50, no. 1, pp. 56–62, Mar. 2004.

[24] *Guideline for Evaluation of Radio Transmission Technology for IMT-2000*, ITU-R M. 1225 Standard, Dec. 1997.

[25] F. Wan, W. Zhu, and M. Swamy, "Semi-blind most significant tap detection for sparse channel estimation of OFDM systems," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 57, no. 3, pp. 703–713, Mar. 2010.

[26] X. Zhou, F. Yang, and J. Song, "Novel transmit diversity scheme for TDS-OFDM system with frequency-shift m-sequence padding," *IEEE Trans. Broadcast.*, vol. 58, no. 2, pp. 317–324, Jun. 2012.

[27] L. Dai, Z. Wang, and S. Chen, "A novel uplink multiple access scheme based on TDS-FDMA," *IEEE Trans. Wireless Commun.*, vol. 10, no. 3, pp. 757–761, Mar. 2011.

[28] L. Dai, Z. Wang, C. Pan, and S. Chen, "Wireless positioning using TDS-OFDM signals in single-frequency networks," *IEEE Trans. Broadcast.*, vol. 58, no. 2, pp. 236–246, Jun. 2012.

[29] S. M. Kay, *Fundamentals of Statistical Signal Processing, Volumn I: Estimation Theory*. Upper Saddle River, NJ, USA: Prentice-Hall, 1993.

[30] J. Haupt, W. Bajwa, G. Raz, and R. Nowak, "Toeplitz compressed sensing matrices with applications to sparse channel estimation," *IEEE Trans. Inf. Theory*, vol. 56, no. 11, pp. 5862–5875, Nov. 2010.

[31] J. Chen, A. Dholakia, E. Eleftheriou, M. Fossorier, and X. Hu, "Reduced-complexity decoding of LDPC codes," *IEEE Trans. Commun.*, vol. 53, no. 8, pp. 1288–1299, Aug. 2005.

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