Diffen

PROJEKT 2B

U2. Härledningar:

a) $u_t = du_{xx} + g(x,t)$ (1) , $0 \leq x \leq 1$, $t \geq 0$

$u(x,0) = f(x)$

$$u^N(x,t) = \sum_{k=-N/2}^{N/2-1} \hat{u}_k(t) e^{2\pi ikx} , \quad g^N(x,t) = \sum_{k=-N/2}^{N/2-1} \hat{g}_k(t) e^{2\pi ik}$$

$$u_{xx}(x,t) = \sum_{k=-N/2}^{N/2-1} \hat{u}_k (2\pi ik)^2 e^{2\pi ikx} = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} -\hat{u}_k (2\pi k)^2 e^{2\pi ikx}$$

$$u_t(x,t) = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{u}_k'(t) e^{2\pi ikx}$$

Givet ekvation (1) får vi nu:

$$\sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{u}_k'(t) e^{2\pi ikx} = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} -\hat{u}_k (2\pi k)^2 d e^{2\pi ikx} + \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{g}_k(t) e^{2\pi ikx}$$

$$\Rightarrow \quad \hat{u}_k' = -(2\pi k)^2 d \hat{u}_k + \hat{g}_k$$

b) $g = 0 \Rightarrow \hat{u}_k' + (2\pi k)^2 d \hat{u}_k = 0 \Rightarrow \hat{u}_k(t) = c e^{-(2\pi k)^2 d \cdot t}$ (2)

Vi har att $f^N(x) = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{f}_k e^{2\pi ikx}$, $u(x,0) = f(x)$, $u(0,t) = u(1,t)$

För godtyckligt definierat t , för $t = 0 \Rightarrow$

$$\Rightarrow u(x,0) = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{u}_k(0) \cdot e^{2\pi ikx} = \sum_{k=-N/2}^{N/2-1} \hat{f}_k e^{2\pi ikx} \Rightarrow$$

$\Rightarrow \hat{u}_k(0) = \hat{f}_k$, insättes i (2) $\Rightarrow \hat{u}_k(0) = c \cdot e^{-(2\pi k)^2 d \cdot 0} = \hat{f}_k \Rightarrow$

$\Rightarrow c = \hat{f}_k \Rightarrow$ $\hat{u}_k(t) = \hat{f}_k \cdot e^{-(2\pi k)^2 dt}$

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b) Insättes i ekvation (5) enligt uppgiftsbeskrivningen:

$$u^N(x,t) = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{f}_k e^{-(2\pi k)^2 dt} \cdot e^{2\pi i k x} = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{f}_k \cdot e^{-(2\pi k) i dt x}$$