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# Exame de Recurso - Parte I solução abreviada

1a)

$$\begin{cases} V_{\beta A} = (R_1 // R_2 // R_3) I_A = 7.5\text{V} \\ I_{1A} = \dfrac{V_{\beta A}}{R_1} = 3.75\text{mA} \end{cases} \quad ou \quad \begin{cases} I_{1A} = \dfrac{R_2 // R_3}{R_1 + R_2 // R_3} I_A = 3.75\text{mA} \\ V_{\beta A} = R_1 I_{1A} = 7.5\text{V} \end{cases} \quad ou \quad \ldots$$

$$I_{1B} = \frac{-V_B}{R_1 + R_2 // R_3} = -2.25\text{mA} \qquad V_{\beta B} = (R_2 // R_3)(-I_{1B}) = 7.5\text{V}$$

$$I_1 = I_{1A} + I_{1B} = 1.5\text{mA} \qquad V_\beta = V_{\beta A} + V_{\beta B} = 15\text{V}$$

1b)

$$P_A = (-V_\alpha) I_A = -(V_\beta - V_B) I_A = -18\text{mW} \quad \Rightarrow \quad \text{fornece energia}$$

$$P_B = V_B (I_1 - I_A) = -54\text{mW} \quad \Rightarrow \quad \text{fornece energia}$$

1c)

$$R_{eq} = R_1 + R_2 // R_3 = \frac{16}{3}\text{k}\Omega \qquad I_{eq} = I_{\beta\alpha} = \frac{R_1}{R_1 + R_2 // R_3}(-I_A) = -2.25\text{mA}$$

2a)

correntes circulação nas malhas (sentido horário): $\{I_E, I_C, I_D\}$

$$\begin{cases} I_E = I_A \\ -V_B + R_2 (I_C - I_D) + R_1 (I_C - I_E) = 0 \\ R_3 I_D + R_2 (I_D - I_C) = 0 \end{cases} \quad \rightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ -(1+k) R_1 & (1+k) R_1 + R_2 & -R_2 \\ 0 & -R_2 & R_2 + R_3 \end{bmatrix} \times \begin{bmatrix} I_E \\ I_C \\ I_D \end{bmatrix} = \begin{bmatrix} I_A \\ 0 \\ 0 \end{bmatrix}$$

Eq. auxiliar: $V_B = kV_\alpha = k\left[R_1 (I_E - I_C)\right]$

2b)

potenciais nodais: $\{V_\alpha, V_\beta\}$

$$\begin{cases} V_\beta - V_\alpha = kV_\alpha \\ I_A = \dfrac{V_\alpha}{R_1} + \dfrac{V_\beta}{R_2} + \dfrac{V_\beta}{R_3} \end{cases} \quad \rightarrow \quad \begin{bmatrix} 1+k & -1 \\ 1/R_1 & 1/R_2 + 1/R_3 \end{bmatrix} \times \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \begin{bmatrix} 0 \\ I_A \end{bmatrix}$$

# Exame de Recurso - Parte I solução abreviada

3a) circuito em regime estacionário

$$\begin{cases} V_C = V_\beta = (R_1//R_2)I_A = 2.4\text{V} \\ I_2 = -I_L = \dfrac{V_\beta}{R_2} = 12\text{mA} \end{cases} \quad ou \quad \begin{cases} I_2 = -I_L = \dfrac{R_1}{R_1+R_2}I_A = 12\text{mA} \\ V_C = V_\beta = R_2 I_2 = 2.4\text{V} \end{cases} \quad ou \quad \ldots$$

$$W_L = \frac{1}{2}LI_L^2 = 2.16\mu\text{J} \qquad W_C = \frac{1}{2}CV_C^2 = 2.88\mu\text{J} \qquad W_T = W_L + W_C = 5.04\mu\text{J}$$

3b)

$$i_1(t) = v_\alpha(t)/R_1 = v_C(t)/R_1$$

$t = 0^-$ regime estacinonário $v_C(0^-) = (R_1//R_2)i_A(0^-) = 2.4\text{V}$

continuidade da tensão no condensador $v_C(0^-) = v_C(0) = v_C(0^+)$

$t = +\infty$ regime estacinonário $v_C(+\infty) = (R_1//R_2)i_A(+\infty) = -1.05\text{V}$

$$v_C(t) = K_1 + K_2 e^{-\frac{t}{\tau}} \quad , \quad t \ge 0s \qquad K_1 = v_C(+\infty) \qquad K_1 + K_2 = v_C(0)$$

$$\tau = (R_1//R_2)C = 150\mu s$$

$$v_C(t) = \begin{cases} 2.4\text{V} & , \quad t<0s \\ -1.05 + 3.45e^{-\frac{t}{1.5\times10^{-4}}}V & , \quad t \ge 0s \end{cases} \qquad i_1(t) = \begin{cases} 4\text{mA} & , \quad t<0s \\ -1.75 + 5.75e^{-\frac{t}{1.5\times10^{-4}}}mA & , \quad t \ge 0s \end{cases}$$

4a)

$$\overline{I_a} = 20e^{-j\pi/4}mA \qquad Z_C = \frac{1}{j\omega C} \qquad Z_L = j\omega L$$

$$\overline{I_{\alpha\beta}} = \frac{R_1}{R_1 + (Z_L + R_2//Z_C)}\overline{I_a} \qquad \overline{V_\beta} = (R_2//Z_C)\overline{I_{\alpha\beta}} = 2.19e^{-j104^\circ\times\pi/180^\circ}\text{V} \qquad v_\beta(t) = 2.19\cos\left(2\pi 10^3 t - 104^\circ\frac{\pi}{180^\circ}\right)\text{V}$$

4b)

$$Z_{eq} = R_1//\left[Z_L + (R_2//Z_C)\right] = 105e^{j42^\circ\pi/180^\circ}\Omega \qquad Z_{eq} = \frac{\overline{V}}{\overline{I}} = \frac{V_M}{I_M}e^{j(\theta_V - \theta_I)} \qquad fp = \cos(\theta_V - \theta_I) = \cos(\theta_Z) = 0.744$$

4c)

$$T(s) = \frac{I_2(s)}{I_a(s)} = \frac{I_2(s)}{I_{\alpha\beta}(s)}\frac{I_{\alpha\beta}(s)}{I_a(s)} = \frac{Z_C}{R_2+Z_C}\frac{R_1}{R_1+Z_L+(R_2//Z_C)} = \frac{R_1}{s^2(R_2LC) + s(L + R_1R_2C) + R_1 + R_2}$$

$$T(0) = \frac{R_1}{R_1+R_2} \qquad T(s\to+\infty) = 0 \qquad \text{Passa-baixo}$$