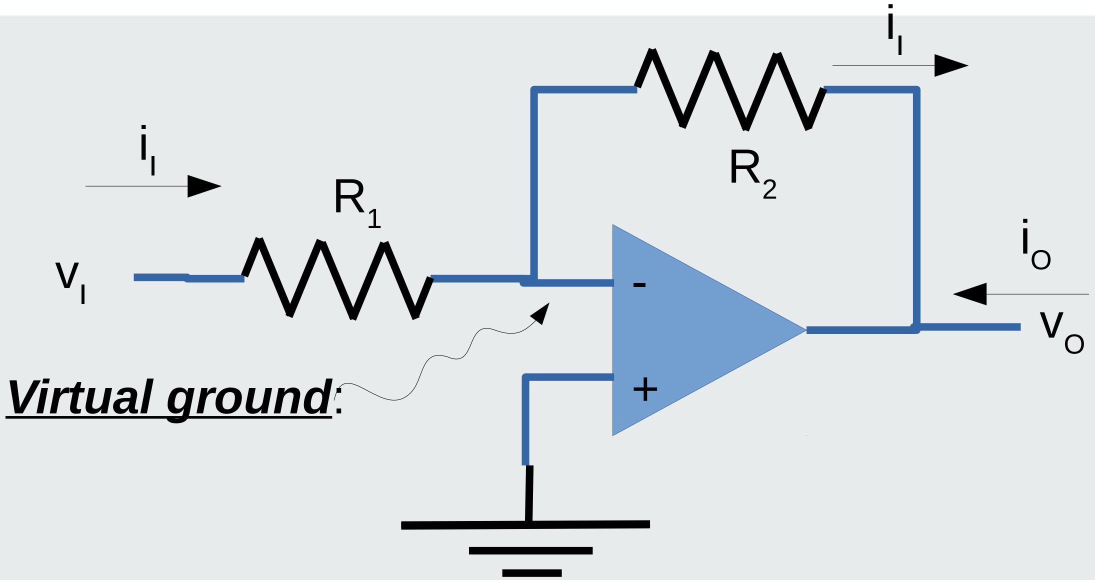
TÉCNICO LISBOA

Circuit Theory and Electronics Fundamentals

Lecture 23: Operational Amplifier Circuits

- Analogue negation circuit
- The addition circuit
- Summation circuits
- Subtraction circuits
- The subtraction of summations circuit

Remember: inverting amplifier with resistive feedback loop

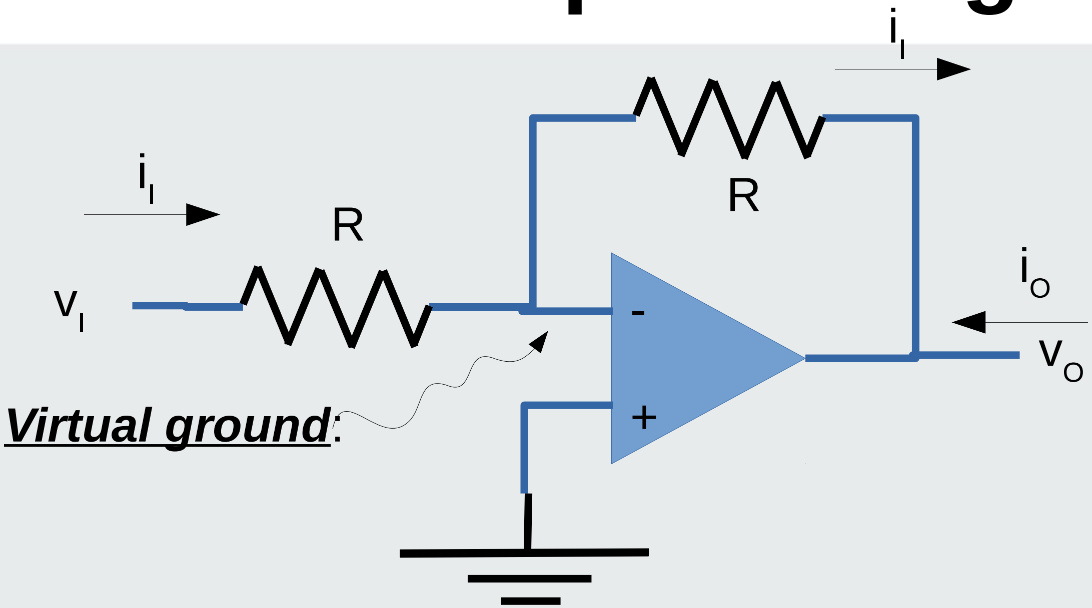


$$\frac{v_O}{v_I} = -\frac{R_2}{R_1} \quad \text{Gain}$$

$$Z_I = \left.\frac{v_I}{i_I}\right|_{Z_L=\infty} = \frac{v_I}{\frac{v_I - 0}{R_1}} = R_1$$

$$Z_O = \left.\frac{v_O}{i_O}\right|_{v_I=0} = R_2 || 0 = 0$$

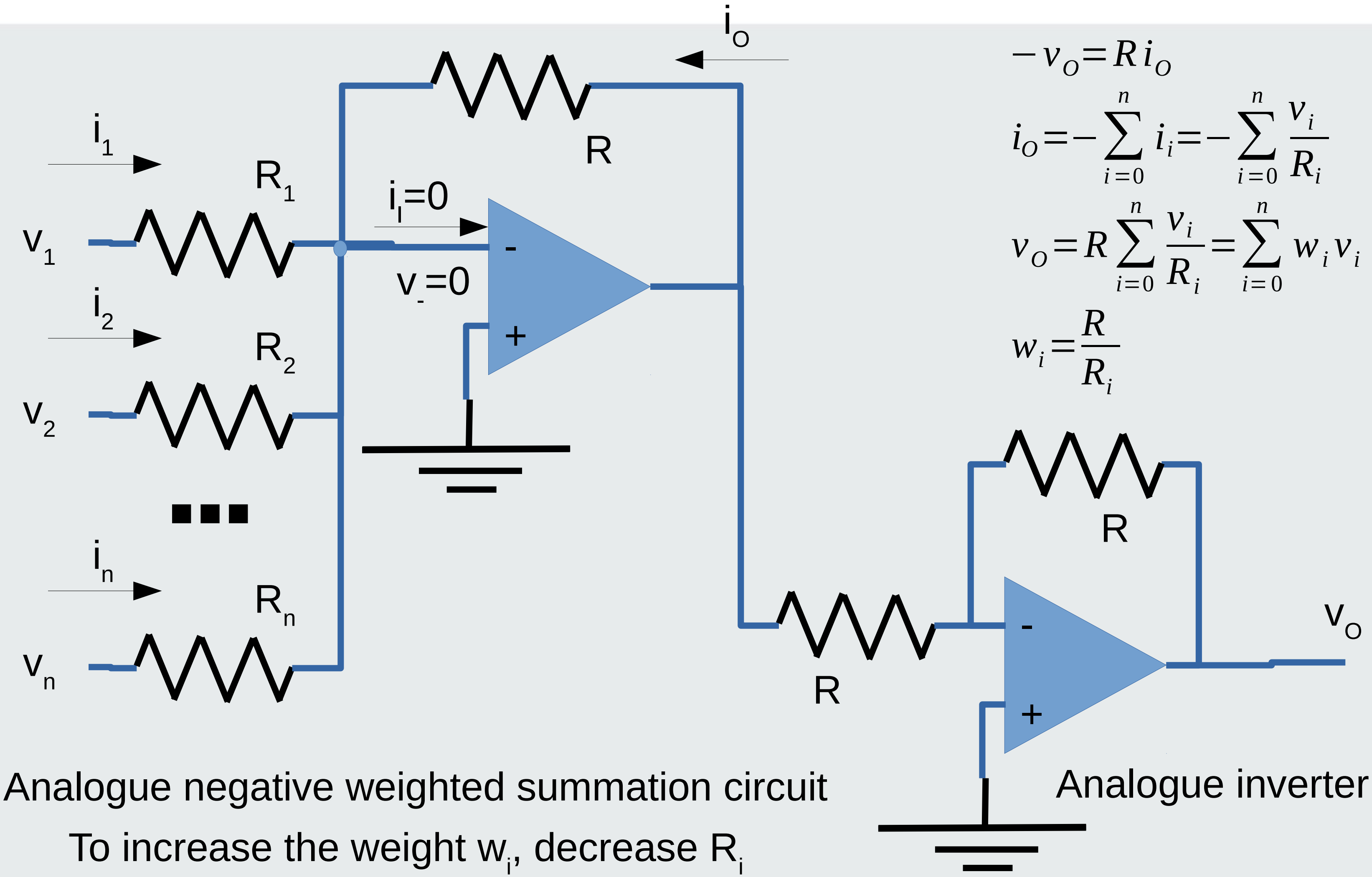
Compute the negative of an input voltage



Just make $R_1 = R_2 = R$

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1} = -\frac{R}{R} = -1$$

Compute a weighted summation of the inputs



$$-v_O = R\, i_O$$

$$i_O = -\sum_{i=0}^{n} i_i = -\sum_{i=0}^{n} \frac{v_i}{R_i}$$

$$v_O = R \sum_{i=0}^{n} \frac{v_i}{R_i} = \sum_{i=0}^{n} w_i v_i$$

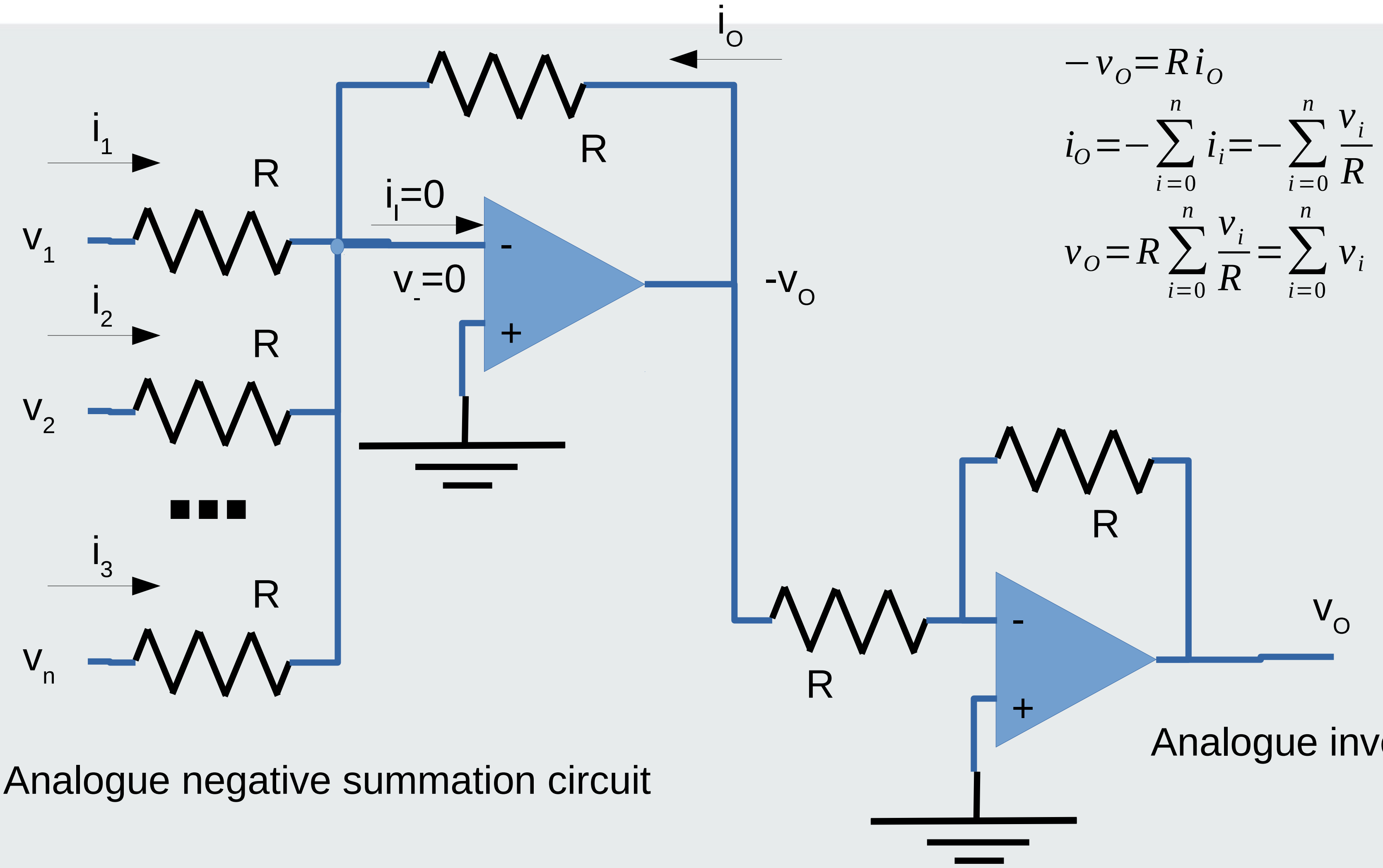
$$w_i = \frac{R}{R_i}$$

Analogue negative weighted summation circuit

Analogue inverter circuit

To increase the weight w_i, decrease R_i

Compute the summation of the inputs: just make $R_i = R$



$$-v_O = R i_O$$

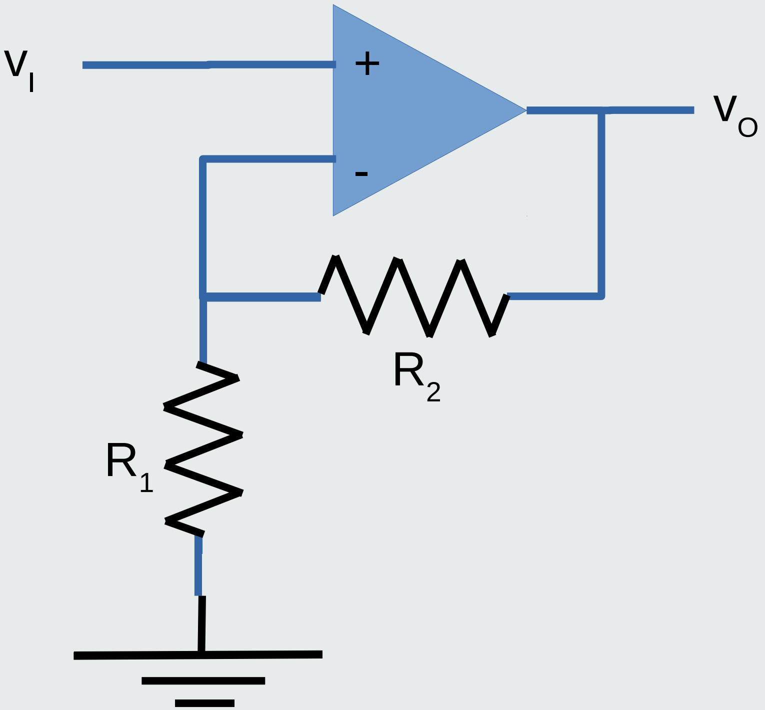
$$i_O = -\sum_{i=0}^{n} i_i = -\sum_{i=0}^{n} \frac{v_i}{R}$$

$$v_O = R \sum_{i=0}^{n} \frac{v_i}{R} = \sum_{i=0}^{n} v_i$$

Analogue negative summation circuit

Analogue inverter

Compute summation using non-inverting amplifier (1)

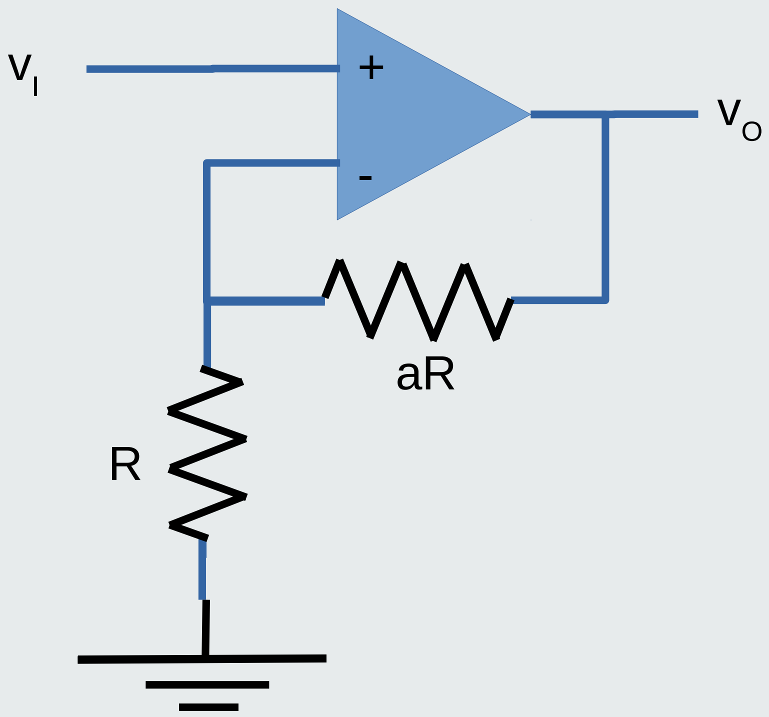


$$v_- = \frac{R_1}{R_1 + R_2} v_O$$

$$v_- = v_+ = v_I$$

$$\frac{v_O}{v_I} = 1 + \frac{R_2}{R_1}$$

Compute summation using non-inverting amplifier (2)



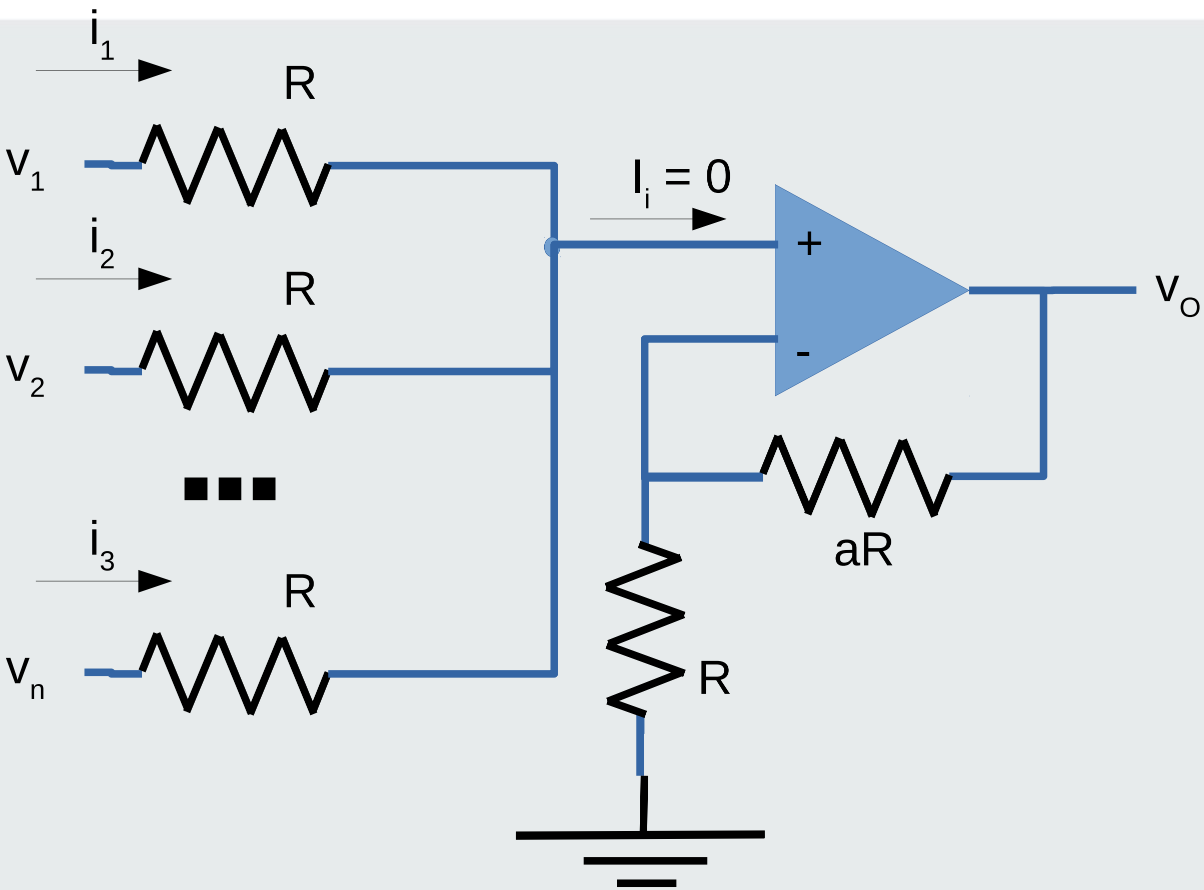
Positive voltage amplifier

$$\frac{v_O}{v_I}=1+\frac{aR}{R}=1+a$$

$$k=1+a$$

$$v_O=k\,v_I,k>1$$

Compute the summation using non-inverting amplifier (3)



$$v_+ = v_-$$

$$v_- = \frac{R}{R+aR} v_O = \frac{1}{1+a} v_o = \frac{v_O}{k}$$

$$k = 1+a > 1$$

$$v_- = \frac{v_O}{k}$$

$$\sum_{i=1}^{n} \frac{v_i - \frac{v_o}{k}}{R} = 0$$

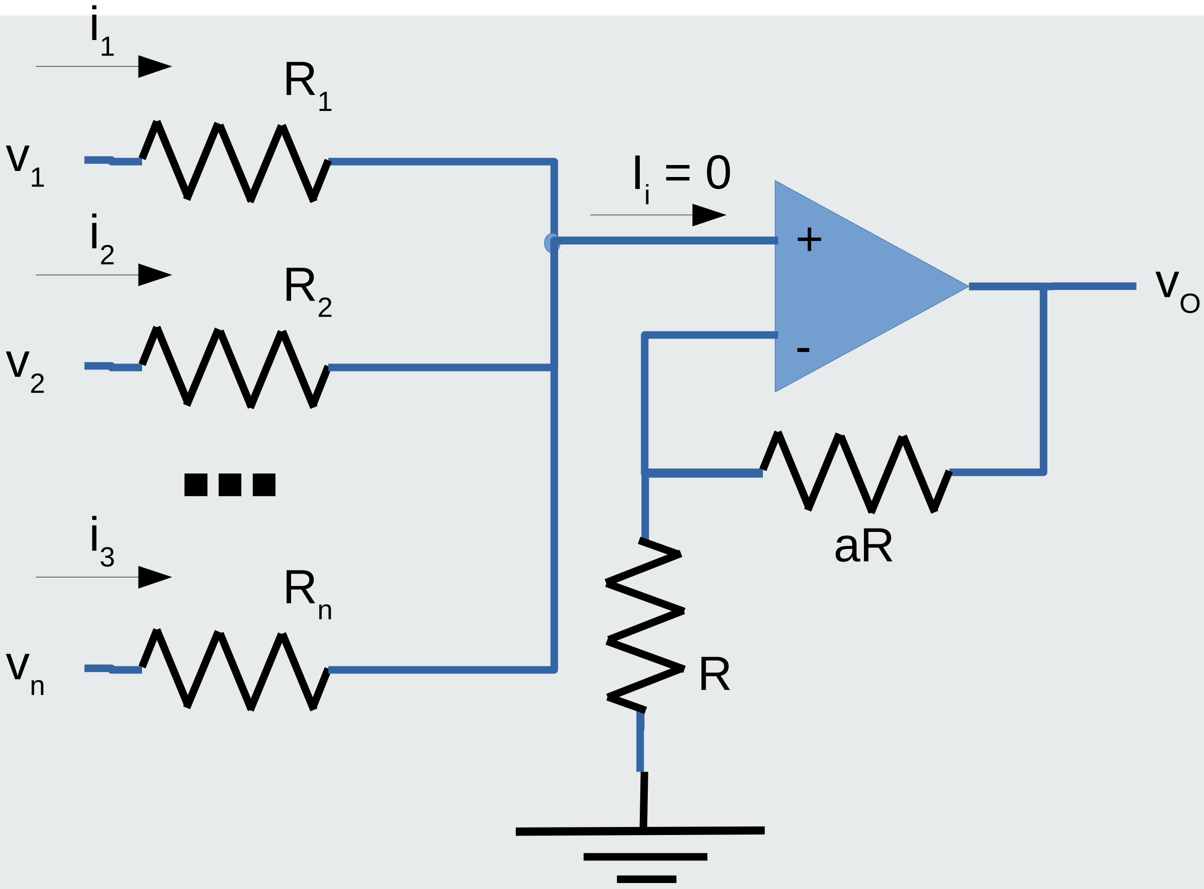
$$\sum_{i=1}^{n} v_i - \frac{n}{k} v_o = 0$$

$$v_O = \frac{k}{n} \sum_{i=1}^{n} v_i$$

$$\frac{k}{n} = 1 \Rightarrow k = n = 1 + a \Rightarrow a = n - 1$$

$$v_O = \sum_{i=1}^{n} v_i$$

Compute weighted summation using non-inverting amplifier



$$\sum_{i=1}^{n} \frac{v_i - \frac{v_o}{k}}{R_i} = 0$$

$$\sum_{i=1}^{n} \frac{v_i}{R_i} - \left(\frac{1}{k} \sum_{i=1}^{n} \frac{1}{R_i}\right) v_o = 0$$

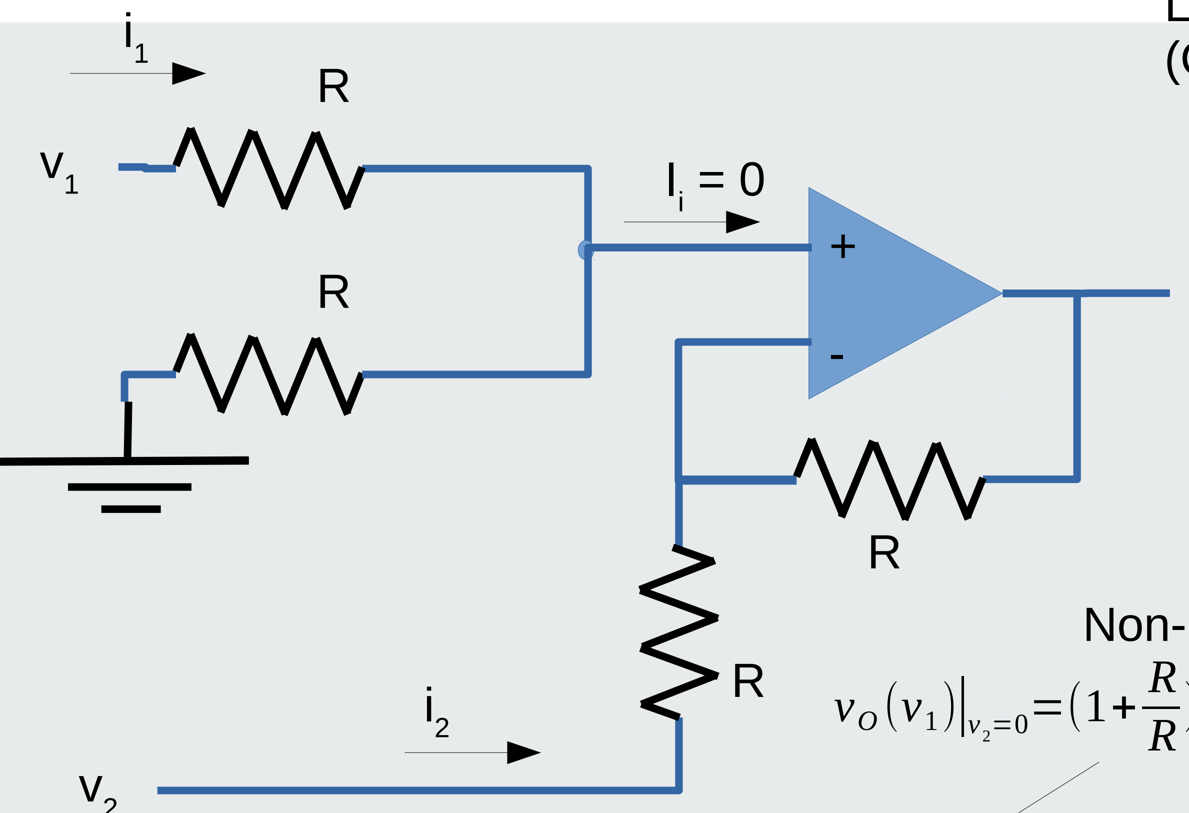
$$v_O = \frac{k}{\sum_{i=1}^{n} \frac{1}{R_i}} \sum_{i=1}^{n} \frac{v_i}{R_i} = \sum_{i=1}^{n} w_i v_i$$

$$w_i = \frac{k}{R_i \sum_{i=1}^{n} \frac{1}{R_i}}$$

$$w_i < k$$

To increase the weight w_i, decrease R_i

Compute the subtraction of two voltages



Let's use the superposition theorem (OPAMPs make linear circuits!)

$$v_O = v_O(v_1)\big|_{v_2=0} + v_O(v_2)\big|_{v_1=0}$$

Non-inverting amp

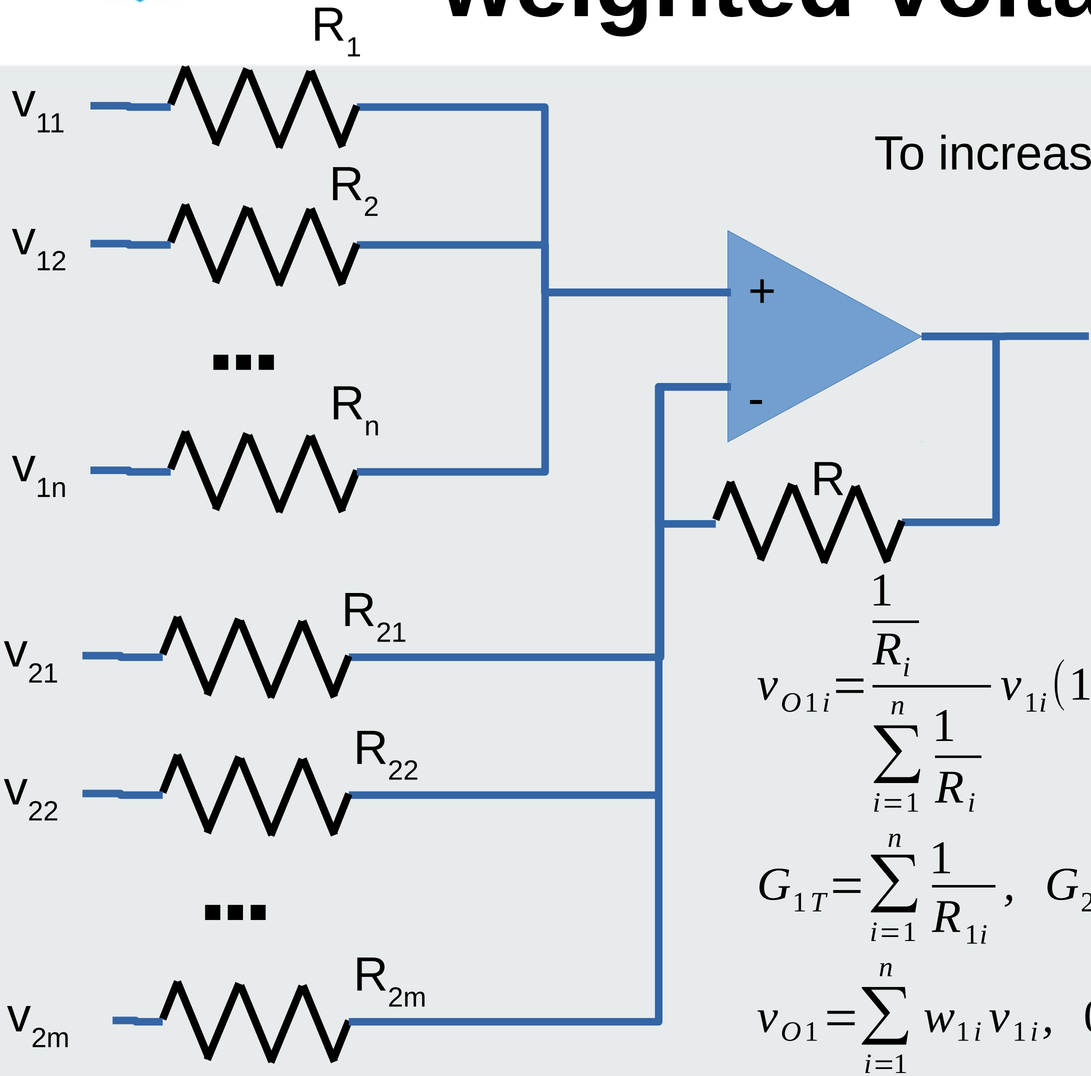
$$v_O(v_1)\big|_{v_2=0} = \left(1+\frac{R}{R}\right)v_+ = \left(1+\frac{R}{R}\right)\left(\frac{R}{R+R}v_1\right) = v_1$$

Inverting amp

$$v_O = v_O(v_2)\big|_{v_1=0} = -\frac{R}{R}v_2 = -v_2$$

$$\boxed{v_O = v_1 - v_2}$$

Compute the subtraction of two weighted voltage summations



To increase the weight w_{ij}, decrease R_{ij}

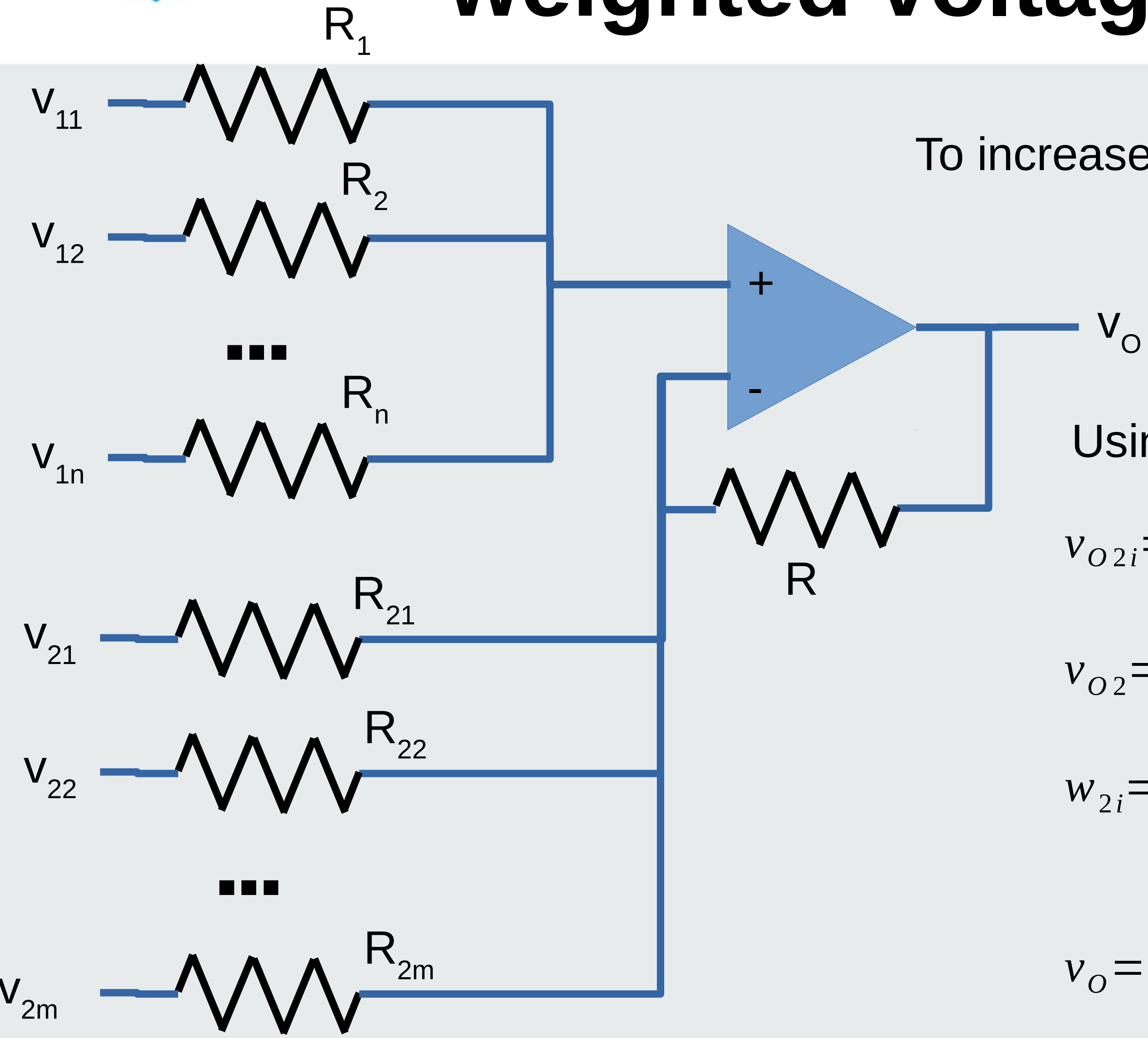
Using the superposition theorem:

$$v_{O1i}=\frac{\frac{1}{R_i}}{\sum_{i=1}^{n}\frac{1}{R_i}}v_{1i}\left(1+R\sum_{i=1}^{m}\frac{1}{R_{2i}}\right)=\frac{G_{1i}}{G_{1T}}\left(1+RG_{2T}\right)v_{1i}$$

$$G_{1T}=\sum_{i=1}^{n}\frac{1}{R_{1i}},\quad G_{2T}=\sum_{i=1}^{m}\frac{1}{R_{2i}},\quad w_{1i}=\frac{G_{1i}}{G_{1T}}\left(1+RG_{2T}\right)$$

$$v_{O1}=\sum_{i=1}^{n}w_{1i}v_{1i},\quad 0<w_{1i}<1+RG_{2T}$$

Compute the subtraction of two weighted voltage summations



To increase the weight w_{ij}, decrease R_{ij}

Using the superposition theorem:

$$v_{O2i} = -\frac{R}{R_i} v_{2i}$$ Inverting amp

$$v_{O2} = -\sum_{i=1}^{m} \frac{R}{R_i} v_{2i} = -\sum_{i=1}^{m} w_{2i} v_{2i}$$

$$w_{2i} = \frac{R}{R_i}, \quad 0 < w_{2i} < \infty$$

$$v_O = \sum_{i=1}^{n} w_{1i} v_{1i} - \sum_{i=1}^{m} w_{2i} v_{2i}$$

Conclusion

- Analogue negation circuit
- The addition circuit
- Summation circuits
- Subtraction circuits
- The subtraction of summations circuit