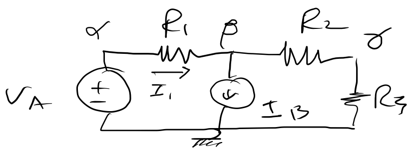
EXAM 22/06/21 — PART I

1.



$v_A = 21\ V$

$R_1 = 1K\Omega$

$R_2 = 10K\Omega$

$R_3 = 3K\Omega$

$I_B = 14mA$

Superposition

$$V_\gamma = \underbrace{\frac{R_3}{R_1+R_2+R_3}}_{\text{voltage divider}} V_A - R_3 \underbrace{\frac{R_1}{R_1+R_2+R_3}}_{\text{current divider}} I_B$$

$$\frac{3}{1+10+3} = \frac{3}{14} \qquad 3\ \frac{1}{14}$$

$$V_\gamma = \frac{3}{14}21 - \frac{3}{14}14 = 1.5\ V$$

$$I_1 = \frac{V_A}{R_1+R_2+R_3} + \frac{R_2+R_3}{R_1+R_2+R_3} I_B$$

$$\frac{21}{14} + \frac{13}{14}14 = \frac{3}{2} + 13 = 14.5\ mA$$

b) $P_A = V_A(-I_1) < 0$ supplying

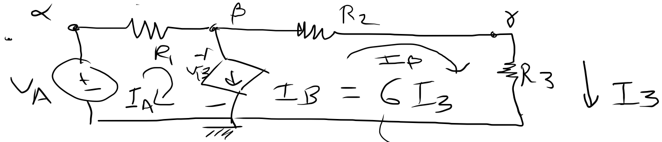
$P_B = v_\beta I_B > 0$ Receiving

$V_\beta = V_A - R_1 I_1 = (21 - 1 \times 14.5)\ V$

c) $V_{eq} = \frac{R_2+R_3}{R_1+R_2+R_3} V_A = \frac{13}{14}\ 21 = 19.5\ V$

$R_{eq} = R_1 \parallel (R_2+R_3) = 1 \parallel 13 = \frac{13}{14}\ K\Omega$

2.



a)

$$-V_A + R_1 I_\alpha + V_B = 0 \qquad (1)$$

$$-V_\beta + R_2 I_\beta + R_3 I_\beta = 0 \qquad (2)$$

$$I_\alpha - I_\beta = 6 I_\beta \qquad (3)$$

Add (1) and (2)

$$-V_A + R_1 I_\alpha + (R_2 + R_3) I_\beta = 0 \qquad (4)$$

$$\begin{bmatrix} R_1 & R_2 + R_3 \\ 1 & -7 \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} = \begin{bmatrix} V_A \\ 0 \end{bmatrix}$$

b)

$$\begin{cases} V_\alpha = V_A \\ \dfrac{V_\beta - V_\alpha}{R_1} + I_B + \dfrac{V_\beta - V_\gamma}{R_2} = 0 \\ \dfrac{V_\gamma - V_\beta}{R_2} + \dfrac{V_\gamma}{R_3} = 0 \end{cases}$$

$$I_B = 3 I_3 = 3 \frac{V_\gamma}{R_3}$$

$$\begin{cases} -\frac{1}{R_1} V_\alpha + \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_\beta + \left(-\frac{1}{R_2} + \frac{3}{R_3}\right) V_\gamma \\ -\frac{1}{R_2} V_\beta + \left(\frac{1}{R_2} + \frac{1}{R_3}\right) V_\gamma = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1/R_1 & 1/R_1 + 1/R_2 & \frac{-1}{R_2} + \frac{3}{R_3} \\ 0 & -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \\ V_\gamma \end{bmatrix} = \begin{bmatrix} V_A \\ 0 \\ 0 \end{bmatrix}$$

3.

$v_A(t) = 10 - 5u(t)$ V

$R_1 = 1\ k\Omega$

$R_3 = 3\ k\Omega$

$C = 20\ nF$

$L = 200\ mH$

a) At $t = -5s$ $V_A = 10V$

L is short-circuit

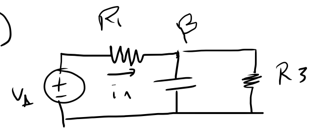
C is open circuit

$V_\beta = \frac{R_3}{R_1+R_3} V_A = \frac{3}{4} 10$ V

$I_3 = \frac{V_A}{R_1+R_3} = \frac{10}{4}$ mA

$W = \frac{1}{2}\left(L I_3^2 + C V_\beta^2\right)$

b)



$R_{eq} = R_1 \| R_3 = 750\Omega$

$\tau = 0.75\times10^{3} \times 20\times10^{-9}$

$= 15\ \mu s$

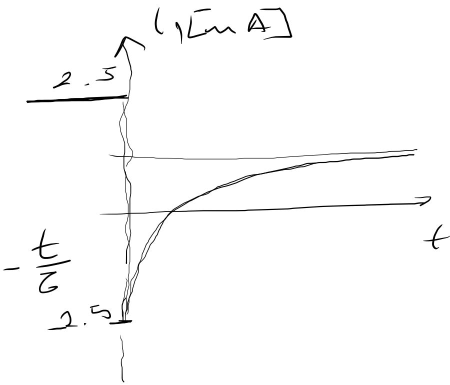
$i_1(t<0) = \frac{V_A}{R_1+R_3} = \frac{10}{4} = 2.5$ mA

$v_\beta(0) = \frac{R_3}{R_1+R_3} 10 = \frac{3\times10}{4} = 7.5$ V

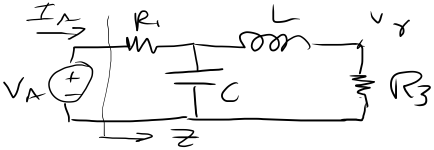
$i_1(0) = \frac{5-7.5}{R_1} = -2.5$ mA

$i_1(\infty) = \frac{5}{R_1+R_2} = 1.25$ mA

$i_1(t>0) = 1.25 + [-2.5 - 1.25]\, e^{-\frac{t}{\tau}}$ mA



4.



$$\tilde{V}_r = \frac{R_3}{R_3 + sL} \cdot \frac{\frac{1}{sC} \parallel (R_3 + sL)}{R_1 + \frac{1}{sC} \parallel (R_3 + sL)} \tilde{V}_A$$

$R_1 = 1\,k\Omega$

$R_3 = 3\,k\Omega$

$L = 200\,mH$

$C = 20\,nF$

$v_A(t) = 30 \sin\left(2\pi \cdot 50 \cdot t + \frac{\pi}{3}\right) V$

$\tilde{V}_A = 30 e^{j\left(\frac{\pi}{3} - \frac{\pi}{2}\right)} \; V$

$s = j\omega, \quad \omega = 2\pi \times 50$ rad/s

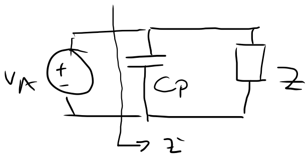
$v_r(t) = |\tilde{V}_r| \cos\left(\omega t + \measuredangle \tilde{V}_r\right) \; V$

$Z = R_1 + \dfrac{1}{sC + \dfrac{1}{sL + R_3}} \; \Omega$ for $s = j\omega$

b) Power factor $= \cos\alpha$

To compensate the P.F. $\alpha = 0$, $PF = 1$

$\alpha = \measuredangle\left(\frac{1}{2} \tilde{V}_A \tilde{I}_A^*\right) = \measuredangle\left(\frac{\tilde{V}_A}{\tilde{I}_A}\right) = \measuredangle Z'$



$Z' = \dfrac{1}{sC_p + Z^{-1}}$ must be real

$Z' = \frac{1}{sC_p} \parallel Z$

$j\omega C_p + j\,\mathrm{Imag}(Z^{-1}) = 0$

Get C_p from above equation