# Circuit Theory and Electronics Fundamentals

Aerospace Engineering, Técnico, University of Lisbon

T1 Laboratory Report

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# 1 Introduction

The objective of this laboratory assignment is to study a circuit containing a voltage source $V_a$, a current source $I_d$, a voltage dependent current source $I_b$, a current dependent voltage source $V_c$ and 7 resistors $R_n$. The circuit can be seen if Figure 1.

In Section 2, a theoretical analysis of the circuit is presented, using both the nodal and mesh methods. In Section 3, the circuit is analysed by means of a ngspice simulation. The results are then compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

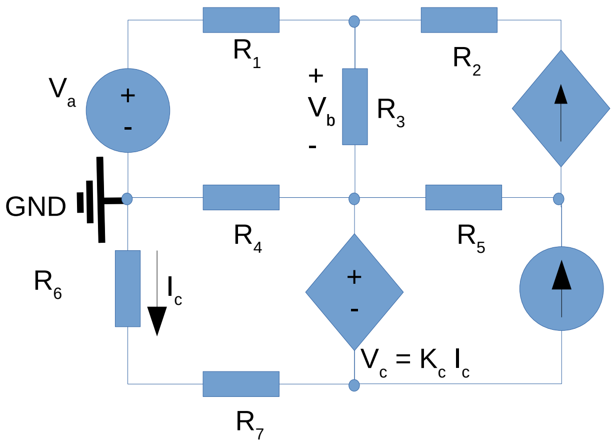


Figure 1: Assigned Circuit.

## 2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, in terms of its time and frequency responses.

### 2.1 Mesh Analysis

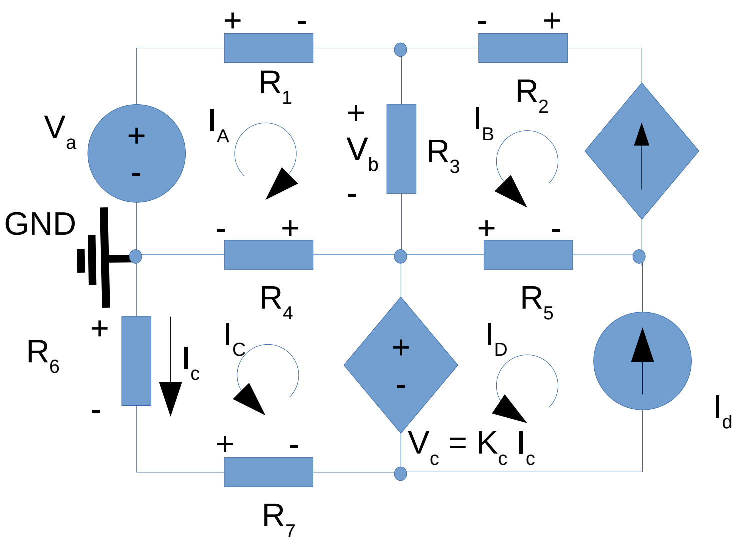


Figure 2: Mesh Analysis

The Figure 2 has both the mesh's currents and the component's admited direction. This reference is indispensable for the Mesh Analysis. Using KVL in each of the 4 meshes, we achieve the following equations:

Malha A:

$$R_1 * I_A + R_3(I_A + I_B) + R_4(I_A + I_C) = V_A$$

Malha B:

$$I_B = K_c * R_3(I_A + I_B)$$

Malha C:

$$R_4(I_A + I_C) + R_6 * I_C + R_7 * I_C = K_c * I_C$$

Malha D:

$$I_D = I_d$$

After doing some algebra, we can get the following system of equations. After that, we can solve it by enquadrating it in a matriz like shown bellow:

$$\begin{bmatrix} R_1 + R_3 + R_4 & R_3 & R_4 & 0 \\ K_b * R_3 & K_b * R_3 - 1 & 0 & 0 \\ R_4 & 0 & R_6 - K_c + R_7 - R_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \\ I_D \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \\ I_d \end{bmatrix}$$

After solving these equations using octave, we get the following results:

| | I(mA) |
|---|---|
| Ia | 0.236456 |
| Ib | -0.247520 |
| Ic | 0.964663 |
| Id | 1.005321 |

To get the current in each resistor, we use the following equations:

$$I_1 = I_A$$
$$I_2 = I_B$$
$$I_3 = I_A + I_B$$
$$I_4 = I_A + I_C$$
$$I_5 = I_B - I_D$$
$$I_6 = I_C$$
$$I_7 = I_C$$

Leading to the results bellow:

| | I(mA) |
|---|---|
| I1 | 0.236456 |
| I2 | -0.247520 |
| I3 | -0.011064 |
| I4 | 1.201119 |
| I5 | -1.252842 |
| I6 | 0.964663 |
| I7 | 0.964663 |

## 2.2 Nodal Analysis

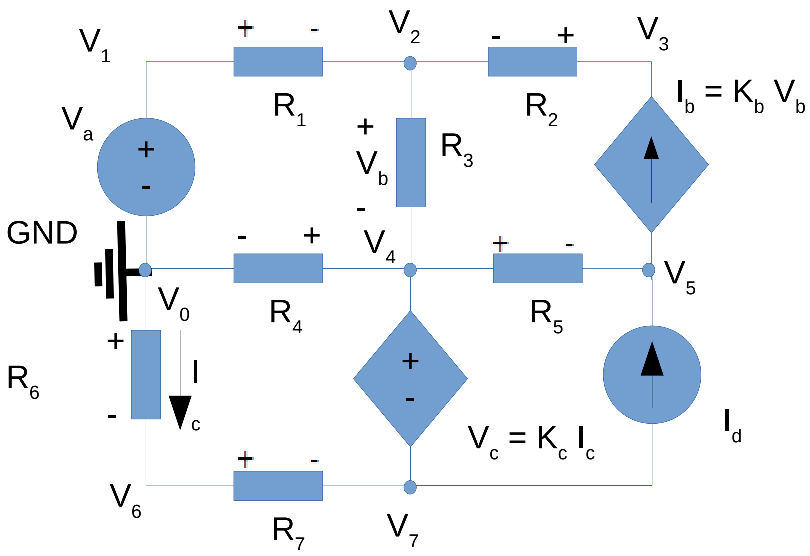


Figure 3: Nodal Analysis

The Figure 3 has the node references appended. This reference is indispensable for the Nodal Analysis. Using KVL in each of the 4 meshes, we achieve the following equations:

Node 2:

$$(V_1 - V_2)G_1 + (V_3 - V_2)G_2 = (V_2 - V_4)G_3$$

Node 3:

$$(V_3 - V_2)G_2 = K_b(V_2 - V_4)$$

Node 5:

$$(V_4 - V_5)G_5 + I_d = K_b(V_2 - V_4)$$

Node 6:

$$(V_0 - V_6)G_6 = (V_6 - V_7)G_7$$

Additional equations:

$$V_0 = 0$$

$$(V_1 - V_0) = V_a$$

$$V_4 - V_7 = K_c(V_0 - V_6)G_6$$

$$(V_0 - V_6)G_6 - (V_4 - V_0)G_4 + (V_1 - V_2)G_1 = 0$$

Similarly to the last example, the algebric manipulation brings us to the following system of equations, and the solution can be found by enquadrating it in a matriz like shown bellow:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & K_c * G_6 & -1 \\
G_1 & -G_1 - G_2 - G_3 & G_2 & G_3 & 0 & 0 & 0 \\
0 & -K_b & 0 & G_5 + K_b & -G_5 & 0 & 0 \\
0 & -K_b & 0 & G_5 + K_b & -G_5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -G_6 - G_7 & G_7 \\
G_1 & -G_1 & 0 & -G_4 & 0 & -G_6 & 0
\end{bmatrix}
\cdot
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix}
=
\begin{bmatrix} V_a \\ 0 \\ 0 \\ 0 \\ -I_d \\ 0 \\ 0 \end{bmatrix}
$$

After solving these equations using octave, we get the following results:

| | V(V) |
|---|---|
| V1 | 5.076389 |
| V2 | 4.828242 |
| V3 | 4.317617 |
| V4 | 4.862303 |
| V5 | 8.665727 |
| V6 | -1.946930 |
| V7 | -2.953630 |

## 2.3 Result Comparison

In order to compare the awnsers from the mesh and node methods, we can use the voltage values we obtained and, using Ohm's law (1), check if the current values are similar.

$$U = RI. \tag{1}$$

| n R | I(mA) | RI(V) | V nodal(V) | Difference(V) |
|---|---|---|---|---|
| 1.000000 | 0.236456 | 0.248147 | 0.248147 | 0.000000 |
| 2.000000 | -0.247520 | -0.510625 | -0.510625 | 0.000000 |
| 3.000000 | -0.011064 | -0.034061 | -0.034061 | 0.000000 |
| 4.000000 | 1.201119 | 4.862303 | 4.862303 | 0.000000 |
| 5.000000 | -1.252842 | -3.803425 | -3.803424 | 0.000001 |
| 6.000000 | 0.964663 | 1.946929 | 1.946930 | 0.000001 |
| 7.000000 | 0.964663 | 1.006700 | 1.006700 | 0.000000 |

The interpretation of the previous table leads us to the conclusion that the values are virtualy equal.

# 3 Simulation Analysis

## 3.1 Operating Point Analysis

Table 1 shows the simulated operating point results for the study case circuit.
Note that V9 is not a real node, but it is needed for ngspice calculations.

| Name | Value [A or V] |
|---|---|
| @gb[i] | -2.47520e-04 |
| @id[current] | 1.005321e-03 |
| @r1[i] | 2.364560e-04 |
| @r2[i] | -2.47520e-04 |
| @r3[i] | -1.10640e-05 |
| @r4[i] | 1.201119e-03 |
| @r5[i] | -1.25284e-03 |
| @r6[i] | 9.646630e-04 |
| @r7[i] | 9.646630e-04 |
| v(1) | 5.076387e+00 |
| v(2) | 4.828240e+00 |
| v(3) | 4.317615e+00 |
| v(4) | 4.862301e+00 |
| v(5) | 8.665725e+00 |
| v(6) | -1.94693e+00 |
| v(7) | -2.95363e+00 |
| v(9) | -1.94693e+00 |

Table 1: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

# 4 Conclusion

Even though the theoretical and simulation models might not always give the same results, in our case of study the outcomes were precisely matched. This was in line with what was expected since we were presented with linear components and, to solve the circuit, ngspice utilized the same methods as we did in the theoretical Octave calculation. In conclusion, we believe that the goals of this report were achieved.