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# Circuit Theory and Electronics Fundamentals

Aerospace Engineering, Técnico, University of Lisbon

T1 Laboratory Report

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# Contents

# 1 Introduction

The objective of this laboratory assignment is to study a circuit containing a voltage source $V_s$, a voltage dependent current source $I_b$, a current dependent voltage source $V_d$, 7 resistors $R_n$ and a capacitor $C$. The circuit can be seen if Figure 1.

In Section 2, a theoretical analysis of the circuit is presented, using both the nodal and mesh methods. In Section 3, the circuit is analysed by means of a ngspice simulation. The results are then compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

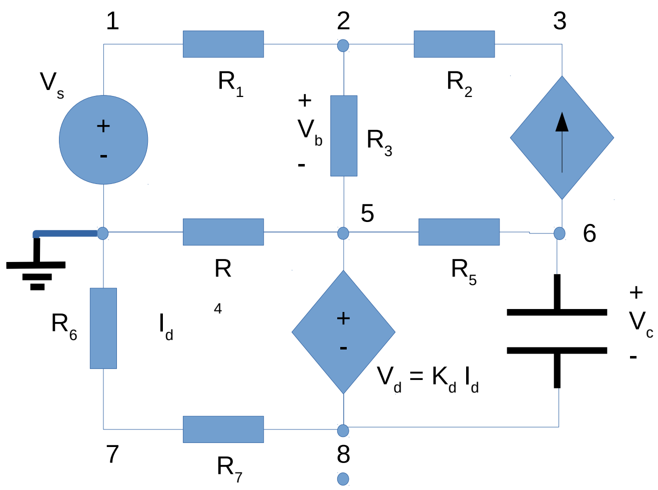


Figure 1: Assigned Circuit.

# 2 Theoretial Analysis

## 2.1 At t<0

For t $<$ 0, we can assume that the capacitor has achieved equilibrium and because of that we can say that the voltage drop in the capacitor is now constant, that is, $dv/dt = 0$. In a capacitor $dv/dt = i/C$, so $0 = i/C \iff i = 0$, which is equivalent to an open circuit. With that being said, we can redesign the circuit like so:

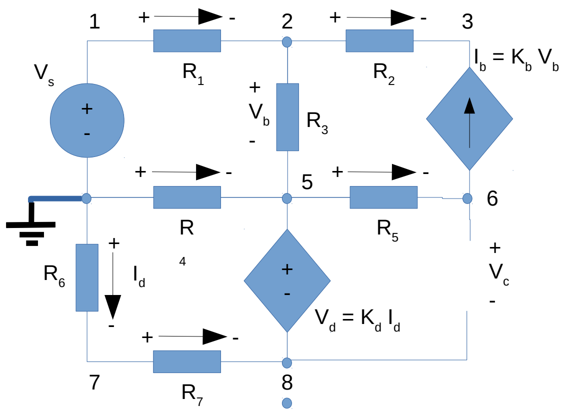


Figure 2: Circuit at t<0

By using node analysis, one can get the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1-G_2-G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & G_2+K_b & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & K_d*G_6 & -1 \\ 0 & -K_b & 0 & G_5+K_b & -G_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -G_6-G_7 & G_7 \\ 0 & G3 & 0 & -G4-G5-G3 & G5 & G7 & -G7 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using octave to resolve it, one gets the following results:

| | V (V) |
|---|---|
| V1 | 5.076387 |
| V2 | 4.828240 |
| V3 | 4.317615 |
| V5 | 4.862301 |
| V6 | 5.613732 |
| V7 | -1.946929 |
| V8 | -2.953629 |

## 2.2 Equivalent Resistance

To get the equilibrium resistor, we take $V_s$ from the circuit and replace the capacitor with a voltage source, while maintaining the voltage drop at its nodes.

The voltage drop at the end of the capacitor can be considered constant because there is continuity in those nodes.

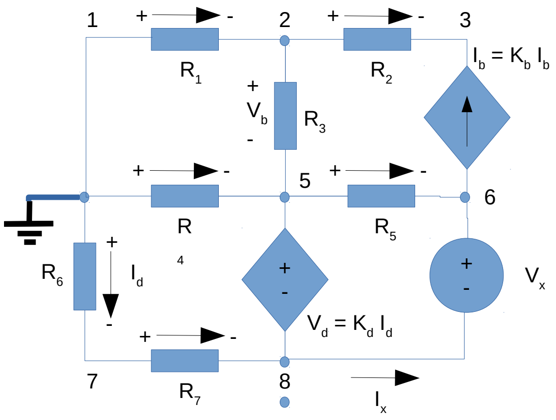


Figure 3: Circuit to calculate $R_{eq}$

Thus, the circuit that will be analysed will be the following:
To calculate the value of $V_x$, it is needed Ohm's Law

$$V_x = R_{eq} * I_x$$

:

To get Ix, it is used KCL in the node 6:

$$I_x = G_5(V_6 - V_5) + K_b * (V_2 - V_5)$$

Using Ohm's Law ($V_x = R_eq * I_x$):

$$R = (V_6 - V_5)/I_x$$

By using node analysis, one can get the following matrix:

$$\begin{bmatrix} -G_1 - G_2 - G_3 & G_2 & G_3 & 0 & 0 & 0 & \\ 0 & G_2 + K_b & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & K_d * G_6 & -1 \\ 0 & 0 & 0 & 1 & 0 & -1 & \\ 0 & 0 & 0 & 0 & -G_6 - G_7 & G_7 & \\ G_3 - K_b & 0 & -G_4 - G_3 + K_b & 0 & G_7 & -G_7 & \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ Vx \\ 0 \\ 0 \end{bmatrix}$$

Using octave to resolve it, one gets the following results:

| | V (V) |
|---|---|
| V2 | 0.000000 |
| V3 | -0.000000 |
| V5 | 0.000000 |
| V6 | 8.567360 |
| V7 | 0.000000 |
| V8 | 0.000000 |

### 2.3 t $>$ 0

**Natural Solution:**

The natural solution is given by:

$$V_x = Ae^{(\frac{-1}{R_{eq}C}t)}$$

Where $A = V_6(t=0) - V_8(t=0)$ due to continuity at t=0

This function can be plotted resulting, resulting in:

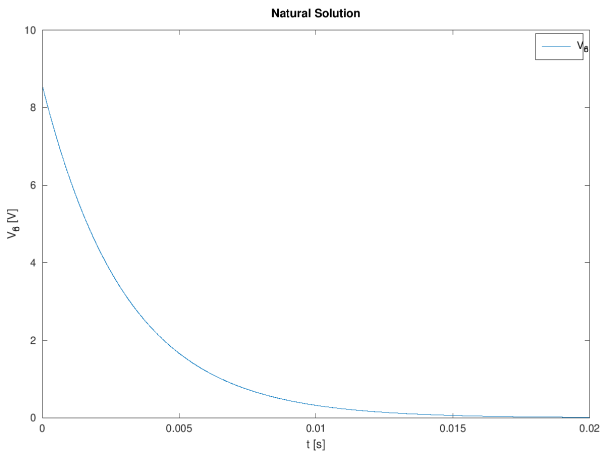


Figure 4: Natural Solution

**Forced Solution:**

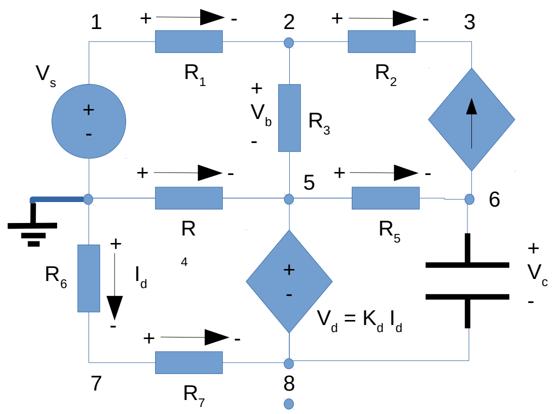


Figure 5: Natural Solution

To get the forced solution it is used complex numbers, impedance and node analysis, and by applying it to the circuit above, one can get the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_1 & -G_1-G_2-G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & G_2+K_b & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & K_d*G_6 & -1 \\ 0 & -K_b & 0 & G_5+K_b & -G_5-j*w*C & 0 & j*w*C \\ 0 & 0 & 0 & 0 & 0 & -G_6-G_7 & G_7 \\ 0 & G3 & 0 & -G4-G5-G3 & G5+j*w*C & G7 & -G7-j*w*C \end{bmatrix}$$

$$\cdot \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_3 \\ \tilde{V}_5 \\ \tilde{V}_6 \\ \tilde{V}_7 \\ \tilde{V}_8 \end{bmatrix} = \begin{bmatrix} \tilde{V}_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

After solving it, to get the forced solution, it is needed to multiply $\tilde{V}_6$ with $e^{i*w*C}$. The result of that is displayed in the plot bellow:

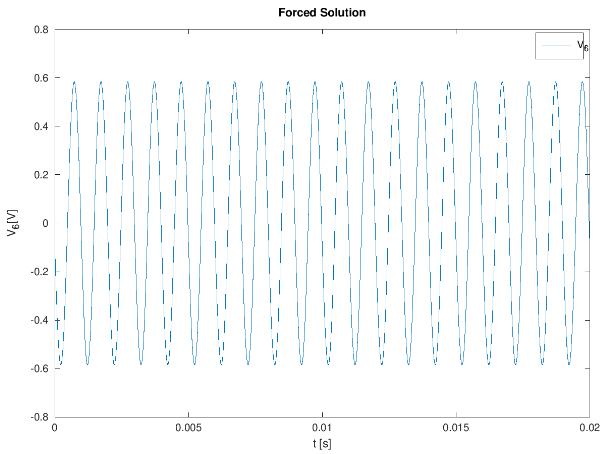


Figure 6: Forced Solution

**Final Total Solution:**

Since

$$V_6 = V_{6n}(t) + V_{6f}(t)$$

the total forced solution is (note that in the plot it is also displayed the voltage for t$<$0):

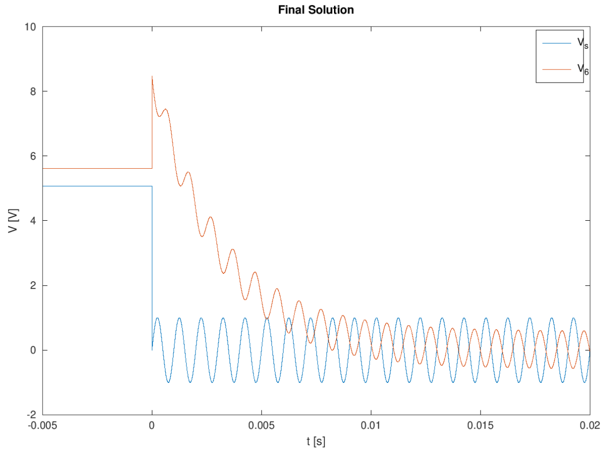


Figure 7: $V_6$

## 2.4 Frequency Response

In this part, the frequency was altered from 0.1 Hz to 1 MHz and the frequency response of $V_6$, $V_s$, $V_c = V_6 - V_8$ were plotted.

The transfer functions calculated for the frequency response are:

$$T_{V_6} = \frac{e^{-i\phi_6}}{e^{-i\frac{\pi}{2}}}$$

$$T_{V_c} = \frac{e^{-i\phi_6} - e^{-i\phi_6}}{e^{-i\frac{\pi}{2}}}$$

$$T_{V_s} = \frac{e^{-i\frac{\pi}{2}}}{e^{-i\frac{\pi}{2}}}$$

The absolute values are plotted here:

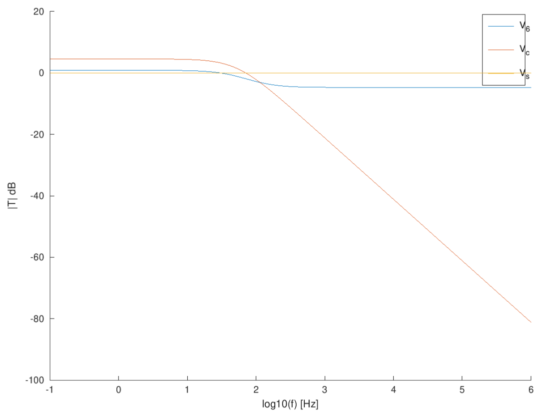


Figure 8: Absolute Value

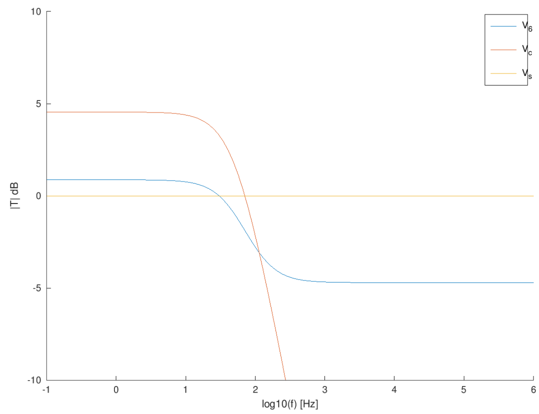


Figure 9: Zoom of Absolute Value

The phases are plotted here:

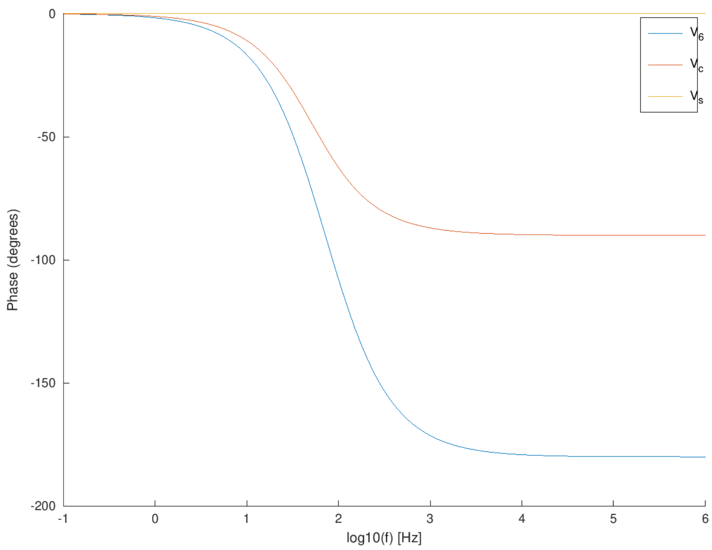


Figure 10: Phase

As you can see from the previous figures, it is possible to conclude that the circuit is analysis is a low pass filter, since for frequencies in the range of 0.1 to 10 Hz the absolute value of the transfer function is approximately 1 and the phase angle is approximately 0, which means that their value are basically the same. On the other hand, for larger frequencies, larger than 1000 Hz, the absolute value of the transfer function is much lower because the nominator is almost 0, while the phase angle is -180$^o$.

## 3 Simulation Analysis

### 3.1 t$<$0

The following tables show the simulated and theoretical operating point results for the study case circuit, for t$<$0.

Note that aldr is not a real node, but it is needed for ngspice calculations.

| Name | Value [V] |
|---|---|
| v(1) | 5.076387e+00 |
| v(2) | 4.828240e+00 |
| v(3) | 4.317615e+00 |
| v(5) | 4.862301e+00 |
| v(6) | 5.613732e+00 |
| v(7) | -1.94693e+00 |
| v(8) | -2.95363e+00 |
| aldr | -1.94693e+00 |

Table 1: Simulation Results

| | V (V) |
|---|---|
| V1 | 5.076387 |
| V2 | 4.828240 |
| V3 | 4.317615 |
| V5 | 4.862301 |
| V6 | 5.613732 |
| V7 | -1.946929 |
| V8 | -2.953629 |

Table 2: Theoretical results.

### 3.2 t $=$ 0

The following tables show the simulated and theoretical operating point results for the study case circuit, for t$=$0. Note that aldr is not a real node, but it is needed for ngspice calculations.

| Name | Value [V] |
|---|---|
| v(1) | 0.000000e+00 |
| v(2) | 2.058976e-15 |
| v(3) | 6.295837e-15 |
| v(5) | 1.776357e-15 |
| v(6) | 8.567360e+00 |
| v(7) | -8.75275e-16 |
| v(8) | -1.77636e-15 |
| aldr | -8.75275e-16 |

Table 3: Simulation Results

| | V (V) |
|---|---|
| V2 | 0.000000 |
| V3 | -0.000000 |
| V5 | 0.000000 |
| V6 | 8.567360 |
| V7 | 0.000000 |
| V8 | 0.000000 |

Table 4: Theoretical results.

To simulate the operating point for $V_s(0)$, the capacitor was replaced with a voltage source $V_x$ = V(6)-V(8), where V(6) and V(8) are the voltages in nodes 6 and 8 obtained in 3.1.. This step is needed because the capacitor will discharge through the resistor but this takes time, therefore: $V_c = V_c(t < 0) = V(6) - V(8)$. This is necessary to calculate the ”base” voltage in the specified nodes, to be used as a boundary condition in the following analyses. Note that even though the results may seem different, they are quite similar. Note that the powers in the simulation results are quite low, so they are approximately zero.

### 3.3 Natural response using 3.2’s boundary conditions

The Table 11 shows the simulated natural solution results for the study case circuit. It is possible to denote that the results are similar. Both cases show V(6).

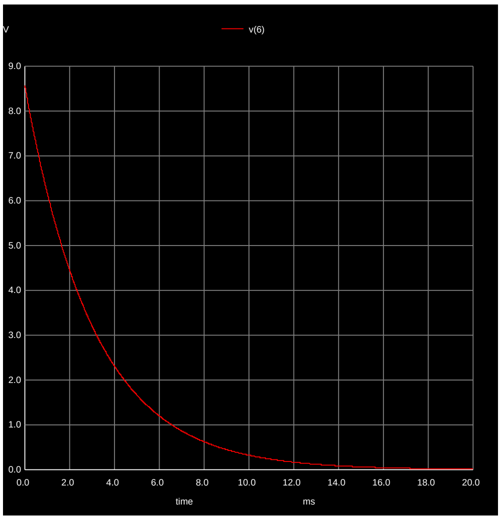


Figure 11: Natural response from ngspice

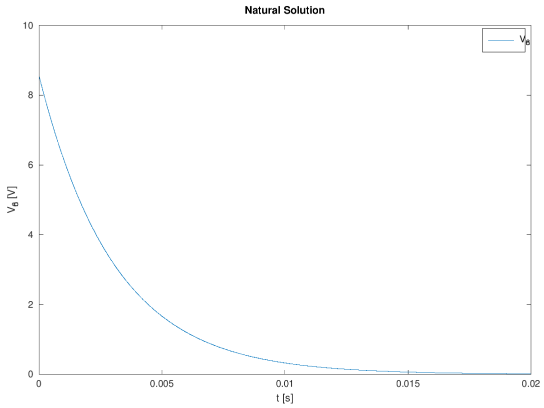


Figure 12: Natural response from matlab

## 3.4 Natural and forced response using 3.2's boundary conditions

Figure 13 shows the simulated Natural and forced response results for the study case circuit, calculated from ngspice.

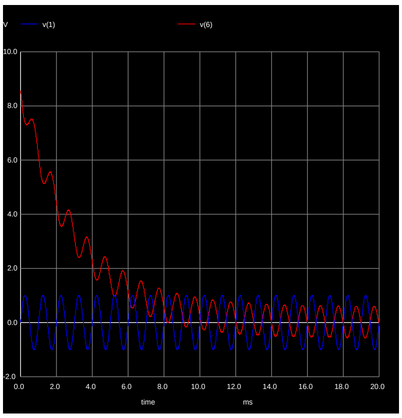


Figure 13: Stimulus and Response from Ngspice

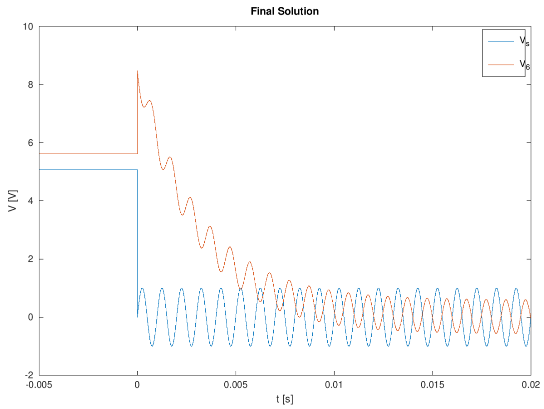


Figure 14: Stimulus and Response from Matlab

### 3.5 Frequency response using 3.2's boundary conditions

As explained in 2.4, the changes in the frequency and phase responses are characteristic of a low passing filters.

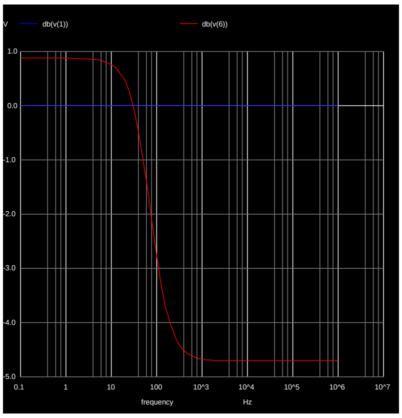

Figure 15: Frequency response from ngspice

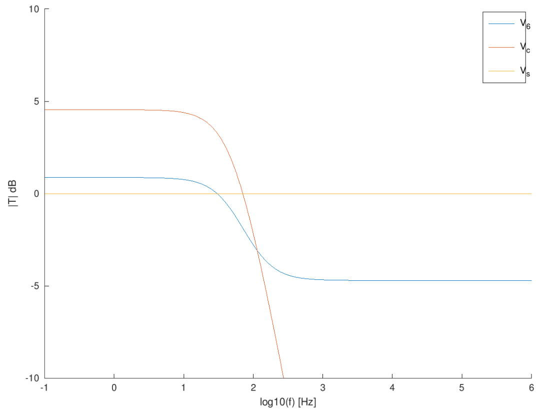

Figure 16: Frequency response from matlab

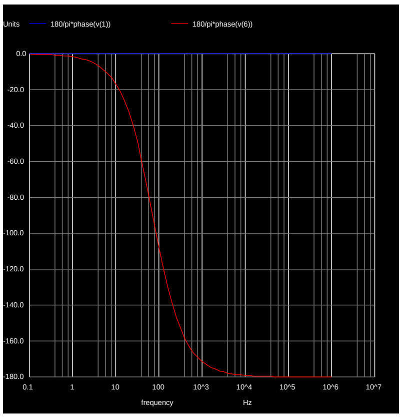

Figure 17: Phase in degrees from ngspice

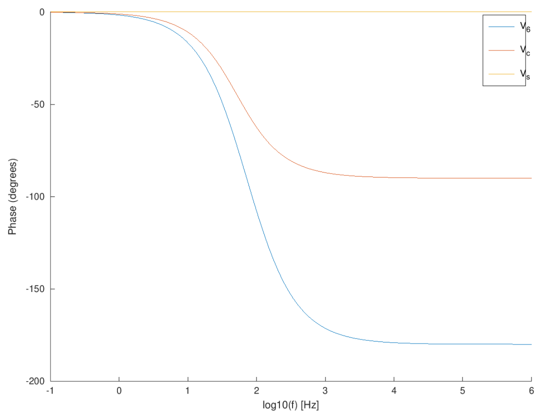

Figure 18: Phase in degrees from matlab

# 4 Conclusion

Even though the theoretical and simulation models might not always give the same results, in our case of study the outcomes were precisely matched. This was in line with what was expected since we were presented with linear components and, to solve the circuit, ngspice utilized the same methods as we did in the theoretical Octave calculation. In conclusion, we believe that the goals of this report were achieved.