# Algorithms

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## $\mathsf{CHAPTER}\ 1$

Computational Geometry

#### 1.1 Convex Hull

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Convex Hull.hpp (1063 bytes, 36 lines)

```
1 #include<bits/stdc++.h>
   using namespace std;
   template<class T>struct ConvexHull{
       struct point{
           T x, y;
           point(T _x,T _y):
               x(_x),y(_y)
           point operator—(point a){
                return point(x-a.x,y-a.y);
11
            T operator*(point a){
                return x*a.y-y*a.x;
           int operator<(point a){</pre>
                return x==a.x?y<a.y:x<a.x;</pre>
            }
       };
       static int check(point a,point b,point c){
           return (a-c)*(b-c)<=0;
21
       static vector<vector<point> >run(vector<point>a){
            sort(a.begin(),a.end());
           vector<point>u,d;
           for(int i=0;i<a.size();u.push_back(a[i++]))</pre>
                while(u.size()>1&&check(a[i],u.back(),u[u.size()-2]))
                    u.pop back();
           for(int i=int(a.size()-1);i \ge 0;d.push back(a[i--]))
                while(d.size()>1&&check(a[i],d.back(),d[d.size()-2]))
                    d.pop back();
           vector<vector<point> >r;
31
            r.push_back(u);
            r.push back(d);
           return r;
       }
```

### 1.2 Delaunay Triangulation

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Delaunay Triangulation.hpp (4889 bytes, 159 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   template < class T > struct DelaunayTriangulation{
       const static double E;
       struct poi{
            T x,y;
            poi(T _x=0,T _y=0):
                x(_x),y(_y)
            poi operator-(poi b){
                return poi(x-b.x,y-b.y);
            int operator<(poi b)const{</pre>
14
                if(fabs(x-b.x)<E)</pre>
                    return y<b.y;</pre>
                return x<b.x;
            }
       };
       int n;
       vector<pair<poi,int> >pts;
       vector<vector<int> >egs;
       T det(poi a,poi b){
            return a.x*b.y-a.y*b.x;
24
       T dot(poi a,poi b){
            return a.x*b.x+a.y*b.y;
       int dir(poi a,poi b,poi c){
            T r=det(c-a,b-a);
            if(r < -E)
                return -1;
            return r>E?1:0;
```

```
int inc(poi a,poi b,poi c,poi d){
34
           a=a-d;
           b=b-d;
           c=c-d;
           T az=a.x*a.x+a.y*a.y,bz=b.x*b.x+b.y*b.y,cz=c.x*c.x+c.y*c.y;
           return a.x*b.y*cz+b.x*c.y*az+c.x*a.y*bz-a.x*bz*c.y-b.x*a.y*cz-c.x*
       b.y*az>E;
       int crs(poi a,poi b,poi c,poi d){
           return dir(a,b,c)*dir(a,b,d)==-1&dir(c,d,a)*dir(c,d,b)==-1;
44
       DelaunayTriangulation():
           n(0),pts(1){
       void add(T x,T y){
           poi a;
           a.x=x;
           a.y=y;
           pts.push_back(make_pair(a,++n));
       poi&pot(int a){
54
           return pts[a].first;
       void con(int a,int b){
           egs[a].push_back(b);
           egs[b].push_back(a);
       void dco(int a,int b){
           egs[a].erase(find(egs[a].begin(),egs[a].end(),b));
           egs[b].erase(find(egs[b].begin(),egs[b].end(),a));
       void dnc(int 1,int r){
64
           if(r==1)
               return;
           if(r==l+1){
               con(1,r);
               return;
           if(r==1+2){
               if(dir(pot(l),pot(l+1),pot(r)))
```

```
con(1,1+1), con(1+1,r), con(1,r);
74
                 else{
                     if(dot(pot(1+1)-pot(1),pot(r)-pot(1))<0)
                         con(l,l+1),con(l,r);
                     else if(dot(pot(1)-pot(1+1),pot(r)-pot(1+1))<0)
                         con(1,1+1),con(1+1,r);
                     else
                         con(l,r),con(l+1,r);}
                 return;
             }
            int m=(1+r)/2,pl=1,pr=r;
84
            dnc(1,m);
            dnc(m+1,r);
            for(int f=0;;f=0){
                 for(int i=0;i<egs[pl].size();++i){</pre>
                     int a=egs[pl][i],d=dir(pot(pl),pot(pr),pot(a));
                     if(d>0||(d==0&&dot(pot(pl)-pot(a),pot(pr)-pot(a))<0)){</pre>
                         pl=a;
                         f=1;
                         break;
                     }
94
                 }
                 for(int i=0;i<egs[pr].size();++i){</pre>
                     int a=egs[pr][i],d=dir(pot(pl),pot(pr),pot(a));
                     if(d>0||(d==0&&dot(pot(pl)-pot(a),pot(pr)-pot(a))<0)){
                         pr=a;
                         f=1;
                         break;
                     }
                 }
                 if(!f)
104
                     break;
             con(pl,pr);
            for(int pn=-1,wh=0;;pn=-1,wh=0){
                 for(int i=0;i<egs[pl].size();++i){</pre>
                     int a=egs[pl][i],d=dir(pot(pl),pot(pr),pot(a));
                     if(d<0\&\&(pn==-1||inc(pot(pl),pot(pr),pot(pn),pot(a))))
                         pn=a;
                 for(int i=0;i<egs[pr].size();++i){</pre>
```

```
int a=egs[pr][i],d=dir(pot(pl),pot(pr),pot(a));
114
                    if(d<0\&\&(pn==-1||inc(pot(pl),pot(pr),pot(pn),pot(a))))
                        pn=a,wh=1;
                if(pn==-1)
                    break;
                vector<int>ne;
                if(!wh){
                    for(int i=0;i<egs[pl].size();++i){</pre>
                        int a=egs[pl][i];
124
                        if(!crs(pot(pn),pot(pr),pot(pl),pot(a)))
                             ne.push back(a);
                        else
                             egs[a].erase(find(egs[a].begin(),egs[a].end(),pl));
                    egs[pl]=ne;
                    con(pr,pn);
                    pl=pn;
                }else{
                    for(int i=0;i<egs[pr].size();++i){</pre>
134
                        int a=egs[pr][i];
                        if(!crs(pot(pn),pot(p1),pot(pr),pot(a)))
                             ne.push_back(a);
                        else
                             egs[a].erase(find(egs[a].begin(),egs[a].end(),pr));
                    egs[pr]=ne;
                    con(pl,pn);
                    pr=pn;
                }
            }
144
        vector<vector<int> >run(){
            egs.resize(n+1);
            sort(pts.begin()+1,pts.end());
            dnc(1,n);
            vector<vector<int> >res(n+1);
            for(int u=1;u<=n;++u)
                for(int i=0;i<egs[u].size();++i){</pre>
                    int v=egs[u][i];
                    res[pts[u].second].push_back(pts[v].second);
154
```

```
}
    return res;
}
};
template<class T>const double DelaunayTriangulation<T>::E=1e-8;
```

## 1.3 Dynamic Convex Hull (Set)

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Dynamic Convex Hull (Set).hpp (2239 bytes, 77 lines)

```
1 #include<bits/stdc++.h>
   using namespace std;
   template<class T>struct DynamicConvexHull{
       struct point{
           Tx,y;
           point(T _x=0,T _y=0):
               x(_x),y(_y)
           point operator-(const point&a)const{
               point p(x-a.x,y-a.y);
               return p;
11
           T operator*(const point&a)const{
               return x*a.y-y*a.x;
           }
       };
       struct node{
           node**nxt;point p;
           node(node**_n,point _p):
               nxt(_n),p(_p){
21
           }
           node(const node&a):
               nxt(new node*(*a.nxt)),p(a.p){
           }
           ~node(){
               delete nxt;
           int operator<(const node&a)const{</pre>
```

```
if(ctp)
                   return p.x==a.p.x?p.y<a.p.y:p.x<a.p.x;</pre>
31
               point p1,p2;
               int f=1;
               if(nxt)
                   p1=*nxt?(*nxt)->p-p:point(0,-1),p2=a.p;
               else
                   f=0,p1=*a.nxt?(*a.nxt)->p-a.p:point(0,-1),p2=p;
               T x=p1*p2;
               return f?x<0:x>0;
           }
       };
       static int ctp;
41
       set<node>nds;
       typedef typename set<node>::iterator P;
       int check(P a,P b,P c){
           return (b->p-a->p)*(c->p-b->p)>=0;
       }
       void next(P a,P b){
           *(a->nxt)=(node*)&*b;
       void insert(T x,T y){
           ctp=1;
51
           node t(new node*(0),point(x,y));
           P it=nds.insert(t).first,itl1=it,itl2,itr1=it,itr2=it;
           if(it!=nds.begin())
               for(next(--itl1,it);itl1!=nds.begin()&&check(--(itl2=itl1),
       itl1, it);)
                   next(itl2,it),nds.erase(itl1),itl1=itl2;
           if(++(itr1=it)!=nds.end())
               next(it,itr1);
           if(itl1!=it&&itr1!=nds.end()&&check(itl1,it,itr1)){
               next(itl1,itr1);
61
               nds.erase(it);
               return;
           if(itr1!=nds.end())
               for(;++(itr2=itr1)!=nds.end()&&check(it,itr1,itr2);)
                   next(it,itr2),nds.erase(itr1),itr1=itr2;
       int size(){
```

```
return nds.size();
}
71    pair<T,T>query(T x,T y){
        ctp=0;
        node t=*nds.lower_bound(node(0,point(x,y)));
        return make_pair(t.p.x,t.p.y);
    }
};
template<class T>int DynamicConvexHull<T>::ctp=0;
```

## 1.4 Dynamic Convex Hull (Treap)

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Dynamic Convex Hull (Treap).hpp (9485 bytes, 327 lines)

```
#include<bits/stdc++.h>
   using namespace std;
3 template<class T>struct DynamicConvexHull{
       struct point{
           Tx,y;
           point(T _x,T _y):
               x(_x),y(_y)
           point operator-(const point&a)const{
               point p(x-a.x,y-a.y);
               return p;
           T operator*(const point&a)const{
13
               return x*a.y-y*a.x;
            int operator<(const point&a)const{</pre>
               return x==a.x?y<a.y:x<a.x;</pre>
            int operator==(const point&a)const{
               return x==a.x&y==a.y;
            }
       };
23
       struct hull{
           point*pt;
```

```
hull*ch[2],*nb[2];
           int sz,fx;
           hull(point* pt):
               pt(_pt),sz(1),fx(rand()*1.0/RAND_MAX*1e9){
               ch[0]=ch[1]=nb[0]=nb[1]=0;
            }
           T check(point p){
               return (nb[1]?*nb[1]->pt-*pt:point(0,-1))*p;
33
           void update(){
               sz=1;
               for(int i=0;i<2;++i)</pre>
                    if(ch[i])
                       sz+=ch[i]->sz;
            }
       };
       static int sz(hull*x){
           return x?x->sz:0;
43
       }
       static point&pt(hull*x){
           return*x->pt;
       }
       static struct memory{
           hull*ps,*pp,**ss,**sp;
           int pm,sm;
           vector<hull*>ns;
           memory():
               ps((hull*)malloc(sizeof(hull))),pp(ps),pm(1),ss((hull**)malloc(
       sizeof(hull*))),sp(ss),sm(1){
53
               ns.push_back(ps);
            }
           ~memory(){
               free(ss);
               for(int i=0;i<ns.size();++i)</pre>
                    free(ns[i]);
           hull*create(const hull&x){
               if(sp!=ss){
                    --sp;
63
                    **sp=x;
                    return*sp;
```

```
}
                 if(pp==ps+pm){
                     pp=ps=(hull*)malloc(sizeof(hull)*(pm<<=1));</pre>
                     ns.push_back(ps);
                 }
                 *pp=x;
                 return pp++;
73
            void destroy(hull*x){
                 if(sp==ss+sm){
                     hull**t=(hull**)malloc(sizeof(hull*)*sm<<1);</pre>
                     memcpy(t,ss,sm*sizeof(hull*));
                     free(ss);
                     sp=(ss=t)+sm;
                     sm<<=1;}
                 *(sp++)=x;
             }
        }me;
83
        struct array{
             hull**ps,**pp;
             int pm;
             array():
                 ps((hull**)malloc(sizeof(hull*))),pp(ps),pm(1){
             }
             ~array(){
                 free(ps);
             int size(){
                 return pp-ps;
93
             hull*operator[](int i){
                 return ps[i];
             }
             void push(hull*x){
                 if(pp==ps+pm){
                     hull**t=(hull**)malloc(sizeof(hull*)*pm<<1);</pre>
                     memcpy(t,ps,pm*sizeof(hull*));
                     free(ps);
103
                     pp=(ps=t)+pm;
                     pm<<=1;
                 }
```

```
*(pp++)=x;
             }
        };
        static hull*link(hull*x,hull*y,hull*lb,hull*rb,int d,array&ns){
             hull*r=me.create(*x);
             if(x==1b||x==rb){
                 r->nb[d]=y;
113
                 if(y)
                     y->nb[!d]=r;
             }else
                 r->ch[d]=link(r->ch[d],y,lb,rb,d,ns);
             r->update();
             ns.push(r);
             return r;
        static hull*merge(hull*x,hull*y,hull*lb,hull*rb,array&ns){
             if(!x)
123
                 return y;
             if(!y)
                 return x;
             int d=x->fx>y->fx;
             hull*r=me.create(d?*x:*y);
             r\rightarrow ch[d]=d?merge(r\rightarrow ch[1],y,lb,rb,ns):merge(x,y\rightarrow ch[0],lb,rb,ns);
             if(d&&x==lb||!d&&y==rb)
                 r\rightarrow ch[d]=link(r\rightarrow ch[d],r,lb,rb,!d,ns);
             r->update();
             ns.push(r);
             return r;
133
        static pair<hull*,hull*>split(hull*x,int k,array&ns){
             if(!x)
                 return make_pair((hull*)0,(hull*)0);
             int t=sz(x->ch[0])+1;
            hull*r=me.create(*x);
             ns.push(r);
             pair<hull*,hull*>s=split(x->ch[k>=t],k-t*(k>=t),ns);
             if(k>=t){
143
                 r->ch[1]=s.first;r->update();
                 return make pair(r,s.second);
             }else{
                 r->ch[0]=s.second;r->update();
```

```
return make pair(s.first,r);
            }
        }
        static void turn(hull*&x,int d,int&k){
            k+=(sz((x=x->ch[d])->ch[!d])+1)*(2*d-1);
        static pair<T,T>range(hull*x){
153
            hull*l=x,*r=x;
            while(1->ch[0])
                l=1->ch[0];
            while(r->ch[1])
                r=r->ch[1];
            return make_pair(pt(1).x,pt(r).x);
        static hull*merge(hull*x,hull*y,array&ns){
            int kp=sz(x->ch[0])+1,kq=sz(y->ch[0])+1,pd[2],qd[2];
            pair<T,T>pr=range(x),qr=range(y);
163
            int pf=1;
            hull*p=x,*q=y;
            if(pr.second==qr.first&&pr.first==pr.second&&p->ch[pf=0])
                turn(p,0,kp);
            for(point pq=pt(q)-pt(p);;pq=pt(q)-pt(p)){
                pd[0]=(p->nb[0]&&(pt(p->nb[0])-pt(p))*pq<=0)*pf;
                qd[1]=(q->nb[1]&&(pt(q->nb[1])-pt(q))*pq<=0);
                pd[1]=(p->nb[1]&&(pt(p->nb[1])-pt(p))*pq<0)*pf;
                qd[0]=(q->nb[0]&&(pt(q->nb[0])-pt(q))*pq<0);
173
                if(!(pd[0]+pd[1]+qd[0]+qd[1])){
                    hull*l=split(x,kp,ns).first,*r=split(y,kq-1,ns).second,*lb=
        1,*rb=r;
                   while(lb->ch[1])
                        lb=lb->ch[1];
                   while(rb->ch[0])
                        rb=rb->ch[0];
                    return merge(1,r,lb,rb,ns);
                }
                if(!(pd[0]+pd[1]))
                    turn(q,qd[1],kq);
183
                if(!(qd[0]+qd[1]))
                    turn(p,pd[1],kp);
                if(pd[0]&&qd[1])
                    turn(p,0,kp),turn(q,1,kq);
```

```
if(pd[1]&&qd[1])
                    turn(q,1,kq);
                if(pd[0]&&qd[0])turn(p,0,kp);
                if(pd[1]&&qd[0]){
                    point vp=pt(p->nb[1])-pt(p), vq=pt(q->nb[0])-pt(q);
                    if(vp.x==0\&vq.x==0)
193
                        turn(p,1,kp),turn(q,0,kq);
                    else if(vp.x==0)
                        turn(p,1,kp);
                    else if(vq.x==0)
                        turn(q,0,kq);
                    else{
                        long double m=pr.second,pb=vp.y*(m-pt(p).x),qb=vq.y*(m-
        pt(q).x);
                        pb=pb/vp.x+pt(p).y;
                        qb=qb/vq.x+pt(q).y;
                        if(qb>pb+1e-8)
203
                            turn(q,0,kq);
                        else if(pb>qb+1e-8)
                            turn(p,1,kp);
                        else if(pt(q->nb[0]).x+pt(p->nb[1]).x<2*m)
                            turn(q,0,kq);
                        else
                            turn(p,1,kp);
                    }
                }
            }
213
        hull*query(hull*x,point p){
            for(hull*y=0;;){
                T d=x->check(p);
                if(d>0)
                    y=x,x=x->ch[0];
                else if(d<0)</pre>
                    x=x->ch[1];
                else
                    y=x;
                if(!d||!x)
223
                    return y;
            }
        }
```

```
struct treap{
            int fx,ct,sz;
            point pt;
            treap*ch[2];
            struct hull*ip,*hu;
            array ns;
233
            treap(point _pt):
                fx(rand()*1.0/RAND MAX*1e9),ct(1),sz(1),pt( pt),ip(me.create(
        hull(&pt))),hu(ip){
                ch[0]=ch[1]=0;
            ~treap(){
                for(hull**i=ns.ps;i!=ns.pp;++i)
                    me.destroy(*i);
                me.destroy(ip);
            }
            void update(){
                for(hull**i=ns.ps;i!=ns.pp;++i)
243
                    me.destroy(*i);
                ns.pp=ns.ps;
                sz=1;
                hu=ip;
                if(ch[0])
                    hu=merge(ch[0]->hu,hu,ns),sz+=ch[0]->sz;
                if(ch[1])
                    hu=merge(hu,ch[1]->hu,ns),sz+=ch[1]->sz;
            }
        }*root;
253
        void rotate(treap*&x,int d){
            treap*y=x->ch[d];
            x->ch[d]=y->ch[!d];
            y \rightarrow ch[!d]=x;
            x=y;
        int insert(treap*&x,point p){
            if(!x)
                x=new treap(p);
            else if(p==x->pt){
263
                ++x->ct;
                return 0;
            }else{
```

```
int d=x->pt<p;</pre>
                if(!insert(x->ch[d],p))
                    return 0;
                if(x->ch[d]->fx>x->fx)
                    rotate(x,d),x->ch[!d]->update();
                x->update();
273
            }
            return 1;
        int erase(treap*&x,point p){
            if(p==x->pt){
                if(x->ct>1){
                    --x->ct;
                    return 0;
                }
                treap*y=x;
                if(!x->ch[0])
283
                    x=x->ch[1],delete y;
                else if(!x->ch[1])
                    x=x->ch[0],delete y;
                else{
                    int d=x->ch[0]->fx<x->ch[1]->fx;
                    rotate(x,d);
                    erase(x->ch[!d],p);
                    x->update();
                }
293
                return 1;
            if(erase(x->ch[x->pt<p],p)){</pre>
                x->update();
                return 1;
            }else{
                --x->sz;
                return 0;
            }
        void clear(treap*x){
303
            if(x)
                clear(x->ch[0]),clear(x->ch[1]),delete x;
        DynamicConvexHull():
```

1.5. GEOMETRY 2D 21

```
root(0){
        }
        ~DynamicConvexHull(){
            clear(root);
        int size(){
313
            return root?root->sz:0;
        void insert(T x,T y){
            insert(root,point(x,y));
        void erase(T x,T y){
            erase(root,point(x,y));
        pair<T,T>query(T x,T y){
            point r=pt(query(root->hu,point(x,y)));
323
            return make_pair(r.x,r.y);
        }
    };
    template<class T>typename DynamicConvexHull<T>::memory DynamicConvexHull<T</pre>
        >::me;
```

### 1.5 Geometry 2D

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Geometry 2D.hpp (5031 bytes, 159 lines)

```
13
               return a<b?-1:1;</pre>
           if(a<b-eps)</pre>
               return -1;
           if(a>b+eps)
               return 1;
           return 0;
       template<class T>struct Point{
           T x, y;
           Point(T _x=0,T _y=0):
23
               x(_x),y(_y)
           Point<T>&operator+=(const Point<T>&a){
               return*this=*this+a;
           }
           Point<T>&operator = (const Point<T>&a){
               return*this=*this-a;
           }
       };
33
       #define Vector Point
       template<class T>Point<T>operator+(const Point<T>&a,const Point<T>&b){
           return Point<T>(a.x+b.x,a.y+b.y);
       }
       template<class T>Point<T>operator-(const Point<T>&a,const Point<T>&b){
           return Point<T>(a.x-b.x,a.y-b.y);
       }
       template<class T>Point<T>operator*(T a,const Point<T>&b){
           return Point<T>(b.x*a,b.y*a);
       template<class T>Point<T>operator*(const Point<T>&a,T b){
43
           return b*a;
       }
       template<class T>Point<T>operator/(const Point<T>&a,T b){
           return Point<T>(a.x/b,a.y/b);
       template<class T>bool operator==(const Point<T>&a,const Point<T>&b){
           return !cmp(a.x,b.x)&&!cmp(a.y,b.y);
       template<class T>bool operator!=(const Point<T>&a,const Point<T>&b){
           return !(a==b);
53
```

1.5. GEOMETRY 2D 23

```
template<class T>bool operator<(const Point<T>&a,const Point<T>&b){
            int t=cmp(a.x,b.x);
            if(t)
                return t<0;
            return cmp(a.y,b.y)<0;</pre>
        template<class T>bool operator>(const Point<T>&a,const Point<T>&b){
            return b<a;
63
        template < class T > Point < T > NaP() {
            T t=numeric limits<T>::max();
            return Point<T>(t,t);
        template<class T>T det(const Point<T>&a,const Point<T>&b){
            return a.x*b.y-a.y*b.x;
        template<class T>T dot(const Point<T>&a,const Point<T>&b){
            return a.x*b.x+a.y*b.y;
73
        template < class T>T abs(const Point < T>&a){
            return sqrt(sqr(a.x)+sqr(a.y));
        template < class T>T dis(const Point < T>&a, const Point < T>&b){
            return abs(a-b);
        template<class T>istream&operator>>(istream&s,Point<T>&a){
            return s>>a.x>>a.y;
        template<class T>ostream&operator<<(ostream&s,const Point<T>&a){
83
            return s<<a.x<<" "<<a.y;</pre>
        template < class T > struct Segment;
        template<class T>struct Line{
            Point<T>u,v;
            Line(const Point<T>& u=Point<T>(),const Point<T>& v=Point<T>()):
                u(\underline{u}), v(\underline{v})
            Line(const Segment<T>&a):
93
                u(a.u),v(a.v){
            }
```

```
};
        template<class T>Point<T>nor(const Line<T>&a){
            Point<T>t=a.v-a.u;
             return Point<T>(t.y,-t.x);
        }
        template < class T > Point < T > dir(const Line < T > &a) {
             return a.v-a.u;
        }
103
        template < class T > int dir(const Line < T > a, const Point < T > b) {
             return cmp(det(b-a.u,a.v-a.u),T(0));
        template<class T>Point<T>operator&(const Line<T>&a,const Line<T>&b){
            T p=det(b.u-a.v,b.v-b.u),q=det(a.u-b.v,b.v-b.u);
            return (a.u*p+a.v*q)/(p+q);
        template<class T>struct Segment{
            Point<T>u,v;
            Segment(const Point<T>&_u=Point<T>(),const Point<T>&_v=Point<T>()):
113
                 u(u),v(v){
        };
        template < class T > Point < T > nor (const Segment < T > & a) {
            Point<T>t=a.v-a.u;
            return Point<T>(t.y,-t.x);
        template < class T > Point < T > dir(const Segment < T > & a) {
             return a.v-a.u;
123
        template < class T > int dir(const Segment < T > a, const Point < T > b) {
             return cmp(b-a.u,a.v-a.u);
        template<class T>Point<T>operator&(const Line<T>&a,const Segment<T>&b){
             if(dir(a,b.u)*dir(a,b.v)<=0)
                 return a&Line<T>(b);
            return NaP<T>();
        template<class T>Point<T>operator&(const Segment<T>&a,const Line<T>&b){
             return b&a;
133
        template<class T>pair<T,T>dis(const Segment<T>&a,const Point<T>&b){
            pair<T,T>d(dis(a.u,b),dis(a.v,b));
```

```
if(d.first>d.second)
                swap(d.first,d.second);
            Point<T>t=Line<T>(b,b+nor(a))&a;
            if(t!=NaP<T>())
                d.first=dis(t,b);
            return d;
143
        template<class T>pair<T,T>dis(const Point<T>&a,const Segment<T>&b){
            return dis(b,a);
        template<class T>struct Circle{
            Point<T>c;
            Tr;
            Circle(const Point<T>&_c=Point<T>(),T _r=0):
                c(_c),r(_r){
            }
        };
        template < class T>T abs(const Circle < T>&a){
153
            return pi*sqr(a.r);
        template<class T>bool col(const Point<T>&a,const Point<T>&b,const Point
        <T>&c){
            return !cmp(det(a-c,b-c),T(0));
        }
    }
```

#### 1.6 Half-Plane Intersection

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Half-Plane Intersection.hpp (1950 bytes, 70 lines)

```
1 #include<bits/stdc++.h>
  using namespace std;
namespace HalfPlaneIntersection{
    const double E=1e-8;
    struct pot{
       pot(double a=0,double b=0):
            x(a),y(b){
       }
```

```
double x,y;
       };
11
       double ag(pot p){
           return atan2(double(p.x),double(p.y));
       }
       pot operator+(pot p,pot q){
           return pot(p.x+q.x,p.y+q.y);
       }
       pot operator-(pot p,pot q){
           return pot(p.x-q.x,p.y-q.y);
       pot operator*(pot p,double q){
           return pot(p.x*q,p.y*q);
21
       pot operator/(pot p,double q){
           return pot(p.x/q,p.y/q);
       double det(pot p,pot q){
           return p.x*q.y-q.x*p.y;
       double dot(pot p,pot q){
           return p.x*q.x+p.y*q.y;
31
       struct lin{
           pot p,q;
           double a;
           lin(pot a,pot b):
               p(a),q(b),a(ag(b-a)){
           }
       };
       pot operator*(lin a,lin b){
           double a1=det(b.p-a.q,b.q-b.p);
           double a2=det(a.p-b.q,b.q-b.p);
41
           return (a.p*a1+a.q*a2)/(a1+a2);
       }
       bool cmp(lin a,lin b){
           if(fabs(a.a-b.a)>E)
               return a.a<b.a;</pre>
           else
               return det(a.q-b.p,b.q-b.p)<-E;</pre>
       }
```

```
bool left(lin a,lin b,lin c){
51
           pot t=a*b;
           return det(t-c.p,c.q-c.p)<-E;</pre>
       deque<lin>run(vector<lin>lns){
           deque<lin>ans;
           sort(lns.begin(),lns.end(),cmp);
           for(int i=0;i<lns.size();++i){</pre>
               while(ans.size()>1&&!left(ans.back(),ans[ans.size()-2],lns[i]))
                   ans.pop_back();
               while(ans.size()>1&&!left(ans[0],ans[1],lns[i]))
                   ans.pop front();
61
               if(ans.empty()||fabs(ans.back().a-lns[i].a)>E)
                   ans.push_back(lns[i]);}
           while(ans.size()>1&&!left(ans[ans.size()-1],ans[ans.size()-2],ans.
       front()))
               ans.pop_back();
           if(ans.size()<3)</pre>
               ans.clear();
           return ans;
       }
   }
```

## $\mathsf{CHAPTER}\ 2$

Data Structures

#### 2.1 Binary Heap

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Binary Heap.hpp (1629 bytes, 73 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   template<class T,class C>struct BinaryHeap{
       struct node{
           node(int _p,T _v):
               p(_p),v(_v){
           int p;
           T v;
10
       };
       vector<node*>a;
       BinaryHeap():
           a(1){}
       }
       ~BinaryHeap(){
           clear();
       void move(int i,int j){
            swap(a[i]->p,a[j]->p);
20
            swap(a[i],a[j]);
       int check(int i,int j){
           if(!j||j>=a.size()||a[i]->v==a[j]->v)
               return 0;
           return a[i]->v<a[j]->v?-1:1;
       int up(int i){
            if(check(i,i>>1)<0){
               move(i,i>>1);
30
               return i>>1;
            }else
               return 0;
       int down(int i){
           if(check(i,i<<1)<=0&&check(i,i<<1^1)<=0)</pre>
```

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```
return a.size();
            if(check(i<<1,i<<1^1)<=0){</pre>
                move(i,i<<1);
                return i<<1;
40
            }else{
                move(i,i<<1^1);
                return i<<1^1;
            }
        void maintain(int i){
            for(int j=up(i);j;i=j,j=up(i));
            for(int j=down(i);j<a.size();i=j,j=down(i));</pre>
        void clear(){
            for(int i=1;i<a.size();++i)</pre>
50
                delete a[i];
            a.resize(1);
        }
        node*push(T v){
            a.push_back(new node(a.size(),v));
            node*r=a.back();
            maintain(a.size()-1);
            return r;
        }
        T top(){
60
            return a[1]->v;
        }
        void pop(){
            move(1,a.size()-1);
            delete a.back();
            a.pop_back();
            maintain(1);
        }
        void modify(node*x,T v){
70
            x \rightarrow v = v;
            maintain(x->p);
        }
    };
```

### 2.2 Dynamic Sequence

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Dynamic Sequence.hpp (4119 bytes, 177 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   template<class T>struct DynamicSequence{
       struct node{
           node(T _i):
               i(_i),v(_i),s(1),r(0){
7
                   c[0]=c[1]=0;
                   static int g;
                   w=g=(214013*g+2531011);
           Ti,v;
           int s,r,w;
           node*c[2];
       }*rt,*s1,*sr;
       struct pool{
           node*ps,*pp,**ss,**sp;
17
           int pm,sm;
           vector<node*>ns;
           pool():
               ps((node*)malloc(sizeof(node))),pp(ps),pm(1),ss((node**)malloc(
       sizeof(node*))),sp(ss),sm(1){
                   ns.push_back(ps);
           ~pool(){
               free(ss);
               for(int i=0;i<ns.size();++i)</pre>
                   free(ns[i]);
27
           node*crt(T a){
               if(sp!=ss){
                    --sp;
                   **sp=node(a);
                   return*sp;
               if(pp==ps+pm){
```

```
pp=ps=(node*)malloc(sizeof(node)*(pm<<=1));</pre>
                     ns.push back(ps);
37
                }
                *pp=node(a);
                return pp++;
            }
            void des(node*x){
                if(sp==ss+sm){
                     node**t=(node**)malloc(sizeof(node*)*sm<<1);</pre>
                     memcpy(t,ss,sm*sizeof(node*));
                     free(ss);
                     sp=(ss=t)+sm;
47
                     sm<<=1;
                *(sp++)=x;
            }
        }me;
        node*bud(T*a,int l,int r){
            if(1>r)
                return 0;
            int m=l+r>>1;
            node*t=me.crt(a[m]);
57
            t->c[0]=bud(a,1,m-1);
            t->c[1]=bud(a,m+1,r);
            pup(t);
            return t;
        }
        void pdw(node*x){
            for(int d=0;d<2&&(x->i>x->v,1);++d)
                if(x->c[d])
                    x->i>x->c[d]->i;
            *x->i;
            *x->v;
67
            if(x->r){
                -x->i;
                for(int d=0;d<2;++d)</pre>
                     if(x->c[d])
                         x\rightarrow c[d]\rightarrow r^=1;
                swap(x->c[0],x->c[1]);
                x->r=0;
            }
```

```
77
          void pup(node*x){
               x->i=x->v;
               x->s=1;
               for(int d=0;d<2;++d)</pre>
                    if(x->c[d])
                         pdw(x\rightarrow c[d]), x\rightarrow s+=x\rightarrow c[d]\rightarrow s, x\rightarrow i=d?x\rightarrow i+x\rightarrow c[d]\rightarrow i:x\rightarrow
          c[d] \rightarrow i+x \rightarrow i;
          void jon(node*x){
               rt=jon(jon(sl,x),sr);
87
          node*jon(node*x,node*y){
               if(!x)
                    return y;
               if(!y)
                    return x;
               pdw(x);
               pdw(y);
               if(x->w<y->w){
                    x\rightarrow c[1]=jon(x\rightarrow c[1],y);
                    pup(x);
 97
                    return x;
               }else{
                    y->c[0]=jon(x,y->c[0]);
                    pup(y);
                    return y;
               }
          }
          node*spt(int l,int r){
               spt(rt,l-1);
               node*t=s1;
               spt(sr,r-l+1);
107
               swap(sl,t);
               return t;
          void spt(node*x,int p){
               if(!x){
                    sl=sr=0;
                    return;
               }
```

```
pdw(x);
             int t=x->c[0]?x->c[0]->s:0;
117
             if(t<p)</pre>
                 spt(x->c[1],p-t-1),x->c[1]=sl,sl=x;
             else
                 spt(x->c[0],p),x->c[0]=sr,sr=x;
            pup(x);
        }
        void clr(node*x){
             if(x)
                 clr(x\rightarrow c[0]), clr(x\rightarrow c[1]), me.des(x);
127
        DynamicSequence(T*a=0,int n=0){
             rt=bud(a,1,n);
        ~DynamicSequence(){
             clr(rt);
        }
        void clear(){
             clr(rt);
             rt=0;
137
        void insert(T a,int p){
             insert(&a-1,1,p);
        void insert(T*a,int n,int p){
             spt(p+1,p);
             jon(bud(a,1,n));
        void erase(int p){
             erase(p,p);
147
        void erase(int l,int r){
             clr(spt(l,r));
             jon(0);
        T query(int p){
             return query(p,p);
        T query(int 1,int r){
             node*t=spt(1,r);
```

```
157
            T i=t->i;
            jon(t);
            return i;
        void modify(T a,int 1){
            modify(a,1,1);
        void modify(T a,int l,int r){
            node*t=spt(1,r);
            a>t->i;
167
            jon(t);
        void reverse(int l,int r){
            node*t=spt(1,r);
            t->r=1;
            jon(t);
        int length(){
            return rt?rt->s:0;
        }
177
    };
```

## 2.3 Fenwick Tree

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Fenwick Tree.hpp (529 bytes, 25 lines)

```
#include<bits/stdc++.h>
using namespace std;

template<class T>struct FenwickTree{
    FenwickTree(int _n):
        n(_n),l(log2(n)),a(n+1){
    }
    void add(int v,T d){
        for(;v<=n;v+=v&-v)
            a[v]+=d;
}
T sum(int v){
    T r=0;</pre>
```

2.4. K-D TREE 37

#### 2.4 K-D Tree

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### K-D Tree.hpp (2467 bytes, 80 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   struct KDTree{
       struct node{
 5
           node(int x0,int x1,int d):
               color(1),cover(0),dir(d){
                   ch[0]=ch[1]=0;
                   x[0]=mi[0]=mx[0]=x0;
                   x[1]=mi[1]=mx[1]=x1;
           }
           node*ch[2];
           int x[2],mi[2],mx[2],color,cover,dir;
       }*root;
       KDTree(pair<int,int>*a,int n){
15
           root=build(a,1,n,0);
       static int direct;
       static int cmp(pair<int,int>a,pair<int,int>b){
           if(direct)
               return make_pair(a.second,a.first)<make_pair(b.second,b.first);</pre>
```

```
return a<b;
       }
       node*build(pair<int,int>*a,int l,int r,int d){
            int m=(r+1)/2;
25
            direct=d;
            nth element(a+l,a+m,a+r+1,cmp);
            node*p=new node((a+m)->first,(a+m)->second,d);
            if(1!=m)
                p\rightarrow ch[0]=build(a,l,m-1,!d);
            if(r!=m)
                p->ch[1]=build(a,m+1,r,!d);
            for(int i=0;i<2;++i)</pre>
                for(int j=0;j<2;++j)</pre>
                    if(p->ch[j]){
35
                        p->mi[i]=min(p->mi[i],p->ch[j]->mi[i]);
                        p->mx[i]=max(p->mx[i],p->ch[j]->mx[i]);
                    }
            return p;
       void down(node*a){
            if(a->cover){
                for(int i=0;i<2;++i)</pre>
                    if(a->ch[i])
                        a->ch[i]->cover=a->cover;
45
                a->color=a->cover;
                a->cover=0;
            }
       void modify(node*a,int mi0,int mx0,int mi1,int mx1,int c){
            if(mi0>a->mx[0]||mx0<a->mi[0]||mi1>a->mx[1]||mx1<a->mi[1])
            if(mi0<=a->mi[0]&&mx0>=a->mx[0]&&mi1<=a->mi[1]&&mx1>=a->mx[1]){
                a->cover=c;
                return;
55
            }
            down(a);
            if(mi0 <= a - x[0] \& mx0 >= a - x[0] \& mi1 <= a - x[1] \& mx1 >= a - x[1])
                a->color=c;
            for(int i=0;i<2;++i)</pre>
                if(a->ch[i])
                    modify(a->ch[i],mi0,mx0,mi1,mx1,c);
```

2.5. LINK-CUT TREE 39

```
void modify(int mi0,int mx0,int mi1,int mx1,int c){
            modify(root,mi0,mx0,mi1,mx1,c);
65
       int query(node*a,int x0,int x1){
            down(a);
            if(x0==a->x[0]&&x1==a->x[1])
                return a->color;
            direct=a->dir;
            if(cmp(make_pair(x0,x1),make_pair(a->x[0],a->x[1])))
                return query(a->ch[0],x0,x1);
            else
                return query(a \rightarrow ch[1], x0, x1);
75
       int query(int x0,int x1){
            return query(root,x0,x1);
       }
    };
    int KDTree::direct=0;
```

### 2.5 Link-Cut Tree

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Link-Cut Tree.hpp (5518 bytes, 215 lines)

10

```
~LinkCutTree(){
             delete ptrs;
        int direct(node*x){
            if(!x->pr)
20
                 return 2;
             if(x==x->pr->ch[0])
                 return 0;
             if(x==x->pr->ch[1])
                 return 1;
             return 2;
        void down(node*x){
             if(x->rev){
                 x->ifo.reverse();
                 swap(x->ch[0],x->ch[1]);
30
                 for(int i=0;i<2;++i)</pre>
                     if(x->ch[i])
                          x->ch[i]->rev^=1;
                 x\rightarrow rev=0;
             }
            x-ifo.down(x-ch[0]?x-ch[0]->ifo:0,x-ch[1]?x-ch[1]->ifo:0);
        void up(node*x){
             for(int i=0;i<2;++i)</pre>
40
                 if(x->ch[i])
                     down(x->ch[i]);
            x\rightarrow ifo.up(x\rightarrow ch[0]?&x\rightarrow ch[0]\rightarrow ifo:0,x\rightarrow ch[1]?&x\rightarrow ch[1]\rightarrow ifo:0);
        void setchild(node*x,node*y,int d){
             x->ch[d]=y;
             if(y)
                 y->pr=x;
             up(x);
        }
        void rotate(node*x){
50
             node*y=x->pr,*z=y->pr;
             int d1=direct(x),d2=direct(y);
             setchild(y,x->ch[!d1],d1);
             setchild(x,y,!d1);
             if(d2<2)
```

2.5. LINK-CUT TREE 41

```
setchild(z,x,d2);
            else
                x->pr=z;
       void release(node*x){
60
            if(direct(x)<2)</pre>
                release(x->pr);
            down(x);
       }
       void splay(node*x){
            for(release(x);direct(x)<2;){</pre>
                node*y=x->pr;
                if(direct(y)==2)
                    rotate(x);
                else if(direct(x)==direct(y))
70
                    rotate(y),rotate(x);
                else
                    rotate(x),rotate(x);
            }
       }
       node*access(node*x){
            node*y=0;
            for(;x;y=x,x=x->pr){
                splay(x);
                setchild(x,y,1);
80
            return y;
       void evert(node*x){
            access(x);
            splay(x);
            x->rev=1;
       }
       void set(int x,T v){
           ptrs[x].ifo=v;
90
       int linked(int a,int b){
            access((ptrs+a));
            node*z=access((ptrs+b));
            return z==access((ptrs+a));
       }
```

```
void link(int a,int b){
            evert((ptrs+b));
            (ptrs+b)->pr=(ptrs+a);
100
        void cut(int a,int b){
            access((ptrs+b));
            node*z=access((ptrs+a));
            if(z==(ptrs+a))
                splay((ptrs+b)),(ptrs+b)->pr=0;
            else
                access((ptrs+b)),splay((ptrs+a)),(ptrs+a)->pr=0;
        int root(int a){
            access((ptrs+a));
110
            splay((ptrs+a));
            node*r=(ptrs+a);
            while(r->ch[1])
                r=r->ch[1];
            return r-ptrs;
        }
        void evert(int a){
            evert((ptrs+a));
120
        int lca(int a,int b){
            access((ptrs+a));
            return access((ptrs+b))-ptrs;
        }
        T query(int a){
            splay((ptrs+a));
            T p=(ptrs+a)->ifo;
            p.up(0,0);
            return p;
        T query(int a,int b){
130
            if((ptrs+a)==(ptrs+b))
                return query((ptrs+a));
            access((ptrs+a));
            node*c=access((ptrs+b));
            T p=c.ifo;
            if(c==(ptrs+b)){
                splay((ptrs+a));
```

2.5. LINK-CUT TREE 43

```
T q=(ptrs+a)->ifo;
                q.reverse();
140
                p.up(&q,0);
                return p;
            }else if(c==(ptrs+a))
                p.up(0,&(ptrs+a)->ch[1]->ifo);
            else{
                splay((ptrs+a));
                T q=(ptrs+a)->ifo;
                q.reverse();
                p.up(&q,&c->ch[1]->ifo);
            }
150
            return p;
        T equery(int a){
            return query(a);
        T equery(int a,int b){
            access((ptrs+a));
            node*c=access((ptrs+b));
            if(c==(ptrs+b)){
                splay((ptrs+a));
                T q=(ptrs+a)->ifo;
160
                q.reverse();
                return q;
            }else if(c==(ptrs+a))
                return (ptrs+a)->ch[1]->ifo;
            else{
                splay((ptrs+a));
                node*t=c->ch[1];
                while(t->ch[0])
                    t=t->ch[0];
170
                splay(t);
                if(t->ch[1])
                    down(t->ch[1]);
                T p=t->ifo,q=(ptrs+a)->ifo;
                q.reverse();
                p.up(&q,t->ch[1]?&t->ch[1]->ifo:0);
                return p;
            }
        }
```

```
template < class F > void modify(int a, F f){
            splay((ptrs+a));
180
            f(&(ptrs+a)->ifo);
            up((ptrs+a));
        }
        template<class F>void modify(int a,int b,F f){
            if((ptrs+a)==(ptrs+b)){
                splay((ptrs+a));
                f(0,&(ptrs+a)->ifo,0);
                up((ptrs+a));
                return;
190
            }
            access((ptrs+a));
            node*c=access((ptrs+b));
            if(c==(ptrs+b))
                splay((ptrs+a)),f(&(ptrs+a)->ifo,&(ptrs+b)->ifo,0);
            else if(c==a)
                f(0,&(ptrs+a)->ifo,&(ptrs+a)->ch[1]->ifo);
            else
                splay(a), f(&(ptrs+a)->ifo,&c->ifo,&c->ch[1]->ifo);
            up(c);
200
        template<class F>void emodify(int a,F f){
            modify(a,f);
        template < class F > void emodify(int a, int b, F f){
            access((ptrs+a));
            node*c=access((ptrs+b));
            if(c==(ptrs+b))
                splay((ptrs+a)),f(&(ptrs+a)->ifo,0);
            else if(c==a)
                f(0,&(ptrs+a)->ch[1]->ifo);
210
            else
                splay(a),f(&(ptrs+a)->ifo,&c->ch[1]->ifo);
            up(c);
        }
    };
```

2.6. MERGEABLE SET 45

# 2.6 Mergeable Set

### Description

Maintain sets of elements whose values are in a given range. Two sets can be merged efficiently. Range query is also supported.

#### Methods

template <class t,class="" u="">Mergea</class>	
Description	construct an object of MergeableSet, it is not a
	set, it maintains sets
Parameters	Description
T	type of range information, should support +,
	+ is applied when two range do not intersect
	or they represent the same leaf
U	type of values of elements
1	minimum value of elements
r	maximum value of elements
Time complexity	$\Theta(1)$
Space complexity	$\Theta(1)$
Return value	an object of MergeableSet
node*insert(node*x,T f,U v);	
Description	insert a element into a set
Parameters	Description
X	root of the set, use 0 to represent empty set
f	information of the element
v	value of the element
Time complexity	$\Theta(\log  r-l )$
Space complexity	$\Theta(\log  r-l )$
Return value	root of the new set
node*erase(node*x,U v);	
Description	erase the element with certain value
Parameters	Description
X	root of the set
v	value of the element
Time complexity	$\Theta(1)$ (amortized)
Space complexity	$\Theta(1)$ (amortized)
Return value	root of the new set

node*merge(node*x,node*y);		
Description	merge two sets	
Parameters	Description	
X	root of one set, use 0 to represent empty set	
у	root of another set, use 0 to represent empty	
m: 1 ·	set (1)	
Time complexity	$\Theta(1)$ (amortized)	
Space complexity	$\Theta(1)$ (amortized)	
Return value	root of the new set	
vector <t>query(node*x,U ql,U qr);</t>		
Description	do range query	
Parameters	Description	
X	root of one set, use 0 to represent empty set	
ql	start of the range, itself is included	
qr	end of the range, itself is included	
Time complexity	$\Theta(log r-l )$	
Space complexity	$\Theta(log r-l )$	
Return value	vector of information, if it is empty means no	
	information in that range other wise the result	
	is it's first element	
void destroy(node*x);		
Description	delete whole set	
Parameters	Description	
X	root of one set, use 0 to represent empty set	
Time complexity	$\Theta(1)$ (amortized)	
Space complexity	$\Theta(1)$ (amortized)	
Return value	none	

### References

Title	Author
线段树的合并——不为人知的实用技巧	黄嘉泰

### Code

Mergeable Set.hpp (2254 bytes, 91 lines)

#include<vector>
using namespace std;
template<class T,class U>struct MergeableSet{

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```
struct node{
              node(T _f):f(_f){
 5
                  c[0]=c[1]=0;
             Tf;
             node*c[2];
         };
         MergeableSet(U 1,U r):v1(1),vr(r){
         void update(node*x){
              if(x->c[0]&&x->c[1])
                  x \rightarrow f = x \rightarrow c[0] \rightarrow f + x \rightarrow c[1] \rightarrow f;
15
              else
                  x\rightarrow f=x\rightarrow c[0]?x\rightarrow c[0]\rightarrow f:x\rightarrow c[1]\rightarrow f;
         node*insert(node*x,T f,U v,U l=0,U r=0){
              if(!1&&!r)
                  1=v1,r=vr;
              if(l==r){
                  if(x)
                       x->f=x->f+f;
25
                  else
                       x=new node(f);
              }else{
                  U m=1+(r-1)/2;
                  int d=v>m;
                  node*y=insert(x?x->c[d]:0,f,v,d?m+1:1,d?r:m);
                  if(!x)
                       x=new node(y->f);
                  x->c[d]=y,update(x);
              }
35
              return x;
         node*erase(node*x,U v,U l=0,U r=0){
              if(!1&&!r)
                  1=v1,r=vr;
              if(l==r){
                  delete x;
                  return 0;
              U m=1+(r-1)/2;
```

```
int d=v>m;
45
             x \rightarrow c[d] = erase(x?x \rightarrow c[d]:0,v,d?m+1:1,d?r:m);
             if(!x->c[0]&&!x->c[1])
                 delete x;
                 return 0;
             }
             update(x);
             return x;
        node*merge(node*x,node*y,U 1=0,U r=0){
             if(!1&&!r)
55
                 l=v1,r=vr;
             if(!x||!y)
                 return x?x:y;
             if(l==r)
                 x->f=x->f+y->f;
             else{
                 U m=1+(r-1)/2;
                 x \rightarrow c[0] = merge(x \rightarrow c[0], y \rightarrow c[0], 1, m);
                 x \rightarrow c[1] = merge(x \rightarrow c[1], y \rightarrow c[1], m+1,r);
65
                 update(x);
             }
             return x;
        }
        vector<T>query(node*x,U ql,U qr,U l=0,U r=0){
             if(!1&&!r)
                 l=v1,r=vr;
             if(!x||ql>r||qr<1)
                 return vector<T>();
             if(ql<=1&&qr>=r)
                 return vector<T>(1,x->f);
75
             U m=1+(r-1)/2;
             vector<T>u=query(x->c[0],ql,qr,l,m),
                 v=query(x->c[1],ql,qr,m+1,r);
             if(v.size()&u.size())
                 u[0]=u[0]+v[0];
             return u.size()?u:v;
        void destroy(node*x){
             if(x)
                 destroy(x->c[0]),
85
```

2.7. PAIRING HEAP

# 2.7 Pairing Heap

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Pairing Heap.hpp (2226 bytes, 102 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   template<class T,class C>struct PairingHeap{
        PairingHeap():
            root(0), siz(0){
        ~PairingHeap(){
            clear(root);
 9
        struct node{
            node(const T&_val):
                val(_val),ch(0),br(0),pr(0){
            T val;
            node*ch,*br,*pr;
        }*root;
        int siz;
        void merge(node*&x,node*y){
19
            if(!x)
                x=y;
            else if(y){
                if(C()(y->val,x->val))
                    swap(x,y);
                y->br=x->ch;
                if(x->ch)
                    x->ch->pr=y;
                y \rightarrow pr = x;
```

```
x->ch=y;
            }
29
        void cut(node*&x,node*y){
            if(x==y)
                x=0;
            else{
                if(y==y->pr->ch)
                    y-pr-ch=y-br;
                else
                    y->pr->br=y->br;
39
                if(y->br)
                    y->br->pr=y->pr;
                y->pr=y->br=0;
            }
        }
        node*split(node*x){
            vector<node*>t;
           for(node*i=x->ch;i;i=i->br)
                t.push_back(i);
           x->ch=0;
49
            node*r=0;
            for(int i=0;i<t.size();++i)</pre>
                t[i]->pr=t[i]->br=0;
            for(int i=0;i+1<t.size();i+=2)</pre>
                merge(t[i],t[i+1]);
            for(int i=0;i<t.size();i+=2)</pre>
                merge(r,t[i]);
            return r;
       void clear(node*x){
59
            if(x){
                clear(x->ch);
                clear(x->br);
                delete x;
            }
        }
        void clear(){
            clear(root);
            root=0;
            siz=0;
```

```
69
       }
       node*push(T a){
           node*r=new node(a);
           merge(root,r);
           ++siz;
           return r;
       void erase(node*x){
           cut(root,x);
           merge(root,split(x));
79
           --siz;
       T top(){
           return root->val;
       void pop(){
           erase(root);
       void merge(PairingHeap<T,C>&a){
           merge(root,a.root);
89
           a.root=0;
           siz+=a.siz;
           a.siz=0;
       }
       void modify(node*x,T v){
           if(C()(x->val,v))
               x->val=v,merge(root,split(x));
           else
               x->val=v,cut(root,x),merge(root,x);
99
       int size(){
           return siz;
       }
    };
```

# 2.8 Red-Black Tree

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Red-Black Tree.hpp (7432 bytes, 307 lines)

```
#include<bits/stdc++.h>
    using namespace std;
    template<class T,class C>struct RedBlackTree{
         struct node{
               node(T _v,node*1,node*r,node*_p,int _b,int _s):
                    v(_v),p(_p),b(_b),s(_s)
                    c[0]=1;
                    c[1]=r;
 8
               }
               T v;
               node*c[2],*p;
               int b,s;
         }*root,*nil;
         void clear(node*x){
               if(x!=nil){
                    clear(x \rightarrow c[0]);
                    clear(x->c[1]);
                    delete x;
18
               }
         void rotate(node*x,int d){
               node*y=x->c[!d];
              x\rightarrow c[!d]=y\rightarrow c[d];
               if(y->c[d]!=nil)
                    y \rightarrow c[d] \rightarrow p = x;
               y \rightarrow p = x \rightarrow p;
               if(x->p==nil)
                    root=y;
28
               else
                    x \rightarrow p \rightarrow c[x!=x \rightarrow p \rightarrow c[0]]=y;
              y->c[d]=x;
              x->p=y;
               y \rightarrow s = x \rightarrow s;
               x \rightarrow s = x \rightarrow c[0] \rightarrow s + x \rightarrow c[1] \rightarrow s + 1;
         void insert fixup(node*z){
               while(!z->p->b){
                    int d=z->p==z->p->c[0];
38
                    node*y=z->p->c[d];
                    if(!y->b)
```

```
z \rightarrow p \rightarrow b=1, y \rightarrow b=1, (z=z \rightarrow p \rightarrow p) \rightarrow b=0;
                    else{
                         if(z==z->p->c[d])
                              rotate(z=z->p,!d);
                         z\rightarrow p\rightarrow b=1;
                         z\rightarrow p\rightarrow p\rightarrow b=0;
                         rotate(z->p->p,d);
48
                    }
               }
               root->b=1;
         void erase(node*z){
               node*y;
               for(y=z;y!=nil;y=y->p)
                    --y->s;
               if(z\rightarrow c[0]==nil||z\rightarrow c[1]==nil)
                    y=z;
               else{
58
                    for(y=z->c[1];y->c[0]!=nil;)
                        y=y->c[0];
                    z\rightarrow v=y\rightarrow v;
                    y=z->c[1];
                    while(y \rightarrow c[0]!=nil)
                         --y->s,y=y->c[0];
               }
               node*x=y->c[y->c[0]==nil];
               x \rightarrow p = y \rightarrow p;
68
               if(y->p==nil)
                    root=x;
               else
                    y->p->c[y!=y->p->c[0]]=x;
               if(y->b)
                    erase_fixup(x);
               delete y;
         }
         void erase_fixup(node*x){
               while(x!=root&&x->b){
                    int d=x==x-p-c[0];
78
                    node*w=x-p-c[d];
                    if(!w->b){
                         w->b=1;
```

```
x->p->b=0;
                        rotate(x->p,!d);
                       w=x-p-c[d];
                   if(w->c[0]->b\&w->c[1]->b)
                       w->b=0, x=x->p;
 88
                   else{
                        if(w->c[d]->b)
                            w \to c[!d] \to b=1, w \to b=0, rotate(w,d), w=x \to p \to c[d];
                       w->b=x->p->b;
                        x->p->b=1;
                       w \rightarrow c[d] \rightarrow b=1;
                        rotate(x->p,!d);
                        x=root;
                   }
              }
 98
              x->b=1;
          node*clone(node*x,node*y){
              if(x.size==0)
                   return nil;
              node*z=new node(*x);
              z\rightarrow c[0]=clone(x\rightarrow c[0],z);
              z\rightarrow c[1]=clone(x\rightarrow c[1],z);
              z \rightarrow p = y;
              return z;
108
          }
          node*precursor(node*x){
              if(x->c[0]->count){
                   for(x=x\rightarrow c[0];x\rightarrow c[1]\rightarrow count;)
                        x=x->c[1];
                   return x;
              }else{
                   node*y=x->p;
                   while(y->count&&x==y->c[0])
                        x=y,y=y-p;
118
                   return y;
              }
          node*successor(node*x){
              if(x->c[1]->count){
```

```
for(x=x->c[1];x->c[0]->count;)
                    x=x->c[0];
                return x;
            }else{
                node*y=x->p;
128
                while(y->count&x==y->c[1])
                    x=y,y=y->p;
                return y;
            }
        }
        RedBlackTree(){
            root=nil=(node*)malloc(sizeof(node));
            nil->b=1;
            nil->s=0;
        RedBlackTree(const RedBlackTree&a){
138
            nil=new node(*a.nil);
            root=clone(a.root,nil);
        }
        ~RedBlackTree(){
            clear(root);
            free(nil);
        RedBlackTree&operator=(const RedBlackTree&a){
            clear(root);
148
            root=clone(a.root,nil);
            return*this;
        }
        node*begin(){
            node*z=root;
            while(z!=nil&&z->c[0]!=nil)
                z=z->c[0];
            return z;
        node*reverse_begin(){
            node*z=root;
158
            while(z!=nil&&z->c[1]!=nil)
                z=z->c[1];
            return z;
        node*end(){
```

```
return nil;
         node*reverse_end(){
             return nil;
168
         }
         void clear(){
             clear(root);
             root=nil;
         void insert(T a){
             node*y=nil,*x=root;
             while(x!=nil)
                  y=x,++x->s,x=x->c[C()(x->v,a)];
             node*z=new node(a,nil,nil,y,0,1);
178
             if(y==nil)
                  root=z;
             else
                 y \rightarrow c[C()(y \rightarrow v, z \rightarrow v)] = z;
             insert_fixup(z);
         }
         void erase(T a){
             node*z=root;
             for(;;)
                  if(C()(a,z->v))
                      z=z->c[0];
188
                  else if(C()(z\rightarrow v,a))
                      z=z->c[1];
                 else
                      break;
             erase(z);
         int count(T a){
             return count_less_equal(a)-count_less(a);
         int count_less(T a){
198
             int r=0;
             node*z=root;
             while(z!=nil)
                  if(C()(z\rightarrow v,a))
                      r+=z->c[0]->s+1,z=z->c[1];
                  else
```

```
z=z->c[0];
             return r;
         int count_less_equal(T a){
208
             int r=0;
             node*z=root;
             while(z!=nil){
                 if(!C()(a,z->v))
                     r+=z->c[0]->s+1,z=z->c[1];
                 else
                     z=z->c[0];
             return r;
218
         int count_greater(T a){
             int r=0;
             node*z=root;
             while(z!=nil)
                 if(C()(a,z->v))
                     r+=z->c[1]->s+1,z=z->c[0];
                 else
                     z=z->c[1];
             return r;
228
         }
         int count greater equal(T a){
             int r=0;
             node*z=root;
             while(z!=nil)
                 if(!C()(z->v,a))
                     r+=z->c[1]->s+1,z=z->c[0];
                 else
                     z=z->c[1];
             return r;
238
         node*nth element(int a){
             node*z=root;
             for(;;)
                 if(z\rightarrow c[0]\rightarrow s>=a)
                     z=z->c[0];
                 else if((z\rightarrow c[0]\rightarrow s+1)<a)
                     a=z->c[0]->s+1,z=z->c[1];
```

```
else
                     return z;
248
        node*precursor(T a){
             node*z=root,*r=nil;
            while(z!=nil)
                 if(C()(z\rightarrow v,a))
                     r=z,z=z->c[1];
                 else
                     z=z->c[0];
             return r;
        }
        node*successor(T a){
258
             node*z=root,*r=nil;
            while(z!=nil)
                 if(C()(a,z->v))
                     r=z,z=z->c[0];
                 else
                     z=z->c[1];
             return r;
        }
        node*find(T a){
             node*z=root,*r=nil;
268
            while(z!=nil)
                 if(C()(a,z->v))
                     z=z->c[0];
                 else if(C()(z\rightarrow v,a))
                     z=z->c[1];
                 else
                     break;
             return r;
        node*lower_bound(T a){
278
             node*z=root,*r=nil;
            while(z!=nil)
                 if(C()(z->v,a))
                     r=z,z=z->c[1];
                 else if(C()(a,z->v))
                     z=z->c[0];
                 else
                     r=z,z=z->c[0];
```

```
return r;
288
        node*upper bound(T a){
            return successor(a);
        pair<node*,node*> equal range(T a){
            return make_pair(lower_bound(a),upper_bound(a));
        int size(){
            return root->s;
298
        int empty(){
            return !root->s;
        T front(){
            return*begin();
        T back(){
            return*reverse begin();
        }
    };
```

# 2.9 Self-Adjusting Top Tree

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Self-Adjusting Top Tree.hpp (12629 bytes, 443 lines)

```
#include<bits/stdc++.h>
using namespace std;

struct SelfAdjustingTopTree{
    const static int inf=~0u>>1;
    static void gmin(int&a,int b){
        a=min(a,b);
    }
    static void gmax(int&a,int b){
        a=max(a,b);
    }
    struct treap{
        SelfAdjustingTopTree*tr;
```

```
treap(struct SelfAdjustingTopTree*a,int n):
13
                tr(a),ns(n){
            }
            struct node{
                node(){
                }
                node(int a,int b,int c,int d,int e){
                    ch[0]=ch[1]=0;
                    val=a;
                    fix=rand();
23
                    add=0;
                    mi=vmi=b;
                    mx=vmx=c;
                    sum=vsum=d;
                    siz=vsiz=e;
                    sam=inf;
                }
                node*ch[2];
                int val,fix,vmi,vmx,vsum,vsiz,mi,mx,sum,siz,add,sam;
            };
33
            vector<node>ns;
            void down(node*a){
                if(a->sam!=inf){
                    a->mi=a->mx=a->vmi=a->vmx=a->sam;
                    a->vsum=a->sam*a->vsiz;
                    a->sum=a->sam*a->siz;
                    (&tr->ns[0]+(a-&ns[0]))->viradd=0;
                    (&tr->ns[0]+(a-&ns[0]))->virsam=a->sam;
                    (&tr->ns[0]+(a-&ns[0]))->add=0;
                    (&tr->ns[0]+(a-&ns[0]))->sam=a->sam;
                    for(int i=0;i<=1;++i)</pre>
43
                        if(a->ch[i])
                            a->ch[i]->add=0,a->ch[i]->sam=a->sam;
                    a->sam=inf;
                }
                if(a->add){
                    a->mi+=a->add;
                    a->mx+=a->add;
                    a->vmi+=a->add;
                    a \rightarrow vmx += a \rightarrow add;
                    a->vsum+=a->add*a->vsiz;
53
```

```
a->sum+=a->add*a->siz;
                     (&tr->ns[0]+(a-&ns[0]))->viradd+=a->add;
                     (&tr->ns[0]+(a-&ns[0]))->add+=a->add;
                     for(int i=0;i<=1;++i)</pre>
                         if(a->ch[i])
                             a\rightarrow ch[i]\rightarrow add+=a\rightarrow add;
                     a->add=0;
                }
            void update(node*a){
63
                for(int i=0;i<=1;++i)</pre>
                     if(a->ch[i])
                         down(a->ch[i]);
                a->mi=a->vmi;
                for(int i=0;i<=1;++i)</pre>
                     if(a->ch[i])
                         gmin(a->mi,a->ch[i]->mi);
                a->mx=a->vmx;
                for(int i=0;i<=1;++i)</pre>
73
                     if(a->ch[i])
                         gmax(a->mx,a->ch[i]->mx);
                a->sum=a->vsum;
                for(int i=0;i<=1;++i)</pre>
                     if(a->ch[i])
                         a->sum+=a->ch[i]->sum;
                a->siz=a->vsiz;
                for(int i=0;i<=1;++i)</pre>
                     if(a->ch[i])
                         a->siz+=a->ch[i]->siz;
83
            void rotate(node*&a,int d){
                node*b=a->ch[d];
                a->ch[d]=b->ch[!d];
                b->ch[!d]=a;
                update(a);
                update(b);
                a=b;
            void insert(node*&a,node*b){
93
                if(!a)
                     a=b;
```

```
else{
                    down(a);
                    int d=b->val>a->val;
                    insert(a->ch[d],b);
                    update(a);
                    if(a->ch[d]->fix<a->fix)
                        rotate(a,d);
                }
103
            }
            void erase(node*&a,int b){
                down(a);
                if(a->val==b){
                    if(!a->ch[0])
                        a=a->ch[1];
                    else if(!a->ch[1])
                        a=a->ch[0];
                    else{
                        int d=a->ch[1]->fix<a->ch[0]->fix;
113
                        down(a->ch[d]);
                        rotate(a,d);
                        erase(a->ch[!d],b);
                        update(a);
                    }
                }else{
                    int d=b>a->val;
                    erase(a->ch[d],b);
                    update(a);
                }
123
            }
        };
        int n;
        SelfAdjustingTopTree(int _n,vector<int>*to,int*we,int rt):
            trp(this,_n+1),ns(_n+1),n(_n){
            build(to,we,rt);
        }
        struct node{
            node(){}
            node(int a,node*b){
133
                ch[0]=ch[1]=0;
                pr=b;
                vir=0;
```

```
val=a;
                mi=mx=a;
                siz=1;
                rev=virsum=add=0;
                virmi=inf;
                virmx=-inf;
                sam=inf;
143
                virsam=inf;
                virsiz=0;
                viradd=0;
            }
            node*ch[2],*pr;
            int val,mi,mx,sum,virmi,virmx,virsum,virsam,viradd,virsiz,rev,sam,
        siz,add;
            treap::node*vir;
        };
        vector<node>ns;
        treap trp;
153
        int direct(node*a){
            if(!a->pr)
                return 3;
            else if(a==a->pr->ch[0])
                return 0;
            else if(a==a->pr->ch[1])
                return 1;
            else
                return 2;
        void down(node*a){
163
            if(a->rev){
                swap(a->ch[0],a->ch[1]);
                for(int i=0;i<=1;++i)</pre>
                     if(a->ch[i])
                         a->ch[i]->rev^=1;
                a\rightarrow rev=0;
            if(a->sam!=inf){
                a->val=a->mi=a->mx=a->sam;
173
                a->sum=a->sam*a->siz;
                for(int i=0;i<=1;++i)</pre>
                     if(a->ch[i])a->ch[i]->sam=a->sam,a->ch[i]->add=0;
```

```
a->sam=inf;
             }
            if(a->add){
                 a->val+=a->add;
                 a->mi+=a->add;
                 a \rightarrow mx += a \rightarrow add;
                 a->sum+=a->add*a->siz;
183
                 for(int i=0;i<=1;++i)</pre>
                     if(a->ch[i])a->ch[i]->add+=a->add;
                 a->add=0;
            if(a->virsam!=inf){
                 if(a->virsiz){
                     a->virmi=a->virmx=a->virsam;
                     a->virsum=a->virsam*a->virsiz;
                     if(a->vir)
                         a->vir->add=0,a->vir->sam=a->virsam;
                     for(int i=0;i<=1;++i)</pre>
193
                         if(a->ch[i])
                             a->ch[i]->viradd=0,a->ch[i]->virsam=a->virsam;
                 }
                 a->virsam=inf;
            if(a->viradd){
                 if(a->virsiz){
                     a->virmi+=a->viradd;
                     a->virmx+=a->viradd;
                     a->virsum+=a->viradd*a->virsiz;
203
                     if(a->vir)a->vir->add+=a->viradd;
                     for(int i=0;i<=1;++i)</pre>
                         if(a->ch[i])
                             a->ch[i]->viradd+=a->viradd;
                 a->viradd=0;
             }
        void update(node*a){
            for(int i=0;i<=1;++i)</pre>
213
                 if(a->ch[i])
                     down(a->ch[i]);
            if(a->vir)
```

```
trp.down(a->vir);
             a->mi=a->val;
            for(int i=0;i<=1;++i)</pre>
                 if(a->ch[i])
                     gmin(a->mi,a->ch[i]->mi);
             a->virmi=inf;
            for(int i=0;i<=1;++i)</pre>
223
                 if(a->ch[i])
                     gmin(a->virmi,a->ch[i]->virmi);
             if(a->vir)
                 gmin(a->virmi,a->vir->mi);
             a->mx=a->val;
             for(int i=0;i<=1;++i)</pre>
                 if(a->ch[i])
                     gmax(a->mx,a->ch[i]->mx);
             a->virmx=-inf;
            for(int i=0;i<=1;++i)</pre>
233
                 if(a->ch[i])
                     gmax(a->virmx,a->ch[i]->virmx);
            if(a->vir)
                 gmax(a->virmx,a->vir->mx);
            a->sum=a->val;
            for(int i=0;i<=1;++i)</pre>
                 if(a->ch[i])
                     a->sum+=a->ch[i]->sum;
             a->virsum=0;
            for(int i=0;i<=1;++i)</pre>
243
                 if(a->ch[i])
                     a->virsum+=a->ch[i]->virsum;
             if(a->vir)
                 a->virsum+=a->vir->sum;
            a->siz=1;
            for(int i=0;i<=1;++i)</pre>
                 if(a->ch[i])
                     a->siz+=a->ch[i]->siz;
             a->virsiz=0;
            for(int i=0;i<=1;++i)</pre>
253
                 if(a->ch[i])
                     a->virsiz+=a->ch[i]->virsiz;
             if(a->vir)
                 a->virsiz+=a->vir->siz;
```

```
void setchd(node*a,node*b,int d){
            a->ch[d]=b;
            if(b)
                b->pr=a;
263
            update(a);
        void connect(node*a,node*b){
            down(a);
            *(&trp.ns[0]+(a-&ns[0]))=treap::node(a-&ns[0],min(a->virmi,a->mi),
        max(a->virmx,a->mx),a->virsum+a->sum,a->virsiz+a->siz);
            trp.insert(b->vir,&trp.ns[0]+(a-&ns[0]));
        void disconnect(node*a,node*b){
            trp.erase(b->vir,a-&ns[0]);
        }
        void rotate(node*a){
273
            node*b=a->pr,*c=a->pr->pr;
            int d1=direct(a),d2=direct(b);
            setchd(b,a->ch[!d1],d1);
            setchd(a,b,!d1);
            if(d2<2)
                setchd(c,a,d2);
            else if(d2==2){
                disconnect(b,c);
                connect(a,c);
283
                a->pr=c;
            }else
                a \rightarrow pr=0;
        void release(node*a){
            if(direct(a)<2)</pre>
                release(a->pr);
            else if(a->pr)
                disconnect(a,a->pr),connect(a,a->pr);
            down(a);
293
        void splay(node*a){
            release(a);
            while(direct(a)<2){</pre>
                node*b=a->pr;
```

```
if(!b->pr||direct(b)>1)
                    rotate(a);
                else if(direct(a)==direct(b))
                    rotate(b),rotate(a);
                else
303
                    rotate(a),rotate(a);
            }
        }
        node*access(node*a){
            node*b=0;
            while(a){
                splay(a);
                if(a->ch[1])
                    connect(a->ch[1],a);
                if(b)
                    disconnect(b,a);
313
                setchd(a,b,1);
                b=a;
                a=a->pr;
            }
            return b;
        void evert(node*a){
            access(a);
            splay(a);
323
            a->rev=1;
        int qchain(node*a,node*b,int d){
            access(a);
            node*c=access(b);
            splay(c);
            splay(a);
            int ret=c->val;
            if(d==1){
                if(a!=c)
333
                    gmin(ret,a->mi);
                if(c->ch[1])
                    down(c->ch[1]),gmin(ret,c->ch[1]->mi);
            }else if(d==2){
                if(a!=c)
                    gmax(ret,a->mx);
```

```
if(c->ch[1])
                       down(c\rightarrow ch[1]), gmax(ret, c\rightarrow ch[1]\rightarrow mx);
              }else if(d==3){
                   if(a!=c)
343
                       ret+=a->sum;
                   if(c->ch[1])
                       down(c\rightarrow ch[1]), ret+=c\rightarrow ch[1]\rightarrow sum;
              }
              return ret;
         void mchain(node*a,node*b,int u,int d){
              access(a);
              node*c=access(b);
              splay(c);
353
              splay(a);
              if(d==1){
                   c->val+=u;
                   if(a!=c)
                       a->add=u,disconnect(a,c),connect(a,c);
                   if(c->ch[1])
                       down(c\rightarrow ch[1]), c\rightarrow ch[1]\rightarrow add=u;
              }else if(d==2){
                   c->val=u;
                   if(a!=c)
                       a->sam=u,disconnect(a,c),connect(a,c);
363
                   if(c->ch[1])
                       down(c\rightarrow ch[1]), c\rightarrow ch[1]\rightarrow sam=u;
              }
              update(c);
         int qtree(node*a,int d){
              access(a);
              splay(a);
              int ret=a->val;
373
              if(d==1){
                   if(a->vir)
                       trp.down(a->vir),gmin(ret,a->vir->mi);
              }else if(d==2){
                   if(a->vir)
                       trp.down(a->vir),gmax(ret,a->vir->mx);
              }else if(d==3){
```

```
if(a->vir)
                    trp.down(a->vir),ret+=a->vir->sum;
            }
383
            return ret;
        }
        void mtree(node*a,int u,int d){
            access(a);
            splay(a);
            if(d==1){
                a->val+=u;
                if(a->vir)
                    trp.down(a->vir),a->vir->add=u;
            }else if(d==2){
393
                a->val=u;
                if(a->vir)
                    trp.down(a->vir),a->vir->sam=u;
            update(a);
        void stparent(node*a,node*b){
            access(b);
            if(access(a)!=a){
                splay(a);
403
                node*c=a->ch[0];
                down(c);
                while(c->ch[1])
                    c=c->ch[1],down(c);
                splay(c);
                c->ch[1]=0;
                update(c);
                access(b);
                splay(b);
                connect(a,b);
413
                a->pr=b;
                update(b);
            }
        void build(vector<int>*to,int*we,int rt){
            vector<int>pr(n);
            vector<int>vec;
            queue<int>qu;
```

7

```
qu.push(rt);
            while(!qu.empty()){
                 int u=qu.front();
423
                 qu.pop();
                 vec.push back(u);
                 for(int i=0;i<to[u].size();++i){</pre>
                     int v=to[u][i];
                     if(v!=pr[u])
                         qu.push(v),pr[v]=u;
                 }
             for(int i=0;i<n;++i){</pre>
                 int u=vec[i];
433
                 ns[u]=node(we[u],pr[u]?&ns[0]+pr[u]:0);
             for(int i=n-1; i>=0; --i){
                 int u=vec[i];
                 update(&ns[0]+u);
                 if(pr[u])
                     connect(&ns[0]+u,&ns[0]+pr[u]);
             }
        }
443
    };
```

# 2.10 Skew Heap

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Skew Heap.hpp (1220 bytes, 61 lines)

```
#include<bits/stdc++.h>
using namespace std;
template<class T,class C>struct SkewHeap{
    SkewHeap():
        root(0),siz(0){
    }
    ~SkewHeap(){
        clear(root);
    }
    struct node{
```

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```
node(T _val):
                val(_val){
                ch[0]=ch[1]=0;
            }
            T val;
            node*ch[2];
17
        }*root;
        int siz;
        node*merge(node*x,node*y){
            if(!x)
                return y;
            if(!y)
                return x;
            if(C()(y->val,x->val))
                swap(x,y);
            swap(x\rightarrow ch[0],x\rightarrow ch[1]=merge(x\rightarrow ch[1],y));
27
            return x;
        void clear(node*x){
            if(x){}
                clear(x->ch[0]);
                clear(x->ch[1]);
                delete x;
            }
        void clear(){
            clear(root);
37
            root=0;
            siz=0;
        void push(T a){
            root=merge(root, new node(a));
            ++siz;
        T top(){
            return root->val;
47
        void pop(){
            root=merge(root->ch[0],root->ch[1]);
            --siz;
        }
```

```
void merge(SkewHeap<T,C>&a){
    root=merge(root,a.root);
    a.root=0;
    siz+=a.siz;
    a.siz=0;

57  }
  int size(){
    return siz;
  }
};
```

# $\mathsf{CHAPTER}\ 3$

Graph Algorithms

## 3.1 Chordality Test

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Chordality Test.hpp (1343 bytes, 42 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   struct ChordalityTest{
       int n,ns;
       vector<vector<int> >to;
       ChordalityTest(int _n):
           n(n),ns(n),to(n+1){
       void add(int u,int v){
 9
           to[u].push_back(v),to[v].push_back(u);
       }
       bool run(){
           vector < int > pos(n+1), idx(n+2), lab(n+1), tab(n+1);
           vector<list<int>>qu(n);
           for(int i=1;i<=n;++i)</pre>
               qu[0].push_back(i);
            for(int b=0,i=1,u=0;i<=n;++i,u=0){</pre>
               for(;u?++b,0:1;--b)
                    for(auto j=qu[b].begin();j!=qu[b].end()&&!u;qu[b].erase(j++)
19
       )
                        if(!pos[*j]&&lab[*j]==b)
                            u=*i;
               pos[u]=ns,idx[ns--]=u;
               for(int v:to[u])
                    if(!pos[v])
                       b=max(b,++lab[v]),qu[lab[v]].push back(v);}
            for(int i=1,u=idx[1],v=-1;i<=n;++i,u=idx[i],v=-1){</pre>
               for(int w:to[u])
                    if(pos[w]>pos[u]&&(v==-1||pos[w]<pos[v]))
29
               if(v!=-1){
                    for(int w:to[v])
                       tab[w]=1;
                    for(int w:to[u])
                        if(pos[w]>pos[u]&&w!=v&&!tab[w])
```

3.2. DOMINATOR TREE 75

### 3.2 Dominator Tree

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Dominator Tree.hpp (2916 bytes, 94 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   struct DominatorTree{
       int n,r;
       vector<vector<int> >to,rto,chd,rsemi;
       vector<int>dfn,res,prt,rdfn,semi,misemi;
       DominatorTree(int n,int r):n(n),r(r),to(n+1),rto(n+1),dfn(n+1),res(
       n+1), prt(n+1), rdfn(1), semi(n+1), misemi(n+1), chd(n+1), rsemi(n+1){
8
       int fd(int a){
           stack<int>stk;
           for(int b=a;prt[b]!=prt[prt[b]];b=prt[b])
               stk.push(b);
           for(int b;stk.empty()?0:(b=stk.top(),stk.pop(),1);){
               if(dfn[semi[misemi[prt[b]]]]<dfn[semi[misemi[b]]])</pre>
                   misemi[b]=misemi[prt[b]];
               prt[b]=prt[prt[b]];
           return prt[a];
18
       void add(int a,int b){
           to[a].push_back(b);
           rto[b].push back(a);
       void dfs(){
```

```
stack<pair<int,int> >stk;
            semi[r]=r;
            for(stk.push(make_pair(r,0));!stk.empty();){
                int a=stk.top().first,i=stk.top().second;
28
                stk.pop();
                if(!i)
                    dfn[a]=rdfn.size(),rdfn.push_back(a);
                if(i<to[a].size()){</pre>
                    stk.push(make_pair(a,i+1));
                    int b=to[a][i];
                    if(!semi[b])
                        semi[b]=a,chd[a].push back(b),
                        stk.push(make_pair(b,0));
38
                }
            semi[r]=0;
        void calcsemi(){
            for(int i=1;i<=n;++i)</pre>
                prt[i]=i,misemi[i]=i;
            for(int i=rdfn.size()-1;i>=1;--i){
                int a=rdfn[i];
                for(int b:rto[a]){
48
                    if(!dfn[b])
                        continue;
                    if(dfn[b]<dfn[a]){</pre>
                        if(dfn[b]<dfn[semi[a]])</pre>
                             semi[a]=b;
                    }else{
                        int c=fd(b);
                        if(dfn[semi[c]]<dfn[semi[a]])</pre>
                             semi[a]=semi[c];
                        if(dfn[semi[misemi[b]]]<dfn[semi[a]])</pre>
58
                             semi[a]=semi[misemi[b]];
                    }
                for(int b:chd[a])
                    prt[b]=a;
            }
        void calcres(){
```

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```
for(int i=1;i<=n;++i)</pre>
                prt[i]=i,misemi[i]=i,rsemi[semi[i]].push_back(i);
            for(int i=rdfn.size()-1;i>=1;--i){
68
                int a=rdfn[i];
                for(int b:rsemi[a]){
                    fd(b);
                    int c=misemi[b];
                    if(dfn[semi[c]]>dfn[semi[prt[b]]])
                        c=prt[b];
                    if(semi[c]==semi[b])
                        res[b]=semi[b];
                    else
                        res[b]=-c;}
78
                for(int b:chd[a])
                    prt[b]=a;
            for(int i=1;i<rdfn.size();++i){</pre>
                int a=rdfn[i];
                if(res[a]<0)
                    res[a]=res[-res[a]];
            }
       vector<int>run(){
88
            dfs();
            calcsemi();
            calcres();
            return res;
       }
   };
```

### 3.3 K Shortest Path

### Description

Find the length of k shortest path between two vertices in a given weighted directed graph. The path does not need to be loopless. But the edge weights must be non-negative.

### Methods

template <class t="">KShortestPath<t>::KShortestPath(int n);</t></class>				
Description	construct an object of KShortestPath			
Parameters	Description			
T	type of edge weights, be careful since the result			
	can be $\Theta(nkC)$			
n	number of vertices			
Time complexity	$\Theta(n)$			
Space complexity	$\Theta(11n)$			
Return value	an object of KShortestPath			
template <class t="">void KShortestPath<t>::add(int a,int b,T c);</t></class>				
Description	add a directed weighted edge to the graph			
Parameters	Description			
а	start vertex of the edge, indexed from one			
Ь	end vertex of the edge, indexed from one			
С	weight of the edge, should be non-negative			
Time complexity	$\Theta(1)$ (amortized)			
Space complexity	$\Theta(1)$ (amortized)			
Return value	none			
template <class t="">T KShortestPath<t>::run(int s,int t,int k);</t></class>				
Description	find the length of k shortest path			
Parameters	Description			
S	start vertex of the path, indexed from one			
t	end vertex of the path, indexed from one			
k	k in 'k shortest path'			
Time complexity	$O((n+m)\log n + k\log(nmk))$			
Space complexity	$O(n\log n + m + k\log(nm))$			
Return value	length of k shortest path from s to t or -1 if it			
	doesn't exist			

### Performance

Problem	Constraints	Time	Memory	Date
JDFZ 2978	$N = 10^4, M =$	324 ms	14968 kB	2016-02-13
	$10^5, K = 10^4$			

### References

Title	Author
堆的可持久化和 k 短路	<b>俞鼎力</b>

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#### Code

#### K Shortest Path.hpp (5105 bytes, 170 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   template<class T>struct KShortestPath{
       KShortestPath(int _n):
           n(n), m(1 << (int)ceil(log2(n)+1e-8)), from(n+1,-1),
           tov(n+1), wev(n+1), to(n+1), we(n+1), inf(numeric limits < T > :: max()),
 6
            sg(2*m, make pair(inf,0)), di(n+1, inf), nxt(n+1), chd(n+1), torev(n+1)
       }
       ~KShortestPath(){
           for(int i=0;i<all.size();++i)</pre>
               free(all[i]);
       void add(int u,int v,T w){
           tov[v].push_back(u);
           wev[v].push_back(w);
           to[u].push_back(v);
16
           we[u].push_back(w);
           torev[v].push_back(to[u].size()-1);
       int upd(T&a,T b,T c){
            if(b!=inf&&c!=inf&&b+c<a){
                a=b+c;
                return 1;
            }
           return 0;
26
       void mod(int u,T d){
           for(sg[u+m-1]=make pair(d,u),u=u+m-1>>1;u;u>>=1)
                sg[u]=min(sg[u<<1],sg[u<<1^1]);</pre>
       template<class T2>struct node{
           node(T2 v):
                v(_v),s(0),l(0),r(0)
            }
           T2 v;
            int s;
36
           node*1,*r;
```

```
};
         template<class T2>node<T2>*merge(node<T2>*a,node<T2>*b){
              if(!a||!b)
                   return a?a:b;
              if(a->v>b->v)
                   swap(a,b);
              a \rightarrow r = merge(a \rightarrow r, b);
              if(|a-\rangle 1||a-\rangle 1-\rangle s\langle a-\rangle r-\rangle s)
46
                   swap(a->1,a->r);
              a \rightarrow s = (a \rightarrow r?a \rightarrow r \rightarrow s:-1)+1;
              return a;
         template<class T2>node<T2>*mak(T2 v){
              node<T2>*t=(node<T2>*)malloc(sizeof(node<T2>));
              *t=node<T2>(v);
              all.push back(t);
              return t;
         }
56
         template<class T2>node<T2>*pmerge(node<T2>*a,node<T2>*b){
              if(!a||!b)
                  return a?a:b;
              if(a->v>b->v)
                   swap(a,b);
              node<T2>*r=mak(a->v);
              r->l=a->l;
              r->r=pmerge(a->r,b);
              if(!r\rightarrow l||r\rightarrow l\rightarrow s< r\rightarrow r\rightarrow s)
                   swap(r->1,r->r);
66
              r->s=(r->r?r->r->s:-1)+1;
              return r;
         }
         struct edge{
              edge(T _1,int _v):
                  1(_1),v(_v){
              bool operator>(const edge&a){
                  return 1>a.1;
76
              T 1;
              int v;
         };
```

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```
struct edgeheap{
            edgeheap(node<edge>*r):
                root(r){
            bool operator>(const edgeheap&a){
                return root->v.l>a.root->v.l;
86
            node<edge>*root;
        };
        edgeheap merge(edgeheap a,edgeheap b){
            return edgeheap(pmerge(a->root,b->root));
        edgeheap popmin(edgeheap a){
            return edgeheap(pmerge(a.root->1,a.root->r));
        node<edgeheap>*popmin(node<edgeheap>*a){
               node<edgeheap>*x=pmerge(a->1,a->r);
96
               a=mak(popmin(a->v));
               if(a->v.root)
                   x=pmerge(x,a);
               return x;
        }
        struct path{
            path(int _vp,int _v,T _1,T _d,node<edgeheap>*_c):
                vp(vp), v(v), l(l), d(d), can(c)
            bool operator<(const path&a)const{</pre>
106
                return 1>a.1;
            int vp,v;
            T 1,d;
            node<edgeheap>*can;
        };
        T run(int s,int t,int k){
            di[t]=0;
            for(int i=1;i<=n;++i)</pre>
                sg[i+m-1]=make_pair(di[i],i);
116
            for(int i=m-1;i>=1;--i)
                sg[i]=min(sg[i<<1],sg[i<<1^1]);</pre>
            for(int u=sg[1].second;sg[1].first!=inf;u=sg[1].second){
                mod(u,inf),tre.push_back(u);
```

```
for(int i=0;i<tov[u].size();++i){</pre>
                    int v=tov[u][i];
                    T w=wev[u][i];
                    if(upd(di[v],di[u],w))
                        mod(v,di[v]),nxt[v]=u,
                        from[v]=torev[u][i];
126
                }
            for(int i=0;i<tre.size();++i){</pre>
                queue<node<edge>*>qu;
                for(int j=0;j<to[tre[i]].size();++j)</pre>
                    if(di[to[tre[i]][j]]!=inf&&j!=from[tre[i]])
                        qu.push(mak(edge(we[tre[i]][j]-di[tre[i]]+di[to[tre[i]][
        j]],to[tre[i]][j])));
                for(node<edge>*x,*y;qu.size()>1;)
                    x=qu.front(),qu.pop(),y=qu.front(),qu.pop(),
                    qu.push(merge(x,y));
136
                if(qu.size())
                    chd[tre[i]]=pmerge(mak(edgeheap(qu.front())),chd[nxt[tre[i
        ]]]);
                else
                    chd[tre[i]]=chd[nxt[tre[i]]];
            priority_queue<path>pth;
            if(di[s]==inf)
                return -1;
            pth.push(path(0,s,di[s],0,0));
            for(int i=1;i<k;++i){</pre>
146
                if(pth.empty())
                    return -1;
                path p=pth.top();
                pth.pop();
                if(p.can){
                    edge t=p.can->v.root->v;
                    pth.push(path(p.vp,t.v,p.l-p.d+t.l,t.l,popmin(p.can)));
                }
                if(chd[p.v]){
                    edge t=chd[p.v]->v.root->v;
                    pth.push(path(p.v,t.v,p.l+t.l,t.l,popmin(chd[p.v])));
156
                }
            }
```

```
return pth.size()?pth.top().1:-1;
}
T inf;
int n,m;
vector<T>di;
vector<int>nxt,tre,from;
vector<void*>all;
vector<node<edgeheap>*>chd;
vector<pair<T,int> >sg;
vector<vector<T> >wev,we;
vector<vector<int> >tov,to,torev;
};
```

## 3.4 Maximal Clique Count

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Maximal Clique Count.hpp (927 bytes, 34 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   template<int N>struct MaximalCliqueCount{
       int n,r;
       vector<bitset<N> >e,rht,msk;
       MaximalCliqueCount(int _n):
           n(n),e(n),rht(n),msk(n),r(0)
       void add(int u,int v){
           e[u-1][v-1]=e[v-1][u-1]=1;
10
       void dfs(int u,bitset<N>cur,bitset<N>can){
           if(cur==can){
               ++r;
               return;
            for(int v=0;v<u;++v)</pre>
               if(can[v]&&!cur[v]&&(e[v]&rht[u]&can)==(rht[u]&can))
                    return;
            for(int v=u+1; v<n; ++v)</pre>
20
               if(can[v])
```

```
dfs(v,cur|msk[v],can&e[v]);
}
int run(){
    for(int i=1;i<=n;++i){
        rht[i-1]=bitset<N>(string(n-i,'1')+string(i,'0'));
        msk[i-1]=bitset<N>(1)<<i-1;
        e[i-1]|=msk[i-1];
}
for(int i=0;i<n;++i)
        dfs(i,msk[i],e[i]);
    return r;
}
};</pre>
```

## 3.5 Maximal Planarity Test

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Maximal Planarity Test.hpp (5195 bytes, 165 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   struct MaximalPlanarityTesting{
       int n,m;
       vector<set<int> >to2;
       vector<vector<int> >to;
6
       vector<int>dec,rmd,mrk,invc,rt;
       vector<list<int>::iterator>dpos,pos;
       bool order(int v1,int v2,int vn){
           rt[0]=v1;
           rt[1]=v2;
           rt[n-1]=vn;
           fill(invc.begin(),invc.end(),0);
           invc[v1]=1;
           invc[v2]=1;
16
           invc[vn]=1;
           list<int>deg;
           dpos[vn]=deg.insert(deg.begin(),vn);
           fill(dec.begin(),dec.end(),0);
           dec[v1]=2;
```

```
dec[v2]=2;
           dec[vn]=2;
           for(int i=n-1;i>=2;--i){
               if(deg.empty())
                   return false;
26
               int v=*deg.begin();
               deg.erase(deg.begin());
               invc[v]=-1;
               rt[i]=v;
               for(int u:to[v]){
                   if(invc[u]==1){
                       if(u!=v1&&u!=v2&&dec[u]==2)
                           deg.erase(dpos[u]);
                       --dec[u];
                       if(u!=v1&&u!=v2&&dec[u]==2)
                           dpos[u]=deg.insert(deg.begin(),u);
36
                   }else if(invc[u]==0)
                       invc[u]=2;
               }
               for(int u:to[v])
                   if(invc[u]==2)
                       for(int w:to[u])
                           if(invc[w]==1){
                               if(w!=v1\&w!=v2\&dec[w]==2)
                                   deg.erase(dpos[w]);
46
                               ++dec[w];
                               if(w!=v1&&w!=v2&&dec[w]==2)
                                   dpos[w]=deg.insert(deg.begin(),w);
                               ++dec[u];
                           }else if(invc[w]==2)
                               ++dec[u];
               for(int u:to[v]){
                   if(invc[u]==2){
                       invc[u]=1;
                       if(dec[u]==2)
                           dpos[u]=deg.insert(deg.begin(),u);
56
                   }
               }
           }
           return true;
       }
```

```
bool embed(){
            list<int>ext;
           int mker=0;
           fill(mrk.begin(),mrk.end(),0);
66
           pos[rt[1]]=ext.insert(ext.begin(),rt[1]);
           pos[rt[2]]=ext.insert(ext.begin(),rt[2]);
           pos[rt[0]]=ext.insert(ext.begin(),rt[0]);
           fill(rmd.begin(),rmd.end(),0);
            rmd[rt[1]]=1;
            rmd[rt[2]]=1;
            rmd[rt[0]]=1;
           for(int i=3;i<n;++i){</pre>
               int v=rt[i];
               rmd[v]=1;
               vector<int>can;
76
               ++mker:
               for(int u:to[v])
                   if(rmd[u])
                       mrk[u]=mker,can.push back(u);
               int start=-1,end=-1;
               for(int u:can){
                   list<int>::iterator it=pos[u];
                   if(it==list<int>::iterator())
                       return false;
                   if(it==ext.begin()){
86
                       if(start!=-1)
                            return false;
                       start=u;
                    }else{
                       list<int>::iterator tmp=it;
                       if(mrk[*(--tmp)]!=mker){
                            if(start!=-1)
                               return false;
                            start=u;
96
                       }
                    }
                   list<int>::iterator tmp=it;++tmp;
                   if(tmp==ext.end()){
                       if(end!=-1)
                            return false;
                       end=u;
```

```
}else{
                        if(mrk[*tmp]!=mker){
                            if(end!=-1)
106
                                return false;
                            end=u;
                        }
                    }
                }
                if(start==-1||end==-1)
                    return false;
                for(int u:can)
                    if(u!=start&&u!=end)
                        ext.erase(pos[u]),pos[u]=list<int>::iterator();
                pos[v]=ext.insert(pos[end],v);
116
            return true;
        bool istri(int u,int v,int w){
            return to2[u].count(v)&&to2[v].count(w)&&to2[w].count(u);
        MaximalPlanarityTesting(int _n):
            n(n), to(n), to(n), m(0), rt(n), invc(n), dec(n), dec(n), pos(n), rmd(n),
        mrk(n){
        }
        void add(int u,int v){
126
            to[u-1].push_back(v-1);
            to[v-1].push_back(u-1);
            to2[u-1].insert(v-1);
            to2[v-1].insert(u-1);++m;
        bool run(){
            if(n==1\&\&m==0)
                return true;
            if(n==2\&\&m==1)
136
                return true;
            if(n==3\&\&m==3)
                return true;
            if(n<=3)
                return false;
            if(m!=3*n-6)
                return false;
```

```
int v1;
             for(v1=0;v1<n;++v1)</pre>
                  if(to[v1].size()<3)</pre>
                      return false;
146
             for(v1=0;v1<n;++v1)</pre>
                  if(to[v1].size()<=5)
                      break;
             if(v1>=n)
                  return false;
             int v2=to[v1].back();
             for(int i=0;i+1<to[v1].size();++i){</pre>
                  int vn=to[v1][i];
                  if(istri(v1,v2,vn)){
                      if(!order(v1,v2,vn))
156
                          continue;
                      if(!embed())
                          continue;
                      return true;
                  }
             }
             return false;
         }
     };
```

## 3.6 Maximum Flow

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Maximum Flow.hpp (2311 bytes, 79 lines)

```
#include<bits/stdc++.h>
using namespace std;
template<class T>struct MaximumFlow{
    struct edge{
5     int v;
    T c,l;
    edge(int _v,T _c):
        v(_v),c(_c),l(_c){
    }
};
```

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```
int n,src,snk;
       vector<edge>egs;
       vector<vector<int> >bge;
       vector<int>hei,gap,cur,frm;
15
       MaximumFlow(int n,int src,int snk):
       υge(_
_snk-1){
}
           bge(_n),hei(_n,_n),gap(_n+1),n(_n),cur(_n),frm(_n),src(_src-1),snk(
       void lab(){
           hei[snk]=0;
           queue<int>qu;
           qu.push(snk);
           for(int u;qu.empty()?0:(u=qu.front(),qu.pop(),1);)
               for(int i=0;i<bge[u].size();++i){</pre>
                   edge&e=egs[bge[u][i]],&ev=egs[bge[u][i]^1];
25
                   if(ev.c>0&&hei[e.v]==n)
                       hei[e.v]=hei[u]+1,qu.push(e.v);
               }
           for(int i=0;i<n;++i)</pre>
               ++gap[hei[i]];
       }
       T aug(){
           T f=0;
           for(int u=snk;u!=src;u=egs[frm[u]^1].v)
               if(f<=0||f>egs[frm[u]].c)
35
                   f=egs[frm[u]].c;
           for(int u=snk;u!=src;u=egs[frm[u]^1].v)
               egs[frm[u]].c-=f,egs[frm[u]^1].c+=f;
           return f;
       void add(int u,int v,T c){
           bge[u-1].push_back(egs.size());
           egs.push_back(edge(v-1,c));
           bge[v-1].push_back(egs.size());
           egs.push back(edge(u-1,0));
45
       }
       T run(){
           lab();
           T r=0;
           for(int u=src;hei[src]!=n;){
               if(u==snk)
```

```
r+=aug(),u=src;
                int f=0;
                for(int i=cur[u];i<bge[u].size();++i){</pre>
                    edge&e=egs[bge[u][i]];
                    if(e.c>0&&hei[u]==hei[e.v]+1){
55
                        f=1;
                        frm[e.v]=bge[u][i];
                        u=e.v;
                        break;
                    }
                }
                if(!f){
                    int mh=n-1;
                    for(int i=0;i<bge[u].size();++i){</pre>
                        edge&e=egs[bge[u][i]];
65
                        if(e.c>0&&mh>hei[e.v])
                             mh=hei[e.v];
                    if(!--gap[hei[u]])
                        break;
                    ++gap[hei[u]=mh+1];
                    cur[u]=0;
                    if(u!=src)
                        u=egs[frm[u]^1].v;
75
                }
            return r;
        }
    };
```

## 3.7 Maximum Matching

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Maximum Matching.hpp (3123 bytes, 112 lines)

```
1 #include<bits/stdc++.h>
  using namespace std;
  struct MaximumMatching{
   int n;
```

3.7. MAXIMUM MATCHING

```
vector<int>res,nxt,mrk,vis,top,prt,rnk;
       vector<vector<int> >to;
       queue<int>qu;
       MaximumMatching(int _n):
            n(n), res(n+1), nxt(n+1), mrk(n+1), vis(n+1), top(n+1), to(n+1), prt(n+1)
       ,rnk(n+1){
11
       int fd(int x){
           return x==prt[x]?x:prt[x]=fd(prt[x]);
       void lk(int x,int y){
            if(rnk[x=fd(x)]>rnk[y=fd(y)])
                prt[y]=x;
            else if(rnk[x]<rnk[y])</pre>
                prt[x]=y;
            else
                prt[x]=y,++rnk[y];
21
       int lca(int x,int y){
            static int t;
            ++t;
            for(;;swap(x,y))
                if(x){}
                    x=top[fd(x)];
                    if(vis[x]==t)
                        return x;
                    vis[x]=t;
31
                if(res[x])
                    x=nxt[res[x]];
                else
                    x=0;
                }
       void uni(int x,int p){
            for(;fd(x)!=fd(p);){
                int y=res[x],z=nxt[y];
                if(fd(z)!=fd(p))
                    nxt[z]=y;
41
                if(mrk[y]==2)
                    mrk[y]=1,qu.push(y);
                if(mrk[z]==2)
```

```
mrk[z]=1,qu.push(z);
                int t=top[fd(z)];
                1k(x,y);
                1k(y,z);
                top[fd(z)]=t;
                x=z;
51
            }
        void aug(int s){
            for(int i=1;i<=n;++i)</pre>
                nxt[i]=0,mrk[i]=0,top[i]=i,prt[i]=i,rnk[i]=0;
            mrk[s]=1;
            qu=queue<int>();
            for(qu.push(s);!qu.empty();){
                int x=qu.front();
                qu.pop();
                for(int i=0;i<to[x].size();++i){</pre>
61
                    int y=to[x][i];
                    if(res[x]==y||fd(x)==fd(y)||mrk[y]==2)
                        continue;
                    if(mrk[y]==1){
                        int z=lca(x,y);
                        if(fd(x)!=fd(z))
                            nxt[x]=y;
                        if(fd(y)!=fd(z))
                            nxt[y]=x;
                        uni(x,z);
71
                        uni(y,z);
                    }else if(!res[y]){
                        for(nxt[y]=x;y;){
                            int z=nxt[y],mz=res[z];
                            res[z]=y;
                            res[y]=z;
                            y=mz;
                        }
                        return;
81
                    }else{
                        nxt[y]=x;
                        mrk[res[y]]=1;
                        qu.push(res[y]);
                        mrk[y]=2;
```

```
}
                  }
             }
         void add(int x,int y){
91
             to[x].push_back(y);
             to[y].push_back(x);
         int run(){
             for(int i=1;i<=n;++i)</pre>
                  if(!res[i])
                      for(int j=0;j<to[i].size();++j)</pre>
                           if(!res[to[i][j]]){
                               res[to[i][j]]=i;
                               res[i]=to[i][j];
                               break;
101
                           }
             for(int i=1;i<=n;++i)</pre>
                  if(!res[i])
                      aug(i);
             int r=0;
             for(int i=1;i<=n;++i)</pre>
                  if(res[i])
                      ++r;
             return r/2;
111
         }
     };
```

## 3.8 Minimum Cost Maximum Flow

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Minimum Cost Maximum Flow.hpp (2278 bytes, 82 lines)

```
#include<bits/stdc++.h>
using namespace std;
template<class F=int,class C=int>struct MinimumCostMaximumFlow{
    struct edge{
        edge(int _v,F _c,C _w):
            v(_v),c(_c),w(_w){
```

```
}
8
           int v;
           F c;
           C w;
       };
       MinimumCostMaximumFlow(int _n,int _src,int _snk,F _all):
           n(n), src(src-1), snk(snk-1), bg(n), vis(n), dis(n), all(all), flow
       (0),cost(0){}
       void add(int u,int v,F c,C w){
           bg[u-1].push_back(eg.size());
           eg.push_back(edge(v-1,c,w));
           bg[v-1].push back(eg.size());
           eg.push back(edge(u-1,0,-w));
18
       int spfa(){
           vector<int>in(n,0);
           queue<int>qu;
           fill(vis.begin(), vis.end(),0);
           dis[src]=0;
           vis[src]=in[src]=1;
           qu.push(src);
           while(!qu.empty()){
28
                int u=qu.front();
                qu.pop();
                in[u]=0;
                for(int i=0;i<bg[u].size();++i){</pre>
                    edge&e=eg[bg[u][i]];
                    if(e.c!=0&&(!vis[e.v]||dis[u]+e.w<dis[e.v])){</pre>
                        dis[e.v]=dis[u]+e.w;
                        vis[e.v]=1;
                        if(!in[e.v]){
                            in[e.v]=1;
38
                            qu.push(e.v);
                        }
                    }
                }
           return vis[snk]&&dis[snk]<0;</pre>
       F dfs(int u,F f){
           if(u==snk)
```

```
return f;
48
            F g=f;
            vis[u]=1;
            for(int i=0;i<bg[u].size();++i){</pre>
                edge&e=eg[bg[u][i]],&ev=eg[bg[u][i]^1];
                if(e.c!=0&&dis[e.v]==dis[u]+e.w&&!vis[e.v]){
                    F t=dfs(e.v,min(g,e.c));
                    g-=t;
                    e.c-=t;
                    ev.c+=t;
                    cost+=t*e.w;
58
                    if(g==0)
                        return f;
                }
            return f-g;
        pair<F,C>run(){
           while(all!=0&&spfa()){
                Ft;
                do{
                    fill(vis.begin(), vis.end(),0);
68
                    flow+=(t=dfs(src,all));
                    all-=t;
                }while(t!=0);
            }
            return make_pair(flow,cost);
        int n,src,snk;
        vector<vector<int> >bg;
        vector<edge>eg;
78
        vector<int>vis;
        vector<C>dis;
        F all, flow;
        C cost;
   };
```

## 3.9 Minimum Spanning Arborescence

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Minimum Spanning Arborescence.hpp (1933 bytes, 64 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   template < class T > struct MinimumSpanningArborescence{
        struct eg{
            int u,v;
            Tw;
        };
8
        int n,rt;
        vector<eg>egs;
        vector<int>vi,in,id;
        vector<T>inw;
        MinimumSpanningArborescence(int _n,int _rt):
            n(_n),rt(_rt),vi(n+1),in(n+1),inw(n+1),id(n+1){
        }
        void add(int u,int v,T w){
            eg e;
            e.u=u;
18
            e.v=v;
            e.w=w;
            egs.push_back(e);
        T run(){
            int nv=0;
            for(T r=0;;n=nv,nv=0,rt=id[rt]){
                for(int i=1;i<=n;++i)</pre>
                    in[i]=-1;
                for(int i=0;i<egs.size();++i)</pre>
                    if(egs[i].u!=egs[i].v&&(in[egs[i].v]==-1||egs[i].w<inw[egs[</pre>
28
       i].v]))
                        in[egs[i].v]=egs[i].u,inw[egs[i].v]=egs[i].w;
                for(int i=1;i<=n;++i)</pre>
                    if(i!=rt&&in[i]==-1)
                        return numeric limits<T>::max();
                for(int i=1;i<=n;++i){</pre>
                    if(i!=rt)
```

```
r+=inw[i];
                     id[i]=-1,vi[i]=0;
                for(int i=1;i<=n;++i)</pre>
38
                     if(i!=rt&&!vi[i]){
                         int u=i;
                         do{
                             vi[u]=i;
                             u=in[u];
                         }while(!vi[u]&&u!=rt);
                         if(u!=rt&&vi[u]==i){
                             int v=u;
                             ++nv;
                             do{
48
                                 id[v]=nv;
                                 v=in[v];
                             }while(v!=u);
                         }
                     }
                if(nv==0)
                     return r;
                for(int i=1;i<=n;++i)</pre>
                     if(id[i]==-1)
58
                         id[i]=++nv;
                for(int i=0;i<egs.size();++i)</pre>
                     egs[i].w-=inw[egs[i].v],egs[i].u=id[egs[i].u],
                     egs[i].v=id[egs[i].v];
            }
        }
   };
```

## 3.10 Minimum Spanning Tree

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Minimum Spanning Tree.hpp (1049 bytes, 44 lines)

```
#include<bits/stdc++.h>
using namespace std;
template<class T,class C=less<T> >struct MinimumSpanningTree{
```

```
struct edge{
            Tw;
 6
            int u,v;
            int operator<(const edge&b)const{</pre>
                return C()(w,b.w);
            }
        };
        int n;
        vector<edge>egs;
        vector<int>pr;
        MinimumSpanningTree(int _n):
            n(n), pr(n+1){
16
        void add(int u,int v,T w){
            edge e;
            e.u=u;
            e.v=v;
            e.w=w;
            egs.push_back(e);
        }
        int fd(int x){
            return x==pr[x]?x:pr[x]=fd(pr[x]);
26
        void lk(int x,int y){
            pr[fd(x)]=y;
        pair<T,vector<edge> >run(){
            vector<edge>ret;
            T sum=0;
            sort(egs.begin(),egs.end());
            for(int i=1;i<=n;++i)</pre>
                pr[i]=i;
            for(int i=0;i<egs.size();++i){</pre>
36
                int u=egs[i].u,v=egs[i].v;
                T w=egs[i].w;
                if(fd(u)!=fd(v))
                    lk(u,v),ret.push_back(egs[i]),sum+=w;
            }
            return make pair(sum, ret);
        }
    };
```

3.11. SHORTEST PATH 99

### 3.11 Shortest Path

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Shortest Path.hpp (1293 bytes, 45 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   template < class T > struct ShortestPath{
       int n,m;
       vector<vector<int> >to;
 6
       vector<vector<T> >we;
       T inf;
       vector<pair<T,int> >sg;
       vector<T>di;
       ShortestPath(int _n):
           n(n), m(1 << (int)ceil(log2(n)+1e-8)), to(n+1), we(n+1), inf(
       numeric_limits<T>::max()),sg(2*m,make_pair(inf,0)),di(n+1,inf){
       void set(int u,T d){
           di[u]=d;
16
       void add(int u,int v,T w){
           to[u].push_back(v);
           we[u].push_back(w);
       int upd(T&a,T b,T c){
            if(b!=inf&&c!=inf&&b+c<a){
               a=b+c;
               return 1;
            }
           return 0;
26
       void mod(int u,T d){
           for(sg[u+m-1]=make_pair(d,u),u=(u+m-1)>>1;u;u>>=1)
               sg[u]=min(sg[u<<1],sg[u<<1^1]);</pre>
       vector<T>run(){
           for(int i=1;i<=n;++i)</pre>
               sg[i+m-1]=make_pair(di[i],i);
           for(int i=m-1;i>=1;--i)
```

### 3.12 Steiner Tree

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Steiner Tree.hpp (1745 bytes, 56 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   template<class T>struct SteinerTree{
       int n,k,z;
       T inf=numeric_limits<T>::max();
5
       vector<vector<T> >wei,dp;
       vector<int>im;
       SteinerTree(int n):
           n(n),k(0),wei(n+1,vector<T>(n+1,inf)),im(n+1){
       }
       void set(int u){
           if(!im[u])
               im[z=u]=++k;
       void add(int u,int v,T w){
15
           wei[u][v]=wei[v][u]=min(w,wei[u][v]);
       int upd(T&a,T b,T c){
           if(b!=inf&&c!=inf&&b+c<a){
               a=b+c;
               return 1;
```

3.13. VIRTUAL TREE 101

```
}
            return 0;
        int ins(int s,int u){
25
            return im[u]&&((s>>im[u]-1)&1);
        T run(){
            for(int l=1;l<=n;++1)</pre>
                for(int i=1;i<=n;++i)</pre>
                    for(int j=1;j<=n;++j)</pre>
                        upd(wei[i][j],wei[i][l],wei[l][j]);
            dp=vector<vector<T> >(1<<k-1, vector<T>(n+1, inf));
            fill(begin(dp[0]),end(dp[0]),0);
            for(int s=1;s<(1<<k-1);++s){</pre>
35
                queue<int>qu;
                vector<int>in(n+1);
                for(int u=1;u<=n;++u){</pre>
                    if(ins(s,u))
                        continue;
                    qu.push((u));
                    in[u]=1;
                    for(int t=(s-1)&s;t;t=(t-1)&s)
                        upd(dp[s][u],dp[t][u],dp[s^t][u]);
45
                    for(int v=1; v<=n;++v)
                        if(ins(s,v))
                            upd(dp[s][u],dp[s^{(1<im[v]-1)}[v],wei[u][v]);
                }
                for(int u;qu.empty()?0:(u=qu.front(),qu.pop(),in[u]=0,1);)
                    for(int v=1; v<=n;++v)
                        if(!ins(s,v)&&upd(dp[s][v],dp[s][u],wei[u][v])&&!in[v])
                            in[v]=1,qu.push(v);
            return k?dp[(1<< k-1)-1][z]:0;
55
        }
    };
```

### 3.13 Virtual Tree

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Virtual Tree.hpp (2375 bytes, 77 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   struct VirtualTree{
       int n,r,1;
4
       vector<vector<int> >to,vto,up;
       vector<int>lst,dp,dfn,edf,imp;
       VirtualTree(int _n,int _r):
            n(n),r(r),l(ceil(log2(n)+le-8)),to(n+1),vto(n+1),up(n+1,vector<
       int>(l+1)),dp(n+1),dfn(n+1),edf(n+1),imp(n+1){
       void add(int u,int v){
           to[u].push_back(v);
           to[v].push back(u);
       void vadd(int u,int v){
14
           vto[u].push back(v);
       int lca(int u,int v){
           if(dp[u]<dp[v])</pre>
               swap(u,v);
           for(int i=0;i<=1;++i)</pre>
               if(((dp[u]-dp[v])>>i)&1)
                    u=up[u][i];
            if(u==v)
24
               return u;
           for(int i=1;i>=0;--i)
               if(up[u][i]!=up[v][i])
                    u=up[u][i],v=up[v][i];
           return up[u][0];
       void dfs(int u){
           dfn[u]=++dfn[0];
           for(int i=1;i<=1;++i)</pre>
               up[u][i]=up[up[u][i-1]][i-1];
            for(int i=0;i<to[u].size();++i){</pre>
34
               int v=to[u][i];
               if(v!=up[u][0])
                    up[v][0]=u,dp[v]=dp[u]+1,dfs(v);
           edf[u]=dfn[0];
```

3.13. VIRTUAL TREE 103

```
}
        void build(){
            dfs(r);
        void run(int*a,int m){
44
            for(int i=0;i<lst.size();++i)</pre>
                imp[lst[i]]=0,vto[lst[i]].clear();
            vector<pair<int,int> >b(m+1);
            for(int i=1;i<=m;++i)</pre>
                imp[a[i]]=1,b[i]=make_pair(dfn[a[i]],a[i]);
            sort(b.begin()+1,b.end());
            vector<int>st(1,r);
            lst=st;
            for(int i=1;i<=m;++i){</pre>
54
                int u=b[i].second,v=st.back();
                if(u==r)
                    continue;
                if(dfn[u]<=edf[v])</pre>
                    st.push back(u);
                else{
                    int w=lca(u,v);
                    while(st.size()>=2&&dp[st[st.size()-2]]>=dp[w]){
                        vadd(st[st.size()-2],*st.rbegin());
                        lst.push_back(*st.rbegin()),st.pop_back();
64
                    if(st.size()>=2&&w!=st[st.size()-1]){
                        vadd(w,*st.rbegin()),lst.push_back(*st.rbegin());
                        st.pop_back(),st.push_back(w);
                    st.push_back(u);
                }
            }
            while(st.size()>=2){
                vadd(st[st.size()-2],*st.rbegin());
74
                lst.push back(*st.rbegin()),st.pop back();
            }
        }
    };
```

# CHAPTER 4

Number Theory

## 4.1 Discrete Logarithm

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Discrete Logarithm.hpp (1819 bytes, 74 lines)

```
#include<bits/stdc++.h>
   using namespace std;
 3 namespace DiscreteLogarithm{
       typedef long long T;
       int ti[1<<16],va[1<<16],mp[1<<16],nx[1<<16],hd[1<<16],tm,nw;</pre>
       void ins(int x,int v){
            int y=x&65535;
            if(ti[y]!=tm)
                ti[y]=tm,hd[y]=0;
            for(int i=hd[y];i;i=nx[i])
                if(va[i]==x){
                    mp[i]=v;
                    return;
13
                }
            va[++nw]=x;
            mp[nw]=v;
           nx[nw]=hd[y];
            hd[y]=nw;
       int get(int x){
            int y=x&65535;
            if(ti[y]!=tm)
23
                ti[y]=tm,hd[y]=0;
            for(int i=hd[y];i;i=nx[i])
                if(va[i]==x){
                    return mp[i];
                }
            return -1;
       T pow(T a,T b,T c){
            T r=1;
            for(;b;b&1?r=r*a%c:0,b>>=1,a=a*a%c);
33
            return r;
       T gcd(T a, T b){
```

```
return b?gcd(b,a%b):a;
        }
        void exg(T a,T b,T&x,T&y){
            if(!b)
                x=1, y=0;
            else
                exg(b,a\%b,y,x),y=a/b*x;
43
        T inv(T a, T b){
            Tx,y;
            exg(a,b,x,y);
            return x+b;
        T bgs(T a,T b,T c){
            ++tm;
            nw=0:
            T m=sqrt(c);
            for(T i=m-1,u=pow(a,i,c),v=inv(a,c);i>=0;--i,u=u*v%c)
53
                ins(u,i);
            for(T i=0,u=1,v=inv(pow(a,m,c),c);i*m<=c;++i,u=u*v%c){</pre>
                T t=u*b%c,j;
                if((j=get(t))!=-1)
                    return i*m+j;
            }
            return -1;
        T run(T a, T b, T c){
            T u=1, t=0;
63
            a=(a\%c+c)\%c;
            b=(b\%c+c)\%c;
            for(int i=0;i<32;++i)</pre>
                if(pow(a,i,c)==b)
                    return i;
            for(T d;(d=gcd(a,c))!=1;++t,u=a/d*u%c,b/=d,c/=d)
                if(b%d)
                    return -1;
            return (u=bgs(a,b*inv(u,c)%c,c))<0?-1:u+t;</pre>
73
        }
    }
```

## 4.2 Integer Factorization (Pollard's Rho Algorithm)

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Integer Factorization (Pollard's Rho Algorithm).hpp (2848 bytes, 93 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   namespace IntegerFactorization{
       template < class T>T mul(T x,T y,T z){
           if(typeid(T)==typeid(int))
               return (long long)x*y%z;
 6
           else if(typeid(T)==typeid(long long))
               return (x*y-(T)(((long double)x*y+0.5)/z)*z+z)%z;
           else
               return x*y%z;
       template < class T>T pow(T a, T b, T c){
           for(;b;b&1?r=mul(r,a,c):0,b>>=1,a=mul(a,a,c));
           return r;
16
       template < class T > int chk(T a, int c=10){
           if(a==2)
               return 1;
           if(a%2==0||a<2)
               return 0;
           static int pi[]={2,7,61},pl
       []={2,325,9375,28178,450775,9780504,1795265022};
           if(typeid(T)==typeid(int))
               c=3;
           else if(typeid(T)==typeid(long long))
26
               c=7;
           T u=a-1,t=0,p=1;
           for(;u%2==0;u/=2,++t);
           for(int i=0;i<c;++i){</pre>
               if(typeid(T)==typeid(int))
                   p=pi[i]%a;
               else if(typeid(T)==typeid(long long))
                   p=pl[i]%a;
               else
```

```
p=(p*29+7)%a;
               if(!p||p==1||p==a-1)
36
                    continue;
               T x=pow(p,u,a);
               if(x==1)
                    continue;
               for(int j=0;x!=a-1&&j<t;++j){}
                    x=mul(x,x,a);
                    if(x==1)
                       return 0;
46
               if(x==a-1)
                    continue;
               return 0;
           return 1;
       template < class T>T gcd(T a, T b){
           if(a<0)
               a=-a;
           if(b<0)
56
               b=-b;
           return b?gcd(b,a%b):a;
       }
       template < class T>T rho(T a, T c){
           T x=double(rand())/RAND_MAX*(a-1),y=x;
           for(int i=1,k=2;;){
               x=(mul(x,x,a)+c)%a;
               T d=\gcd(y-x,a);
               if(d!=1&&d!=a)
                    return d;
               if(y==x)
66
                    return a;
               if(++i==k)
                   y=x, k=2*k;
            }
       template<class T>vector<pair<T,int> >run(T a){
           if(a==1)
               return vector<pair<T,int> >();
           if(chk(a))
```

```
76
               return vector<pair<T,int> >(1,make pair(a,1));
           T b=a;
           while((b=rho(b,T(double(rand())/RAND MAX*(a-1))))==a);
           vector<pair<T,int> >u=run(b),v=run(a/b),r;
           for(int pu=0,pv=0;pu<u.size()||pv<v.size();){</pre>
               if(pu==u.size())
                   r.push back(v[pv++]);
               else if(pv==v.size())
                   r.push back(u[pu++]);
               else if(u[pu].first==v[pv].first)
                   r.push_back(make_pair(u[pu].first,(u[pu].second+v[pv].second
86
       ))),++pu,++pv;
               else if(u[pu].first>v[pv].first)
                   r.push back(v[pv++]);
                   r.push_back(u[pu++]);}
           return r;
       }
   }
```

# 4.3 Integer Factorization (Shanks' Square Forms Factorization)

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Integer Factorization (Shanks' Square Forms Factorization).hpp (4675 bytes, 147 lines)

```
#include<bits/stdc++.h>
using namespace std;
namespace IntegerFactorization{
    typedef long long ll;
    typedef unsigned long long ull;
    ll lim=368934881469425832611;

ull srt(const ull&a){
    ull b=sqrt(a);
    b-=b*b>a;
    return b+=(b+1)*(b+1)<=a;
}
int sqr(const ull&a,ll&b){</pre>
```

```
b=srt(a);
           return b*b==a;
       }
       ull gcd(const ull&a,const ull&b){
           return b?gcd(b,a%b):a;
17
       11 amb(11 a,const 11&B,const 11&dd,const 11&D){
           for(11 q=(dd+B/2)/a,b=q*a*2-B,c=(D-b*b)/4/a,qc,qcb,a0=a,b0=a,b1=b,
       c0=c;;b1=b,c0=c){
               if(c0>dd)
                   qcb=c0-b, b=c0+qcb, c=a-qcb;
               else{
                   q=(dd+b/2)/c0;
                   if(q==1)
                       qcb=c0-b,b=c0+qcb,c=a-qcb;
                   else
27
                       qc=q*c0,qcb=qc-b,b=qc+qcb,c=a-q*qcb;
               if(a=c0,b==b1)
                   break;
               if(b==b0&&a==a0)
                   return 0;
           return a&1?a:a>>1;
37
       ull fac(const ull&n){
           if(n&1^1)
               return 2;
           if(n%3==0)
               return 3;
           if(n\%5==0)
               return 5;
           if(srt(n)*srt(n)==n)
               return srt(n);
           static ll d1,d2,a1,b1,c1,dd1,L1,a2,b2,c2,dd2,L2,a,q,c,qc,qcb,D1,D2,
       bl1[1<<19],bl2[1<<19];
47
           int p1=0,p2=0,ac1=1,ac2=1,j,nm4=n&3;
           if(nm4==1)
               D1=n,D2=5*n,d2=srt(D2),dd2=d2/2+d2%2,b2=(d2-1)|1;
           else
               D1=3*n,D2=4*n,dd2=srt(D2),d2=dd2*2,b2=d2;
```

```
d1=srt(D1), b1=(d1-1) | 1, c1=(D1-b1*b1) / 4, c2=(D2-b2*b2) / 4, L1=srt(d1),
       L2=srt(d2), dd1=d1/2+d1%2;
           for(int i=a1=a2=1;ac1||ac2;++i){
                #define m(t)\
                if(ac##t){\
                    c=c##t;\
                    q=c>dd##t?1:(dd##t+b##t/2)/c;\
57
                    if(q==1)\
                       qcb=c-b##t,b##t=c+qcb,c##t=a##t-qcb;\
                    else\
                        qc=q*c,qcb=qc-b##t,b##t=qc+qcb,c##t=a##t-q*qcb;\
                    if((a##t=c)<=L##t)\
                       bl##t[p##t++]=a##t;\
                }
                m(1)m(2)
                if(i&1)
                    continue;
67
                #define m(t)\
                if((ac##t=ac##t&a##t!=1)&&sqr(a##t,a)){\
                    if(a<=L##t)\
                       for(j=0;j<p##t;j++)\</pre>
                            if(a==bl##t[j]){\
                                a=0;\
                                break;\
                            }\
                    if(a>0){\
                       if((q=gcd(a,b##t))>1)\
77
                            return q*q;\
                       q=amb(a,b##t,dd##t,D##t);\
                        if(nm4==5-2*t&&(q=amb(a,b##t,dd##t,D##t))%(2*t+1)==0)
                            q/=2*t+1;\
                       if(q>1)\
                            return q;\
                    }\
                }
                m(1)m(2)
                #undef m
87
           for(int i=3;;i+=2)
                if(n\%i==0)
                    return i;
```

```
11 mul(const 11&x,const 11&y,const 11&z){
            return(x*y-(11)(((long double)x*y+0.5)/z)*z+z)%z;
        11 pow(11 a,11 b,const 11&c){
97
            ll r=1;
            for(;b;b&1?r=mul(r,a,c):0,b>>=1,a=mul(a,a,c));
            return r;
        int chk(const 11&a){
            if(a==2)
                return 1;
            if(a\%2==0||a<2)
                return 0;
            static int pf[]={2,325,9375,28178,450775,9780504,1795265022};
107
            11 u=a-1,t=0,p;
            for(;u%2==0;u/=2,++t);
            for(int i=0;i<7;++i){</pre>
                p=pf[i]%a;
                if(!p||p==a-1)
                    continue;
                11 x=pow(p,u,a);
                if(x==1)
                    continue;
                for(int j=0;x!=a-1&&j<t;++j){</pre>
117
                    x=mul(x,x,a);
                    if(x==1)
                        return 0;
                if(x==a-1)
                    continue;
                return 0;
            }
            return 1;
        }
        vector<pair<ll,int> >run(const ll&a){
127
            if(a==1)
                return vector<pair<ll,int> >();
            if(chk(a))
                return vector<pair<11,int> >(1,make pair(a,1));
            11 b=fac(a);
```

```
vector<pair<11,int> >u=run(b),v=run(a/b),r;
            for(int pu=0,pv=0;pu<u.size()||pv<v.size();){</pre>
                if(pu==u.size())
                    r.push_back(v[pv++]);
                else if(pv==v.size())
137
                    r.push back(u[pu++]);
                else if(u[pu].first==v[pv].first)
                    r.push back(make pair(u[pu].first,(u[pu].second+v[pv].second
        ))),++pu,++pv;
                else if(u[pu].first>v[pv].first)
                    r.push_back(v[pv++]);
                else
                    r.push_back(u[pu++]);}
            return r;
        }
147
```

## 4.4 Modular Integer

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Modular Integer.hpp (2886 bytes, 98 lines)

```
#include<bits/stdc++.h>
   using namespace std;
3 template<class T>struct ModularInteger{
       ModularInteger(T t=0):
           v(t){
           if(v<0||v>=p)
               v=(v\%p+p)\%p;
       ModularInteger<T>&operator=(T a){
           v=a;
           if(v<0||v>=p)
               v%=p;
           return*this;
13
       ModularInteger<T>operator-(){
           return v?p-v:0;
       }
```

4.4. MODULAR INTEGER 115

```
ModularInteger<T>&operator+=(ModularInteger<T>a){
           return*this=*this+a;
       }
       ModularInteger<T>&operator = (ModularInteger<T>a){
           return*this=*this-a;
23
       ModularInteger<T>&operator*=(ModularInteger<T>a){
           return*this=*this*a;
       ModularInteger<T>&operator/=(ModularInteger<T>a){
           return*this=*this/a;
       }
       Tν;
       static T p;
   };
   template<class T>ModularInteger<T>pow(ModularInteger<T>a,long long b){
33
       ModularInteger<T>r(1);
       for(;b;b>>=1,a=a*a)
           if(b&1)
               r=r*a;
       return r;
   }
   template<class T>ModularInteger<T>inv(ModularInteger<T>a){
       return pow(a,a.p-2);
43 template<class T>vector<ModularInteger<T> >sqrt(ModularInteger<T>a){
       vector<ModularInteger<T> >r;
       if(!a.v)
           r.push back(ModularInteger<T>(0));
       else if(pow(a,a.p-1>>1).v==1){
           int s=a.p-1,t=0;
           ModularInteger<T>b=1;
           for(;pow(b,a.p-1>>1).v!=a.p-1;b=rand()*1.0/RAND MAX*(a.p-1));
           for(;s%2==0;++t,s/=2);
           ModularInteger<T>x=pow(a,(s+1)/2),e=pow(a,s);
           for(int i=1;i<t;++i,e=x*x/a)</pre>
53
               if(pow(e,1<<t-i-1).v!=1)
                   x=x*pow(b,(1<<i-1)*s);
           r.push back(x);
           r.push back(-x);
       }
```

```
return r;
   }
   template<class T>ModularInteger<T>operator+(ModularInteger<T>a,
       ModularInteger<T>b){
       ModularInteger<T>c(a.v+b.v);
63
       if(c.v>=a.p)
           c.v-=a.p;
       return c;
   }
   template<class T>ModularInteger<T>operator - (ModularInteger<T>a,
       ModularInteger<T>b){
       ModularInteger<T>c(a.v-b.v);
       if(c.v<0)
           c.v+=a.p;
       return c;
   }
73 template<class T>ModularInteger<T>operator*(ModularInteger<T>a,
       ModularInteger<T>b){
       if(typeid(T)!=typeid(int))
           return ModularInteger<T>((a.v*b.v-(long long)(((long double)a.v*b.v
       +0.5)/a.p)*a.p+a.p)%a.p);
       else
           return ModularInteger<T>((long long)a.v*b.v%a.p);
   }
   template<class T>ModularInteger<T>operator/(ModularInteger<T>a,
       ModularInteger<T>b){
       return a*inv(b);
   template<class T>bool operator==(ModularInteger<T>a,ModularInteger<T>b){
       return a.v==b.v;
83
   template<class T>bool operator!=(ModularInteger<T>a,ModularInteger<T>b){
       return a.v!=b.v;
   template<class T>istream&operator>>(istream&s,ModularInteger<T>&a){
       s>>a.v;
       return s;
   template<class T>ostream&operator<<(ostream&s,ModularInteger<T>a){
93
       s<<a.v;
       if(a.v<0||a.v>=a.p)
```

4.5. MÖBIUS FUNCTION 117

```
a.v%=a.p;
return s;
}
template<class T>T ModularInteger<T>::p=1e9+7;
```

#### 4.5 Möbius Function

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Möbius Function.hpp (534 bytes, 21 lines)

```
#include<bits/stdc++.h>
2 using namespace std;
   namespace MobiusFunction{
        vector<int>run(int n){
            vector<int>p,ntp(n+1),u(n+1);
            ntp[1]=1;
            u[1]=1;
            for(int i=2;i<=n;++i){</pre>
                if(!ntp[i])
                    p.push_back(i),u[i]=-1;
                for(int j=0;j<p.size()&&p[j]*i<=n;++j){</pre>
                    ntp[p[j]*i]=1;
12
                    if(i%p[j]==0)
                        break;
                    else
                        u[p[j]*i]=-u[i];
                }
            return u;
        }
   }
```

## 4.6 Primality Test

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Primality Test.hpp (1509 bytes, 52 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   namespace PrimalityTest{
       template < class T>T mul(T x,T y,T z){
           if(typeid(T)==typeid(int))
               return (long long)x*y%z;
           else if(typeid(T)==typeid(long long))
               return (x*y-(T)(((long double)x*y+0.5)/z)*z+z)%z;
           else
9
               return x*y%z;
       template < class T>T pow(T a, T b, T c){
           for(;b;b&1?r=mul(r,a,c):0,b>>=1,a=mul(a,a,c));
           return r;
       template < class T > int run(T a, int c=10){
           if(a==2)
19
               return 1;
           if(a\%2==0||a<2)
               return 0;
           static int pi[]={2,7,61},pl
       []={2,325,9375,28178,450775,9780504,1795265022};
           if(typeid(T)==typeid(int))
               c=3;
           else if(typeid(T)==typeid(long long))
               c=7;
           T u=a-1,t=0,p=1;
           for(;u%2==0;u/=2,++t);
29
           for(int i=0;i<c;++i){</pre>
               if(typeid(T)==typeid(int))
                   p=pi[i]%a;
               else if(typeid(T)==typeid(long long))
                   p=pl[i]%a;
               else
                   p=(p*29+7)%a;
               if(|p||p=1||p=a-1)
                   continue;
               T x=pow(p,u,a);
39
               if(x==1)
```

4.7. PRIME NUMBER 119

```
continue;
    for(int j=0;x!=a-1&&j<t;++j){
        x=mul(x,x,a);
        if(x==1)
            return 0;
     }
     if(x==a-1)
        continue;
     return 0;

49     }
    return 1;
    }
}</pre>
```

## 4.7 Prime Number

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Prime Number.hpp (473 bytes, 18 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   namespace PrimeNumber{
        pair<vector<int>,vector<int> >run(int n){
            vector<int>p,ntp(n+1);
            ntp[1]=1;
            for(int i=2;i<=n;++i){</pre>
8
                if(!ntp[i])
                    p.push_back(i);
                for(int j=0;j<p.size()&&p[j]*i<=n;++j){</pre>
                    ntp[p[j]*i]=1;
                    if(i%p[j]==0)
                        break;
                }
            return make_pair(p,ntp);
        }
18
```

#### 4.8 Primitive Root

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Primitive Root.hpp (3256 bytes, 106 lines)

```
#include<bits/stdc++.h>
 2 using namespace std;
   namespace PrimitiveRoot{
        template<class T>T mul(T x,T y,T z){
            if(typeid(T)==typeid(int))
                return (long long)x*y%z;
            else
                return (x*y-(T)(((long double)x*y+0.5)/z)*z+z)%z;
        template < class T>T pow(T a, T b, T c){
            T r=1;
            for(;b;b&1?r=mul(r,a,c):0,b>>=1,a=mul(a,a,c));
12
            return r;
        }
        template < class T > bool chk(T a, int c=10){
            if(a==1)
                return false;
            T u=a-1,t=0;
            for(;u%2==0;u/=2,++t);
            for(int i=0;i<c;++i){</pre>
                T x=pow(T(rand()*1.0/RAND MAX*(a-2)+1),u,a),y;
                for(int j=0;j<t;++j){</pre>
22
                    y=x;
                    x=mul(x,x,a);
                    if(x==1\&&y!=1\&&y!=a-1)
                        return false;
                if(x!=1)
                    return false;
            }
            return true;
32
        template < class T>T gcd(T a,T b){
            if(a<0)
                a=-a;
```

4.8. PRIMITIVE ROOT 121

```
if(b<0)
               b=-b;
           return b?gcd(b,a%b):a;
       template < class T>T rho(T a, T c){
           T x=double(rand())/RAND MAX*(a-1),y=x;
           for(int i=1,k=2;;){
42
               x=(mul(x,x,a)+c)%a;
               T d=gcd(y-x,a);
               if(d!=1&&d!=a)
                   return d;
               if(y==x)
                   return a;
               if(++i==k)
                   y=x, k=2*k;
           }
52
       template<class T>vector<pair<T,int> >fac(T a){
           if(a==1)
               return vector<pair<T,int> >();
           if(chk(a))
               return vector<pair<T,int> >(1,make pair(a,1));
           while((b=rho(b,T(double(rand())/RAND_MAX*(a-1))))==a);
           vector<pair<T,int> >u=fac(b),v=fac(a/b),r;
           for(int pu=0,pv=0;pu<u.size()||pv<v.size();){</pre>
               if(pu==u.size())
62
                   r.push_back(v[pv++]);
               else if(pv==v.size())
                   r.push back(u[pu++]);
               else if(u[pu].first==v[pv].first)
                   r.push back(make pair(u[pu].first,(u[pu].second+v[pv].second
       ))),++pu,++pv;
               else if(u[pu].first>v[pv].first)
                   r.push back(v[pv++]);
               else
                   r.push_back(u[pu++]);}
72
           return r;
       template<class T>void dfs(vector<pair<T,int> >&f,int i,T now,vector<T>&
       r){
```

```
if(i==f.size()){
                 r.push back(now);
                 return;
            for(int j=0;j<=f[i].second;++j,now*=f[i].first)</pre>
                 dfs(f,i+1,now,r);
82
        template < class T>T run(T a){
            vector<pair<T,int> >fa=fac(a),fpa;
            if(fa.size()==0||fa.size()>2)
                 return -1;
            if(fa.size()==1&&fa[0].first==2&&fa[0].second>2)
                 return -1;
            if(fa.size()==2&&fa[0]!=make_pair(T(2),1))
                 return -1;
            T pa=a:
            for(int i=0;i<fa.size();++i)</pre>
                pa=pa/fa[i].first*(fa[i].first-1);
92
            fpa=fac(pa);
            vector<T>fs;
            dfs(fpa,0,1,fs);
            for(T g=1,f=0;;++g,f=0){
                 for(int i=0;i<fs.size();++i)</pre>
                     if(fs[i]!=pa&&pow(g,fs[i],a)==1){
                         f=1;
                         break;
                     }
                if(!f)
102
                    return g;
            }
        }
    }
```

4.9. SEQUENCE 123

## 4.9 Sequence

#### Numbers n such that a Hadamard matrix of order n exists.

1, 2, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100, 104, 108, 112, 116, 120, 124, 128, 132, 136, 140, 144, 148, 152, 156, 160, 164, 168, 172, 176, 180, 184, 188, 192, 196, 200, 204, 208, 212, 216, 220, 224, 228, 232, 236, 240, ...

Catalan numbers:  $C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$ . Also called Segner numbers.

#### Bell or exponential numbers: number of ways to partition a set of n labeled elements.

1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975, 678570, 4213597, 27644437, 190899322, 1382958545, 10480142147, 82864869804, 682076806159, 5832742205057, 51724158235372, 474869816156751, 4506715738447323, 44152005855084346, 445958869294805289, 4638590332229999353, 49631246523618756274, ...

## CHAPTER 5

Numerical Algorithms

## 5.1 Convolution (Fast Fourier Transform)

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Convolution (Fast Fourier Transform).hpp (1300 bytes, 39 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   namespace Convolution{
        typedef complex<double>T;
 4
        void fft(vector<T>&a,int n,double s,vector<int>&rev){
            T im(0,1);
            double pi=acos(-1);
            for(int i=0;i<n;++i)</pre>
                if(i<rev[i])</pre>
                    swap(a[i],a[rev[i]]);
            for(int i=1, m=2; (1<<i)<=n; ++i, m<<=1){</pre>
                T wm=exp(s*im*2.0*pi/double(m)),w;
                for(int j=(w=1,0);j<n;j+=m,w=1)</pre>
                    for(int k=0;k<(m>>1);++k,w*=wm){
14
                        T u=a[j+k],v=w*a[j+k+(m>>1)];
                        a[j+k]=u+v;
                        a[i+k+(m>>1)]=u-v;
                    }
            }
        }
        vector<double>run(const vector<double>&a,const vector<double>&b){
            int l=ceil(log2(a.size()+b.size()-1)),n=1<<1;</pre>
            vector<int>rv;
            for(int i=(rv.resize(n),0);i<n;++i)</pre>
24
                rv[i]=(rv[i>>1]>>1)|((i&1)<<(1-1));
            vector<T>ta(n),tb(n);
            copy(a.begin(),a.end(),ta.begin());
            copy(b.begin(),b.end(),tb.begin());
            fft(ta,n,1,rv);
            fft(tb,n,1,rv);
            for(int i=0;i<n;++i)</pre>
                ta[i]*=tb[i];
            fft(ta,n,-1,rv);
            vector<double>c(a.size()+b.size()-1);
34
            for(int i=0;i<c.size();++i)</pre>
```

```
c[i]=real(ta[i])/n;
    return c;
}
```

## 5.2 Convolution (Karatsuba Algorithm)

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Convolution (Karatsuba Algorithm).hpp (1416 bytes, 43 lines)

```
1 #include<bits/stdc++.h>
   using namespace std;
   namespace Convolution{
        template<class T>void kar(T*a,T*b,int n,int 1,T**r){
            T*rl=r[1],*rll=r[1-1];
            for(int i=0;i<2*n;++i)</pre>
                *(rl+i)=0;
            if(n<=30){
                for(int i=0;i<n;++i)</pre>
                    for(int j=0;j<n;++j)</pre>
11
                        *(rl+i+j)+=*(a+i)**(b+j);
                return;
            }
            kar(a,b,n>>1,l-1,r);
            for(int i=0;i<n;++i)</pre>
                *(rl+i)+=*(rll+i),*(rl+i+(n>>1))+=*(rll+i);
            kar(a+(n)>1),b+(n)>1),n>>1,l-1,r);
            for(int i=0;i<n;++i)</pre>
                *(rl+i+n)+=*(rll+i),*(rl+i+(n>>1))+=*(rll+i);
            for(int i=0;i<(n>>1);++i){
                *(rl+(n<<1)+i)=*(a+(n>>1)+i)-*(a+i);
21
                *(rl+i+(n>>1)*5)=*(b+i)-*(b+(n>>1)+i);
            kar(rl+(n<<1),rl+(n>>1)*5,n>>1,l-1,r);
            for(int i=0;i<n;++i)</pre>
                *(rl+i+(n>>1))+=*(rll+i);}
        template < class T > vector < T > run(vector < T > a, vector < T > b) {
            int l=ceil(log2(max(a.size(),b.size()))+1e-8);
            vector<T>rt(a.size()+b.size()-1);
```

```
a.resize(1<<1);
             b.resize(1<<1);</pre>
31
             T**r=new T*[1+1];
             for(int i=0;i<=1;++i)</pre>
                 r[i]=new T[(1<< i)*3];
             kar(&a[0],&b[0],1<<1,1,r);
             for(int i=0;i<rt.size();++i)</pre>
                 rt[i]=*(r[1]+i);
             for(int i=0;i<=1;++i)</pre>
                 delete r[i];
             delete r;
41
             return rt;
        }
    }
```

## 5.3 Convolution (Number Theoretic Transform)

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Convolution (Number Theoretic Transform).hpp (1620 bytes, 51 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   namespace Convolution{
        typedef long long T;
        T pow(T a, T b, T c){
            T r=1;
7
            for(;b;b&1?r=r*a%c:0,b>>=1,a=a*a%c);
            return r;
        void ntt(vector<T>&a,int n,int s,vector<int>&rev,T p,T g){
            g=s==1?g:pow(g,p-2,p);
            vector<T>wm;
            for(int i=0;1<<i<=n;++i)</pre>
                wm.push_back(pow(g,(p-1)>>i,p));
            for(int i=0;i<n;++i)</pre>
                if(i<rev[i])</pre>
17
                    swap(a[i],a[rev[i]]);
            for(int i=1, m=2;1<<i<=n;++i, m<<=1){</pre>
                vector<T>wmk(1,1);
```

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```
for(int k=1;k<(m>>1);++k)
                    wmk.push back(wmk.back()*wm[i]%p);
                for(int j=0;j<n;j+=m)</pre>
                    for(int k=0;k<(m>>1);++k){
                        T u=a[j+k], v=wmk[k]*a[j+k+(m>>1)]%p;
                        a[j+k]=u+v;
                        a[j+k+(m>>1)]=u-v+p;
27
                        if(a[j+k]>=p)
                            a[j+k]-=p;
                        if(a[j+k+(m>>1)]>=p)
                            a[j+k+(m>>1)]-=p;
                    }
            }
        vector<T>run(vector<T>a, vector<T>b, T p=15*(1<<27)+1, T g=31){</pre>
            int tn,l=ceil(log2(tn=a.size()+b.size()-1)),n=1<<1;</pre>
            vector<int>rv;
            for(int i=(rv.resize(n),0);i<n;++i)</pre>
37
                rv[i]=(rv[i>>1]>>1)|((i&1)<<(1-1));
            a.resize(n);
            b.resize(n);
            ntt(a,n,1,rv,p,g);
            ntt(b,n,1,rv,p,g);
            for(int i=0;i<n;++i)</pre>
                a[i]=a[i]*b[i]%p;
            ntt(a,n,-1,rv,p,g);
            n=pow(n,p-2,p);
            for(T&v:a)
47
                v=v*n%p;
            return a.resize(tn),a;
        }
    }
```

## 5.4 Fraction

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Fraction.hpp (2217 bytes, 100 lines)

```
using namespace std;
   template<class T>struct Fraction{
       T p,q;
       int s;
       T gcd(T a,T b){
           return b?gcd(b,a%b):a;
9
       void reduce(){
           T d=gcd(p,q);
           p/=d;
           q/=d;
           if(p==0)
               s=0;
       Fraction(int _s=0,T _p=0,T _q=1):
           s(_s),p(_p),q(_q){
           reduce();
19
       }
       Fraction(string a){
           if(a[0]=='-'){
               s=-1;
               a=a.substr(1,a.size()-1);
           }else if(a[0]=='+'){
               s=1;
               a=a.substr(1,a.size()-1);
           }else
               s=1;
29
           stringstream ss;
           char tc;
           ss<<a;
           ss>>p>>tc>>q;
           reduce();
       Fraction(const char*a){
           *this=Fraction(string(a));
       Fraction<T>&operator=(string a){
           return*this=Fraction<T>(a);
39
       Fraction<T>&operator=(const char*a){
           return*this=Fraction<T>(a);
```

5.4. FRACTION 131

```
}
   };
   template<class T>ostream&operator<<(ostream&s,const Fraction<T>&a){
       if(a.s==-1)
           s<<'-';
       return s<<a.p<<'/'<<a.q;</pre>
49
   template<class T>istream&operator>>(istream&s,Fraction<T>&a){
       string t;
       s>>t;
       a=t;
       return s;
   }
   template<class T>vector<string>real(const Fraction<T>&a){
       vector<string>r;
       stringstream ss;
59
       string st;
       if(a.s<0)
           r.push_back("-");
       else
           r.push_back("+");
       T p=a.p,q=a.q;
       ss<<p/q;
       ss>>st;
       r.push back(st);
       p\%=q;
69
       st.clear();
       map<T,int>mp;
       while(true){
           if(p==0){
               r.push_back(st);
               r.push_back("");
               return r;
           if(mp.count(p)){
               r.push_back(st.substr(0,mp[p]));
               r.push_back(st.substr(mp[p],st.size()-mp[p]));
79
               return r;
           }
           p*=10;
           mp[p/10]=st.size();
```

```
st.push_back('0'+p/q);
           p%=q;
       return r;
89 template<class T>string decimal(const Fraction<T>&a){
       string r;
       vector<string>t=real(a);
       if(t[0]=="-")
           r.push_back('-');
       r+=t[1];
       if(t[2].size()||t[3].size())
           r+="."+t[2];
       if(t[3].size())
           r+="("+t[3]+")";
99
       return r;
   }
```

## 5.5 Integer

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Integer.hpp (6378 bytes, 269 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   struct Integer operator+(Integer a,Integer b);
   Integer operator+(Integer a,int b);
   Integer operator—(Integer a,Integer b);
   Integer operator*(Integer a,Integer b);
   Integer operator*(Integer a,Integer b);
   Integer operator/(Integer a,Integer b);
   Integer operator%(Integer a,Integer b);
10 Integer operator%(Integer a,int b);
   Integer operator%(Integer a,long long b);
   bool operator!=(Integer a,int b);
   bool operator<=(Integer a,int b);</pre>
   struct Integer{
       operator bool(){
           return *this!=0;
```

5.5. INTEGER 133

```
Integer(long long a=0){
           if(a<0){
20
               s=-1;
               a=-a;
           }else
               s=a!=0;
           do{
               d.push_back(a%B);
               a/=B;
           }while(a);
       Integer(string a){
           s=(a[0]=='-')?-1:(a!="0");
30
           for(int i=a.size()-1;i>=(a[0]=='-');i-=L){
               int t=0,j=max(i-L+1,int(a[0]=='-'));
               for(int k=j;k<=i;++k)</pre>
                   t=t*10+a[k]-'0';
               d.push back(t);
           }
       }
       Integer(const Integer&a){
           d=a.d;
40
           s=a.s;
       Integer&operator=(long long a){
           return*this=Integer(a);
       Integer&operator+=(Integer a){
           return*this=*this+a;
       Integer&operator—=(Integer a){
           return*this=*this-a;
50
       Integer&operator*=(Integer a){
           return*this=*this*a;
       }
       Integer&operator/=(Integer a){
           return*this=*this/a;
       Integer&operator%=(Integer a){
```

```
return*this=*this%a;
        Integer&operator++(){
60
            return*this=*this+1;
        }
        operator string()const{
            string r;
            for(int i=0;i<d.size();++i){</pre>
                stringstream ts;
                ts<<d[i];
                string tt;
                ts>>tt;
                reverse(tt.begin(),tt.end());
70
                while(i+1!=d.size()&&tt.size()<L)</pre>
                    tt.push_back('0');
                r+=tt;
            }
            reverse(r.begin(),r.end());
            return r;
        }
        int s;
        vector<int>d;
80
        static const int B=1e8,L=8;
    };
    string str(const Integer&a){
        return string(a);
   bool operator<(Integer a,Integer b){</pre>
        if(a.s!=b.s)
            return a.s<b.s;</pre>
        if(a.d.size()!=b.d.size())
            return (a.s!=1)^(a.d.size()<b.d.size());</pre>
        for(int i=a.d.size()-1;i \ge 0;--i)
90
            if(a.d[i]!=b.d[i])
                return (a.s!=1)^(a.d[i]<b.d[i]);</pre>
        return false;
   bool operator>(Integer a,Integer b){
        return b<a;
   bool operator<=(Integer a,Integer b){</pre>
```

5.5. INTEGER 135

```
return !(a>b);
100
    bool operator>=(Integer a,Integer b){
        return !(a<b);</pre>
    bool operator==(Integer a,Integer b){
        return !(a<b)&&!(a>b);
    bool operator!=(Integer a,Integer b){
        return !(a==b);
110 istream&operator>>(istream&s,Integer&a){
        string t;
        s>>t;
        a=Integer(t);
        return s;
    }
    ostream&operator<<(ostream&s,Integer a){</pre>
        if(a.s==-1)
             s<<'-';
        for(int i=a.d.size()-1;i \ge 0;--i){
120
            if(i!=a.d.size()-1)
                 s<<setw(Integer::L)<<setfill('0');</pre>
             s<<a.d[i];
        s<<setw(0)<<setfill(' ');</pre>
        return s;
    void dzero(Integer&a){
        while(a.d.size()>1&&a.d.back()==0)
            a.d.pop_back();
130
    Integer operator—(Integer a){
        a.s*=-1;
        if(a.d.size()==1&&a.d[0]==0)
             a.s=1;
        return a;
    }
    Integer operator+(Integer a,int b){
        return a+Integer(b);
    }
```

```
Integer operator*(Integer a,int b){
        return a*Integer(b);
    }
    Integer operator%(Integer a,int b){
        return a%Integer(b);
    Integer operator%(Integer a,long long b){
        return a%Integer(b);
    bool operator!=(Integer a,int b){
150
        return a!=Integer(b);
    bool operator<=(Integer a,int b){</pre>
        return a<=Integer(b);</pre>
    Integer operator+(Integer a,Integer b){
        if(a.s*b.s!=-1){
             Integer c;c.s=a.s?a.s:b.s;
             c.d.resize(max(a.d.size(),b.d.size())+1);
             for(int i=0;i<c.d.size()-1;++i){</pre>
                 if(i<a.d.size())</pre>
160
                     c.d[i]+=a.d[i];
                 if(i<b.d.size())</pre>
                     c.d[i]+=b.d[i];
                 if(c.d[i]>=Integer::B){
                     c.d[i]-=Integer::B;
                     ++c.d[i+1];
                 }
             }
            dzero(c);
170
             return c;
        return a-(-b);
    Integer operator—(Integer a,Integer b){
        if(a.s*b.s==1){
             if(a.s==-1)
                 return (-b)-(-a);
             if(a<b)</pre>
                 return -(b-a);
180
             if(a==b)
```

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```
return 0;
            for(int i=0;i<b.d.size();++i){</pre>
                a.d[i]-=b.d[i];
                if(a.d[i]<0){
                     a.d[i]+=Integer::B;
                     --a.d[i+1];
                }
             }
            dzero(a);
190
            return a;
        return a+(-b);
    Integer operator*(Integer a,Integer b){
        vector<long long>t(a.d.size()+b.d.size());
        for(int i=0;i<a.d.size();++i)</pre>
             for(int j=0;j<b.d.size();++j)</pre>
                t[i+j]+=(long long)a.d[i]*b.d[j];
        for(int i=0;i<t.size()-1;++i){</pre>
            t[i+1]+=t[i]/Integer::B;
200
            t[i]%=Integer::B;
        }
        Integer c;
        c.s=a.s*b.s;c.d.resize(t.size());
        copy(t.begin(),t.end(),c.d.begin());
        dzero(c);
        return c;
    }
    Integer div2(Integer a){
210
        for(int i=a.d.size()-1;i>=0;--i){
             if(i)
                a.d[i-1]+=(a.d[i]&1)*Integer::B;
            a.d[i]>>=1;
        dzero(a);
        if(a.d.size()==1&&a.d[0]==0)
            a.s=0;
        return a;
220 Integer operator/(Integer a,Integer b){
        if(!a.s)
```

```
return 0;
         if(a.s<0)
             return-((-a)/b);
         if(a<b)</pre>
             return 0;
         Integer l=1,r=1;
        while(r*b<=a)</pre>
             r=r*2;
230
        while(l+1<r){</pre>
             Integer m=div2(l+r);
             if(m*b>a)
                 r=m;
             else
                 1=m;
         return 1;
    Integer operator%(Integer a,Integer b){
         return a-a/b*b;
240
    Integer gcd(Integer a,Integer b){
         Integer r=1;
         while(a!=0&&b!=0){
             if(!(a.d[0]&1)&&!(b.d[0]&1)){
                 a=div2(a);
                 b=div2(b);
                 r=r*2;
             }else if(!(a.d[0]&1))
250
                 a=div2(a);
             else if(!(b.d[0]&1))
                 b=div2(b);
             else{
                 if(a<b)</pre>
                     swap(a,b);
                 a=div2(a-b);
             }
         if(a!=0)
260
             return r*a;
         return r*b;
    }
```

```
int length(Integer a){
    a.s=1;
    return string(a).size();
}
int len(Integer a){
    return length(a);
}
```

## 5.6 Linear Programming

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Linear Programming.hpp (2522 bytes, 89 lines)

```
1 #include<bits/stdc++.h>
   using namespace std;
   struct LinearProgramming{
       const double E;
       int n,m,p;
       vector<int>mp,ma,md;
       vector<vector<double> >a;
       vector<double>res;
       LinearProgramming(int _n,int _m):
            n(_n),m(_m),p(0),a(n+2,vector<double>(m+2)),mp(n+1),ma(m+n+2),md(m+n+2)
       +2), res(m+1), E(1e-8){
11
       void piv(int 1,int e){
            swap(mp[1],md[e]);
            ma[mp[1]]=1;
            ma[md[e]]=-1;
            double t=-a[1][e];
            a[1][e]=-1;
            vector<int>qu;
            for(int i=0;i<=m+1;++i)</pre>
                if(fabs(a[1][i]/=t)>E)
                    qu.push_back(i);
21
            for(int i=0;i<=n+1;++i)</pre>
                if(i!=1&&fabs(a[i][e])>E){
                    t=a[i][e];
                    a[i][e]=0;
```

```
for(int j=0;j<qu.size();++j)</pre>
                         a[i][qu[j]]+=a[l][qu[j]]*t;
                }
            if(-p==1)
                p=e;
31
            else if(p==e)
                p=-1;
        int opt(int d){
            for(int l=-1,e=-1;;piv(l,e),l=-1,e=-1){
                for(int i=1;i<=m+1;++i)</pre>
                     if(a[d][i]>E){
                         e=i;
                         break;
                if(e==-1)
41
                     return 1;
                double t;
                for(int i=1;i<=n;++i)</pre>
                     if(a[i][e] < -E&&(l==-1||a[i][0]/-a[i][e] < t))
                         t=a[i][0]/-a[i][e],l=i;
                if(1==-1)
                     return 0;
            }
51
        double&at(int x,int y){
            return a[x][y];
        vector<double>run(){
            for(int i=1;i<=m+1;++i)</pre>
                ma[i]=-1,md[i]=i;
            for(int i=m+2;i<=m+n+1;++i)</pre>
                ma[i]=i-(m+1), mp[i-(m+1)]=i;
            double t;
            int l=-1;
            for(int i=1;i<=n;++i)</pre>
61
                if(l==-1||a[i][0]<t)
                     t=a[i][0],l=i;
            if(t<-E){
                for(int i=1;i<=n;++i)</pre>
                     a[i][m+1]=1;
```

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```
a[n+1][m+1]=-1;
                p=m+1;
                piv(1,m+1);
                if(!opt(n+1)||fabs(a[n+1][0])>E)
                     return vector<double>();
71
                if(p<0)
                     for(int i=1;i<=m;++i)</pre>
                         if(fabs(a[-p][i])>E){
                             piv(-p,i);
                             break:
                for(int i=0;i<=n;++i)</pre>
                     a[i][p]=0;
            if(!opt(0))
81
                return vector<double>();
            res[0]=a[0][0];
            for(int i=1;i<=m;++i)</pre>
                if(ma[i]!=−1)
                     res[i]=a[ma[i]][0];
            return res;
        }
    };
```

## 5.7 Linear System

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Linear System.hpp (1477 bytes, 56 lines)

```
1 #include<bits/stdc++.h>
using namespace std;
template<class T>struct LinearSystem{
   int n;
   vector<vector<T> >a;
   vector<iint>main,pos;
   vector<T> ans;
   int cmp(T a){
      if(typeid(T)==typeid(double)||typeid(T)==typeid(long double)||
      typeid(T)==typeid(float)){
```

```
if(a<-1e-8)
                    return -1;
11
                if(a>1e-8)
                    return 1;
                return 0;
            }
            if(a<0)
                return -1;
            if(a>0)
                return 1;
            return 0;
21
        T&at(int i,int j){
            return a[i][j];
        vector<T>&at(int i){
            return a[i];
        }
        LinearSystem(int _n):
            n(_n),a(n+1,vector<T>(n+1)),main(n+1),pos(n+1),ans(n){
        vector<T>run(){
31
            for(int i=1;i<=n;++i){</pre>
                int j=1;
                for(;j<=n&&!cmp(a[i][j]);++j);</pre>
                if(j<=n){
                    main[i]=j;
                    pos[j]=i;
                    T t=a[i][j];
                    for(int k=0;k<=n;++k)</pre>
                         a[i][k]/=t;
                    for(int k=1;k<=n;++k)</pre>
41
                        if(k!=i&&cmp(a[k][j])){
                             t=a[k][j];
                             for(int 1=0;1<=n;++1)
                                 a[k][l]-=a[i][l]*t;
                        }
                }
            for(int i=1;i<=n;++i){</pre>
                if(!pos[i])
```

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#### 5.8 Matrix

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Matrix.hpp (1457 bytes, 51 lines)

```
#include<bits/stdc++.h>
    template<class T,int N>struct Matrix{
        Matrix(T t=0){
 4
            for(int i=0;i<N;++i)</pre>
                for(int j=0;j<N;++j)</pre>
                    u[i][i]=i==j?t:0;
        T u[N][N];
    };
    template<class T,int N>Matrix<T,N>operator+(const Matrix<T,N>&a,const
       Matrix<T,N>&b){
        Matrix<T,N>c;
        for(int i=0;i<N;++i)</pre>
            for(int j=0;j<N;++j)</pre>
14
                c.u[i][j]=a.u[i][j]+b.u[i][j];
        return c;
    }
    template<class T,int N>Matrix<T,N>operator*(const Matrix<T,N>&a,const
       Matrix<T,N>&b){
        Matrix<T,N>c;
        for(int i=0;i<N;++i)</pre>
            for(int j=0;j<N;++j)</pre>
                for(int k=0;k<N;++k)</pre>
                    c.u[i][j]+=a.u[i][k]*b.u[k][j];
        return c;
24
    template<class T,int N>Matrix<T,N>operator*(const Matrix<T,N>&a,const T&b){
```

```
Matrix<T,N>c=a;
        for(int i=0;i<N;++i)</pre>
            for(int j=0;j<N;++j)</pre>
                c.u[i][j]*=b;
        return c;
    }
   template<class T,int N>Matrix<T,N>operator/(const Matrix<T,N>&a,const T&b){
        Matrix<T,N>c=a;
34
        for(int i=0;i<N;++i)</pre>
            for(int j=0;j<N;++j)</pre>
                c.u[i][j]/=b;
        return c;
    }
   template<class T,int N>Matrix<T,N>pow(Matrix<T,N>a,long long b){
        Matrix<T,N>r(1);
        for(;b;a=a*a,b>>=1)
            if(b&1)
                r=r*a;
44
        return r;
    }
   template<class T,int N>ostream&operator<<(ostream&s,const Matrix<T,N>a){
        for(int i=0;i<N;++i)</pre>
            for(int j=0;j<N;++j)</pre>
                s<<a.u[i][j]<<(j+1==N?'\n':' ');</pre>
        return s;
    }
```

## 5.9 Polynomial Interpolation

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Polynomial Interpolation.hpp (372 bytes, 15 lines)

```
#include<bits/stdc++.h>
using namespace std;
template<class T>T PolynomialInterpolation(vector<T>x,vector<T>y,T x0){
   T r=0;
   for(int i=0;i<x.size();++i){
        T p=1,q=1;
        for(int j=0;j<x.size();++j)</pre>
```

# CHAPTER 6

String Algorithms

### 6.1 Aho-Corasick Automaton

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Aho-Corasick Automaton.hpp (1369 bytes, 50 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   struct AhoCorasickAutomaton{
       struct node{
           node(int m):
 5
               tr(m),fail(0),cnt(0){
           vector<node*>tr;
           node*fail;
           int cnt;
       };
       int m;
       node*root;
       vector<node*>all;
       AhoCorasickAutomaton(int _m):
15
           m(_m),root(new node(m)),all(1,root){
       ~AhoCorasickAutomaton(){
           for(int i=0;i<all.size();++i)</pre>
               delete all[i];
       node*insert(int*s){
           node*p;
           for(p=root;*s!=-1;p=p->tr[*(s++)])
               if(!p->tr[*s])
25
                    p->tr[*s]=new node(m);
           return p;
       void build(){
           queue<node*>qu;
            for(int i=0;i<m;++i)</pre>
               if(!root->tr[i])
                    root->tr[i]=root;
               else
                    root->tr[i]->fail=root,qu.push(root->tr[i]);
35
```

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### 6.2 Factor Oracle

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Factor Oracle.hpp (569 bytes, 16 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   template < class T, int N, int M, T D>struct FactorOracle{
       void insert(T*s,int n){
            memset(tr,(lrs[0]=0,sp[0]=-1),4*M);
            for(int i=0,j,c=s[i]-D,u,v;i<n;c=s[++i]-D){</pre>
                memset(tr+i+1,(lrs[i+1]=0)-1,4*M);
                for(j=i;j>-1&&tr[j][c]<0;tr[j][c]=i+1,j=sp[u=j]);</pre>
                if(v=sp[i+1]=j<0?0:tr[j][c]){</pre>
10
                    for(v=v-1==sp[u]?u:v-1;sp[u]!=sp[v];v=sp[v]);
                    lrs[i+1]=min(lrs[u],lrs[v])+1;
                }
            }
       int sp[N+1],lrs[N+1],tr[N+1][M];
   };
```

## 6.3 Longest Common Substring

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Longest Common Substring.hpp (1181 bytes, 28 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   template<class T,int N,int M,T D>struct LongestCommonSubstring{
       void ins(int c){
4
           memset(tr+i+1,(lrs[i+1]=0)-1,4*M);
           for(j=i;j>-1&&((v=tr[j][c])>=l1+2&&v<=l1+lb+1||v<0);tr[j][c]=i+1+lb</pre>
       ,j=sp[u=j]);
           if(v=sp[i+1]=j<0?0:tr[j][c]-(tr[j][c]>l1+1)*lb){
               for(v=v-1==sp[u]?u:v-1;sp[u]!=sp[v];v=sp[v]);
               lrs[i+1]=min(lrs[u],lrs[v])+1;
           if(sp[i+1]<=11)
               tm[sp[i+1]]=max(tm[sp[i+1]],lrs[i+1]);
       int run(vector<pair<int,T*> >s){
14
           swap(s[0],*min element(s.begin(),s.end()));
           l1=s[k=lb=0].first;
           memset(mi,63,4*N+4);
           memset(tr,(lrs[0]=0,sp[0]=-1),4*M+4);
           for(i=0;i<11;ins(*(s[0].second+i)-D),++i);</pre>
           for(k=1,ins(M);k<s.size();lb+=s[k++].first){</pre>
               memset(tm,0,4*N+4);
               for(i=11+1;i-11-1<s[k].first;ins(*(s[k].second+i-11-1)-D),++i)
       ;
               for(i=11;i;mi[i]=min(mi[i],tm[i]),tm[sp[i]]=max(tm[sp[i]],lrs[i
       |*!!tm[i]),--i);
24
           return min(*max_element(mi+1,mi+l1+1),l1);
       int sp[2*N+2],1rs[2*N+2],tr[2*N+2][M+1],mi[N+1],tm[N+1],11,1b,i,j,k,u,v
   };
```

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### 6.4 Palindromic Tree

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Palindromic Tree.hpp (1327 bytes, 50 lines)

```
#include<bits/stdc++.h>
 2 using namespace std;
   template<class T>struct PalindromicTree{
       struct node{
           node(int m, node*f, int 1):
               nxt(m),fail(f),len(1){
           }
           vector<node*>nxt;
           node*fail;
           T val;
           int len;
12
       }*root;
       int m;
       vector<int>str;
       vector<node*>all;
       PalindromicTree(int _m):
           m(m){
           node*n0=new node(m,0,-2),*n1=new node(m,n0,-1),*n2=new node(m,n1,0)
       ;
           all.push back(n0);
           all.push back(n1);
           all.push_back(n2);
22
           fill(n0->nxt.begin(),n0->nxt.end(),n2);
           root=n1;
       }
       ~PalindromicTree(){
           for(int i=0;i<all.size();++i)</pre>
               delete all[i];
       node*find(node*x){
           while(x->fail&&str[str.size()-x->len-2]!=str[str.size()-1])
               x=x->fail;
32
           return x;
       node*insert(node*p,int c,T v){
```

```
if(p==root)
               str=vector < int > (1,-1);
           str.push_back(c);
           p=find(p);
           if(!p->nxt[c]){
               node*np=(p->nxt[c]=new node(m,find(p->fail)->nxt[c],p->len+2))
       ;
               all.push back(np);
42
           p->nxt[c]->val+=v;
           return p->nxt[c];
       void count(){
           for(int i=all.size()-1;i>=1;--i)
               all[i]->fail->val+=all[i]->val;
       }
   };
```

## 6.5 String Matching

#### Description

Find the occurrences of a pattern in a text using KMP algorithm. The prefix array is also provided.

#### **Methods**

template <class t="">StringMatching<t>::StringMatching(T*p,int t=1);</t></class>		
Description	construct an object of SuffixMatching for a giv-	
	en pattern	
Parameters	Description	
T	type of character	
t	whether to optimize the prefix array, do not	
	turn it on if you want to use the prefix array	
p	pattern, indexed from one, ended by zero	
Time complexity	$\Theta( p )$	
Space complexity	$\Theta( p )$	
Return value	an object of StringMatching	

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template <class t="">int StringMatching<t>::run(T*t,int k=0);</t></class>		
Description	given an occurence of the pattern in a text, find	
	the next occurrence	
Parameters	Description	
t	text, indexed from one, ended by zero	
k	start index of the last occurence of the pattern,	
	use zero if there is none	
Time complexity	O( t )	
Space complexity	$\Theta(1)$	
Return value	start index of the next occurence of the pattern	

#### **Fields**

template <class t="">vector<int>StringMatching<t>::f;</t></int></class>	
<b>Description</b> prefix array of KMP algorithm, indexed from	
one	

#### **Performance**

Problem	Constraints	Time	Memory	Date
POJ 3461	$ p  = 10^4,  t  =$	141 ms	1340 kB	2016-02-14
	$10^{6}$			

#### References

Title	Author
Fast Pattern Matching in Strings	Donald E. Knuth, James H. Morris, Vaughan R.
	Pratt

#### Code

### String Matching.hpp (686 bytes, 25 lines)

```
#include<vector>
using namespace std;
template < class T > struct StringMatching{
    StringMatching(T*p,int t=1):
        b(2,p[1]),f(2),1(2){
        for(int i=0;p[1]?1:(--1,0);b.push_back(p[1++])){
            for(;i&&p[i+1]!=p[1];i=f[i]);
            f.push_back(i=i+(p[i+1]==p[1]));
        }
```

```
for(int i=2;t&&i<1;++i)</pre>
10
               if(p[f[i]+1]==p[i+1])
                    f[i]=f[f[i]];
       int run(T*t,int k=0){
           for(int i=k?k+l:1,j=k?f[l]:0;t[i];++i){
               for(;j&&b[j+1]!=t[i];j=f[j]);
                if((j+=b[j+1]==t[i])==1)
                    return i-l+1;
            }
           return 0;
20
       int 1;
       vector<T>b;
       vector<int>f;
   };
```

## 6.6 Suffix Array (DC3 Algorithm)

### Description

Construct a suffix array and it's height array from a given string using DC3 algorithm.

#### **Methods**

template <class d="" m,t="" t,int="">SuffixArray<t,m,d>::SuffixArray(T*s,int n);</t,m,d></class>		
Description	construct an object of SuffixArray and in the	
	mean time construct the suffix array and	
	height array	
Parameters	Description	
T	type of character, usually char	
M	size of alphabet	
D	offset of alphabet, use 'a' for lowercase letters	
S	string from which to build a suffix array, in-	
	dexed from one	
n	length of s	
Time complexity	$\Theta(n+M)$	
Space complexity	$\Theta(10n+M)$	
Return value	an object of SuffixArray	

#### **Fields**

template <class d="" m,t="" t,int="">int*SuffixArray<t,m,d>::sa;</t,m,d></class>		
Description         suffix array, indexed from one		
template <class d="" m,t="" t,int="">int*SuffixArray<t,m,d>::ht;</t,m,d></class>		
<b>Description</b> height array, indexed from one		

#### **Performance**

Problem	Constraints	Time	Memory	Date
UOJ 35	$N = 10^5, M =$	416 ms (18+ cas-	4248 kB	2016-02-14
	26	es)		

#### References

Title	Author
后缀数组——处理字符串的有力工具	罗穗骞

#### Code

#### Suffix Array (DC3 Algorithm).hpp (2656 bytes, 82 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   template<class T,int M,int D>struct SuffixArray{
       int*sa,*ht,*rk,*ts,*ct,*st;
5
       SuffixArray(T*s,int n){
           crt(st,n),crt(sa,n),crt(ht,n);
           crt(rk,n),crt(ts,n),crt(ct,max(n,M));
           for(int i=1;i<=n;++i)st[i]=s[i]-D+1;</pre>
           dc3(st,n,M,sa,rk);
           for(int i=1;i<=n;++i){</pre>
               if(rk[i]==1){ht[1]=0;continue;}
               int&d=ht[rk[i]]=max(i==1?0:ht[rk[i-1]]-1,0);
               for(;i+d<=n&&sa[rk[i]-1]+d<=n</pre>
                   &&st[i+d]==st[sa[rk[i]-1]+d];++d);
15
            }
       ~SuffixArray(){
           del(sa),del(ht),del(rk);
           del(ts),del(ct),del(st);
```

```
}
       void crt(int*&a,int n){
           a=new int[n+1];
       void del(int*a){
25
           delete a;
       }
       #define fc(i)(p0[i]+d>n||!p0[i]?0:s[p0[i]+d])
       int cmp(int*p0,int i,int*s,int n){
           for(int d=0;d<3;++d)
                if(fc(i)!=fc(i-1))return 1;
           return 0;
       }
       void sot(int*p0,int n0,int*s,int n,int m,int d){
           memset(ct,0,(m+1)*4);
           for(int i=1;i<=n0;++i)++ct[fc(i)];</pre>
35
            for(int i=1;i<=m;++i)ct[i]+=ct[i-1];</pre>
            for(int i=n0;i>=1;--i)ts[ct[fc(i)]--]=p0[i];
           memcpy(p0+1,ts+1,n0*4);
       }
       #define fc(d)\
           if(s[i+d]!=s[j+d])return s[i+d]<s[j+d];\
            if(i==n-d)|_{j==n-d}return i==n-d;
       bool cmp(int*s,int n,int*r,int i,int j){
           fc(0)
45
           if(j%3==1)return r[i+1]<r[j+1];</pre>
           fc(1)
           return r[i+2]<r[j+2];
       }
       #undef fc
       void dc3(int*s,int n,int m,int*a,int*r){
            int n0=n-(n/3)+1, *a0, *s0, i, j=0, k=n/3+bool(n%3)+1,1;
            crt(s0,n0),s0[k]=1,crt(a0,n0+1),a0[k]=0;
            for(i=1;i<=n;i+=3)a0[++j]=i,a0[j+k]=i+1;</pre>
           for(i=2;i>=0;--i)sot(a0,n0,s,n,m,i);
           for(r[a0[1]]=1,i=2;i<=n0;++i)</pre>
55
                r[a0[i]]=r[a0[i-1]]+cmp(a0,i,s,n);
           for(i=1, j=0; i<=n; i+=3)
                s0[++j]=r[i],s0[j+k]=r[i+1];
           if(r[a0[n0]]==n0){
                memcpy(r+1, s0+1, n0*4);
```

```
for(i=1;i<=n0;++i)a0[a[i]=r[i]]=i;</pre>
            }else
                dc3(s0,n0,r[a0[n0]],a0,a);
            for(i=1,j=0;i<=n;i+=3)</pre>
                r[i]=a[++j],r[i+1]=a[j+k];
65
            if(j=0,n%3==0)
                s0[++j]=n;
            for(i=1;i<=n0;++i)</pre>
                if(a0[i]>=k)
                     a0[i]=(a0[i]-k)*3-1;
                else
                     if((a0[i]=3*a0[i]-2)!=1)s0[++j]=a0[i]-1;
           sot(s0,j,s,n,m,0);
           for(i=1,k=2,l=0;i<=j||k<=n0;)</pre>
                if(k>n0||i<=j&&cmp(s,n,r,s0[i],a0[k]))</pre>
75
                     a[++1]=s0[i++];
                else
                     a[++1]=a0[k++];
            for(i=1;i<=n;++i)r[a[i]]=i;</pre>
            del(a0),del(s0);
        }
    };
```

## 6.7 Suffix Array (Factor Oracle)

### Description

Use a factor oracle to construct a suffix array and it's height array from a given string. It is theoretically slow, but usually fast in practice. Object of it should be static since it has large data members.

### Methods

template <class d="" m,t="" n,int="" t,int="">SuffixArray<t,n,m,d>::SuffixArray();</t,n,m,d></class>		
Description	construct an object of SuffixArray	
Parameters	Description	
T	type of character, usually char	
N	maximum length of input string	
M	size of alphabet	
D	offset of alphabet, use 'a' for lowercase letters	
Time complexity	$\Theta(1)$	
Space complexity	$\Theta((M+13)N)$	
Return value	an object of SuffixArray	

template <class d="" m,t="" n,int="" t,int="">void SuffixArray<t,n,m,d>::build(T*s,int n);</t,n,m,d></class>		
Description	build suffix array and height array	
Parameters	Description	
S	string from which to build a suffix array, in-	
	dexed from zero	
n	length of s	
Time complexity	O((M+n)n)	
Space complexity	$\Theta(n)$	
Return value	none	

## **Fields**

template <class d="" m,t="" t,int="">int SuffixArray<t,m,d>::sa[N+1];</t,m,d></class>		
<b>Description</b> suffix array, indexed from one		
template <class d="" m,t="" t,int="">int SuffixArray<t,m,d>::ht[N+1];</t,m,d></class>		
template <class d="" m,t="" t,int="">int SuffixArra</class>	ay <t,m,d>::ht[N+1];</t,m,d>	

### Performance

Problem	Constraints	Time	Memory	Date
Tyvj 1860	N =	1154 ms (10 cas-	33012 kB	2016-02-14
	$2 \times 10^5, M = 26$	es)		

#### References

Title	Author
Factor Oracle, Suffix Oracle	Cyril Allauzen, Maxime Crochemore, Mathieu
	Raffinot
Computing Repeated Factors with a Factor Or-	Arnaud Lefebvre, Thierry Lecroq
acle	

#### Code

#### Suffix Array (Factor Oracle).hpp (2640 bytes, 71 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   template<class T,int N,int M,T D>struct SuffixArray{
       int val(int i,int d){
           return d<0?(d>-2?lrs[i]:n-1-lrs[i]):s[n-i+lrs[i]+d]-D;
       void sort(int*a,int*b,int m,int d){
           static int c[N];
8
           memset(c,0,4*(d>=0?M:n));
           for(i=1;i<=m;++c[val(a[i],d)],++i);</pre>
           for(i=1;i<(d>=0?M:n);c[i]+=c[i-1],++i);
           for(i=m;i>=1;b[c[val(a[i],d)]--]=a[i],--i);
       void sort(int a,int b,int d,int 1){
           sort(z+a-1,t,b-a+1,d);
           memcpy(z+a,t+1,(b-a+1)*4);
           for(i=a,j;i<=b;i=j+1){</pre>
18
               for(j=i;j+1<=b&&val(z[j],d)==val(z[j+1],d);++j);</pre>
               if(j-i)
                   sort(i,j,d+1,l);
           }
       void add(int&b,int v){
           cv[++cp]=v,cn[cp]=b,b=cp;
       void dfs(int u){
           #define m(p,q)\
               for(int i=p##b[u],j;i;){\
28
                   for(*z=0,j=i;cn[j]&&lrs[cv[j]]==lrs[cv[cn[j]]];z[++z[0]]=cv[
       j],j=cn[j]);\
```

```
z[++z[0]]=cv[j],sort(1,*z,0,q);\
                   for(z[0]=1;i!=cn[j];cv[i]=z[z[0]++],i=cn[i]);\
               }
           m(1,0)
           for(int i=lb[u];i;dfs(cv[i]),i=cn[i]);
           sa[++*sa]=n+1-u,*sa-=!u;
           m(r,1)
           for(int i=rb[u];i;dfs(cv[i]),i=cn[i]);
38
       void build(T* s,int n){
           n=_n,s=_s,memset(tr,(cp=*sa=*vl=*vr=*lb=*rb=*lrs=0,*z=-1),4*M);
           for(int i=0,c=s[n-1-i]-D,u,v;i<n;c=s[n-1-++i]-D){
               memset(tr+i+1,(lb[i+1]=rb[i+1]=lrs[i+1]=0)-1,4*M);
               for(j=i;j>-1&&tr[j][c]<0;tr[j][c]=i+1,j=z[u=j]);</pre>
               if(v=z[i+1]=j<0?0:tr[j][c]){</pre>
                   for(v=v-1==z[u]?u:v-1;z[u]!=z[v];v=z[v]);
                   lrs[i+1]=min(lrs[u],lrs[v])+1;
               }
48
               for(j=0;n-(z[i+1]-lrs[i+1]-j)<n&&s[n-(z[i+1]-lrs[i+1]-j)]==s[
       n-1-i+lrs[i+1]+j];++j);
               if(n-(z[i+1]-lrs[i+1]-j)< n\&\&s[n-(z[i+1]-lrs[i+1]-j)]> s[n-1-i]
       +lrs[i+1]+j])
                   v1[++*v1]=i+1;
               else
                   vr[++*vr]=i+1;
           sort(vl,t,*vl,-1), sort(vr,vl,*vr,-2);
           for(i=*vl;i;add(lb[z[t[i]]],t[i]),--i);
           for(i=*vr;i;add(rb[z[vl[i]]],vl[i]),--i);
           dfs(0);
           for(i=1;i<=n;++i)</pre>
58
               rk[sa[i]]=i;
           for(i=1;i<=n;++i){</pre>
               if(rk[i]==1){
                   ht[1]=0;
                   continue;
               }
               int&d=ht[rk[i]]=max(i==1?0:ht[rk[i-1]]-1,0);
               for(;i+d<=n&&sa[rk[i]-1]+d<=n&&s[i+d-1]==s[sa[rk[i]-1]+d-1];++</pre>
       d);
           }
```

```
68     }
     T*s;
     int n,sa[N+1],ht[N+1],rk[N+1],lrs[N+1],tr[N+1][M],i,j,lb[N+1],rb[N+1],
     cv[N+1],cn[N+1],cp,vl[N+1],vr[N+1],t[N+1],z[N+1];
};
```

## 6.8 Suffix Array (Prefix-Doubling Algorithm)

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Suffix Array (Prefix-Doubling Algorithm).hpp (1357 bytes, 55 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   struct SuffixArray{
       int*a,*h,*r,*t,*c,n,m;
       #define lp(u,v)for(int i=u;i<=v;++i)</pre>
       #define rp(u,v)for(int i=u;i>=v;--i)
       void sort(){
           memset(c+1,0,m*4);
9
           lp(1,n)
                ++c[r[t[i]]];
           1p(2,m)
                c[i]+=c[i-1];
            rp(n,1)
                a[c[r[t[i]]]--]=t[i];
       }
       SuffixArray(int*s){
           for(n=m=0;s[n+1];m=max(m,s[++n]));
           a=new int[4*n+max(n,m)+3];
19
           h=a+n;
           r=h+n+1;
           t=r+n+1;
           c=t+n;
           lp(1,n)
               t[i]=i,r[i]=s[i];
            sort();
           for(int l=1;l<=n;l<<=1,r[a[n]]==n?l=n+1:m=r[a[n]]){</pre>
                t[0]=0;
                lp(n-l+1,n)
```

```
29
                   t[++t[0]]=i;
               lp(1,n)
                   if(a[i]>1)
                       t[++t[0]]=a[i]-1;
               sort();
               swap(r,t);
               r[a[1]]=1;
               1p(2,n)
                   r[a[i]]=r[a[i-1]]+(t[a[i]]!=t[a[i-1]]||a[i]+1>n||a[i-1]+1>n
       ||t[a[i]+l]!=t[a[i-1]+l]);
39
           int 1=0;
           a[0]=n+1;
            lp(1,n){
               if(r[i]==1)
                   1=0:
               1-=(1>0);
               int j=a[r[i]-1];
               for(;s[i+1]==s[j+1];++1);
               h[r[i]]=1;
            }
49
       #undef lp
       #undef rp
       ~SuffixArray(){
           delete a;
       }
   };
```

## 6.9 Suffix Array (Suffix Tree)

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Suffix Array (Suffix Tree).hpp (2849 bytes, 115 lines)

```
#include<bits/stdc++.h>
using namespace std;
template<class T,int N,int M,T D>struct SuffixTree{
    struct node;
5    struct edge{
```

```
edge():
               1(0),r(0),t(0){
           int length(){
               return r-1;
           T*1,*r;
           node*t;
       }pe[2*N],*ep=pe;
15
       edge*newedge(T*1,T*r,node*t){
           ep->1=1;
           ep->r=r;
           ep->t=t;
           return ep++;
       struct node{
           node():
               s(0),c({0}){
            }
           node*s;
25
           edge*c[M+1];
       }pn[2*N+1],*np=pn;
       SuffixTree():
           root(np++),ct(0){
       void extend(T*s){
           for(;ae&&al>=ae->length();){
               s+=ae->length();
               al-=ae->length();
35
               an=ae->t;
               ae=al?an->c[*s-D]:0;
            }
       bool extend(int c){
           if(ae){
               if(*(ae->l+al)-D-c)
                   return true;
               ++al;
            }else{
45
               if(!an->c[c])
                   return true;
```

```
ae=an->c[c];
               al=1;
               if(pr)
                   pr->s=an;
            }
           extend(ae->1);
           return false;
55
       void dfs(node*u,int d){
            int t=0,s=0;
           for(int i=0;i<M+1;++i)</pre>
               if(u->c[i]){
                   if(!t)
                       t=1;
                   else if(!s){
                       s=1;
                       *sp++=d;
                   dfs(u->c[i]->t,d+u->c[i]->length());
65
               }
           if(s)
                --sp;
           else if(!t&&sp!=sk){
               *hp++=*(sp-1);
               *fp++=ct-d+1;
            }
       }
       void build(T*s,int n){
75
            s[n++]=M+D;
           ct+=n;
           an=root;
           ae=al=0;
           for(T*p=s;p!=s+n;++p)
               for(pr=0;extend(*p-D);){
                   edge*x=newedge(p,s+n,np++);
                   if(!ae)
                       an->c[*p-D]=x;
                   else{
85
                       edge*&y=an->c[*ae->l-D];
                       y=newedge(ae->1,ae->1+a1,np++);
                       y->t->c[*(ae->l+=al)-D]=ae;
```

```
y->t->c[*p-D]=x;
                        ae=y;
                    }
                    if(pr)
                        pr->s=ae?ae->t:an;
                    pr=ae?ae->t:an;
                    int r=1;
95
                    if(an==root&&!al)
                        break;
                    if(an==root)
                        --al;
                    else{
                        an=an->s?an->s:root;
                        r=0;
                    if(al){
                        T*t=ae->l+(an==root)*r;
                        ae=an->c[*t-D];
105
                        extend(t);
                    }else
                        ae=0;
            dfs(root,0);
        }
        edge*ae;
        node*root,*an,*pr;
        int al,ct,sk[N],*sp=sk,ht[N],*hp=ht,sa[N],*fp=sa;
115
    };
```

## 6.10 Suffix Array (Treap)

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Suffix Array (Treap).hpp (3803 bytes, 147 lines)

```
#include<bits/stdc++.h>
using namespace std;
template<class T>struct SuffixArray{
    struct node{
5    node*c[2],*p;
```

```
T v;
               int f,s,l,h,m;
               double t;
               node(node*_p,T _v,int _1):
                     f(rand()*1.0/RAND_MAX*1e9),p(_p),v(_v),s(1),l(_1),h(0),m(0),t(5)
          e8){
                     c[0]=c[1]=0;
               }
          }*root;
          vector<T>a;
15
          SuffixArray():
                root(new node(0,0,0)),a(1){}
          ~SuffixArray(){
               clear(root);
          }
          void relabel(node*x,double 1,double r){
               x->t=(1+r)/2;
               if(x->c[0])
                     relabel(x \rightarrow c[0], 1, x \rightarrow t);
25
               if(x->c[1])
                     relabel(x \rightarrow c[1], x \rightarrow t, r);
          void update(node*x){
               x \rightarrow s=1;
               x->m=x->h;
               for(int i=0;i<2;++i)</pre>
                     if(x->c[i])
                          x \rightarrow s + = x \rightarrow c[i] \rightarrow s, x \rightarrow m = min(x \rightarrow m, x \rightarrow c[i] \rightarrow m);
          void rotate(node*&x,int d){
35
               node*y=x->c[d];
               x\rightarrow c[d]=y\rightarrow c[!d];
               y \rightarrow c[!d]=x;
               y \rightarrow s = x \rightarrow s;
               y \rightarrow m = x \rightarrow m;
               update(x);
               x=y;
          void clear(node*x){
               if(!x)
45
```

```
return;
            clear(x->c[0]);
            clear(x->c[1]);
            delete x;
        }
        node*insert(node*&x,node*p,T v,node*1,node*r){
            int d=x->v!=v?x->v<v:x->p->t<p->t;
            double tl=1?1->t:0,tr=r?r->t:1e9;
            node*y;
            if(d)
55
                1=x;
            else
                r=x:
            if(!x->c[d]){
                y=new node(p,v,p->l+1);
                y->t=((1?1->t:0)+(r?r->t:1e9))/2;
                y->m=y->h=1->v==y->v?lcp(1->p,y->p)+1:0;
                if(r)
                     r->h=r->v==y->v?lcp(r->p,y->p)+1:0;
65
                x \rightarrow c[d] = y;
            }else
                y=insert(x->c[d],p,v,l,r);
            update(x);
            if(x\rightarrow c[d]\rightarrow f\rightarrow x\rightarrow f)
                rotate(x,d),relabel(x,tl,tr);
            return y;
        }
        node*insert(node*p,T v){
            a.push back(v);
75
            return insert(root,p,v,0,0);
        void erase(node*&x,node*y){
            if(x==y){
                if(!x->c[0]){
                    x=x->c[1];
                    delete y;
                else if(!x->c[1])
                    x=x->c[0];
                    delete y;
85
                }else{
                    int d=x->c[0]->f< x->c[1]->f;
```

```
rotate(x,d);
                     erase(x->c[!d],y);
                     --x->s;
            }else
                 erase(x \rightarrow c[x \rightarrow t < y \rightarrow t], y), update(x);
        void erase(node*y){
95
            erase(root,y);
             a.pop_back();
        bool check(node*x,T*y,node*&p,int&l){
            if(p){
                 int t=x-c[p-t>x-t]?x-c[p-t>x-t]-m:~0u>>1;
                 if(p->t>x->t)
                     t=min(t,p->h);
                 else
                     t=min(t,x->h);
105
                 if(t<1)
                     return x->t<p->t;
            for(p=x;l+1<=x->1&&y[1+1];++1)
                 if(a[x->l-1]!=y[l+1])
                     return a[x->1-1]<y[1+1];
            return y[1+1]!=0;
        int count(node*x,T*y){
             int r=0,1=0;
115
            for(node*p=0;x;)
                 if(check(x,y,p,1))
                     r+=(x->c[0]?x->c[0]->s:0)+1,x=x->c[1];
                 else
                     x=x->c[0];
            return r;
        }
        int count(T*y){
            T*t=y;
            while(*(t+1))
125
                 ++t;
            int r=-count(root,y);
            ++*t;
```

```
r+=count(root,y);
            --*t;
            return r;
        int lcp(node*x,double u,double v,double l,double r){
            if(v<1||u>r||!x)
                return ~0u>>1;
135
            if(u<1&&v>=r)
                return x->m;
            int t=u(x-)t\&v=x-)t?x-h:~0u>>1;
            t=min(t,lcp(x->c[0],u,v,l,x->t));
            t=min(t,lcp(x->c[1],u,v,x->t,r));
            return t;
        int lcp(node*x,node*y){
            if(x->t>y->t)
                swap(x,y);
            return lcp(root,x->t,y->t,0,1e9);
145
        }
    };
```

## 6.11 Suffix Automaton

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Suffix Automaton.hpp (1694 bytes, 59 lines)

```
#include<bits/stdc++.h>
    using namespace std;

template<class T>struct SuffixAutomaton{
    struct node{
        node(vector<node*>&all,int m,node*_pr=0,int _ln=0,T _va=T()):
            pr(_pr),tr(m),ln(_ln),va(_va){
            all.push_back(this);
        }
        T va;
        int ln;
        node*pr;
        vector<node*>tr;

}
```

```
SuffixAutomaton(int m):
           root(new node(all,m)),m(_m){
       ~SuffixAutomaton(){
           for(int i=0;i<all.size();++i)</pre>
               delete all[i];
       node*insert(node*lst,int c,T v){
           node*p=lst,*np=p->tr[c]?0:new node(all,m,0,lst->ln+1,v);
23
           for(;p&&!p->tr[c];p=p->pr)
               p->tr[c]=np;
           if(!p)np->pr=root;
           else{
               node*q=p->tr[c];
               if(p==lst)
                   np=q;
               if(q->ln==p->ln+1)
                   p==1st?(q->va+=v):(np->pr=q,0);
               else{
33
                   node*nq=new node(all,m,q->pr,p->ln+1,p==lst?v:T());
                   nq->tr=q->tr;
                   q->pr=np->pr=nq;
                   if(p==lst)
                       np=nq;
                   for(;p&&p->tr[c]==q;p=p->pr)
                       p->tr[c]=nq;
               }
           }
           return np;
43
       void count(){
           vector<int>cnt(all.size());
           vector<node*>tmp=all;
           for(int i=0;i<tmp.size();++i)</pre>
               ++cnt[tmp[i]->ln];
           for(int i=1;i<cnt.size();++i)</pre>
               cnt[i]+=cnt[i-1];
           for(int i=0;i<tmp.size();++i)</pre>
               all[--cnt[tmp[i]->ln]]=tmp[i];
53
           for(int i=int(all.size())-1;i>0;--i)
               all[i]->pr->va+=all[i]->va;
```

```
}
int m;
node*root;
vector<node*>all;
};
```

# 6.12 Suffix Tree (Suffix Automaton)

## Description

Use a suffix automaton to build a suffix tree. It has large data members, make its object static.

### Methods

template <class d="" m,t="" n,int="" t,int="">SuffixTree<t,n,m,d>::SuffixTree();</t,n,m,d></class>		
Description	construct an object of SuffixTree	
Parameters	Description	
T	type of character, usually char	
N	maximum length of string	
M	size of alphabet	
D	offset of alphabet, use 'a' for lowercase letters	
Time complexity	$\Theta(1)$	
Space complexity	$\Theta(8NM)$	
Return value	an object of SuffixTree	
template <class d="" m,t="" n,int="" t,int="">void SuffixTree<t,n,m,d>::build(const T*s,int n);</t,n,m,d></class>		
<b>Description</b> build suffix tree for a given string		
Parameters Description		
S	string from which to build a suffix tree, in-	
	dexed from zero	
n	length of s	
Time complexity	$\Theta(nM)$	
Space complexity	$\Theta(1)$	
Return value	an object of SuffixTree	

#### **Fields**

template <class d="" m,t="" n,int="" t,int="">int SuffixTree<t,n,m,d>::nc;</t,n,m,d></class>		
Description	number of nodes in suffix tree, they are labeled	
	from one to $nc$ , note that $nc$ can be almost $2* s $	

template <class d="" m,t="" n,int="" t,int="">int SuffixTree<t,n,m,d>::pr[2*N];</t,n,m,d></class>			
<b>Description</b> parent array of the suffix tree			
template <class d="" m,t="" n,int="" t,int="">int SuffixTree<t,n,m,d>::ch[2*N][M];</t,n,m,d></class>			
<b>Description</b> children array of the suffix tree			
template <class d="" m,t="" n,int="" t,int="">const T*SuffixTree<t,n,m,d>::el[2*N][M];</t,n,m,d></class>			
Description	the start pointer of the string on children edge		
template <class d="" m,t="" n,int="" t,int="">const T*SuffixTree<t,n,m,d>::er[2*N][M];</t,n,m,d></class>			
Description	the end pointer of the string on children edge,		
	itself is not included		
template <class d="" m,t="" n,int="" t,int="">int SuffixTree<t,n,m,d>::tr[2*N][M];</t,n,m,d></class>			
Description	$tr[u][i]$ is the node that represents $\{(D+i)+s \mid$		
	$u$ represents $s$ }		
template <class d="" m,t="" n,int="" t,int="">int SuffixTree<t,n,m,d>::dp[2*N];</t,n,m,d></class>			
Description	depth array of the suffix tree		
template <class d="" m,t="" n,int="" t,int="">int SuffixTree<t,n,m,d>::id[2*N];</t,n,m,d></class>			
Description	id[u] is the start of a postion where the strings		
	u represents occur		
template <class d="" m,t="" n,int="" t,int="">int SuffixTree<t,n,m,d>::sf[2*N];</t,n,m,d></class>			
Description	sf[u] means whether $u$ represents a suffix		

#### References

Title	Author
后缀自动机	陈立杰

#### Code

#### Suffix Tree (Suffix Automaton).hpp (1010 bytes, 29 lines)

```
for(c=s[i]-D;p&&!tr[p][c];p=pr[p])
                   tr[p][c]=np;
               if(p&&dp[q=tr[p][c]]!=dp[p]+1){
                   dp[nq=node()]=dp[p]+1,pr[nq]=pr[q],id[nq]=i+1;
                   memcpy(tr[pr[q]=pr[np]=nq],tr[q],4*M);
                   for(;p&&tr[p][c]==q;p=pr[p])
                       tr[p][c]=nq;
               }else
                   pr[np]=p?q:1;
21
           for(int i=2,j,c;i<=nc;++i)</pre>
               c=s[id[i]+dp[j=pr[i]]-1]-D,
               el[j][c]=s+id[i]+dp[j]-1,
               er[j][c]=s+id[i]+dp[ch[j][c]=i]-1;
       const T*el[2*N][M],*er[2*N][M];
       int nc,pr[2*N],tr[2*N][M],dp[2*N],id[2*N],sf[2*N],ch[2*N][M];
   };
```

## 6.13 Suffix Tree (Ukkonen's Algorithm)

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Suffix Tree (Ukkonen's Algorithm).hpp (2296 bytes, 94 lines)

```
ep->1=1;
           ep->r=r;
           ep->t=t;
           return ep++;
       }
21
       struct node{
           node():
               s(0),c({0}){
           node*s;
           edge*c[M];
       }pn[2*N+1],*np=pn;
       SuffixTree():
           root(np++),ct(0){
31
       void extend(T*s){
           for(;ae&&al>=ae->length();){
               s+=ae->length();
               al-=ae->length();
               an=ae->t;
               ae=al?an->c[*s-D]:0;
           }
       bool extend(int c){
           if(ae){
41
               if(*(ae->l+al)-D-c)
                   return true;
               ++al;
           }else{
               if(!an->c[c])
                   return true;
               ae=an->c[c];
               al=1;
               if(pr)
                   pr->s=an;
51
           }
           extend(ae->1);
           return false;
       void insert(T*s,int n){
           ct+=n;
```

```
an=root;
           ae=al=0;
           for(T*p=s;p!=s+n;++p)
               for(pr=0;extend(*p-D);){
                   edge*x=newedge(p,s+n,np++);
61
                   if(!ae)
                       an->c[*p-D]=x;
                   else{
                       edge*&y=an->c[*ae->l-D];
                       y=newedge(ae->1,ae->1+a1,np++);
                       y->t->c[*(ae->l+=al)-D]=ae;
                       y->t->c[*p-D]=x;
                       ae=y;
                   }
                   if(pr)
71
                       pr->s=ae?ae->t:an;
                   pr=ae?ae->t:an;
                   int r=1;
                   if(an==root&&!al)
                       break;
                   if(an==root)
                       --al;
                   else{
                       an=an->s?an->s:root;
                       r=0;
81
                   if(al){
                       T*t=ae->l+(an==root)*r;
                       ae=an->c[*t-D];
                       extend(t);
                   }else
                       ae=0;
               }
       edge*ae;
91
       int al,ct;
       node*root,*an,*pr;
   };
```

# CHAPTER 7

Utility Tools

## 7.1 Checker

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Checker.bat (166 bytes, 7 lines)

```
:again
generator > input.txt
program1 < input.txt > output1.txt
program2 < input.txt > output2.txt
fc output1.txt output2.txt
6 if errorlevel 1 pause
goto again
```

### 7.2 Date

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Date.hpp (3596 bytes, 145 lines)

```
#include<bits/stdc++.h>
   using namespace std;
3 struct Date{
       int y,m,d,w;
       Date&operator++(){
           return*this=*this+1;
       bool leap(int a)const{
           return a%400==0||(a%4==0&&a%100!=0);
       int month_sum(int a,int b)const{
           if(b==0)
13
               return 0;
           if(b==1)
               return 31;
           return 59+leap(a)+30*(b-2)+(b+1)/2-1+(b>=8&&b%2==0);
       string month_name(int a)const{
           if(a==1)
               return"January";
```

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```
if(a==2)
               return"February";
23
           if(a==3)
               return"March";
           if(a==4)
               return"April";
           if(a==5)
               return"May";
           if(a==6)
               return"June";
           if(a==7)
               return"July";
33
           if(a==8)
               return"August";
           if(a==9)
               return"September";
           if(a==10)
               return"October";
           if(a==11)
               return"November";
           if(a==12)
               return"December";
43
       string day_name(int a)const{
           if(a==0)
               return"Sunday";
           if(a==1)
               return"Monday";
           if(a==2)
               return"Tuesday";
           if(a==3)
               return"Wednesday";
           if(a==4)
53
               return"Thursday";
           if(a==5)
               return"Friday";
           if(a==6)
               return"Saturday";
       operator int()const{
           int t=(y-1)*365+(y-1)/4-(y-1)/100+(y-1)/400+month_sum(y,m-1)+d;
```

```
if(y==1752\&m>=9\&d>2||y>1752)
63
                t-=11;
            t=min(y-1,1700)/400-min(y-1,1700)/100;
            if(y<=1700&&y%400!=0&&y%100==0&&m>2)
                ++t;
            return t;
        Date(int y,int m,int d):
            y(_y),m(_m),d(_d),w((int(*this)+5)%7){
        Date(int a){
73
            int yl=0,yr=1e7;
            while(yl+1<yr){</pre>
                int ym=(yl+yr)/2;
                if(int(Date(ym,12,31))<a)</pre>
                    yl=ym;
                else
                    yr=ym;
            }
            y=yr;
            int ml=0,mr=12;
83
            while(ml+1<mr){</pre>
                int mm=(ml+mr)/2,mt;
                if(mm==2){
                    if(y < 1700)
                         mt=28+(y\%4==0);
                    else
                         mt=28+(y\%4==0\&8y\%100!=0||y\%400==0);
                }else if(mm<=7)</pre>
                    mt=30+mm%2;
                else
93
                    mt=31-mm%2;
                if(int(Date(y,mm,mt))<a)</pre>
                    ml=mm;
                else
                    mr=mm;
            }
            m=mr;
            for(int i=1;;++i){
                if(y==1752&&m==9&&i>2&&i<14)
                    continue;
```

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```
if(int(Date(y,m,i))==a){
103
                     d=i;
                     break;
                 }
             }
             w=(5+a)\%7;
        operator string()const{
             stringstream s;
             string t;
             s<<day_name(w)+", "+month_name(m)+" "<<d<<", "<<y;</pre>
113
             getline(s,t);
             return t;
        }
    };
    ostream&operator<<(ostream&s,const Date&a){</pre>
        return s<<string(a);</pre>
    int operator-(const Date&a,const Date&b){
        return int(a)-int(b);
123
    Date operator+(const Date&a,int b){
        return Date(int(a)+b);
    Date operator-(const Date&a,int b){
        return Date(int(a)-b);
    bool operator<(const Date&a,const Date&b){</pre>
        if(a.y==b.y&&a.m==b.m)
             return a.d<b.d;</pre>
133
        if(a.y==b.y)
             return a.m<b.m;</pre>
        return a.y<b.y;</pre>
    bool operator>(const Date&a,const Date&b){
        return b<a;
    bool operator!=(const Date&a,const Date&b){
        return a.y!=b.y||a.m!=b.m||a.d!=b.d;
143 bool operator==(const Date&a,const Date&b){
```

```
return !(a!=b);
}
```

## 7.3 Fast Reader

warning: old style will be replaced ... see Suffix Array (DC3) for new style

#### Fast Reader.hpp (1251 bytes, 61 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   struct FastReader{
       FILE*f;
       char*p,*e;
 5
       vector<char>v;
       void ipt(){
            for(int i=1,t;;i<<=1){</pre>
                v.resize(v.size()+i);
                if(i!=(t=fread(&v[0]+v.size()-i,1,i,f))){
                    p=&v[0],e=p+v.size()-i+t;
                    break;
                }
            }
15
       void ign(){
           while(p!=e&&isspace(*p))
       int isc(){
            return p!=e&&!isspace(*p);
       int isd(){
            return p!=e&&isdigit(*p);
25
       FastReader(FILE*_f):
            f(_f)
            ipt();
       FastReader(string _f):
            f(fopen(_f.c_str(), "r")){
```

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```
ipt();
       ~FastReader(){
            fclose(f);
35
       template<class T>FastReader&operator>>(T&a){
            int n=1;
            ign();
            if(*p=='-')
               n=-1,++p;
            for(a=0;isd();)
                a=a*10+*p++-'0';
           a*=n;
45
            return*this;
       FastReader&operator>>(char&a){
            ign();
            a=*p++;
            return*this;
       FastReader&operator>>(char*a){
            for(ign();isc();)
                *a++=*p++;
55
            *a=0;
            return*this;
       char get(){
            return*p++;
       }
    };
```

## 7.4 Fast Writer

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Fast Writer.hpp (866 bytes, 39 lines)

```
#include<bits/stdc++.h>
using namespace std;
struct FastWriter{
```

```
FILE*f;
        vector<char>p;
        FastWriter(FILE*_f):
            f(_f){
        }
        FastWriter(string _f):
9
            f(fopen(_f.c_str(),"w")){
        }
        ~FastWriter(){
            if(p.size())
                fwrite(&p[0],1,p.size(),f);
            fclose(f);
        }
        FastWriter&operator<<(char a){</pre>
            p.push_back(a);
            return*this;
19
        FastWriter&operator<<(const char*a){
            while(*a)
                p.push_back(*a++);
            return*this;
        template < class T > FastWriter & operator < < (T a) {</pre>
            if(a<0)
                p.push_back('-'),a=-a;
29
            static char t[19];
            char*q=t;
            do{
                T b=a/10;
                *q++=a-b*10+'0',a=b;
            }while(a);
            while(q>t)
                p.push_back(*--q);
            return*this;
        }
39
   };
```

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## 7.5 Large Stack

### **Description**

Make system stack larger. Simply put this code before main function, and the system stack will be enlarged.

#### **Fields**

#define STACK_SIZE 64	
Description	the size of system stack in MB

#### Code

#### Large Stack.hpp (845 bytes, 32 lines)

```
1 #include<cstdlib>
   using namespace std;
   #define STACK_SIZE 64
   #if GNUC
       #if __x86_64__||__ppc64__
           extern int _main(void)__asm__("_main");
       #else
           extern int _main(void)__asm__("__main");
       #endif
       int __main();
       int _main() {
11
           __main();
           exit(0);
       int main(){
                   __volatile__(
           __asm__
                    __x86_64__||__ppc64__
                   "movq %0,%%rsp\n"
                   "pushq $exit\n"
                   "jmp _main\n"
21
               #else
                   "movl %0,%%esp\n"
                   "pushl $_exit\n"
                   "jmp __main\n"
```

## 7.6 Number Speller

warning: old style will be replaced ... see Suffix Array (DC3) for new style

Number Speller.hpp (2143 bytes, 72 lines)

```
#include<bits/stdc++.h>
   using namespace std;
   namespace NumberSpeller{
       template < class T > string run(T a){
           map<T,string>m;
           m[0]="zero";
           m[1]="one";
           m[2]="two";
           m[3]="three";
9
           m[4]="four";
           m[5]="five";
           m[6]="six";
           m[7]="seven";
           m[8]="eight";
           m[9]="nine";
           m[10]="ten";
           m[11]="eleven";
           m[12]="twelve";
           m[13]="thirteen";
19
           m[14]="fourteen";
           m[15]="fifteen";
           m[16]="sixteen";
           m[17]="seventeen";
           m[18]="eighteen";
           m[19]="nineteen";
```

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```
m[20]="twenty";
           m[30]="thirty";
           m[40]="forty";
29
           m[50]="fifty";
           m[60]="sixty";
           m[70]="seventy";
           m[80]="eighty";
           m[90]="ninety";
           if(a<0)
               return"minus "+run(-a);
           if(m.count(a))
               return m[a];
           if(a<100)
               return run(a/10*10)+"-"+run(a%10);
39
           if(a<1000&&a%100==0)
               return run(a/100)+" hundred";
           if(a<1000)
               return run(a/100*100)+" and "+run(a%100);
           vector<string>t;
           t.push back("thousand");
           t.push_back("million");
           t.push back("billion");
           t.push back("trillion");
49
           t.push_back("quadrillion");
           t.push back("quintillion");
           t.push back("sextillion");
           t.push_back("septillion");
           t.push_back("octillion");
           t.push back("nonillion");
           t.push back("decillion");
           t.push back("undecillion");
           t.push_back("duodecillion");
           t.push back("tredecillion");
           t.push_back("quattuordecillion");
59
           t.push back("quindecillion");
           string r=a%1000?run(a%1000):"";
           a/=1000;
           for(int i=0;a;++i,a/=1000)
               if(a%1000){
                   if(!i&&r.find("and")==string::npos&&r.find("hundred")==
       string::npos&&r.size())
```