Optimization and Algorithms Project

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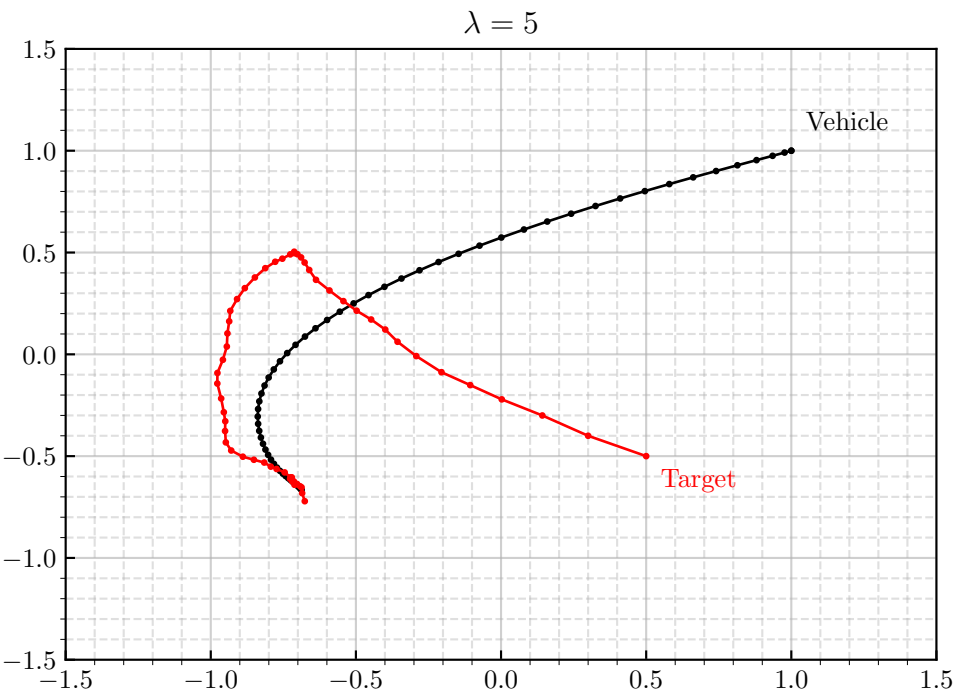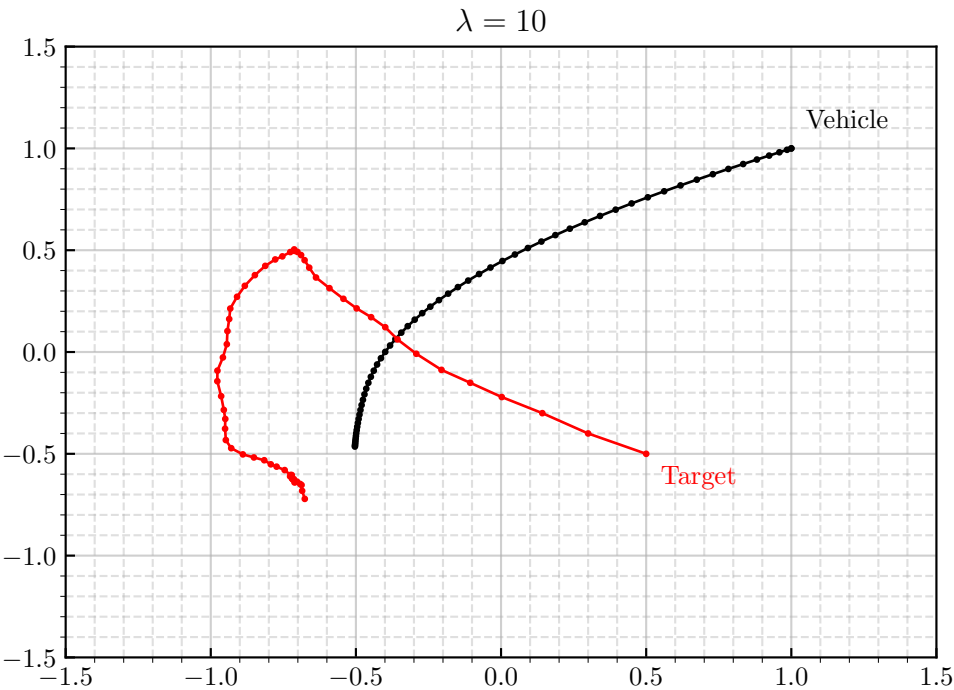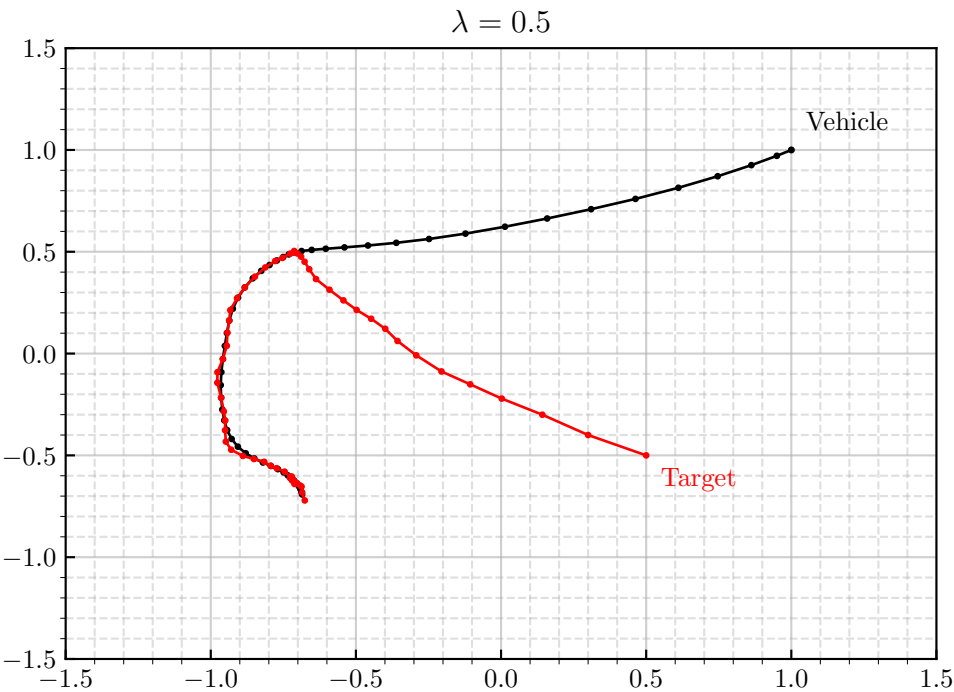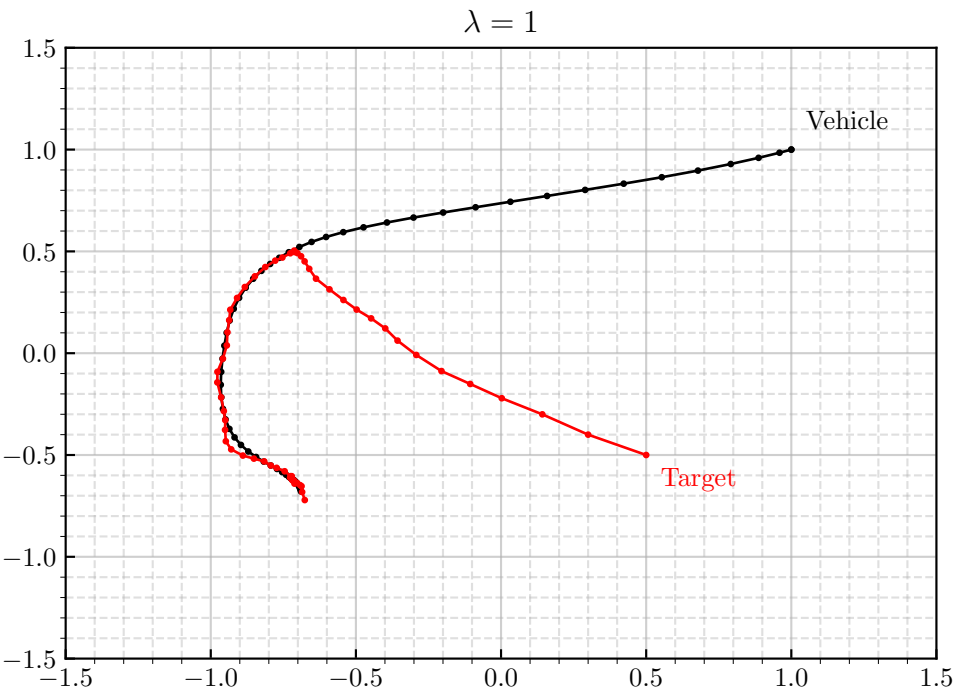João De Assis Marcos Soares Nabais (965259)

Group 8

2022/2023 – 1º Semester, P1

Contents

1 Task 1

$\lambda = 10$

$\lambda = 5$

$\lambda = 1$

$\lambda = 0.5$

$\lambda = 0.1$

$\lambda = 0.05$

$\lambda = 0.01$

$\lambda = 0.005$

$$\lambda = 0.001$$
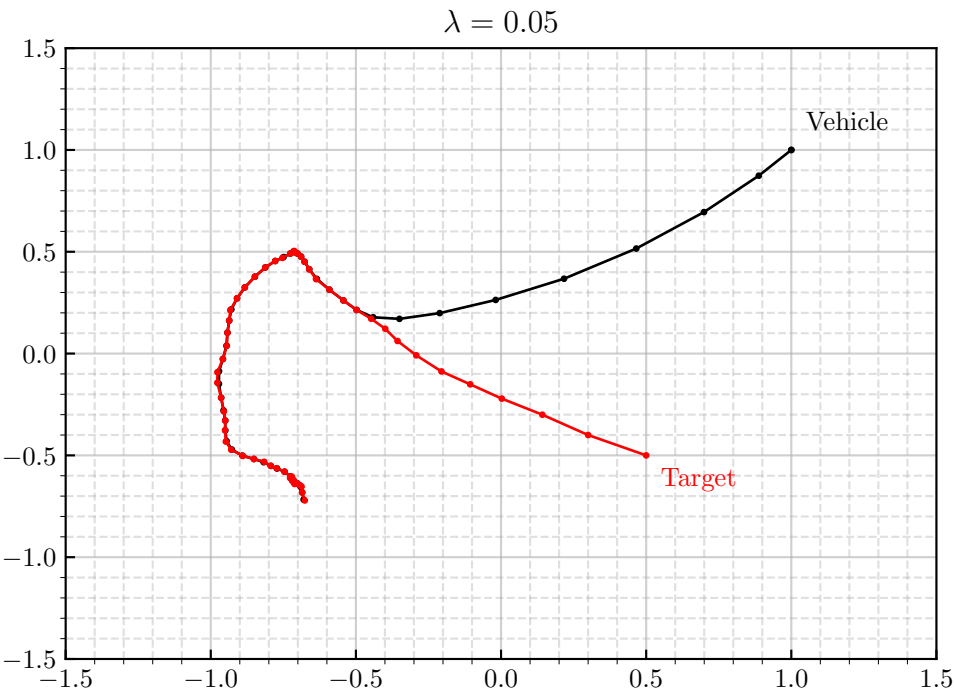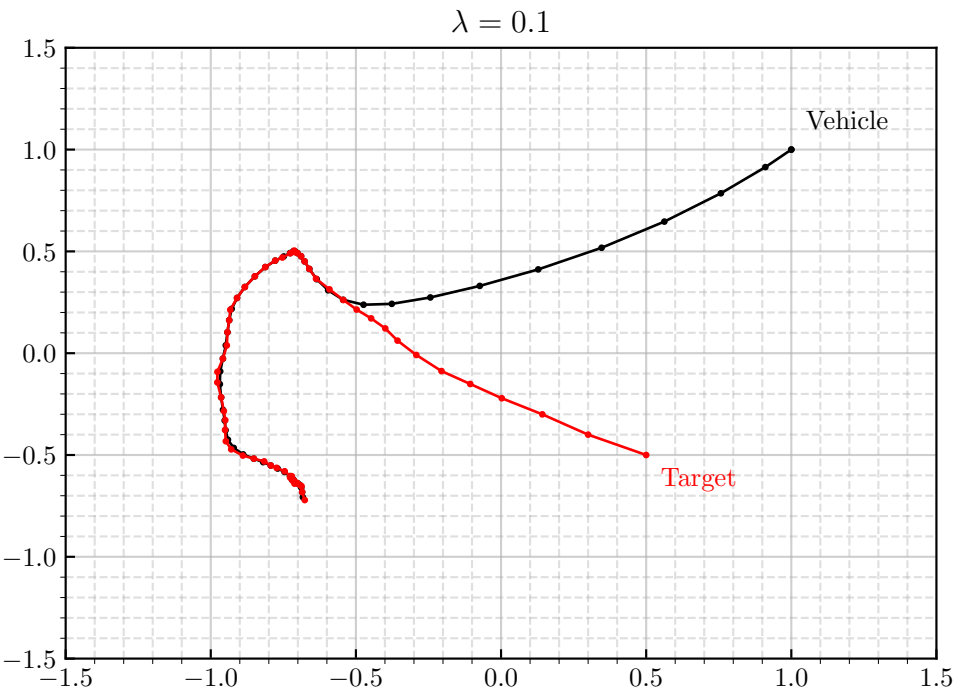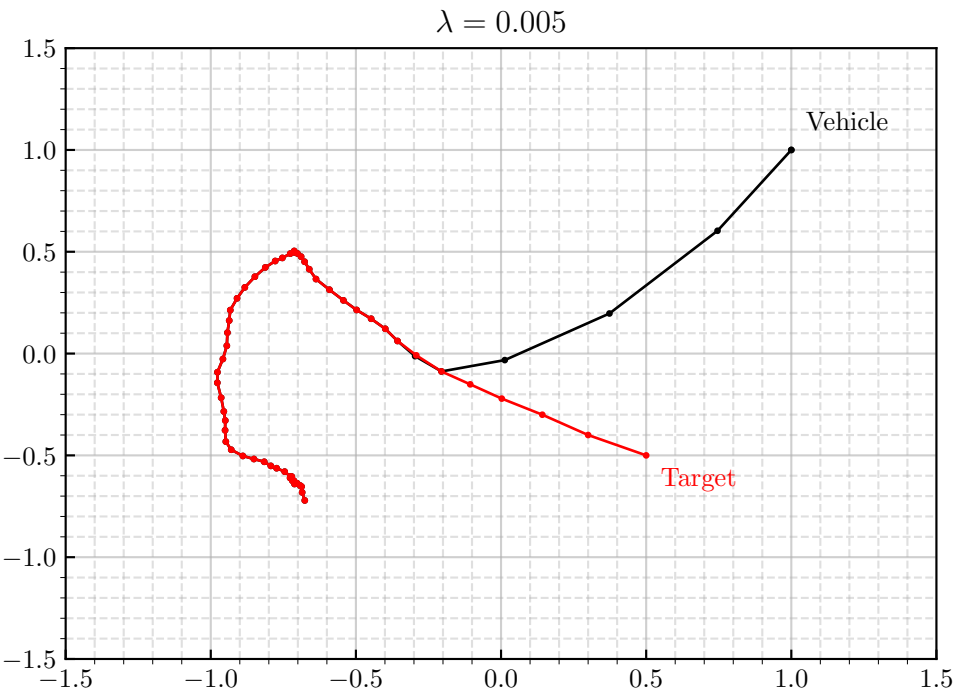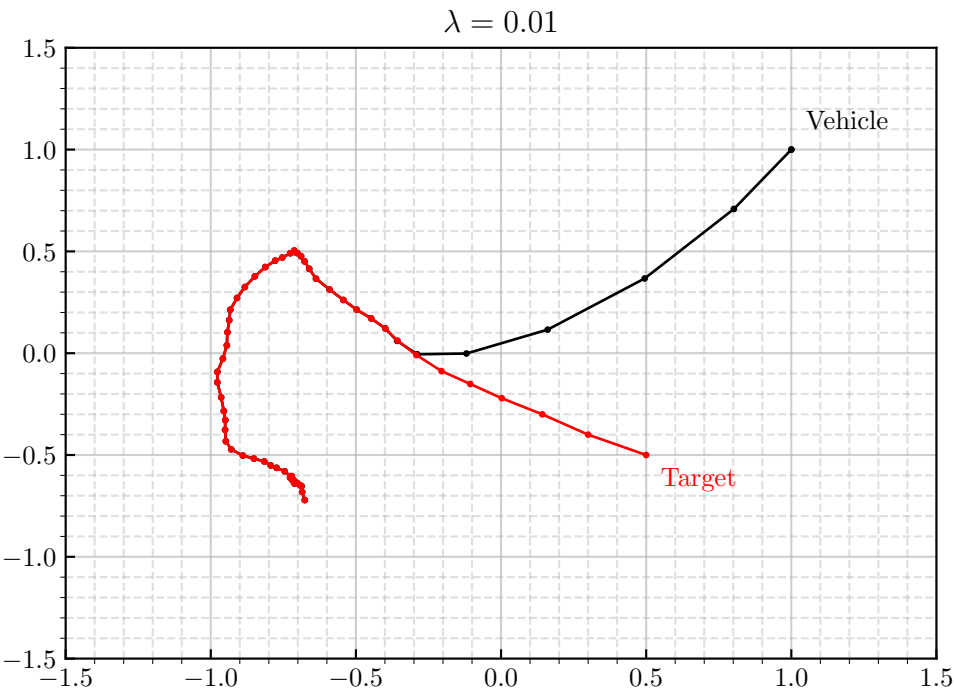
When the parameter λ increases, the Tracking Error and Control Effort associated with the optimal solution of the resulting optimization problem increases and decreases, respectively.

2   Task 2

Let (x_a, u_a) and (x_b, u_b) denote the minimizers obtained after solving the given optimization problem for $\lambda = \lambda_a$ and $\lambda = \lambda_b$, respectively. Let $\text{TE}(x, u)$ and $\text{CE}(x, u)$ denote the Tracking Error and Control Effort for a given value of $(x, u) = (x(1), ..., x(T), u(1), ..., u(T-1))$, respectively. Then, suppose that $\text{TE}(x_a, u_a) \leq \text{TE}(x_b, u_b)$.
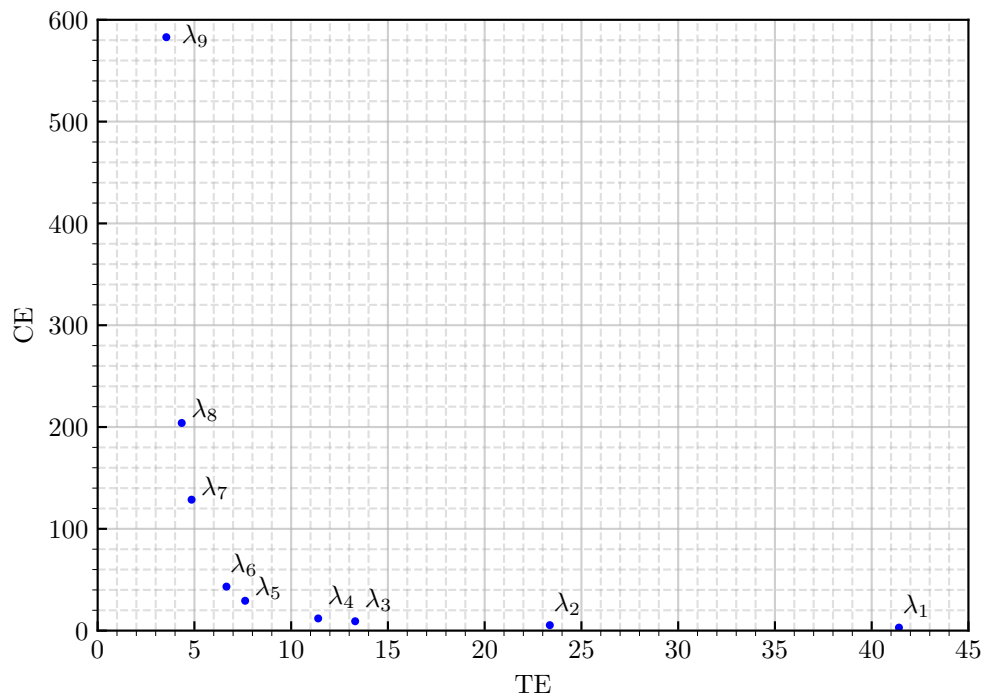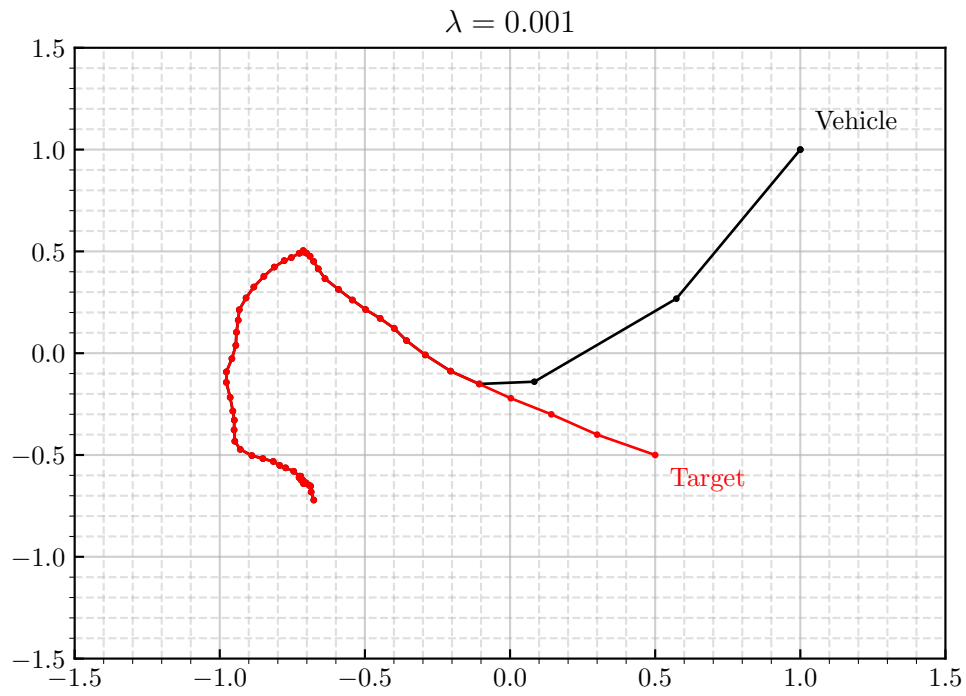
Since (x_b, u_b) minimizes the given optimization problem for $\lambda = \lambda_b$, it follows that:

$$\text{TE}(x_b, u_b) + \lambda_b \text{CE}(x_b, u_b) \leq \text{TE}(x_a, u_a) + \lambda_b \text{CE}(x_a, u_a) \tag{1}$$
$$\leq \text{TE}(x_b, u_b) + \lambda_b \text{CE}(x_a, u_a) \tag{2}$$

where we used the hypothesis that $\text{TE}(x_a, u_a) \leq \text{TE}(x_b, u_b)$ do derive (2) from (1). In particular, we have that:

$$\text{TE}(x_b, u_b) + \lambda_b \text{CE}(x_b, u_b) \leq \text{TE}(x_b, u_b) + \lambda_b \text{CE}(x_a, u_a)$$
$$\Leftrightarrow \lambda_b \text{CE}(x_b, u_b) \leq \lambda_b \text{CE}(x_a, u_a)$$
$$\Leftrightarrow \text{CE}(x_b, u_b) \leq \text{CE}(x_a, u_a)$$

with the last inequality coming from the fact that $\lambda_b > 0$. We have thus proven the desired result. $\qquad\square$

Appendices

A   First appendix

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B   Second appendix

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