EletroMagnetismo

MEFT 2020-2021

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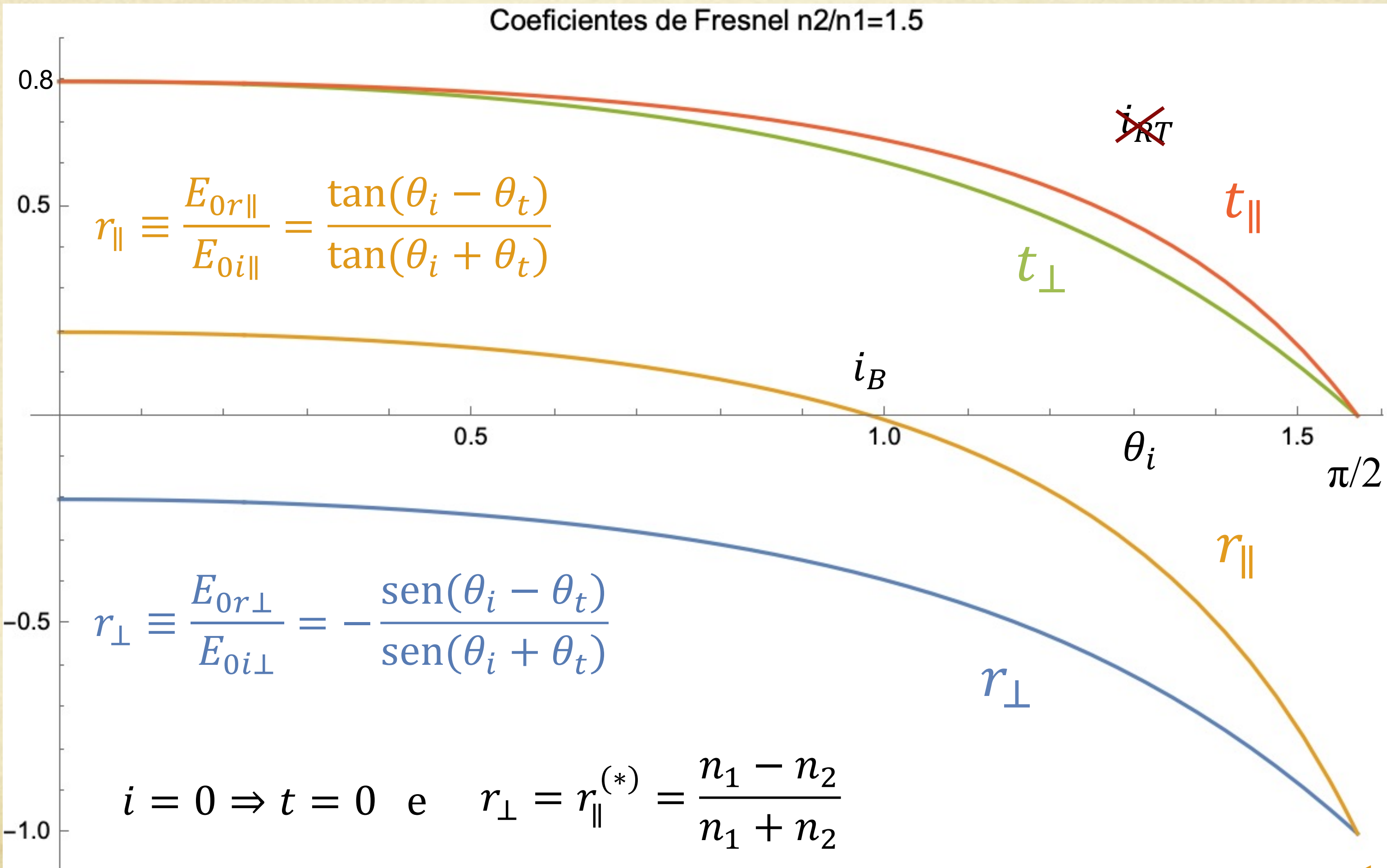
22ª Aula

Resumo das equações de Fresnel e coeficientes *r* e *t* ;
Troca de fase na reflexão;
Pode uma onda não ser refletida nem transmitida?
Energia refletida e Reflectância R;
Energia transmitida e Transmitância T;
Conservação da Energia;
Expressão de R,T em função de γ_i, $R_\perp$,$R_\parallel$, $T_\perp$,$T_\parallel$.

Há duas atitudes para encarar a Natureza: como se não houvessem milagres, e como se tudo fosse um milagre.

Albert Einstein [Prémio Nobel 1921]

Coeficientes de Fresnel $n_2 > n_1$

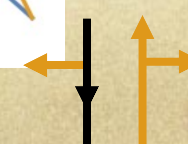


$$r_\parallel \equiv \frac{E_{0r\parallel}}{E_{0i\parallel}} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

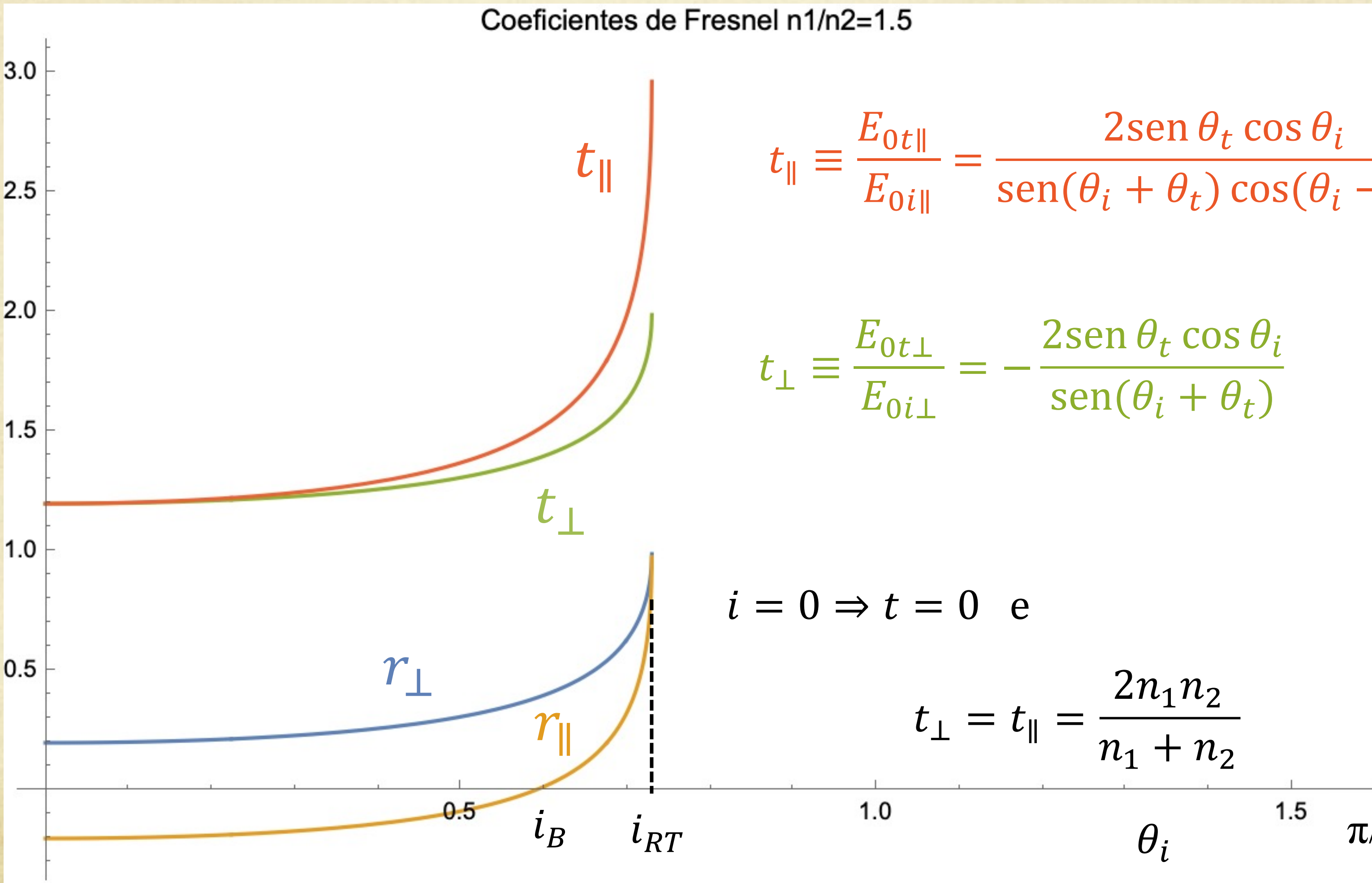
$$r_\perp \equiv \frac{E_{0r\perp}}{E_{0i\perp}} = -\frac{\mathrm{sen}(\theta_i - \theta_t)}{\mathrm{sen}(\theta_i + \theta_t)}$$

$$i = 0 \Rightarrow t = 0 \quad \text{e} \quad r_\perp = r_\parallel^{(*)} = \frac{n_1 - n_2}{n_1 + n_2}$$

$^{(*)}E_{0r\parallel} = -E_{0i\parallel}$ na definição para $i = 0$



Coeficientes de Fresnel $n_1 > n_2$



$$t_\parallel \equiv \frac{E_{0t\parallel}}{E_{0i\parallel}} = \frac{2\operatorname{sen}\theta_t \cos\theta_i}{\operatorname{sen}(\theta_i+\theta_t)\cos(\theta_i-\theta_t)}$$

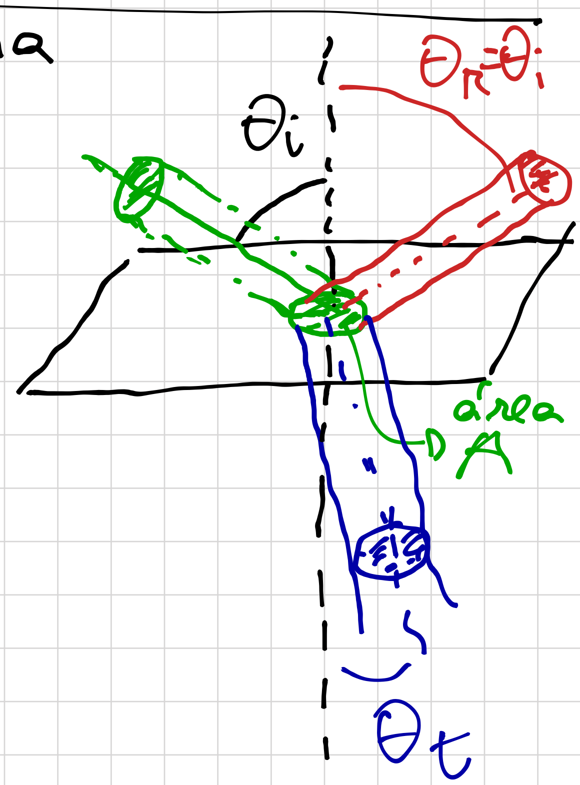
$$t_\perp \equiv \frac{E_{0t\perp}}{E_{0i\perp}} = -\frac{2\operatorname{sen}\theta_t \cos\theta_i}{\operatorname{sen}(\theta_i+\theta_t)}$$

$i = 0 \Rightarrow t = 0$ e

$$t_\perp = t_\parallel = \frac{2n_1 n_2}{n_1 + n_2}$$

22ª Aula Teórica, 2ªF, 24/Maio, #38+

$$\vec{\Sigma} \equiv \vec{E} \times \vec{H} = \frac{1}{\mu} \vec{E} \times \vec{B} \overset{\text{nesta disciplina}}{=} \frac{1}{\mu_0} \vec{E} \times \vec{B}$$



$$\vec{\Sigma} = |\vec{\Sigma}|\, \vec{e}_k = \underbrace{v\,\varepsilon}\, \underbrace{E_0^2 \cos^2(\ldots)}_{\text{depende da polariz.}}\, \vec{e}_k$$

$v = \frac{c}{n}$, $\varepsilon = \underbrace{\varepsilon_R}_{n^2} \varepsilon_0$

$v\varepsilon = n c \varepsilon_0$

$$\left(E_{0x}^2 + E_{0y}^2 + E_{0z}^2\right)$$

$$I = \langle |\vec{\Sigma}| \rangle$$

$P_i \equiv$ Potência incidente $= A \cdot I_i \underbrace{\cos\theta_i}_{\vec{e}_k \cdot \vec{n}}$ $\left(= \langle \iint \vec{\Sigma} \cdot \vec{n}\, dS \rangle \right)$

$P_R \equiv$ " refletida $= A \cdot I_R \cos\theta_R$

$P_t \equiv$ " transmitida $= A \cdot I_t \cos\theta_t$

DEFINE-SE:

REFLETÂNCIA $\equiv \boxed{R = \frac{P_R}{P_i} = \frac{A I_R \cos\theta_R}{A I_i \cos\theta_i} = \frac{I_R}{I_i} = \frac{E_{0R}^2}{E_{0i}^2} = r^2}$

TRANSMITÂNCIA $\equiv T \equiv \frac{P_t}{P_i} = \frac{A I_t \cos\theta_t}{A I_i \cos\theta_i} = \frac{I_t \cos\theta_t}{I_i \cos\theta_i} = \frac{n_2 \cos\theta_t\, E_{0t}^2}{n_1 \cos\theta_i\, E_{0i}^2}$

c/ $\frac{E_{0t}^2}{E_{0i}^2} = t^2 \Leftrightarrow T = t^2 \cdot \frac{n_2 \cos\theta_t}{n_1 \cos\theta_i}$

Se $\theta_i > 0$ ($e < \theta_{i_{RT}}$) $\Rightarrow \frac{n_2}{n_1} = \frac{\operatorname{sen}\theta_i}{\operatorname{sen}\theta_t}$

e $\boxed{T = t^2 \cdot \frac{\tan\theta_i}{\tan\theta_t}}$ $\left(\begin{array}{l} \text{Se} \\ \theta_i = 0 \end{array} \Rightarrow T = t^2 \frac{n_2}{n_1} \right)$

$$t \equiv \frac{E_{0t}}{E_{0i}} \qquad r \equiv \frac{E_{0r}}{E_{0i}} \qquad \underset{(r,t)}{E_{0i}} = \sqrt{E_{0ix}^2 + E_{0iy}^2 + E_{0iz}^2}$$

(r,t)

Existindo plano de incidência ($\theta_i > 0$)

$$t_{//} \equiv \frac{E_{0t//}}{E_{0i//}}, \quad t_{\perp} \equiv \frac{E_{0t\perp}}{E_{0i\perp}}, \quad r_{//} \equiv \frac{E_{0r//}}{E_{0i//}}, \quad r_{\perp} \equiv \frac{E_{0r\perp}}{E_{0i\perp}}$$

$$\Rightarrow \quad R_{//} \equiv r_{//}^2, \quad R_{\perp} \equiv r_{\perp}^2, \quad \underset{(\perp)}{T_{//}} = \underset{(\perp)}{t_{//}^2} \frac{\tan\theta_i}{\tan\theta_t}$$

$$\underbrace{P_i = P_r + P_t}_{\text{(CONSERVAÇÃO DA ENERGIA)}} \Leftrightarrow \cancel{A} \underbrace{I_i \cos\theta_i} = \cancel{A} I_r \cos\theta_r + \cancel{A} I_t \cos\theta_t$$

$$\Leftrightarrow \qquad 1 = R + T$$

(CONSERVAÇÃO DA ENERGIA)

$$\boxed{R + T = 1}, \quad R_{\perp} + T_{\perp} = 1$$

$$R_{//} + T_{//} = 1$$

$$\cancel{R = R_{//} + R_{\perp}}$$

$$\cancel{T = T_{//} + T_{\perp}}$$

Se $\theta_i = 0$, ($\Rightarrow \theta_t = 0$), $\quad R \equiv R_{\perp} = R_{//} = r_{//}^2 = r_{\perp}^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$

$$T \equiv T_{\perp} = T_{//} = t_{//}^2 \frac{n_2}{n_1} = t_{\perp}^2 \frac{n_2}{n_1} = \frac{4 n_2 n_1}{(n_1 + n_2)^2}$$

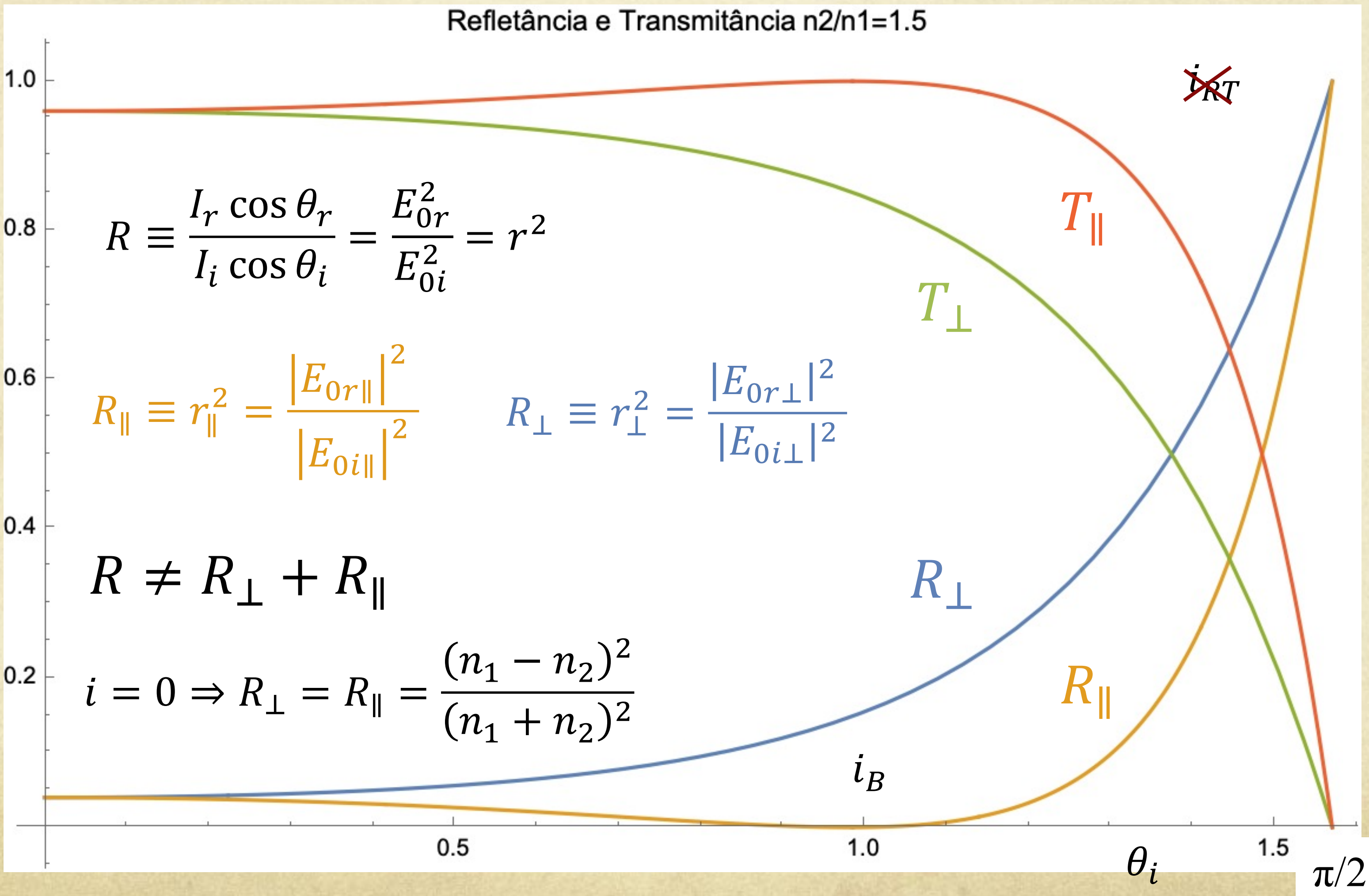
$$t_{//}^2 = t_{\perp}^2 = \left(\frac{2 n_1}{(n_1 + n_2)}\right)^2$$

Em geral, $R \neq R_{\perp} + R_{//}$ (e $R^2 \neq R_{\perp}^2 + R_{//}^2$)

$T \neq T_{\perp} + T_{//}$ (e $T^2 \neq T_{\perp}^2 + T_{//}^2$)

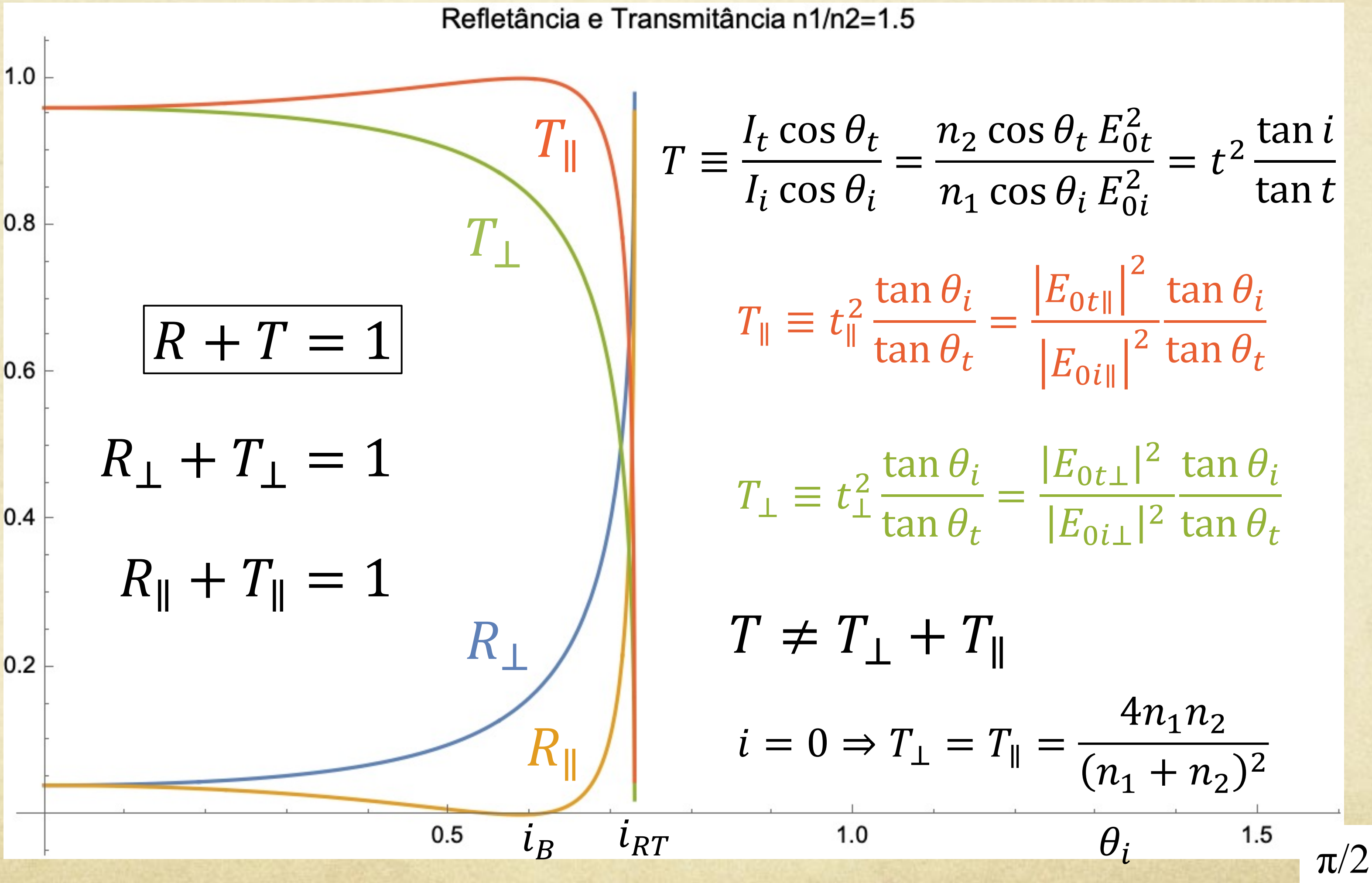
Refletância e Transmitância

$n_2 > n_1$



Refletância e Transmitância

$n_1 > n_2$



$$\boxed{R + T = 1}$$

$$R_\perp + T_\perp = 1$$

$$R_\parallel + T_\parallel = 1$$

$$T \equiv \frac{I_t \cos\theta_t}{I_i \cos\theta_i} = \frac{n_2 \cos\theta_t \, E_{0t}^2}{n_1 \cos\theta_i \, E_{0i}^2} = t^2 \frac{\tan i}{\tan t}$$

$$T_\parallel \equiv t_\parallel^2 \frac{\tan\theta_i}{\tan\theta_t} = \frac{\left|E_{0t\parallel}\right|^2}{\left|E_{0i\parallel}\right|^2} \frac{\tan\theta_i}{\tan\theta_t}$$

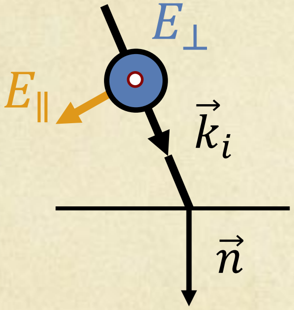
$$T_\perp \equiv t_\perp^2 \frac{\tan\theta_i}{\tan\theta_t} = \frac{|E_{0t\perp}|^2}{|E_{0i\perp}|^2} \frac{\tan\theta_i}{\tan\theta_t}$$

$$T \neq T_\perp + T_\parallel$$

$$i = 0 \Rightarrow T_\perp = T_\parallel = \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

Refletância e Transmitância

Lembrete: $R \neq R_\perp + R_\parallel$ $\qquad$ $T \neq T_\perp + T_\parallel$



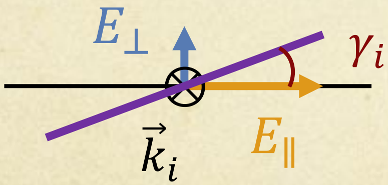
mas com $\gamma_i = \arctan \dfrac{E_{0i\perp}}{E_{0i\parallel}}$ $\qquad$ $\left(0 < \theta_i < \dfrac{\pi}{2}\right)$

Plano de incidência

$(\vec{k}_i, \vec{n})$

$$\boxed{R = R_\parallel \cos^2 \gamma_i + R_\perp \operatorname{sen}^2 \gamma_i}$$

Plano de Polarização



$$\boxed{T = T_\parallel \cos^2 \gamma_i + T_\perp \operatorname{sen}^2 \gamma_i}$$

(mas se $\theta_i = 0$, $R = R_\perp = R_\parallel$, $T = T_\perp = T_\parallel$,

se $E_{0i\parallel} = 0$, $\cancel{r_\parallel}, \cancel{t_\parallel}$ $R = R_\perp$, $T = T_\perp$

$E_{0i\perp} = 0$, $\cancel{r_\perp}, \cancel{t_\perp}$, $R = R_\parallel$, $T = T_\parallel$)

Existindo $\gamma_i \equiv \arctan \dfrac{E_{0i\perp}}{E_{0i\parallel}}$ $\left(0 \leq \gamma_i \leq \frac{\pi}{2}\right)$

$$R = \frac{E_{0R}^2}{E_{0i}^2} = \frac{E_{0R\perp}^2 + E_{0R\parallel}^2}{E_{0i\perp}^2 + E_{0i\parallel}^2} = \frac{r_\perp^2 E_{0i\perp}^2 + r_\parallel^2 E_{0i\parallel}^2}{E_{0i\perp}^2 + E_{0i\parallel}^2} \underset{\cdot \frac{1}{E_{0i\parallel}^2}}{=}$$

$$= \frac{r_\parallel^2 \cdot 1 + r_\perp^2 \tan^2\gamma_i}{1 + \tan^2\gamma_i} = \overset{R_\perp}{r_\parallel^2} \cos^2\gamma_i + \overset{R_\parallel}{r_\perp^2} \operatorname{sen}^2\gamma_i$$

$$1 + \tan^2\gamma_i = 1 + \frac{\operatorname{sen}^2\gamma_i}{\cos^2\gamma_i} = \frac{1}{\cos^2\gamma_i}$$

$$R = R_\parallel \cos^2\gamma_i + R_\perp \operatorname{sen}^2\gamma_i$$

$$T = \frac{\tan\theta_i}{\tan\theta_t} \frac{E_{0t\parallel}^2 + E_{0t\perp}^2}{E_{0i\parallel}^2 + E_{0i\perp}^2} \underset{\times \frac{1}{E_{0i\parallel}^2}}{=} \frac{\tan\theta_i}{\tan\theta_t} \cdot \frac{t_\parallel^2 + t_\perp^2 \tan^2\gamma_i}{1 + \tan^2\gamma_i}$$

$$= \frac{\tan\theta_i}{\tan\theta_t}\left(t_\parallel^2 \cos^2\gamma_i + t_\perp^2 \operatorname{sen}^2\gamma_i\right) \Leftrightarrow$$

$$T = T_\parallel \cos^2\gamma_i + T_\perp \operatorname{sen}^2\gamma_i$$