

# A Novel Channel Estimation Method for OFDM Mobile Communication Systems Based on Pilot Signals and Transform-Domain Processing

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**Abstract** — A method of estimating the channel transfer function is presented for Orthogonal Frequency Division Multiplexing (OFDM) mobile communication systems working under time-variant radio channel conditions. The proposed method employs lowpass filtering in a transform domain so that intercarrier interference and additive white Gaussian noise components in the received pilot signals are significantly reduced. The cutoff frequency of the transform-domain filter is dynamically selected by tracking the received pilot signals. The channel transfer function for all the subcarriers is obtained by a high-resolution interpolation realized by zero-padding and DFT/IDFT. The proposed method is applicable for all linear modulation OFDM systems. It is demonstrated with a 16QAM-OFDM system which includes both amplitude and phase modulations.

## I. INTRODUCTION

A wideband radio channel is normally frequency selective and time variant. For an Orthogonal Frequency Division Multiplexing (OFDM) [1] mobile communication system, the channel transfer function at different subcarriers appears unequal in both frequency and time domains. In the case that multi-amplitude signals (e.g., 16QAM) are modulated on subcarriers, which gives higher bandwidth efficiency, a dynamic estimation of the channel is necessary for the signal correction.

Pilot-based approaches are widely used to estimate the channel properties and correct the received signal. Some methods have been developed under the assumption of a slow fading channel, where the channel transfer function for the previous OFDM data block is used as the transfer function for the present data block [2,3,4].

In practice, a wideband radio channel is time-variant, frequency-selective and noisy. The estimation of its transfer function becomes rather difficult. First, the slow fading assumption does not always hold. Thus the transfer function might have significant changes even for adjacent OFDM data blocks. Therefore, it is preferable to estimate channel based on the pilots signals in each individual OFDM data block. Secondly, the pilot signals are also corrupted by intercarrier interference (ICI), due to the fast variation of the mobile channel. In addition, additive white Gaussian noise (AWGN) always exists. ICI and AWGN components in the received pilot signals strongly affect the accuracy of the estimation.

In this paper, we propose a novel channel estimation method for OFDM systems applied in mobile communications with fast- or slowly-fading noisy radio channels. Comb-type pilot

subcarrier arrangement is adopted. The key points of the proposed method are as follows. The ICI and AWGN in the pilot subcarriers are reduced by lowpass filtering in a transform domain. The passband of the filter is determined dynamically from the received pilot signals. The channel transfer function for all the subcarriers is obtained by the high-resolution interpolation realized by zero padding and DFT/IDFT.

It is worth noting that the proposed method is applicable to all linear modulation OFDM systems. A 16QAM-OFDM system is chosen in this paper for demonstration because it includes both amplitude modulation and phase modulation.

The paper is organized as follows. In Section II, the pilot-based OFDM system is described, the available methods are reviewed and remarked. Section III discusses the proposed estimation method in detail. Simulation results are given in Section IV, which show BER improvements achieved with the proposed method, as compared to the linear interpolation method [5]. Section V discusses the proposed method and concludes the paper.

## II. PILOT-BASED SIGNAL CORRECTION

### A. Baseband Model of OFDM System

A baseband model of an OFDM system using pilot-based signal correction is shown in Fig. 1. Binary information data are encoded into multi-amplitude-multiphase signals (16QAM). Then pilot signals are uniformly inserted into the information data sequence. The function of the IFFT block is to convert the data sequence of length  $N$  into  $N$  parallel data, modulate them on  $N$  subcarriers, and sum them. After conventional lowpass filtering, the signals are (modulated on carrier frequency and then) transmitted to channel. At the receiver side, the pilot-based signal correction is performed after FFT (parallel-to-serial conversion and subcarrier-demodulation), followed by decoding.

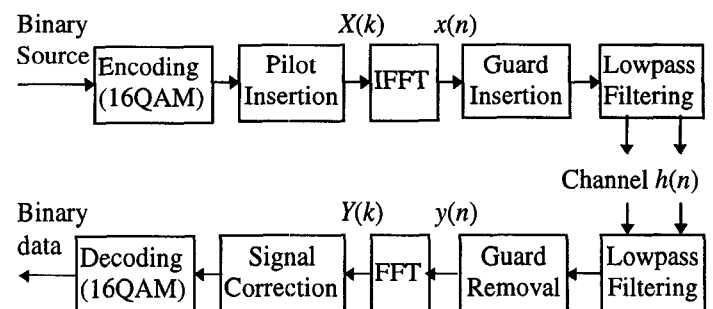


Fig. 1 Baseband model of the OFDM system with pilot-based signal correction.

The total  $N$  subcarriers of the OFDM system are arranged as follows. Adjacent  $L$  ( $L < N$ ) subcarriers are grouped together, without overlapping between adjacent groups. In each group, the first subcarrier is used to transmit pilot signal and thus is called the pilot subcarrier. The rest of the subcarriers bear information data and thus are called information subcarriers. Therefore, there are total  $M=N/L$  pilot subcarriers and  $N-M$  information subcarriers.

Through the whole paper,  $n \in [0, N-1]$  denotes the index in discrete time domain, and  $k \in [0, N-1]$  denotes the index in discrete frequency domain.  $k$  is further expressed as  $k = mL + l$ , with integers  $l \in [0, L-1]$  and  $m \in [0, M-1]$ .

Assume that all the pilot signals have an equal complex value  $c$ , then the OFDM signal modulated on the  $k$ th subcarrier can be expressed as

$$X(k) = X(mL + l) = \begin{cases} c, & l = 0, \\ \text{information data}, & l = 1, \dots, L-1. \end{cases} \quad (1)$$

The corresponding time-domain signal  $x(n)$  is obtained by IFFT.

The channel is assumed to be a time-variant multipath propagation channel. During an OFDM symbol period  $T$ , the discrete-time channel impulse response function can be expressed as [6,7]

$$h(n) = \sum_{i=0}^{r-1} h_i \exp\left(j \frac{2\pi}{N} f_{Di} T n\right) \delta(\lambda - \tau_i), \quad (2)$$

$$0 \leq n \leq N-1,$$

where  $r$  is the total number of propagation paths,  $h_i$  the complex impulse response of the  $i$ th path,  $f_{Di}$  the  $i$ th-path Doppler frequency shift which causes ICI of the received signals,  $\lambda$  the delay spread index, and  $\tau_i$  the  $i$ th-path delay time normalized by sampling time. For simplicity,  $\lambda$  and  $\tau_i$  are assumed to be integers.

The received time-domain OFDM signal  $y(n)$  is a function of the transmitted signal, the channel transfer function and AWGN  $w(n)$ . It can be expressed as

$$y(n) = x(n) * h(n) + w(n), \quad 0 \leq n \leq N-1, \quad (4)$$

where “\*” denotes convolution.

The received frequency-domain signal  $Y(k)$  is the Fourier transform of  $y(n)$ . After some derivations,  $Y(k)$  can be expressed as

$$Y(k) = X(k)H(k) + I(k) + W(k), \quad (4)$$

where  $W(k)$  is the Fourier transform of  $w(n)$ , and

$$H(k) = \sum_{i=0}^{r-1} h_i e^{j\pi f_{Di} T} \frac{\sin(\pi f_{Di} T)}{\pi f_{Di} T} e^{-j \frac{2\pi \tau_i}{N} k}, \quad (5)$$

$$I(k) = \frac{1}{N} \sum_{i=0}^{r-1} \sum_{\substack{K=0 \\ K \neq k}}^{N-1} h_i X(K) \frac{1 - e^{j2\pi(f_{Di} T - k + K)}}{1 - e^{j \frac{2\pi}{N}(f_{Di} T - k + K)}} e^{-j \frac{2\pi \tau_i}{N} K}. \quad (6)$$

$H(k)$  is recognized as accurate channel transfer function at the  $k$ th subcarrier, which is independent from transmitted signals  $X(k)$ .

On the other hand,  $H(k)$  is a sinusoidal function of  $k$ , the changing rate depends on  $\tau_i/N$ . A smaller  $\tau_i/N$  gives slower variation of  $H(k)$ . When the channel bandwidth is 2 MHz,  $\tau_i$  is normally 10 for typical Rayleigh channel in urban area and 1 for Rician channel.  $N$  is chosen as 1024. Therefore,  $\tau_i \ll N$ , and  $H(k)$  changes rather slowly with respect to  $k$ .  $I(k)$  is the ICI component in the received signal at the  $k$ th subcarrier, depending on the signal values  $X(k)$  modulated on all the other subcarriers. Since the information data  $X(k)$  are zero-mean random variables,  $I(k)$  appears as fast-changing function of  $k$ . These relations and properties still stand when  $\tau_i$  is not an integer, although the mathematical derivation becomes more complicated.

Our goal is to estimate  $H(k)$  from the received pilot signals. Since the value of the pilot signals is known as  $c$ , a *rough estimate* of the channel transfer function at the pilot subcarriers can be obtained as

$$\hat{H}_N(mL) = \frac{Y_N(mL)}{c} = H_N(mL) + \frac{1}{c} (I_N(mL) + W_N(mL)), \quad (7)$$

$$m = 0, 1, \dots, M-1.$$

From here to the end of this paper, the subscript  $N$  or  $M$  indicates the length of the sequence, a capital letter with a dot on it denotes a noisy sequence, and a capital letter with a hat represents an estimate sequence. In (7),  $\hat{H}_N(mL)$  denotes the noisy transfer function at  $M$  pilot subcarriers ( $k = mL$ ). A more delicate channel estimate over the whole band ( $k = 0, \dots, N-1$ ) can then be obtained by interpolation.

## B. Available Estimation Methods

One available method [5] is based on the idea mentioned above, under the assumption of a noiseless channel. Let us analyze the two inherent drawbacks of this method with the following example. If AWGN noise is also considered and a first-order interpolation is used, we have the following linear interpolation relation

$$\begin{aligned} \hat{H}_N(k) &= \hat{H}_N(mL + l) = \left(1 - \frac{l}{L}\right) \hat{H}_N(mL) + \frac{l}{L} \hat{H}_N(mL + L) \\ &= \left[ \left(1 - \frac{l}{L}\right) H_N(mL) + \frac{l}{L} H_N(mL + L) \right] \\ &\quad + \left[ \left(1 - \frac{l}{L}\right) \frac{I_N(mL) + W_N(mL)}{c} + \frac{l}{L} \frac{I_N(mL + L) + W_N(mL + L)}{c} \right] \end{aligned} \quad (8)$$

where  $\hat{H}_N(k)$  is the value of the transfer function at the  $k$ th subcarrier,  $\hat{H}_N(mL)$  and  $\hat{H}_N(mL + L)$  are the values of the transfer function at the two closest pilot subcarriers defined in (7). The term in the first square brackets of (8) is the desired estimate, provided that every segment of the channel transfer function between adjacent pilot subcarriers is ramp-like. Unfortunately, this assumption does not hold for most of the cases. In fact, any fixed-parameter (order or coefficients) interpolation does not always fit to the channel properties. This model mismatch forms a part of the estimation error. The term in the second square brackets in (8) represents another part of the

estimation error, which comes from the ICI and AWGN components in the received pilot signals. For the sake of performance comparisons, this linear interpolation method is employed in this paper as a reference. The simulation results obtained by linear interpolation method and the proposed method are compared in Section IV.

Another type of methods employ a different pilot-signal arrangement. The pilot signals are modulated on all subcarriers for certain OFDM data block (training block), so that the values of the channel transfer function at all subcarriers for that block can be obtained at the receiver side [2,3,4]. Under the assumption of a quasistationary channel, the channel transfer function for the following blocks can be derived from that for the training block by matrix decomposition, in minimum mean squared error (MMSE) or least mean squares (LMS) sense. These methods are applicable to communications through slowly time-variant channels.

### III. PROPOSED CHANNEL ESTIMATION METHOD

In this paper we assume that the channel is time-variant. Therefore, the transfer function for the present data block should be obtained independently of that for the previous data block. The subcarrier arrangement in (1) is then adopted. The proposed channel estimation method based on pilot signals and transform-domain processing is depicted in Fig. 2. The three key points mentioned in Section I are discussed in detail in the following.

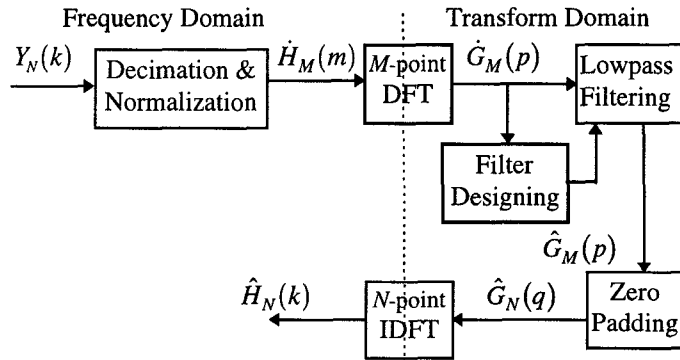


Fig. 2 Channel estimation approach.

#### A. ICI and AWGN Reduction

By  $L:1$  downsampling the received sequence  $Y_N(k)$ , the samples at pilot subcarriers are picked up and a sequence of length  $M$  is obtained. Normalizing this sequence to the pilot signal value  $c$  gives the noisy channel transfer function  $\dot{H}_M(m)$  which is identical to the rough estimate  $\dot{H}_N(mL)$  in (7), that is

$$\dot{H}_M(m) = H_M(m) + [I_M(m) + W_M(m)]/c, \quad (9)$$

$$m = 0, \dots, M-1.$$

Since the ICI and AWGN in time domain are zero mean random processes [7,8], it can be derived that the noise component  $[I_M(m) + W_M(m)]/c$  in (9) is a random process with zero mean and Gaussian distribution [9].

As analyzed in the previous section, variation of the actual

transfer function  $H_M(m)$  is very slow with respect to the pilot subcarrier index  $m$ , while the noise component  $[I_M(m) + W_M(m)]/c$  changes very fast. Therefore, they are separable. The key is to find a relevant strategy.

Considering that the channel parameters are unknown and changing from time to time, it is difficult to reduce the noise component in frequency domain by normal curve-fitting algorithm based on a fixed low-order polynomial assumption. On the contrary, a lowpass filtering in a transform domain is straightforward and feasible.

We define the transform domain so that any sequence in this domain is the DFT of its counterpart in the frequency domain. Therefore, a sequence in the transform domain is the "spectral sequence" of its counterpart in the frequency domain. The argument  $p$  of the transform domain can be viewed as the "frequency" which reflects the variation speed of a frequency-domain function.

The transform domain representation of  $\dot{H}_M(m)$  is then

$$\dot{G}_M(p) = \sum_{m=0}^{M-1} \dot{H}_M(m) \exp\left(-j \frac{2\pi}{M} mp\right), \quad (10)$$

where  $p$  is the transform-domain index and  $p \in [0, M-1]$ . As expected, the signal component in  $\dot{G}_M(p)$  is located at the lower "frequency" (around  $p=0$  and  $p=M-1$ ) region, while the noise component is spread over whole "frequency" region ( $p = 0, \dots, M-1$ ). Fig. 3 gives an example.

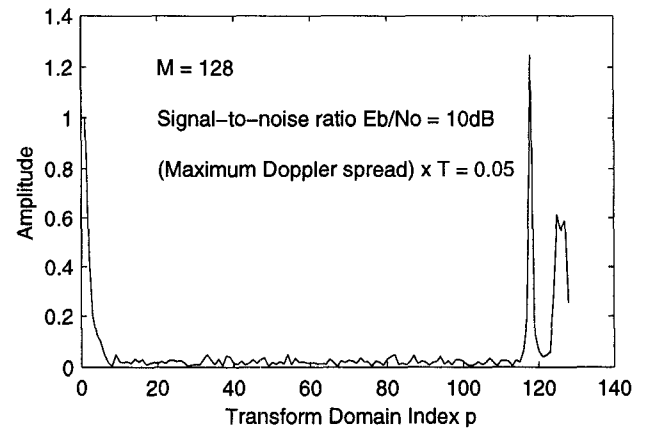


Fig. 3 An example of  $|\dot{G}_M(p)|$ .

The lowpass filtering can be realized simply by setting the samples in the "high frequency" region to zero, that is

$$\hat{G}_M(p) = \begin{cases} \dot{G}_M(p), & 0 \leq p \leq p_c, M - p_c \leq p \leq M-1, \\ 0, & \text{otherwise.} \end{cases} \quad (11)$$

where  $p_c$  is the "cutoff frequency" of the filter in the transform domain. After the filtering, the noise component is reduced to  $2p_c/M$  of its original value.

#### B. Interpolation Approach

According to the slow-variation property, the channel transfer function can be viewed as the sum of several sinusoidal functions with respect to  $k$ . However, the number and the "frequencies" of

the sinusoids vary due to the changing in the mobile radio channel. To avoid the model mismatch problem, we do not transform  $\hat{G}_M(p)$  back to frequency domain and then perform interpolation. Instead, a high-resolution interpolation approach based on zero-padding and DFT/IDFT [10] is used.

First, the  $M$ -sample transform-domain sequence  $\hat{G}_M(p)$  is extended to an  $N$ -sample sequence  $\hat{G}_N(q)$  by padding with  $N-M$  zero samples at the "high frequency" region around  $p=M/2$ , resulting in

$$\hat{G}_N(q) = \begin{cases} \hat{G}_M(q), & 0 \leq q \leq p_c, \\ 0, & p_c < q < N - p_c, \\ \hat{G}_M(q - N + M), & N - p_c \leq q \leq N - 1. \end{cases} \quad (12)$$

This  $N$ -sample sequence  $\hat{G}_N(q)$ , in its physical meaning, is the Fourier transform of the desired estimate of the channel transfer function. By performing an  $N$ -point IDFT, the estimated transfer function, with lower noise levels at all subcarriers, is obtained as

$$\hat{H}(k) = a \cdot \sum_{q=0}^{N-1} \hat{G}_N(q) \exp\left(-j \frac{2\pi}{N} qk\right), \quad 0 \leq k \leq N-1. \quad (13)$$

Noticing that the  $M$ -point FFT and  $N$ -point IFFT are performed between frequency domain and transform domain, a constant  $a$  is needed for calibration.

This approach provides an accurate estimate because 1) no assumption on the signal variation speed is made, thus there is no model-mismatch problem; 2) all the desired "frequency" components in the transform domain are reserved and transformed to frequency domain. In addition, this approach is easy to realize.

### C. Dynamic Selection of Cutoff Frequency

The "cutoff frequency"  $p_c$  of the transform-domain lowpass filter is an important parameter which affects the accuracy of the channel estimation, as shown in (11). Its value changes continuously due to the variation of the mobile radio channel. Therefore, an approach is needed to select  $p_c$  dynamically by tracking the received signals.

Let us observe the energy distribution in the transform domain with respect to different values of  $p$ . As shown in Fig. 3, most of the energy concentrates at the "low frequency" region where the desired components are located. Therefore,  $p_c$  is determined from the following relation

$$\left[ \sum_{p=0}^{p_c} |\bar{G}_M(p)|^2 + \sum_{p=M-p_c}^{M-1} |\bar{G}_M(p)|^2 \right] / \sum_{p=0}^{M-1} |\bar{G}_M(p)|^2 = R \quad (14)$$

where the numerator is the energy in "passband", the denominator the total energy,  $R$  a value within 0.9~0.95, and  $\bar{G}_M(p)$  the average of  $\hat{G}_M(p)$  of the present data block and 10 previous data blocks.

The simple algorithm for searching the cutoff frequency  $p_c$  starts from "low frequency" toward "high frequency" region. If

the cumulated energy in "low frequency" region is larger than  $R$  (certain percentage of the total energy), the corresponding  $p$  is taken as "cutoff frequency"  $p_c$ . This tracking procedure is shown in Fig. 2 as "filter designing".

## IV. SIMULATIONS

The simulations are concentrated on comparison between the linear-interpolation channel estimation method and proposed method. The bit error rate (BER) performances for both Rayleigh and Rician fading channels are examined.

A 16QAM-OFDM system with carrier frequency of 1 GHz and bandwidth of 2 MHz is used. The total number of all subcarriers is  $N=1024$ , and the number of uniformly-distributed pilot subcarriers is  $M=128$  (i.e.,  $L=8$ ). The channel models are Rayleigh and Rician as recommended by GSM Recommendations 05.05 [12], with parameters shown in Table 1. The guard intervals is assumed to be longer than the maximum delay spread of the channel.

Table 1. Parameters of channels.

Path Number	Rayleigh channel		Rician channel	
	Average Power (dB)	Delay ( $\mu$ s)	Average Power (dB)	Delay ( $\mu$ s)
1	-3.0	0.0	0.0	0.0 (line-of-sight)
2	0.0	0.2	-4.0	0.1
3	-2.0	0.5	-8.0	0.2
4	-6.0	1.6	-12.0	0.3
5	-8.0	2.3	-16.8	0.4
6	-10.0	5.0	-20.0	0.5

Simulations are carried out for different noise and ICI levels, and the results are shown in Fig. 4. The horizontal variable is chosen as the signal energy per bit-to-noise power density ratio ( $E_b/N_0$ ). It should be pointed out that  $N_0$  only reflects AWGN level. The ICI influence can be distinguished by choosing different Doppler spreads, or vehicle speeds.

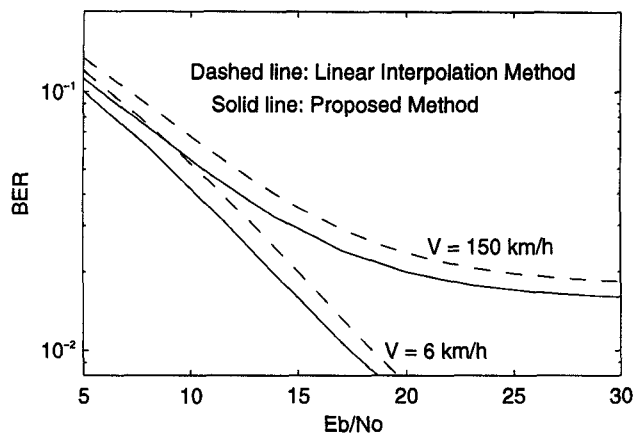
First, we choose a low vehicle speed  $V = 6$  km/h, corresponding to the maximum Doppler spread (normalized to the subcarrier separation  $1/T$ ) 0.003. Thus, the ICI can be neglected. The proposed method provides about 1 dB improvement in  $E_b/N_0$  for the same BER values.

Then let  $V = 150$  km/h, i.e., the normalized maximum Doppler spread is 0.075. Thus the system performances are strongly affected by ICI. The lowpass filtering in transform domain also reduces ICI, therefore more obvious improvement up to 3 dB can be obtained.

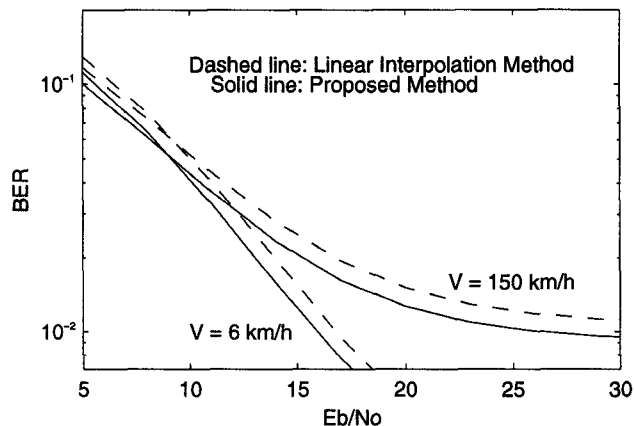
## V. CONCLUSIONS

A new method for estimating channel transfer function has been proposed in this paper.

This method has wider applicability than the MMSE or LMS methods [2,3,4] which are applicable for slowly time-varying channel only. It enables OFDM systems to work in mobile communications where the channels are time-variant and



(a) Rayleigh channel



(b) Rician channel

Fig. 4 Comparisons of BERs in different channels.

frequency-selective, because no knowledge of the channel transfer function for the previous OFDM data blocks is needed.

This method provides more accurate estimate of the channel transfer function, as compared to the normal interpolation method [5]. The lowpass filtering in the transform domain reduces AWGN and ICI significantly. The high-resolution interpolation approach suits well to the mathematical model of a mobile channel.

It is suggested to choose the number of pilot signals as a power of two so that the efficient radix-2 FFT/IFFT algorithms can be adopted, although other numbers can also be used.

When the Doppler spread is large, the proposed method still works better than other methods, but not as well as expected. The reason is the existence of the severe ICI caused by the Doppler frequency shifts. The severe ICI components in the received signals affect the accuracy of the signal correction even if the estimate of the channel transfer function is accurate. An ICI self-cancellation approach has been proposed to combat ICI in OFDM mobile systems [11] but the corresponding channel estimation method needs to be worked out. Besides, the prediction of the bandwidth of the transform-domain lowpass filter could be employed, so that  $p_c$  could follow the change of the channel characteristics better. Our future work will focus on these two topics.

## ACKNOWLEDGMENT

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