· Normalização à densidade

$$\int d^3p \, \exp\left(-Ap^2\right) = \int_0^{+\infty} 4\pi p^2 \exp\left(-Ap^2\right) dp$$

integrando em coordenadas esféricas

Mas $\displaystyle\int_0^{+\infty} x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$

Pois

$$\underbrace{\int_0^{+\infty} x^2 e^{-x^2} dx}_{v\,du} = \underbrace{\left[-\frac{1}{2} x e^{-x^2}\right]_{x=0}^{+\infty}}_{=0} + \underbrace{\frac{1}{2}\int_0^{+\infty} e^{-x^2} dx}_{u\,dv}$$

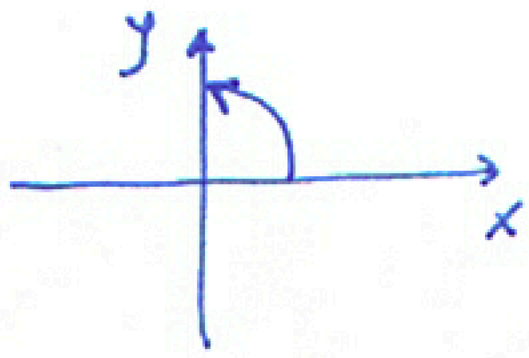
$$\begin{cases} u = e^{-x^2} \\ du = -2x e^{-x^2} dx \end{cases} \qquad \begin{cases} v = -\frac{1}{2} x \\ dv = -\frac{1}{2} dx \end{cases}$$

e

$$\left[\int_0^{+\infty} e^{-x^2} dx\right]^2 = \int_0^{+\infty} e^{-x^2} dx \int_0^{+\infty} e^{-y^2} dy = \int_0^{+\infty}\int_0^{+\infty} e^{-(x^2+y^2)} dx\,dy$$

$$= \int_0^{\pi/2}\int_0^{+\infty} e^{-r^2} r\,dr\,d\varphi = \frac{\pi}{2}\left[-\frac{e^{-r^2}}{2}\right]_0^{+\infty} = \frac{\pi}{4}$$

coordenadas polares



logo $\displaystyle\int_0^{+\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ e $\displaystyle\int_0^{+\infty} x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$

V.S.F.F. →

Assim, $\displaystyle\int_0^{+\infty} 4\pi p^2 \exp(-A p^2)\, dp = \frac{4\pi}{\sqrt{A}} \int_0^{+\infty} \frac{x^2}{A} \exp(-x^2)\, dx =$

$$x = A^{1/2} p$$

$$dx = A^{1/2} dp$$

$$\begin{cases} p = 0, & x = 0 \\ p = \infty, & x = \infty \end{cases}$$

$$= \frac{4\pi}{A^{3/2}} \int_0^{+\infty} x^2 \exp(-x^2)\, dx = \frac{4\pi}{A^{3/2}} \times \frac{\sqrt{\pi}}{4} = \left(\frac{\pi}{A}\right)^{3/2}$$

Finalmente, $\displaystyle n = C \int d^3p \exp(-A p^2) = C \left(\frac{\pi}{A}\right)^{3/2}$ ✓

· <u>Integral na energia</u>

já sabemos que $\int_0^{+\infty} p^2 \exp(-Ap^2)\, dp = \left(\frac{\pi}{A}\right)^{3/2} \frac{1}{4\pi} =$

Queremos agora calcular $\int_0^{+\infty} \underbrace{p^4 \exp(-Ap^2)\, dp}_{u\,dv} = \underbrace{\left[-p^3 \frac{1}{2A} \exp(-Ap^2)\right]_0^{\infty}}_{uv} + \int_0^{+\infty} \underbrace{\frac{1}{2A} 3 p^2 \exp(-Ap^2) dp}_{-v\,du}$

$$\begin{cases} u = p^3 \\ du = 3p^2\, dp \end{cases} \qquad \begin{cases} v = -\frac{1}{2A} \exp(-Ap^2) \\ dv = p \exp(-Ap^2) \end{cases}$$

$$= 0 + \frac{3}{2A} \underbrace{\int_0^{+\infty} p^2 \exp(-Ap^2)\, dp}_{\left(\frac{\pi}{A}\right)^{3/2} \frac{1}{4\pi}}$$

$$\varepsilon = \frac{2\pi C}{n m} \int_0^{+\infty} p^4 \exp(-Ap^2)\, dp = \frac{2\pi C}{n m} \times \frac{3}{2A} \left(\frac{\pi}{A}\right)^{3/2} \frac{1}{4\pi}$$

$$= \frac{C\,3}{4 n m} \left(\frac{\pi}{A}\right)^{3/2} \frac{1}{A} = \frac{3}{4mA}$$

$$C = \left(\frac{A}{\pi}\right)^{3/2} n$$

$$A = \frac{3}{4m\varepsilon} \quad , \quad C = n \left(\frac{3}{4\pi m \varepsilon}\right)^{3/2}$$