ELETROSTÁTICA

Formulário EM/MEFT 2018/19
v0.0 (1/2/2019), Pedro Abreu

$\boxed{\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})}$ $\quad \vec{E} = k_e \frac{Q}{r^2}\hat{e}_r \quad \vec{E} = -\vec{\nabla}\phi \quad \phi(\vec{r}) = -\int_{\mathcal{R}}^{\vec{r}} \vec{E}\cdot d\vec{l} \quad \phi(\mathcal{R}) \equiv 0 \quad (\mathcal{R} \to \infty)$

$k_e = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \mathrm{N.C^{-2}.m^2} \quad \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \quad (\vec{E}_2 - \vec{E}_1)\cdot\vec{n} = \frac{\sigma}{\varepsilon_0} \quad \vec{\nabla}\times\vec{E} = 0 \quad \oint \vec{E}\cdot d\vec{l} = 0$

$u_E = \frac{1}{2}\sum_i q_i\phi_i = \frac{1}{2}\varepsilon_0 E^2 \quad \iiint \vec{\nabla}\cdot\vec{E}\,dv = \oiint \vec{E}\cdot\vec{n}dS = \frac{Q_{\mathrm{int}}}{\varepsilon_0} \quad \nabla^2\phi = -\frac{\rho}{\varepsilon_0} \quad \vec{\nabla}\cdot\vec{D} = \rho$

$\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon_0(1+\chi_E)\vec{E} = \varepsilon\vec{E} \quad \iiint \vec{\nabla}\cdot\vec{D}\,dv = \oiint \vec{D}\cdot\vec{n}dS = Q_{\mathrm{int}} \quad (\vec{D}_2 - \vec{D}_1)\cdot\vec{n} = \sigma$

$\rho' = -\vec{\nabla}\cdot\vec{P} \quad \sigma' = \vec{P}\cdot\vec{n}_{\mathrm{ext}} \quad C = \frac{Q}{V} = \frac{Q}{\phi_2 - \phi_1} \quad U_E = \frac{1}{2}CV^2 = \frac{Q^2}{2C} \quad C_{\mathrm{eq}}(\parallel) = C_1 + C_2$

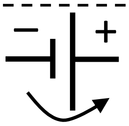
$u_E = \frac{1}{2}\vec{E}\cdot\vec{D} \quad \vec{E}_{\mathrm{int.condut.eq.el.}} = 0 \quad (\vec{E}_2 - \vec{E}_1)\times\vec{n} = 0 \quad (\vec{D}_2 - \vec{D}_1)\times\vec{n} = (\vec{P}_2 - \vec{P}_1)\times\vec{n} \quad C_{\mathrm{eq}}^{-1}(\text{série}) = C_1^{-1} + C_2^{-1}$

CORRENTE ELÉTRICA

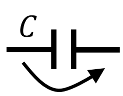
$\vec{J} = \rho\vec{v} \quad \vec{K} = \sigma\vec{v} \quad \vec{I} = \lambda\vec{v} \quad I = \frac{dQ}{dt} = \iint_{S\perp} \vec{J}\cdot\vec{n}dS$

$\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0 \quad \vec{J} = \sigma_{(C)}\vec{E} \quad V = \int \vec{E}\cdot d\vec{l} = \int \frac{1}{\sigma}\vec{J}\cdot d\vec{l} = I\int\frac{dl}{\sigma S} = RI \quad R_{\mathrm{eq}}(\text{série}) = R_1 + R_2$

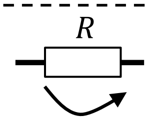
Lei dos nós: $\sum_k I_k = 0$ Lei das malhas: $\sum_k V_k = 0$ $\quad R_{\mathrm{eq}}^{-1}(\parallel) = R_1^{-1} + R_2^{-1}$



$\varepsilon \quad V_k = -\varepsilon$



$V_k = \frac{Q}{C} = \frac{1}{C}\int I dt$



$V_k = RI \quad P = VI = RI^2$

descarga do condensador: $V_C(t) = \frac{Q_0}{C}e^{-t/(RC)}$ carga do condensador: $V_C(t) = \varepsilon\left(1 - e^{-t/(RC)}\right)$

MAGNETOSTÁTICA

$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{Id\vec{l}\times\hat{e}_r}{r^2} = \frac{\mu_0}{4\pi}\int\frac{\vec{J}\times\hat{e}_r}{r^2}dv = \frac{\mu_0}{4\pi}\int\frac{\vec{K}\times\hat{e}_r}{r^2}dS = \vec{\nabla}\times\vec{A}$

$\vec{F} = \int I d\vec{l}\times\vec{B} \quad \oiint B\cdot\vec{n}dS = 0 \quad \vec{\nabla}\cdot\vec{B} = 0 \quad (\vec{B}_2 - \vec{B}_1)\cdot\vec{n} = 0 \quad (\vec{B}_2 - \vec{B}_1) = \mu_0\vec{K}\times\vec{n} \quad \vec{\nabla}\times\vec{B} = \mu_0\vec{J} \quad \oint\vec{B}\cdot d\vec{l} = \mu_0 I_{\mathrm{int}}$

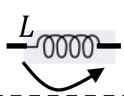
$\vec{B} = \mu_0(\vec{H} + \vec{M}) \quad \vec{\mu} = \vec{m} = IA\vec{n} \quad \vec{M} = \chi_m\vec{H} \quad \vec{B} = \mu_0(1+\chi_m)\vec{H} = \mu\vec{H} \quad \vec{\nabla}\times\vec{M} = \vec{J}_M \quad \vec{M}\times\vec{n} = \vec{K}_M \quad \oint\vec{M}\cdot d\vec{l} = I_{\mathrm{Magn}}$

$\oint\vec{H}\cdot d\vec{l} = I_{\mathrm{int}} \quad (\mu_2\vec{H}_2 - \mu_1\vec{H}_1)\cdot\vec{n} = 0 \quad (\vec{H}_2 - \vec{H}_1) = \vec{K}\times\vec{n} \quad \vec{\nabla}\times\vec{H} = \vec{J} \quad \vec{\nabla}\cdot\vec{H} = -\vec{\nabla}\cdot\vec{M}$

$\oint\vec{H}\cdot d\vec{l} = \iint_{S\perp}\vec{J}\cdot\vec{n}dS + \int_C \vec{K}\cdot d\vec{l} \quad \oint\vec{B}\cdot d\vec{l} = \mu_0\iint_{S\perp}(\vec{J} + \vec{J}_M)\cdot\vec{n}dS + \mu_0\int_C(\vec{K} + \vec{K}_M)\cdot d\vec{l}$

$\Phi_\alpha = \iint_S \vec{B}\cdot\vec{n}dS = \sum_\beta L_{\alpha\beta}I_\beta \quad L_{\alpha\beta} = \frac{1}{4\pi}\int_{\Gamma_\alpha}\int_{\Gamma_\beta}\mu\frac{d\vec{l}_\alpha\cdot d\vec{l}_\beta}{|\vec{r}_\alpha - \vec{r}_\beta|} \quad u_M = \frac{1}{2}\vec{B}\cdot\vec{H} \quad U_M = \frac{1}{2}LI^2$

$U_M(1+2) = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 + MI_1I_2 = \frac{1}{2}\sum_k\Phi_kI_k$



$V_k = -L\frac{dI}{dt} \quad L_{\mathrm{eq}}(\text{série}) = L_1 + L_2 \quad L_{\mathrm{eq}}^{-1}(\parallel) = L_1^{-1} + L_2^{-1}$

descarga do solenóide: $I(t) = I_0e^{-Rt/L}$ carga do solenóide: $I(t) = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)$

DF DEPARTAMENTO DE FÍSICA TÉCNICO LISBOA

LIP

MATEMÁTICA para E.M.

Formulário EM/MEFT 2018/19
v0.0 (1/2/2019), Pedro Abreu

$$\int \frac{dr}{\sqrt{a^2+r^2}} = \log\left(\sqrt{a^2+r^2}+r\right)+C \qquad \int \frac{dr}{(a^2+r^2)^{3/2}} = \frac{r}{a^2\sqrt{a^2+r^2}}+C$$

$$\int \frac{rdr}{(a^2+r^2)^{3/2}} = -\frac{1}{\sqrt{a^2+r^2}}+C \qquad \int \frac{dr}{r} = \log r + C \qquad \int \frac{dr}{r^2} = -\frac{1}{r}+C \qquad \iiint_{-\infty}^{+\infty} \vec{C}(\vec{r}\,')\delta^3(\vec{r}-\vec{r}\,')\,dv' = 4\pi\vec{C}(\vec{r})$$

Coordenadas cilíndricas

$$\begin{cases} x = R\cos\varphi \\ y = R\sin\varphi \\ z = z \end{cases} \qquad \begin{cases} \vec{e}_x = \cos\varphi\,\vec{e}_R - \sin\varphi\vec{e}_\varphi \\ \vec{e}_y = \sin\varphi\,\vec{e}_R + \cos\varphi\vec{e}_\varphi \\ \vec{e}_z = \vec{e}_z \end{cases} \qquad \begin{cases} R = \sqrt{x^2+y^2} \\ \varphi = \arctan\frac{y}{x} \\ z = z \end{cases} \qquad \begin{cases} \vec{e}_R = \cos\varphi\,\vec{e}_x + \sin\varphi\vec{e}_y \\ \vec{e}_\varphi = -\sin\varphi\,\vec{e}_x + \cos\varphi\vec{e}_y \\ \vec{e}_z = \vec{e}_z \end{cases}$$

$$d\vec{l} = dR\vec{e}_R + Rd\varphi\vec{e}_\varphi + dz\vec{e}_z \qquad dS = RdRd\varphi, dRdz, Rdzd\varphi \qquad dv = RdRd\varphi dz$$

$$\vec{r} = R\vec{e}_R + z\vec{e}_z$$

$$\vec{C} = C_R\vec{e}_R + C_\varphi\vec{e}_\varphi + C_z\vec{e}_z \qquad \vec{\nabla}T = \frac{\partial T}{\partial R}\vec{e}_R + \frac{1}{R}\frac{\partial T}{\partial\varphi}\vec{e}_\varphi + \frac{\partial T}{\partial z}\vec{e}_z \qquad \vec{\nabla}\cdot\vec{C} = \frac{1}{R}\frac{\partial(RC_R)}{\partial R} + \frac{1}{R}\frac{\partial C_\varphi}{\partial\varphi} + \frac{\partial C_z}{\partial z}$$

$$\vec{\nabla}\times\vec{C} = \left(\frac{1}{R}\frac{\partial(C_z)}{\partial\varphi} - \frac{\partial(C_\varphi)}{\partial z}\right)\vec{e}_R + \left(\frac{\partial(C_R)}{\partial z} - \frac{\partial(C_z)}{\partial R}\right)\vec{e}_\varphi + \frac{1}{R}\left(\frac{\partial(RC_\varphi)}{\partial R} - \frac{\partial(C_R)}{\partial\varphi}\right)\vec{e}_z$$

$$\nabla^2\vec{C} = \vec{e}_R\left(\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial C_R}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2 C_R}{\partial\varphi^2} + \frac{\partial^2 C_R}{\partial z^2}\right) + \vec{e}_\varphi\,\mathrm{lap}\,C_\varphi + \vec{e}_z\,\mathrm{lap}\,C_z$$

Coordenadas esféricas

$$\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases} \qquad \left\{ r = \sqrt{x^2+y^2+z^2}, \quad \theta = \arccos\frac{z}{r}, \quad \varphi = \arctan\frac{y}{x} \right.$$

$$\begin{cases} \vec{e}_x = \sin\theta\cos\varphi\,\vec{e}_r + \cos\theta\cos\varphi\,\vec{e}_\theta - \sin\varphi\,\vec{e}_\varphi \\ \vec{e}_y = \sin\theta\sin\varphi\,\vec{e}_r + \cos\theta\sin\varphi\,\vec{e}_\theta + \cos\varphi\,\vec{e}_\varphi \\ \vec{e}_z = \cos\theta\,\vec{e}_r - \sin\theta\,\vec{e}_\theta \end{cases} \qquad \begin{cases} \vec{e}_r = \sin\theta\cos\varphi\,\vec{e}_x + \sin\theta\sin\varphi\,\vec{e}_y + \cos\theta\,\vec{e}_z \\ \vec{e}_\theta = \cos\theta\cos\varphi\,\vec{e}_x + \cos\theta\sin\varphi\,\vec{e}_y - \sin\theta\,\vec{e}_z \\ \vec{e}_\varphi = -\sin\varphi\,\vec{e}_x + \cos\varphi\vec{e}_y \end{cases}$$

$$d\vec{l} = dr\vec{e}_R + rd\theta\vec{e}_\theta + r\sin\theta\,d\varphi\vec{e}_\varphi \qquad dS = rdrd\theta, r\sin\theta drd\varphi, r^2\sin\theta d\theta d\varphi \qquad dv = r^2\sin\theta drd\theta d\varphi$$

$$\vec{r} = r\vec{e}_r \qquad \vec{C} = C_r\vec{e}_r + C_\theta\vec{e}_\theta + C_\varphi\vec{e}_\varphi \qquad \vec{\nabla}T = \frac{\partial T}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial T}{\partial\theta}\vec{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial T}{\partial\varphi}\vec{e}_\varphi$$

$$\vec{\nabla}r = \vec{e}_r \qquad \vec{\nabla}\cdot\frac{\vec{e}_r}{r^2} = 4\pi\delta^3(\vec{r}) = \nabla^2\frac{1}{r} \qquad \vec{\nabla}\cdot\vec{C} = \frac{1}{r^2}\frac{\partial(r^2C_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial\sin\theta\,C_\theta}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial C_\varphi}{\partial\varphi}$$

$$\vec{\nabla}\times\vec{C} = \left(\frac{1}{r\sin\theta}\frac{\partial(\sin\theta\,C_\varphi)}{\partial\theta} - \frac{\partial(\sin\theta\,C_\theta)}{\partial\varphi}\right)\vec{e}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial(C_r)}{\partial\varphi} - \frac{\partial(rC_\varphi)}{\partial r}\right)\vec{e}_\theta + \frac{1}{r}\left(\frac{\partial(rC_\theta)}{\partial r} - \frac{\partial(C_r)}{\partial\theta}\right)\vec{e}_\varphi$$

$$\nabla^2\vec{C} = \vec{e}_r\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C_r}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial C_r}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 C_r}{\partial\varphi^2}\right) + \vec{e}_\theta\,\mathrm{lap}\,C_\theta + \vec{e}_\varphi\,\mathrm{lap}\,C_\varphi$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B}) \qquad \vec{\nabla}\cdot(T\vec{C}) = T\vec{\nabla}\cdot\vec{C} + \vec{C}\cdot\vec{\nabla}T \qquad \vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

$$\vec{\nabla}(TU) = T\vec{\nabla}U + U\vec{\nabla}T \qquad \vec{\nabla}\times(T\vec{C}) = T\vec{\nabla}\times\vec{C} - \vec{C}\times\vec{\nabla}T \qquad \vec{\nabla}(\vec{A}\cdot\vec{C}) = \vec{A}\times(\vec{\nabla}\times\vec{C}) + \vec{C}\times(\vec{\nabla}\times\vec{A}) + (\vec{A}\cdot\vec{\nabla})\vec{C} + (\vec{C}\cdot\vec{\nabla})\vec{A}$$

$$\vec{\nabla}\cdot(\vec{A}\times\vec{C}) = \vec{C}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{C}) \qquad \vec{\nabla}\cdot(\vec{\nabla}\times\vec{C}) = 0 \qquad \vec{\nabla}\times(\vec{A}\times\vec{C}) = (\vec{C}\cdot\vec{\nabla})\vec{A} - (\vec{A}\cdot\vec{\nabla})\vec{C} + (\vec{\nabla}\cdot\vec{C})\vec{A} - (\vec{\nabla}\cdot\vec{A})\vec{C}$$

$$\vec{\nabla}\times(\vec{\nabla}T) = 0 \qquad \vec{\nabla}\times(\vec{\nabla}\times\vec{C}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{C}) - \nabla^2\vec{C} = \mathrm{grad}(\mathrm{div}\,\vec{C}) - \mathrm{lap}\,\vec{C}$$

$$\int_a^b \vec{\nabla}T\cdot d\vec{l} = T(b) - T(a) \qquad \iint \vec{\nabla}\times\vec{C}\cdot\vec{n}dS = \oint \vec{C}\cdot d\vec{l} \qquad (\vec{A}\cdot\vec{\nabla})\vec{C} = A_x\frac{\partial\vec{C}}{\partial x} + A_y\frac{\partial\vec{C}}{\partial y} + A_z\frac{\partial\vec{C}}{\partial z}$$

$$\oint \vec{\nabla}T\cdot d\vec{l} = 0 \qquad \oiint \vec{\nabla}\times\vec{C}\cdot\vec{n}dS = 0 \qquad \iiint \vec{\nabla}\cdot\vec{C}dv = \oiint \vec{C}\cdot\vec{n}dS$$