Simulating a disk inside a DM halo

Técnicas de simulacion numérica

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Introduction

The main objective of this project is to model a disk galaxy rotating within a Dark Matter (DM) halo. To do this, we start integrating the following equations of motion for each particle of the disk:

$$\frac{\mathrm{d}\vec{x}}{\mathrm{dt}} = \vec{v}$$

$$\frac{\mathrm{d}\vec{v}}{\mathrm{dt}} = -\frac{GM_{\mathrm{enclosed}}}{r^3}\vec{x}$$

where $\vec{x}$ is the position of the particle, $\vec{v}$ the velocity and r the distance between the particle and the center of the galaxy.

For the first part of the project, we are going to solve this system of equations using two different integration methods, Euler and 4th Order Runge-Kutta (RK4). Later, as RK4 returns a better result than Euler's method, we will use it for the second part.

The schemes of the integration method use to solve this system can be write as follow:

- **Euler's Method**

$$x_i = x_{i-1} + \mathrm{dt} \cdot f_x(v_{i-1})$$
$$v_i = v_{i-1} + \mathrm{dt} \cdot f_v(x_{i-1})$$

- **4th Order Runge-Kutta Method**

$$k_{1x} = f_x(v_{i-1}) \;\; ; \;\; k_{1v} = f_v(x_{i-1})$$

$$k_{2x} = f_x(v_{i-1} + \frac{\mathrm{dt}}{2} k_{1v}) \;\; ; \;\; k_{2v} = f_v(x_{i-1} + \frac{\mathrm{dt}}{2} k_{1x})$$

$$k_{3x} = f_x(v_{i-1} + \frac{\mathrm{dt}}{2} k_{2v}) \;\; ; \;\; k_{3v} = f_v(x_{i-1} + \frac{\mathrm{dt}}{2} k_{2x})$$

$$k_{4x} = f_x(v_{i-1} + \mathrm{dt} \cdot k_{3v}) \;\; ; \;\; k_{4v} = f_v(x_{i-1} + \mathrm{dt} \cdot k_{3x})$$

$$x_i = x_{i-1} + \frac{\mathrm{dt}}{6} \left(k_{1x} + 2k_{2x} + 2k_{3x} + k_{4x}\right)$$

$$v_i = v_{i-1} + \frac{\mathrm{dt}}{6} \left(k_{1v} + 2k_{2v} + 2k_{3v} + k_{4v}\right)$$

Where $f_{\vec{x}} = \vec{v}$ and $f_{\vec{v}} = -\frac{GM_T}{r^3}\vec{x}$.

For the first part of this work, we make the assumption that the mass of the Sun is much smaller that the DM halo, which of course is correct, but we take into account only the mass of the DM halo without any star mass.

Secondly, we are going to consider more stars particles for our simulation. However, there will not be any interaction between them, only the potential of the DM halo for each star.

Afterwards, we will simulate again the same amount of stars but this time with self-gravity, i.e, taking into account the gravitational interaction between themselves. To do this, we will need to introduce the concept of gravitational softening.

Finally, in order to develop a faster simulation, we will take into account only the gravitational interaction between the nearest neighbours for each particle through a tree algorithm.

We can found the code make for this work in the follow link:

Part 1: The Sun revolving around the Centre of the Galaxy

We start by modeling the orbit of the Sun around the Centre of the Milky Way. To do this, we are going to assume that the gravitational potential is provided only by the DM halo. To model the halo we are going to use the analytic Navarro-Frenk-White (NFW) profile.

Essentially, we need determine the mass of the halo within the radius of the Sun according to the NFW profile and assume that the whole mass is at the center of the galaxy. This technique is similar to a planetary system where all the relevant mass is at the center of the system.

The NFW profile is given by:

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}$$

To find the mass enclosed within any radius we just need to integrate the density profile:

$$M_{\text{enclosed}} = \int_0^{R_{\max}} 4\pi r^2 \rho(r) \mathrm{dr} = 4\pi\rho_0 R_s^3 \left[\ln\left(\frac{R_s + R_{\max}}{R_s}\right) - \frac{R_{\max}}{R_s + R_{\max}}\right]$$

where $R_{\max}$ is the radius of the star and R_s the scale radius.

Once we determine the mass of the DM halo within the radius of the Sun, we have all the parameters that we need to determine the orbit.

We set the parameters and the initial conditions as follow:

$$x_0 = 8\text{kpc} \ \ ; \ \ y_0, z_0 = 0$$

$$v_x, v_z = 0 \ \ ; \ \ v_y = 200\,\text{km/s}$$

$$R_s = 20\,\text{kpc} \ \ ; \ \ \rho_0 = 5932371\,\text{M}_\odot\text{kpc}^3$$

as we can see, we use the same units employed in Gadget. For this reason, $G = 4.302 \cdot 10^{-6}\text{kpc}(\text{km/s})^2\text{M}_\odot^{-1}$.

We are going to solve the system of equations for the Sun under this condition using the two different integration methods, Euler and RK4. Note that we have to solve the DM halo mass within the Sun radius for each time step.

In figure (1), we can see how the orbit of the Sun is not closed in any case. This is because we are only taking into account the DM halo mass. Although the halo corresponds to around 80% of the galaxy mass, in the inner part of the disk we can find a huge concentration of stars that have a gravitational influence on the orbit of the Sun. This means the star density in the inner part of the disk is not negligible and we have to consider the mass given by the nearest stars to the Sun to have a closed orbit.

In the following sections, we will only use the RK4 method, because it is more accurate than Euler one since it uses higher order calculations.

Part 2: A Disk revolving around the Centre of the Galaxy

Now, we are going to simulate the disk of the Milky Way for three different number of particles: 10, 100 and 1000 for which we have been provided with initial conditions.

First, we solve the system only taking into account the NFW DM potential, without gravitational interaction between stars. Because each star particle is located at a different radius, we need to calculate the mass of the DM halo enclosed within that radius for each star and time step.

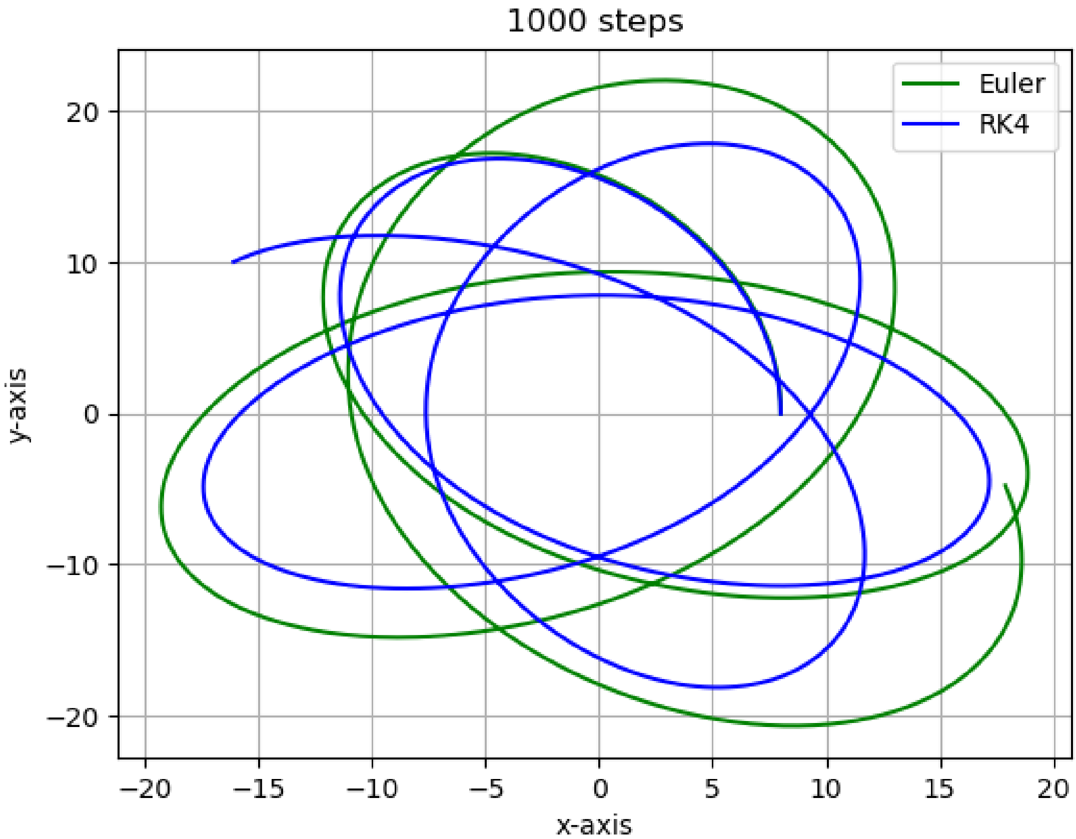


Figure (1) Orbit of the Sun solved for Euler and RK4 method.

In figure (2), we can see how the running time scales with the number of particles like $N^{1.16}$.

Part 3: A Disk revolving around the Centre of the Galaxy with self-gravity

Although DM represents more than 80% of the mass of the Milky Way, in the inner regions of the disk, the stars are actually gravitationally dominant. There is approximately equal mass of stars and DM within the radius of the Sun. The DM is only dominant in the outer parts. Due to the importance of the stellar mass in the inner regions, we should include the mass effect of the stars on the orbits of other stars.

In this section, we are going to add the gravitational interaction between each star. To do this, we only need to change the second equation in our system and add the gravitational interaction between particles as:

$$\frac{\mathrm{d}\vec{v}_j}{\mathrm{dt}} = -\frac{GM_{\text{enclosed}}}{r_j^3}\vec{x}_j - \sum_{i\neq j}^{N_{\text{stars}}} \frac{GM_i}{\left(\sqrt{r_{j,i}^2 + \epsilon^2}\right)^3}\vec{x}_{j,i}$$

where ϵ is the gravitational softening length. These equation reduce to the standard equations of motion when $\epsilon = 0$. The main reason for adding this parameters is to suppress the $1/r$ singularity in the Newtonian potential.

In figure (2), when we use all the gravitational interaction the running time scales with the number of particles like $N^{1.33}$. This method takes much more time than in the previous section, when gravitational interaction between stars was not considered. This result is reasonable, the time grows because we have increased the interaction for each time step.

To get a physically correct result, we need to choose a proper softening value. We set ϵ value of 1 kpc for 10 particles, 0.6 kpc for 100 and 0.3 kpc for 1000. The softening is needed to decrease with the number of particles because when we increase the density, the gravitational potential due to the particles grows. This can be done by taking a smaller value for the gravitational softening.[1].

OPTIONAL EXTRA. Self gravity with KD-tree method

In order to decrease the running scale of the simulation we can consider a weaker gravitational interaction between the particles and keep only the nearest neighbours to each star. To do this, we are going to implement a tree algorithm to find the neighbours. Then, we will follow a similar procedure to the one used in the last section. In this case, we only consider the gravitational interaction of the nearest neighbours and the DM halo enclosed within the radius of the stars.

In figure (2), we see how when we use a tree algorithm the running time decreases. These differences between the running time when we use all the particles for the gravity interactions or only the nearest neighbours becomes particularly important when we have a huge number of particles. Besides, we can note how if we increase the number of neighbours, the running time also increases.

[1]For a better result, in the EXTRA section, we take a softening value of 0.8 for 1000 particle.

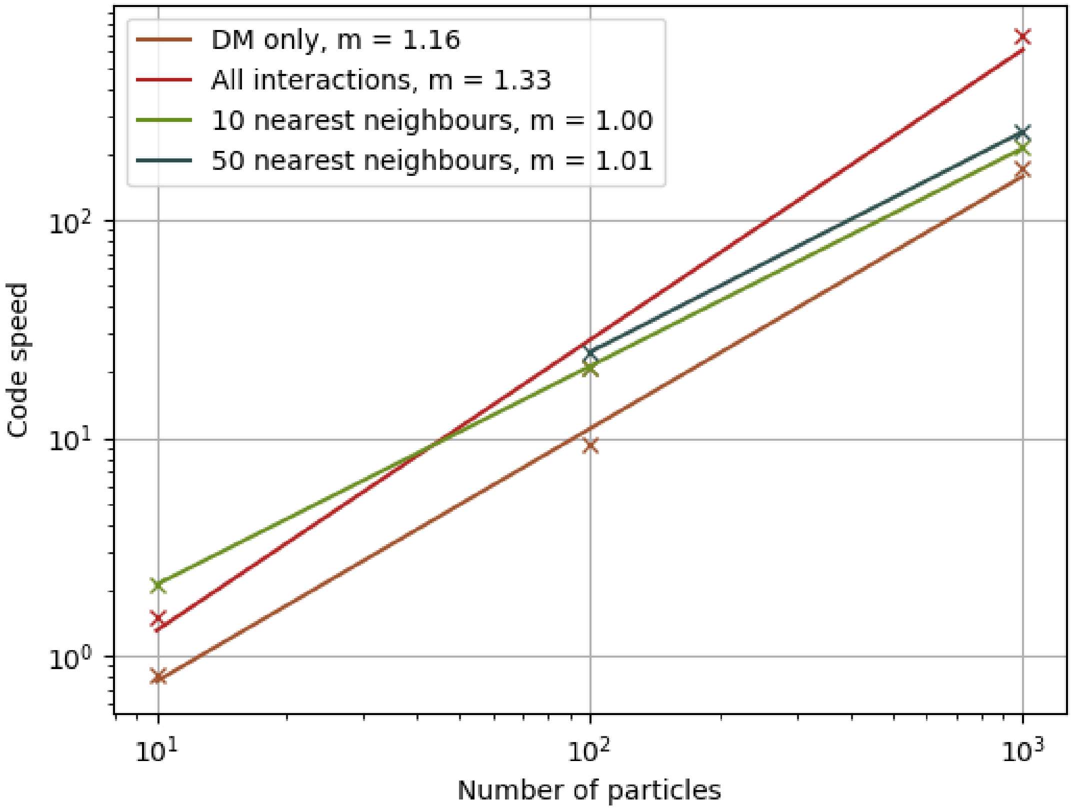
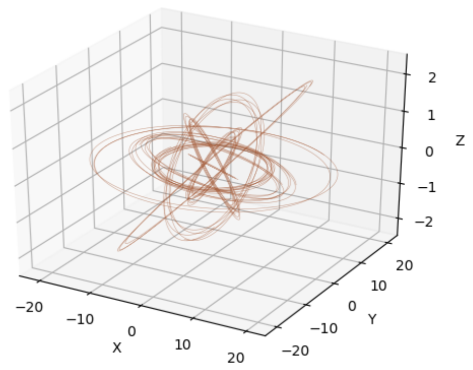


Figure (2) Running scale for each simulation using in this work.

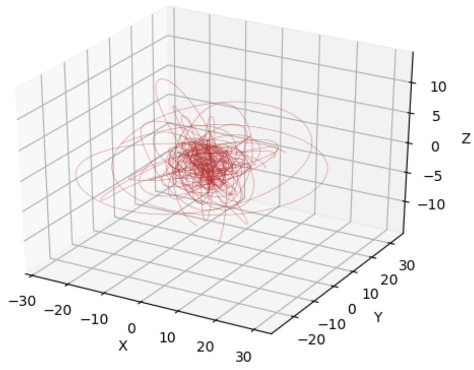
When we use only 10 neighbours for each particle, we can see in Figure (6) some particles escaping from the galaxy. This can be solved by increasing the number of neighbours. Since considering a greater number of neighbours increases the running scales we should be cautious with this decision.

ANNEX.

In this section we can see the result for each method. In Figure (3) the differences is evident when we consider all the interactions between the particles and we only the DM potential is considered. The orbits of the stars when we take into account only the DM halo potential are much distinguishable than the real case when we consider all the gravitational interaction of the particles.

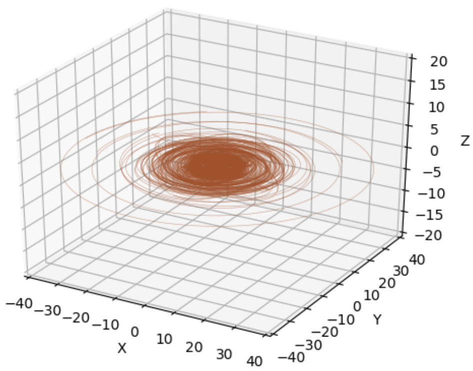


(a) Dark Matter gravitational potential only.

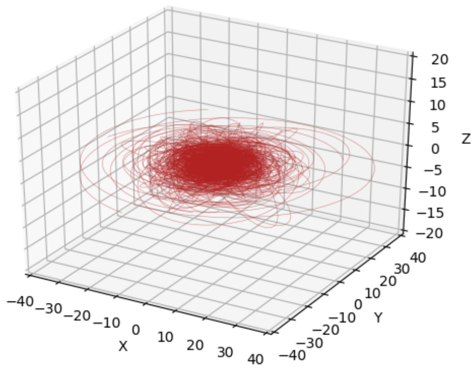


(b) DM potential + gravitational interaction between the particles.

Figure (3) A disk of 10 particles revolving around the Centre of the Galaxy. In the left hand we consider only the DM gravitational potential. In the right hand we consider all the interaction between the particles too with a value of 1.0 kpc for the gravitational softening length.

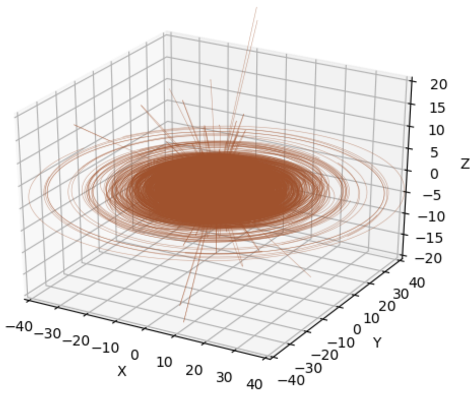


(a) Dark Matter gravitational potential only.

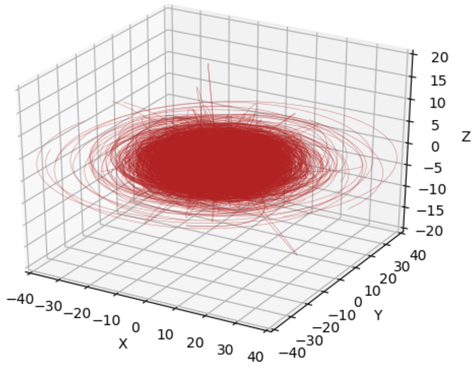


(b) DM potential + gravitational interaction between the particles.

Figure (4) A disk of 100 particles revolving around the Centre of the Galaxy. In the left hand we consider only the DM gravitational potential. In the right hand we consider all the interaction between the particles too with a value of 0.6 kpc for the gravitational softening length.

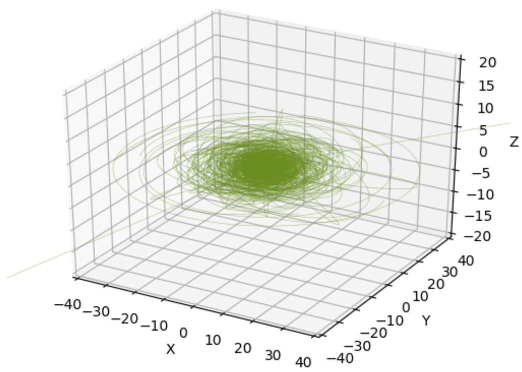


(a) Dark Matter gravitational potential only.

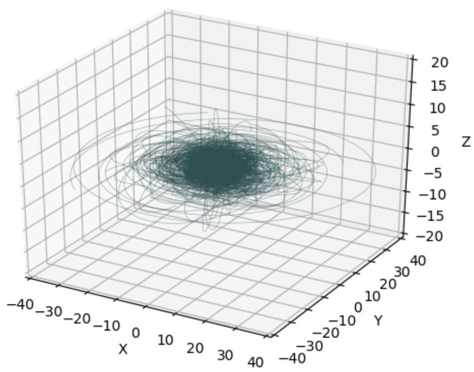


(b) DM potential + gravitational interaction between the particles.

Figure (5) A disk of 1000 particles revolving around the Centre of the Galaxy. In the left hand we consider only the DM gravitational potential. In the right hand we consider all the interaction between the particles too with a value of 0.3 kpc for the gravitational softening length.

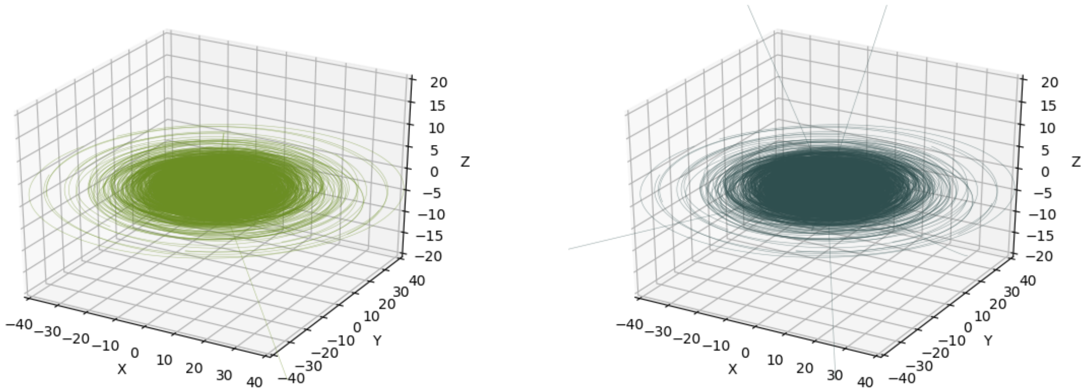


(a) 10 neighbours.



(b) 50 neighbours.

Figure (6) A disk of 100 particles revolving around the Centre of the Galaxy solved with a tree algorithm with a value of 0.6 kpc for the gravitational softening length. In the left hand we have the simulation for 10 nearest neighbours, in the right hand we have the simulation for 50 nearest neighbours.



(a) 10 neighbours.

(b) 50 neighbours.

Figure (7) A disk of 100 particles revolving around the Centre of the Galaxy solved with a tree algorithm with a value of 0.8 kpc for the gravitational softening length. In the left hand we have the simulation for 10 nearest neighbours, in the right hand we have the simulation for 50 nearest neighbours.