Optimization and Algorithms Project

Authors:

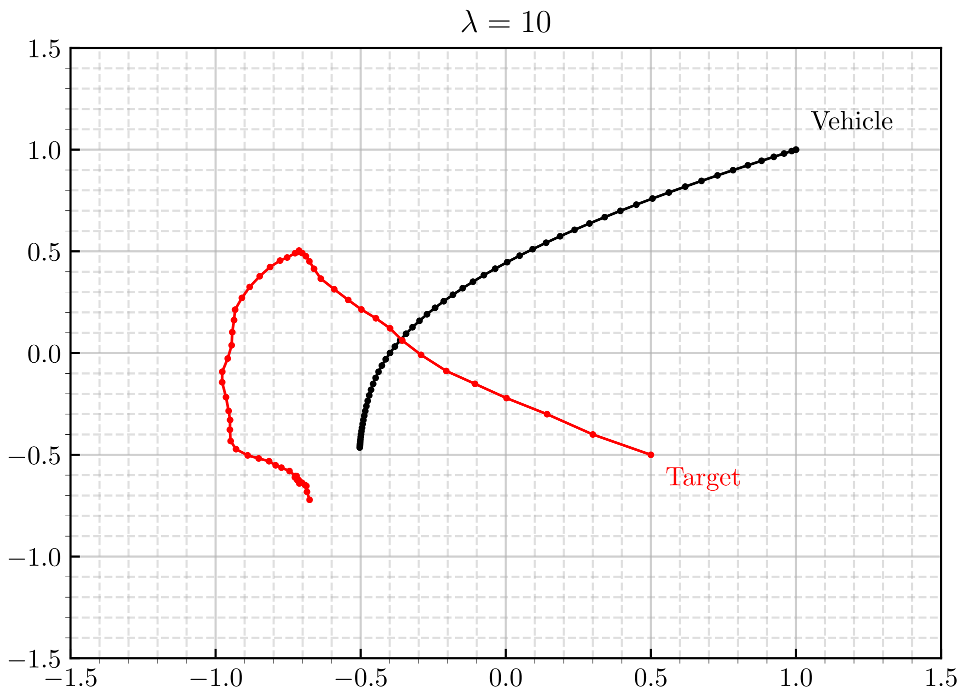
Duarte Calado de Almeida (95565)
Francisco Manuel Leal Mithá Ribeiro (95578)
João De Assis Marcos Soares Nabais (97349)
João De Assis Marcos Soares Nabais (965259)

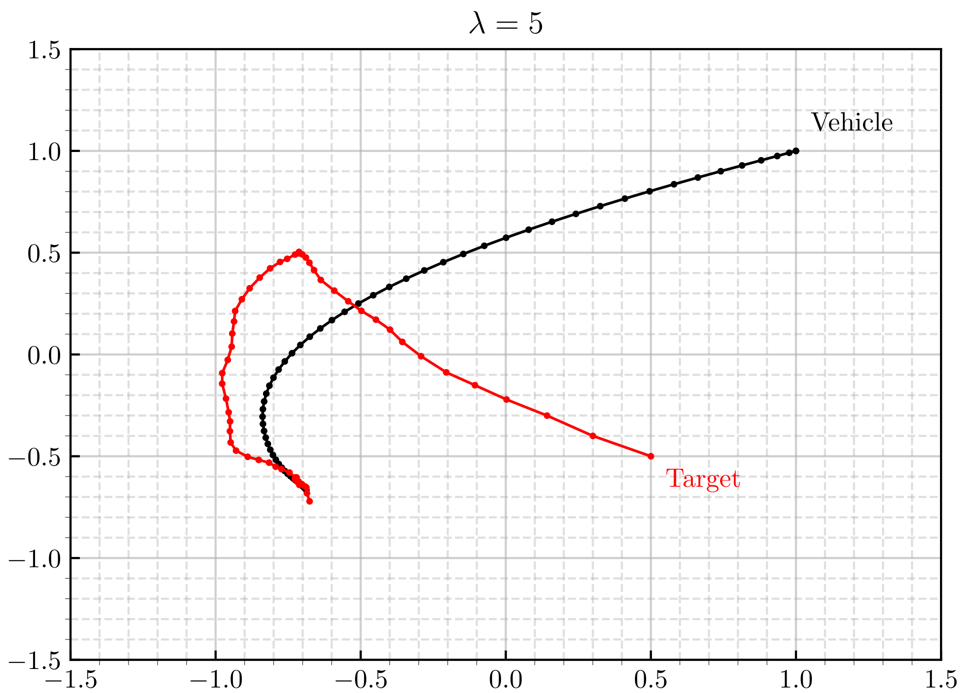
Group 8

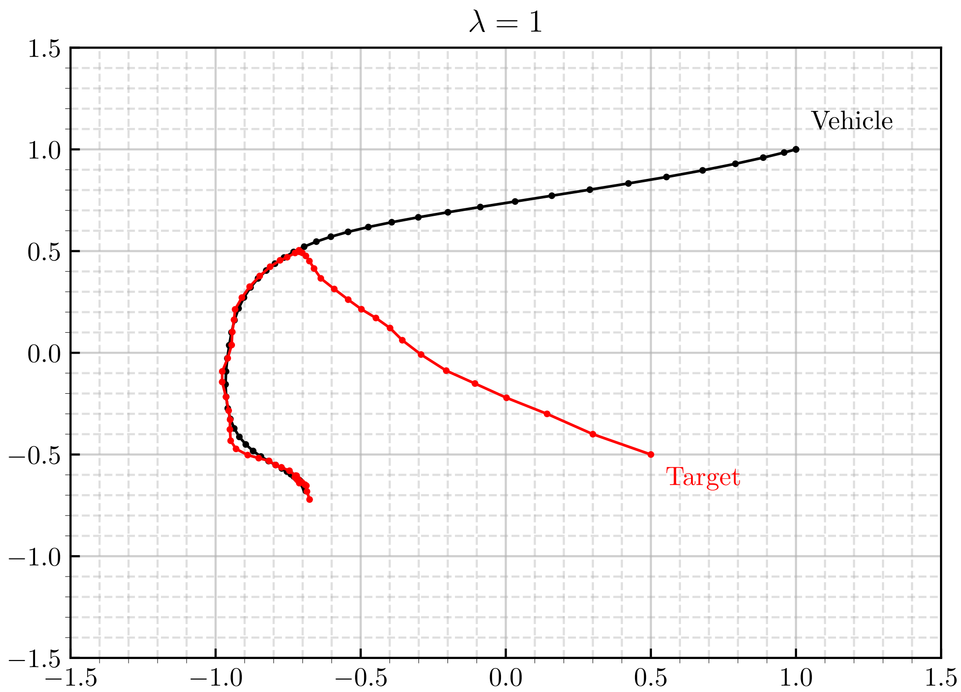
2022/2023 – 1º Semester, P1

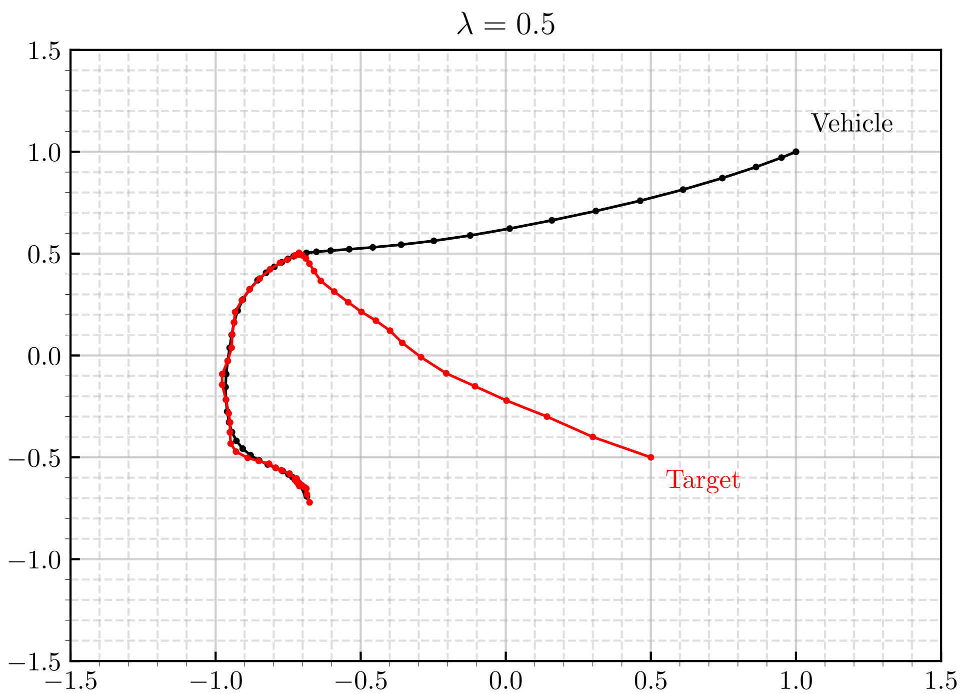
Contents

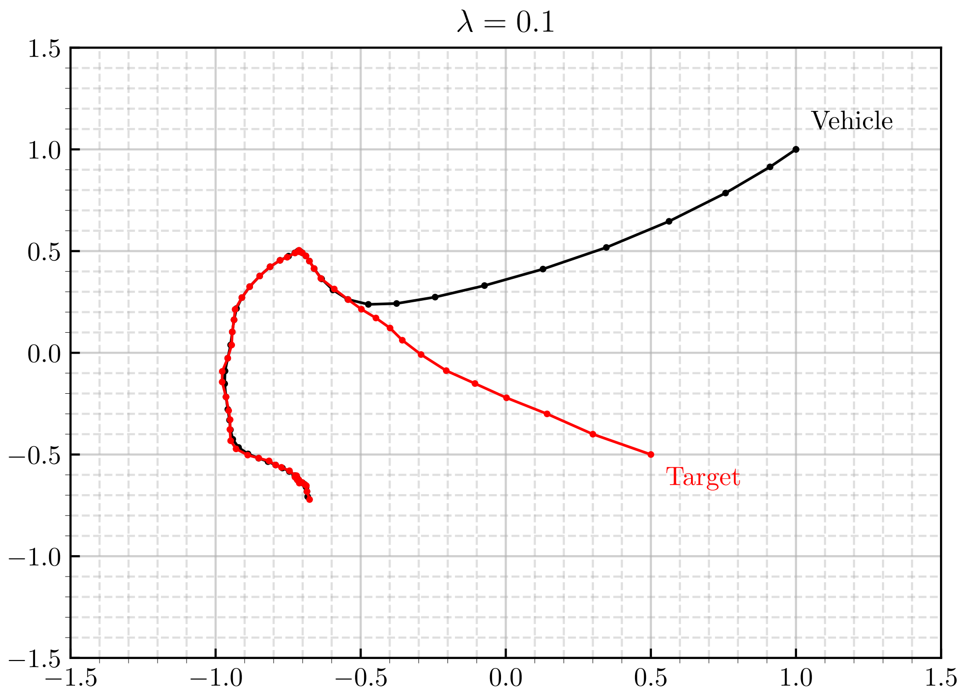
1 Task 1

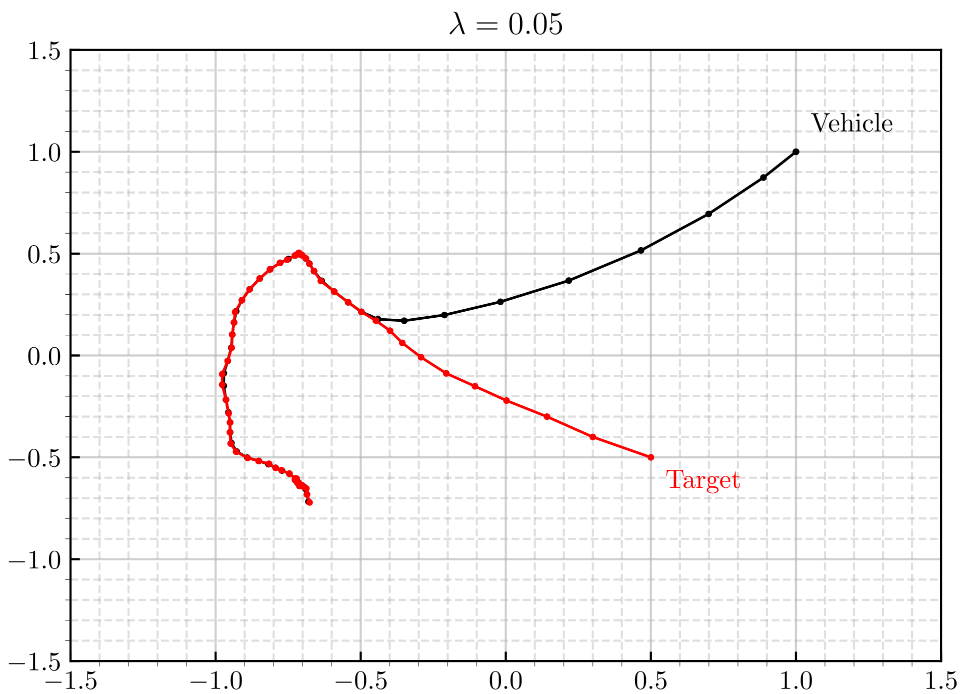

$\lambda = 10$
Vehicle
Target
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5 −1.0 −0.5 0.0 0.5 1.0 1.5

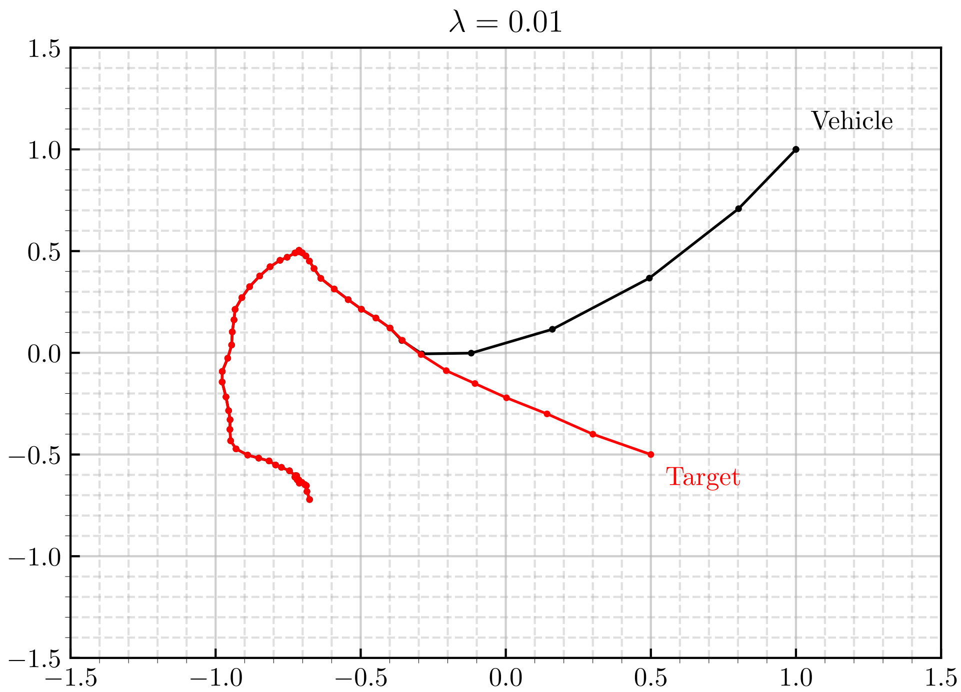

$\lambda = 5$
Vehicle
Target
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5 −1.0 −0.5 0.0 0.5 1.0 1.5

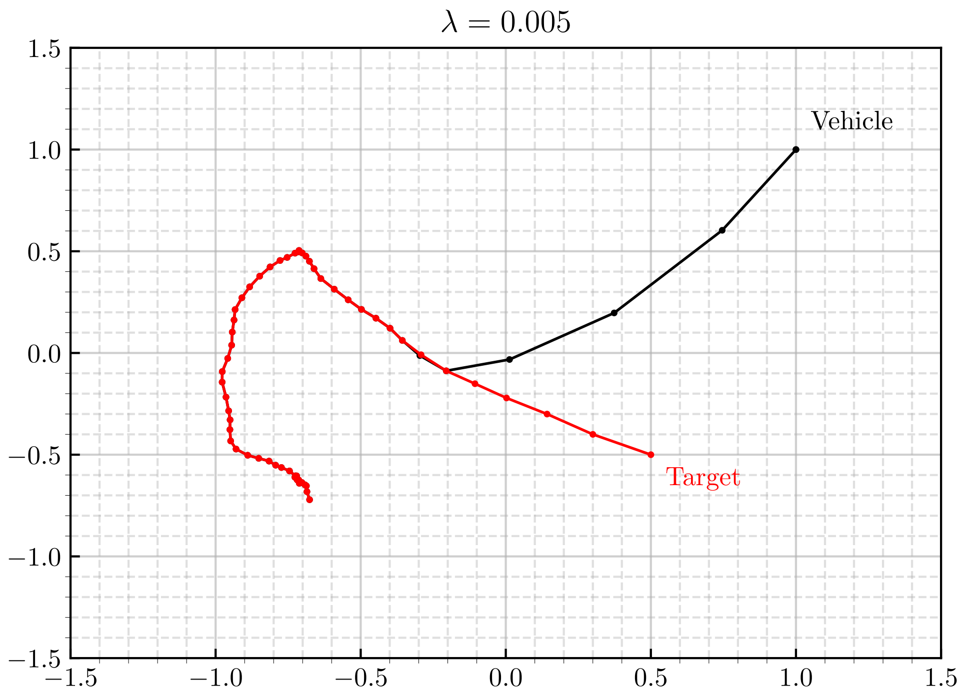

$\lambda = 1$
Vehicle
Target
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

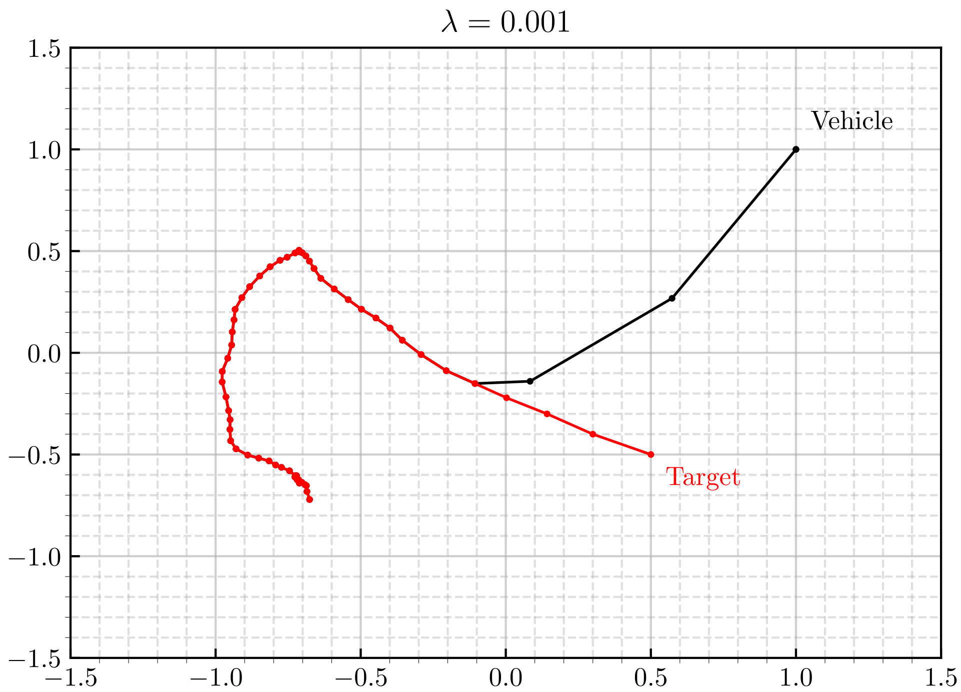

$\lambda = 0.5$
Vehicle
Target
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

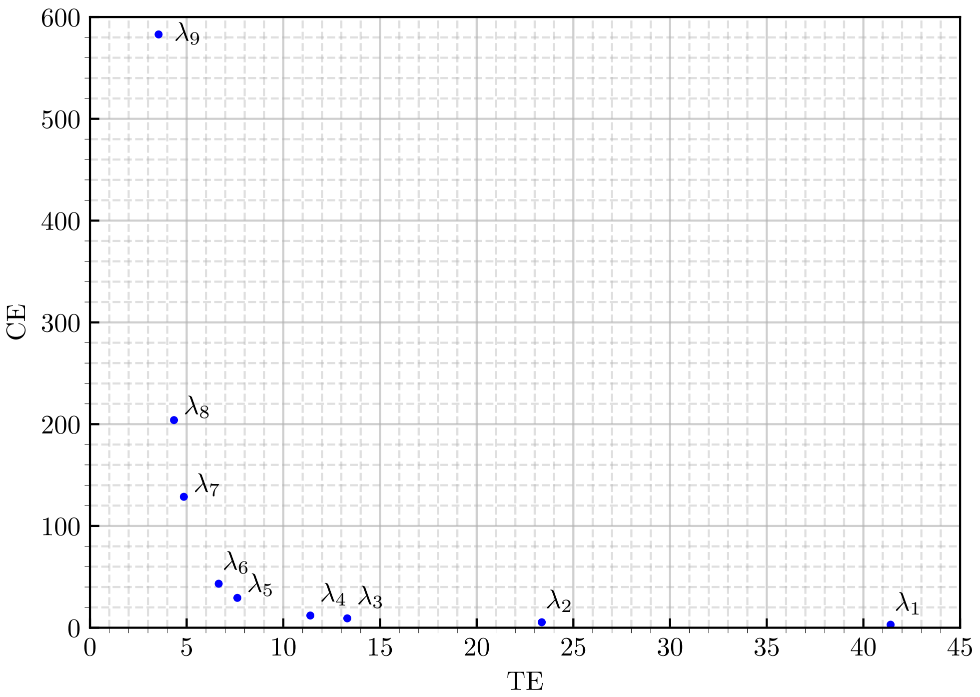

$\lambda = 0.1$
Vehicle
Target


$\lambda = 0.05$
Vehicle
Target


$\lambda = 0.01$
Vehicle
Target
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5 −1.0 −0.5 0.0 0.5 1.0 1.5


$\lambda = 0.005$
Vehicle
Target
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5 −1.0 −0.5 0.0 0.5 1.0 1.5





When the parameter λ increases, the Tracking Error and Control Effort associated with the optimal solution of the resulting optimization problem increases and decreases, respectively.

2 Task 2

Let (x_a, u_a) and (x_b, u_b) denote the minimizers obtained after solving the given optimization problem for $\lambda = \lambda_a$ and $\lambda = \lambda_b$, respectively. Let $\mathrm{TE}(x, u)$ and $\mathrm{CE}(x, u)$ denote the Tracking Error and Control Effort for a given value of $(x, u) = (x(1), ..., x(T), u(1), ..., u(T-1))$, respectively. Then, suppose that $\mathrm{TE}(x_a, u_a) \leq \mathrm{TE}(x_b, u_b)$.

Since (x_b, u_b) minimizes the given optimization problem for $\lambda = \lambda_b$, it follows that:

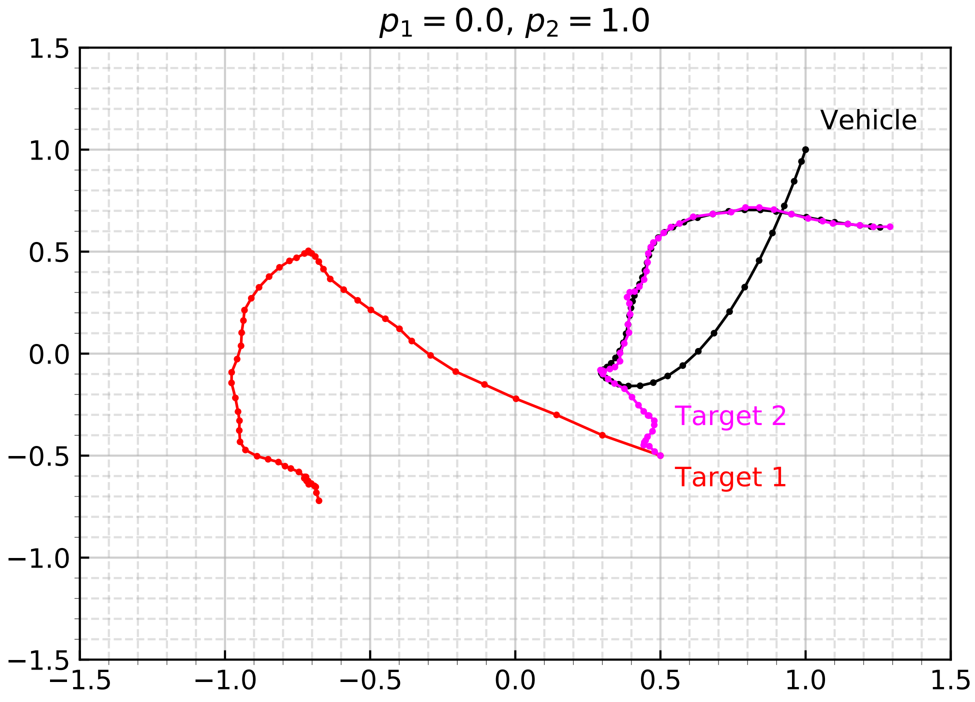
$$\begin{align} \mathrm{TE}(x_b, u_b) + \lambda_b \mathrm{CE}(x_b, u_b) &\leq \mathrm{TE}(x_a, u_a) + \lambda_b \mathrm{CE}(x_a, u_a) \tag{1} \\ &\leq \mathrm{TE}(x_b, u_b) + \lambda_b \mathrm{CE}(x_a, u_a) \tag{2} \end{align}$$

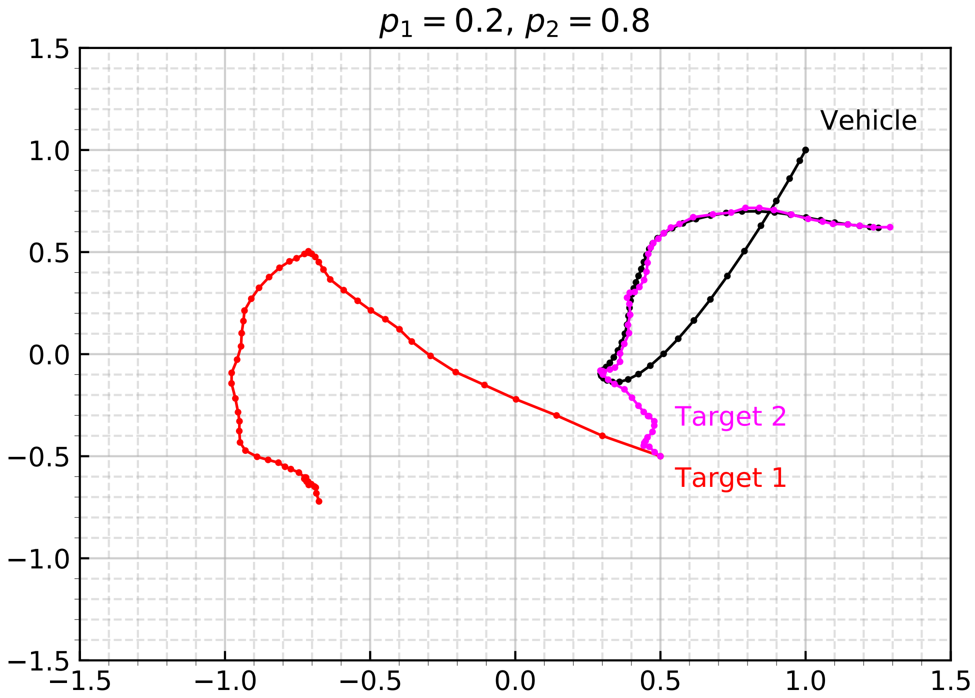
where we used the hypothesis that $\mathrm{TE}(x_a, u_a) \leq \mathrm{TE}(x_b, u_b)$ do derive (2) from (1). In particular, we have that:

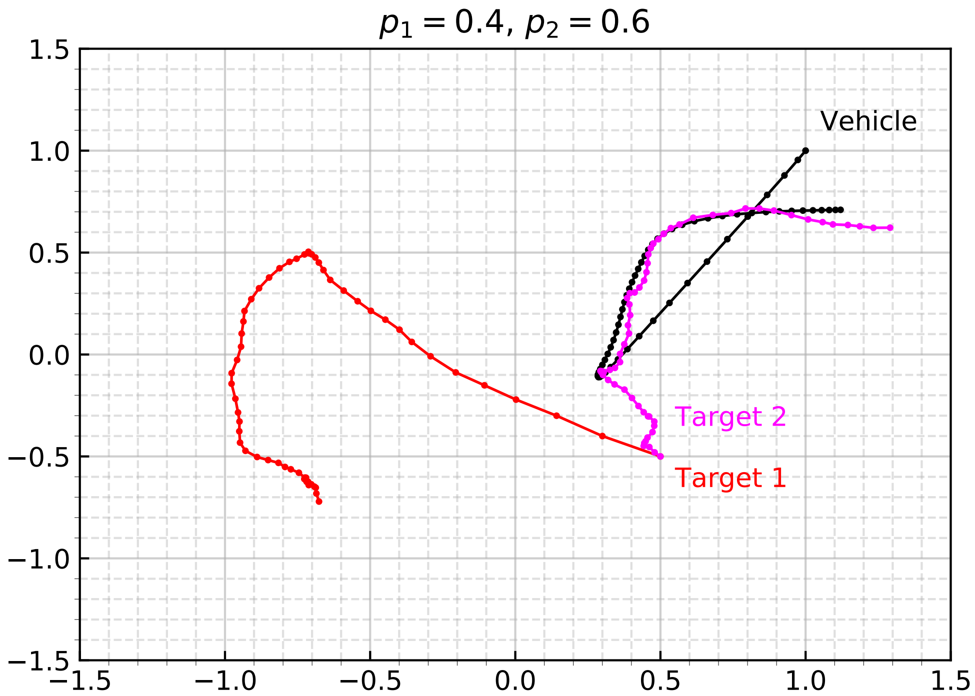
$$\begin{aligned} \mathrm{TE}(x_b, u_b) + \lambda_b \mathrm{CE}(x_b, u_b) &\leq \mathrm{TE}(x_b, u_b) + \lambda_b \mathrm{CE}(x_a, u_a) \\ \Leftrightarrow \lambda_b \mathrm{CE}(x_b, u_b) &\leq \lambda_b \mathrm{CE}(x_a, u_a) \\ \Leftrightarrow \mathrm{CE}(x_b, u_b) &\leq \mathrm{CE}(x_a, u_a) \end{aligned}$$

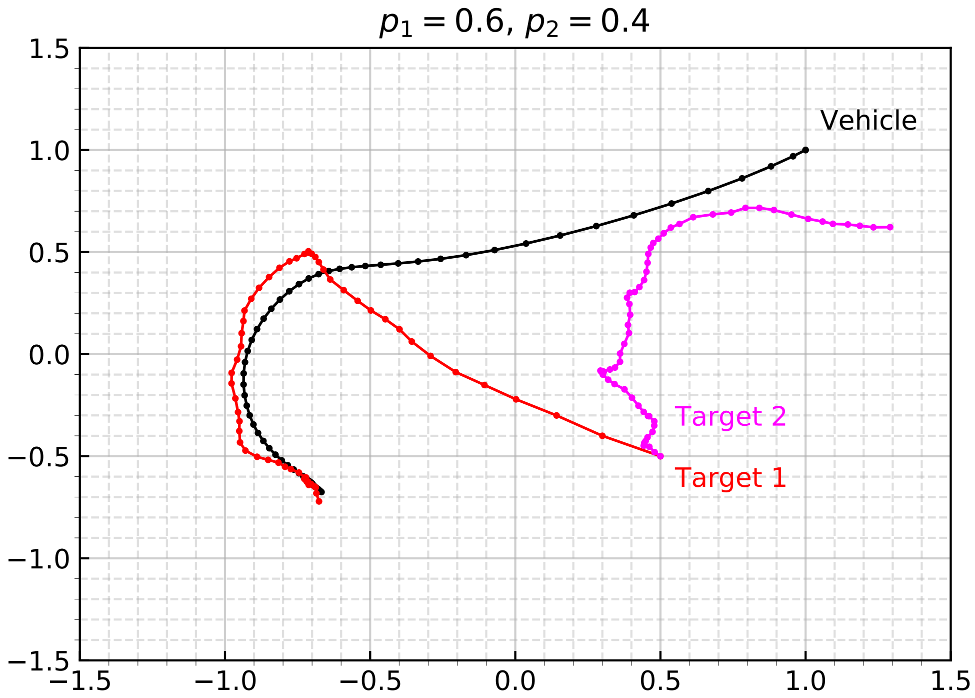
with the last inequality coming from the fact that $\lambda_b > 0$. We have thus proven the desired result. $\square$

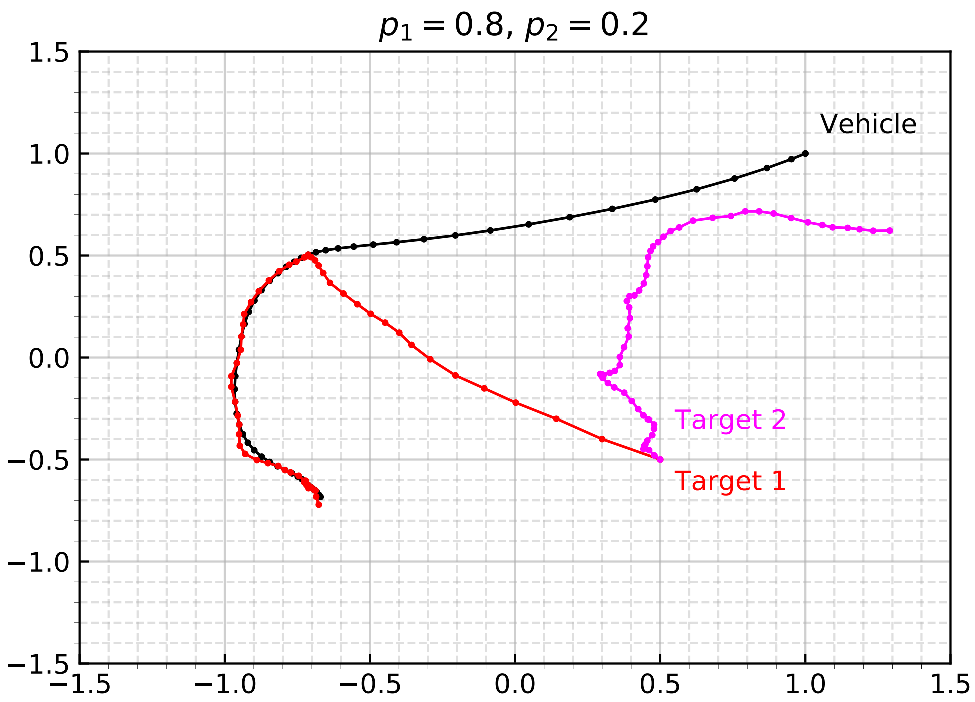
3 Task 4

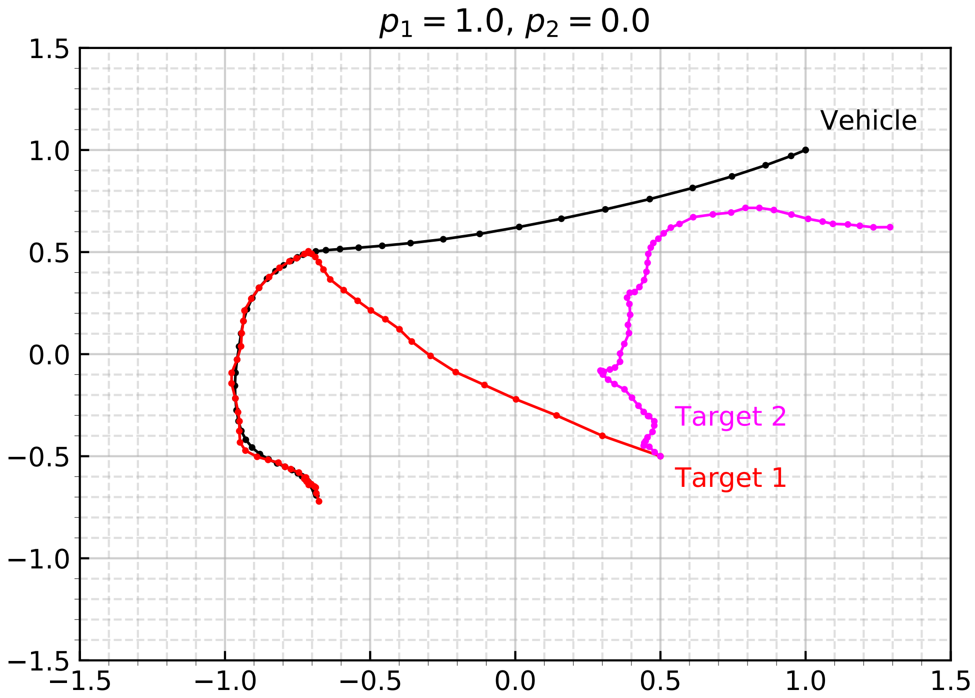



$p_1 = 0.2$, $p_2 = 0.8$
Vehicle
Target 2
Target 1
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


$p_1 = 0.4$, $p_2 = 0.6$
Vehicle
Target 2
Target 1
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


$p_1 = 0.6$, $p_2 = 0.4$
Vehicle
Target 2
Target 1


$p_1 = 0.8$, $p_2 = 0.2$
Vehicle
Target 2
Target 1



From the results, we can see that the tracker chooses the trajectory with the highest probability. However, if the probability is not 100%, it is noticeable an "hesitation", the tracker prefers to follow the one with highest probability, but deviated towards the other.

4 Task 5

The given formulation doesn't include that, until $t = 34$, the tracker will simply follow the minimum cost path between trajectory 1 and 2. Only at $t = 35$ will the tracker follow the true target. Also, there is no coupling between (x_1, u_1) and (x_2, u_2).

5 Task 6

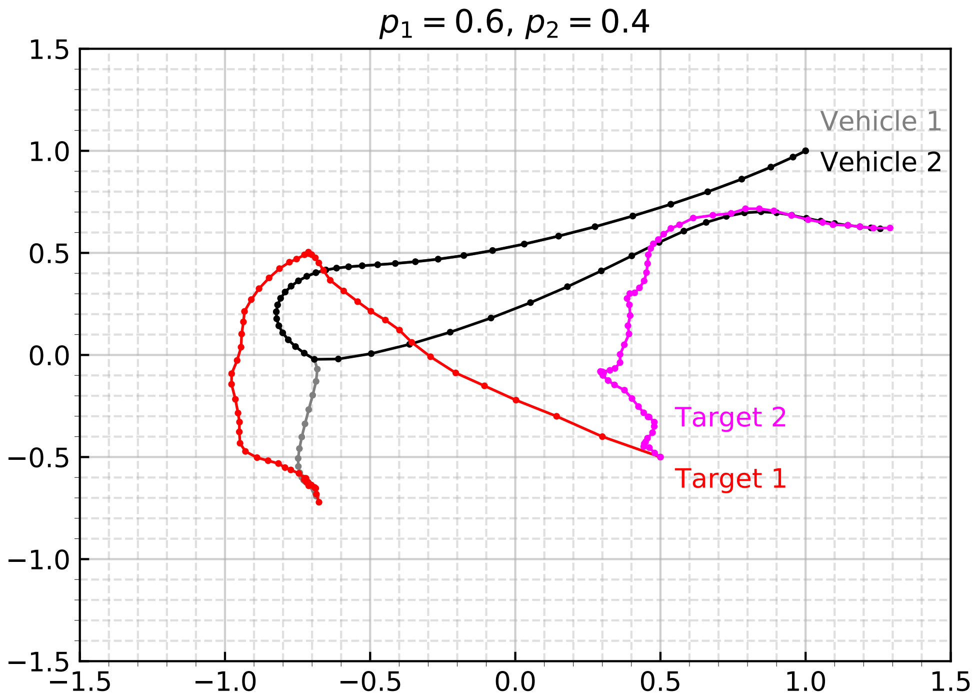
The following constraint solves the questions raised in the previous task:

$$u_1(t) = u_2(t), \quad \text{for } 1 \leq t \leq 34 \tag{3}$$

Note that x_1 and x_2 become coupled (for $1 \leq t \leq 34$) by these constraints together with (3):

$$x_1(t+1) = Ax_1(t) + Bu_1(t), \quad \text{for } 1 \leq t \leq T-1$$
$$x_2(t+1) = Ax_2(t) + Bu_2(t), \quad \text{for } 1 \leq t \leq T-1$$

6 Task 7


$p_1 = 0.6$, $p_2 = 0.4$
Vehicle 1
Vehicle 2
Target 2
Target 1
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

Appendices

A First appendix

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

B Second appendix

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.