Exercise: Reject option in classifiers

(Source: (**?**, Q2.13).)

In many classification problems one has the option either of assigning $\mathbf{x}$ to class j or, if you are too uncertain, of choosing the **reject option**. If the cost for rejects is less than the cost of falsely classifying the object, it may be the optimal action. Let α_i mean you choose action i, for $i = 1 : C + 1$, where C is the number of classes and $C + 1$ is the reject action. Let $Y = j$ be the true (but unknown) **state of nature**. Define the loss function as follows

$$\lambda(\alpha_i|Y = j) = \begin{cases} 0 & \text{if } i = j \text{ and } i, j \in \{1, \ldots, C\} \\ \lambda_r & \text{if } i = C + 1 \\ \lambda_s & \text{otherwise} \end{cases} \tag{1}$$

In otherwords, you incur 0 loss if you correctly classify, you incur λ_r loss (cost) if you choose the reject option, and you incur λ_s loss (cost) if you make a substitution error (misclassification).

1. Show that the minimum risk is obtained if we decide $Y = j$ if $p(Y = j|\mathbf{x}) \geq p(Y = k|\mathbf{x})$ for all k (i.e., j is the most probable class) *and* if $p(Y = j|\mathbf{x}) \geq 1 - \frac{\lambda_r}{\lambda_s}$; otherwise we decide to reject.

2. Describe qualitatively what happens as λ_r/λ_s is increased from 0 to 1 (i.e., the relative cost of rejection increases).