Solution: MLE for the univariate Gaussian

Starting with the mean, we have

$$\begin{aligned}
\frac{\partial \ell}{\partial \mu} &= \frac{2}{2\sigma^2} \sum_i (x_i - \mu) = 0 && (3) \\
\hat{\mu} &= \frac{1}{N} \sum_{i=1}^{N} x_i = \overline{x} && (4)
\end{aligned}$$

We can derive $\hat{\sigma}^2$ as follows.

$$\begin{aligned}
\frac{\partial \ell}{\partial \sigma^2} &= \frac{1}{2}\sigma^{-4} \sum_i (x_i - \hat{\mu})^2 - \frac{n}{2\sigma^2} = 0 && (5) \\
\hat{\sigma}^2 &= \frac{1}{N} \sum_{i=1}^{N} (x_i - \hat{\mu})^2 && (6) \\
&= \frac{1}{N} \left[\sum_i x_i^2 + \sum_i \hat{\mu}^2 - 2 \sum_i x_i \hat{\mu} \right] = \frac{1}{N} [\sum_i x_i^2 + N\hat{\mu}^2 - 2N\hat{\mu}^2] && (7) \\
&= \left(\frac{1}{N} \sum_{i=1}^{N} x_i^2 \right) - (\overline{x})^2 && (8)
\end{aligned}$$