TÉCNICO LISBOA

# Optimization and Algorithms Project

**Authors:**

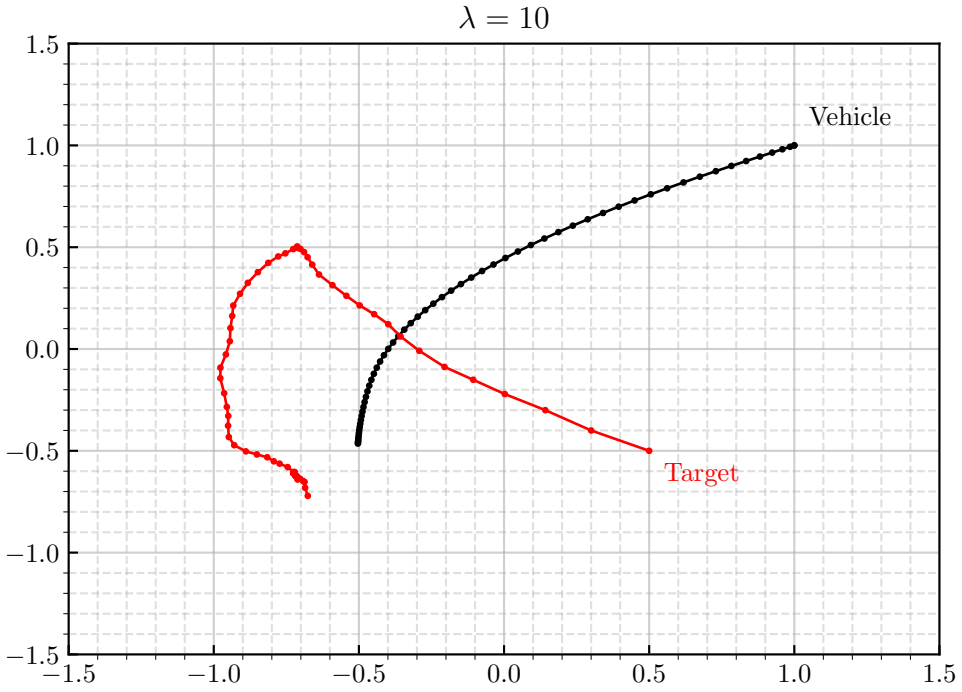
Duarte Calado de Almeida (95565)
Francisco Manuel Leal Mitha Ribeiro (95578)
João De Assis Marcos Soares Nabais (97349)
João De Assis Marcos Soares Nabais (965259)

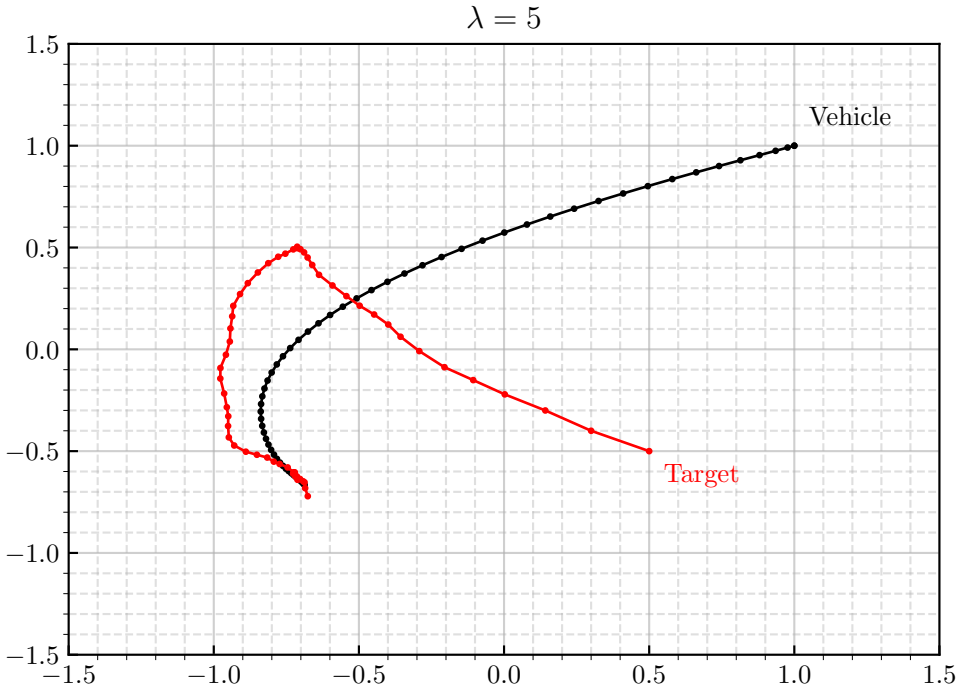
**Group** 8

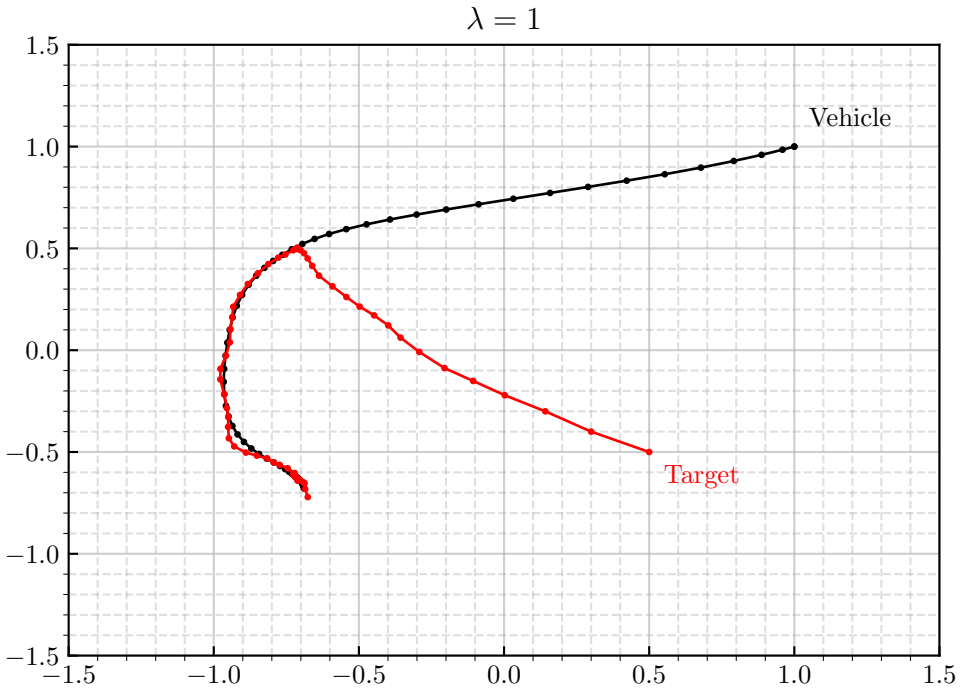
**2022/2023 – 1º Semester, P1**

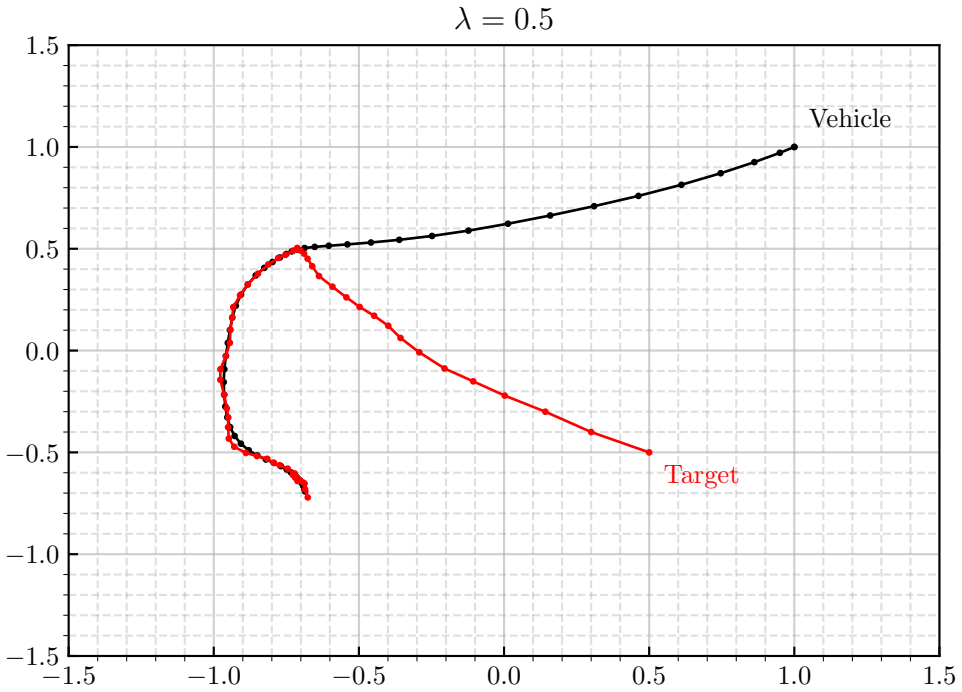
# Contents

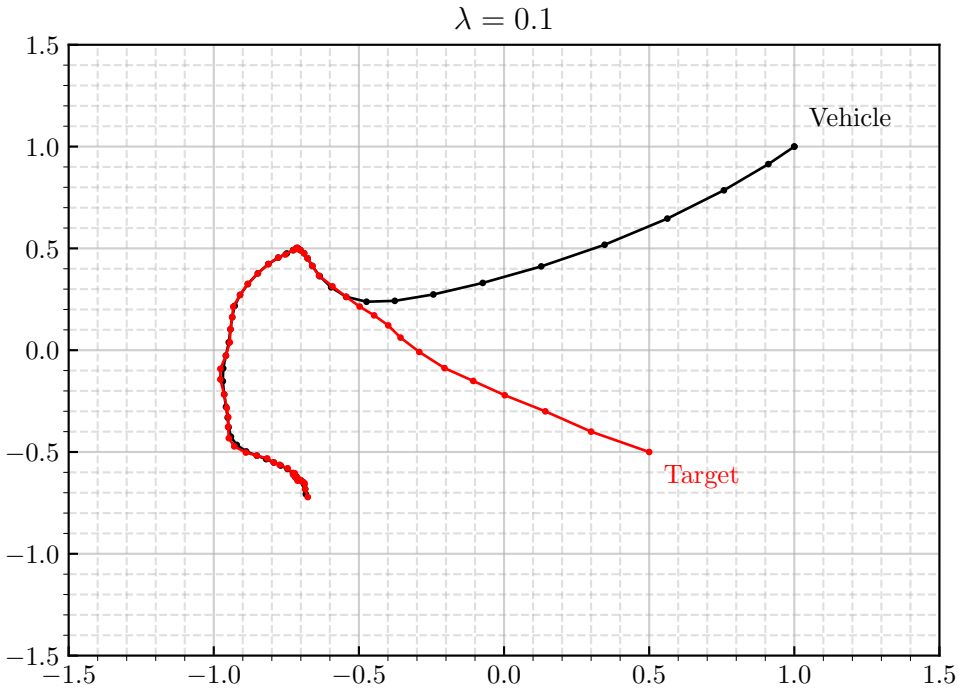
# 1 Task 1

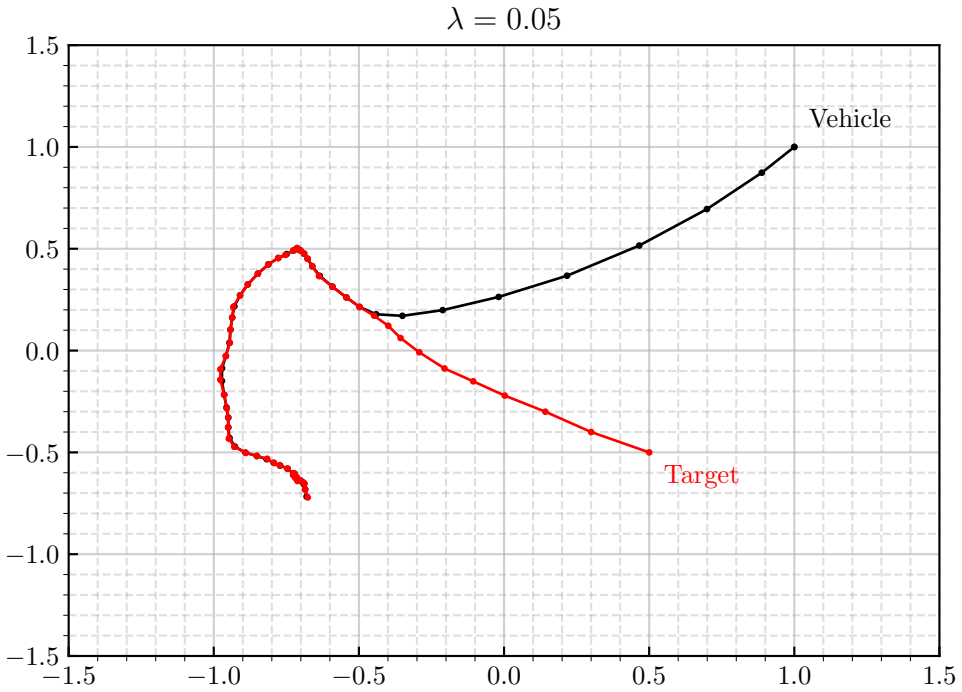

$\lambda = 10$
Vehicle
Target
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

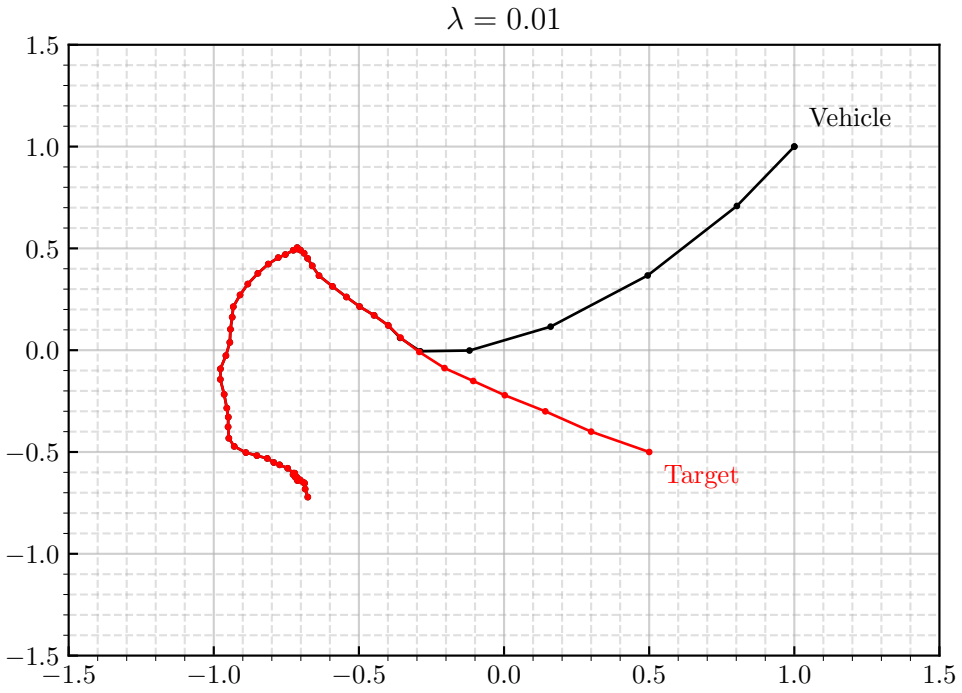

$\lambda = 5$
Vehicle
Target
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

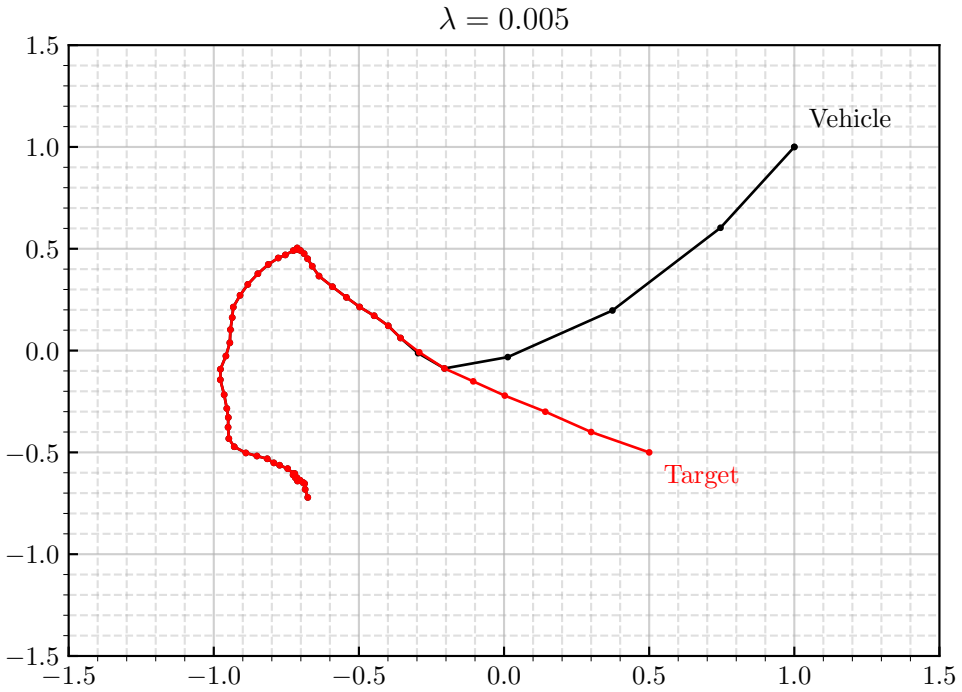

$\lambda = 1$
Vehicle
Target
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

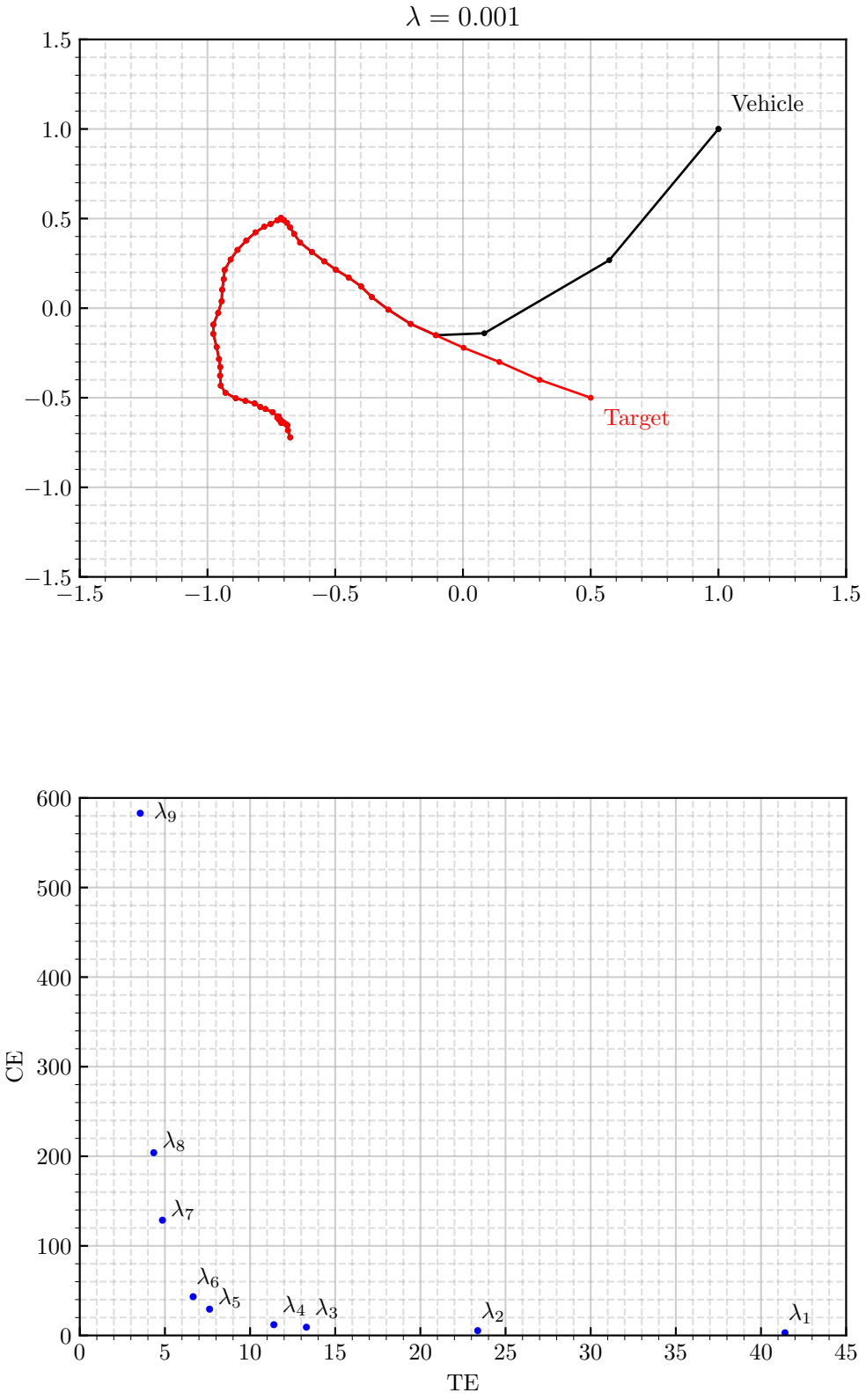

$\lambda = 0.5$
Vehicle
Target
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


$\lambda = 0.1$
Vehicle
Target
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


$\lambda = 0.05$
Vehicle
Target
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


$\lambda = 0.01$
Vehicle
Target
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


$\lambda = 0.005$
Vehicle
Target
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5




When the parameter $\lambda$ increases, the Tracking and Control Efforts associated with the optimal solution of the resulting optimization problem increases and decreases, respectively.

# 2 Task 2

Let $(x_a, u_a)$ and $(x_b, u_b)$ denote the minimizers obtained after solving the given optimization problem for $\lambda = \lambda_a$ and $\lambda = \lambda_b$, respectively. Let $\text{TE}(x, u)$ and $\text{CE}(x, u)$ denote the Tracking and Control Efforts for a given value of $(x, u) = (x(1), ..., x(T), u(1), ..., u(T-1))$, respectively. Then, suppose that $\text{TE}(x_a, u_a) \leq \text{TE}(x_b, u_b)$.

Since $(x_b, u_b)$ minimizes the given optimization problem for $\lambda = \lambda_b$, it follows that:

$$\begin{aligned}
\text{TE}(x_b, u_b) + \lambda_b \text{CE}(x_b, u_b) &\leq \text{TE}(x_a, u_a) + \lambda_b \text{CE}(x_a, u_a) && (1)\\
&= \text{TE}(x_a, u_a) + \lambda_a \text{CE}(x_a, u_a) + (\lambda_b - \lambda_a)\text{CE}(x_a, u_a) && (2)\\
&\leq \text{TE}(x_b, u_b) + \lambda_a \text{CE}(x_a, u_a) + (\lambda_b - \lambda_a)\text{CE}(x_a, u_a) && (3)\\
&= \text{TE}(x_b, u_b) + \lambda_b \text{CE}(x_a, u_a) && (4)
\end{aligned}$$

where we used the hypothesis that $\text{TE}(x_a, u_a) \leq \text{TE}(x_b, u_b)$ do derive (3) from (2). In particular, we have that:

$$\begin{aligned}
\text{TE}(x_b, u_b) + \lambda_b \text{CE}(x_b, u_b) &\leq \text{TE}(x_b, u_b) + \lambda_b \text{CE}(x_a, u_a) && (5)\\
\Leftrightarrow \lambda_b \text{CE}(x_b, u_b) &\leq \lambda_b \text{CE}(x_a, u_a) && (6)\\
\Leftrightarrow \text{CE}(x_b, u_b) &\leq \text{CE}(x_a, u_a) && (7)
\end{aligned}$$

with the last inequality coming from the fact that $\lambda_b > 0$. We have thus proven the desired result. □

# Appendices

## A First appendix

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## B Second appendix

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