TÉCNICO LISBOA

Circuit Theory and Electronics Fundamentals

Lecture 12: Non-Linear Components: piecewise linear and incremental models

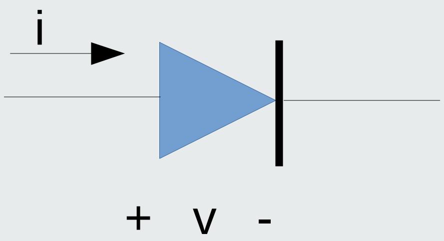
- Ideal diode model
- Voltage source diode model
- Voltage source plus resistor diode model
- Piecewise linear diode model
- Incremental analysis
- Incremental diode model
- Exercise

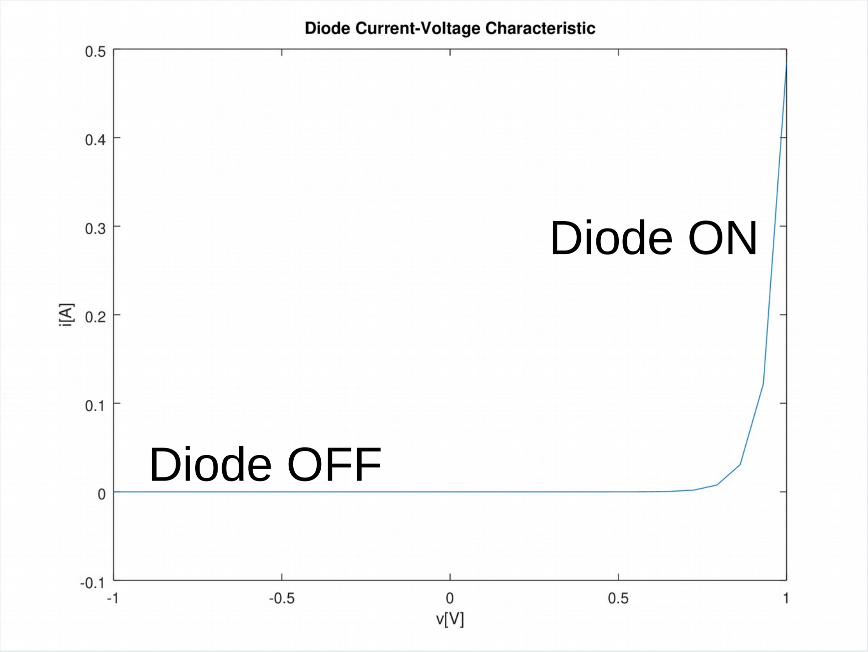
Diode I-V Plots

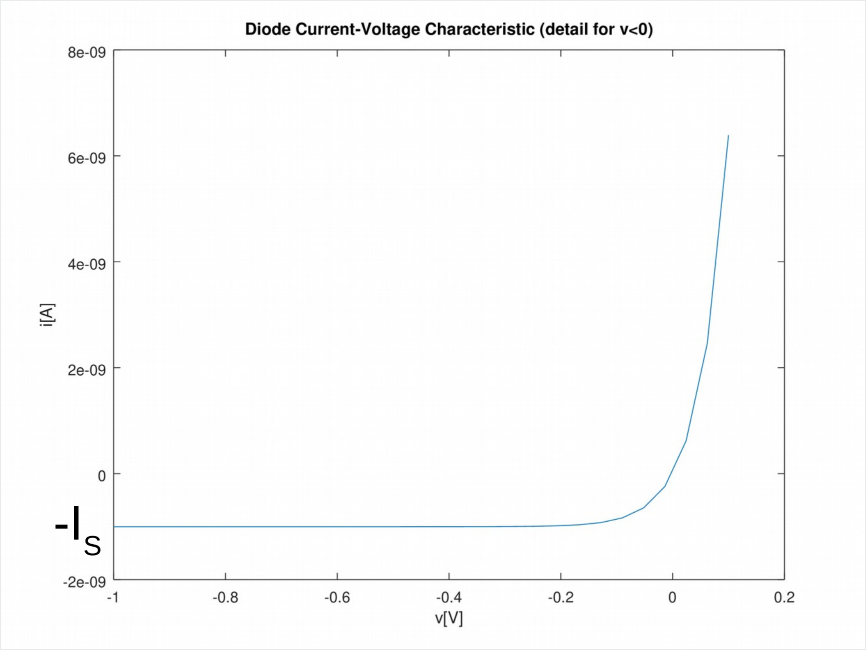
$$i = I_S\left(e^{\frac{v}{\eta V_T}} - 1\right)$$

$$I_S = 1\,nA$$

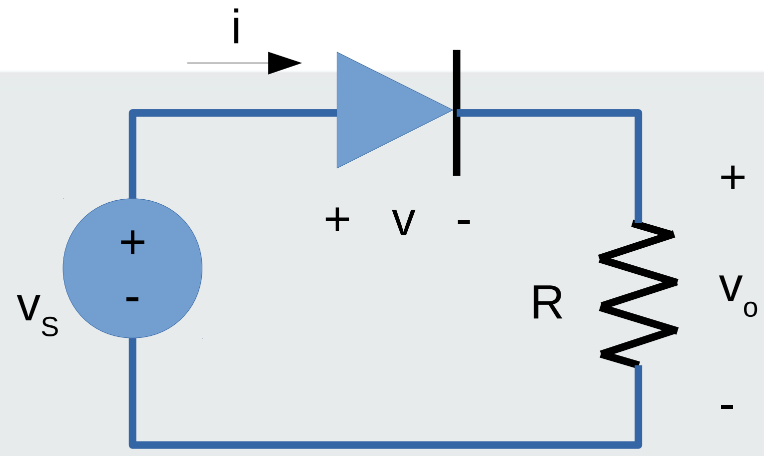
$$\eta = 2$$ (Silicon)







Example diode circuit



$$v+Ri-v_s=0$$ KVL

$$v+RI_S\left(e^{\frac{v}{\eta V_T}}-1\right)-v_s=0$$

Non-Linear Equation!

$$f(v)=v+RI_S\left(e^{\frac{v}{\eta V_T}}-1\right)-v_s$$ Define error function

$$f(v)=0$$ Solve non-linear equation for v

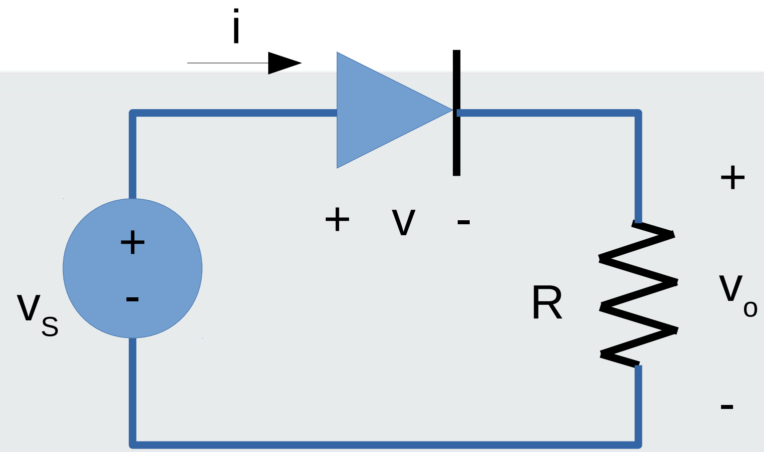
$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$ Use **Newton Raphson's iterative method**

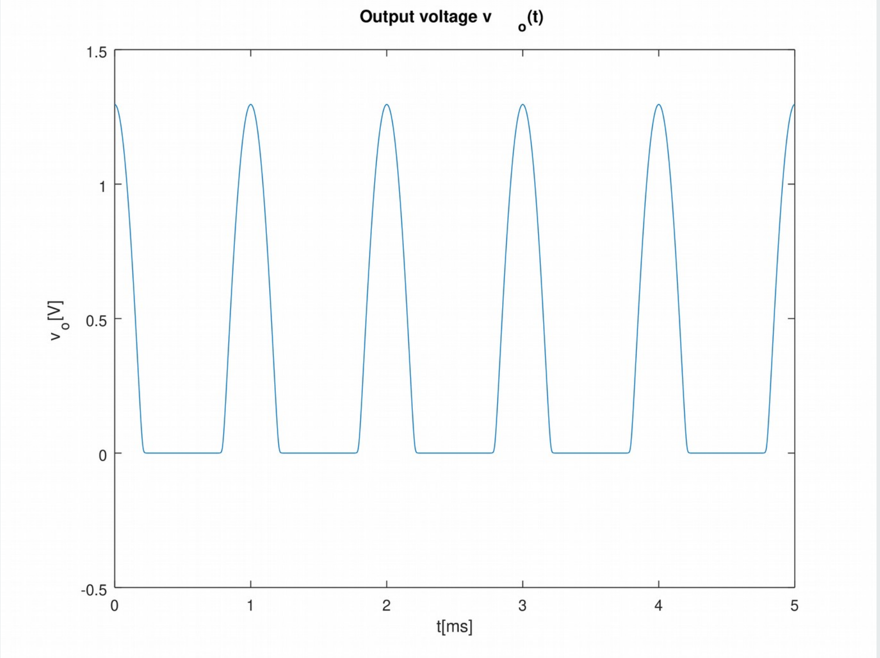
$$|x_{n+1}-x_n|<\delta$$ Stop condition: δ is the desired error

Example diode circuit solution

$$v_s(t) = 2\cos(2\pi f t)$$
$$I_S = 1\,nA$$
$$R = 1\,k\Omega$$
$$f = 1\,kHz$$
$$v_o = v_s - v\,;\, v_o = 0$$
$$i = \frac{v_o}{R}\,;\, i = 0$$

(ON; OFF)





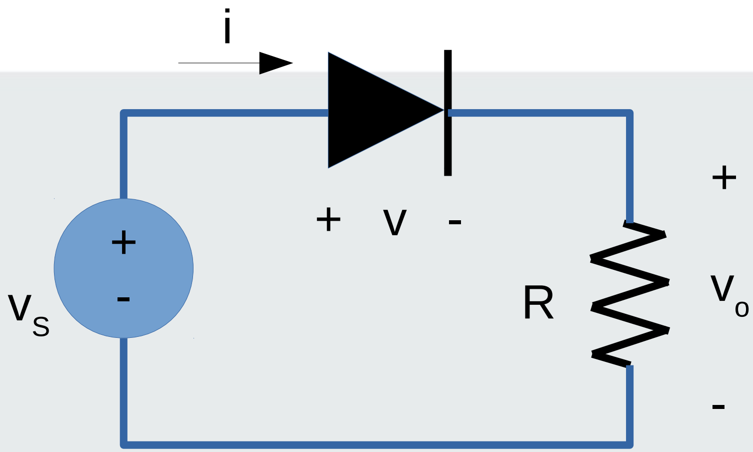
Octave script l11.m solves 1 non-linear equation for v, for each time instant t

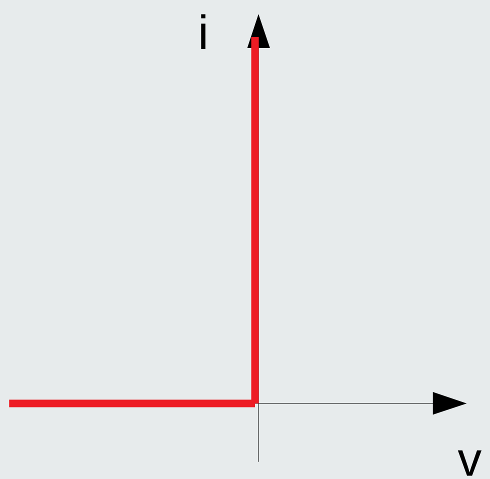
HALF WAVE RECTIFIER CIRCUIT

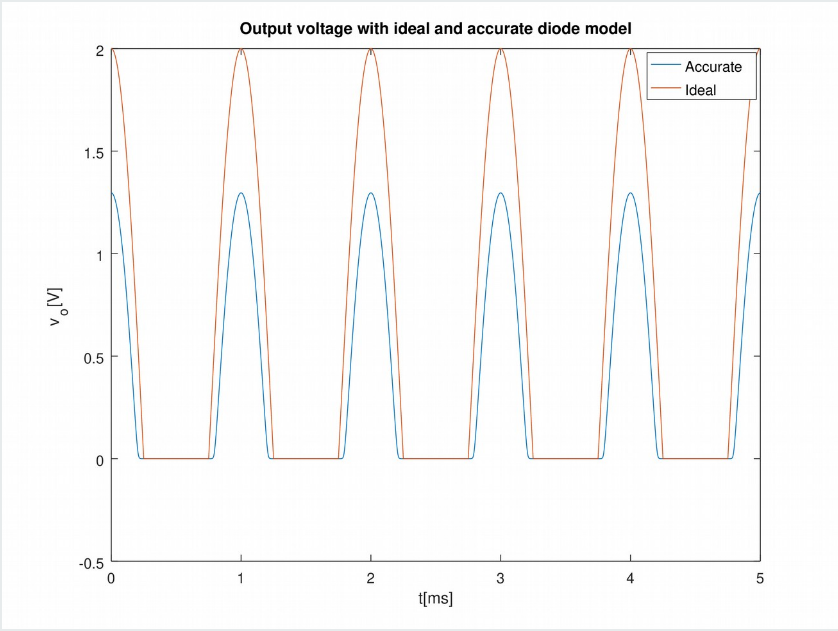
Ideal diode model

$$\begin{cases} i=0, & v<0 \\ v=0, & i\geq 0 \end{cases}$$

$$\begin{cases} v_o=0, & v_s<0 \\ v_o=v_s, & v_s\geq 0 \end{cases}$$



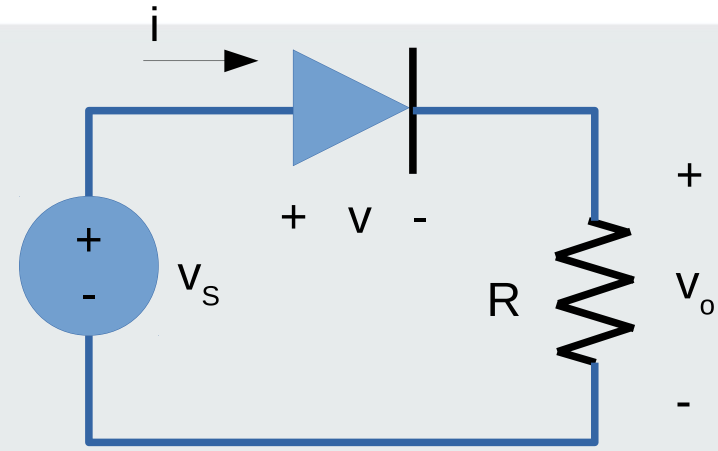


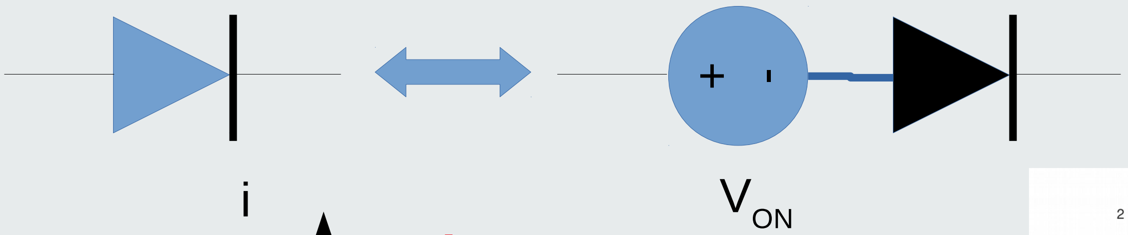


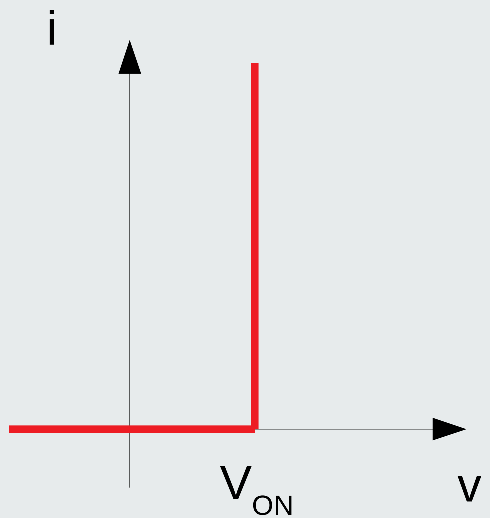
Diode model with ideal diode+voltage source

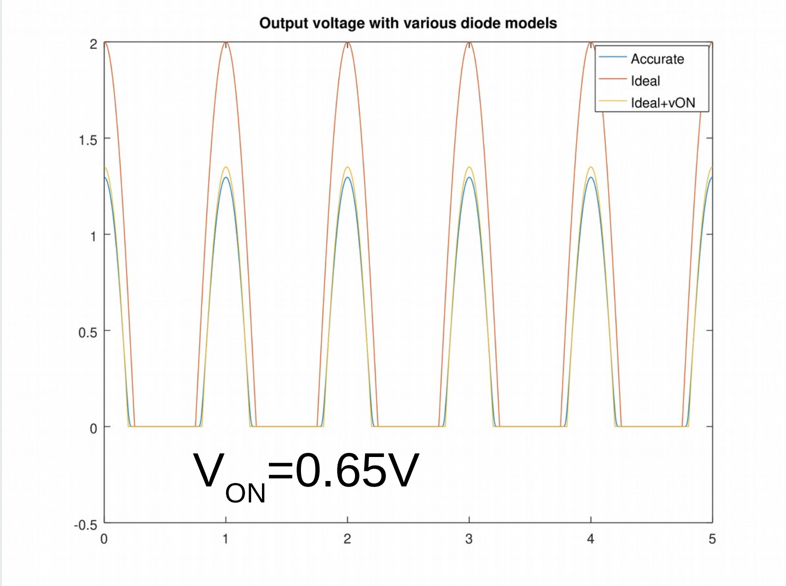
$$\begin{cases} i=0, & v<V_{ON} \\ v=V_{ON}, & i\geq 0 \end{cases}$$

$$\begin{cases} v_o=0, & v_s<V_{ON} \\ v_o=v_s-V_{ON}, & v_s\geq V_{ON} \end{cases}$$



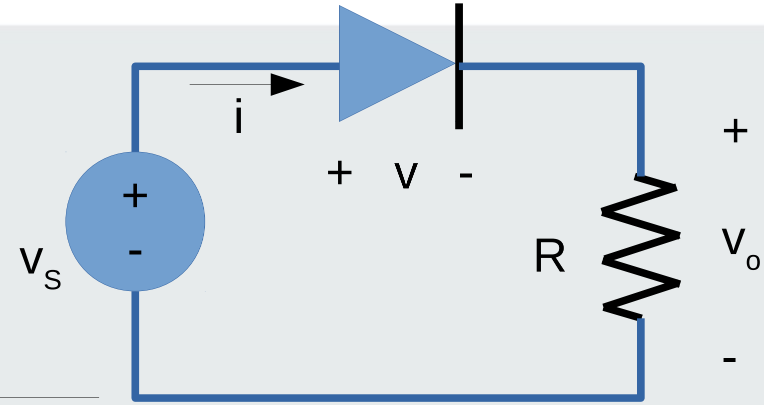


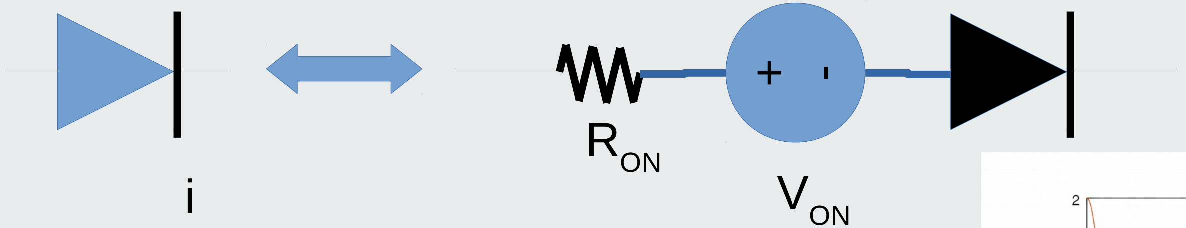


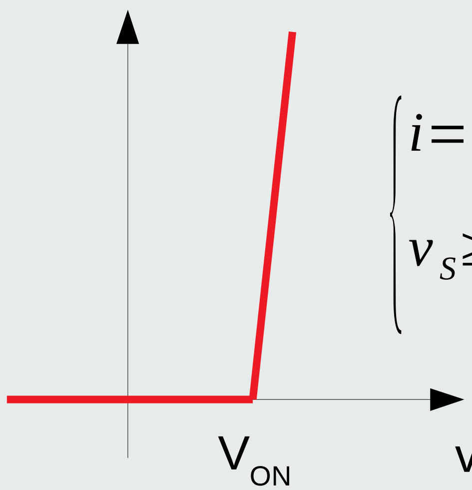


Diode model with ideal diode + voltage source + resistor

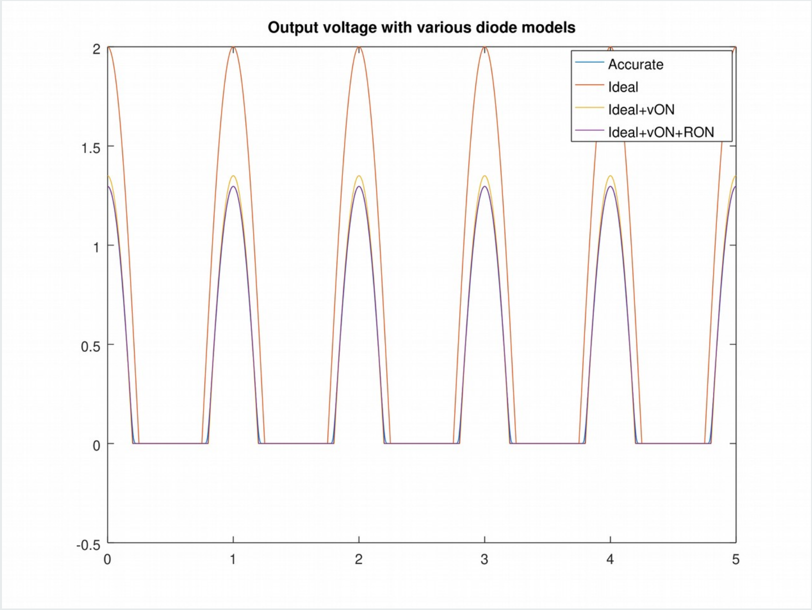
$$\begin{cases} v_o = 0, & v_s < V_{ON} \\ v_o = R\,i, & v_s \geq V_{ON} \end{cases}$$



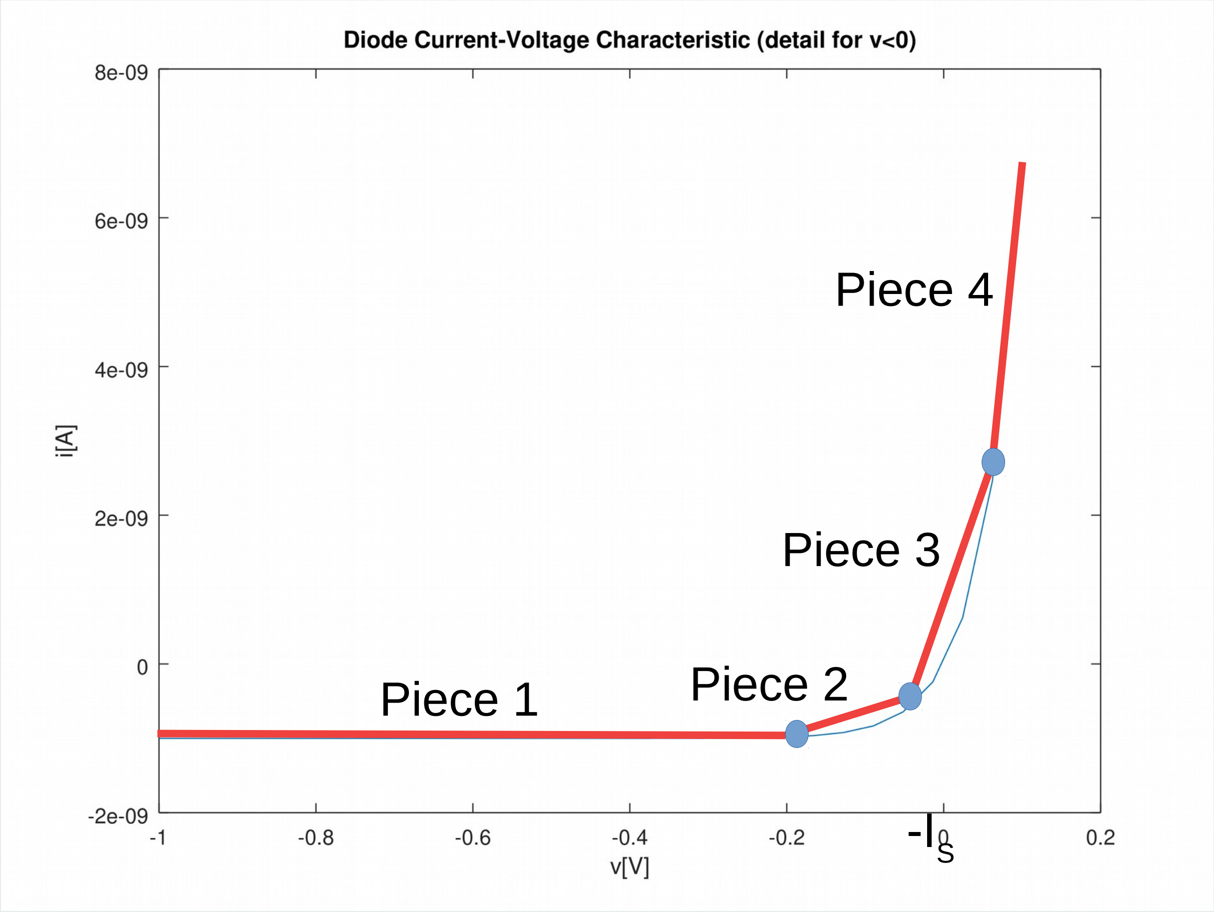


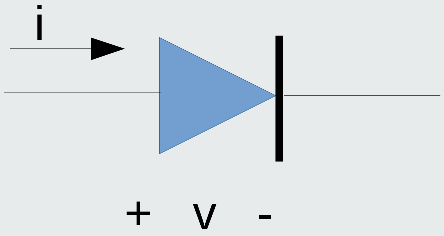


$$\begin{cases} i = 0, & v_S < V_{ON} \\ v_S \geq V_{ON}, & i = \dfrac{v_S - V_{ON}}{R_{ON} + R} \end{cases}$$



Piece-Wise Linear Diode Model





This example model needs:

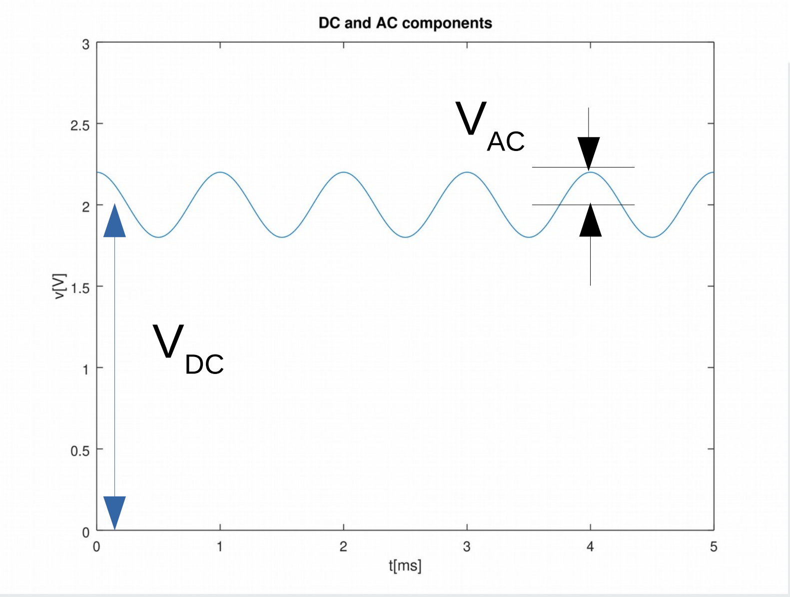
- I_s
- Three (I,V) points
- Slope for piece 4

8-parameter model…

Is this faster than non-linear model? Maybe...

Incremental Analysis

- In many practical situations, the input voltage has a DC component and an AC component
- The DC component dictates the circuit **<u>operating point</u>**
- The AC component contains the energy or information (the message) delivered
- If the DC component has no information, only the AC component needs to be analysed
- If the amplitude of the AC component is small, the distortion caused by non-linear components such as diodes is small …
- This is incremental analysis



$$v_S(t) = V_{DC} + V_{AC}\cos(2\pi f t)$$

$$v_S(t) = V_{DC}\cos(0t) + V_{AC}\cos(2\pi f t)$$

V_{DC} is zero frequency component

V_{AC} is frequency f component

Incremental Analysis Notation

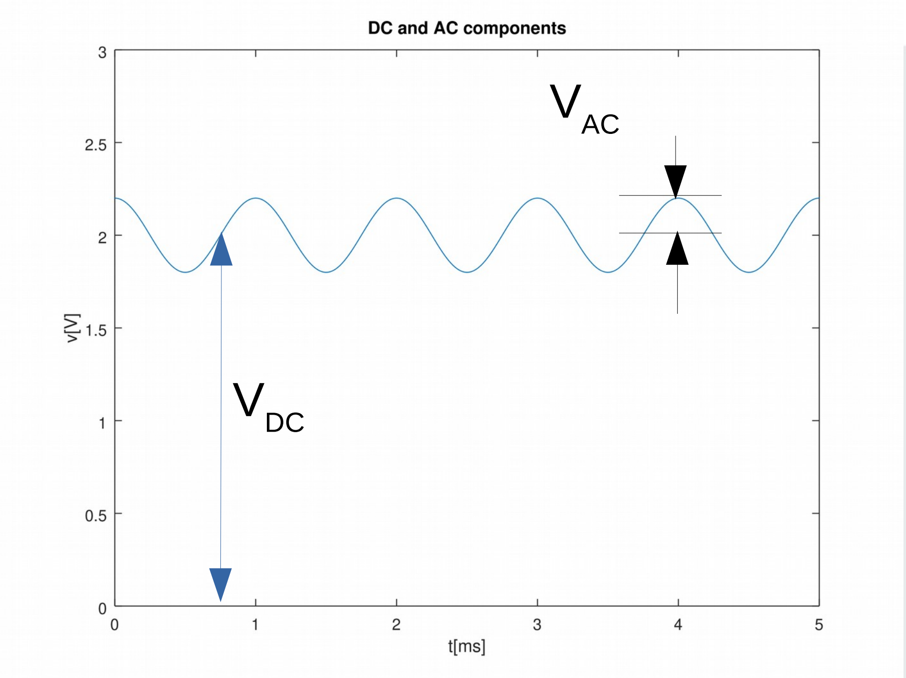
$$v_S(t) = V_{DC} + V_{AC}\cos(2\pi f t)$$

$$v_S(t) = V_S + v_s(t)$$

AC increment: variable and index lower case

DC: variable and index upper case

Total DC+increment: variable lower case and index upper case



$$v_s = \Delta v_S$$

v_s is considered an infinitesimal increment in incremental analysis.

DC and incremental components be analysed separately!

Operating point

$$v_D + R i_D - v_s = 0$$ KVL

$$(V_D + v_d) + R(I_D + i_d) - (V_S + v_s) = 0$$

$$(V_D + R I_D - V_S) + \cancel{(v_d + R i_d - v_s)} = 0$$

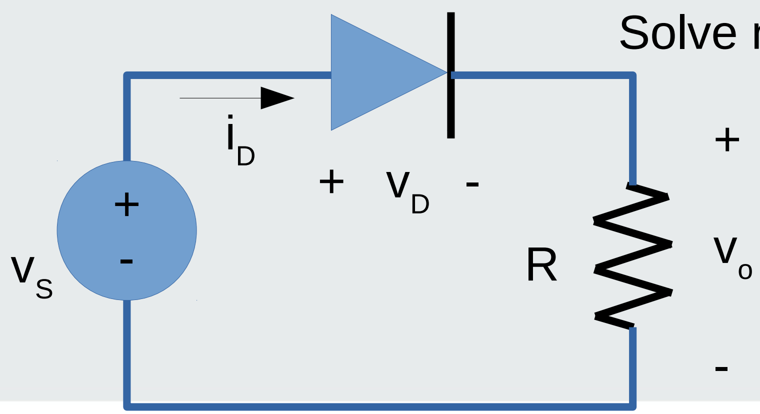
Separate DC and increments

$$v_s = 0 \Rightarrow v_d = 0, \quad i_d = 0$$

The cause of variations is v_S

$$V_D + R I_D - V_S = 0$$ → DC or operating point analysis

Solve non-linear equation to get V_D and I_D (DC values)



Diode incremental model

$$i_D(v_D) = I_S\left(e^{\frac{v_D}{\eta V_T}} - 1\right)$$ Diode equation

Taylor series expansion

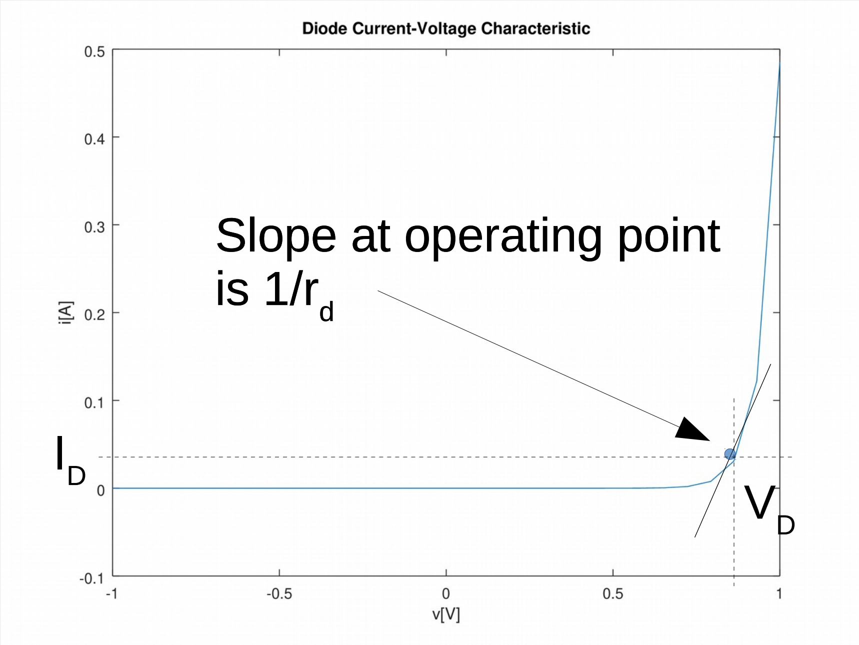
$$i_D(V_D + v_d) = i_D(V_D) + \frac{1}{1!}\frac{di_D}{dv_D}(V_D)\,v_d + \frac{1}{2!}\frac{d^2 i_D}{dv_D^2}(V_D)\,v_d^2 + \ldots$$

First order approximation for small v_d signal

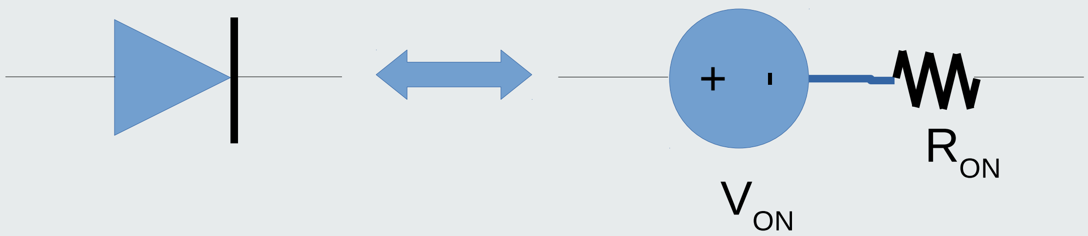
$$\cancel{I_D} + i_d \approx \cancel{I_D} + \frac{di_D}{dv_D}(V_D)\,v_d$$

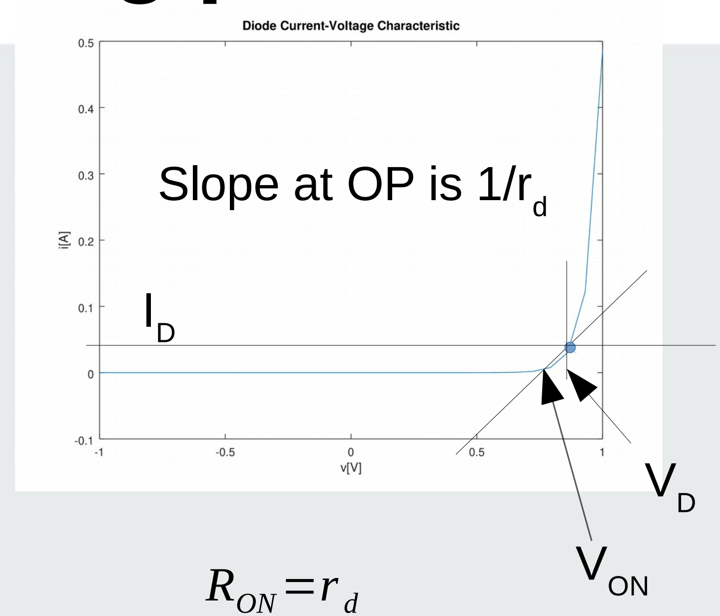
$$i_d \approx \frac{v_d}{r_d}$$ The diode incremental model is approximately a resistor!

$$r_d = \frac{dv_D}{di_D}(V_D)$$ The diode incremental resistor



Diode complete model at given operating point





- Attention: model only valid for given operating point!
- DC voltage V_D varies with the operating point (V_D, I_D)!
- Incremental resistor r_d varies with the operating point!
- Model only valid for OP and ***small*** variations around it!

$$V_D = V_D(V_S)$$

$$r_d = r_d(V_D) = r_d(V_S)$$

$$R_{ON} = r_d$$

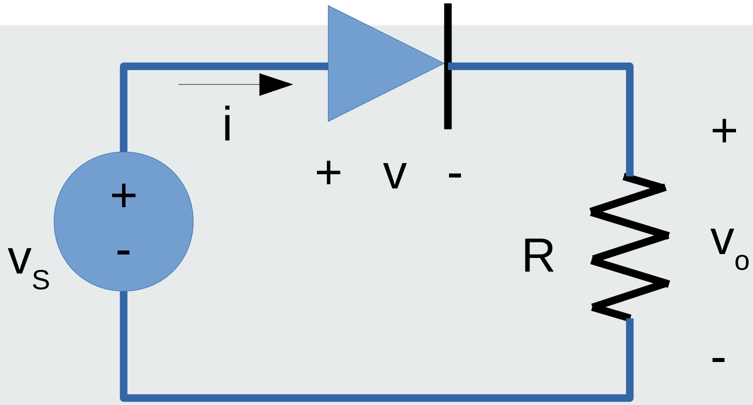
$$r_d = \frac{\eta V_T}{I_S e^{\frac{V_D}{\eta V_T}}}$$

$$V_{ON} = V_D - r_d I_D$$

$$i_D = \begin{cases} 0, & v_D < V_{ON} \\ \dfrac{v_d - V_{ON}}{R_{ON}}, & v_D \geq V_{ON} \end{cases}$$

Exercise: operating point and incremental analysis

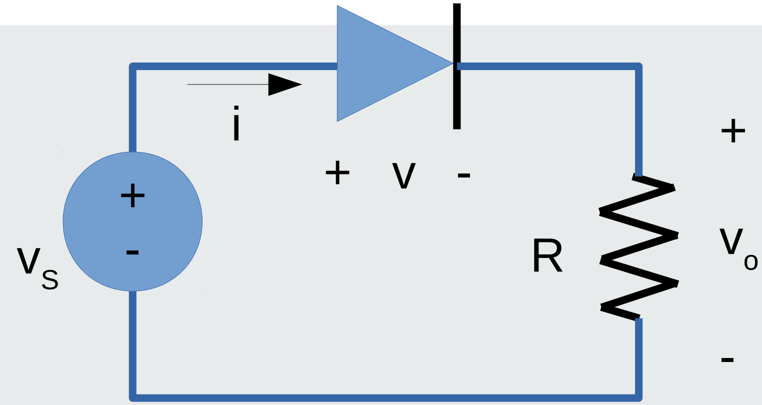
$$v_s(t) = 2 + A\cos(2\pi f t)$$
$$I_S = 1\,nA$$
$$R = 1\,k\Omega$$
$$f = 1\,kHz$$



1. Using the diode equation, plot v_o for A=20mV, 200mV and 1.5V, for 5ms
2. Using incremental analysis, plot v_o for A=20mV, 200mV and 1.5V
3. Compare the plots of 1. and 2.

Exercise resolution (1)

$$v_s(t)=2+A\cos(2\pi f t)$$
$$I_S=1\,nA$$
$$R=1\,k\Omega$$
$$f=1\,kHz$$



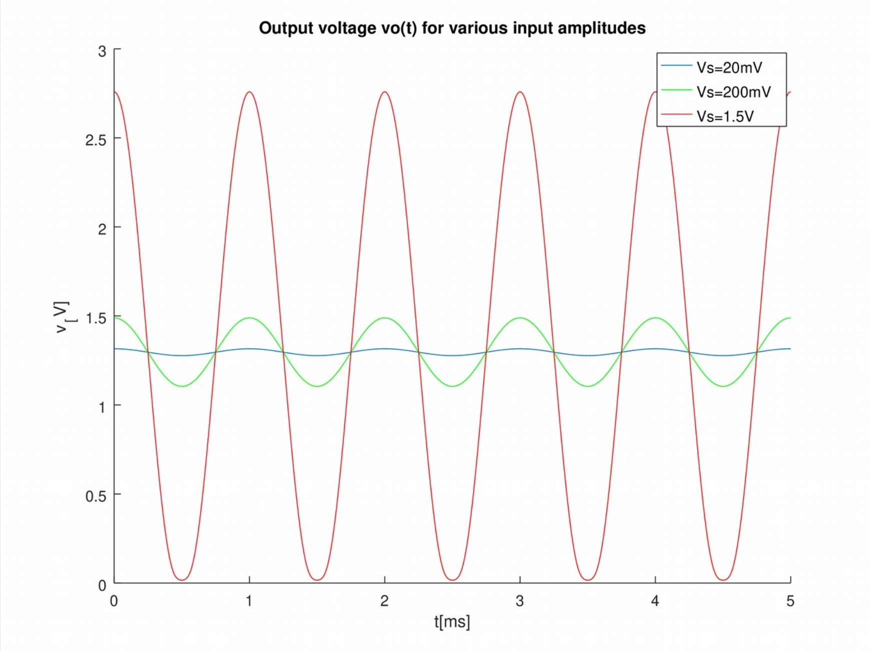
1. *Using the diode equation, plot v_o for A=20mV, 200mV and 1.5V, for 5ms*

The diode equation was taught in Lecture 11. Using the same Octave functions, the v_O is plotted for the 3 values of A (see Octave script l12_exercise.m)

For A=1.5V the distortion is visible (maxima sharper than minima)

For A=200mV distortion is not visible but it is there. Spectral analysis would reveal it.

For A=20mV the distortion is very low and the waveform is almost a pure tone.



Exercise resolution (2)

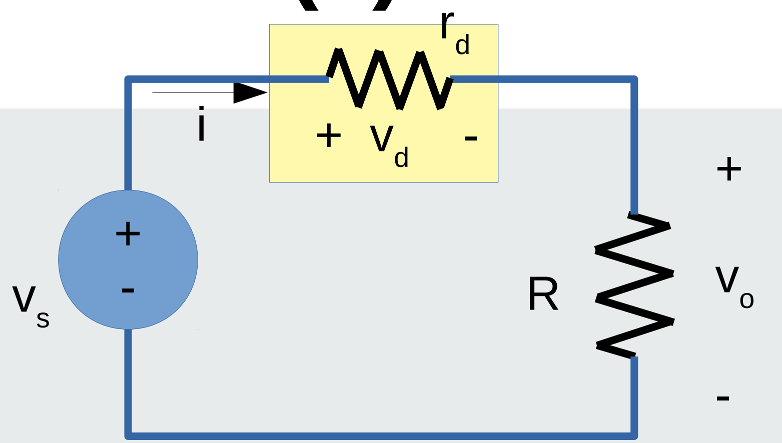
$$v_s(t) = 2 + A\cos(2\pi f t)$$

$$I_S = 1\,nA, R = 1\,k\Omega, f = 1\,kHz$$

$$v_o = \frac{R}{R + r_d} v_s$$ Incremental output voltage

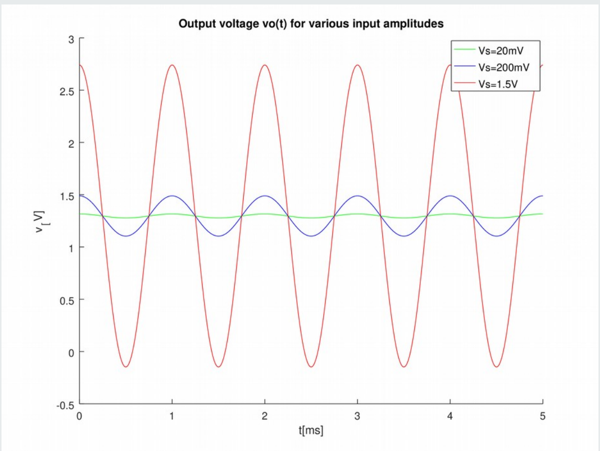
$$V_O = R I_D$$ DC output voltage

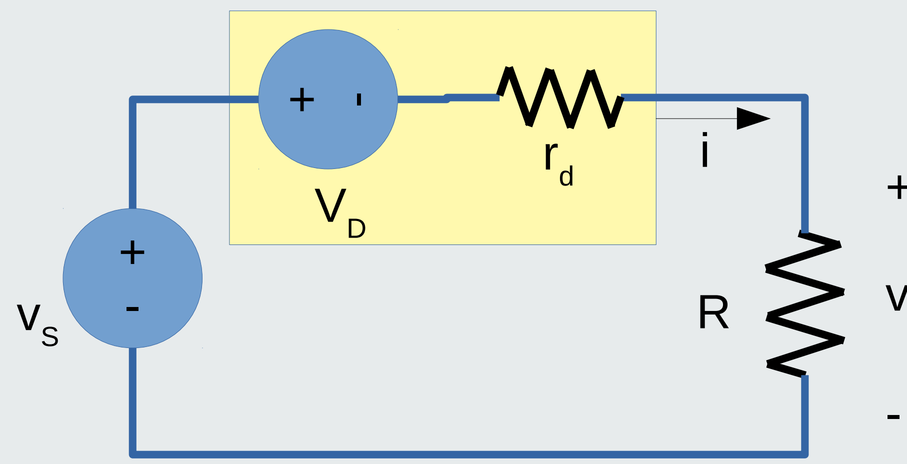
$$v_O = V_O + v_o$$



Incremental circuit model: LINEAR!

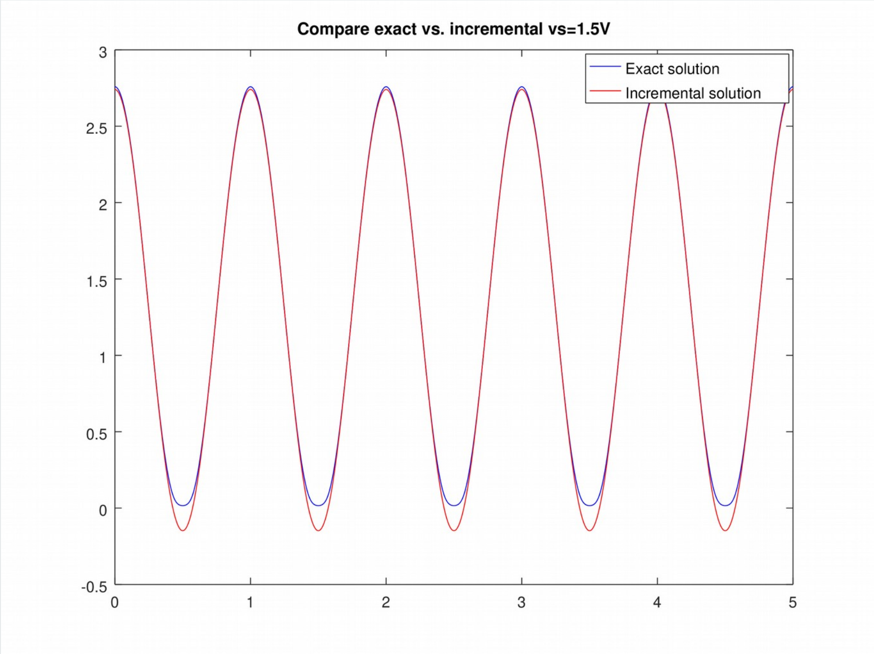
Total output voltage

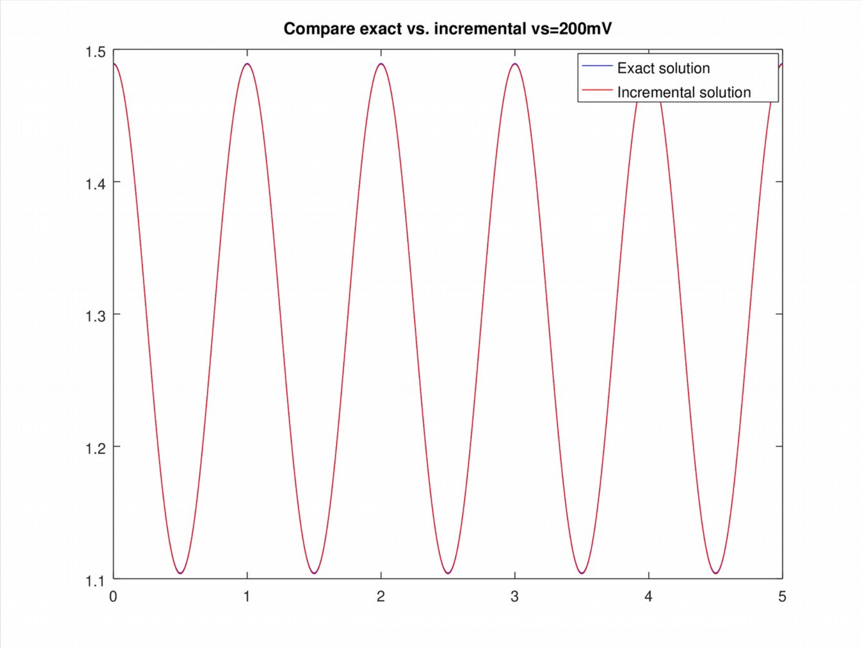




Complete circuit model via incremental analysis: LINEAR!

Exercise resolution (3)





Non-linear and incremental solutions differ visibly

Incremental solution bad for large signals!

Exact and incremental solutions differ little

Incremental solution good for small signals!

Even better for v_s=20mV though not shown

Conclusion

- Ideal diode model
- Voltage source diode model
- Voltage source plus resistor diode model
- Piecewise linear diode model
- Incremental diode model
- Exercise