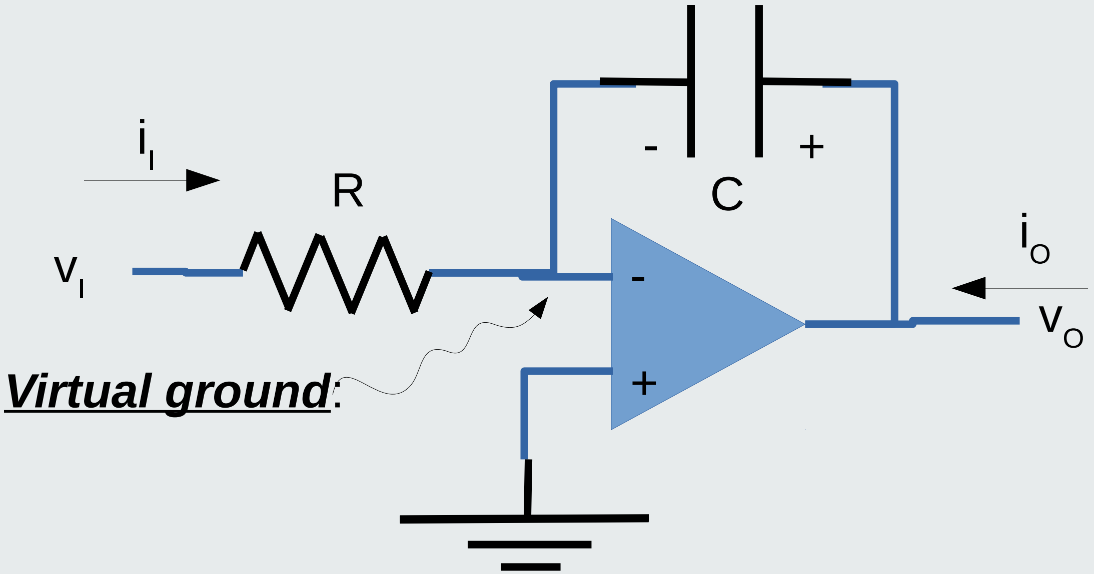
Circuit Theory and Electronics Fundamentals

Lecture 24: Operational Amplifier Integrator and Differentiator Circuits

- Integrator circuit
- Lossy integrator circuit
- Differentiator circuit

Integrator OP-AMP Circuit



$$i_I = -C \frac{d v_o}{dt}$$

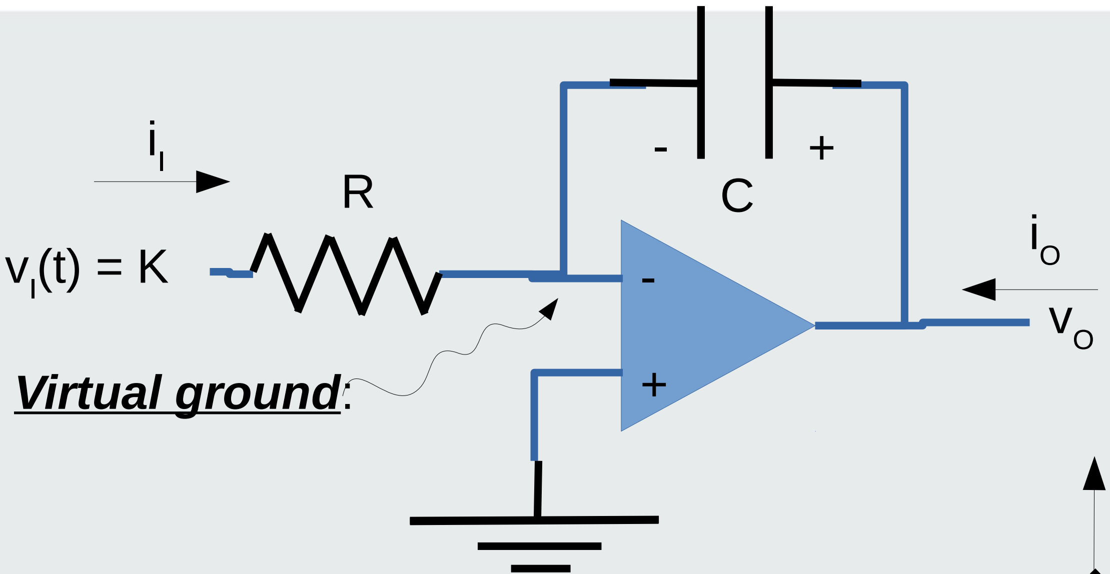
$$d v_O = -\frac{v_I}{RC} dt$$

$$\int_{v_O(0)}^{v_O(t)} d v_O = -\frac{1}{RC} \int_0^t v_i(t') dt'$$

$$v_O(t) = v_O(0) - \frac{1}{RC} \int_0^t v_i(t') dt'$$

Inverting amp with input resistor and feedback capacitor

Integrator time response for constant input

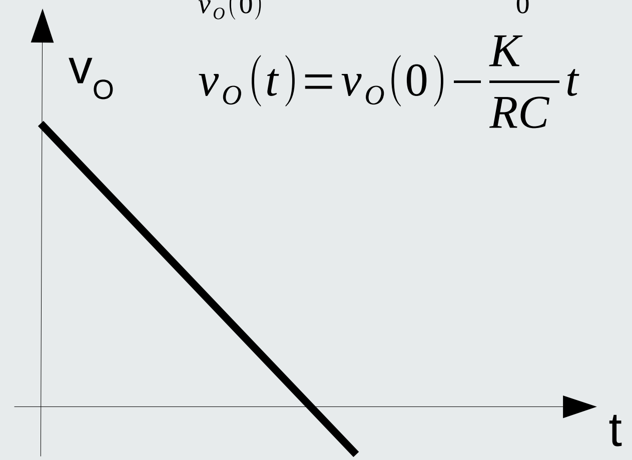


$$i_I = -C\frac{dv_o}{dt}$$

$$dv_O = -\frac{K}{RC}\,dt$$

$$\int_{v_O(0)}^{v_O(t)} dv_O = -\frac{1}{RC}\int_0^t K\,dt'$$

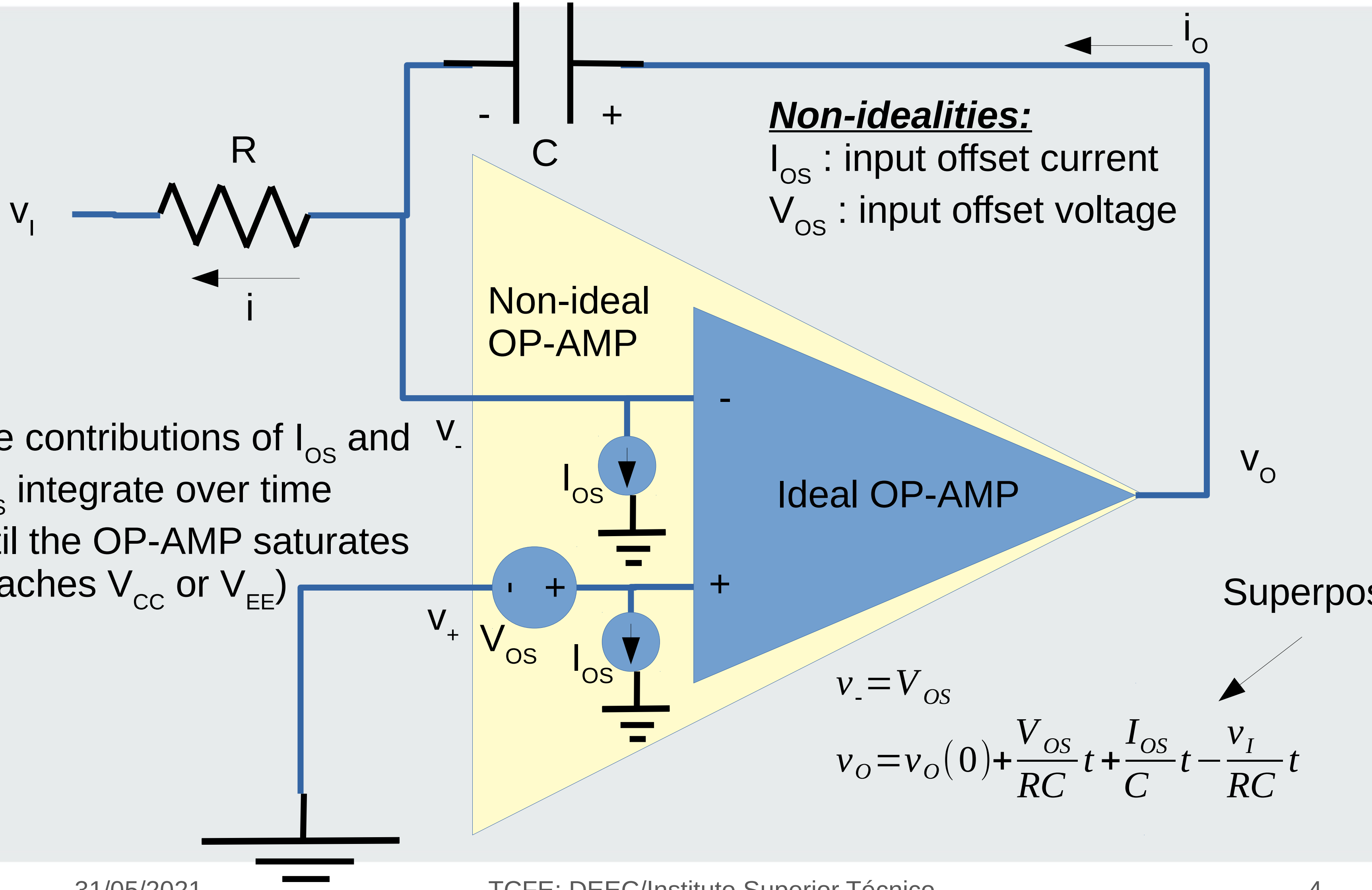
$$v_O(t) = v_O(0) - \frac{K}{RC}t$$



Capacitor forced to charge on constant current
V_O varies linearly in time!

Since constants are integrated to infinity, the integrator circuit **does not work** in practice due to OP-AMP non-idealities called ***offset input current*** and ***offset input voltage***

OP-AMP non-idealities effect on integrator circuit



Non-idealities:

I_{OS} : input offset current

V_{OS} : input offset voltage

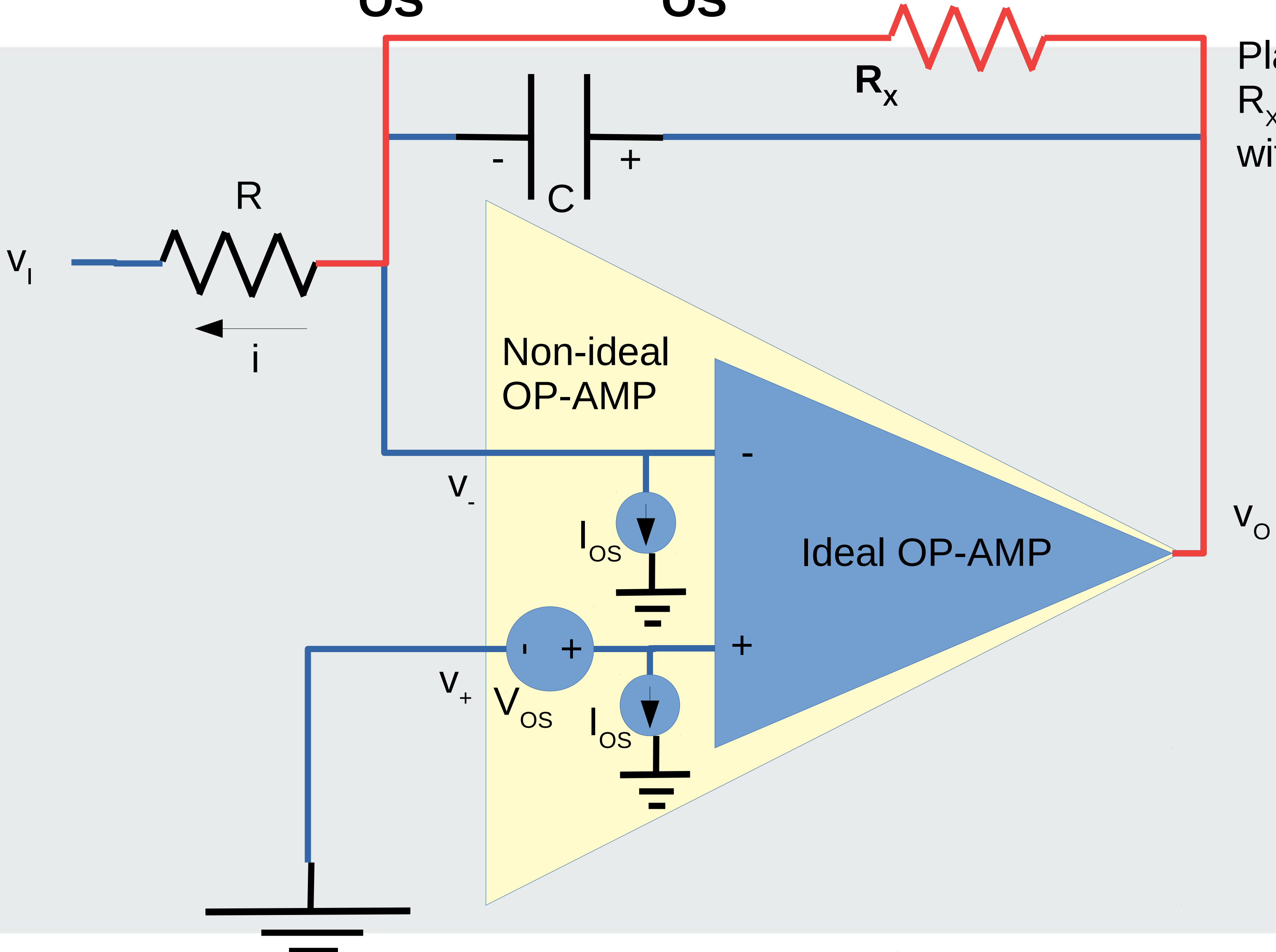
The contributions of I_{OS} and V_{OS} integrate over time until the OP-AMP saturates (reaches V_{CC} or V_{EE})

Superposition

$$v_- = V_{OS}$$

$$v_O = v_O(0) + \frac{V_{OS}}{RC} t + \frac{I_{OS}}{C} t - \frac{v_I}{RC} t$$

Lossy Integrator: solution to I_{OS} and V_{OS} non-idealities



Place resistor R_X in parallel with C

Integrator vs. Lossy Integrator: transfer function and frequency response

Integrator transfer function and frequency response:

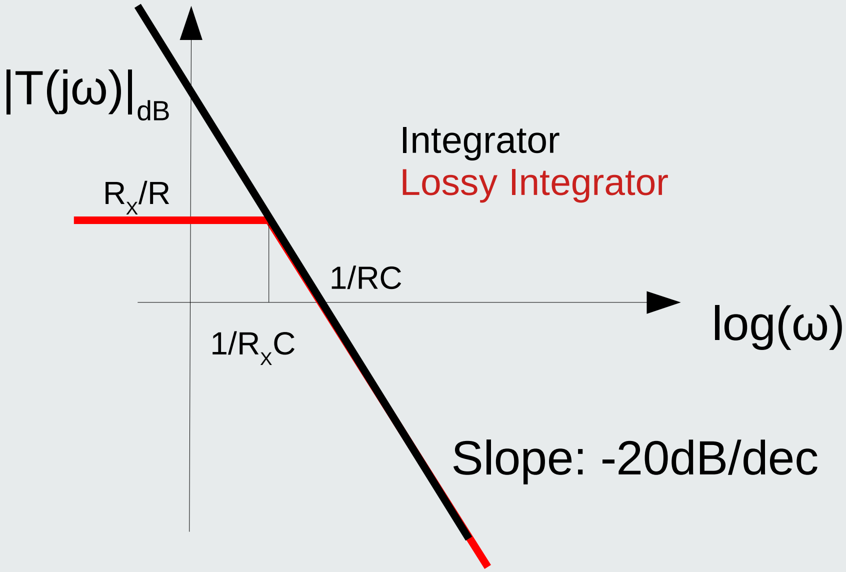
$$T(s) = -\frac{Z_2(s)}{Z_1(s)} = -\frac{\frac{1}{sC}}{R} = -\frac{1}{sRC}$$

$$T(j\omega) = -\frac{1}{j\omega RC}$$

Lossy integrator transfer function and frequency response:

$$T(s) = -\frac{Z_2(s)}{Z_1(s)} = -\frac{\frac{1}{\frac{1}{R_X}+sC}}{R} = -\frac{R_X}{R}\frac{1}{1+sR_XC}$$

$$T(j\omega) = -\frac{R_X}{R}\frac{1}{1+j\omega R_XC}$$



Lossy Integrator Advantages:

- Saturation is avoided
- Good performance at medium to high frequencies

Lossy Integrator Disadvantages:

- The low frequency gain is limited at R_X/R
- The integration is imperfect at low frequencies

Lossy Integrator output DC offset voltage and current

At DC the capacitor voltage is an open circuit

Compute the DC output voltage for $V_I = 0$ in the presence of V_{OS} and I_{OS}

$$v_I = 0, \;\; i_C = 0$$

$$V_O = \left(1 + \frac{R_X}{R}\right) V_{OS} + R_X I_{OS}$$

DC output offset at output

V_{OS} sees non inverting amp.

I_{OS} sees only the R_X resistor

R gets zero current because its voltage drop is 0: both v_I and v_- are 0 V.

Superposition of V_{OS} and I_{OS} effects

Lossy Integrator step response

$v_I = u(t)$ Heaviside step function

$$u(t)=\begin{cases} 1, & t \geq 0 \\ 0, & t<0 \end{cases}$$

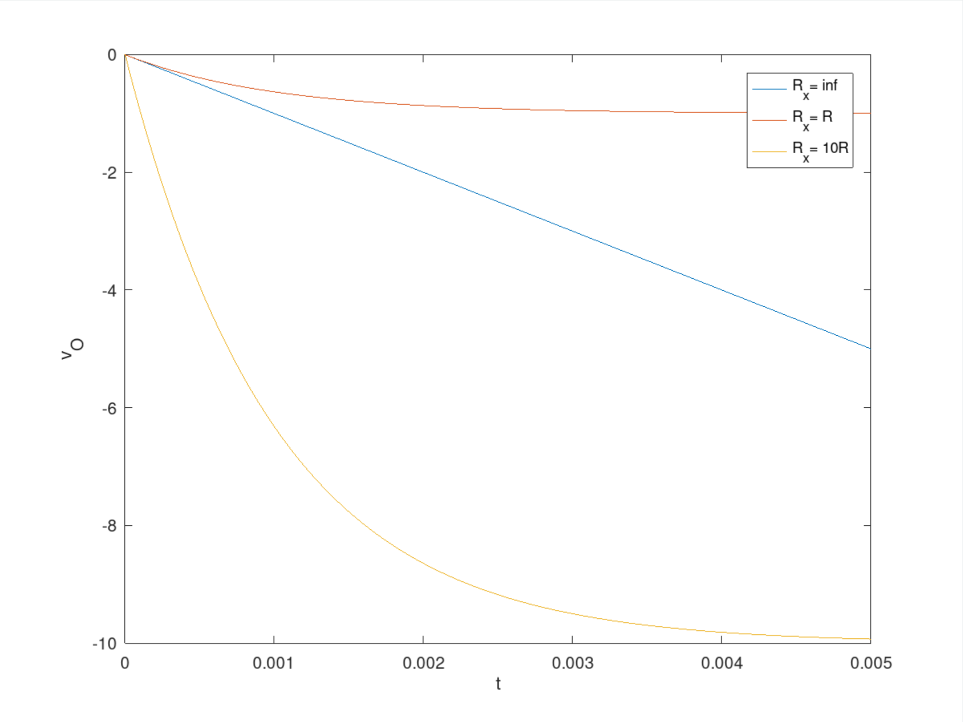
$$v_O = v_O(0) - \frac{1}{RC}\int_0^t 1\, dt' = v_O(0) - \frac{t}{RC}$$

$$v_{O\,lossy} = v_O(\infty) + (v_O(0) - v_O(\infty))\, e^{-\frac{t}{RC}}$$

$$v_{O\,lossy} = -\frac{R_X}{R} + \left(v_O(0) + \frac{R_X}{R}\right) e^{-\frac{t}{RC}}$$

$$v_{O\,lossy} = -\frac{R_X}{R} + \left(v_O(0) + \frac{R_X}{R}\right)\left(1 - \frac{t}{RC} + \ldots\right)$$

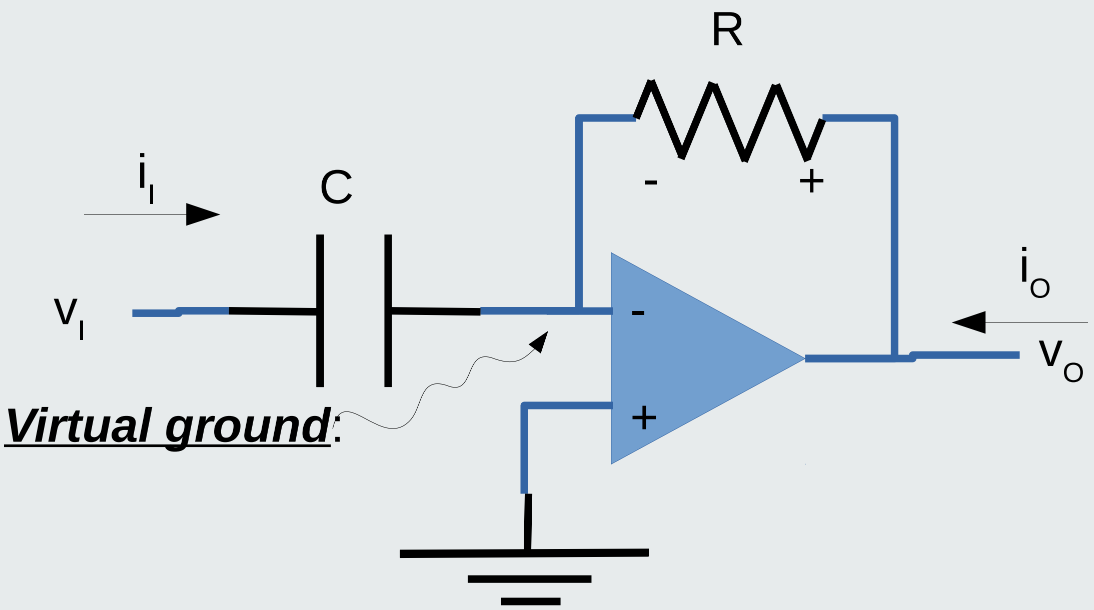
(Taylor expansion)



R_X=R: looks like normal integrator time response for a short while

RX>R: looks like normal integrator time response for longer but has a gain

Differentiator OP-AMP Circuit

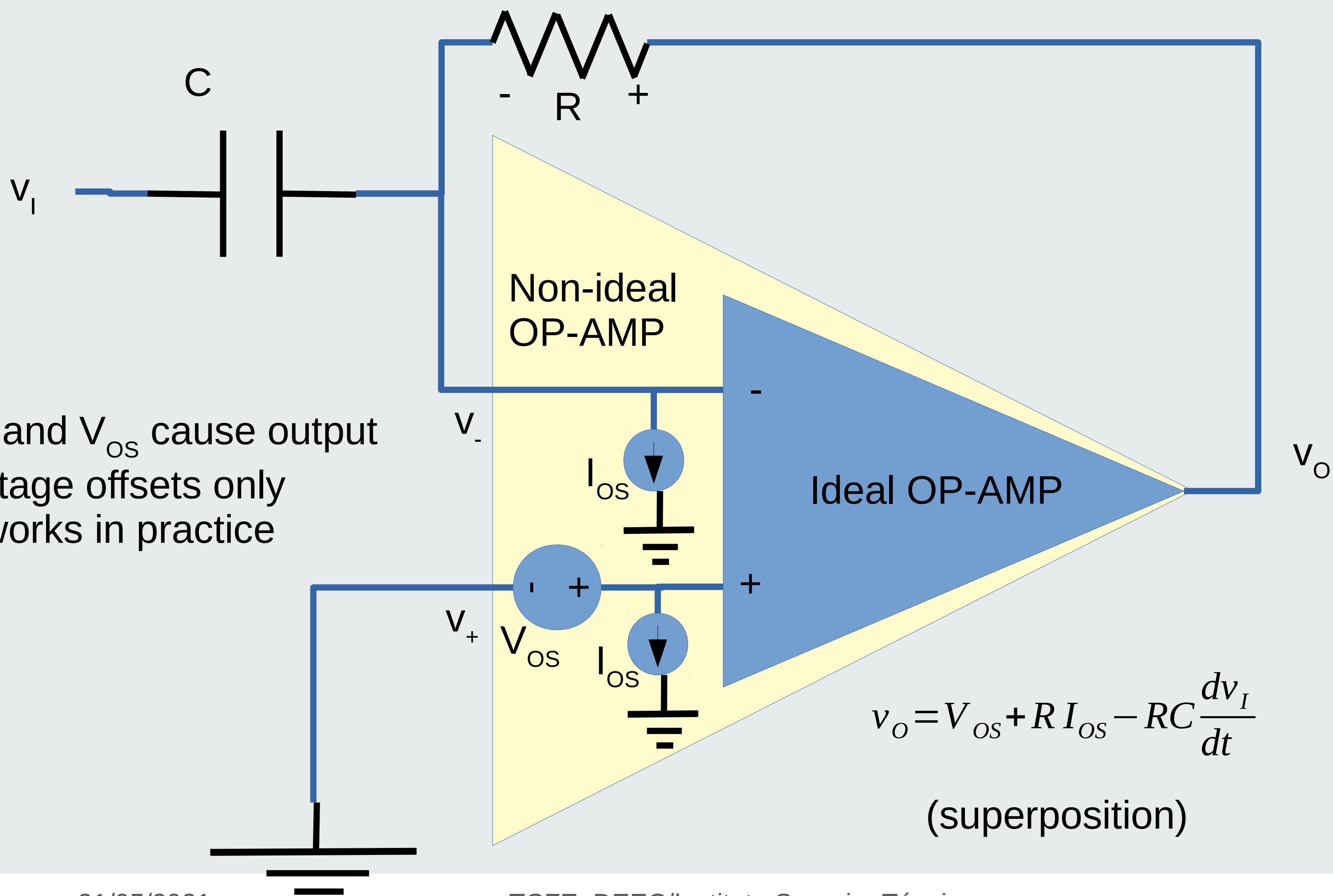


$$i_I = C\frac{dv_I}{dt}$$

$$v_O = -R\,i_I = -RC\frac{dv_I}{dt}$$

Inverting amp with input capacitor and feedback resistor

OP-AMP non-idealities effect on differentiator circuit

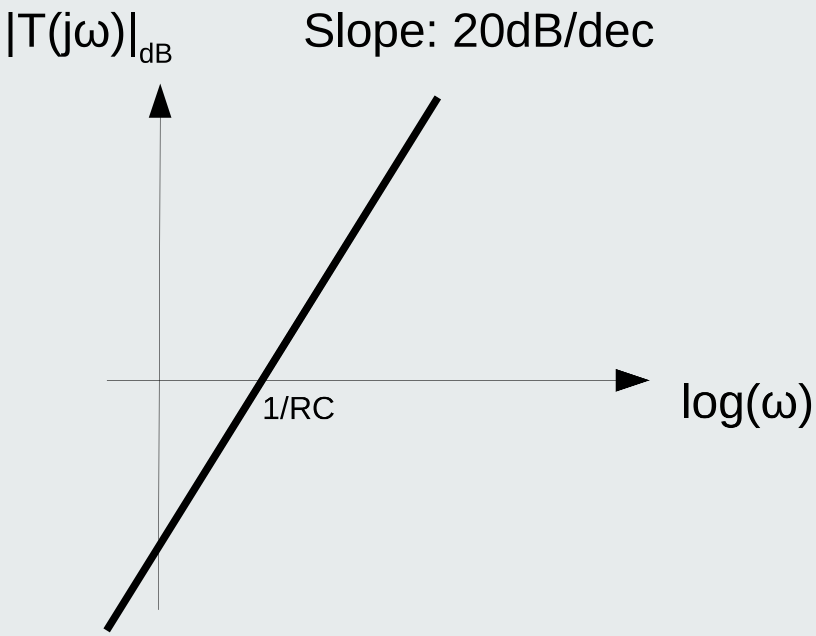


I_{OS} and V_{OS} cause output voltage offsets only – works in practice

$$v_O = V_{OS} + R\,I_{OS} - RC\frac{dv_I}{dt}$$

(superposition)

Differentiator response



Transfer function and frequency response:

$$T(s) = -\frac{Z_2(s)}{Z_1(s)} = -\frac{R}{\frac{1}{sC}} = -sRC$$

$$T(j\omega) = -j\omega RC$$

Time step response

$$v_I = u(t)$$

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$v_O = -RC\frac{dv_I}{dt} = \begin{cases} -RC \times \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

$v_O = -RC\,\delta(t)$ Dirac delta function!

Conclusion

- Integrator circuit
- Lossy integrator circuit
- Differentiator circuit