Homework

Group 76

$$\begin{aligned}\text{class}^* &= \underset{h\in\{0,1\}}{\operatorname{argmax}}\, p(\text{class}=h|x_{new}) = \underset{h\in\{0,1\}}{\operatorname{argmax}}\, \frac{p(x_{new}|\,\text{class}=h)p(\text{class}=h)}{p(x_{new})} \\ &= \underset{h\in\{0,1\}}{\operatorname{argmax}}\, p(y_1,y_2,y_3,y_4|\,\text{class}=h)p(\text{class}=h) \\ &= \underset{h\in\{0,1\}}{\operatorname{argmax}}\, p(y_1|\,\text{class}=h)p(y_2|\,\text{class}=h)p(y_3,y_4|\,\text{class}=h)p(\text{class}=h) \end{aligned} \tag{1}$$

$$\mu_{y_1|\,\text{class}=1} = \frac{\sum_{i=1}^n y_{1i}[\text{class}_i=1]}{\sum_{i=1}^n [\text{class}_i=1]} = 0.05 \tag{2}$$

$$p(y_2|\,\text{class}) = p_A^{[y_2=A]}p_B^{[y_2=B]}p_C^{[y_2=C]}$$

$$y_1|\,\text{class} \sim \mathcal{N}(\mu_{y_1|\,\text{class}}, \sigma^2_{y_1|\,\text{class}}) \tag{3}$$

$$y_3,y_4|\,\text{class} \sim \mathcal{N}(\mu_{y_3,y_4|\,\text{class}}, \Sigma_{y_3,y_4|\,\text{class}}) \tag{4}$$

$$\sigma^2_{y1|\,\text{class}=1} = \frac{1}{(\sum_{i=1}^n [\text{class}_i=1])-1}\sum_{i=1}^n \Big(\big(y_{1i}-\mu_{y1|\,\text{class}=0}\big)^2\cdot[\text{class}=1]\Big) = 0.083 \tag{5}$$

$$\mu_{y_3,y_4|\,\text{class}=1} = \frac{\sum_{i=1}^n \begin{bmatrix}y_{3i}\\y_{4i}\end{bmatrix}[\text{class}_i=1]}{\sum_{i=1}^n [\text{class}_i=1]} = \begin{bmatrix}0.117\\0.083\end{bmatrix} \tag{6}$$

$$\Sigma_{y_3,y_4|\,\text{class}=0} = \frac{1}{(\sum_{i=1}^n [\text{class}_i=0])-1}\sum_{i=1}^n \left(\begin{bmatrix}y_{3i}\\y_{4i}\end{bmatrix}-\mu_{y_3,y_4|\,\text{class}=0}\right)\left(\begin{bmatrix}y_{3i}\\y_{4i}\end{bmatrix}-\mu_{y_3,y_4|\,\text{class}=0}\right)^T[\text{class}_i=0] = \begin{bmatrix}0.18 & 0.18\\0.18 & 0.25\end{bmatrix}$$

where $\mu_{y_3,y_4|\,\text{class}} \in \mathbb{R}^2$ and $\Sigma_{y_3,y_4|\,\text{class}} \in (\mathbb{R}^+)^{2\times 2}$

$$p_{C|\,\text{class}=1} = \frac{\sum_{i=1}^n [y_{2i}=C][\text{class}_i=1]}{\sum_{i=1}^n [\text{class}_i=1]} = 0.5$$

$$p_{C|\,\text{class}=0} = 1 - p_{A|\,\text{class}=0} - p_{B|\,\text{class}=0} = 0.25$$

$$p_{C|\,\text{class}=1} = 1 - p_{A|\,\text{class}=1} - p_{B|\,\text{class}=1} = 0.5$$

$$p_1 = \frac{\sum_{i=1}^n [\text{class}_i=1]}{n} = 0.6$$

$$p_1 = 1 - p_0 = 0.6$$

$$p(x_1,\text{class}=1) = p(y_{11}|\,\text{class}=1)p(y_{21}|\,\text{class}=1)p(y_{31},y_{41}|\,\text{class}=1)p(\text{class}=1) = 0.2239\cdot 0.167\cdot 1.2119\cdot 0.6 = 0.027$$

$$p(x_{10},\text{class}=0) = 0.2807\cdot 0.25\cdot 1.0804\cdot 0.4 = 0.030 \qquad p(x_{10},\text{class}=1) = 0.9503\cdot 0.5\cdot 1.1252\cdot 0.6 = 0.321$$

$$\text{Sensitivity} = \frac{\text{TP}}{\text{TP} + \text{FN}} = \frac{5}{5+1} = 0.833 \quad \text{Precision} = \frac{\text{TP}}{\text{TP} + \text{FP}} = \frac{5}{5+2} = 0.714$$

$$\text{F1 Score} = \frac{2 \cdot \text{Sensitivity} \cdot \text{Precision}}{\text{Sensitivity} + \text{Precision}} = \frac{2 \cdot 0.714 \cdot 0.833}{0.833 + 0.714} = 0.769$$

$$p(\text{class} = 1 | x_i) = \frac{p(\text{class} = 1, x_i)}{p(\text{class} = 0, x_i) + p(\text{class} = 1, x_i)}$$

k	0	1	2	3	4	5	6	7	8	9	10
Accuracy	0.6	0.7	0.8	0.7	0.6	0.7	0.6	0.7	0.6	0.5	0.4

p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
0.165	0.805	0.241	0.541	0.544	0.928	0.936	0.533	0.301	0.914

$$\mathcal{N}(y3, y4 | \,\text{class}) = \frac{1}{(2\pi)^{m/2} |\Sigma_{y_3, y_4 | \,\text{class}}|^{1/2}} \exp\left(\frac{1}{2} \left(\begin{bmatrix} y_3 \\ y_4 \end{bmatrix} - \mu_{y_3, y_4 | \,\text{class}} \right) \Sigma^{-1}_{y_3, y_4 | \,\text{class}} \left(\begin{bmatrix} y_3 \\ y_4 \end{bmatrix} - \mu_{y_3, y_4 | \,\text{class}} \right)^T \right)$$

$$\begin{aligned} \Sigma^{-1}_{y_3, y_4 | \,\text{class}=0} &= \frac{1}{|\Sigma_{y_3, y_4 | \,\text{class}=0}|} \text{adj}\left(\Sigma^{-1}_{y_3, y_4 | \,\text{class}=0} \right) = \frac{1}{0.18 \cdot 0.25 - 0.18 \cdot 0.18} \begin{bmatrix} 0.25 & -0.18 \\ -0.18 & 0.18 \end{bmatrix} \\ &= \frac{1}{0.0126} \begin{bmatrix} 0.25 & -0.18 \\ -0.18 & 0.18 \end{bmatrix} = \begin{bmatrix} 19.841 & -14.286 \\ -14.286 & 14.286 \end{bmatrix} \end{aligned}$$

$$\Sigma^{-1}_{y_3, y_4 | \,\text{class}=1} = \frac{1}{0.0087} \begin{bmatrix} 0.214 & 0.122 \\ 0.122 & 0.110 \end{bmatrix} = \begin{bmatrix} 24.723 & -14.094 \\ -14.094 & 12.708 \end{bmatrix}$$

$$\text{Specificity} = \frac{\text{TN}}{\text{TN} + \text{FP}} = \frac{2}{2+2} = 0.5$$