

Algorithms

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First version on November 28, 2015
Latest version on February 10, 2016

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CHAPTER 1

Computational Geometry

1.1 Convex Hull

Convex Hull.hpp (1063 bytes, 36 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>struct ConvexHull{
4      struct point{
5          T x,y;
6          point(T _x,T _y):
7              x(_x),y(_y){
8              }
9          point operator-(point a){
10             return point(x-a.x,y-a.y);
11         }
12         T operator*(point a){
13             return x*a.y-y*a.x;
14         }
15         int operator<(point a){
16             return x==a.x?y<a.y:x<a.x;
17         }
18     };
19     static int check(point a,point b,point c){
20         return (a-c)*(b-c)<=0;
21     }
22     static vector<vector<point> >run(vector<point>a){
23         sort(a.begin(),a.end());
24         vector<point>u,d;
25         for(int i=0;i<a.size();u.push_back(a[i++]))
26             while(u.size()>1&&check(a[i],u.back(),u[u.size()-2]))
27                 u.pop_back();
28         for(int i=int(a.size()-1);i>=0;d.push_back(a[i--]))
29             while(d.size()>1&&check(a[i],d.back(),d[d.size()-2]))
30                 d.pop_back();
31         vector<vector<point> >r;
32         r.push_back(u);
33         r.push_back(d);
34         return r;
35     }
36 };

```


1.2 Delaunay Triangulation

Delaunay Triangulation.hpp (4889 bytes, 159 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>struct DelaunayTriangulation{
4      const static double E;
5      struct poi{
6          T x,y;
7          poi(T _x=0,T _y=0):
8              x(_x),y(_y){
9              }
10         poi operator-(poi b){
11             return poi(x-b.x,y-b.y);
12         }
13         int operator<(poi b)const{
14             if(fabs(x-b.x)<E)
15                 return y<b.y;
16             return x<b.x;
17         }
18     };
19     int n;
20     vector<pair<poi,int> >pts;
21     vector<vector<int> >egs;
22     T det(poi a,poi b){
23         return a.x*b.y-a.y*b.x;
24     }
25     T dot(poi a,poi b){
26         return a.x*b.x+a.y*b.y;
27     }
28     int dir(poi a,poi b,poi c){
29         T r=det(c-a,b-a);
30         if(r<-E)
31             return -1;
32         return r>E?1:0;
33     }
34     int inc(poi a,poi b,poi c,poi d){
35         a=a-d;
36         b=b-d;
37         c=c-d;

```

```

38         T az=a.x*a.x+a.y*a.y,bz=b.x*b.x+b.y*b.y,cz=c.x*c.x+c.y*c.y;
39         return a.x*b.y*cz+b.x*c.y*az+c.x*a.y*bz-a.x*bz*c.y-b.x*a.y*cz-c.x*
        b.y*az>E;
40     }
41     int crs(poi a,poi b,poi c,poi d){
42         return dir(a,b,c)*dir(a,b,d)==-1&&dir(c,d,a)*dir(c,d,b)==-1;
43     }
44     DelaunayTriangulation():
45         n(0),pts(1){
46     }
47     void add(T x,T y){
48         poi a;
49         a.x=x;
50         a.y=y;
51         pts.push_back(make_pair(a,++n));
52     }
53     poi&pot(int a){
54         return pts[a].first;
55     }
56     void con(int a,int b){
57         egs[a].push_back(b);
58         egs[b].push_back(a);
59     }
60     void dco(int a,int b){
61         egs[a].erase(find(egs[a].begin(),egs[a].end(),b));
62         egs[b].erase(find(egs[b].begin(),egs[b].end(),a));
63     }
64     void dnc(int l,int r){
65         if(r==l)
66             return;
67         if(r==l+1){
68             con(l,r);
69             return;
70         }
71         if(r==l+2){
72             if(dir(pot(l),pot(l+1),pot(r)))
73                 con(l,l+1),con(l+1,r),con(l,r);
74             else{
75                 if(dot(pot(l+1)-pot(l),pot(r)-pot(l))<0)
76                     con(l,l+1),con(l,r);
77                 else if(dot(pot(l)-pot(l+1),pot(r)-pot(l+1))<0)

```

```

78         con(l,l+1),con(l+1,r);
79     else
80         con(l,r),con(l+1,r);}
81     return;
82 }
83 int m=(l+r)/2,p1=l,pr=r;
84 dnc(l,m);
85 dnc(m+1,r);
86 for(int f=0;;f=0){
87     for(int i=0;i<egs[p1].size();++i){
88         int a=egs[p1][i],d=dir(pot(p1),pot(pr),pot(a));
89         if(d>0||(d==0&&dot(pot(p1)-pot(a),pot(pr)-pot(a))<0)){
90             p1=a;
91             f=1;
92             break;
93         }
94     }
95     for(int i=0;i<egs[pr].size();++i){
96         int a=egs[pr][i],d=dir(pot(p1),pot(pr),pot(a));
97         if(d>0||(d==0&&dot(pot(p1)-pot(a),pot(pr)-pot(a))<0)){
98             pr=a;
99             f=1;
100            break;
101        }
102    }
103    if(!f)
104        break;
105 }
106 con(p1,pr);
107 for(int pn=-1,wh=0;;pn=-1,wh=0){
108     for(int i=0;i<egs[p1].size();++i){
109         int a=egs[p1][i],d=dir(pot(p1),pot(pr),pot(a));
110         if(d<0&&(pn===-1||inc(pot(p1),pot(pr),pot(pn),pot(a))))
111             pn=a;
112     }
113     for(int i=0;i<egs[pr].size();++i){
114         int a=egs[pr][i],d=dir(pot(p1),pot(pr),pot(a));
115         if(d<0&&(pn===-1||inc(pot(p1),pot(pr),pot(pn),pot(a))))
116             pn=a,wh=1;
117     }
118     if(pn===-1)

```

```

119         break;
120     vector<int> ne;
121     if(!wh){
122         for(int i=0;i<egs[p1].size();++i){
123             int a=egs[p1][i];
124             if(!crs(pot(pn),pot(pr),pot(p1),pot(a)))
125                 ne.push_back(a);
126             else
127                 egs[a].erase(find(egs[a].begin(),egs[a].end(),p1));
128         }
129         egs[p1]=ne;
130         con(pr,pn);
131         p1=pn;
132     }else{
133         for(int i=0;i<egs[pr].size();++i){
134             int a=egs[pr][i];
135             if(!crs(pot(pn),pot(p1),pot(pr),pot(a)))
136                 ne.push_back(a);
137             else
138                 egs[a].erase(find(egs[a].begin(),egs[a].end(),pr));
139         }
140         egs[pr]=ne;
141         con(p1,pn);
142         pr=pn;
143     }
144 }
145 }
146 vector<vector<int> >run(){
147     egs.resize(n+1);
148     sort(pts.begin()+1,pts.end());
149     dnc(1,n);
150     vector<vector<int> >res(n+1);
151     for(int u=1;u<=n;++u)
152         for(int i=0;i<egs[u].size();++i){
153             int v=egs[u][i];
154             res[pts[u].second].push_back(pts[v].second);
155         }
156     return res;
157 }
158 };
159 template<class T>const double DelaunayTriangulation<T>::E=1e-8;

```

1.3 Dynamic Convex Hull (Set)

Dynamic Convex Hull (Set).hpp (2239 bytes, 77 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>struct DynamicConvexHull{
4      struct point{
5          T x,y;
6          point(T _x=0,T _y=0):
7              x(_x),y(_y){
8              }
9          point operator-(const point&a)const{
10             point p(x-a.x,y-a.y);
11             return p;
12         }
13         T operator*(const point&a)const{
14             return x*a.y-y*a.x;
15         }
16     };
17     struct node{
18         node**nxt;point p;
19         node(node**_n,point _p):
20             nxt(_n),p(_p){
21         }
22         node(const node&a):
23             nxt(new node*(*a.nxt)),p(a.p){
24         }
25         ~node(){
26             delete nxt;
27         }
28         int operator<(const node&a)const{
29             if(ctp)
30                 return p.x==a.p.x?p.y<a.p.y:p.x<a.p.x;
31             point p1,p2;
32             int f=1;
33             if(nxt)
34                 p1=*nxt?(*nxt)->p-p:point(0,-1),p2=a.p;
35             else
36                 f=0,p1=*a.nxt?(*a.nxt)->p-a.p:point(0,-1),p2=p;
37             T x=p1*p2;

```

```

38         return f?x<0:x>0;
39     }
40 };
41 static int ctp;
42 set<node>nds;
43 typedef typename set<node>::iterator P;
44 int check(P a,P b,P c){
45     return (b->p-a->p)*(c->p-b->p)>=0;
46 }
47 void next(P a,P b){
48     *(a->nxt)=(node*)&*b;
49 }
50 void insert(T x,T y){
51     ctp=1;
52     node t(new node*(0),point(x,y));
53     P it=nds.insert(t).first,itl1=it,itl2,ittr1=it,ittr2=it;
54     if(it!=nds.begin())
55         for(next(--itl1,it);itl1!=nds.begin()&&check(--(itl2=itl1),
56 itl1,it);)
57             next(itl2,it),nds.erase(itl1),itl1=itl2;
58     if(++(ittr1=it)!=nds.end())
59         next(it,ittr1);
60     if(itl1!=it&&ittr1!=nds.end()&&check(itl1,it,ittr1)){
61         next(itl1,ittr1);
62         nds.erase(it);
63         return;
64     }
65     if(ittr1!=nds.end())
66         for(;++(ittr2=ittr1)!=nds.end()&&check(it,ittr1,ittr2);)
67             next(it,ittr2),nds.erase(ittr1),ittr1=ittr2;
68 }
69 int size(){
70     return nds.size();
71 }
72 pair<T,T>query(T x,T y){
73     ctp=0;
74     node t=*nds.lower_bound(node(0,point(x,y)));
75     return make_pair(t.p.x,t.p.y);
76 }
77 };
78 template<class T>int DynamicConvexHull<T>::ctp=0;

```

1.4 Dynamic Convex Hull (Treap)

Dynamic Convex Hull (Treap).hpp (9485 bytes, 327 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>struct DynamicConvexHull{
4      struct point{
5          T x,y;
6          point(T _x,T _y):
7              x(_x),y(_y){
8              }
9          point operator-(const point&a)const{
10             point p(x-a.x,y-a.y);
11             return p;
12         }
13         T operator*(const point&a)const{
14             return x*a.y-y*a.x;
15         }
16         int operator<(const point&a)const{
17             return x==a.x?y<a.y:x<a.x;
18         }
19         int operator==(const point&a)const{
20             return x==a.x&&y==a.y;
21         }
22     };
23     struct hull{
24         point*pt;
25         hull*ch[2],*nb[2];
26         int sz,fx;
27         hull(point*_pt):
28             pt(_pt),sz(1),fx(rand()*1.0/RAND_MAX*1e9){
29             ch[0]=ch[1]=nb[0]=nb[1]=0;
30         }
31         T check(point p){
32             return (nb[1]?*nb[1]->pt-*pt:point(0,-1))*p;
33         }
34         void update(){
35             sz=1;
36             for(int i=0;i<2;++i)
37                 if(ch[i])

```

```

38         sz+=ch[i]->sz;
39     }
40 };
41 static int sz(hull*x){
42     return x->sz:0;
43 }
44 static point&pt(hull*x){
45     return*x->pt;
46 }
47 static struct memory{
48     hull*ps,*pp,**ss,**sp;
49     int pm,sm;
50     vector<hull*>ns;
51     memory():
52         ps((hull*)malloc(sizeof(hull))),pp(ps),pm(1),ss((hull**)malloc(
53 sizeof(hull*)),sp(ss),sm(1){
54         ns.push_back(ps);
55     }
56     ~memory(){
57         free(ss);
58         for(int i=0;i<ns.size();++i)
59             free(ns[i]);
60     }
61     hull*create(const hull&x){
62         if(sp!=ss){
63             --sp;
64             **sp=x;
65             return*sp;
66         }
67         if(pp==ps+pm){
68             pp=ps=(hull*)malloc(sizeof(hull)*(pm<=1));
69             ns.push_back(ps);
70         }
71         *pp=x;
72         return pp++;
73     }
74     void destroy(hull*x){
75         if(sp==ss+sm){
76             hull**t=(hull**)malloc(sizeof(hull*)*sm<1);
77             memcpy(t,ss,sm*sizeof(hull*));
78             free(ss);

```



```

78         sp=(ss=t)+sm;
79         sm<=1;}
80     *(sp++)=x;
81 }
82 }me;
83 struct array{
84     hull**ps,**pp;
85     int pm;
86     array():
87         ps((hull**)malloc(sizeof(hull*))),pp(ps),pm(1){
88     }
89     ~array(){
90         free(ps);
91     }
92     int size(){
93         return pp-ps;
94     }
95     hull*operator[](int i){
96         return ps[i];
97     }
98     void push(hull*x){
99         if(pp==ps+pm){
100             hull**t=(hull**)malloc(sizeof(hull*)*pm<1);
101             memcpy(t,ps,pm*sizeof(hull*));
102             free(ps);
103             pp=(ps=t)+pm;
104             pm<=1;
105         }
106         *(pp++)=x;
107     }
108 };
109 static hull*link(hull*x,hull*y,hull*lb,hull*rb,int d,array&ns){
110     hull*r=me.create(*x);
111     if(x==lb||x==rb){
112         r->nb[d]=y;
113         if(y)
114             y->nb[!d]=r;
115     }else
116         r->ch[d]=link(r->ch[d],y,lb,rb,d,ns);
117     r->update();
118     ns.push(r);

```

```

119     return r;
120 }
121 static hull*merge(hull*x,hull*y,hull*lb,hull*rb,array&ns){
122     if(!x)
123         return y;
124     if(!y)
125         return x;
126     int d=x->fx>y->fx;
127     hull*r=me.create(d?*x:*y);
128     r->ch[d]=d?merge(r->ch[1],y,lb,rb,ns):merge(x,y->ch[0],lb,rb,ns);
129     if(d&&x==lb||!d&&y==rb)
130         r->ch[d]=link(r->ch[d],r,lb,rb,!d,ns);
131     r->update();
132     ns.push(r);
133     return r;
134 }
135 static pair<hull*,hull*>split(hull*x,int k,array&ns){
136     if(!x)
137         return make_pair((hull*)0,(hull*)0);
138     int t=sz(x->ch[0])+1;
139     hull*r=me.create(*x);
140     ns.push(r);
141     pair<hull*,hull*>s=split(x->ch[k>=t],k-t*(k>=t),ns);
142     if(k>=t){
143         r->ch[1]=s.first;r->update();
144         return make_pair(r,s.second);
145     }else{
146         r->ch[0]=s.second;r->update();
147         return make_pair(s.first,r);
148     }
149 }
150 static void turn(hull*&x,int d,int&k){
151     k+=(sz((x=x->ch[d])->ch[!d])+1)*(2*d-1);
152 }
153 static pair<T,T>range(hull*x){
154     hull*l=x,*r=x;
155     while(l->ch[0])
156         l=l->ch[0];
157     while(r->ch[1])
158         r=r->ch[1];
159     return make_pair(pt(l).x,pt(r).x);

```

```

160 }
161 static hull*merge(hull*x,hull*y,array&ns){
162     int kp=sz(x->ch[0])+1,kq=sz(y->ch[0])+1,pd[2],qd[2];
163     pair<T,T>pr=range(x),qr=range(y);
164     int pf=1;
165     hull*p=x,*q=y;
166     if(pr.second==qr.first&&pr.first==pr.second&&p->ch[pf=0])
167         turn(p,0,kp);
168     for(point pq=pt(q)-pt(p);;pq=pt(q)-pt(p)){
169         pd[0]=(p->nb[0]&&(pt(p->nb[0])-pt(p))*pq<=0)*pf;
170         qd[1]=(q->nb[1]&&(pt(q->nb[1])-pt(q))*pq<=0);
171         pd[1]=(p->nb[1]&&(pt(p->nb[1])-pt(p))*pq<0)*pf;
172         qd[0]=(q->nb[0]&&(pt(q->nb[0])-pt(q))*pq<0);
173         if(!(pd[0]+pd[1]+qd[0]+qd[1])){
174             hull*l=split(x,kp,ns).first,*r=split(y,kq-1,ns).second,*lb=
175             l,*rb=r;
176             while(lb->ch[1])
177                 lb=lb->ch[1];
178             while(rb->ch[0])
179                 rb=rb->ch[0];
180             return merge(l,r,lb,rb,ns);
181         }
182         if(!(pd[0]+pd[1]))
183             turn(q,qd[1],kq);
184         if(!(qd[0]+qd[1]))
185             turn(p,pd[1],kp);
186         if(pd[0]&&qd[1])
187             turn(p,0,kp),turn(q,1,kq);
188         if(pd[1]&&qd[1])
189             turn(q,1,kq);
190         if(pd[0]&&qd[0])turn(p,0,kp);
191         if(pd[1]&&qd[0]){
192             point vp=pt(p->nb[1])-pt(p),vq=pt(q->nb[0])-pt(q);
193             if(vp.x==0&&vq.x==0)
194                 turn(p,1,kp),turn(q,0,kq);
195             else if(vp.x==0)
196                 turn(p,1,kp);
197             else if(vq.x==0)
198                 turn(q,0,kq);
199             else{
199                 long double m=pr.second,pb=vp.y*(m-pt(p).x),qb=vq.y*(m-

```

```

pt(q).x);
200         pb=pb/vp.x+pt(p).y;
201         qb=qb/vq.x+pt(q).y;
202         if(qb>pb+1e-8)
203             turn(q,0,kq);
204         else if(pb>qb+1e-8)
205             turn(p,1,kp);
206         else if(pt(q->nb[0]).x+pt(p->nb[1]).x<2*m)
207             turn(q,0,kq);
208         else
209             turn(p,1,kp);
210     }
211 }
212 }
213 }
214 hull*query(hull*x,point p){
215     for(hull*y=0;;){
216         T d=x->check(p);
217         if(d>0)
218             y=x,x=x->ch[0];
219         else if(d<0)
220             x=x->ch[1];
221         else
222             y=x;
223         if(!d||!x)
224             return y;
225     }
226 }
227 struct treap{
228     int fx,ct,sz;
229     point pt;
230     treap*ch[2];
231     struct hull*ip,*hu;
232     array ns;
233     treap(point _pt):
234         fx(rand()*1.0/RAND_MAX*1e9),ct(1),sz(1),pt(_pt),ip(me.create(
235         hull(&pt))),hu(ip){
236         ch[0]=ch[1]=0;
237     }
238     ~treap(){
239         for(hull**i=ns.ps;i!=ns.pp;++i)

```

```

239         me.destroy(*i);
240     me.destroy(ip);
241 }
242 void update(){
243     for(hull**i=ns.ps;i!=ns.pp;++i)
244         me.destroy(*i);
245     ns.pp=ns.ps;
246     sz=1;
247     hu=ip;
248     if(ch[0])
249         hu=merge(ch[0]->hu,hu,ns),sz+=ch[0]->sz;
250     if(ch[1])
251         hu=merge(hu,ch[1]->hu,ns),sz+=ch[1]->sz;
252 }
253 }*root;
254 void rotate(treap*&x,int d){
255     treap*y=x->ch[d];
256     x->ch[d]=y->ch[!d];
257     y->ch[!d]=x;
258     x=y;
259 }
260 int insert(treap*&x,point p){
261     if(!x)
262         x=new treap(p);
263     else if(p==x->pt){
264         ++x->ct;
265         return 0;
266     }else{
267         int d=x->pt<p;
268         if(!insert(x->ch[d],p))
269             return 0;
270         if(x->ch[d]->fx>x->fx)
271             rotate(x,d),x->ch[!d]->update();
272         x->update();
273     }
274     return 1;
275 }
276 int erase(treap*&x,point p){
277     if(p==x->pt){
278         if(x->ct>1){
279             --x->ct;

```

```

280         return 0;
281     }
282     treap*y=x;
283     if(!x->ch[0])
284         x=x->ch[1], delete y;
285     else if(!x->ch[1])
286         x=x->ch[0], delete y;
287     else{
288         int d=x->ch[0]->fx<x->ch[1]->fx;
289         rotate(x,d);
290         erase(x->ch[!d],p);
291         x->update();
292     }
293     return 1;
294 }
295 if(erase(x->ch[x->pt<p],p)){
296     x->update();
297     return 1;
298 }else{
299     --x->sz;
300     return 0;
301 }
302 }
303 void clear(treap*x){
304     if(x)
305         clear(x->ch[0]), clear(x->ch[1]), delete x;
306 }
307 DynamicConvexHull():
308     root(0){
309 }
310 ~DynamicConvexHull(){
311     clear(root);
312 }
313 int size(){
314     return root?root->sz:0;
315 }
316 void insert(T x,T y){
317     insert(root,point(x,y));
318 }
319 void erase(T x,T y){
320     erase(root,point(x,y));

```

```

321     }
322     pair<T,T>query(T x,T y){
323         point r=pt(query(root->hu,point(x,y)));
324         return make_pair(r.x,r.y);
325     }
326 };
327 template<class T>typename DynamicConvexHull<T>::memory DynamicConvexHull<T
    >::me;

```

1.5 Geometry 2D

Geometry 2D.hpp (5120 bytes, 164 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  namespace Geometry2D{
4      double eps=1e-8;
5      long double pi=acos((long double)-1);
6      template<class T>T sqr(T a){
7          return a*a;
8      }
9      template<class T>int cmp(T a,T b){
10         if(typeid(T)==typeid(int)||typeid(T)==typeid(long long)){
11             if(a==b)
12                 return 0;
13             return a<b?-1:1;
14         }
15         if(a<b-eps)
16             return -1;
17         if(a>b+eps)
18             return 1;
19         return 0;
20     }
21     template<class T>struct Point{
22         T x,y;
23         Point(T _x=0,T _y=0):
24             x(_x),y(_y){
25         }
26         Point<T>&operator+=(const Point<T>&a){

```

```

27         return*this=*this+a;
28     }
29     Point<T>&operator+=(const Point<T>&a){
30         return*this=*this+a;
31     }
32 };
33 #define Vector Point
34 template<class T>Point<T>operator+(const Point<T>&a,const Point<T>&b){
35     return Point<T>(a.x+b.x,a.y+b.y);
36 }
37 template<class T>Point<T>operator-(const Point<T>&a,const Point<T>&b){
38     return Point<T>(a.x-b.x,a.y-b.y);
39 }
40 template<class T>Point<T>operator*(T a,const Point<T>&b){
41     return Point<T>(b.x*a,b.y*a);
42 }
43 template<class T>Point<T>operator*(const Point<T>&a,T b){
44     return b*a;
45 }
46 template<class T>Point<T>operator/(const Point<T>&a,T b){
47     return Point<T>(a.x/b,a.y/b);
48 }
49 template<class T>bool operator==(const Point<T>&a,const Point<T>&b){
50     return !cmp(a.x,b.x)&&!cmp(a.y,b.y);
51 }
52 template<class T>bool operator!=(const Point<T>&a,const Point<T>&b){
53     return !(a==b);
54 }
55 template<class T>bool operator<(const Point<T>&a,const Point<T>&b){
56     int t=cmp(a.x,b.x);
57     if(t)
58         return t<0;
59     return cmp(a.y,b.y)<0;
60 }
61 template<class T>bool operator>(const Point<T>&a,const Point<T>&b){
62     return b<a;
63 }
64 template<class T>Point<T>NaP(){
65     T t=numeric_limits<T>::max();
66     return Point<T>(t,t);
67 }

```



```

68     template<class T>T det(const Point<T>&a,const Point<T>&b){
69         return a.x*b.y-a.y*b.x;
70     }
71     template<class T>T dot(const Point<T>&a,const Point<T>&b){
72         return a.x*b.x+a.y*b.y;
73     }
74     template<class T>T abs(const Point<T>&a){
75         return sqrt(sqr(a.x)+sqr(a.y));
76     }
77     template<class T>T dis(const Point<T>&a,const Point<T>&b){
78         return abs(a-b);
79     }
80     template<class T>istream&operator>>(istream&s,Point<T>&a){
81         return s>>a.x>>a.y;
82     }
83     template<class T>ostream&operator<<(ostream&s,const Point<T>&a){
84         return s<<a.x<<" "<<a.y;
85     }
86     template<class T>struct Segment;
87     template<class T>struct Line{
88         Point<T>u,v;
89         Line(const Point<T>&_u=Point<T>(),const Point<T>&_v=Point<T>()):
90             u(_u),v(_v){
91         }
92         Line(const Segment<T>&a):
93             u(a.u),v(a.v){
94         }
95     };
96     template<class T>Point<T>nor(const Line<T>&a){
97         Point<T>t=a.v-a.u;
98         return Point<T>(t.y,-t.x);
99     }
100    template<class T>Point<T>dir(const Line<T>&a){
101        return a.v-a.u;
102    }
103    template<class T>int dir(const Line<T>a,const Point<T>b){
104        return cmp(det(b-a.u,a.v-a.u),T(0));
105    }
106    template<class T>Point<T>operator&(const Line<T>&a,const Line<T>&b){
107        T p=det(b.u-a.v,b.v-b.u),q=det(a.u-b.v,b.v-b.u);
108        return (a.u*p+a.v*q)/(p+q);

```

```

109 }
110 template<class T>struct Segment{
111     Point<T>u,v;
112     Segment(const Point<T>&_u=Point<T>(),const Point<T>&_v=Point<T>()):
113         u(_u),v(_v){
114     }
115 };
116 template<class T>Point<T>nor(const Segment<T>&a){
117     Point<T>t=a.v-a.u;
118     return Point<T>(t.y,-t.x);
119 }
120 template<class T>Point<T>dir(const Segment<T>&a){
121     return a.v-a.u;
122 }
123 template<class T>int dir(const Segment<T>a,const Point<T>b){
124     return cmp(b-a.u,a.v-a.u);
125 }
126 template<class T>Point<T>operator&(const Line<T>&a,const Segment<T>&b){
127     if(dir(a,b.u)*dir(a,b.v)<=0)
128         return a&Line<T>(b);
129     return NaP<T>();
130 }
131 template<class T>Point<T>operator&(const Segment<T>&a,const Line<T>&b){
132     return b&a;
133 }
134 template<class T>pair<T,T>dis(const Segment<T>&a,const Point<T>&b){
135     pair<T,T>d(dis(a.u,b),dis(a.v,b));
136     if(d.first>d.second)
137         swap(d.first,d.second);
138     Point<T>t=Line<T>(b,b+nor(a))&a;
139     if(t!=NaP<T>())
140         d.first=dis(t,b);
141     return d;
142 }
143 template<class T>pair<T,T>dis(const Point<T>&a,const Segment<T>&b){
144     return dis(b,a);
145 }
146 template<class T>struct Circle{
147     Point<T>c;
148     T r;
149     Circle(const Point<T>&_c=Point<T>(),T _r=0):

```

```

150         c(_c),r(_r){
151     }
152 };
153 template<class T>T abs(const Circle<T>&a){
154     return pi*sqr(a.r);
155 }
156 template<class T>bool col(const Point<T>&a,const Point<T>&b,const Point
157 <T>&c){
158     return !cmp(det(a-c,b-c),T(0));
159 }
160 template<class T>T read(){
161     T t;
162     cin>>t;
163     return t;
164 }

```

1.6 Half-Plane Intersection

Half-Plane Intersection.hpp (1950 bytes, 70 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  namespace HalfPlaneIntersection{
4      const double E=1e-8;
5      struct pot{
6          pot(double a=0,double b=0):
7              x(a),y(b){
8          }
9          double x,y;
10     };
11     double ag(pot p){
12         return atan2(double(p.x),double(p.y));
13     }
14     pot operator+(pot p,pot q){
15         return pot(p.x+q.x,p.y+q.y);
16     }
17     pot operator-(pot p,pot q){
18         return pot(p.x-q.x,p.y-q.y);

```

```

19     }
20     pot operator*(pot p, double q){
21         return pot(p.x*q, p.y*q);
22     }
23     pot operator/(pot p, double q){
24         return pot(p.x/q, p.y/q);
25     }
26     double det(pot p, pot q){
27         return p.x*q.y - q.x*p.y;
28     }
29     double dot(pot p, pot q){
30         return p.x*q.x + p.y*q.y;
31     }
32     struct lin{
33         pot p, q;
34         double a;
35         lin(pot a, pot b):
36             p(a), q(b), a(ag(b-a)){
37         }
38     };
39     pot operator*(lin a, lin b){
40         double a1 = det(b.p - a.q, b.q - b.p);
41         double a2 = det(a.p - b.q, b.q - b.p);
42         return (a.p*a1 + a.q*a2)/(a1+a2);
43     }
44     bool cmp(lin a, lin b){
45         if(fabs(a.a - b.a) > E)
46             return a.a < b.a;
47         else
48             return det(a.q - b.p, b.q - b.p) < -E;
49     }
50     bool left(lin a, lin b, lin c){
51         pot t = a*b;
52         return det(t - c.p, c.q - c.p) < -E;
53     }
54     deque<lin> run(vector<lin> lns){
55         deque<lin> ans;
56         sort(lns.begin(), lns.end(), cmp);
57         for(int i=0; i<lns.size(); ++i){
58             while(ans.size() > 1 && !left(ans.back(), ans[ans.size()-2], lns[i]))
59                 ans.pop_back();

```

```
60         while(ans.size()>1&&!left(ans[0],ans[1],lns[i]))
61             ans.pop_front();
62         if(ans.empty()||fabs(ans.back().a-lns[i].a)>E)
63             ans.push_back(lns[i]);}
64     while(ans.size()>1&&!left(ans[ans.size()-1],ans[ans.size()-2],ans.
front()))
65         ans.pop_back();
66     if(ans.size()<3)
67         ans.clear();
68     return ans;
69 }
70 }
```


CHAPTER 2

Data Structures

2.1 Binary Heap

BinaryHeap.hpp (1629 bytes, 73 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T, class C> struct BinaryHeap{
4      struct node{
5          node(int _p, T _v):
6              p(_p), v(_v){
7              }
8          int p;
9          T v;
10     };
11     vector<node*> a;
12     BinaryHeap():
13         a(1){
14     }
15     ~BinaryHeap(){
16         clear();
17     }
18     void move(int i, int j){
19         swap(a[i]->p, a[j]->p);
20         swap(a[i], a[j]);
21     }
22     int check(int i, int j){
23         if(!j || j>=a.size() || a[i]->v==a[j]->v)
24             return 0;
25         return a[i]->v<a[j]->v?-1:1;
26     }
27     int up(int i){
28         if(check(i, i>>1)<0){
29             move(i, i>>1);
30             return i>>1;
31         }else
32             return 0;
33     }
34     int down(int i){
35         if(check(i, i<<1)<=0&&check(i, i<<1^1)<=0)
36             return a.size();
37         if(check(i<<1, i<<1^1)<=0){

```



```
38         move(i,i<<1);
39         return i<<1;
40     }else{
41         move(i,i<<1^1);
42         return i<<1^1;
43     }
44 }
45 void maintain(int i){
46     for(int j=up(i);j;i=j,j=up(i));
47     for(int j=down(i);j<a.size();i=j,j=down(i));
48 }
49 void clear(){
50     for(int i=1;i<a.size();++i)
51         delete a[i];
52     a.resize(1);
53 }
54 node*push(T v){
55     a.push_back(new node(a.size(),v));
56     node*r=a.back();
57     maintain(a.size()-1);
58     return r;
59 }
60 T top(){
61     return a[1]->v;
62 }
63 void pop(){
64     move(1,a.size()-1);
65     delete a.back();
66     a.pop_back();
67     maintain(1);
68 }
69 void modify(node*x,T v){
70     x->v=v;
71     maintain(x->p);
72 }
73 };
```

2.2 Dynamic Sequence

Dynamic Sequence.hpp (4119 bytes, 177 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>struct DynamicSequence{
4      struct node{
5          node(T _i):
6              i(_i),v(_i),s(1),r(0){
7              c[0]=c[1]=0;
8              static int g;
9              w=g=(214013*g+2531011);
10         }
11         T i,v;
12         int s,r,w;
13         node*c[2];
14     }*rt,*sl,*sr;
15     struct pool{
16         node*ps,*pp,**ss,**sp;
17         int pm,sm;
18         vector<node*>ns;
19         pool():
20             ps((node*)malloc(sizeof(node))),pp(ps),pm(1),ss((node**)malloc(
21             sizeof(node*))),sp(ss),sm(1){
22             ns.push_back(ps);
23         }
24         ~pool(){
25             free(ss);
26             for(int i=0;i<ns.size();++i)
27                 free(ns[i]);
28         }
29         node*crt(T a){
30             if(sp!=ss){
31                 --sp;
32                 **sp=node(a);
33                 return*sp;
34             }
35             if(pp==ps+pm){
36                 pp=ps=(node*)malloc(sizeof(node)*(pm<=1));
37                 ns.push_back(ps);

```

```

37     }
38     *pp=node(a);
39     return pp++;
40 }
41 void des(node*x){
42     if(sp==ss+sm){
43         node**t=(node**)malloc(sizeof(node*)*sm<<1);
44         memcpy(t,ss,sm*sizeof(node*));
45         free(ss);
46         sp=(ss=t)+sm;
47         sm<=<=1;
48     }
49     *(sp++)=x;
50 }
51 }me;
52 node*bud(T*a,int l,int r){
53     if(l>r)
54         return 0;
55     int m=l+r>>1;
56     node*t=me.crt(a[m]);
57     t->c[0]=bud(a,l,m-1);
58     t->c[1]=bud(a,m+1,r);
59     pup(t);
60     return t;
61 }
62 void pdw(node*x){
63     for(int d=0;d<2&&(x->i>x->v,1);++d)
64         if(x->c[d])
65             x->i>x->c[d]->i;
66     *x->i;
67     *x->v;
68     if(x->r){
69         -x->i;
70         for(int d=0;d<2;++d)
71             if(x->c[d])
72                 x->c[d]->r^=1;
73         swap(x->c[0],x->c[1]);
74         x->r=0;
75     }
76 }
77 void pup(node*x){

```

```

78     x->i=x->v;
79     x->s=1;
80     for(int d=0;d<2;++d)
81         if(x->c[d])
82             pdw(x->c[d]),x->s+=x->c[d]->s,x->i=d?x->i+x->c[d]->i:x->
c[d]->i+x->i;
83     }
84     void jon(node*x){
85         rt=jon(jon(sl,x),sr);
86     }
87     node*jon(node*x,node*y){
88         if(!x)
89             return y;
90         if(!y)
91             return x;
92         pdw(x);
93         pdw(y);
94         if(x->w<y->w){
95             x->c[1]=jon(x->c[1],y);
96             pup(x);
97             return x;
98         }else{
99             y->c[0]=jon(x,y->c[0]);
100            pup(y);
101            return y;
102        }
103    }
104    node*spt(int l,int r){
105        spt(rt,l-1);
106        node*t=sl;
107        spt(sr,r-l+1);
108        swap(sl,t);
109        return t;
110    }
111    void spt(node*x,int p){
112        if(!x){
113            sl=sr=0;
114            return;
115        }
116        pdw(x);
117        int t=x->c[0]?x->c[0]->s:0;

```

```

118         if(t<p)
119             spt(x->c[1],p-t-1),x->c[1]=s1,s1=x;
120         else
121             spt(x->c[0],p),x->c[0]=sr,sr=x;
122         pup(x);
123     }
124     void clr(node*x){
125         if(x)
126             clr(x->c[0]),clr(x->c[1]),me.des(x);
127     }
128     DynamicSequence(T*a=0,int n=0){
129         rt=bud(a,1,n);
130     }
131     ~DynamicSequence(){
132         clr(rt);
133     }
134     void clear(){
135         clr(rt);
136         rt=0;
137     }
138     void insert(T a,int p){
139         insert(&a-1,1,p);
140     }
141     void insert(T*a,int n,int p){
142         spt(p+1,p);
143         jon(bud(a,1,n));
144     }
145     void erase(int p){
146         erase(p,p);
147     }
148     void erase(int l,int r){
149         clr(spt(l,r));
150         jon(0);
151     }
152     T query(int p){
153         return query(p,p);
154     }
155     T query(int l,int r){
156         node*t=spt(l,r);
157         T i=t->i;
158         jon(t);

```

```

159     return i;
160 }
161 void modify(T a,int l){
162     modify(a,l,l);
163 }
164 void modify(T a,int l,int r){
165     node*t=spt(l,r);
166     a>t->i;
167     jon(t);
168 }
169 void reverse(int l,int r){
170     node*t=spt(l,r);
171     t->r=1;
172     jon(t);
173 }
174 int length(){
175     return rt?rt->s:0;
176 }
177 };

```

2.3 Fenwick Tree

Fenwick Tree.hpp (529 bytes, 25 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>struct FenwickTree{
4      FenwickTree(int _n):
5          n(_n),l(log2(n)),a(n+1){
6      }
7      void add(int v,T d){
8          for(;v<=n;v+=v&-v)
9              a[v]+=d;
10     }
11     T sum(int v){
12         T r=0;
13         for(;v>=1;v-=v&-v)
14             r+=a[v];
15         return r;

```

```

16     }
17     int kth(T k,int r=0){
18         for(int i=1<<1;i;i>=1)
19             if(r+i<=n&&a[r+i]<k)
20                 k-=a[r+=i];
21         return r+1;
22     }
23     int n,l;
24     vector<T>a;
25 };

```

2.4 K-D Tree

K-D Tree.hpp (2467 bytes, 80 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  struct KDTree{
4      struct node{
5          node(int x0,int x1,int d):
6              color(1),cover(0),dir(d){
7              ch[0]=ch[1]=0;
8              x[0]=mi[0]=mx[0]=x0;
9              x[1]=mi[1]=mx[1]=x1;
10         }
11         node*ch[2];
12         int x[2],mi[2],mx[2],color,cover,dir;
13     }*root;
14     KDTree(pair<int,int>*a,int n){
15         root=build(a,1,n,0);
16     }
17     static int direct;
18     static int cmp(pair<int,int>a,pair<int,int>b){
19         if(direct)
20             return make_pair(a.second,a.first)<make_pair(b.second,b.first);
21         return a<b;
22     }
23     node*build(pair<int,int>*a,int l,int r,int d){
24         int m=(r+l)/2;

```

```

25     direct=d;
26     nth_element(a+l,a+m,a+r+1,cmp);
27     node*p=new node((a+m)->first,(a+m)->second,d);
28     if(l!=m)
29         p->ch[0]=build(a,l,m-1,!d);
30     if(r!=m)
31         p->ch[1]=build(a,m+1,r,!d);
32     for(int i=0;i<2;++i)
33         for(int j=0;j<2;++j)
34             if(p->ch[j]){
35                 p->mi[i]=min(p->mi[i],p->ch[j]->mi[i]);
36                 p->mx[i]=max(p->mx[i],p->ch[j]->mx[i]);
37             }
38     return p;
39 }
40 void down(node*a){
41     if(a->cover){
42         for(int i=0;i<2;++i)
43             if(a->ch[i])
44                 a->ch[i]->cover=a->cover;
45         a->color=a->cover;
46         a->cover=0;
47     }
48 }
49 void modify(node*a,int mi0,int mx0,int mi1,int mx1,int c){
50     if(mi0>a->mx[0] || mx0<a->mi[0] || mi1>a->mx[1] || mx1<a->mi[1])
51         return;
52     if(mi0<=a->mi[0]&&mx0>=a->mx[0]&&mi1<=a->mi[1]&&mx1>=a->mx[1]){
53         a->cover=c;
54         return;
55     }
56     down(a);
57     if(mi0<=a->x[0]&&mx0>=a->x[0]&&mi1<=a->x[1]&&mx1>=a->x[1])
58         a->color=c;
59     for(int i=0;i<2;++i)
60         if(a->ch[i])
61             modify(a->ch[i],mi0,mx0,mi1,mx1,c);
62 }
63 void modify(int mi0,int mx0,int mi1,int mx1,int c){
64     modify(root,mi0,mx0,mi1,mx1,c);
65 }

```



```

66     int query(node*a,int x0,int x1){
67         down(a);
68         if(x0==a->x[0]&& x1==a->x[1])
69             return a->color;
70         direct=a->dir;
71         if(cmp(make_pair(x0,x1),make_pair(a->x[0],a->x[1])))
72             return query(a->ch[0],x0,x1);
73         else
74             return query(a->ch[1],x0,x1);
75     }
76     int query(int x0,int x1){
77         return query(root,x0,x1);
78     }
79 };
80 int KDTree::direct=0;

```

2.5 Link-Cut Tree

Link-Cut Tree.hpp (5518 bytes, 215 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>struct LinkCutTree{
4      struct node{
5          node():
6              ch({0,0}),pr(0),rev(0){
7              }
8              node*ch[2],*pr;
9              T ifo;
10             int rev;
11         }*ptrs;
12         LinkCutTree(int n):
13             ptrs(new node[n]-1){
14         }
15         ~LinkCutTree(){
16             delete ptrs;
17         }
18         int direct(node*x){
19             if(!x->pr)

```

```

20         return 2;
21     if(x==x->pr->ch[0])
22         return 0;
23     if(x==x->pr->ch[1])
24         return 1;
25     return 2;
26 }
27 void down(node*x){
28     if(x->rev){
29         x->ifo.reverse();
30         swap(x->ch[0],x->ch[1]);
31         for(int i=0;i<2;++i)
32             if(x->ch[i])
33                 x->ch[i]->rev^=1;
34         x->rev=0;
35     }
36     x->ifo.down(x->ch[0]?&x->ch[0]->ifo:0,x->ch[1]?&x->ch[1]->ifo:0);
37 }
38 void up(node*x){
39     for(int i=0;i<2;++i)
40         if(x->ch[i])
41             down(x->ch[i]);
42     x->ifo.up(x->ch[0]?&x->ch[0]->ifo:0,x->ch[1]?&x->ch[1]->ifo:0);
43 }
44 void setchild(node*x,node*y,int d){
45     x->ch[d]=y;
46     if(y)
47         y->pr=x;
48     up(x);
49 }
50 void rotate(node*x){
51     node*y=x->pr,*z=y->pr;
52     int d1=direct(x),d2=direct(y);
53     setchild(y,x->ch[!d1],d1);
54     setchild(x,y,!d1);
55     if(d2<2)
56         setchild(z,x,d2);
57     else
58         x->pr=z;
59 }
60 void release(node*x){

```

```

61         if(direct(x)<2)
62             release(x->pr);
63         down(x);
64     }
65     void splay(node*x){
66         for(release(x);direct(x)<2;){
67             node*y=x->pr;
68             if(direct(y)==2)
69                 rotate(x);
70             else if(direct(x)==direct(y))
71                 rotate(y),rotate(x);
72             else
73                 rotate(x),rotate(x);
74         }
75     }
76     node*access(node*x){
77         node*y=0;
78         for(;x;y=x,x=x->pr){
79             splay(x);
80             setchild(x,y,1);
81         }
82         return y;
83     }
84     void evert(node*x){
85         access(x);
86         splay(x);
87         x->rev=1;
88     }
89     void set(int x,T v){
90         ptrs[x].ifo=v;
91     }
92     int linked(int a,int b){
93         access((ptrs+a));
94         node*z=access((ptrs+b));
95         return z==access((ptrs+a));
96     }
97     void link(int a,int b){
98         evert((ptrs+b));
99         (ptrs+b)->pr=(ptrs+a);
100     }
101     void cut(int a,int b){

```

```

102     access((ptrs+b));
103     node*z=access((ptrs+a));
104     if(z==(ptrs+a))
105         splay((ptrs+b)),(ptrs+b)->pr=0;
106     else
107         access((ptrs+b)),splay((ptrs+a)),(ptrs+a)->pr=0;
108 }
109 int root(int a){
110     access((ptrs+a));
111     splay((ptrs+a));
112     node*r=(ptrs+a);
113     while(r->ch[1])
114         r=r->ch[1];
115     return r-ptrs;
116 }
117 void evert(int a){
118     evert((ptrs+a));
119 }
120 int lca(int a,int b){
121     access((ptrs+a));
122     return access((ptrs+b))-ptrs;
123 }
124 T query(int a){
125     splay((ptrs+a));
126     T p=(ptrs+a)->ifo;
127     p.up(0,0);
128     return p;
129 }
130 T query(int a,int b){
131     if((ptrs+a)==(ptrs+b))
132         return query((ptrs+a));
133     access((ptrs+a));
134     node*c=access((ptrs+b));
135     T p=c.ifo;
136     if(c==(ptrs+b)){
137         splay((ptrs+a));
138         T q=(ptrs+a)->ifo;
139         q.reverse();
140         p.up(&q,0);
141         return p;
142     }else if(c==(ptrs+a))

```

```

143         p.up(0,&(ptrs+a)->ch[1]->if0);
144     else{
145         splay((ptrs+a));
146         T q=(ptrs+a)->if0;
147         q.reverse();
148         p.up(&q,&c->ch[1]->if0);
149     }
150     return p;
151 }
152 T equery(int a){
153     return query(a);
154 }
155 T equery(int a,int b){
156     access((ptrs+a));
157     node*c=access((ptrs+b));
158     if(c==(ptrs+b)){
159         splay((ptrs+a));
160         T q=(ptrs+a)->if0;
161         q.reverse();
162         return q;
163     }else if(c==(ptrs+a))
164         return (ptrs+a)->ch[1]->if0;
165     else{
166         splay((ptrs+a));
167         node*t=c->ch[1];
168         while(t->ch[0])
169             t=t->ch[0];
170         splay(t);
171         if(t->ch[1])
172             down(t->ch[1]);
173         T p=t->if0,q=(ptrs+a)->if0;
174         q.reverse();
175         p.up(&q,t->ch[1]?&t->ch[1]->if0:0);
176         return p;
177     }
178 }
179 template<class F>void modify(int a,F f){
180     splay((ptrs+a));
181     f(&(ptrs+a)->if0);
182     up((ptrs+a));
183 }

```

```

184 template<class F>void modify(int a,int b,F f){
185     if((ptrs+a)==(ptrs+b)){
186         splay((ptrs+a));
187         f(0,&(ptrs+a)->ifo,0);
188         up((ptrs+a));
189         return;
190     }
191     access((ptrs+a));
192     node*c=access((ptrs+b));
193     if(c==(ptrs+b))
194         splay((ptrs+a)),f(&(ptrs+a)->ifo,&(ptrs+b)->ifo,0);
195     else if(c==a)
196         f(0,&(ptrs+a)->ifo,&(ptrs+a)->ch[1]->ifo);
197     else
198         splay(a),f(&(ptrs+a)->ifo,&c->ifo,&c->ch[1]->ifo);
199     up(c);
200 }
201 template<class F>void emodify(int a,F f){
202     modify(a,f);
203 }
204 template<class F>void emodify(int a,int b,F f){
205     access((ptrs+a));
206     node*c=access((ptrs+b));
207     if(c==(ptrs+b))
208         splay((ptrs+a)),f(&(ptrs+a)->ifo,0);
209     else if(c==a)
210         f(0,&(ptrs+a)->ch[1]->ifo);
211     else
212         splay(a),f(&(ptrs+a)->ifo,&c->ch[1]->ifo);
213     up(c);
214 }
215 };

```

2.6 Pairing Heap

Pairing Heap.hpp (2226 bytes, 102 lines)

```

1 #include<bits/stdc++.h>
2 using namespace std;

```

```

3  template<class T, class C> struct PairingHeap{
4      PairingHeap():
5          root(0), siz(0){
6      }
7      ~PairingHeap(){
8          clear(root);
9      }
10     struct node{
11         node(const T& val):
12             val(_val), ch(0), br(0), pr(0){
13         }
14         T val;
15         node*ch, *br, *pr;
16     }*root;
17     int siz;
18     void merge(node*&x, node*&y){
19         if(!x)
20             x=y;
21         else if(y){
22             if(C()(y->val, x->val))
23                 swap(x, y);
24             y->br=x->ch;
25             if(x->ch)
26                 x->ch->pr=y;
27             y->pr=x;
28             x->ch=y;
29         }
30     }
31     void cut(node*&x, node*&y){
32         if(x==y)
33             x=0;
34         else{
35             if(y==y->pr->ch)
36                 y->pr->ch=y->br;
37             else
38                 y->pr->br=y->br;
39             if(y->br)
40                 y->br->pr=y->pr;
41             y->pr=y->br=0;
42         }
43     }

```

```

44 node*split(node*x){
45     vector<node*>t;
46     for(node*i=x->ch;i;i=i->br)
47         t.push_back(i);
48     x->ch=0;
49     node*r=0;
50     for(int i=0;i<t.size();++i)
51         t[i]->pr=t[i]->br=0;
52     for(int i=0;i+1<t.size();i+=2)
53         merge(t[i],t[i+1]);
54     for(int i=0;i<t.size();i+=2)
55         merge(r,t[i]);
56     return r;
57 }
58 void clear(node*x){
59     if(x){
60         clear(x->ch);
61         clear(x->br);
62         delete x;
63     }
64 }
65 void clear(){
66     clear(root);
67     root=0;
68     siz=0;
69 }
70 node*push(T a){
71     node*r=new node(a);
72     merge(root,r);
73     ++siz;
74     return r;
75 }
76 void erase(node*x){
77     cut(root,x);
78     merge(root,split(x));
79     --siz;
80 }
81 T top(){
82     return root->val;
83 }
84 void pop(){

```



```

85     erase(root);
86 }
87 void merge(PairingHeap<T,C>&a){
88     merge(root,a.root);
89     a.root=0;
90     siz+=a.siz;
91     a.siz=0;
92 }
93 void modify(node*x,T v){
94     if(C()(x->val,v))
95         x->val=v,merge(root,split(x));
96     else
97         x->val=v,cut(root,x),merge(root,x);
98 }
99 int size(){
100     return siz;
101 }
102 };

```

2.7 Red-Black Tree

Red-Black Tree.hpp (7432 bytes, 307 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T,class C>struct RedBlackTree{
4      struct node{
5          node(T _v,node*l,node*r,node*_p,int _b,int _s):
6              v(_v),p(_p),b(_b),s(_s){
7              c[0]=l;
8              c[1]=r;
9          }
10         T v;
11         node*c[2],*p;
12         int b,s;
13     }*root,*nil;
14     void clear(node*x){
15         if(x!=nil){
16             clear(x->c[0]);

```

```

17         clear(x->c[1]);
18         delete x;
19     }
20 }
21 void rotate(node*x,int d){
22     node*y=x->c[!d];
23     x->c[!d]=y->c[d];
24     if(y->c[d]!=nil)
25         y->c[d]->p=x;
26     y->p=x->p;
27     if(x->p==nil)
28         root=y;
29     else
30         x->p->c[x!=x->p->c[0]]=y;
31     y->c[d]=x;
32     x->p=y;
33     y->s=x->s;
34     x->s=x->c[0]->s+x->c[1]->s+1;
35 }
36 void insert_fixup(node*z){
37     while(!z->p->b){
38         int d=z->p==z->p->p->c[0];
39         node*y=z->p->p->c[d];
40         if(!y->b)
41             z->p->b=1,y->b=1,(z=z->p->p)->b=0;
42         else{
43             if(z==z->p->c[d])
44                 rotate(z=z->p,!d);
45             z->p->b=1;
46             z->p->p->b=0;
47             rotate(z->p->p,d);
48         }
49     }
50     root->b=1;
51 }
52 void erase(node*z){
53     node*y;
54     for(y=z;y!=nil;y=y->p)
55         --y->s;
56     if(z->c[0]==nil||z->c[1]==nil)
57         y=z;

```

```

58     else{
59         for(y=z->c[1];y->c[0]!=nil;)
60             y=y->c[0];
61         z->v=y->v;
62         y=z->c[1];
63         while(y->c[0]!=nil)
64             --y->s,y=y->c[0];
65     }
66     node*x=y->c[y->c[0]==nil];
67     x->p=y->p;
68     if(y->p==nil)
69         root=x;
70     else
71         y->p->c[y!=y->p->c[0]]=x;
72     if(y->b)
73         erase_fixup(x);
74     delete y;
75 }
76 void erase_fixup(node*x){
77     while(x!=root&&x->b){
78         int d=x==x->p->c[0];
79         node*w=x->p->c[d];
80         if(!w->b){
81             w->b=1;
82             x->p->b=0;
83             rotate(x->p,!d);
84             w=x->p->c[d];
85         }
86         if(w->c[0]->b&&w->c[1]->b)
87             w->b=0,x=x->p;
88         else{
89             if(w->c[d]->b)
90                 w->c[!d]->b=1,w->b=0,rotate(w,d),w=x->p->c[d];
91             w->b=x->p->b;
92             x->p->b=1;
93             w->c[d]->b=1;
94             rotate(x->p,!d);
95             x=root;
96         }
97     }
98     x->b=1;

```

```

99     }
100 node*clone(node*x,node*y){
101     if(x.size==0)
102         return nil;
103     node*z=new node(*x);
104     z->c[0]=clone(x->c[0],z);
105     z->c[1]=clone(x->c[1],z);
106     z->p=y;
107     return z;
108 }
109 node*precursor(node*x){
110     if(x->c[0]->count){
111         for(x=x->c[0];x->c[1]->count;)
112             x=x->c[1];
113         return x;
114     }else{
115         node*y=x->p;
116         while(y->count&&x==y->c[0])
117             x=y,y=y->p;
118         return y;
119     }
120 }
121 node*successor(node*x){
122     if(x->c[1]->count){
123         for(x=x->c[1];x->c[0]->count;)
124             x=x->c[0];
125         return x;
126     }else{
127         node*y=x->p;
128         while(y->count&&x==y->c[1])
129             x=y,y=y->p;
130         return y;
131     }
132 }
133 RedBlackTree(){
134     root=nil=(node*)malloc(sizeof(node));
135     nil->b=1;
136     nil->s=0;
137 }
138 RedBlackTree(const RedBlackTree&a){
139     nil=new node(*a.nil);

```

```

140     root=clone(a.root,nil);
141 }
142 ~RedBlackTree(){
143     clear(root);
144     free(nil);
145 }
146 RedBlackTree&operator=(const RedBlackTree&a){
147     clear(root);
148     root=clone(a.root,nil);
149     return*this;
150 }
151 node*begin(){
152     node*z=root;
153     while(z!=nil&&z->c[0]!=nil)
154         z=z->c[0];
155     return z;
156 }
157 node*reverse_begin(){
158     node*z=root;
159     while(z!=nil&&z->c[1]!=nil)
160         z=z->c[1];
161     return z;
162 }
163 node*end(){
164     return nil;
165 }
166 node*reverse_end(){
167     return nil;
168 }
169 void clear(){
170     clear(root);
171     root=nil;
172 }
173 void insert(T a){
174     node*y=nil,*x=root;
175     while(x!=nil)
176         y=x,++x->s,x=x->c[C()](x->v,a)];
177     node*z=new node(a,nil,nil,y,0,1);
178     if(y==nil)
179         root=z;
180     else

```

```

181         y->c[C() (y->v, z->v)] = z;
182     insert_fixup(z);
183 }
184 void erase(T a){
185     node*z=root;
186     for(;;)
187         if(C() (a, z->v))
188             z=z->c[0];
189         else if(C() (z->v, a))
190             z=z->c[1];
191         else
192             break;
193     erase(z);
194 }
195 int count(T a){
196     return count_less_equal(a)-count_less(a);
197 }
198 int count_less(T a){
199     int r=0;
200     node*z=root;
201     while(z!=nil)
202         if(C() (z->v, a))
203             r+=z->c[0]->s+1, z=z->c[1];
204         else
205             z=z->c[0];
206     return r;
207 }
208 int count_less_equal(T a){
209     int r=0;
210     node*z=root;
211     while(z!=nil){
212         if(!C() (a, z->v))
213             r+=z->c[0]->s+1, z=z->c[1];
214         else
215             z=z->c[0];
216     }
217     return r;
218 }
219 int count_greater(T a){
220     int r=0;
221     node*z=root;

```

```

222     while(z!=nil)
223         if(C()(a,z->v))
224             r+=z->c[1]->s+1, z=z->c[0];
225         else
226             z=z->c[1];
227     return r;
228 }
229 int count_greater_equal(T a){
230     int r=0;
231     node*z=root;
232     while(z!=nil)
233         if(!C()(z->v,a))
234             r+=z->c[1]->s+1, z=z->c[0];
235         else
236             z=z->c[1];
237     return r;
238 }
239 node*nth_element(int a){
240     node*z=root;
241     for(;;)
242         if(z->c[0]->s>=a)
243             z=z->c[0];
244         else if((z->c[0]->s+1)<a)
245             a-=z->c[0]->s+1, z=z->c[1];
246         else
247             return z;
248 }
249 node*precursor(T a){
250     node*z=root,*r=nil;
251     while(z!=nil)
252         if(C()(z->v,a))
253             r=z, z=z->c[1];
254         else
255             z=z->c[0];
256     return r;
257 }
258 node*successor(T a){
259     node*z=root,*r=nil;
260     while(z!=nil)
261         if(C()(a,z->v))
262             r=z, z=z->c[0];

```

```

263         else
264             z=z->c[1];
265     return r;
266 }
267 node*find(T a){
268     node*z=root,*r=nil;
269     while(z!=nil)
270         if(C()(a,z->v))
271             z=z->c[0];
272         else if(C()(z->v,a))
273             z=z->c[1];
274         else
275             break;
276     return r;
277 }
278 node*lower_bound(T a){
279     node*z=root,*r=nil;
280     while(z!=nil)
281         if(C()(z->v,a))
282             r=z,z=z->c[1];
283         else if(C()(a,z->v))
284             z=z->c[0];
285         else
286             r=z,z=z->c[0];
287     return r;
288 }
289 node*upper_bound(T a){
290     return successor(a);
291 }
292 pair<node*,node*> equal_range(T a){
293     return make_pair(lower_bound(a),upper_bound(a));
294 }
295 int size(){
296     return root->s;
297 }
298 int empty(){
299     return !root->s;
300 }
301 T front(){
302     return*begin();
303 }

```



```

304     T back(){
305         return*reverse_begin();
306     }
307 };

```

2.8 Self-Adjusting Top Tree

Self-Adjusting Top Tree.hpp (12629 bytes, 443 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  struct SelfAdjustingTopTree{
4      const static int inf=~0u>>1;
5      static void gmin(int&a,int b){
6          a=min(a,b);
7      }
8      static void gmax(int&a,int b){
9          a=max(a,b);
10     }
11     struct treap{
12         SelfAdjustingTopTree*tr;
13         treap(struct SelfAdjustingTopTree*a,int n):
14             tr(a),ns(n){
15             }
16         struct node{
17             node(){
18             }
19             node(int a,int b,int c,int d,int e){
20                 ch[0]=ch[1]=0;
21                 val=a;
22                 fix=rand();
23                 add=0;
24                 mi=vmi=b;
25                 mx=vmx=c;
26                 sum=vsum=d;
27                 siz=vsiz=e;
28                 sam=inf;
29             }
30             node*ch[2];

```

```

31         int val, fix, vmi, vmx, vsum, vsiz, mi, mx, sum, siz, add, sam;
32     };
33     vector<node>ns;
34     void down(node*a){
35         if(a->sam!=inf){
36             a->mi=a->mx=a->vmi=a->vmx=a->sam;
37             a->vsum=a->sam*a->vsiz;
38             a->sum=a->sam*a->siz;
39             (&tr->ns[0]+(a-&ns[0]))->viradd=0;
40             (&tr->ns[0]+(a-&ns[0]))->virsam=a->sam;
41             (&tr->ns[0]+(a-&ns[0]))->add=0;
42             (&tr->ns[0]+(a-&ns[0]))->sam=a->sam;
43             for(int i=0;i<=1;++i)
44                 if(a->ch[i])
45                     a->ch[i]->add=0,a->ch[i]->sam=a->sam;
46             a->sam=inf;
47         }
48         if(a->add){
49             a->mi+=a->add;
50             a->mx+=a->add;
51             a->vmi+=a->add;
52             a->vmx+=a->add;
53             a->vsum+=a->add*a->vsiz;
54             a->sum+=a->add*a->siz;
55             (&tr->ns[0]+(a-&ns[0]))->viradd+=a->add;
56             (&tr->ns[0]+(a-&ns[0]))->add+=a->add;
57             for(int i=0;i<=1;++i)
58                 if(a->ch[i])
59                     a->ch[i]->add+=a->add;
60             a->add=0;
61         }
62     }
63     void update(node*a){
64         for(int i=0;i<=1;++i)
65             if(a->ch[i])
66                 down(a->ch[i]);
67         a->mi=a->vmi;
68         for(int i=0;i<=1;++i)
69             if(a->ch[i])
70                 gmin(a->mi,a->ch[i]->mi);
71         a->mx=a->vmx;

```

```

72         for(int i=0;i<=1;++i)
73             if(a->ch[i])
74                 gmax(a->mx,a->ch[i]->mx);
75         a->sum=a->vsum;
76         for(int i=0;i<=1;++i)
77             if(a->ch[i])
78                 a->sum+=a->ch[i]->sum;
79         a->siz=a->vsiz;
80         for(int i=0;i<=1;++i)
81             if(a->ch[i])
82                 a->siz+=a->ch[i]->siz;
83     }
84     void rotate(node*&a,int d){
85         node*b=a->ch[d];
86         a->ch[d]=b->ch[!d];
87         b->ch[!d]=a;
88         update(a);
89         update(b);
90         a=b;
91     }
92     void insert(node*&a,node*b){
93         if(!a)
94             a=b;
95         else{
96             down(a);
97             int d=b->val>a->val;
98             insert(a->ch[d],b);
99             update(a);
100            if(a->ch[d]->fix<a->fix)
101                rotate(a,d);
102        }
103    }
104     void erase(node*&a,int b){
105         down(a);
106         if(a->val==b){
107             if(!a->ch[0])
108                 a=a->ch[1];
109             else if(!a->ch[1])
110                 a=a->ch[0];
111             else{
112                 int d=a->ch[1]->fix<a->ch[0]->fix;

```

```

113         down(a->ch[d]);
114         rotate(a,d);
115         erase(a->ch[!d],b);
116         update(a);
117     }
118     }else{
119         int d=b>a->val;
120         erase(a->ch[d],b);
121         update(a);
122     }
123 }
124 };
125 int n;
126 SelfAdjustingTopTree(int _n,vector<int>*to,int*we,int rt):
127     trp(this,_n+1),ns(_n+1),n(_n){
128     build(to,we,rt);
129 }
130 struct node{
131     node(){}
132     node(int a,node*b){
133         ch[0]=ch[1]=0;
134         pr=b;
135         vir=0;
136         val=a;
137         mi=mx=a;
138         siz=1;
139         rev=virsum=add=0;
140         virmi=inf;
141         virmx=-inf;
142         sam=inf;
143         virsam=inf;
144         virsiz=0;
145         viradd=0;
146     }
147     node*ch[2],*pr;
148     int val,mi,mx,sum,virmi,virmx,virsum,virsam,viradd,virsiz,rev,sam,
149     siz,add;
150     treap::node*vir;
151 };
152 vector<node>ns;
153 treap trp;

```

```

153     int direct(node*a){
154         if(!a->pr)
155             return 3;
156         else if(a==a->pr->ch[0])
157             return 0;
158         else if(a==a->pr->ch[1])
159             return 1;
160         else
161             return 2;
162     }
163     void down(node*a){
164         if(a->rev){
165             swap(a->ch[0],a->ch[1]);
166             for(int i=0;i<=1;++i)
167                 if(a->ch[i])
168                     a->ch[i]->rev^=1;
169             a->rev=0;
170         }
171         if(a->sam!=inf){
172             a->val=a->mi=a->mx=a->sam;
173             a->sum=a->sam*a->siz;
174             for(int i=0;i<=1;++i)
175                 if(a->ch[i])a->ch[i]->sam=a->sam,a->ch[i]->add=0;
176             a->sam=inf;
177         }
178         if(a->add){
179             a->val+=a->add;
180             a->mi+=a->add;
181             a->mx+=a->add;
182             a->sum+=a->add*a->siz;
183             for(int i=0;i<=1;++i)
184                 if(a->ch[i])a->ch[i]->add+=a->add;
185             a->add=0;
186         }
187         if(a->virsam!=inf){
188             if(a->virsiz){
189                 a->virmi=a->virmx=a->virsam;
190                 a->virsum=a->virsam*a->virsiz;
191                 if(a->vir)
192                     a->vir->add=0,a->vir->sam=a->virsam;
193                 for(int i=0;i<=1;++i)

```

```

194         if(a->ch[i])
195             a->ch[i]->viradd=0,a->ch[i]->virsam=a->virsam;
196     }
197     a->virsam=inf;
198 }
199 if(a->viradd){
200     if(a->virsiz){
201         a->virmi+=a->viradd;
202         a->virmx+=a->viradd;
203         a->virsum+=a->viradd*a->virsiz;
204         if(a->vir)a->vir->add+=a->viradd;
205         for(int i=0;i<=1;++i)
206             if(a->ch[i])
207                 a->ch[i]->viradd+=a->viradd;
208     }
209     a->viradd=0;
210 }
211 }
212 void update(node*a){
213     for(int i=0;i<=1;++i)
214         if(a->ch[i])
215             down(a->ch[i]);
216     if(a->vir)
217         trp.down(a->vir);
218     a->mi=a->val;
219     for(int i=0;i<=1;++i)
220         if(a->ch[i])
221             gmin(a->mi,a->ch[i]->mi);
222     a->virmi=inf;
223     for(int i=0;i<=1;++i)
224         if(a->ch[i])
225             gmin(a->virmi,a->ch[i]->virmi);
226     if(a->vir)
227         gmin(a->virmi,a->vir->mi);
228     a->mx=a->val;
229     for(int i=0;i<=1;++i)
230         if(a->ch[i])
231             gmax(a->mx,a->ch[i]->mx);
232     a->virmx=-inf;
233     for(int i=0;i<=1;++i)
234         if(a->ch[i])

```

```

235         gmax(a->virmx,a->ch[i]->virmx);
236     if(a->vir)
237         gmax(a->virmx,a->vir->mx);
238     a->sum=a->val;
239     for(int i=0;i<=1;++i)
240         if(a->ch[i])
241             a->sum+=a->ch[i]->sum;
242     a->virsum=0;
243     for(int i=0;i<=1;++i)
244         if(a->ch[i])
245             a->virsum+=a->ch[i]->virsum;
246     if(a->vir)
247         a->virsum+=a->vir->sum;
248     a->siz=1;
249     for(int i=0;i<=1;++i)
250         if(a->ch[i])
251             a->siz+=a->ch[i]->siz;
252     a->virsiz=0;
253     for(int i=0;i<=1;++i)
254         if(a->ch[i])
255             a->virsiz+=a->ch[i]->virsiz;
256     if(a->vir)
257         a->virsiz+=a->vir->siz;
258 }
259 void setchd(node*a,node*b,int d){
260     a->ch[d]=b;
261     if(b)
262         b->pr=a;
263     update(a);
264 }
265 void connect(node*a,node*b){
266     down(a);
267     *(&trp.ns[0]+(a-&ns[0]))=treap::node(a-&ns[0],min(a->virmi,a->mi),
max(a->virmx,a->mx),a->virsum+a->sum,a->virsiz+a->siz);
268     trp.insert(b->vir,&trp.ns[0]+(a-&ns[0]));
269 }
270 void disconnect(node*a,node*b){
271     trp.erase(b->vir,a-&ns[0]);
272 }
273 void rotate(node*a){
274     node*b=a->pr,*c=a->pr->pr;

```

```

275     int d1=direct(a),d2=direct(b);
276     setchd(b,a->ch[!d1],d1);
277     setchd(a,b,!d1);
278     if(d2<2)
279         setchd(c,a,d2);
280     else if(d2==2){
281         disconnect(b,c);
282         connect(a,c);
283         a->pr=c;
284     }else
285         a->pr=0;
286 }
287 void release(node*a){
288     if(direct(a)<2)
289         release(a->pr);
290     else if(a->pr)
291         disconnect(a,a->pr),connect(a,a->pr);
292     down(a);
293 }
294 void splay(node*a){
295     release(a);
296     while(direct(a)<2){
297         node*b=a->pr;
298         if(!b->pr||direct(b)>1)
299             rotate(a);
300         else if(direct(a)==direct(b))
301             rotate(b),rotate(a);
302         else
303             rotate(a),rotate(a);
304     }
305 }
306 node*access(node*a){
307     node*b=0;
308     while(a){
309         splay(a);
310         if(a->ch[1])
311             connect(a->ch[1],a);
312         if(b)
313             disconnect(b,a);
314         setchd(a,b,1);
315         b=a;

```



```

316         a=a->pr;
317     }
318     return b;
319 }
320 void evert(node*a){
321     access(a);
322     splay(a);
323     a->rev=1;
324 }
325 int qchain(node*a,node*b,int d){
326     access(a);
327     node*c=access(b);
328     splay(c);
329     splay(a);
330     int ret=c->val;
331     if(d==1){
332         if(a!=c)
333             gmin(ret,a->mi);
334         if(c->ch[1])
335             down(c->ch[1]),gmin(ret,c->ch[1]->mi);
336     }else if(d==2){
337         if(a!=c)
338             gmax(ret,a->mx);
339         if(c->ch[1])
340             down(c->ch[1]),gmax(ret,c->ch[1]->mx);
341     }else if(d==3){
342         if(a!=c)
343             ret+=a->sum;
344         if(c->ch[1])
345             down(c->ch[1]),ret+=c->ch[1]->sum;
346     }
347     return ret;
348 }
349 void mchain(node*a,node*b,int u,int d){
350     access(a);
351     node*c=access(b);
352     splay(c);
353     splay(a);
354     if(d==1){
355         c->val+=u;
356         if(a!=c)

```

```

357         a->add=u,disconnect(a,c),connect(a,c);
358     if(c->ch[1])
359         down(c->ch[1]),c->ch[1]->add=u;
360 }else if(d==2){
361     c->val=u;
362     if(a!=c)
363         a->sam=u,disconnect(a,c),connect(a,c);
364     if(c->ch[1])
365         down(c->ch[1]),c->ch[1]->sam=u;
366 }
367 update(c);
368 }
369 int qtree(node*a,int d){
370     access(a);
371     splay(a);
372     int ret=a->val;
373     if(d==1){
374         if(a->vir)
375             trp.down(a->vir),gmin(ret,a->vir->mi);
376     }else if(d==2){
377         if(a->vir)
378             trp.down(a->vir),gmax(ret,a->vir->mx);
379     }else if(d==3){
380         if(a->vir)
381             trp.down(a->vir),ret+=a->vir->sum;
382     }
383     return ret;
384 }
385 void mtree(node*a,int u,int d){
386     access(a);
387     splay(a);
388     if(d==1){
389         a->val+=u;
390         if(a->vir)
391             trp.down(a->vir),a->vir->add=u;
392     }else if(d==2){
393         a->val=u;
394         if(a->vir)
395             trp.down(a->vir),a->vir->sam=u;
396     }
397     update(a);

```

```

398     }
399     void stparent(node*a,node*b){
400         access(b);
401         if(access(a)!=a){
402             splay(a);
403             node*c=a->ch[0];
404             down(c);
405             while(c->ch[1])
406                 c=c->ch[1],down(c);
407             splay(c);
408             c->ch[1]=0;
409             update(c);
410             access(b);
411             splay(b);
412             connect(a,b);
413             a->pr=b;
414             update(b);
415         }
416     }
417     void build(vector<int>*to,int*we,int rt){
418         vector<int>pr(n);
419         vector<int>vec;
420         queue<int>qu;
421         qu.push(rt);
422         while(!qu.empty()){
423             int u=qu.front();
424             qu.pop();
425             vec.push_back(u);
426             for(int i=0;i<to[u].size();++i){
427                 int v=to[u][i];
428                 if(v!=pr[u])
429                     qu.push(v),pr[v]=u;
430             }
431         }
432         for(int i=0;i<n;++i){
433             int u=vec[i];
434             ns[u]=node(we[u],pr[u]?&ns[0]+pr[u]:0);
435         }
436         for(int i=n-1;i>=0;--i){
437             int u=vec[i];
438             update(&ns[0]+u);

```

```

439         if(pr[u])
440             connect(&ns[0]+u,&ns[0]+pr[u]);
441     }
442 }
443 };

```

2.9 Skew Heap

Skew Heap.hpp (1220 bytes, 61 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T,class C>struct SkewHeap{
4      SkewHeap():
5          root(0),siz(0){
6      }
7      ~SkewHeap(){
8          clear(root);
9      }
10     struct node{
11         node(T _val):
12             val(_val){
13             ch[0]=ch[1]=0;
14         }
15         T val;
16         node*ch[2];
17     }*root;
18     int siz;
19     node*merge(node*x,node*y){
20         if(!x)
21             return y;
22         if(!y)
23             return x;
24         if(C()(y->val,x->val))
25             swap(x,y);
26         swap(x->ch[0],x->ch[1]=merge(x->ch[1],y));
27         return x;
28     }
29     void clear(node*x){

```

```
30         if(x){
31             clear(x->ch[0]);
32             clear(x->ch[1]);
33             delete x;
34         }
35     }
36     void clear(){
37         clear(root);
38         root=0;
39         siz=0;
40     }
41     void push(T a){
42         root=merge(root,new node(a));
43         ++siz;
44     }
45     T top(){
46         return root->val;
47     }
48     void pop(){
49         root=merge(root->ch[0],root->ch[1]);
50         --siz;
51     }
52     void merge(SkewHeap<T,C>&a){
53         root=merge(root,a.root);
54         a.root=0;
55         siz+=a.siz;
56         a.siz=0;
57     }
58     int size(){
59         return siz;
60     }
61 };
```


CHAPTER 3

Graph Algorithms

3.1 Chordality Test

Chordality Test.hpp (1343 bytes, 42 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  struct ChordalityTest{
4      int n,ns;
5      vector<vector<int> >to;
6      ChordalityTest(int _n):
7          n(_n),ns(n),to(n+1){
8      }
9      void add(int u,int v){
10         to[u].push_back(v),to[v].push_back(u);
11     }
12     bool run(){
13         vector<int>pos(n+1),idx(n+2),lab(n+1),tab(n+1);
14         vector<list<int>>>qu(n);
15         for(int i=1;i<=n;++i)
16             qu[0].push_back(i);
17         for(int b=0,i=1,u=0;i<=n;++i,u=0){
18             for(;u?++b,0:1;--b)
19                 for(auto j=qu[b].begin();j!=qu[b].end()&&!u;qu[b].erase(j++))
20                     if(!pos[*j]&&lab[*j]==b)
21                         u=*j;
22             pos[u]=ns,idx[ns--]=u;
23             for(int v:to[u])
24                 if(!pos[v])
25                     b=max(b,++lab[v]),qu[lab[v]].push_back(v);}
26         for(int i=1,u=idx[1],v=-1;i<=n;++i,u=idx[i],v=-1){
27             for(int w:to[u])
28                 if(pos[w]>pos[u]&&(v==-1||pos[w]<pos[v]))
29                     v=w;
30             if(v!=-1){
31                 for(int w:to[v])
32                     tab[w]=1;
33                 for(int w:to[u])
34                     if(pos[w]>pos[u]&&w!=v&&!tab[w])
35                         return false;
36                 for(int w:to[v])

```



```

37         tab[w]=0;
38     }
39 }
40 return true;
41 }
42 };

```

3.2 Dominator Tree

Dominator Tree.hpp (2916 bytes, 94 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  struct DominatorTree{
4      int n,r;
5      vector<vector<int> >to,rto,chd,rsemi;
6      vector<int>dfn,res,prt,rdfn,semi,misemi;
7      DominatorTree(int _n,int _r):n(_n),r(_r),to(n+1),rto(n+1),dfn(n+1),res(
n+1),prt(n+1),rdfn(1),semi(n+1),misemi(n+1),chd(n+1),rsemi(n+1){
8  }
9      int fd(int a){
10         stack<int>stk;
11         for(int b=a;prt[b]!=prt[prt[b]];b=prt[b])
12             stk.push(b);
13         for(int b;stk.empty()?0:(b=stk.top(),stk.pop(),1);){
14             if(dfn[semi[misemi[prt[b]]]]<dfn[semi[misemi[b]]])
15                 misemi[b]=misemi[prt[b]];
16             prt[b]=prt[prt[b]];
17         }
18         return prt[a];
19     }
20     void add(int a,int b){
21         to[a].push_back(b);
22         rto[b].push_back(a);
23     }
24     void dfs(){
25         stack<pair<int,int> >stk;
26         semi[r]=r;
27         for(stk.push(make_pair(r,0));!stk.empty();){

```

```

28         int a=stk.top().first,i=stk.top().second;
29         stk.pop();
30         if(!i)
31             dfn[a]=rdfn.size(),rdfn.push_back(a);
32         if(i<to[a].size()){
33             stk.push(make_pair(a,i+1));
34             int b=to[a][i];
35             if(!semi[b])
36                 semi[b]=a, chd[a].push_back(b),
37                 stk.push(make_pair(b,0));
38         }
39     }
40     semi[r]=0;
41 }
42 void calcsemi(){
43     for(int i=1;i<=n;++i)
44         prt[i]=i, misemi[i]=i;
45     for(int i=rdfn.size()-1;i>=1;--i){
46         int a=rdfn[i];
47         for(int b:rto[a]){
48             if(!dfn[b])
49                 continue;
50             if(dfn[b]<dfn[a]){
51                 if(dfn[b]<dfn[semi[a]])
52                     semi[a]=b;
53             }else{
54                 int c=fd(b);
55                 if(dfn[semi[c]]<dfn[semi[a]])
56                     semi[a]=semi[c];
57                 if(dfn[semi[misemi[b]]]<dfn[semi[a]])
58                     semi[a]=semi[misemi[b]];
59             }
60         }
61         for(int b:chd[a])
62             prt[b]=a;
63     }
64 }
65 void calcres(){
66     for(int i=1;i<=n;++i)
67         prt[i]=i, misemi[i]=i, rsemi[semi[i]].push_back(i);
68     for(int i=rdfn.size()-1;i>=1;--i){

```

```

69         int a=rdfn[i];
70         for(int b:rsemi[a]){
71             fd(b);
72             int c=misemi[b];
73             if(dfn[semi[c]]>dfn[semi[prt[b]]])
74                 c=prt[b];
75             if(semi[c]==semi[b])
76                 res[b]=semi[b];
77             else
78                 res[b]=-c;}
79         for(int b:chd[a])
80             prt[b]=a;
81     }
82     for(int i=1;i<rdfn.size();++i){
83         int a=rdfn[i];
84         if(res[a]<0)
85             res[a]=res[-res[a]];
86     }
87 }
88 vector<int>run(){
89     dfs();
90     calcsemi();
91     calcres();
92     return res;
93 }
94 };

```

3.3 Maximal Clique Count

Maximal Clique Count.hpp (927 bytes, 34 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<int N>struct MaximalCliqueCount{
4      int n,r;
5      vector<bitset<N> >e,rht,msk;
6      MaximalCliqueCount(int _n):
7          n(_n),e(n),rht(n),msk(n),r(0){
8      }

```

```

9  void add(int u,int v){
10     e[u-1][v-1]=e[v-1][u-1]=1;
11 }
12 void dfs(int u,bitset<N>cur,bitset<N>can){
13     if(cur==can){
14         ++r;
15         return;
16     }
17     for(int v=0;v<u;++v)
18         if(can[v]&&!cur[v]&&(e[v]&rht[u]&can)==(rht[u]&can))
19             return;
20     for(int v=u+1;v<n;++v)
21         if(can[v])
22             dfs(v,cur|msk[v],can&e[v]);
23 }
24 int run(){
25     for(int i=1;i<=n;++i){
26         rht[i-1]=bitset<N>(string(n-i,'1')+string(i,'0'));
27         msk[i-1]=bitset<N>(1)<<i-1;
28         e[i-1]|=msk[i-1];
29     }
30     for(int i=0;i<n;++i)
31         dfs(i,msk[i],e[i]);
32     return r;
33 }
34 };

```

3.4 Maximal Planarity Test

Maximal Planarity Test.hpp (5195 bytes, 165 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  struct MaximalPlanarityTesting{
4      int n,m;
5      vector<set<int> >to2;
6      vector<vector<int> >to;
7      vector<int>dec,rmd,mrk,invc,rt;
8      vector<list<int>::iterator>dpos,pos;

```

```

9      bool order(int v1,int v2,int vn){
10          rt[0]=v1;
11          rt[1]=v2;
12          rt[n-1]=vn;
13          fill(inv.begin(),inv.end(),0);
14          inv[v1]=1;
15          inv[v2]=1;
16          inv[vn]=1;
17          list<int>deg;
18          dpos[vn]=deg.insert(deg.begin(),vn);
19          fill(dec.begin(),dec.end(),0);
20          dec[v1]=2;
21          dec[v2]=2;
22          dec[vn]=2;
23          for(int i=n-1;i>=2;--i){
24              if(deg.empty())
25                  return false;
26              int v=*deg.begin();
27              deg.erase(deg.begin());
28              inv[v]=-1;
29              rt[i]=v;
30              for(int u:to[v]){
31                  if(inv[u]==1){
32                      if(u!=v1&&u!=v2&&dec[u]==2)
33                          deg.erase(dpos[u]);
34                      --dec[u];
35                      if(u!=v1&&u!=v2&&dec[u]==2)
36                          dpos[u]=deg.insert(deg.begin(),u);
37                  }else if(inv[u]==0)
38                      inv[u]=2;
39              }
40              for(int u:to[v])
41                  if(inv[u]==2)
42                      for(int w:to[u])
43                          if(inv[w]==1){
44                              if(w!=v1&&w!=v2&&dec[w]==2)
45                                  deg.erase(dpos[w]);
46                              ++dec[w];
47                              if(w!=v1&&w!=v2&&dec[w]==2)
48                                  dpos[w]=deg.insert(deg.begin(),w);
49                              ++dec[u];

```

```

50         }else if(invc[w]==2)
51             ++dec[u];
52     for(int u:to[v]){
53         if(invc[u]==2){
54             invc[u]=1;
55             if(dec[u]==2)
56                 dpos[u]=deg.insert(deg.begin(),u);
57         }
58     }
59 }
60 return true;
61 }
62 bool embed(){
63     list<int>ext;
64     int mker=0;
65     fill(mrk.begin(),mrk.end(),0);
66     pos[rt[1]]=ext.insert(ext.begin(),rt[1]);
67     pos[rt[2]]=ext.insert(ext.begin(),rt[2]);
68     pos[rt[0]]=ext.insert(ext.begin(),rt[0]);
69     fill(rmd.begin(),rmd.end(),0);
70     rmd[rt[1]]=1;
71     rmd[rt[2]]=1;
72     rmd[rt[0]]=1;
73     for(int i=3;i<n;++i){
74         int v=rt[i];
75         rmd[v]=1;
76         vector<int>can;
77         ++mker;
78         for(int u:to[v])
79             if(rmd[u])
80                 mrk[u]=mker,can.push_back(u);
81         int start=-1,end=-1;
82         for(int u:can){
83             list<int>::iterator it=pos[u];
84             if(it==list<int>::iterator())
85                 return false;
86             if(it==ext.begin()){
87                 if(start!=-1)
88                     return false;
89                 start=u;
90             }else{

```

```

91         list<int>::iterator tmp=it;
92         if(mrk[*(--tmp)]!=mker){
93             if(start!=-1)
94                 return false;
95             start=u;
96         }
97     }
98     list<int>::iterator tmp=it;++tmp;
99     if(tmp==ext.end()){
100         if(end!=-1)
101             return false;
102         end=u;
103     }else{
104         if(mrk[*tmp]!=mker){
105             if(end!=-1)
106                 return false;
107             end=u;
108         }
109     }
110 }
111 if(start==--1||end==--1)
112     return false;
113 for(int u:can)
114     if(u!=start&&u!=end)
115         ext.erase(pos[u]),pos[u]=list<int>::iterator();
116 pos[v]=ext.insert(pos[end],v);
117 }
118 return true;
119 }
120 bool istri(int u,int v,int w){
121     return to2[u].count(v)&&to2[v].count(w)&&to2[w].count(u);
122 }
123 MaximalPlanarityTesting(int _n):
124     n(_n),to(n),to2(n),m(0),rt(n),invc(n),dec(n),dpos(n),pos(n),rmd(n),
mrk(n){
125 }
126 void add(int u,int v){
127     to[u-1].push_back(v-1);
128     to[v-1].push_back(u-1);
129     to2[u-1].insert(v-1);
130     to2[v-1].insert(u-1);++m;

```

```

131 }
132 bool run(){
133     if(n==1&&m==0)
134         return true;
135     if(n==2&&m==1)
136         return true;
137     if(n==3&&m==3)
138         return true;
139     if(n<=3)
140         return false;
141     if(m!=3*n-6)
142         return false;
143     int v1;
144     for(v1=0;v1<n;++v1)
145         if(to[v1].size(<3)
146             return false;
147     for(v1=0;v1<n;++v1)
148         if(to[v1].size(<=5)
149             break;
150     if(v1>=n)
151         return false;
152     int v2=to[v1].back();
153     for(int i=0;i+1<to[v1].size();++i){
154         int vn=to[v1][i];
155         if(istri(v1,v2,vn)){
156             if(!order(v1,v2,vn))
157                 continue;
158             if(!embed())
159                 continue;
160             return true;
161         }
162     }
163     return false;
164 }
165 };

```

3.5 Maximum Flow

Maximum Flow.hpp (2311 bytes, 79 lines)


```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>struct MaximumFlow{
4      struct edge{
5          int v;
6          T c,l;
7          edge(int _v,T _c):
8              v(_v),c(_c),l(_c){
9          }
10     };
11     int n,src,snk;
12     vector<edge>egs;
13     vector<vector<int>> >bge;
14     vector<int>hei,gap,cur,frm;
15     MaximumFlow(int _n,int _src,int _snk):
16         bge(_n),hei(_n,_n),gap(_n+1),n(_n),cur(_n),frm(_n),src(_src-1),snk(
17         _snk-1){
18     }
19     void lab(){
20         hei[snk]=0;
21         queue<int>qu;
22         qu.push(snk);
23         for(int u;qu.empty()?0:(u=qu.front(),qu.pop(),1);){
24             for(int i=0;i<bge[u].size();++i){
25                 edge&e=egs[bge[u][i]],&ev=egs[bge[u][i]^1];
26                 if(ev.c>0&&hei[e.v]==n)
27                     hei[e.v]=hei[u]+1,qu.push(e.v);
28             }
29             for(int i=0;i<n;++i)
30                 ++gap[hei[i]];
31         }
32     T aug(){
33         T f=0;
34         for(int u=snk;u!=src;u=egs[frm[u]^1].v)
35             if(f<=0||f>egs[frm[u]].c)
36                 f=egs[frm[u]].c;
37         for(int u=snk;u!=src;u=egs[frm[u]^1].v)
38             egs[frm[u]].c-=f,egs[frm[u]^1].c+=f;
39         return f;
40     }

```

```

40 void add(int u,int v,T c){
41     bge[u-1].push_back(egs.size());
42     egs.push_back(edge(v-1,c));
43     bge[v-1].push_back(egs.size());
44     egs.push_back(edge(u-1,0));
45 }
46 T run(){
47     lab();
48     T r=0;
49     for(int u=src;hei[src]!=n;){
50         if(u==snk)
51             r+=aug(),u=src;
52         int f=0;
53         for(int i=cur[u];i<bge[u].size();++i){
54             edge&e=egs[bge[u][i]];
55             if(e.c>0&&hei[u]==hei[e.v]+1){
56                 f=1;
57                 frm[e.v]=bge[u][i];
58                 u=e.v;
59                 break;
60             }
61         }
62         if(!f){
63             int mh=n-1;
64             for(int i=0;i<bge[u].size();++i){
65                 edge&e=egs[bge[u][i]];
66                 if(e.c>0&&mh>hei[e.v])
67                     mh=hei[e.v];
68             }
69             if(!--gap[hei[u]])
70                 break;
71             ++gap[hei[u]=mh+1];
72             cur[u]=0;
73             if(u!=src)
74                 u=egs[frm[u]^1].v;
75         }
76     }
77     return r;
78 }
79 };

```

3.6 Maximum Matching

Maximum Matching.hpp (3123 bytes, 112 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  struct MaximumMatching{
4      int n;
5      vector<int>res,nxt,mrk,vis,top,prt,rnk;
6      vector<vector<int>> >to;
7      queue<int>qu;
8      MaximumMatching(int _n):
9          n(_n),res(n+1),nxt(n+1),mrk(n+1),vis(n+1),top(n+1),to(n+1),prt(n+1)
10         ,rnk(n+1){
11     }
12     int fd(int x){
13         return x==prt[x]?x:prt[x]=fd(prt[x]);
14     }
15     void lk(int x,int y){
16         if(rnk[x=fd(x)]>rnk[y=fd(y)])
17             prt[y]=x;
18         else if(rnk[x]<rnk[y])
19             prt[x]=y;
20         else
21             prt[x]=y,++rnk[y];
22     }
23     int lca(int x,int y){
24         static int t;
25         ++t;
26         for(;;swap(x,y))
27             if(x){
28                 x=top[fd(x)];
29                 if(vis[x]==t)
30                     return x;
31                 vis[x]=t;
32             }
33         if(res[x])
34             x=nxt[res[x]];
35         else
36             x=0;
37     }
38 }
```

```

37 void uni(int x,int p){
38     for(;fd(x)!=fd(p);){
39         int y=res[x],z=nxt[y];
40         if(fd(z)!=fd(p))
41             nxt[z]=y;
42         if(mrk[y]==2)
43             mrk[y]=1,qu.push(y);
44         if(mrk[z]==2)
45             mrk[z]=1,qu.push(z);
46         int t=top[fd(z)];
47         lk(x,y);
48         lk(y,z);
49         top[fd(z)]=t;
50         x=z;
51     }
52 }
53 void aug(int s){
54     for(int i=1;i<=n;++i)
55         nxt[i]=0,mrk[i]=0,top[i]=i,prt[i]=i,rnk[i]=0;
56     mrk[s]=1;
57     qu=queue<int>();
58     for(qu.push(s);!qu.empty();){
59         int x=qu.front();
60         qu.pop();
61         for(int i=0;i<to[x].size();++i){
62             int y=to[x][i];
63             if(res[x]==y||fd(x)==fd(y)||mrk[y]==2)
64                 continue;
65             if(mrk[y]==1){
66                 int z=lca(x,y);
67                 if(fd(x)!=fd(z))
68                     nxt[x]=y;
69                 if(fd(y)!=fd(z))
70                     nxt[y]=x;
71                 uni(x,z);
72                 uni(y,z);
73             }else if(!res[y]){
74                 for(nxt[y]=x;y;){
75                     int z=nxt[y],mz=res[z];
76                     res[z]=y;
77                     res[y]=z;

```

```

78             y=mz;
79         }
80         return;
81     }else{
82         nxt[y]=x;
83         mrk[res[y]]=1;
84         qu.push(res[y]);
85         mrk[y]=2;
86     }
87 }
88 }
89 }
90 void add(int x,int y){
91     to[x].push_back(y);
92     to[y].push_back(x);
93 }
94 int run(){
95     for(int i=1;i<=n;++i)
96         if(!res[i])
97             for(int j=0;j<to[i].size();++j)
98                 if(!res[to[i][j]]){
99                     res[to[i][j]]=i;
100                     res[i]=to[i][j];
101                     break;
102                 }
103     for(int i=1;i<=n;++i)
104         if(!res[i])
105             aug(i);
106     int r=0;
107     for(int i=1;i<=n;++i)
108         if(res[i])
109             ++r;
110     return r/2;
111 }
112 };

```

3.7 Minimum Cost Maximum Flow

Minimum Cost Maximum Flow.hpp (2278 bytes, 82 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class F=int,class C=int>struct MinimumCostMaximumFlow{
4      struct edge{
5          edge(int _v,F _c,C _w):
6              v(_v),c(_c),w(_w){
7              }
8          int v;
9          F c;
10         C w;
11     };
12     MinimumCostMaximumFlow(int _n,int _src,int _snk,F _all):
13         n(_n),src(_src-1),snk(_snk-1),bg(_n),vis(n),dis(n),all(_all),flow
14         (0),cost(0){}
15     void add(int u,int v,F c,C w){
16         bg[u-1].push_back(eg.size());
17         eg.push_back(edge(v-1,c,w));
18         bg[v-1].push_back(eg.size());
19         eg.push_back(edge(u-1,0,-w));
20     }
21     int spfa(){
22         vector<int>in(n,0);
23         queue<int>qu;
24         fill(vis.begin(),vis.end(),0);
25         dis[src]=0;
26         vis[src]=in[src]=1;
27         qu.push(src);
28         while(!qu.empty()){
29             int u=qu.front();
30             qu.pop();
31             in[u]=0;
32             for(int i=0;i<bg[u].size();++i){
33                 edge&e=eg[bg[u][i]];
34                 if(e.c!=0&&(!vis[e.v]||dis[u]+e.w<dis[e.v])){
35                     dis[e.v]=dis[u]+e.w;
36                     vis[e.v]=1;
37                     if(!in[e.v]){
38                         in[e.v]=1;
39                         qu.push(e.v);
40                     }
41                 }
42             }
43         }
44     }
45 };

```

```

40         }
41     }
42 }
43     return vis[snk]&&dis[snk]<0;
44 }
45 F dfs(int u,F f){
46     if(u==snk)
47         return f;
48     F g=f;
49     vis[u]=1;
50     for(int i=0;i<bg[u].size();++i){
51         edge&e=eg[bg[u][i]],&ev=eg[bg[u][i]^1];
52         if(e.c!=0&&dis[e.v]==dis[u]+e.w&&!vis[e.v]){
53             F t=dfs(e.v,min(g,e.c));
54             g-=t;
55             e.c-=t;
56             ev.c+=t;
57             cost+=t*e.w;
58             if(g==0)
59                 return f;
60         }
61     }
62     return f-g;
63 }
64 pair<F,C>run(){
65     while(all!=0&&spfa()){
66         F t;
67         do{
68             fill(vis.begin(),vis.end(),0);
69             flow+=(t=dfs(src,all));
70             all-=t;
71         }while(t!=0);
72     }
73     return make_pair(flow,cost);
74 }
75 int n,src,snk;
76 vector<vector<int>>>bg;
77 vector<edge>eg;
78 vector<int>vis;
79 vector<C>dis;
80 F all,flow;

```

```

81     C cost;
82 };

```

3.8 Minimum Spanning Arborescence

Minimum Spanning Arborescence.hpp (1933 bytes, 64 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>struct MinimumSpanningArborescence{
4      struct eg{
5          int u,v;
6          T w;
7      };
8      int n,rt;
9      vector<eg>egs;
10     vector<int>vi,in,id;
11     vector<T>inw;
12     MinimumSpanningArborescence(int _n,int _rt):
13         n(_n),rt(_rt),vi(n+1),in(n+1),inw(n+1),id(n+1){
14     }
15     void add(int u,int v,T w){
16         eg e;
17         e.u=u;
18         e.v=v;
19         e.w=w;
20         egs.push_back(e);
21     }
22     T run(){
23         int nv=0;
24         for(T r=0;;n=nv,nv=0,rt=id[rt]){
25             for(int i=1;i<=n;++i)
26                 in[i]=-1;
27             for(int i=0;i<egs.size();++i)
28                 if(egs[i].u!=egs[i].v&&(in[egs[i].v]==-1||egs[i].w<inw[egs[
29 i].v]))
30                     in[egs[i].v]=egs[i].u,inw[egs[i].v]=egs[i].w;
31             for(int i=1;i<=n;++i)
32                 if(i!=rt&&in[i]==-1)

```



```

32         return numeric_limits<T>::max();
33     for(int i=1;i<=n;++i){
34         if(i!=rt)
35             r+=inw[i];
36         id[i]=-1,vi[i]=0;
37     }
38     for(int i=1;i<=n;++i)
39         if(i!=rt&&!vi[i]){
40             int u=i;
41             do{
42                 vi[u]=i;
43                 u=in[u];
44             }while(!vi[u]&&u!=rt);
45             if(u!=rt&&vi[u]==i){
46                 int v=u;
47                 ++nv;
48                 do{
49                     id[v]=nv;
50                     v=in[v];
51                 }while(v!=u);
52             }
53     }
54     if(nv==0)
55         return r;
56     for(int i=1;i<=n;++i)
57         if(id[i]==-1)
58             id[i]=++nv;
59     for(int i=0;i<egs.size();++i)
60         egs[i].w-=inw[egs[i].v],egs[i].u=id[egs[i].u],
61         egs[i].v=id[egs[i].v];
62     }
63 }
64 };

```

3.9 Minimum Spanning Tree

Minimum Spanning Tree.hpp (1049 bytes, 44 lines)

```
1 #include<bits/stdc++.h>
```

```

2 using namespace std;
3 template<class T, class C=less<T> > struct MinimumSpanningTree{
4     struct edge{
5         T w;
6         int u,v;
7         int operator<(const edge&b) const{
8             return C()(w,b.w);
9         }
10    };
11    int n;
12    vector<edge>egs;
13    vector<int>pr;
14    MinimumSpanningTree(int _n):
15        n(_n),pr(n+1){
16    }
17    void add(int u,int v,T w){
18        edge e;
19        e.u=u;
20        e.v=v;
21        e.w=w;
22        egs.push_back(e);
23    }
24    int fd(int x){
25        return x==pr[x]?x:pr[x]=fd(pr[x]);
26    }
27    void lk(int x,int y){
28        pr[fd(x)]=y;
29    }
30    pair<T,vector<edge> >run(){
31        vector<edge>ret;
32        T sum=0;
33        sort(egs.begin(),egs.end());
34        for(int i=1;i<=n;++i)
35            pr[i]=i;
36        for(int i=0;i<egs.size();++i){
37            int u=egs[i].u,v=egs[i].v;
38            T w=egs[i].w;
39            if(fd(u)!=fd(v))
40                lk(u,v),ret.push_back(egs[i]),sum+=w;
41        }
42        return make_pair(sum,ret);

```

```

43     }
44 };

```

3.10 Shortest Path

Shortest Path.hpp (1279 bytes, 44 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>struct ShortestPath{
4      int n,m;
5      vector<vector<int> >to;
6      vector<vector<T> >we;
7      T inf;
8      vector<pair<T,int> >sg;
9      vector<T>di;
10     ShortestPath(int _n):
11         n(_n),m(1<<((int)ceil(log2(n)+1e-8)),to(n+1),we(n+1),inf(
numeric_limits<T>::max()),sg(2*m,make_pair(inf,0)),di(n+1,inf){
12     }
13     void set(int u,T d){
14         di[u]=d;
15     }
16     void add(int u,int v,T w){
17         to[u].push_back(v);
18         we[u].push_back(w);
19     }
20     int upd(T&a,T b,T c){
21         if(b!=inf&&c!=inf&&b+c<a){
22             a=b+c;
23             return 1;
24         }
25         return 0;
26     }
27     void mod(int u,T d){
28         for(sg[u+m-1]=make_pair(d,u),u=(u+m-1)>>1;u>=1)
29             sg[u]=min(sg[u<<1],sg[u<<1^1]);
30     }
31     vector<T>run(){

```

```

32     for(int i=1;i<=n;++i)
33         sg[i+m-1]=make_pair(di[i],i);
34     for(int i=m-1;i>=1;--i)
35         sg[i]=min(sg[i<<1],sg[i<<1^1]);
36     for(int u=sg[1].second;sg[1].first!=inf?(mod(u,inf),1):0;u=sg[1].
second)
37         for(int i=0;i<to[u].size();++i){
38             int v=to[u][i];
39             T w=we[u][i];
40             if(upd(di[v],di[u],w))
41                 mod(v,di[v]);}
42     return di;
43 }
44 };

```

3.11 Steiner Tree

Steiner Tree.hpp (1745 bytes, 56 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>struct SteinerTree{
4      int n,k,z;
5      T inf=numeric_limits<T>::max();
6      vector<vector<T> >wei,dp;
7      vector<int>im;
8      SteinerTree(int _n):
9          n(_n),k(0),wei(n+1,vector<T>(n+1,inf)),im(n+1){
10     }
11     void set(int u){
12         if(!im[u])
13             im[z=u]=++k;
14     }
15     void add(int u,int v,T w){
16         wei[u][v]=wei[v][u]=min(w,wei[u][v]);
17     }
18     int upd(T&a,T b,T c){
19         if(b!=inf&&c!=inf&&b+c<a){
20             a=b+c;

```

```

21         return 1;
22     }
23     return 0;
24 }
25 int ins(int s,int u){
26     return im[u]&&((s>>im[u]-1)&1);
27 }
28 T run(){
29     for(int l=1;l<=n;++l)
30         for(int i=1;i<=n;++i)
31             for(int j=1;j<=n;++j)
32                 upd(wei[i][j],wei[i][l],wei[l][j]);
33     dp=vector<vector<T>>>(1<<k-1,vector<T>(n+1,inf));
34     fill(begin(dp[0]),end(dp[0]),0);
35     for(int s=1;s<(1<<k-1);++s){
36         queue<int>qu;
37         vector<int>in(n+1);
38         for(int u=1;u<=n;++u){
39             if(ins(s,u))
40                 continue;
41             qu.push((u));
42             in[u]=1;
43             for(int t=(s-1)&s;t=(t-1)&s)
44                 upd(dp[s][u],dp[t][u],dp[s^t][u]);
45             for(int v=1;v<=n;++v)
46                 if(ins(s,v))
47                     upd(dp[s][u],dp[s^(1<<im[v]-1)][v],wei[u][v]);
48         }
49         for(int u;qu.empty()?0:(u=qu.front(),qu.pop(),in[u]=0,1);)
50             for(int v=1;v<=n;++v)
51                 if(!ins(s,v)&&upd(dp[s][v],dp[s][u],wei[u][v])&&!in[v])
52                     in[v]=1,qu.push(v);
53     }
54     return k?dp[(1<<k-1)-1][z]:0;
55 }
56 };

```

3.12 Virtual Tree

Virtual Tree.hpp (2375 bytes, 77 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  struct VirtualTree{
4      int n,r,l;
5      vector<vector<int> >to,vto,up;
6      vector<int>lst,dp,dfn,edf,imp;
7      VirtualTree(int _n,int _r):
8          n(_n),r(_r),l(ceil(log2(n)+1e-8)),to(n+1),vto(n+1),up(n+1,vector<
9          int>(l+1)),dp(n+1),dfn(n+1),edf(n+1),imp(n+1){
10     }
11     void add(int u,int v){
12         to[u].push_back(v);
13         to[v].push_back(u);
14     }
15     void vadd(int u,int v){
16         vto[u].push_back(v);
17     }
18     int lca(int u,int v){
19         if(dp[u]<dp[v])
20             swap(u,v);
21         for(int i=0;i<=l;++i)
22             if(((dp[u]-dp[v])>>i)&1)
23                 u=up[u][i];
24         if(u==v)
25             return u;
26         for(int i=l;i>=0;--i)
27             if(up[u][i]!=up[v][i])
28                 u=up[u][i],v=up[v][i];
29         return up[u][0];
30     }
31     void dfs(int u){
32         dfn[u]=++dfn[0];
33         for(int i=1;i<=l;++i)
34             up[u][i]=up[up[u][i-1]][i-1];
35         for(int i=0;i<to[u].size();++i){
36             int v=to[u][i];
37             if(v!=up[u][0])

```

```

37         up[v][0]=u,dp[v]=dp[u]+1,dfs(v);
38     }
39     edf[u]=dfn[0];
40 }
41 void build(){
42     dfs(r);
43 }
44 void run(int*a,int m){
45     for(int i=0;i<lst.size();++i)
46         imp[lst[i]]=0,vto[lst[i]].clear();
47     vector<pair<int,int> >b(m+1);
48     for(int i=1;i<=m;++i)
49         imp[a[i]]=1,b[i]=make_pair(dfn[a[i]],a[i]);
50     sort(b.begin()+1,b.end());
51     vector<int>st(1,r);
52     lst=st;
53     for(int i=1;i<=m;++i){
54         int u=b[i].second,v=st.back();
55         if(u==r)
56             continue;
57         if(dfn[u]<=edf[v])
58             st.push_back(u);
59         else{
60             int w=lca(u,v);
61             while(st.size()>=2&&dp[st[st.size()-2]]>=dp[w]){
62                 vadd(st[st.size()-2],*st.rbegin());
63                 lst.push_back(*st.rbegin()),st.pop_back();
64             }
65             if(st.size()>=2&&w!=st[st.size()-1]){
66                 vadd(w,*st.rbegin()),lst.push_back(*st.rbegin());
67                 st.pop_back(),st.push_back(w);
68             }
69             st.push_back(u);
70         }
71     }
72     while(st.size()>=2){
73         vadd(st[st.size()-2],*st.rbegin());
74         lst.push_back(*st.rbegin()),st.pop_back();
75     }
76 }
77 };

```


CHAPTER 4

Number Theory

4.1 Discrete Logarithm

DiscreteLogarithm.hpp (1819 bytes, 74 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  namespace DiscreteLogarithm{
4      typedef long long T;
5      int ti[1<<16],va[1<<16],mp[1<<16],nx[1<<16],hd[1<<16],tm,nw;
6      void ins(int x,int v){
7          int y=x&65535;
8          if(ti[y]!=tm)
9              ti[y]=tm,hd[y]=0;
10         for(int i=hd[y];i;i=nx[i])
11             if(va[i]==x){
12                 mp[i]=v;
13                 return;
14             }
15         va[++nw]=x;
16         mp[nw]=v;
17         nx[nw]=hd[y];
18         hd[y]=nw;
19     }
20     int get(int x){
21         int y=x&65535;
22         if(ti[y]!=tm)
23             ti[y]=tm,hd[y]=0;
24         for(int i=hd[y];i;i=nx[i])
25             if(va[i]==x){
26                 return mp[i];
27             }
28         return -1;
29     }
30     T pow(T a,T b,T c){
31         T r=1;
32         for(;b;b&1?r=r*a%c:0,b>>=1,a=a*a%c);
33         return r;
34     }
35     T gcd(T a,T b){
36         return b?gcd(b,a%b):a;
37     }

```

```

38 void exg(T a,T b,T&x,T&y){
39     if(!b)
40         x=1,y=0;
41     else
42         exg(b,a%b,y,x),y-=a/b*x;
43 }
44 T inv(T a,T b){
45     T x,y;
46     exg(a,b,x,y);
47     return x+b;
48 }
49 T bgs(T a,T b,T c){
50     ++tm;
51     nw=0;
52     T m=sqrt(c);
53     for(T i=m-1,u=pow(a,i,c),v=inv(a,c);i>=0;--i,u=u*v%c)
54         ins(u,i);
55     for(T i=0,u=1,v=inv(pow(a,m,c),c);i*m<=c;++i,u=u*v%c){
56         T t=u*b%c,j;
57         if((j=get(t))!=-1)
58             return i*m+j;
59     }
60     return -1;
61 }
62 T run(T a,T b,T c){
63     T u=1,t=0;
64     a=(a%c+c)%c;
65     b=(b%c+c)%c;
66     for(int i=0;i<32;++i)
67         if(pow(a,i,c)==b)
68             return i;
69     for(T d;(d=gcd(a,c))!=1;++t,u=a/d*u%c,b/=d,c/=d)
70         if(b%d)
71             return -1;
72     return (u=bgs(a,b*inv(u,c)%c,c))<0?-1:u+t;
73 }
74 }

```

4.2 Integer Factorization

Integer Factorization.hpp (2848 bytes, 93 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  namespace IntegerFactorization{
4      template<class T>T mul(T x,T y,T z){
5          if(typeid(T)==typeid(int))
6              return (long long)x*y%z;
7          else if(typeid(T)==typeid(long long))
8              return (x*y-(T)(((long double)x*y+0.5)/z)*z+z)%z;
9          else
10             return x*y%z;
11     }
12     template<class T>T pow(T a,T b,T c){
13         T r=1;
14         for(;b;b&1?r=mul(r,a,c):0,b>>=1,a=mul(a,a,c));
15         return r;
16     }
17     template<class T>int chk(T a,int c=10){
18         if(a==2)
19             return 1;
20         if(a%2==0 || a<2)
21             return 0;
22         static int pi[]={2,7,61},p1
23         [=]{2,325,9375,28178,450775,9780504,1795265022};
24         if(typeid(T)==typeid(int))
25             c=3;
26         else if(typeid(T)==typeid(long long))
27             c=7;
28         T u=a-1,t=0,p=1;
29         for(;u%2==0;u/=2,++t);
30         for(int i=0;i<c;++i){
31             if(typeid(T)==typeid(int))
32                 p=pi[i]%a;
33             else if(typeid(T)==typeid(long long))
34                 p=p1[i]%a;
35             else
36                 p=(p*29+7)%a;
37             if(!p || p==1 || p==a-1)

```

```

37         continue;
38         T x=pow(p,u,a);
39         if(x==1)
40             continue;
41         for(int j=0;x!=a-1&&j<t;++j){
42             x=mul(x,x,a);
43             if(x==1)
44                 return 0;
45         }
46         if(x==a-1)
47             continue;
48         return 0;
49     }
50     return 1;
51 }
52 template<class T>T gcd(T a,T b){
53     if(a<0)
54         a=-a;
55     if(b<0)
56         b=-b;
57     return b?gcd(b,a%b):a;
58 }
59 template<class T>T rho(T a,T c){
60     T x=double(rand())/RAND_MAX*(a-1),y=x;
61     for(int i=1,k=2;;){
62         x=(mul(x,x,a)+c)%a;
63         T d=gcd(y-x,a);
64         if(d!=1&&d!=a)
65             return d;
66         if(y==x)
67             return a;
68         if(++i==k)
69             y=x,k=2*k;
70     }
71 }
72 template<class T>vector<pair<T,int> >run(T a){
73     if(a==1)
74         return vector<pair<T,int> >();
75     if(chk(a))
76         return vector<pair<T,int> >(1,make_pair(a,1));
77     T b=a;

```

```

78     while((b=rho(b,T(double(rand())/RAND_MAX*(a-1))))==a);
79     vector<pair<T,int> >u=run(b),v=run(a/b),r;
80     for(int pu=0,pv=0;pu<u.size()||pv<v.size();){
81         if(pu==u.size())
82             r.push_back(v[pv++]);
83         else if(pv==v.size())
84             r.push_back(u[pu++]);
85         else if(u[pu].first==v[pv].first)
86             r.push_back(make_pair(u[pu].first,(u[pu].second+v[pv].second
87 ))),++pu,++pv;
88         else if(u[pu].first>v[pv].first)
89             r.push_back(v[pv++]);
90         else
91             r.push_back(u[pu++]);}
92     return r;
93 }

```

4.3 Modular Integer

Modular Integer.hpp (2923 bytes, 96 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T,T P>struct ModularInteger{
4      ModularInteger(T t=0):
5          v(t){
6              if(v<0||v>=P)
7                  v=(v%P+P)%P;
8          }
9      ModularInteger<T,P>&operator=(T a){
10         v=a;
11         if(v<0||v>=P)
12             v%=P;
13         return*this;
14     }
15     ModularInteger<T,P>operator-(){
16         return v?P-v:0;
17     }

```

```

18     ModularInteger<T,P>&operator+=(ModularInteger<T,P>a){
19         return*this=*this+a;
20     }
21     ModularInteger<T,P>&operator--=(ModularInteger<T,P>a){
22         return*this=*this-a;
23     }
24     ModularInteger<T,P>&operator*=(ModularInteger<T,P>a){
25         return*this=*this*a;
26     }
27     ModularInteger<T,P>&operator/=(ModularInteger<T,P>a){
28         return*this=*this/a;
29     }
30     T v;
31 };
32 template<class T,T P>ModularInteger<T,P>pow(ModularInteger<T,P>a,long long
    b){
33     ModularInteger<T,P>r(1);
34     for(;b>=1,a=a*a)
35         if(b&1)
36             r=r*a;
37     return r;
38 }
39 template<class T,T P>ModularInteger<T,P>inv(ModularInteger<T,P>a){
40     return pow(a,P-2);
41 }
42 template<class T,T P>vector<ModularInteger<T,P> >sqrt(ModularInteger<T,P>a)
    {
43     vector<ModularInteger<T,P> >r;
44     if(!a.v)
45         r.push_back(ModularInteger<T,P>(0));
46     else if(pow(a,P-1>>1).v==1){
47         int s=P-1,t=0;
48         ModularInteger<T,P>b=1;
49         for(;pow(b,P-1>>1).v!=P-1;b=rand()*1.0/RAND_MAX*(P-1));
50         for(;s%2==0;++t,s/=2);
51         ModularInteger<T,P>x=pow(a,(s+1)/2),e=pow(a,s);
52         for(int i=1;i<t;++i,e=x*x/a)
53             if(pow(e,1<<t-i-1).v!=1)
54                 x=x*pow(b,(1<<i-1)*s);
55         r.push_back(x);
56         r.push_back(-x);

```

```

57     }
58     return r;
59 }
60 template<class T,T P>ModularInteger<T,P>operator+(ModularInteger<T,P>a,
    ModularInteger<T,P>b){
61     ModularInteger<T,P>c(a.v+b.v);
62     if(c.v>=P)
63         c.v-=P;
64     return c;
65 }
66 template<class T,T P>ModularInteger<T,P>operator-(ModularInteger<T,P>a,
    ModularInteger<T,P>b){
67     ModularInteger<T,P>c(a.v-b.v);
68     if(c.v<0)
69         c.v+=P;
70     return c;
71 }
72 template<class T,T P>ModularInteger<T,P>operator*(ModularInteger<T,P>a,
    ModularInteger<T,P>b){
73     if(typeid(T)!=typeid(int))
74         return ModularInteger<T,P>((a.v*b.v-(long long)(((long double)a.v*b
    .v+0.5)/P)*P+P)%P);
75     else
76         return ModularInteger<T,P>((long long)a.v*b.v%P);
77 }
78 template<class T,T P>ModularInteger<T,P>operator/(ModularInteger<T,P>a,
    ModularInteger<T,P>b){
79     return a*inv(b);
80 }
81 template<class T,T P>bool operator==(ModularInteger<T,P>a,ModularInteger<T,
    P>b){
82     return a.v==b.v;
83 }
84 template<class T,T P>bool operator!=(ModularInteger<T,P>a,ModularInteger<T,
    P>b){
85     return a.v!=b.v;
86 }
87 template<class T,T P>istream&operator>>(istream&s,ModularInteger<T,P>&a){
88     s>>a.v;
89     return s;
90 }

```



```

91 template<class T,T P>ostream&operator<<(ostream&s,ModularInteger<T,P>a){
92     s<<a.v;
93     if(a.v<0||a.v>=P)
94         a.v%=P;
95     return s;
96 }

```

4.4 Möbius Function

Möbius Function.hpp (534 bytes, 21 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  namespace MobiusFunction{
4      vector<int>run(int n){
5          vector<int>p,ntp(n+1),u(n+1);
6          ntp[1]=1;
7          u[1]=1;
8          for(int i=2;i<=n;++i){
9              if(!ntp[i])
10                 p.push_back(i),u[i]=-1;
11                 for(int j=0;j<p.size()&&p[j]*i<=n;++j){
12                     ntp[p[j]*i]=1;
13                     if(i%p[j]==0)
14                         break;
15                     else
16                         u[p[j]*i]=-u[i];
17                 }
18             }
19             return u;
20         }
21     }

```

4.5 Number

Number.hpp (7837 bytes, 345 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>T add(T a,T b,T p){
4      return a+b<p?a+b:a+b-p;
5  }
6  template<class T>T sub(T a,T b,T p){
7      return a-b<0?a-b+p:a-b;
8  }
9  template<class T>T mul(T a,T b,T p){
10     if(typeid(T)==typeid(int))
11         return (long long)a*b%p;
12     else
13         return (a*b-(T)((((long double)a*b+0.5)/p)*p+p)%p;
14 }
15 template<class T>T pow(T a,T b,T p){
16     T r=1;
17     for(;b;b&1?r=mul(r,a,p):0,b>>=1,a=mul(a,a,p));
18     return r;
19 }
20 template<class T>T inv(T a,T p){
21     return pow(a,p-2,p);
22 }
23 template<class T>T div(T a,T b,T p){
24     return mul(a,inv(b,p),p);
25 }
26 template<class T>T gcd(T a,T b){
27     if(a<0)
28         return gcd(-a,b);
29     if(b<0)
30         return gcd(a,-b);
31     return b?gcd(b,a%b):a;
32 }
33 template<class T>pair<T,T>exgcd(T a,T b){
34     if(!b)
35         return make_pair(T(1),T(0));
36     pair<T,T>t=exgcd(b,a%b);
37     swap(t.first,t.second);
38     t.second-=a/b*t.first;
39     return t;
40 }
41 template<class T>vector<T>divisor(T a){

```

```

42     vector<T>t;
43     for(T d=1;d*d<=a;++d)
44         if(a%d==0){
45             t.push_back(d);
46             if(d*d!=a)
47                 t.push_back(a/d);
48         }
49     return t;
50 }
51 template<class T>vector<pair<T,T> >factor(T a){
52     vector<pair<T,T> >t;
53     T b=a;
54     for(T d=2;d*d<=a;++d)
55         if(b%d==0)
56             for(t.push_back(make_pair(d,T(0)));b%d==0;b/=d,++t.back().second
57 );
58     if(b!=1)
59         t.push_back(make_pair(b,T(1)));
60     return t;
61 }
62 template<class T>bool is_prime_number(T a){
63     if(a<2)
64         return false;
65     for(T d=2;d*d<=a;++d)
66         if(a%d==0)
67             return false;
68     return true;
69 }
70 template<class T>bool is_palindromic_number(T a){
71     string b;
72     stringstream s;
73     s<<a;
74     s>>b;
75     for(int i=0;i<b.size();++i)
76         if(b[i]!=b[b.size()-i-1])
77             return false;
78     return true;
79 }
80 template<class T>T pow(T a,T b){
81     T s=1;
82     for(;b/=2,a*=a)

```

```

82         if(b%2)
83             s*=a;
84         return s;
85     }
86     template<class T>T power_sum(T n,T k){
87         T r=0;
88         for(T i=1;i<=n;++i)
89             r+=pow(i,k);
90         return r;
91     }
92     template<class T>T sqr(T a){
93         return a*a;
94     }
95     int nth_prime(int n){
96         int r=1;
97         for(int i=1;i<=n;++i)
98             for(++r;!is_prime_number(r);++r);
99         return r;
100     }
101     vector<vector<int> >pythagorean_triple(int n){
102         vector<vector<int> >t;
103         for(int i=1;i*i<=n;++i)
104             for(int j=1;j<i&& i*i+j*j<=n;++j){
105                 vector<int>u;
106                 u.push_back(i*i-j*j);
107                 u.push_back(2*i*j);
108                 u.push_back(i*i+j*j);
109                 t.push_back(u);
110             }
111         return t;
112     }
113     vector<vector<int> >primitive_pythagorean_triple(int n){
114         vector<vector<int> >t;
115         for(int i=1;i*i<=n;++i)
116             for(int j=1;j<i&& i*i+j*j<=n;++j)
117                 if((i-j)%2&&gcd(i,j)==1){
118                     vector<int>u;
119                     u.push_back(i*i-j*j);
120                     u.push_back(2*i*j);
121                     u.push_back(i*i+j*j);
122                     t.push_back(u);

```

```
123     }
124     return t;
125 }
126 template<class T>nth_triangular_number(T n){
127     if(n%2==0)
128         return n/2*(n+1);
129     else
130         return (n+1)/2*n;
131 }
132 template<class T>nth_pentagonal_number(T n){
133     if(n%2==0)
134         return n/2*(3*n-1);
135     else
136         return (3*n-1)/2*n;
137 }
138 template<class T>nth_hexagonal_number(T n){
139     return n*(2*n-1);
140 }
141 template<class T>vector<T>collatz_sequence(T a){
142     vector<T>t;
143     do{
144         t.push_back(a);
145         if(a==1)
146             return t;
147         if(a%2==0)
148             a/=2;
149         else
150             a=3*a+1;
151     }while(1);
152 }
153 template<class T>T factorial(T n){
154     T r=1;
155     for(T i=1;i<=n;++i)
156         r*=i;
157     return r;
158 }
159 template<class T>T product(T a,T b){
160     T r=1;
161     for(T i=a;i<=b;++i)
162         r*=i;
163     return r;
```

```

164 }
165 template<class T>T C(T n,T k){
166     return factorial(n)/factorial(k)/factorial(n-k);
167 }
168 template<class T>T P(T n,T k){
169     return factorial(n)/factorial(n-k);
170 }
171 vector<int>prime(int n){
172     vector<int>p,ntp(n+1);
173     ntp[1]=1;
174     for(int i=2;i<=n;++i){
175         if(!ntp[i])
176             p.push_back(i);
177         for(int j=0;j<p.size()&&p[j]*i<=n;++j){
178             ntp[p[j]*i]=1;
179             if(i%p[j]==0)
180                 break;
181         }
182     }
183     return p;
184 }
185 template<class T>T digit_sum(T a){
186     T r=0;
187     for(;a;r+=a%10,a/=10);
188     return r;
189 }
190 template<class T>T digit_power_sum(T a,T b){
191     T r=0;
192     for(;a;r+=pow(a%10,b),a/=10);
193     return r;
194 }
195 template<class T>T divisor_sum(T a){
196     vector<T>d=divisor(a);
197     T s=0;
198     for(int i=0;i<d.size();++i)
199         s+=d[i];
200     return s;
201 }
202 template<class T>bool is_perfect(T a){
203     return a*2==divisor_sum(a);
204 }

```

```
205 template<class T>bool is_deficient(T a){
206     return a*2>divisor_sum(a);
207 }
208 template<class T>bool is_abundant(T a){
209     return a*2<divisor_sum(a);
210 }
211 template<class T>set<int>digit_set(T a){
212     set<int>r;
213     for(;a;r.insert(a%10),a/=10);
214     return r;
215 }
216
217 template<class T>multiset<int>digit_multiset(T a){
218     multiset<int>r;
219     for(;a;r.insert(a%10),a/=10);
220     return r;
221 }
222 template<class T>int digit_count(T a){
223     int r=0;
224     if(!a)
225         ++r;
226     for(;a;++r,a/=10);
227     return r;
228 }
229 template<class T>T digit_factorial_sum(T a){
230     T r=0;
231     for(;a;r+=factorial(a%10),a/=10);
232     return r;
233 }
234 template<class T>bool has_distinct_digit(T a){
235     return digit_count(a)==digit_set(a).size();
236 }
237 template<class T>bool has_zero(T a){
238     if(!a)
239         return true;
240     while(a){
241         if(a%10==0)
242             return true;
243         a/=10;
244     }
245     return false;
```

```
246 }
247 template<class T>T right_circular_shift(T a){
248     stringstream ss;
249     ss<<a;
250     string t;
251     ss>>t;
252     t=t.substr(t.size()-1,1)+t.substr(0,t.size()-1);
253     ss<<t;
254     ss>>a;
255     return a;
256 }
257 template<class T>bool is_circular_prime(T a){
258     for(int i=digit_count(a);i-->0,a=right_circular_shift(a))
259         if(!is_prime_number(a))
260             return false;
261     return true;
262 }
263 template<class T>string to_binary(T a){
264     string r;
265     while(a){
266         r.push_back(a%2+'0');
267         a/=2;
268     }
269     reverse(r.begin(),r.end());
270     return r;
271 }
272 template<class T>T digit_reverse(T a){
273     stringstream ss;
274     ss<<a;
275     string t;
276     ss>>t;
277     reverse(t.begin(),t.end());
278     stringstream ss2;
279     ss2<<t;
280     ss2>>a;
281     return a;
282 }
283 template<class T>bool is_truncatable_prime(T a){
284     T b=digit_reverse(a);
285     while(a){
286         if(!is_prime_number(a))
```



```
287         return false;
288     a/=10;
289 }
290 a=b;
291 while(a){
292     if(!is_prime_number(digit_reverse(a)))
293         return false;
294     a/=10;
295 }
296 return true;
297 }
298 template<class T>bool is_triangle_number(T a){
299     if(a<1)
300         return false;
301     T l=1,r=1;
302     while(nth_triangular_number(r)<=a)
303         r*=2;
304     while(l+1<r){
305         T m=l+(r-1)/2;
306         if(nth_triangular_number(m)<=a)
307             l=m;
308         else
309             r=m;
310     }
311     return a==nth_triangular_number(l);
312 }
313 template<class T>bool is_pentagonal_number(T a){
314     if(a<1)
315         return false;
316     T l=1,r=1;
317     while(nth_pentagonal_number(r)<=a)
318         r*=2;
319     while(l+1<r){
320         T m=l+(r-1)/2;
321         if(nth_pentagonal_number(m)<=a)
322             l=m;
323         else
324             r=m;
325     }
326     return a==nth_pentagonal_number(l);
327 }
```

```

328 template<class T>bool is_hexagonal_number(T a){
329     if(a<1)
330         return false;
331     T l=1,r=1;
332     while(nth_hexagonal_number(r)<=a)
333         r*=2;
334     while(l+1<r){
335         T m=l+(r-1)/2;
336         if(nth_hexagonal_number(m)<=a)
337             l=m;
338         else
339             r=m;
340     }
341     return a==nth_hexagonal_number(l);
342 }
343 template<class T>bool is_square_number(T a){
344     return sqr(T(round(sqrt(a))))==a;
345 }

```

4.6 Primality Test

Primality Test.hpp (1509 bytes, 52 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  namespace PrimalityTest{
4      template<class T>T mul(T x,T y,T z){
5          if(typeid(T)==typeid(int))
6              return (long long)x*y%z;
7          else if(typeid(T)==typeid(long long))
8              return (x*y-(T)(((long double)x*y+0.5)/z)*z+z)%z;
9          else
10             return x*y%z;
11     }
12     template<class T>T pow(T a,T b,T c){
13         T r=1;
14         for(;b;b>>=1,a=mul(a,a,c));
15         return r;
16     }

```

```

17  template<class T>int run(T a,int c=10){
18      if(a==2)
19          return 1;
20      if(a%2==0 || a<2)
21          return 0;
22      static int pi[]={2,7,61},p1
[]={2,325,9375,28178,450775,9780504,1795265022};
23      if(typeid(T)==typeid(int))
24          c=3;
25      else if(typeid(T)==typeid(long long))
26          c=7;
27      T u=a-1,t=0,p=1;
28      for(;u%2==0;u/=2,++t);
29      for(int i=0;i<c;++i){
30          if(typeid(T)==typeid(int))
31              p=pi[i]%a;
32          else if(typeid(T)==typeid(long long))
33              p=p1[i]%a;
34          else
35              p=(p*29+7)%a;
36          if(!p || p==1 || p==a-1)
37              continue;
38          T x=pow(p,u,a);
39          if(x==1)
40              continue;
41          for(int j=0;x!=a-1&&j<t;++j){
42              x=mul(x,x,a);
43              if(x==1)
44                  return 0;
45          }
46          if(x==a-1)
47              continue;
48          return 0;
49      }
50      return 1;
51  }
52  }

```

4.7 Prime Number

Prime Number.hpp (473 bytes, 18 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  namespace PrimeNumber{
4      pair<vector<int>,vector<int> >run(int n){
5          vector<int>p,ntp(n+1);
6          ntp[1]=1;
7          for(int i=2;i<=n;++i){
8              if(!ntp[i])
9                  p.push_back(i);
10             for(int j=0;j<p.size()&&p[j]*i<=n;++j){
11                 ntp[p[j]*i]=1;
12                 if(i%p[j]==0)
13                     break;
14             }
15         }
16         return make_pair(p,ntp);
17     }
18 }
```

4.8 Primitive Root

Primitive Root.hpp (3256 bytes, 106 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  namespace PrimitiveRoot{
4      template<class T>T mul(T x,T y,T z){
5          if(typeid(T)==typeid(int))
6              return (long long)x*y%z;
7          else
8              return (x*y-(T)((((long double)x*y+0.5)/z)*z+z)%z;
9      }
10     template<class T>T pow(T a,T b,T c){
11         T r=1;
12         for(;b;b&1?r=mul(r,a,c):0,b>>=1,a=mul(a,a,c));
```

```

13     return r;
14 }
15 template<class T>bool chk(T a,int c=10){
16     if(a==1)
17         return false;
18     T u=a-1,t=0;
19     for(;u%2==0;u/=2,++t);
20     for(int i=0;i<c;++i){
21         T x=pow(T(rand()*1.0/RAND_MAX*(a-2)+1),u,a),y;
22         for(int j=0;j<t;++j){
23             y=x;
24             x=mul(x,x,a);
25             if(x==1&&y!=1&&y!=a-1)
26                 return false;
27         }
28         if(x!=1)
29             return false;
30     }
31     return true;
32 }
33 template<class T>T gcd(T a,T b){
34     if(a<0)
35         a=-a;
36     if(b<0)
37         b=-b;
38     return b?gcd(b,a%b):a;
39 }
40 template<class T>T rho(T a,T c){
41     T x=double(rand())/RAND_MAX*(a-1),y=x;
42     for(int i=1,k=2;;){
43         x=(mul(x,x,a)+c)%a;
44         T d=gcd(y-x,a);
45         if(d!=1&&d!=a)
46             return d;
47         if(y==x)
48             return a;
49         if(++i==k)
50             y=x,k=2*k;
51     }
52 }
53 template<class T>vector<pair<T,int> >fac(T a){

```

```

54     if(a==1)
55         return vector<pair<T,int> >();
56     if(chk(a))
57         return vector<pair<T,int> >(1,make_pair(a,1));
58     T b=a;
59     while((b=rho(b,T(double(rand())/RAND_MAX*(a-1))))==a);
60     vector<pair<T,int> >u=fac(b),v=fac(a/b),r;
61     for(int pu=0,pv=0;pu<u.size()||pv<v.size();){
62         if(pu==u.size())
63             r.push_back(v[pv++]);
64         else if(pv==v.size())
65             r.push_back(u[pu++]);
66         else if(u[pu].first==v[pv].first)
67             r.push_back(make_pair(u[pu].first,(u[pu].second+v[pv].second
68 ))),++pu,++pv;
69         else if(u[pu].first>v[pv].first)
70             r.push_back(v[pv++]);
71         else
72             r.push_back(u[pu++]);}
73     return r;
74 }
75 template<class T>void dfs(vector<pair<T,int> >&f,int i,T now,vector<T>&
76 r){
77     if(i==f.size()){
78         r.push_back(now);
79         return;
80     }
81     for(int j=0;j<=f[i].second;++j,now*=f[i].first)
82         dfs(f,i+1,now,r);
83 }
84 template<class T>T run(T a){
85     vector<pair<T,int> >fa=fac(a),fpa;
86     if(fa.size()==0||fa.size()>2)
87         return -1;
88     if(fa.size()==1&&fa[0].first==2&&fa[0].second>2)
89         return -1;
90     if(fa.size()==2&&fa[0]!=make_pair(T(2),1))
91         return -1;
92     T pa=a;
93     for(int i=0;i<fa.size();++i)
94         pa=pa/fa[i].first*(fa[i].first-1);

```

```

93     fpa=fac(pa);
94     vector<T>fs;
95     dfs(fpa,0,1,fs);
96     for(T g=1,f=0;;++g,f=0){
97         for(int i=0;i<fs.size();++i)
98             if(fs[i]!=pa&&pow(g,fs[i],a)==1){
99                 f=1;
100                break;
101            }
102        if(!f)
103            return g;
104    }
105 }
106 }

```

4.9 Sequence

Sequence.txt (1134 bytes, 8 lines)

Numbers n such that a Hadamard matrix of order n exists.

1, 2, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100, 104, 108, 112, 116, 120, 124, 128, 132, 136, 140, 144, 148, 152, 156, 160, 164, 168, 172, 176, 180, 184, 188, 192, 196, 200, 204, 208, 212, 216, 220, 224, 228, 232, 236, 240

Catalan numbers: $C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$. Also called Segner numbers.

1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670, 129644790, 477638700, 1767263190, 6564120420, 24466267020, 91482563640, 343059613650, 1289904147324, 4861946401452, 18367353072152, 69533550916004, 263747951750360, 1002242216651368, 3814986502092304

Bell or exponential numbers: number of ways to partition a set of n labeled elements.

1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975, 678570, 4213597, 27644437, 190899322, 1382958545, 10480142147, 82864869804, 682076806159, 5832742205057, 51724158235372, 474869816156751, 4506715738447323, 44152005855084346, 445958869294805289, 4638590332229999353, 49631246523618756274

CHAPTER 5

Numerical Algorithms

5.1 Convolution (Fast Fourier Transform)

Convolution (Fast Fourier Transform).hpp (1300 bytes, 39 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  namespace Convolution{
4      typedef complex<double>T;
5      void fft(vector<T>&a,int n,double s,vector<int>&rev){
6          T im(0,1);
7          double pi=acos(-1);
8          for(int i=0;i<n;++i)
9              if(i<rev[i])
10                 swap(a[i],a[rev[i]]);
11          for(int i=1,m=2;(1<i)<=n;++i,m<=1){
12              T wm=exp(s*im*2.0*pi/double(m)),w;
13              for(int j=(w=1,0);j<n;j+=m,w=1)
14                  for(int k=0;k<(m>>1);++k,w*=wm){
15                      T u=a[j+k],v=w*a[j+k+(m>>1)];
16                      a[j+k]=u+v;
17                      a[j+k+(m>>1)]=u-v;
18                  }
19          }
20      }
21      vector<double>run(const vector<double>&a,const vector<double>&b){
22          int l=ceil(log2(a.size()+b.size()-1)),n=1<<l;
23          vector<int>rv;
24          for(int i=(rv.resize(n),0);i<n;++i)
25              rv[i]=(rv[i>>1]>>1)|((i&1)<<(l-1));
26          vector<T>ta(n),tb(n);
27          copy(a.begin(),a.end(),ta.begin());
28          copy(b.begin(),b.end(),tb.begin());
29          fft(ta,n,1,rv);
30          fft(tb,n,1,rv);
31          for(int i=0;i<n;++i)
32              ta[i]*=tb[i];
33          fft(ta,n,-1,rv);
34          vector<double>c(a.size()+b.size()-1);
35          for(int i=0;i<c.size();++i)
36              c[i]=real(ta[i])/n;
37          return c;

```

```

38     }
39 }

```

5.2 Convolution (Karatsuba Algorithm)

Convolution (Karatsuba Algorithm).hpp (1416 bytes, 43 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  namespace Convolution{
4      template<class T>void kar(T*a,T*b,int n,int l,T**r){
5          T*r1=r[l],*r11=r[l-1];
6          for(int i=0;i<2*n;++i)
7              *(r1+i)=0;
8          if(n<=30){
9              for(int i=0;i<n;++i)
10                 for(int j=0;j<n;++j)
11                     *(r1+i+j)+=(a+i)**(b+j);
12             return;
13         }
14         kar(a,b,n>>1,l-1,r);
15         for(int i=0;i<n;++i)
16             *(r1+i)+=(r11+i),*(r1+i+(n>>1))+=(r11+i);
17         kar(a+(n>>1),b+(n>>1),n>>1,l-1,r);
18         for(int i=0;i<n;++i)
19             *(r1+i+n)+=(r11+i),*(r1+i+(n>>1))+=(r11+i);
20         for(int i=0;i<(n>>1);++i){
21             *(r1+(n<<1)+i)=(a+(n>>1)+i)-(a+i);
22             *(r1+i+(n>>1)*5)=(b+i)-(b+(n>>1)+i);
23         }
24         kar(r1+(n<<1),r1+(n>>1)*5,n>>1,l-1,r);
25         for(int i=0;i<n;++i)
26             *(r1+i+(n>>1))+=(r11+i);}
27     template<class T>vector<T>run(vector<T>a,vector<T>b){
28         int l=ceil(log2(max(a.size(),b.size()))+1e-8);
29         vector<T>rt(a.size()+b.size()-1);
30         a.resize(1<<l);
31         b.resize(1<<l);
32         T**r=new T*[l+1];

```

```

33     for(int i=0;i<=l;++i)
34         r[i]=new T[(1<<i)*3];
35     kar(&a[0],&b[0],1<<l,1,r);
36     for(int i=0;i<rt.size();++i)
37         rt[i]=*(r[l]+i);
38     for(int i=0;i<=l;++i)
39         delete r[i];
40     delete r;
41     return rt;
42 }
43 }

```

5.3 Convolution (Number Theoretic Transform)

Convolution (Number Theoretic Transform).hpp (1620 bytes, 51 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  namespace Convolution{
4      typedef long long T;
5      T pow(T a,T b,T c){
6          T r=1;
7          for(;b;b&=1)r=r*a%c;0,b>=1,a=a*a%c);
8          return r;
9      }
10 void ntt(vector<T>&a,int n,int s,vector<int>&rev,T p,T g){
11     g=s==1?g:pow(g,p-2,p);
12     vector<T>wm;
13     for(int i=0;1<<i<=n;++i)
14         wm.push_back(pow(g,(p-1)>>i,p));
15     for(int i=0;i<n;++i)
16         if(i<rev[i])
17             swap(a[i],a[rev[i]]);
18     for(int i=1,m=2;1<<i<=n;++i,m<=1){
19         vector<T>wmk(1,1);
20         for(int k=1;k<(m>>1);++k)
21             wmk.push_back(wmk.back()*wm[i]%p);
22         for(int j=0;j<n;j+=m)
23             for(int k=0;k<(m>>1);++k){

```

```

24         T u=a[j+k],v=wmk[k]*a[j+k+(m>>1)]%p;
25         a[j+k]=u+v;
26         a[j+k+(m>>1)]=u-v+p;
27         if(a[j+k]>=p)
28             a[j+k]-=p;
29         if(a[j+k+(m>>1)]>=p)
30             a[j+k+(m>>1)]=-p;
31     }
32 }
33 }
34 vector<T>run(vector<T>a,vector<T>b,T p=15*(1<<27)+1,T g=31){
35     int tn,l=ceil(log2(tn=a.size()+b.size()-1)),n=1<<l;
36     vector<int>rv;
37     for(int i=(rv.resize(n),0);i<n;++i)
38         rv[i]=(rv[i>>1]>>1)|((i&1)<<(l-1));
39     a.resize(n);
40     b.resize(n);
41     ntt(a,n,1,rv,p,g);
42     ntt(b,n,1,rv,p,g);
43     for(int i=0;i<n;++i)
44         a[i]=a[i]*b[i]%p;
45     ntt(a,n,-1,rv,p,g);
46     n=pow(n,p-2,p);
47     for(T&v:a)
48         v=v*n%p;
49     return a.resize(tn),a;
50 }
51 }

```

5.4 Fraction

Fraction.hpp (2217 bytes, 100 lines)

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 template<class T>struct Fraction{
4     T p,q;
5     int s;
6     T gcd(T a,T b){

```

```

7         return b?gcd(b,a%b):a;
8     }
9     void reduce(){
10         T d=gcd(p,q);
11         p/=d;
12         q/=d;
13         if(p==0)
14             s=0;
15     }
16     Fraction(int _s=0,T _p=0,T _q=1):
17         s(_s),p(_p),q(_q){
18         reduce();
19     }
20     Fraction(string a){
21         if(a[0]=='-'){
22             s=-1;
23             a=a.substr(1,a.size()-1);
24         }else if(a[0]=='+'){
25             s=1;
26             a=a.substr(1,a.size()-1);
27         }else
28             s=1;
29         stringstream ss;
30         char tc;
31         ss<<a;
32         ss>>p>>tc>>q;
33         reduce();
34     }
35     Fraction(const char*a){
36         *this=Fraction(string(a));
37     }
38     Fraction<T>&operator=(string a){
39         return*this=Fraction<T>(a);
40     }
41     Fraction<T>&operator=(const char*a){
42         return*this=Fraction<T>(a);
43     }
44 };
45 template<class T>ostream&operator<<(ostream&s,const Fraction<T>&a){
46     if(a.s== -1)
47         s<<'-';

```

```

48     return s<<a.p<<'/'<<a.q;
49 }
50 template<class T>istream&operator>>(istream&s,Fraction<T>&a){
51     string t;
52     s>>t;
53     a=t;
54     return s;
55 }
56 template<class T>vector<string>real(const Fraction<T>&a){
57     vector<string>r;
58     stringstream ss;
59     string st;
60     if(a.s<0)
61         r.push_back("-");
62     else
63         r.push_back("+");
64     T p=a.p,q=a.q;
65     ss<<p/q;
66     ss>>st;
67     r.push_back(st);
68     p%=q;
69     st.clear();
70     map<T,int>mp;
71     while(true){
72         if(p==0){
73             r.push_back(st);
74             r.push_back("");
75             return r;
76         }
77         if(mp.count(p)){
78             r.push_back(st.substr(0,mp[p]));
79             r.push_back(st.substr(mp[p],st.size()-mp[p]));
80             return r;
81         }
82         p*=10;
83         mp[p/10]=st.size();
84         st.push_back('0'+p/q);
85         p%=q;
86     }
87     return r;
88 }

```

```

89 template<class T>string decimal(const Fraction<T>&a){
90     string r;
91     vector<string>t=real(a);
92     if(t[0]=="-")
93         r.push_back('-');
94     r+=t[1];
95     if(t[2].size()||t[3].size())
96         r+="."+t[2];
97     if(t[3].size())
98         r+="("+t[3]+")";
99     return r;
100 }

```

5.5 Integer

Integer.hpp (6378 bytes, 269 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  struct Integer operator+(Integer a,Integer b);
4  Integer operator+(Integer a,int b);
5  Integer operator-(Integer a,Integer b);
6  Integer operator*(Integer a,Integer b);
7  Integer operator*(Integer a,Integer b);
8  Integer operator/(Integer a,Integer b);
9  Integer operator%(Integer a,Integer b);
10 Integer operator%(Integer a,int b);
11 Integer operator%(Integer a,long long b);
12 bool operator!=(Integer a,int b);
13 bool operator<=(Integer a,int b);
14 struct Integer{
15     operator bool(){
16         return *this!=0;
17     }
18     Integer(long long a=0){
19         if(a<0){
20             s=-1;
21             a=-a;
22         }else

```



```

23         s=a!=0;
24     do{
25         d.push_back(a%B);
26         a/=B;
27     }while(a);
28 }
29 Integer(string a){
30     s=(a[0]=='-')?-1:(a!="0");
31     for(int i=a.size()-1;i>=(a[0]=='-');i-=L){
32         int t=0,j=max(i-L+1,int(a[0]=='-'));
33         for(int k=j;k<=i;++k)
34             t=t*10+a[k]-'0';
35         d.push_back(t);
36     }
37 }
38 Integer(const Integer&a){
39     d=a.d;
40     s=a.s;
41 }
42 Integer&operator=(long long a){
43     return*this=Integer(a);
44 }
45 Integer&operator+=(Integer a){
46     return*this=*this+a;
47 }
48 Integer&operator-=(Integer a){
49     return*this=*this-a;
50 }
51 Integer&operator*=(Integer a){
52     return*this=*this*a;
53 }
54 Integer&operator/=(Integer a){
55     return*this=*this/a;
56 }
57 Integer&operator%=(Integer a){
58     return*this=*this%a;
59 }
60 Integer&operator++(){
61     return*this=*this+1;
62 }
63 operator string()const{

```

```

64     string r;
65     for(int i=0;i<d.size();++i){
66         stringstream ts;
67         ts<<d[i];
68         string tt;
69         ts>>tt;
70         reverse(tt.begin(),tt.end());
71         while(i+1!=d.size()&&tt.size()<L)
72             tt.push_back('0');
73         r+=tt;
74     }
75     reverse(r.begin(),r.end());
76     return r;
77 }
78 int s;
79 vector<int>d;
80 static const int B=1e8,L=8;
81 };
82 string str(const Integer&a){
83     return string(a);
84 }
85 bool operator<(Integer a,Integer b){
86     if(a.s!=b.s)
87         return a.s<b.s;
88     if(a.d.size()!=b.d.size())
89         return (a.s!=1)^(a.d.size()<b.d.size());
90     for(int i=a.d.size()-1;i>=0;--i)
91         if(a.d[i]!=b.d[i])
92             return (a.s!=1)^(a.d[i]<b.d[i]);
93     return false;
94 }
95 bool operator>(Integer a,Integer b){
96     return b<a;
97 }
98 bool operator<=(Integer a,Integer b){
99     return !(a>b);
100 }
101 bool operator>=(Integer a,Integer b){
102     return !(a<b);
103 }
104 bool operator==(Integer a,Integer b){

```

```

105     return !(a<b)&&!(a>b);
106 }
107 bool operator!=(Integer a,Integer b){
108     return !(a==b);
109 }
110 istream&operator>>(istream&s,Integer&a){
111     string t;
112     s>>t;
113     a=Integer(t);
114     return s;
115 }
116 ostream&operator<<(ostream&s,Integer a){
117     if(a.s== -1)
118         s<<"-";
119     for(int i=a.d.size()-1;i>=0;--i){
120         if(i!=a.d.size()-1)
121             s<<setw(Integer::L)<<setfill('0');
122         s<<a.d[i];
123     }
124     s<<setw(0)<<setfill(' ');
125     return s;
126 }
127 void dzero(Integer&a){
128     while(a.d.size()>1&&a.d.back()==0)
129         a.d.pop_back();
130 }
131 Integer operator-(Integer a){
132     a.s*=-1;
133     if(a.d.size()==1&&a.d[0]==0)
134         a.s=1;
135     return a;
136 }
137 Integer operator+(Integer a,int b){
138     return a+Integer(b);
139 }
140 Integer operator*(Integer a,int b){
141     return a*Integer(b);
142 }
143 Integer operator%(Integer a,int b){
144     return a%Integer(b);
145 }

```

```

146 Integer operator%(Integer a,long long b){
147     return a%Integer(b);
148 }
149 bool operator!=(Integer a,int b){
150     return a!=Integer(b);
151 }
152 bool operator<=(Integer a,int b){
153     return a<=Integer(b);
154 }
155 Integer operator+(Integer a,Integer b){
156     if(a.s*b.s!=-1){
157         Integer c;c.s=a.s?a.s:b.s;
158         c.d.resize(max(a.d.size(),b.d.size()+1));
159         for(int i=0;i<c.d.size()-1;++i){
160             if(i<a.d.size())
161                 c.d[i]+=a.d[i];
162             if(i<b.d.size())
163                 c.d[i]+=b.d[i];
164             if(c.d[i]>=Integer::B){
165                 c.d[i]-=Integer::B;
166                 ++c.d[i+1];
167             }
168         }
169         dzero(c);
170         return c;
171     }
172     return a-(-b);
173 }
174 Integer operator-(Integer a,Integer b){
175     if(a.s*b.s==1){
176         if(a.s==-1)
177             return (-b)-(-a);
178         if(a<b)
179             return -(b-a);
180         if(a==b)
181             return 0;
182         for(int i=0;i<b.d.size();++i){
183             a.d[i]-=b.d[i];
184             if(a.d[i]<0){
185                 a.d[i]+=Integer::B;
186                 --a.d[i+1];

```

```

187         }
188     }
189     dzero(a);
190     return a;
191 }
192 return a+(-b);
193 }
194 Integer operator*(Integer a,Integer b){
195     vector<long long>t(a.d.size()+b.d.size());
196     for(int i=0;i<a.d.size();++i)
197         for(int j=0;j<b.d.size();++j)
198             t[i+j]+=(long long)a.d[i]*b.d[j];
199     for(int i=0;i<t.size()-1;++i){
200         t[i+1]+=t[i]/Integer::B;
201         t[i]%=Integer::B;
202     }
203     Integer c;
204     c.s=a.s*b.s;c.d.resize(t.size());
205     copy(t.begin(),t.end(),c.d.begin());
206     dzero(c);
207     return c;
208 }
209 Integer div2(Integer a){
210     for(int i=a.d.size()-1;i>=0;--i){
211         if(i)
212             a.d[i-1]+=(a.d[i]&1)*Integer::B;
213         a.d[i]>>=1;
214     }
215     dzero(a);
216     if(a.d.size()==1&&a.d[0]==0)
217         a.s=0;
218     return a;
219 }
220 Integer operator/(Integer a,Integer b){
221     if(!a.s)
222         return 0;
223     if(a.s<0)
224         return -((-a)/b);
225     if(a<b)
226         return 0;
227     Integer l=1,r=1;

```

```

228     while(r*b<=a)
229         r=r*2;
230     while(l+1<r){
231         Integer m=div2(l+r);
232         if(m*b>a)
233             r=m;
234         else
235             l=m;
236     }
237     return l;
238 }
239 Integer operator%(Integer a,Integer b){
240     return a-a/b*b;
241 }
242 Integer gcd(Integer a,Integer b){
243     Integer r=1;
244     while(a!=0&&b!=0){
245         if(!(a.d[0]&1)&&!(b.d[0]&1)){
246             a=div2(a);
247             b=div2(b);
248             r=r*2;
249         }else if(!(a.d[0]&1))
250             a=div2(a);
251         else if(!(b.d[0]&1))
252             b=div2(b);
253         else{
254             if(a<b)
255                 swap(a,b);
256             a=div2(a-b);
257         }
258     }
259     if(a!=0)
260         return r*a;
261     return r*b;
262 }
263 int length(Integer a){
264     a.s=1;
265     return string(a).size();
266 }
267 int len(Integer a){
268     return length(a);

```

269 | }

5.6 Linear Programming

Linear Programming.hpp (2522 bytes, 89 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  struct LinearProgramming{
4      const double E;
5      int n,m,p;
6      vector<int>mp,ma,md;
7      vector<vector<double>> >a;
8      vector<double>res;
9      LinearProgramming(int _n,int _m):
10         n(_n),m(_m),p(0),a(n+2,vector<double>(m+2)),mp(n+1),ma(m+n+2),md(m
+2),res(m+1),E(1e-8){
11     }
12     void piv(int l,int e){
13         swap(mp[l],md[e]);
14         ma[mp[l]]=1;
15         ma[md[e]]=-1;
16         double t=-a[l][e];
17         a[l][e]=-1;
18         vector<int>qu;
19         for(int i=0;i<=m+1;++i)
20             if(fabs(a[l][i]/t)>E)
21                 qu.push_back(i);
22         for(int i=0;i<=n+1;++i)
23             if(i!=l&&fabs(a[i][e])>E){
24                 t=a[i][e];
25                 a[i][e]=0;
26                 for(int j=0;j<qu.size();++j)
27                     a[i][qu[j]]+=a[l][qu[j]]*t;
28             }
29         if(-p==1)
30             p=e;
31         else if(p==e)
32             p=-1;

```

```

33     }
34     int opt(int d){
35         for(int l=-1,e=-1;;piv(l,e),l=-1,e=-1){
36             for(int i=1;i<=m+1;++i)
37                 if(a[d][i]>E){
38                     e=i;
39                     break;
40                 }
41             if(e== -1)
42                 return 1;
43             double t;
44             for(int i=1;i<=n;++i)
45                 if(a[i][e]<-E&&(l== -1 || a[i][0]/-a[i][e]<t))
46                     t=a[i][0]/-a[i][e],l=i;
47             if(l== -1)
48                 return 0;
49         }
50     }
51     double&at(int x,int y){
52         return a[x][y];
53     }
54     vector<double>run(){
55         for(int i=1;i<=m+1;++i)
56             ma[i]=-1,md[i]=i;
57         for(int i=m+2;i<=m+n+1;++i)
58             ma[i]=i-(m+1),mp[i-(m+1)]=i;
59         double t;
60         int l=-1;
61         for(int i=1;i<=n;++i)
62             if(l== -1 || a[i][0]<t)
63                 t=a[i][0],l=i;
64         if(t<-E){
65             for(int i=1;i<=n;++i)
66                 a[i][m+1]=1;
67             a[n+1][m+1]=-1;
68             p=m+1;
69             piv(l,m+1);
70             if(!opt(n+1) || fabs(a[n+1][0])>E)
71                 return vector<double>();
72             if(p<0)
73                 for(int i=1;i<=m;++i)

```



```

74         if(fabs(a[-p][i])>E){
75             piv(-p,i);
76             break;
77         }
78         for(int i=0;i<=n;++i)
79             a[i][p]=0;
80     }
81     if(!opt(0))
82         return vector<double>();
83     res[0]=a[0][0];
84     for(int i=1;i<=m;++i)
85         if(ma[i]!=-1)
86             res[i]=a[ma[i]][0];
87     return res;
88 }
89 };

```

5.7 Linear System

Linear System.hpp (1477 bytes, 56 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>struct LinearSystem{
4      int n;
5      vector<vector<T> >a;
6      vector<int>main,pos;
7      vector<T>ans;
8      int cmp(T a){
9          if(typeid(T)==typeid(double)||typeid(T)==typeid(long double)||
10             typeid(T)==typeid(float)){
11              if(a<-1e-8)
12                  return -1;
13              if(a>1e-8)
14                  return 1;
15              return 0;
16          }
17          if(a<0)
18              return -1;

```

```

18         if(a>0)
19             return 1;
20         return 0;
21     }
22     T&at(int i,int j){
23         return a[i][j];
24     }
25     vector<T>&at(int i){
26         return a[i];
27     }
28     LinearSystem(int _n):
29         n(_n),a(n+1,vector<T>(n+1)),main(n+1),pos(n+1),ans(n){
30     }
31     vector<T>run(){
32         for(int i=1;i<=n;++i){
33             int j=1;
34             for(;j<=n&&!cmp(a[i][j]);++j);
35             if(j<=n){
36                 main[i]=j;
37                 pos[j]=i;
38                 T t=a[i][j];
39                 for(int k=0;k<=n;++k)
40                     a[i][k]/=t;
41                 for(int k=1;k<=n;++k)
42                     if(k!=i&&cmp(a[k][j])){
43                         t=a[k][j];
44                         for(int l=0;l<=n;++l)
45                             a[k][l]-=a[i][l]*t;
46                     }
47             }
48         }
49         for(int i=1;i<=n;++i){
50             if(!pos[i])
51                 return vector<T>();
52             ans[i-1]=a[pos[i]][0];
53         }
54         return ans;
55     }
56 };

```

5.8 Polynomial Interpolation

Polynomial Interpolation.hpp (372 bytes, 15 lines)

```
1 #include<bits/stdc++.h>
2 using namespace std;
3 template<class T>T PolynomialInterpolation(vector<T>x,vector<T>y,T x0){
4     T r=0;
5     for(int i=0;i<x.size();++i){
6         T p=1,q=1;
7         for(int j=0;j<x.size();++j)
8             if(j!=i){
9                 p*=(x0-x[j]);
10                q*=(x[i]-x[j]);
11            }
12        r+=p/q*y[i];
13    }
14    return r;
15 }
```


CHAPTER 6

String Algorithms

6.1 Aho-Corasick Automaton

Aho-Corasick Automaton.hpp (1369 bytes, 50 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  struct AhoCorasickAutomaton{
4      struct node{
5          node(int m):
6              tr(m),fail(0),cnt(0){
7              }
8          vector<node*>tr;
9          node*fail;
10         int cnt;
11     };
12     int m;
13     node*root;
14     vector<node*>all;
15     AhoCorasickAutomaton(int _m):
16         m(_m),root(new node(m)),all(1,root){
17     }
18     ~AhoCorasickAutomaton(){
19         for(int i=0;i<all.size();++i)
20             delete all[i];
21     }
22     node*insert(int*s){
23         node*p;
24         for(p=root;*s!=-1;p=p->tr[*s++])
25             if(!p->tr[*s])
26                 p->tr[*s]=new node(m);
27         return p;
28     }
29     void build(){
30         queue<node*>qu;
31         for(int i=0;i<m;++i)
32             if(!root->tr[i])
33                 root->tr[i]=root;
34             else
35                 root->tr[i]->fail=root,qu.push(root->tr[i]);
36         for(node*u;qu.size();(u=qu.front(),qu.pop(),all.push_back(u),1):0;)
37             for(int i=0;i<m;++i)

```

```

38         if(!u->tr[i])
39             u->tr[i]=u->fail->tr[i];
40         else
41             u->tr[i]->fail=u->fail->tr[i],qu.push(u->tr[i]);
42     }
43     void run(int*s){
44         for(node*p=root;*s!=-1;++(p=p->tr[*s]))->cnt);
45     }
46     void count(){
47         for(int i=all.size()-1;i>=1;--i)
48             all[i]->fail->cnt+=all[i]->cnt;
49     }
50 };

```

6.2 Factor Oracle

Factor Oracle.hpp (597 bytes, 18 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T,int N,int M,T D>struct FactorOracle{
4      FactorOracle(){
5          memset(tr,(lrs[0]=0,sp[0]=-1),4*M);
6      }
7      void insert(T*s,int n){
8          for(int i=0,j,c=s[i]-D,u,v;i<n;c=s[++i]-D){
9              memset(tr+i+1,(lrs[i+1]=0)-1,4*M);
10             for(j=i;j>-1&&tr[j][c]<0;tr[j][c]=i+1,j=sp[u=j]);
11             if(v=sp[i+1]=j<0?0:tr[j][c]){
12                 for(v=v-1==sp[u]?u:v-1;sp[u]!=sp[v];v=sp[v]);
13                 lrs[i+1]=min(lrs[u],lrs[v])+1;
14             }
15         }
16     }
17     int sp[N+1],lrs[N+1],tr[N+1][M];
18 };

```

6.3 Longest Common Substring

Longest Common Substring.hpp (1181 bytes, 28 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T,int N,int M,T D>struct LongestCommonSubstring{
4      void ins(int c){
5          memset(tr+i+1,(lrs[i+1]=0)-1,4*M);
6          for(j=i;j>-1&&((v=tr[j][c])>=l1+2&&v<=l1+lb+1||v<0);tr[j][c]=i+1+lb
, j=sp[u=j]);
7          if(v=sp[i+1]=j<0?0:tr[j][c]-(tr[j][c]>l1+1)*lb){
8              for(v=v-1==sp[u]?u:v-1;sp[u]!=sp[v];v=sp[v]);
9              lrs[i+1]=min(lrs[u],lrs[v])+1;
10         }
11         if(sp[i+1]<=l1)
12             tm[sp[i+1]]=max(tm[sp[i+1]],lrs[i+1]);
13     }
14     int run(vector<pair<int,T*> >s){
15         swap(s[0],*min_element(s.begin(),s.end()));
16         l1=s[k=lb=0].first;
17         memset(mi,63,4*N+4);
18         memset(tr,(lrs[0]=0,sp[0]=-1),4*M+4);
19         for(i=0;i<l1;ins(*(s[0].second+i)-D),++i);
20         for(k=1,ins(M);k<s.size();lb+=s[k++].first){
21             memset(tm,0,4*N+4);
22             for(i=l1+1;i-l1-1<s[k].first;ins(*(s[k].second+i-l1-1)-D),++i)
;
23             for(i=l1;i;mi[i]=min(mi[i],tm[i]),tm[sp[i]]=max(tm[sp[i]],lrs[i
]*!!tm[i]),--i);
24         }
25         return min(*max_element(mi+1,mi+l1+1),l1);
26     }
27     int sp[2*N+2],lrs[2*N+2],tr[2*N+2][M+1],mi[N+1],tm[N+1],l1,lb,i,j,k,u,v
;
28 };

```


6.4 Palindromic Tree

Palindromic Tree.hpp (1327 bytes, 50 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>struct PalindromicTree{
4      struct node{
5          node(int m,node*f,int l):
6              nxt(m),fail(f),len(l){
7              }
8          vector<node*>nxt;
9          node*fail;
10         T val;
11         int len;
12     }*root;
13     int m;
14     vector<int>str;
15     vector<node*>all;
16     PalindromicTree(int _m):
17         m(_m){
18         node*n0=new node(m,0,-2),*n1=new node(m,n0,-1),*n2=new node(m,n1,0)
19         ;
20         all.push_back(n0);
21         all.push_back(n1);
22         all.push_back(n2);
23         fill(n0->nxt.begin(),n0->nxt.end(),n2);
24         root=n1;
25     }
26     ~PalindromicTree(){
27         for(int i=0;i<all.size();++i)
28             delete all[i];
29     }
30     node*find(node*x){
31         while(x->fail&&str[str.size()-x->len-2]!=str[str.size()-1])
32             x=x->fail;
33         return x;
34     }
35     node*insert(node*p,int c,T v){
36         if(p==root)
37             str=vector<int>(1,-1);

```

```

37     str.push_back(c);
38     p=find(p);
39     if(!p->nxt[c]){
40         node*np=(p->nxt[c]=new node(m,find(p->fail)->nxt[c],p->len+2))
41     ;
42         all.push_back(np);
43     }
44     p->nxt[c]->val+=v;
45     return p->nxt[c];
46 }
47 void count(){
48     for(int i=all.size()-1;i>=1;--i)
49         all[i]->fail->val+=all[i]->val;
50 };

```

6.5 String Searching

String Searching.hpp (682 bytes, 25 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>struct StringSearching{
4      StringSearching(T*a):
5          b(2,a[1]),f(2),l(2){
6          for(int i=0;a[1]?1:(--l,0);b.push_back(a[l++])){
7              for(;i&&a[i+1]!=a[l];i=f[i]);
8              f.push_back(i=i+(a[i+1]==a[l]));
9          }
10         for(int i=2;i<l;++i)
11             if(a[f[i]+1]==a[i+1])
12                 f[i]=f[f[i]];
13     }
14     int run(T*a,int p){
15         for(int i=p?p+1:1,j=p?f[l]:0;a[i];++i){
16             for(;j&&b[j+1]!=a[i];j=f[j]);
17             if((j+=b[j+1]==a[i])==1)
18                 return i-l+1;
19         }

```

```
20     return 0;
21 }
22 int l;
23 vector<T>b;
24 vector<int>f;
25 };
```

6.6 String

String.hpp (987 bytes, 45 lines)

```
1 #include<bits/stdc++.h>
2 using namespace std;
3 string read_all(){
4     string t;
5     getline(cin,t,char(EOF));
6     return t;
7 }
8 string delete_all(string a,char b){
9     string r;
10    for(int i=0;i<a.size();++i)
11        if(a[i]!=b)
12            r.push_back(a[i]);
13    return r;
14 }
15 string substr(string a,int l,int r){
16    return a.substr(l,r-l+1);
17 }
18 vector<string>split(string a,char b){
19    vector<string>r;
20    string t;
21    for(int i=0;i<a.size();++i)
22        if(a[i]!=b)
23            t.push_back(a[i]);
24        else{
25            r.push_back(t);
26            t="";
27        }
28    r.push_back(t);
```

```

29     return r;
30 }
31 int letter_order(char a){
32     return a>='a'&&a<='z'?a-'a'+1:a-'A'+1;
33 }
34 int letter_sum(string a){
35     int r=0;
36     for(int i=0;i<a.size();++i)
37         r+=letter_order(a[i]);
38     return r;
39 }
40 bool is_palindromic_string(string a){
41     for(int i=0;i<a.size();++i)
42         if(a[i]!=a[a.size()-i-1])
43             return false;
44     return true;
45 }

```

6.7 Suffix Array (DC3 Algorithm)

Suffix Array (DC3 Algorithm).hpp (2952 bytes, 107 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  struct SuffixArray{
4      int*sa,*ht,*rk,*ts,*ct,ln;
5      SuffixArray(int*s){
6          int m=0;
7          for(ln=0;s[ln+1];)
8              m=max(m,s[++ln]);
9          crt(sa,ln);
10         crt(ht,ln);
11         crt(rk,ln);
12         crt(ts,ln);
13         crt(ct,max(ln,m));
14         dc3(s,ln,m,sa,rk);
15         for(int i=1;i<=ln;++i){
16             if(rk[i]==1){
17                 ht[1]=0;

```

```

18         continue;
19     }
20     int&d=ht[rk[i]]=max(i==1?0:ht[rk[i-1]]-1,0);
21     for(;i+d<=ln&&sa[rk[i]-1]+d<=ln&&s[i+d]==s[sa[rk[i]-1]+d];++d);
22 }
23 }
24 ~SuffixArray(){
25     del(sa);
26     del(ht);
27     del(rk);
28     del(ts);
29     del(ct);
30 }
31 void crt(int*&a,int n){
32     a=new int[n+1];
33 }
34 void del(int*a){
35     delete a;
36 }
37 #define fc(i)(p0[i]+d>n||!p0[i]?0:s[p0[i]+d])
38 int cmp(int*p0,int i,int*s,int n){
39     for(int d=0;d<3;++d)
40         if(fc(i)!=fc(i-1))
41             return 1;
42     return 0;
43 }
44 void sot(int*p0,int n0,int*s,int n,int m,int d){
45     memset(ct,0,(m+1)*4);
46     for(int i=1;i<=n0;++i)
47         ++ct[fc(i)];
48     for(int i=1;i<=m;++i)
49         ct[i]+=ct[i-1];
50     for(int i=n0;i>=1;--i)
51         ts[ct[fc(i)]--]=p0[i];
52     memcpy(p0+1,ts+1,n0*4);
53 }
54 #define fc(d)if(s[i+d]!=s[j+d])return s[i+d]<s[j+d];if(i==n-d||j==n-d)
55 return i==n-d;
56 bool cmp(int*s,int n,int*r,int i,int j){
57     fc(0)
58     if(j%3==1)

```

```

58         return r[i+1]<r[j+1];
59     fc(1)
60     return r[i+2]<r[j+2];
61 }
62 #undef fc
63 void dc3(int*s,int n,int m,int*a,int*r){
64     int n0=n-(n/3)+1,*a0,*s0,i,j=0,k=n/3+bool(n%3)+1,l;
65     crt(s0,n0);
66     s0[k]=1;
67     crt(a0,n0+1);
68     a0[k]=0;
69     for(i=1;i<=n;i+=3)
70         a0[++j]=i,a0[j+k]=i+1;
71     for(i=2;i>=0;--i)
72         sot(a0,n0,s,n,m,i);
73     r[a0[1]]=1;
74     for(i=2;i<=n0;++i)
75         r[a0[i]]=r[a0[i-1]]+cmp(a0,i,s,n);
76     for(i=1,j=0;i<=n;i+=3)
77         s0[++j]=r[i],s0[j+k]=r[i+1];
78     if(r[a0[n0]]==n0){
79         memcpy(r+1,s0+1,n0*4);
80         for(i=1;i<=n0;++i)
81             a0[a[i]=r[i]]=i;
82     }else
83         dc3(s0,n0,r[a0[n0]],a0,a);
84     for(i=1,j=0;i<=n;i+=3)
85         r[i]=a[++j],r[i+1]=a[j+k];
86     j=0;
87     if(n%3==0)
88         s0[++j]=n;
89     for(i=1;i<=n0;++i)
90         if(a0[i]<k){
91             a0[i]=3*a0[i]-2;
92             if(a0[i]!=1)
93                 s0[++j]=a0[i]-1;
94         }else
95             a0[i]=(a0[i]-k)*3-1;
96     sot(s0,j,s,n,m,0);
97     for(i=1,k=2,l=0;i<=j||k<=n0;)
98         if(k>n0||i<=j&&cmp(s,n,r,s0[i],a0[k]))

```

```

99         a[++l]=s0[i++];
100     else
101         a[++l]=a0[k++];
102     for(i=1;i<=n;++i)
103         r[a[i]]=i;
104     del(a0);
105     del(s0);
106 }
107 };

```

6.8 Suffix Array (Prefix-Doubling Algorithm)

Suffix Array (Prefix-Doubling Algorithm).hpp (1357 bytes, 55 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  struct SuffixArray{
4      int*a,*h,*r,*t,*c,n,m;
5      #define lp(u,v)for(int i=u;i<=v;++i)
6      #define rp(u,v)for(int i=u;i>=v;--i)
7      void sort(){
8          memset(c+1,0,m*4);
9          lp(1,n)
10             ++c[r[t[i]]];
11          lp(2,m)
12             c[i]+=c[i-1];
13          rp(n,1)
14             a[c[r[t[i]]]--]=t[i];
15      }
16      SuffixArray(int*s){
17          for(n=m=0;s[n+1];m=max(m,s[+n]));
18          a=new int[4*n+max(n,m)+3];
19          h=a+n;
20          r=h+n+1;
21          t=r+n+1;
22          c=t+n;
23          lp(1,n)
24             t[i]=i,r[i]=s[i];
25          sort();

```

```

26     for(int l=1;l<=n;l<=1,r[a[n]]==n?l=n+1:m=r[a[n]]){
27         t[0]=0;
28         lp(n-l+1,n)
29         t[++t[0]]=i;
30         lp(1,n)
31         if(a[i]>l)
32             t[++t[0]]=a[i]-1;
33         sort();
34         swap(r,t);
35         r[a[1]]=1;
36         lp(2,n)
37         r[a[i]]=r[a[i-1]]+(t[a[i]]!=t[a[i-1]]||a[i]+l>n||a[i-1]+l>n
||t[a[i]+l]!=t[a[i-1]+l]);
38     }
39     int l=0;
40     a[0]=n+1;
41     lp(1,n){
42         if(r[i]==1)
43             l=0;
44             l--=(l>0);
45         int j=a[r[i]-1];
46         for(;s[i+1]==s[j+1];++l);
47         h[r[i]]=l;
48     }
49 }
50 #undef lp
51 #undef rp
52 ~SuffixArray(){
53     delete a;
54 }
55 };

```

6.9 Suffix Array (Treap)

Suffix Array (Treap).hpp (3803 bytes, 147 lines)

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 template<class T>struct SuffixArray{

```



```

4      struct node{
5          node*c[2],*p;
6          T v;
7          int f,s,l,h,m;
8          double t;
9          node(node*_p,T _v,int _l):
10             f(rand()*1.0/RAND_MAX*1e9),p(_p),v(_v),s(1),l(_l),h(0),m(0),t(5
e8){
11             c[0]=c[1]=0;
12         }
13     }*root;
14     vector<T>a;
15     SuffixArray():
16         root(new node(0,0,0)),a(1){
17     }
18     ~SuffixArray(){
19         clear(root);
20     }
21     void relabel(node*x,double l,double r){
22         x->t=(l+r)/2;
23         if(x->c[0])
24             relabel(x->c[0],l,x->t);
25         if(x->c[1])
26             relabel(x->c[1],x->t,r);
27     }
28     void update(node*x){
29         x->s=1;
30         x->m=x->h;
31         for(int i=0;i<2;++i)
32             if(x->c[i])
33                 x->s+=x->c[i]->s,x->m=min(x->m,x->c[i]->m);
34     }
35     void rotate(node*&x,int d){
36         node*y=x->c[d];
37         x->c[d]=y->c[!d];
38         y->c[!d]=x;
39         y->s=x->s;
40         y->m=x->m;
41         update(x);
42         x=y;
43     }

```

```

44 void clear(node*x){
45     if(!x)
46         return;
47     clear(x->c[0]);
48     clear(x->c[1]);
49     delete x;
50 }
51 node*insert(node*&x,node*p,T v,node*l,node*r){
52     int d=x->v!=v?x->v<v:x->p->t<p->t;
53     double tl=l?l->t:0,tr=r?r->t:1e9;
54     node*y;
55     if(d)
56         l=x;
57     else
58         r=x;
59     if(!x->c[d]){
60         y=new node(p,v,p->l+1);
61         y->t=((l?l->t:0)+(r?r->t:1e9))/2;
62         y->m=y->h=l->v==y->v?lcp(l->p,y->p)+1:0;
63         if(r)
64             r->h=r->v==y->v?lcp(r->p,y->p)+1:0;
65         x->c[d]=y;
66     }else
67         y=insert(x->c[d],p,v,l,r);
68     update(x);
69     if(x->c[d]->f>x->f)
70         rotate(x,d),relabel(x,tl,tr);
71     return y;
72 }
73 node*insert(node*p,T v){
74     a.push_back(v);
75     return insert(root,p,v,0,0);
76 }
77 void erase(node*&x,node*y){
78     if(x==y){
79         if(!x->c[0]){
80             x=x->c[1];
81             delete y;
82         }else if(!x->c[1]){
83             x=x->c[0];
84             delete y;

```

```

85         }else{
86             int d=x->c[0]->f<x->c[1]->f;
87             rotate(x,d);
88             erase(x->c[!d],y);
89             --x->s;
90         }
91     }else
92         erase(x->c[x->t<y->t],y),update(x);
93 }
94 void erase(node*y){
95     erase(root,y);
96     a.pop_back();
97 }
98 bool check(node*x,T*y,node*&p,int&l){
99     if(p){
100         int t=x->c[p->t>x->t]?x->c[p->t>x->t]->m:~0u>>1;
101         if(p->t>x->t)
102             t=min(t,p->h);
103         else
104             t=min(t,x->h);
105         if(t<l)
106             return x->t<p->t;
107     }
108     for(p=x;l+1<=x->l&&y[l+1];++l)
109         if(a[x->l-1]!=y[l+1])
110             return a[x->l-1]<y[l+1];
111     return y[l+1]!=0;
112 }
113 int count(node*x,T*y){
114     int r=0,l=0;
115     for(node*p=0;x;){
116         if(check(x,y,p,l))
117             r+=(x->c[0]?x->c[0]->s:0)+1,x=x->c[1];
118         else
119             x=x->c[0];
120     }
121     return r;
122 }
123 int count(T*y){
124     T*t=y;
125     while(*(t+1))
126         ++t;

```

```

126     int r=-count(root,y);
127     ++*t;
128     r+=count(root,y);
129     --*t;
130     return r;
131 }
132 int lcp(node*x,double u,double v,double l,double r){
133     if(v<l||u>r||!x)
134         return ~0u>>1;
135     if(u<l&&v>=r)
136         return x->m;
137     int t=u<x->t&&v>=x->t?x->h:~0u>>1;
138     t=min(t,lcp(x->c[0],u,v,l,x->t));
139     t=min(t,lcp(x->c[1],u,v,x->t,r));
140     return t;
141 }
142 int lcp(node*x,node*y){
143     if(x->t>y->t)
144         swap(x,y);
145     return lcp(root,x->t,y->t,0,1e9);
146 }
147 };

```

6.10 Suffix Automaton

Suffix Automaton.hpp (1694 bytes, 59 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T>struct SuffixAutomaton{
4      struct node{
5          node(vector<node*>&all,int m,node*_pr=0,int _ln=0,T _va=T()):
6              pr(_pr),tr(m),ln(_ln),va(_va){
7              all.push_back(this);
8          }
9          T va;
10         int ln;
11         node*pr;
12         vector<node*>tr;

```

```

13     };
14     SuffixAutomaton(int _m):
15         root(new node(all,m)),m(_m){
16     }
17     ~SuffixAutomaton(){
18         for(int i=0;i<all.size();++i)
19             delete all[i];
20     }
21     node*insert(node*lst,int c,T v){
22         node*p=lst,*np=p->tr[c]?0:new node(all,m,0,lst->ln+1,v);
23         for(;p&&!p->tr[c];p=p->pr)
24             p->tr[c]=np;
25         if(!p)np->pr=root;
26         else{
27             node*q=p->tr[c];
28             if(p==lst)
29                 np=q;
30             if(q->ln==p->ln+1)
31                 p==lst?(q->va+=v):(np->pr=q,0);
32             else{
33                 node*nq=new node(all,m,q->pr,p->ln+1,p==lst?v:T());
34                 nq->tr=q->tr;
35                 q->pr=np->pr=nq;
36                 if(p==lst)
37                     np=nq;
38                 for(;p&&p->tr[c]==q;p=p->pr)
39                     p->tr[c]=nq;
40             }
41         }
42         return np;
43     }
44     void count(){
45         vector<int>cnt(all.size());
46         vector<node*>tmp=all;
47         for(int i=0;i<tmp.size();++i)
48             ++cnt[tmp[i]->ln];
49         for(int i=1;i<cnt.size();++i)
50             cnt[i]+=cnt[i-1];
51         for(int i=0;i<tmp.size();++i)
52             all[--cnt[tmp[i]->ln]]=tmp[i];
53         for(int i=int(all.size())-1;i>0;--i)

```

```

54         all[i]→pr→va+=all[i]→va;
55     }
56     int m;
57     node*root;
58     vector<node*>all;
59 };

```

6.11 Suffix Tree

Suffix Tree.hpp (2899 bytes, 116 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  template<class T=char,int N=int(1e5),int M=27,T D='a'-1>struct SuffixTree{
4      struct node;
5      struct edge{
6          edge():
7              l(0),r(0),t(0){
8              }
9          int length(){
10             return r-l;
11         }
12         T*l,*r;
13         node*t;
14     }pe[2*N],*ep=pe;
15     edge*newedge(T*l,T*r,node*t){
16         ep→l=l;
17         ep→r=r;
18         ep→t=t;
19         return ep++;
20     }
21     struct node{
22         node():
23             s(0),c({0}){
24             }
25         node*s;
26         edge*c[M];
27     }pn[2*N+1],*np=pn;
28     SuffixTree():

```

```

29     root(np++),ct(0){
30 }
31 void extend(T*s){
32     for(;ae&&al>=ae->length();){
33         s+=ae->length();
34         al-=ae->length();
35         an=ae->t;
36         ae=al?an->c[*s-D]:0;
37     }
38 }
39 bool extend(int c){
40     if(ae){
41         if(*(ae->l+al)-D-c)
42             return true;
43         ++al;
44     }else{
45         if(!an->c[c])
46             return true;
47         ae=an->c[c];
48         al=1;
49         if(pr)
50             pr->s=an;
51     }
52     extend(ae->l);
53     return false;
54 }
55 void insert(T*s,int n){
56     ct+=n;
57     an=root;
58     ae=0;
59     al=0;
60     for(T*p=s;p!=s+n;++p)
61         for(pr=0;extend(*p-D);){
62             edge*x=newedge(p,s+n,np++);
63             if(!ae)
64                 an->c[*p-D]=x;
65             else{
66                 edge*&y=an->c[*ae->l-D];
67                 y=newedge(ae->l,ae->l+al,np++);
68                 y->t->c[* (ae->l+al)-D]=ae;
69                 y->t->c[*p-D]=x;

```

```

70         ae=y;
71     }
72     if(pr)
73         pr->s=ae?ae->t:an;
74     pr=ae?ae->t:an;
75     int r=1;
76     if(an==root&&!a1)
77         break;
78     if(an==root)
79         --a1;
80     else{
81         an=an->s?an->s:root;
82         r=0;
83     }
84     if(a1){
85         T*t=ae->l+(an==root)*r;
86         ae=an->c[*t-D];
87         extend(t);
88     }else
89         ae=0;
90 }
91
92 void build(node*u=0,int d=0){
93     if(!u)
94         u=root;
95     int t=0,s=0;
96     for(int i=0;i<M;++i)
97         if(u->c[i]){
98             if(!t)
99                 t=1;
100             else if(!s){
101                 s=1;
102                 *sp++=d;
103             }
104             build(u->c[i]->t,d+u->c[i]->length());
105         }
106     if(s)
107         --sp;
108     else if(!t&&sp!=sk){
109         *hp++=(sp-1);
110         *fp++=ct-d+1;

```



```
111     }
112   }
113   edge*ae;
114   node*root,*an,*pr;
115   int al,ct,sk[N],*sp=sk,height[N],*hp=height,suffix[N],*fp=suffix;
116 };
```


CHAPTER 7

Utility Tools

7.1 Checker

Checker.bat (166 bytes, 7 lines)

```
1 :again
2 generator > input.txt
3 program1 < input.txt > output1.txt
4 program2 < input.txt > output2.txt
5 fc output1.txt output2.txt
6 if errorlevel 1 pause
7 goto again
```

7.2 Date

Date.hpp (3596 bytes, 145 lines)

```
1 #include<bits/stdc++.h>
2 using namespace std;
3 struct Date{
4     int y,m,d,w;
5     Date&operator++(){
6         return*this=*this+1;
7     }
8     bool leap(int a)const{
9         return a%400==0|| (a%4==0&&a%100!=0);
10    }
11    int month_sum(int a,int b)const{
12        if(b==0)
13            return 0;
14        if(b==1)
15            return 31;
16        return 59+leap(a)+30*(b-2)+(b+1)/2-1+(b>=8&&b%2==0);
17    }
18    string month_name(int a)const{
19        if(a==1)
20            return"January";
21        if(a==2)
22            return"February";
23        if(a==3)
```

```
24         return "March";
25     if(a==4)
26         return "April";
27     if(a==5)
28         return "May";
29     if(a==6)
30         return "June";
31     if(a==7)
32         return "July";
33     if(a==8)
34         return "August";
35     if(a==9)
36         return "September";
37     if(a==10)
38         return "October";
39     if(a==11)
40         return "November";
41     if(a==12)
42         return "December";
43 }
44 string day_name(int a) const {
45     if(a==0)
46         return "Sunday";
47     if(a==1)
48         return "Monday";
49     if(a==2)
50         return "Tuesday";
51     if(a==3)
52         return "Wednesday";
53     if(a==4)
54         return "Thursday";
55     if(a==5)
56         return "Friday";
57     if(a==6)
58         return "Saturday";
59 }
60 operator int() const {
61     int t=(y-1)*365+(y-1)/4-(y-1)/100+(y-1)/400+month_sum(y,m-1)+d;
62     if(y==1752&& m>=9&& d>2 || y>1752)
63         t-=11;
64     t-=min(y-1,1700)/400-min(y-1,1700)/100;
```

```

65         if(y<=1700&&y%400!=0&&y%100==0&&m>2)
66             ++t;
67         return t;
68     }
69     Date(int _y,int _m,int _d):
70         y(_y),m(_m),d(_d),w((int)(*this)+5)%7){
71     }
72     Date(int a){
73         int y1=0,yr=1e7;
74         while(y1+1<yr){
75             int ym=(y1+yr)/2;
76             if(int(Date(ym,12,31))<a)
77                 y1=ym;
78             else
79                 yr=ym;
80         }
81         y=yr;
82         int m1=0,mr=12;
83         while(m1+1<mr){
84             int mm=(m1+mr)/2,mt;
85             if(mm==2){
86                 if(y<=1700)
87                     mt=28+(y%4==0);
88                 else
89                     mt=28+(y%4==0&&y%100!=0 | y%400==0);
90             }else if(mm<=7)
91                 mt=30+mm%2;
92             else
93                 mt=31-mm%2;
94             if(int(Date(y,mm,mt))<a)
95                 m1=mm;
96             else
97                 mr=mm;
98         }
99         m=mr;
100         for(int i=1; ; ++i){
101             if(y==1752&&m==9&&i>2&&i<14)
102                 continue;
103             if(int(Date(y,m,i))==a){
104                 d=i;
105                 break;

```

```

106         }
107     }
108     w=(5+a)%7;
109 }
110 operator string()const{
111     stringstream s;
112     string t;
113     s<<day_name(w)+", "+month_name(m)+" "<<d<<"", "<<y;
114     getline(s,t);
115     return t;
116 }
117 };
118 ostream&operator<<(ostream&s,const Date&a){
119     return s<<string(a);
120 }
121 int operator-(const Date&a,const Date&b){
122     return int(a)-int(b);
123 }
124 Date operator+(const Date&a,int b){
125     return Date(int(a)+b);
126 }
127 Date operator-(const Date&a,int b){
128     return Date(int(a)-b);
129 }
130 bool operator<(const Date&a,const Date&b){
131     if(a.y==b.y&&a.m==b.m)
132         return a.d<b.d;
133     if(a.y==b.y)
134         return a.m<b.m;
135     return a.y<b.y;
136 }
137 bool operator>(const Date&a,const Date&b){
138     return b<a;
139 }
140 bool operator!=(const Date&a,const Date&b){
141     return a.y!=b.y||a.m!=b.m||a.d!=b.d;
142 }
143 bool operator==(const Date&a,const Date&b){
144     return !(a!=b);
145 }

```

7.3 Fast Reader

Fast Reader.hpp (1251 bytes, 61 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  struct FastReader{
4      FILE*f;
5      char*p,*e;
6      vector<char>v;
7      void ipt(){
8          for(int i=1,t;;i<=1){
9              v.resize(v.size()+i);
10             if(i!=(t=fread(&v[0]+v.size()-i,1,i,f))){
11                 p=&v[0],e=p+v.size()-i+t;
12                 break;
13             }
14         }
15     }
16     void ign(){
17         while(p!=e&&isspace(*p))
18             ++p;
19     }
20     int isc(){
21         return p!=e&&!isspace(*p);
22     }
23     int isd(){
24         return p!=e&&isdigit(*p);
25     }
26     FastReader(FILE*_f):
27         f(_f){
28             ipt();
29         }
30     FastReader(string _f):
31         f(fopen(_f.c_str(),"r")){
32             ipt();
33         }
34     ~FastReader(){
35         fclose(f);
36     }
37     template<class T>FastReader&operator>>(T&a){

```



```

38     int n=1;
39     ign();
40     if(*p=='-')
41         n=-1,++p;
42     for(a=0;isd();)
43         a=a*10+*p++-'0';
44     a*=n;
45     return*this;
46 }
47 FastReader&operator>>(char&a){
48     ign();
49     a=*p++;
50     return*this;
51 }
52 FastReader&operator>>(char*a){
53     for(ign();isc();)
54         *a++=*p++;
55     *a=0;
56     return*this;
57 }
58 char get(){
59     return*p++;
60 }
61 };

```

7.4 Fast Writer

Fast Writer.hpp (866 bytes, 39 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  struct FastWriter{
4      FILE*f;
5      vector<char>p;
6      FastWriter(FILE*_f):
7          f(_f){
8      }
9      FastWriter(string _f):
10         f(fopen(_f.c_str(),"w")){

```

```

11     }
12     ~FastWriter(){
13         if(p.size())
14             fwrite(&p[0],1,p.size(),f);
15         fclose(f);
16     }
17     FastWriter&operator<<(char a){
18         p.push_back(a);
19         return*this;
20     }
21     FastWriter&operator<<(const char*a){
22         while(*a)
23             p.push_back(*a++);
24         return*this;
25     }
26     template<class T>FastWriter&operator<<(T a){
27         if(a<0)
28             p.push_back('-'),a=-a;
29         static char t[19];
30         char*q=t;
31         do{
32             T b=a/10;
33             *q++=a-b*10+'0',a=b;
34         }while(a);
35         while(q>t)
36             p.push_back(*--q);
37         return*this;
38     }
39 };

```

7.5 Number Speller

Number Speller.hpp (2143 bytes, 72 lines)

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 namespace NumberSpeller{
4     template<class T>string run(T a){
5         map<T,string>m;

```

```
6         m[0]="zero";
7         m[1]="one";
8         m[2]="two";
9         m[3]="three";
10        m[4]="four";
11        m[5]="five";
12        m[6]="six";
13        m[7]="seven";
14        m[8]="eight";
15        m[9]="nine";
16        m[10]="ten";
17        m[11]="eleven";
18        m[12]="twelve";
19        m[13]="thirteen";
20        m[14]="fourteen";
21        m[15]="fifteen";
22        m[16]="sixteen";
23        m[17]="seventeen";
24        m[18]="eighteen";
25        m[19]="nineteen";
26        m[20]="twenty";
27        m[30]="thirty";
28        m[40]="forty";
29        m[50]="fifty";
30        m[60]="sixty";
31        m[70]="seventy";
32        m[80]="eighty";
33        m[90]="ninety";
34        if(a<0)
35            return"minus "+run(-a);
36        if(m.count(a))
37            return m[a];
38        if(a<100)
39            return run(a/10*10)+"-"+run(a%10);
40        if(a<1000&&a%100==0)
41            return run(a/100)+" hundred";
42        if(a<1000)
43            return run(a/100*100)+" and "+run(a%100);
44        vector<string>t;
45        t.push_back("thousand");
46        t.push_back("million");
```

```

47     t.push_back("billion");
48     t.push_back("trillion");
49     t.push_back("quadrillion");
50     t.push_back("quintillion");
51     t.push_back("sextillion");
52     t.push_back("septillion");
53     t.push_back("octillion");
54     t.push_back("nonillion");
55     t.push_back("decillion");
56     t.push_back("undecillion");
57     t.push_back("duodecillion");
58     t.push_back("tredecillion");
59     t.push_back("quattuordecillion");
60     t.push_back("quindecillion");
61     string r=a%1000?run(a%1000):"";
62     a/=1000;
63     for(int i=0;a;++i,a/=1000)
64         if(a%1000){
65             if(!i&&r.find("and")==string::npos&&r.find("hundred")==
string::npos&&r.size())
66                 r=run(a%1000)+" "+t[i]+" and "+r;
67             else
68                 r=run(a%1000)+" "+t[i]+(r.size()?"," : "")+r;
69         }
70     return r;
71 }
72 }

```

7.6 Utility

Utility.hpp (4146 bytes, 167 lines)

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  typedef long long ll;
4  typedef long double ld;
5  #define lp(i,a,b)for(auto i=a;i<=b;++i)
6  void set_read(string a){
7      freopen(a.c_str(),"r",stdin);

```

```

8 }
9 void set_write(string a){
10     freopen(a.c_str(),"w",stdout);
11 }
12 template<class T>void write(T a){
13     cout<<a;
14 }
15 template<class T>void writeln(T a){
16     cout<<a<<endl;
17 }
18 template<class T>T read(){
19     T a;
20     cin>>a;
21     return a;
22 }
23 void sync(int t){
24     ios::sync_with_stdio(t);
25 }
26 template<class T1,class T2>T2 convert(T1 a){
27     stringstream s;
28     s<<a;
29     T2 t;
30     s>>t;
31     return t;
32 }
33
34 template<class T1,class T2>ostream&operator<<(ostream&s,const pair<T1,T2>&a
35 ){
36     return s<<a.first<<" "<<a.second;
37 }
38 template<class T1,class T2,class T3>struct triple{
39     triple(T1 a,T2 b,T3 c):
40         first(a),second(b),third(c){
41     }
42     T1 first;
43     T2 second;
44     T3 third;
45 };
46 template<class T1,class T2,class T3>ostream&operator<<(ostream&s,const
47     triple<T1,T2,T3>&a){
48     return s<<a.first<<" "<<a.second<<" "<<a.third;

```

```

47 }
48 template<class T1,class T2,class T3>triple<T1,T2,T3>make_triple(T1 a,T2 b,
    T3 c){
49     return triple<T1,T2,T3>(a,b,c);
50 }
51 template<class T>T sum(triple<T,T,T>a){
52     return a.first+a.second+a.third;
53 }
54 template<class T>T product(triple<T,T,T>a){
55     return a.first*a.second*a.third;
56 }
57 template<class T>vector<T>sort(vector<T>a){
58     sort(a.begin(),a.end());
59     return a;
60 }
61 template<class T,class F>vector<T>foreach(vector<T>a,F f){
62     for(int i=0;i<a.size();++i)
63         f(a[i]);
64     return a;
65 }
66 template<class T>T sum(const vector<T>&a){
67     T r=0;
68     for(int i=0;i<a.size();++i)
69         r+=a[i];
70     return r;
71 }
72 template<class T>T sum(const set<T>&a){
73     T r=0;
74     for(typename set<T>::iterator i=a.begin();i!=a.end();++i)
75         r+=*i;
76     return r;
77 }
78 template<class T>ostream&operator<<(ostream&s,const vector<T>&a){
79     for(int i=0;i<a.size();++i){
80         if(i)
81             cout<<' ';
82         cout<<a[i];
83     }
84     return s;
85 }
86 template<class T>vector<T>unique(vector<T>a){

```

```
87     sort(a.begin(),a.end());
88     a.erase(unique(a.begin(),a.end()),a.end());
89     return a;
90 }
91 template<class T>vector<T>combination(T a){
92     vector<T>r;
93     for(int i=0;i<(1<<a.size());++i){
94         T t;
95         for(int j=0;j<a.size();++j)
96             if((i>>j)&1)
97                 t.push_back(a[j]);
98         r.push_back(t);
99     }
100    return r;
101 }
102 template<class T>vector<T>permutation(T a){
103     sort(a.begin(),a.end());
104     vector<T>r;
105     for(int i=0;i<(1<<a.size());++i){
106         T t;
107         for(int j=0;j<a.size();++j)
108             if((i>>j)&1)
109                 t.push_back(a[j]);
110         do{
111             r.push_back(t);
112         }while(next_permutation(t.begin(),t.end()));
113     }
114     return r;
115 }
116 template<class T>vector<T>permutation(T a,int b){
117     sort(a.begin(),a.end());
118     vector<T>r;
119     for(int i=0;i<(1<<a.size());++i){
120         T t;
121         for(int j=0;j<a.size();++j)
122             if((i>>j)&1)
123                 t.push_back(a[j]);
124         if(t.size()!=b)
125             continue;
126         do{
127             r.push_back(t);
```

```

128         }while(next_permutation(t.begin(),t.end()));
129     }
130     return r;
131 }
132 template<class T>set<T>operator+(set<T>a,set<T>b){
133     for(typename set<T>::iterator i=a.begin();i!=a.end();++i)
134         b.insert(*i);
135     return b;
136 }
137 template<class T>vector<T>operator+(vector<T>a,vector<T>b){
138     for(int i=0;i<b.size();++i)
139         a.push_back(b[i]);
140     return a;
141 }
142 template<class T>set<T>operator&(set<T>a,set<T>b){
143     set<T>c;
144     for(typename set<T>::iterator i=a.begin();i!=a.end();++i)
145         if(b.count(*i))
146             c.insert(*i);
147     return c;
148 }
149 string str(int a){
150     return convert<int,string>(a);
151 }
152 string str(ll a){
153     return convert<ll,string>(a);
154 }
155 int to_int(string a){
156     return convert<string,int>(a);
157 }
158 ll to_ll(string a){
159     return convert<string,ll>(a);
160 }
161 template<class T,class F>vector<T>delete_if(vector<T>a,F b){
162     vector<T>c;
163     for(int i=0;i<a.size();++i)
164         if(!b(a[i]))
165             c.push_back(a[i]);
166     return c;
167 }

```