TÉCNICO LISBOA

Circuit Theory and Electronics Fundamentals

Lecture 10: Filter Circuits

- Solving any order circuits (continued from last lesson)
- Introduction to filters and an example filter
- Ideal transformer models (they have not been forgotten!)
- Filter qualitative and quantitative analyses
- Filter transfer function in various RLC configurations
- Second order Bode plots

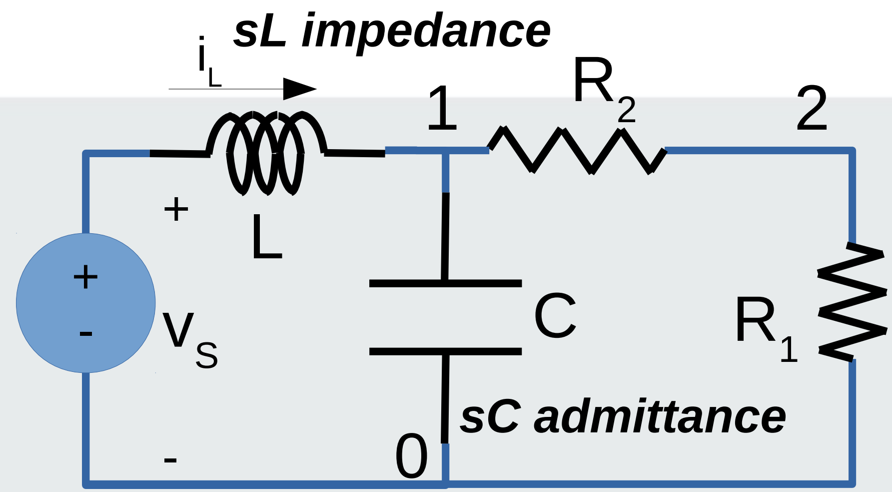
Any order circuits general solution

Nodal analysis:

$$\begin{bmatrix} \frac{1}{sL}+sC+G_2 & -G_2 \\ -G_2 & G_1+G_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} \frac{1}{sL}+sC+G_2 & -G_2 \\ -G_2 & G_1+G_2 \end{vmatrix} = 0$$

$$\left(\frac{1}{sL}+sC+G_2\right)(G_1+G_2)-G_2^2=0$$



Second order RLC circuit

Characteristic equation: assume 2 real roots s_1 and s_2

$$\begin{bmatrix} \frac{1}{s_1 L}+s_1 C+G_2 & -G_2 \\ -G_2 & G_1+G_2 \end{bmatrix} \begin{bmatrix} p_1 \\ q_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Eigen vector E_1

$$q_1 = \frac{R_1}{R_1+R_2} p_1$$

$$\begin{bmatrix} \frac{1}{s_2 L}+s_2 C+G_2 & -G_2 \\ -G_2 & G_1+G_2 \end{bmatrix} \begin{bmatrix} p_2 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

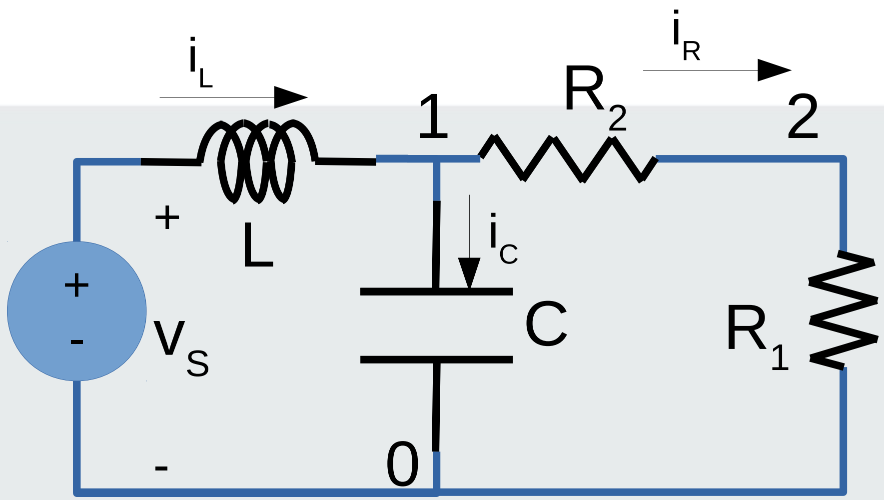
Eigen vector E_2

$E_1 = E_2 = E$

General solution:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{R_1}{R_1+R_2} \end{bmatrix} \left(A e^{s_1 t} + B e^{s_2 t}\right)$$

Any order circuits <u>particular solution</u>



Second order RLC circuit

Solve for A and B

$$\begin{bmatrix} v_1(0) \\ v_2(0) \end{bmatrix} = E\left(A e^{s_1 t} + B e^{s_2 t}\right)$$

$$v_1(0) = v_c(0)$$

$$v_2(0) = \frac{R_1}{R_1 + R_2} v_c(0)$$

Same equation, can't solve!

Need to take into account $i_L(0)$

$$i_L(t) = i_C(t) + i_R(t) \qquad \text{KCL}$$

$$i_c(t) = C\frac{dv_c}{dt} = C\frac{dv_1}{dt} = C\left(A s_1 e^{s_1 t} + B s_2 e^{s_2 t}\right)$$

$$i_R(t) = \frac{v_C}{R_1 + R_2} = \frac{A e^{s_1 t} + B e^{s_2 t}}{R_1 + R_2}$$

$$\begin{cases} i_L(0) = C\left(A S_1 + B s_2\right) + \dfrac{A + B}{R_1 + R_2} \\ v_C(0) = A + B \end{cases}$$

Now we have two independent equations to compute A and B!

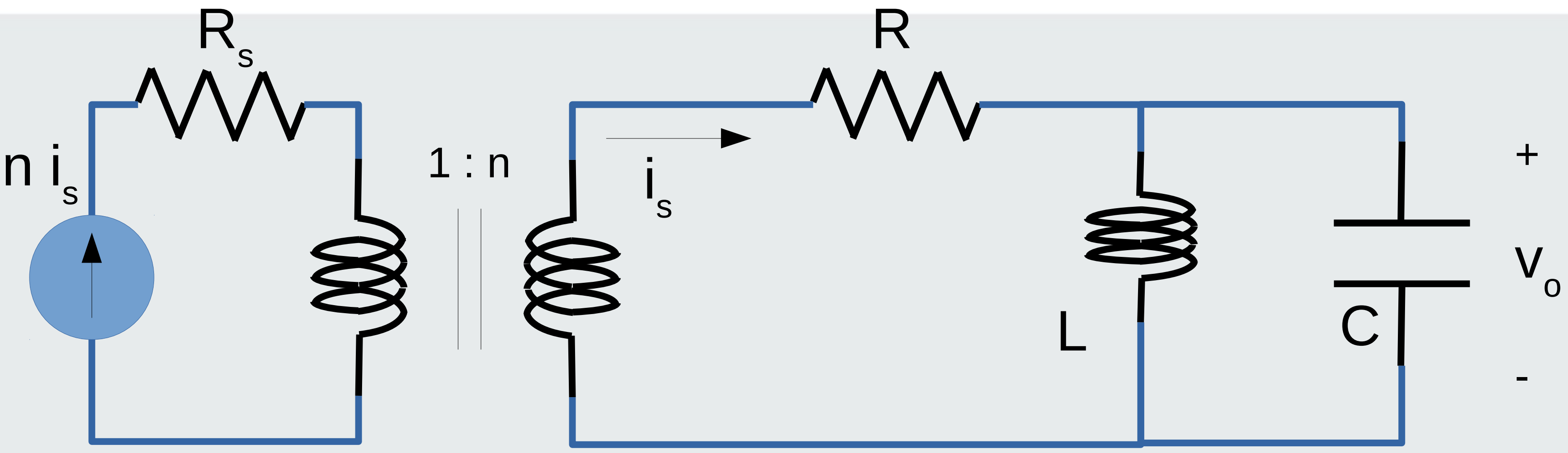
What are filters?

- Filters are circuits that modify the frequency spectrum of an input signal
- Filters are used for
  - Removing the DC component
    - DC component may be bad for transmission for example
  - Removing high frequency (HF) components
    - HF components may be just noise with no info or energy content
  - Isolating the DC component
    - To stabilise a voltage or current source for example
  - Equalising a signal
    - Amplify some frequencies and attenuate others to adapt to an irregular physical channel
  - Many applications

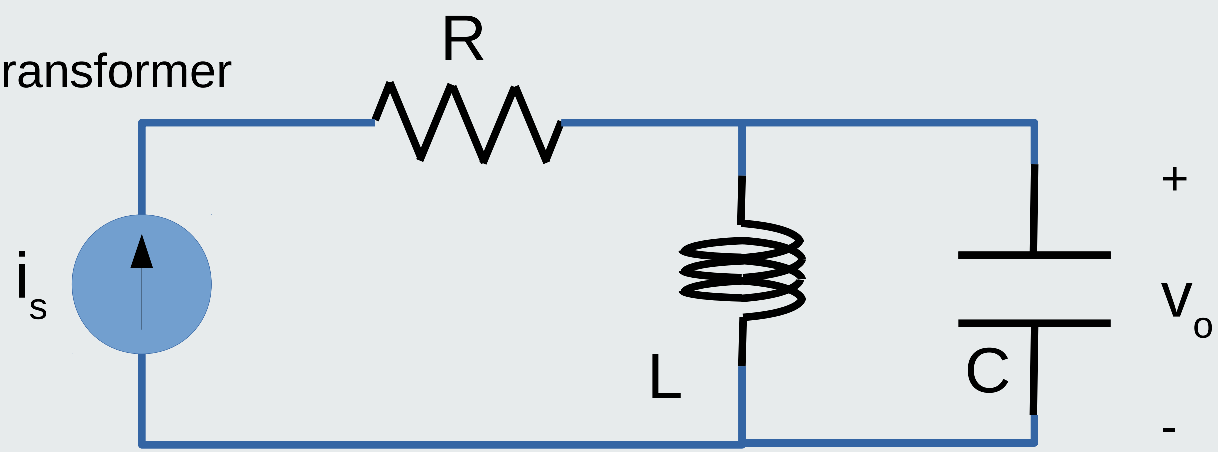
Passive filters

- Filters filters that have passive components: R, L, C and transformers
- In this course we consider passive filters that have
  - One input voltage or current independent source
  - One output voltage or current
  - Any number of resistors
  - 2 complex impedance components
    - 2 capacitive components
    - 2 inductive components
    - 1 capacitive component and 1 inductive component

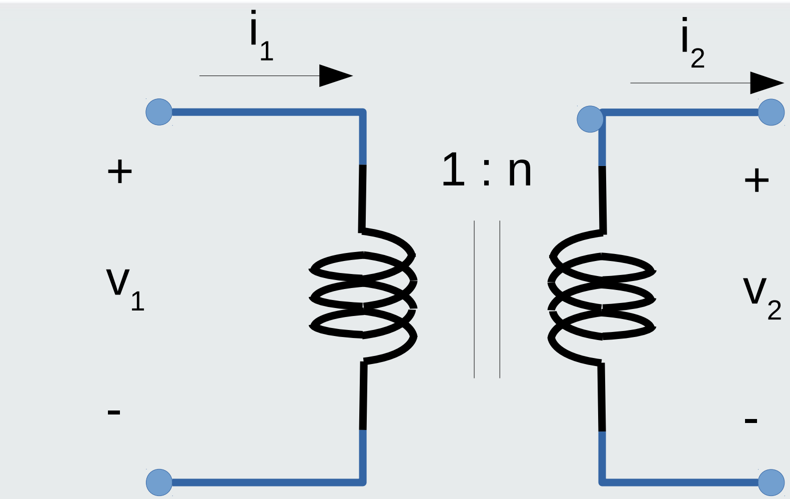
Example filter system

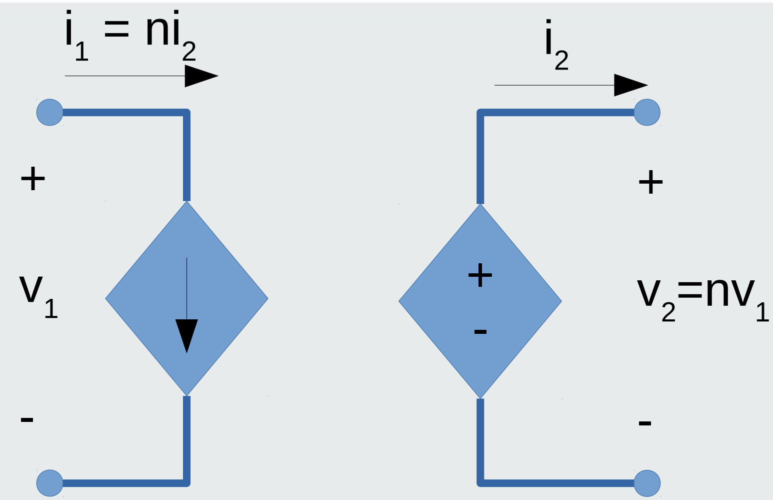


Get rid of transformer

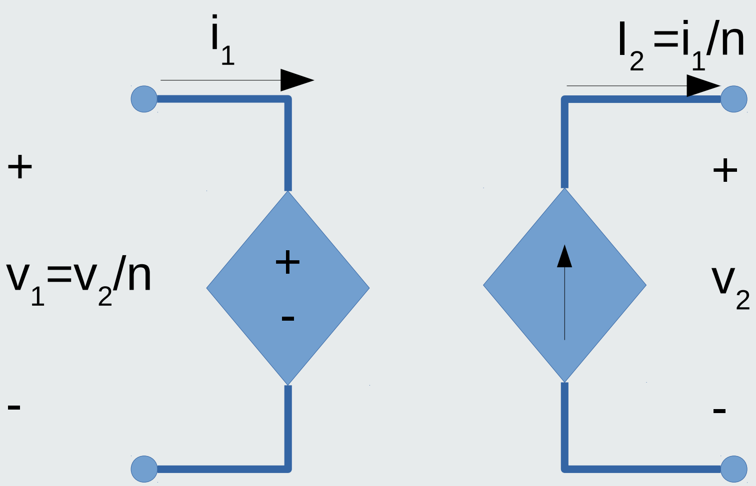


Ideal transformer models





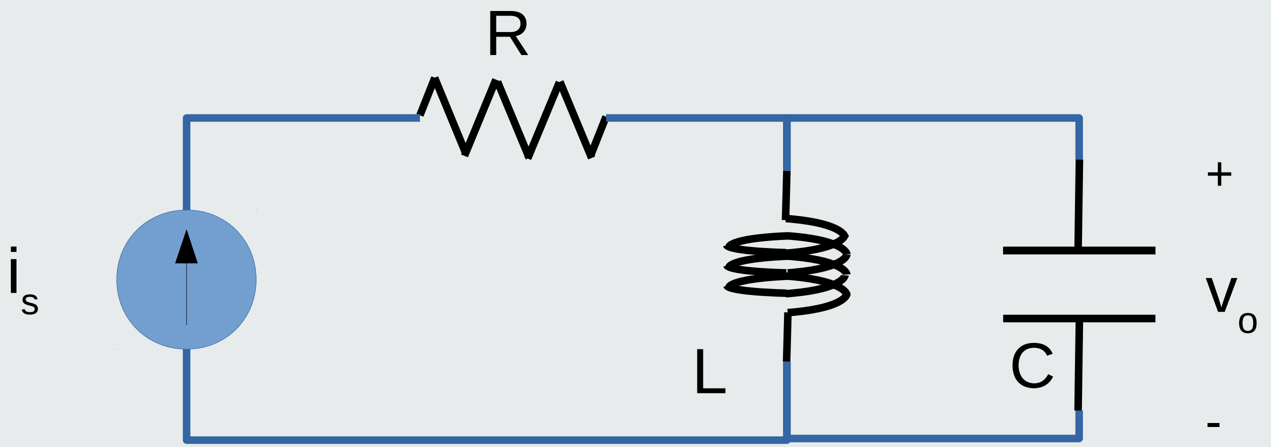
Use if v_1 is known



Use if i_1 is known

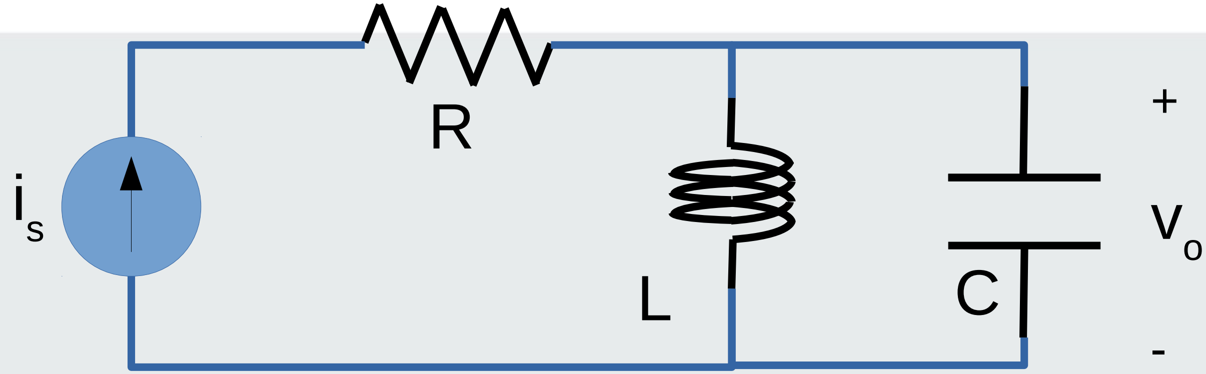
Use this one for circuit in previous slide!

Filter qualitative analysis



- For low frequency the inductor impedance (jωL) is low (short-circuit), and v_o is small – low frequencies are blocked
- For high frequency the capacitor impedance (1/jωC) is low (short-circuit), and v_o is small – high frequencies are also blocked
- For intermediate frequencies, the LC parallel has non-zero impedance Z, and $\tilde{V}_o = Z\,\tilde{I}_s$ will also be non-zero

Filter quantitative analysis



$$\tilde{V}_o = Z \tilde{I}_s$$

$$Z = Z_L \| Z_C = \frac{1}{\frac{1}{j\omega L} + j\omega C}$$

$$\frac{\tilde{V}_o}{\tilde{I}_s} = \frac{j\omega L}{1 - \omega^2 LC}$$

Frequency response = Equivalent impedance

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow Z = \infty \Rightarrow \tilde{V}_o = \infty !!$$

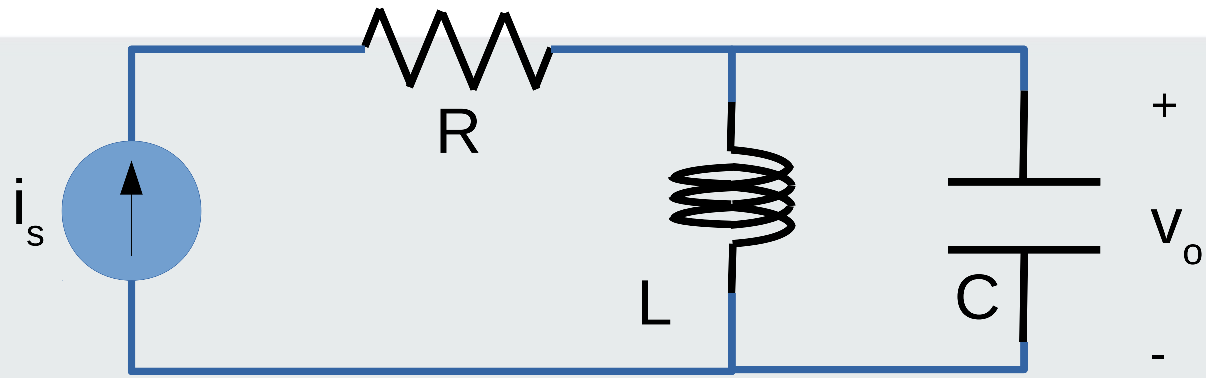
Resonant frequency

- For low frequency v_o is small (L is short-circuit) – low frequencies are blocked
- For high frequency v_o is small (C is short-circuit) – high frequencies are also blocked
- For intermediate frequencies, v_o is non-zero (Z is non-zero)
- For the resonant frequency the voltage magnitude is infinite (V=ZI, with Z large)!!
- Why? Because the ***swing*** is pushed by the source at its natural frequency!!

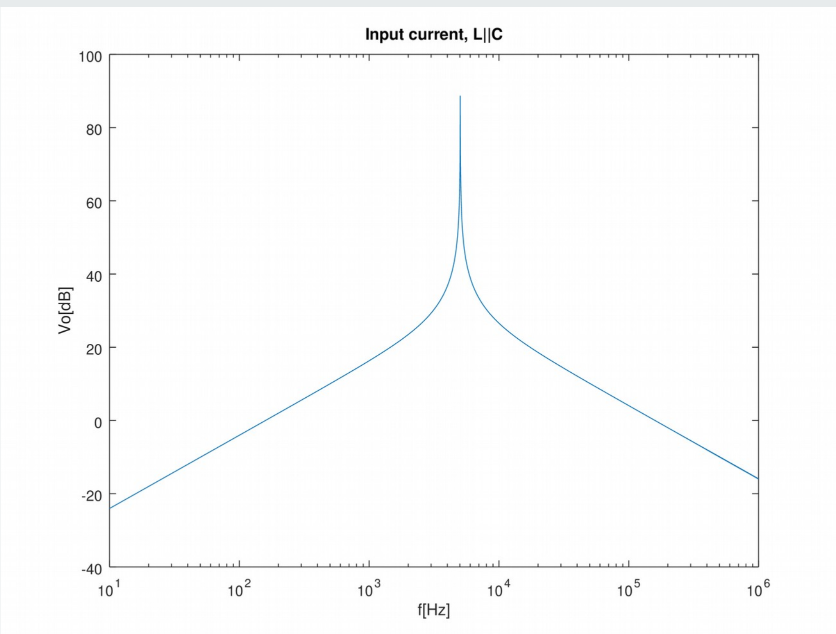
Filter graphical analysis

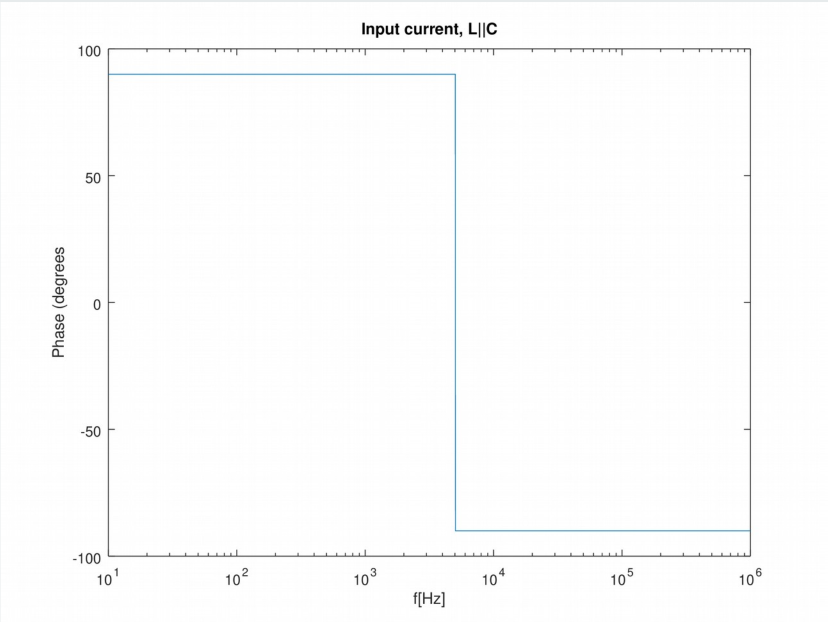
Frequency response

$$\frac{\tilde{V}_o}{\tilde{I}_s} = \frac{j\omega L}{1-\omega^2 LC}$$



Very *selective* Band-Pass Filter

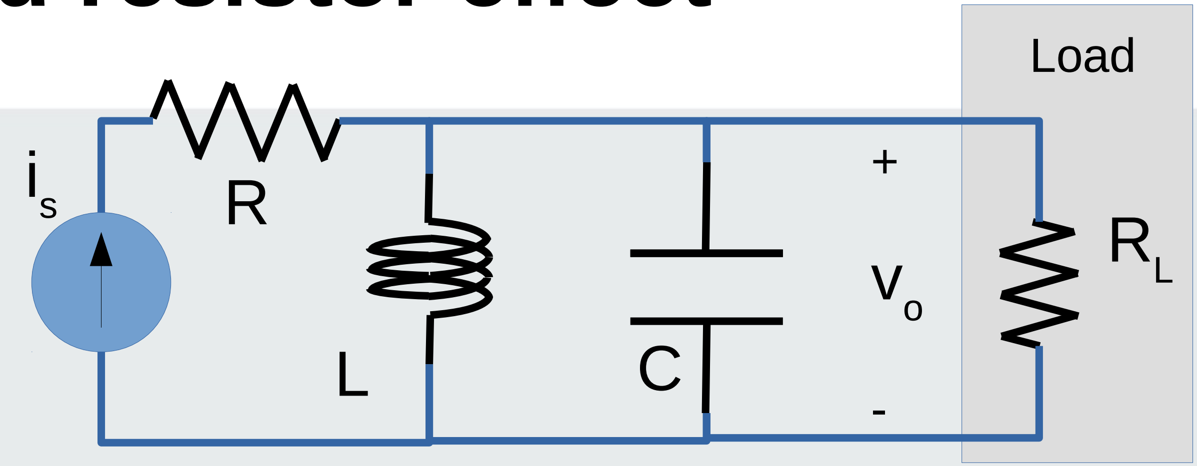




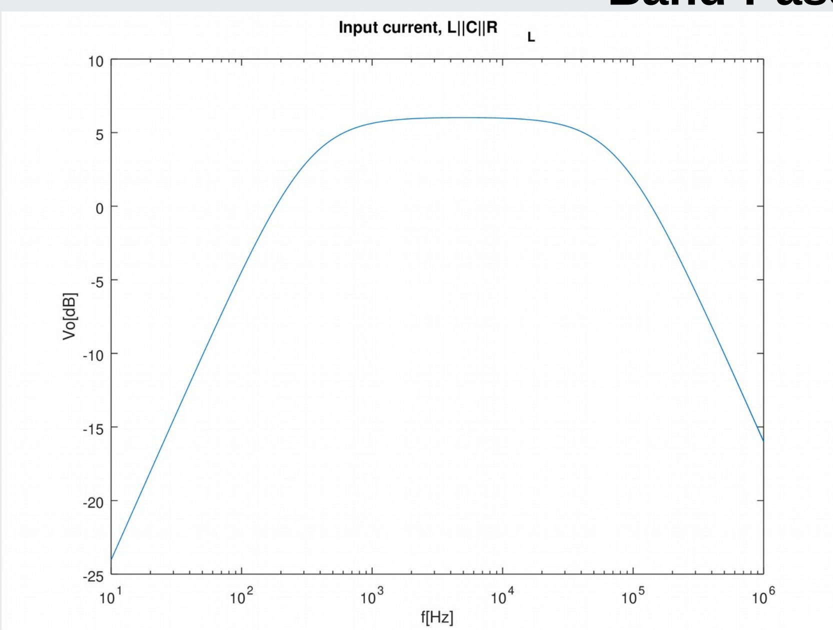
Load resistor effect

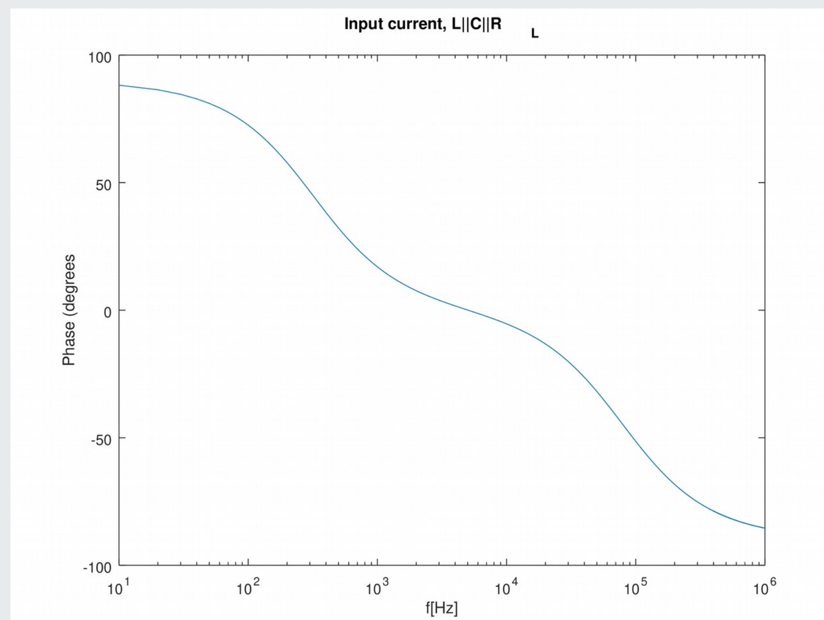
Frequency response

$$\frac{\tilde{V}_o}{\tilde{I}_s} = \frac{1}{\frac{1}{R_L} + \frac{1}{j\,\omega L} + j\,\omega C}$$



Band-Pass Filter



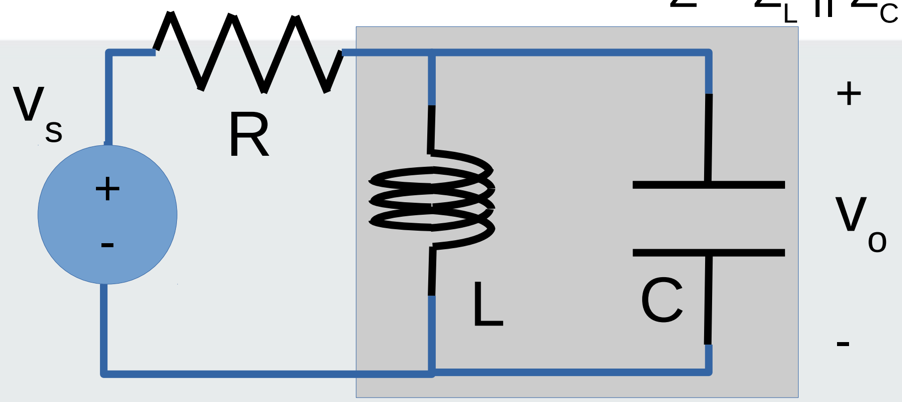


Swap input current source with voltage input source

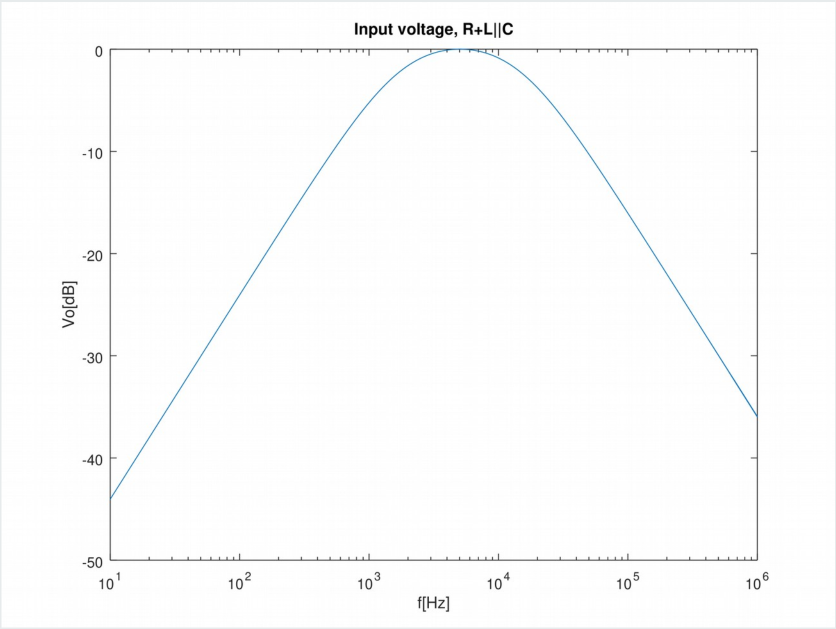
$$\frac{\tilde{V}_o}{\tilde{V}_s}=\frac{\dfrac{1}{\dfrac{1}{j\omega L}+j\omega C}}{R+\dfrac{1}{\dfrac{1}{j\omega L}+j\omega C}}$$

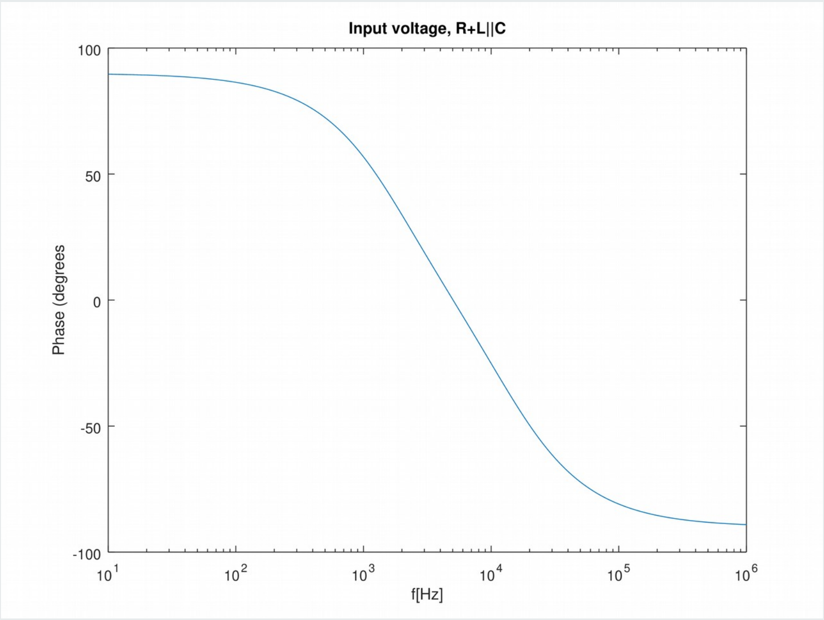
Voltage Divider

$Z = Z_L \,||\, Z_C$



Band-Pass Filter

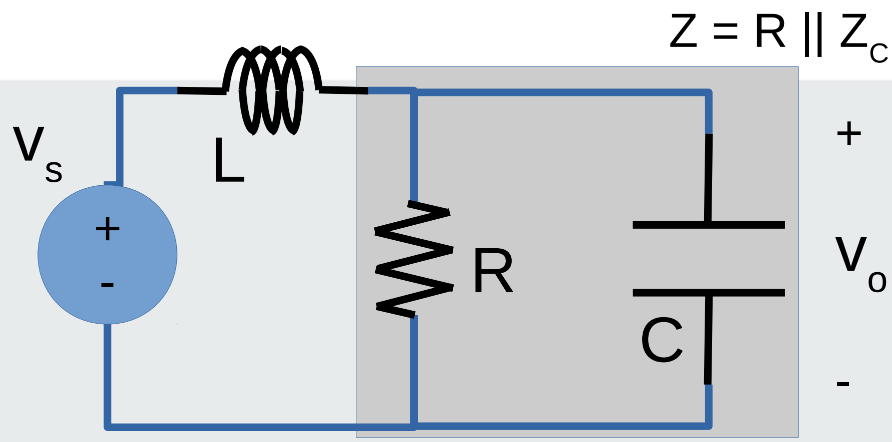




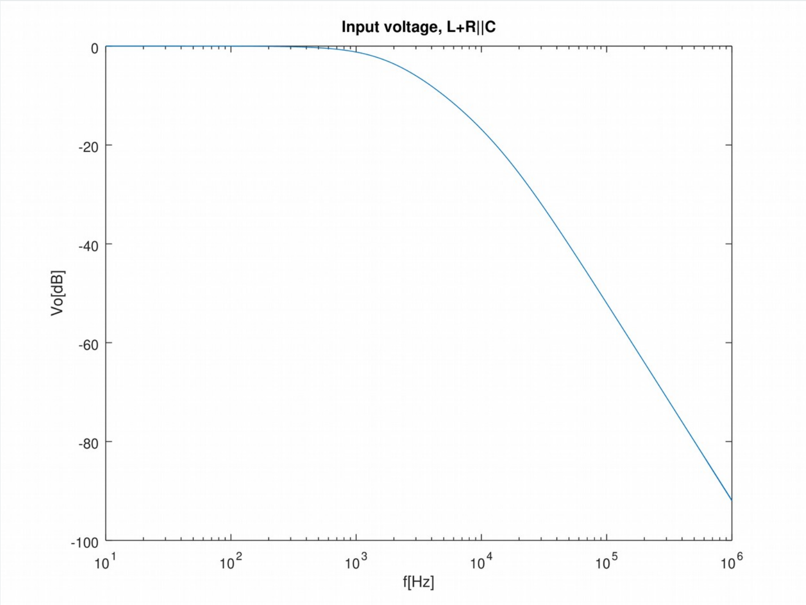
Swap R with L

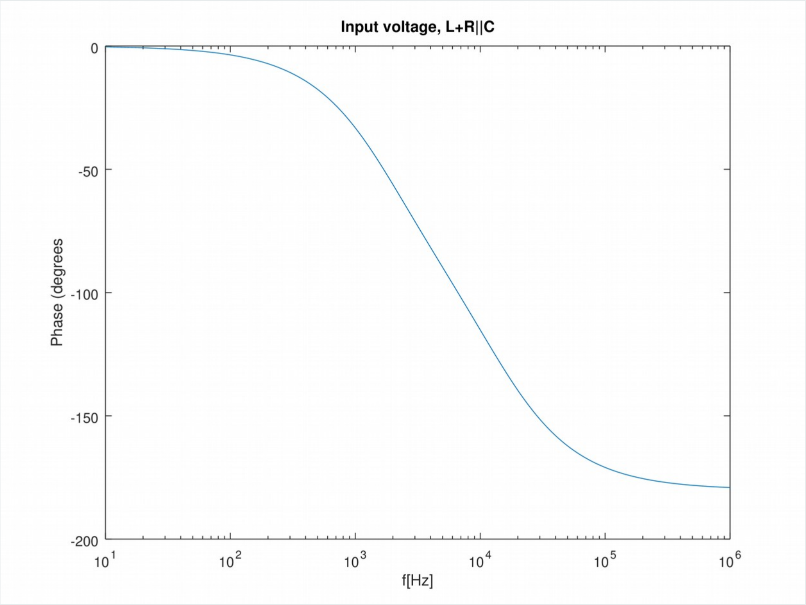
$$\frac{\tilde{V}_o}{\tilde{V}_s}=\frac{\dfrac{1}{\dfrac{1}{R}+j\,\omega C}}{j\,\omega L+\dfrac{1}{\dfrac{1}{R}+j\,\omega C}}$$

Voltage Divider



Low-Pass Filter

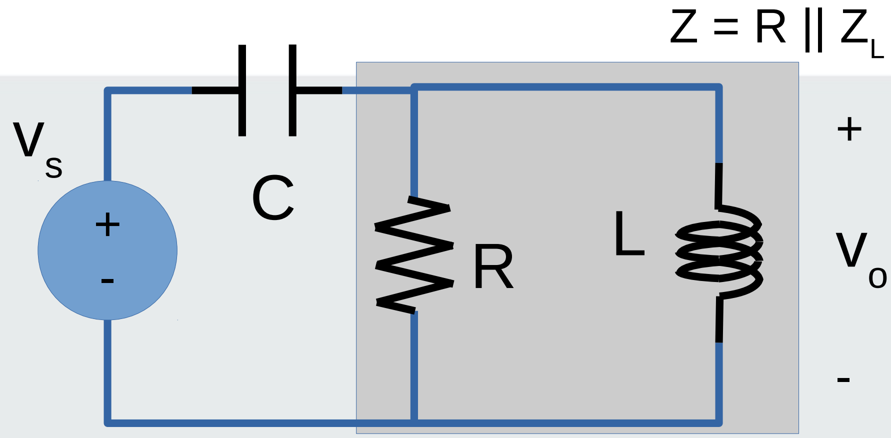




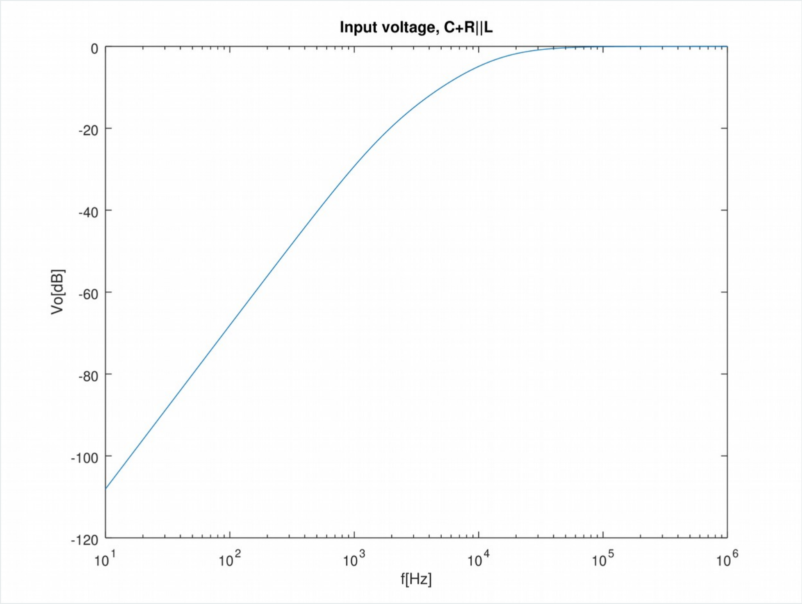
Swap L with C

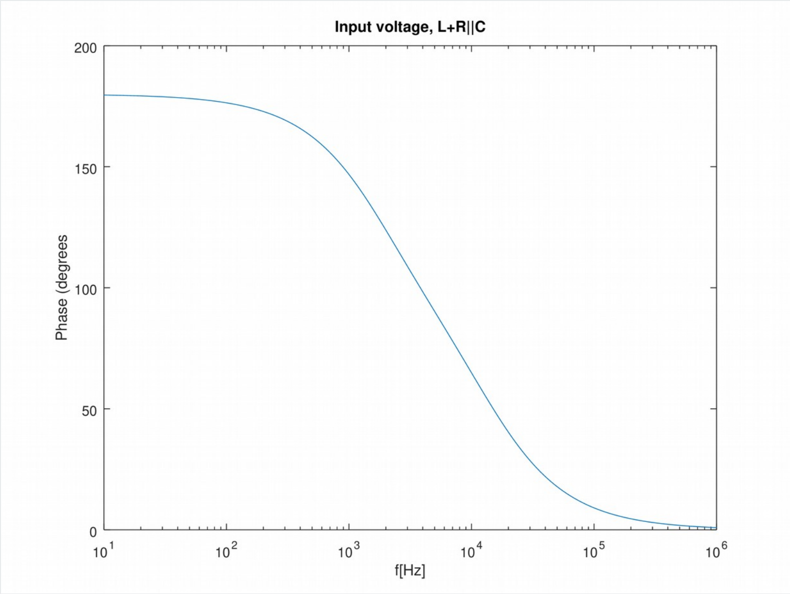
$$\frac{\tilde{V}_o}{\tilde{V}_s} = \frac{\dfrac{1}{\dfrac{1}{R}+\dfrac{1}{j\omega L}}}{\dfrac{1}{j\omega C}+\dfrac{1}{\dfrac{1}{R}+\dfrac{1}{j\omega L}}}$$

Voltage Divider



High-Pass Filter





Bode plots for the LC band-pass filter

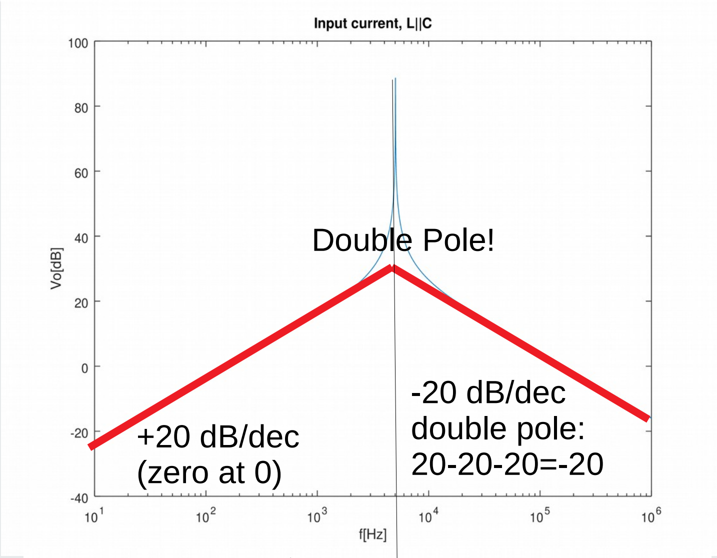
$$T(s) = \frac{V_o(s)}{I_s(s)} = \frac{1}{C}\frac{s}{s^2 + \frac{1}{LC}}$$

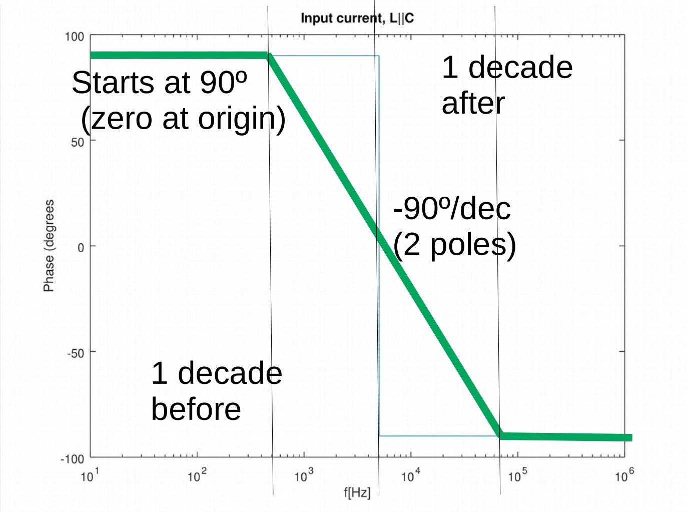
$s = 0$ → **Zero: root of T(s) numerator**

$s = \pm \frac{j}{\sqrt{LC}}$ → **Pole: root of T(s) denominator**

$|s| = \frac{1}{\sqrt{LC}}$ **Pole frequency is the modulus**

- Each Zero adds 20dB/dec to $|T|_{dB}$ slope
- Each Pole adds -20dB/dec to $|T|_{dB}$ slope
- Each Zero adds 45º/dec to the T phase slope in the zero's +/- 1 decade interval
- Each Pole adds -45º/dec to the T phase slope in the pole's +/- 1 decade interval
- Effects are cumulative!



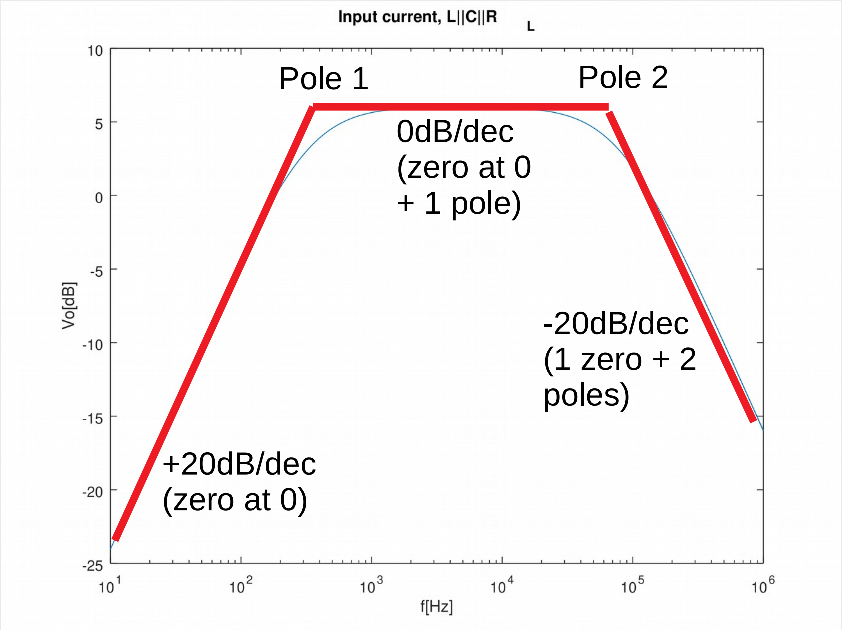


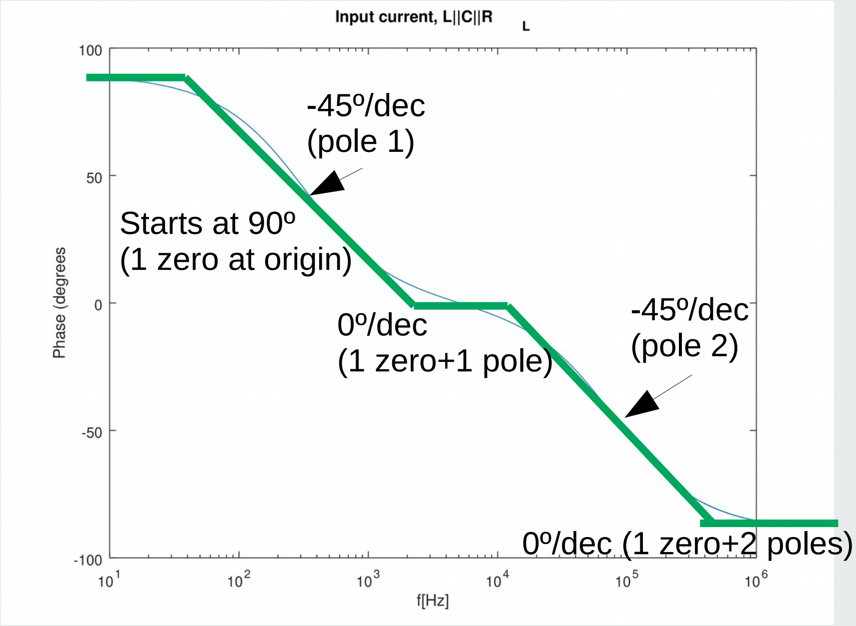
Bode plots for the RLC band-pass filter

$$T(s)\frac{V_o(s)}{I_s(s)}=\frac{1}{C}\frac{s}{s^2+\frac{1}{RC}s+1}$$

Transfer Function:

- Zero at origin
- Two real negative Poles





Conclusion

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- Introduction to filters and example filter
- Ideal transformer models (they have not been forgotten!)
- Filter qualitative and quantitative analyses
- Filter transfer function in various RLC configurations
- Second order Bode plots