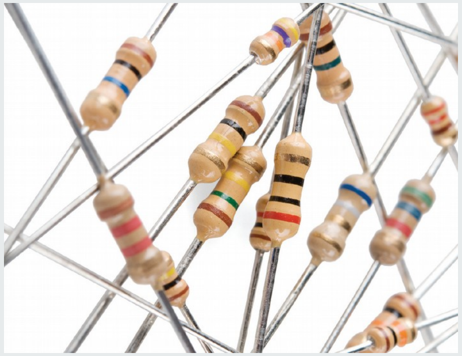
Circuit Theory and Electronics Fundamentals

Lecture 3: RC and RL circuits

- Resistor images
- Capacitors and inductors
- Capacitors and inductors in series and in parallel
- RC circuits
- RL circuits

Resistor Images

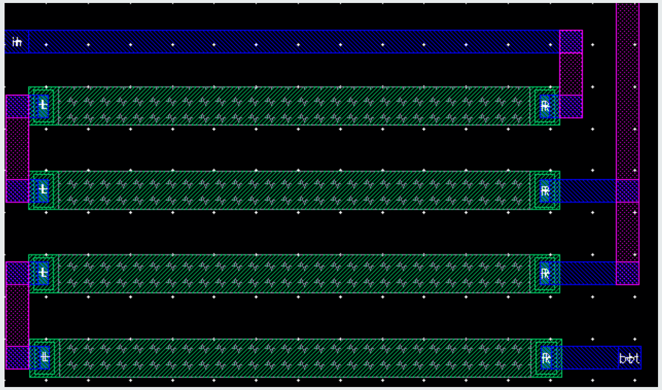


Discrete Resistors

Discrete SMD Resistors

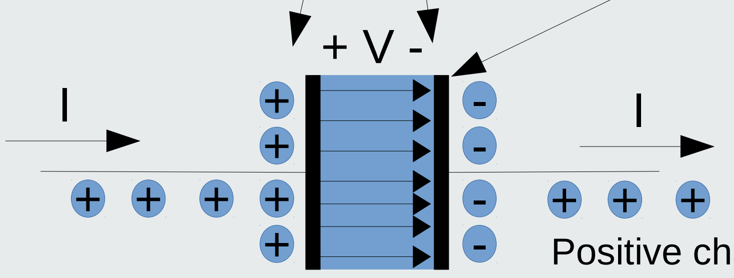
Voltage and Current sources may be complex circuits! Not shown as yet.

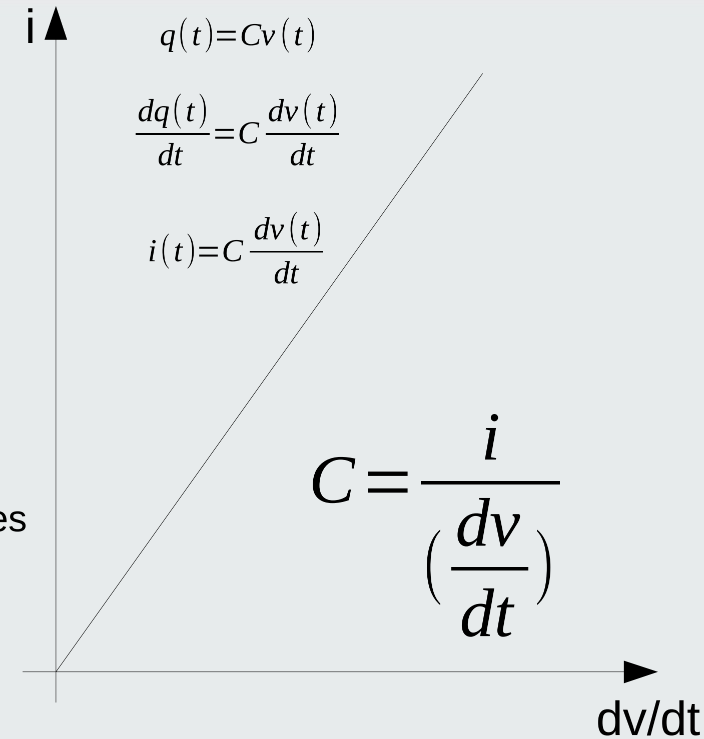
SMD resistors mounted on Printed Circuit Board (PCB)



Integrated resistor (nanometric size: 10^{-9} meter)

Component: Capacitor





$$q(t) = Cv(t)$$

$$\frac{dq(t)}{dt} = C\frac{dv(t)}{dt}$$

$$i(t) = C\frac{dv(t)}{dt}$$

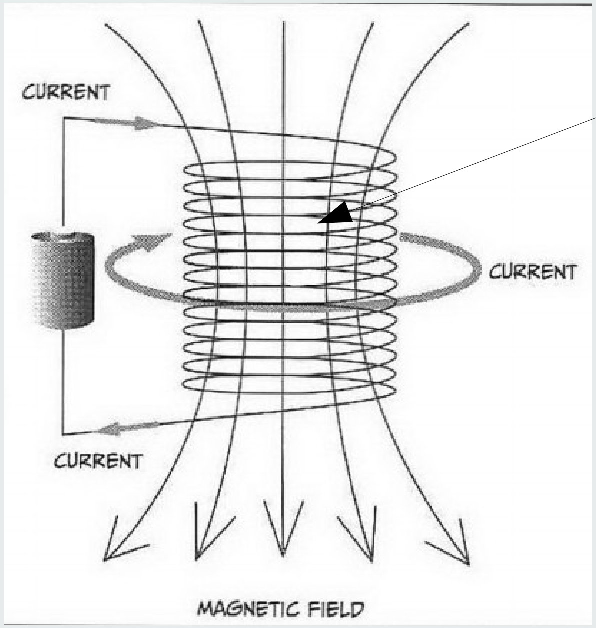
$$C = \frac{i}{\left(\frac{dv}{dt}\right)}$$

- Linear Capacitor: Q = CV
- C is capacitance expressed in Farad: F = C/V

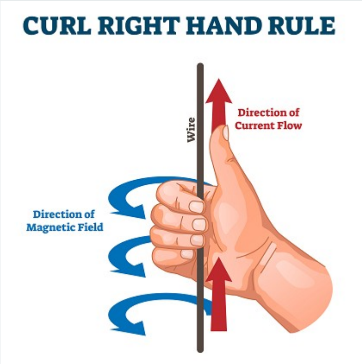
Analysis methods are the same but generate *linear differential equations* instead of *algebraic equations*

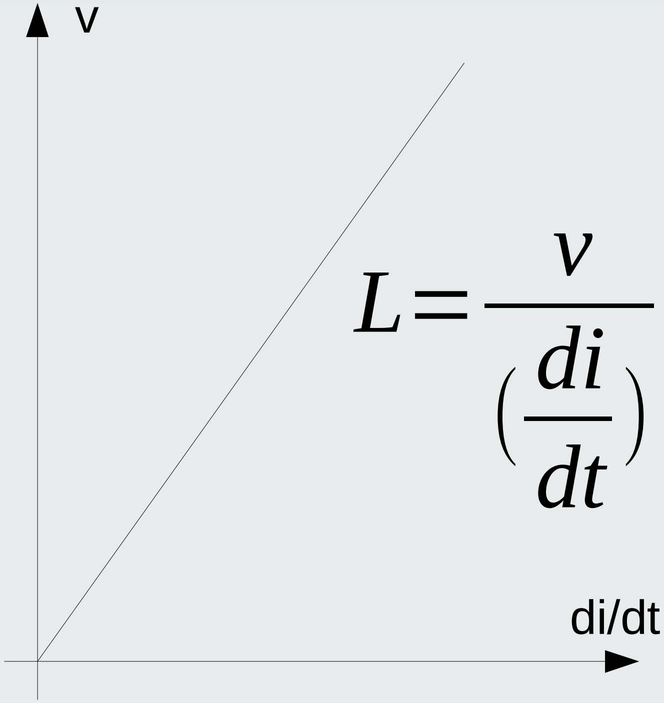
Solutions are time functions (*lower case* notation)

Component: inductor



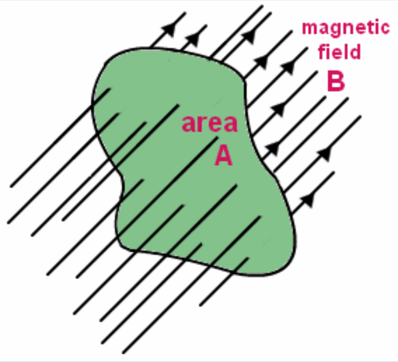
Inductor core (permeability μ)
Storing magnetic field: energy!





$$L = \frac{v}{\left(\frac{di}{dt}\right)}$$

Current keeps magnetic field on, and magnetic field keeps current going



$$\Phi = \iint_A \vec{B} \cdot \vec{n}\, dA$$

$$\Phi = Li$$

$$\frac{d\Phi}{dt} = L\frac{di}{dt}$$

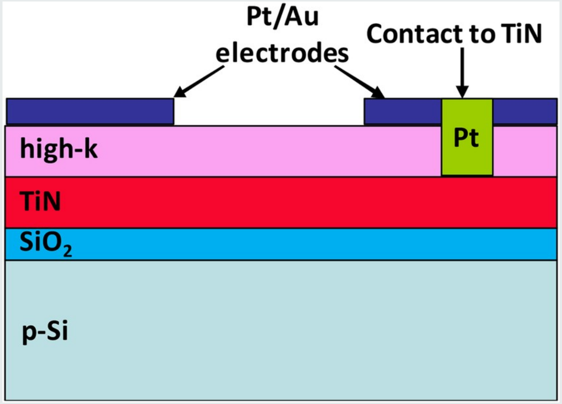
$$v(t) = L\frac{di(t)}{dt}$$

L given Henry: H = Vs^2/C

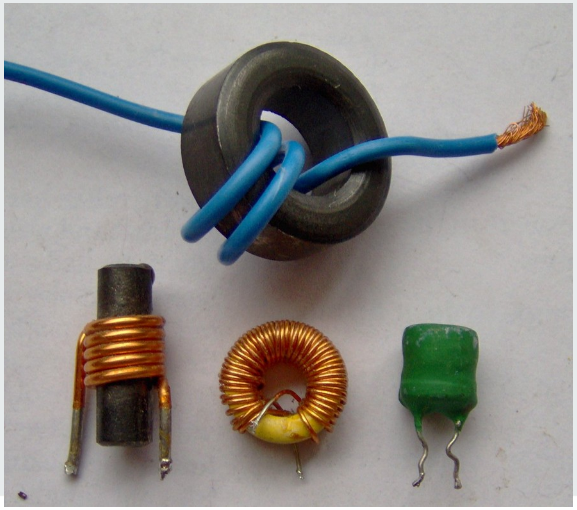
Capacitor and inductors images

Discrete capacitors

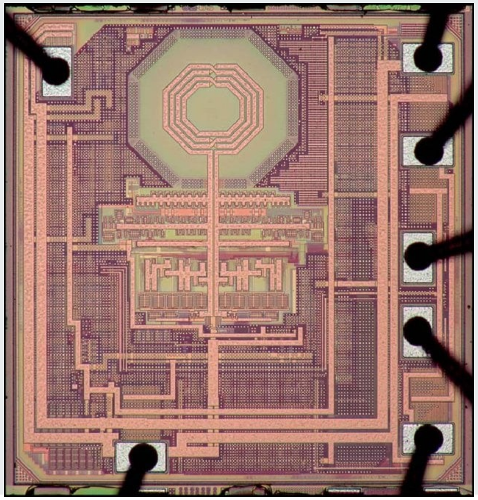
Integrated capacitor



Discrete inductors



Integrated inductor



Parallel of Capacitors

$$i = \sum_{i=1}^{n} i_i$$

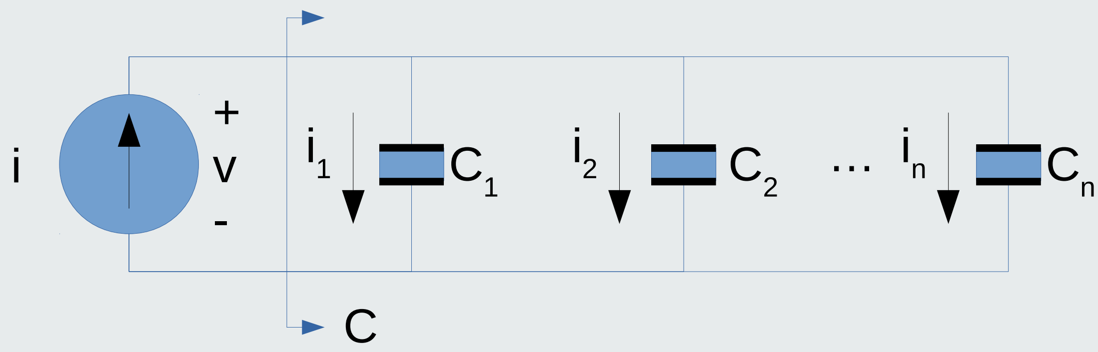
KCL

$$i = \sum_{i=1}^{n} C_i \frac{dv}{dt}$$

All Cs have the same voltage v

$$i = \left(\sum_{i=1}^{n} C_i\right) \frac{dv}{dt}$$

$$C = \sum_{i=1}^{n} C_i$$



Similar to ***<u>series</u>*** of resistors

Series of Capacitors

$$v = \sum_{i=1}^{n} v_i$$

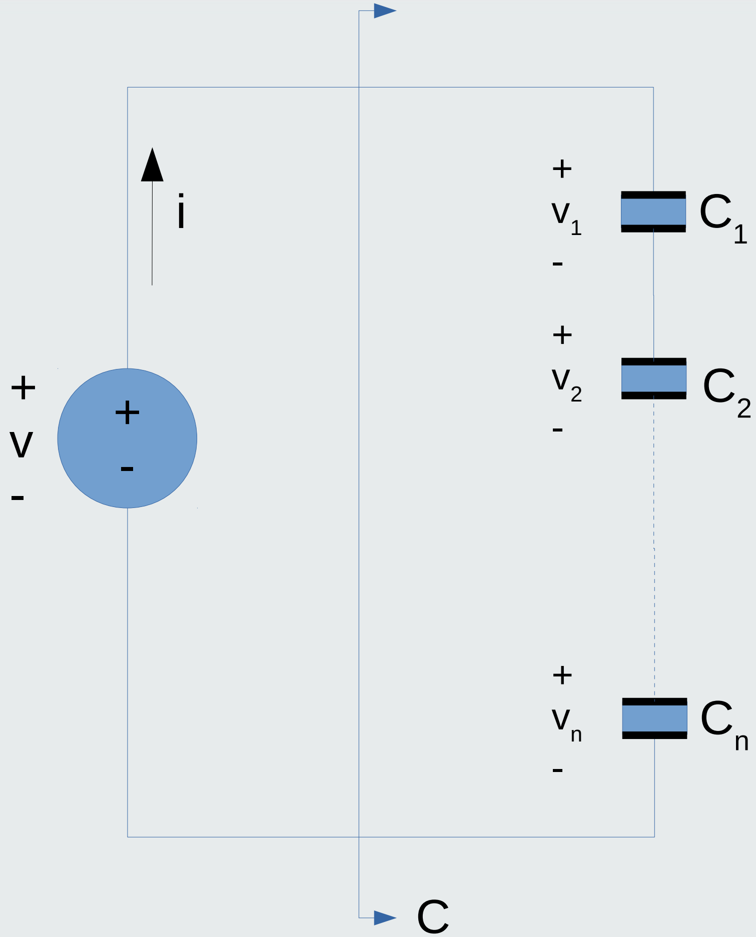
KVL

$$\frac{dv}{dt} = \sum_{i=1}^{n} \frac{dv_i}{dt}$$

$$\frac{dv}{dt} = \sum_{i=1}^{n} \frac{i}{C_i}$$

All Cs have the same current *i*

$$C = \frac{1}{\sum_{i=1}^{n} \frac{1}{C_i}}$$



Similar to ***parallel*** of resistors

Parallel of Inductors

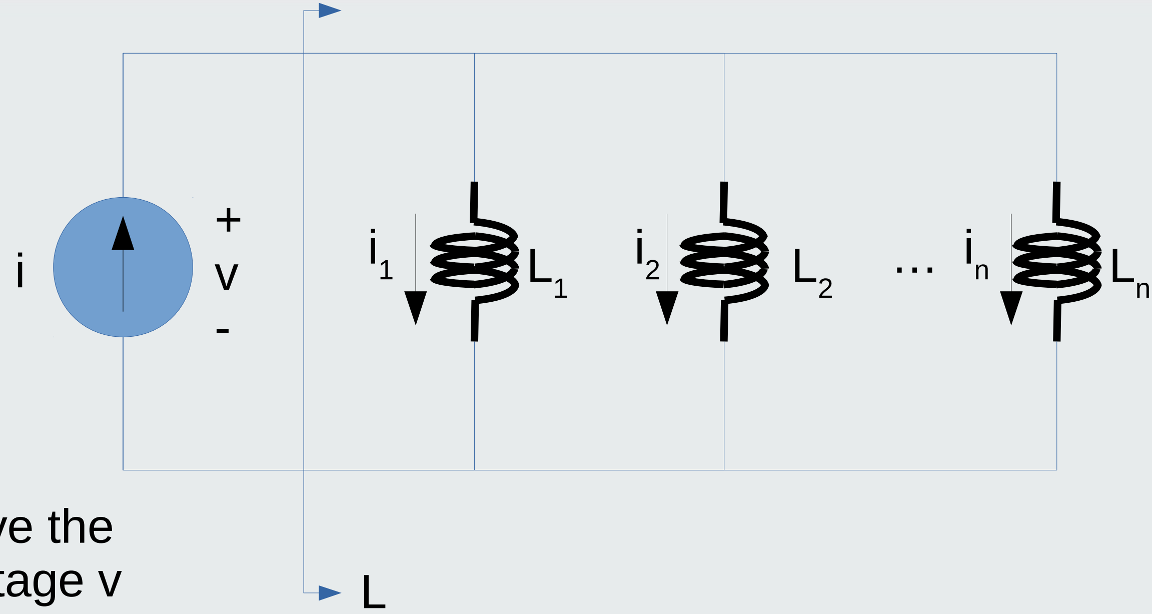
$$i = \sum_{i=1}^{n} i_i$$

KCL

$$\frac{di}{dt} = \sum_{i=1}^{n} \frac{di_i}{dt}$$

$$\frac{di}{dt} = \sum_{i=1}^{n} \frac{v}{L_i}$$

All Ls have the same voltage v



$$L = \frac{1}{\sum_{i=1}^{n} \frac{1}{L_i}}$$

Similar to ***parallel*** of resistors

Series of Inductors

$$v=\sum_{i=1}^{n} v_i$$

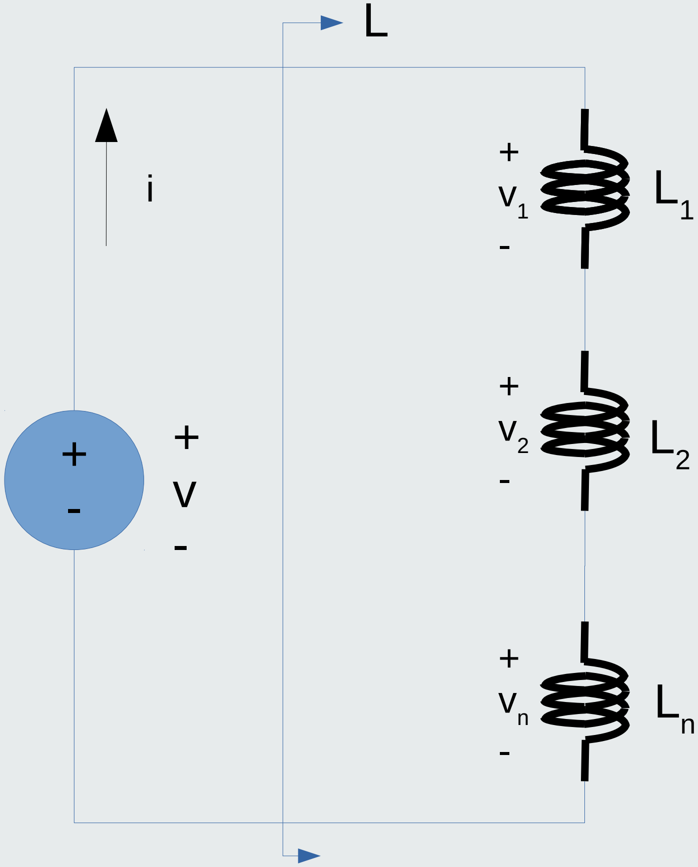
KVL

$$v=\sum_{i=1}^{n} L_i \frac{di}{dt}$$

All Ls have the same current *i*

$$v=\left(\sum_{i=1}^{n} L_i\right)\frac{di}{dt}$$

$$L=\sum_{i=1}^{n} L_i$$



Similar to ***series*** of resistors

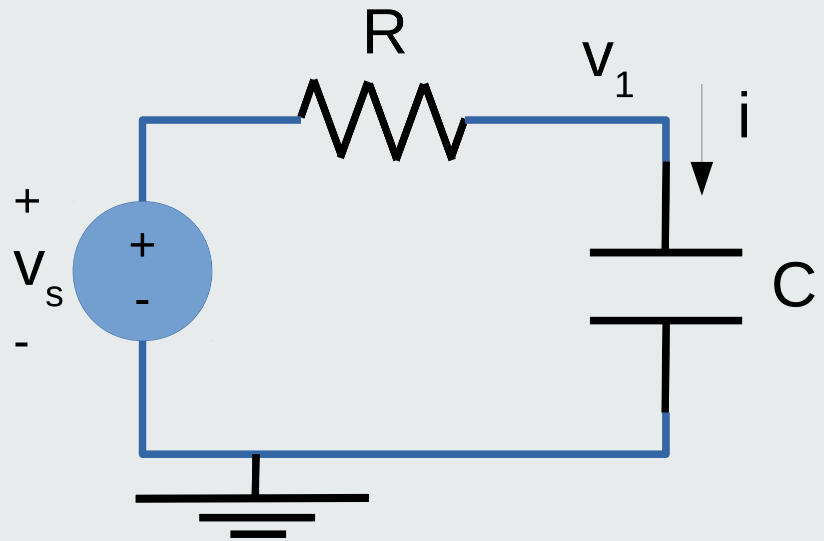
RC series: circuit analysis

$$v_s = R\,i + v_1$$ KVL

$$i = C\frac{dv_1}{dt}$$

$$RC\frac{dv_1}{dt} + v_1 = v_s$$

1st order linear differential equation!



Solution:

$$v_1(t) = v_{1n}(t) + v_{1f}(t)$$

Forced solution: depends on voltage source $v_s(t)$ and R, C

Natural solution: depends on initial charge (voltage) and R, C

RC series: natural solution

$$RC\frac{dv_1}{dt}+v_1=0$$

Remove voltage source

$$\left(RC\frac{d}{dt}+1\right)v_1=0$$

Note *d/dt* is a linear operator

$$RC\frac{d}{dt}+1=0 \vee v_1=0$$

Two solutions, $v_1 = 0$ is a trivial (uninteresting) solution

$$RCs+1=0$$

$$s=-\frac{1}{RC}$$

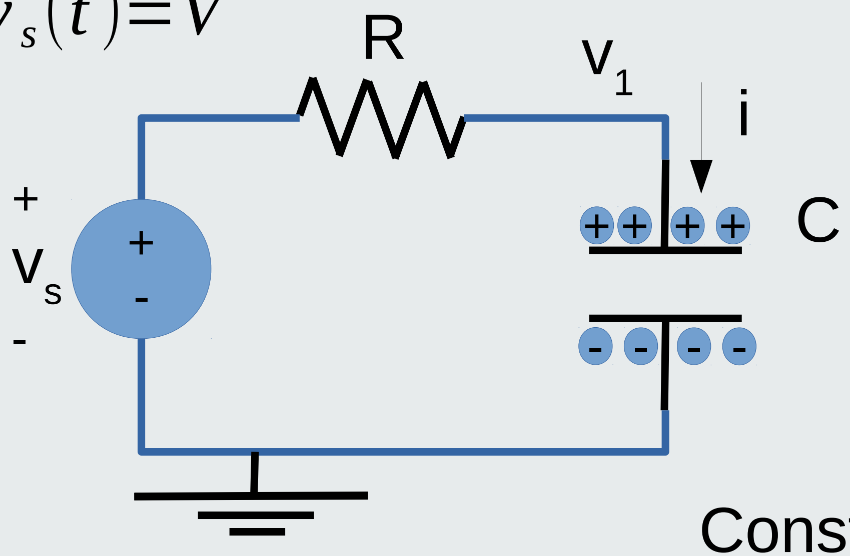
Characteristic equation: replace *d/dt* with *s*, aka complex frequency. Solve for *s*. RC is called “time constant”

$$v_{1n}(t)=Ae^{st}=Ae^{-\frac{t}{RC}}$$

Natural solution, A is a constant to be determined

RC series: forced solution with constant excitation

$$v_s(t)=V$$



Voltage source drives constant voltage V

Eventually C charges up, v_1 reaches $v_1 = V$, and current stops: $i=0$

Constant excitation => constant forced solution!

$$v_{1f}(t)=V$$

Capacitor behaves like an *open-circuit* after charged!

RC series: final solution

$$v_1(t) = v_{1n}(t) + v_{1f}(t)$$

Superimpose natural and forced solutions

$$v_1(t) = Ae^{-\frac{t}{RC}} + V$$

Constant *A* can now be determined by a boundary condition: charge (voltage) of C at some instant. A common boundary condition is the initial capacitor voltage at t=0, here assumed 0. Constant *A* is thus determined

$$v_1(0) = 0$$

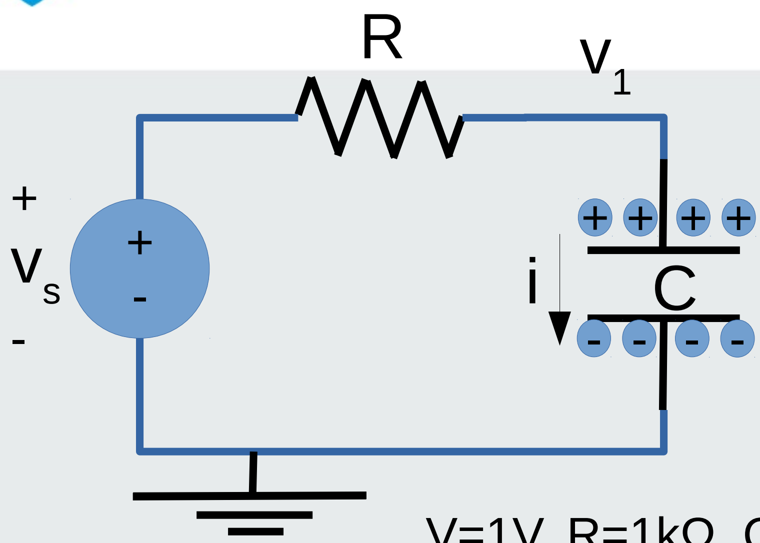
$$v_1(0) = 0 \Rightarrow A = -V$$

$$v_1(t) = V\left(1 - e^{-\frac{t}{RC}}\right)$$

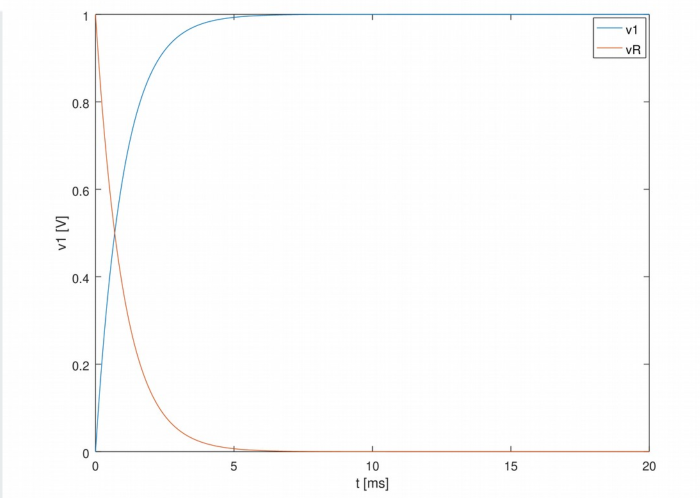
$$i(t) = C\frac{dv_1}{dt} = \frac{V}{R}e^{-\frac{t}{RC}}$$

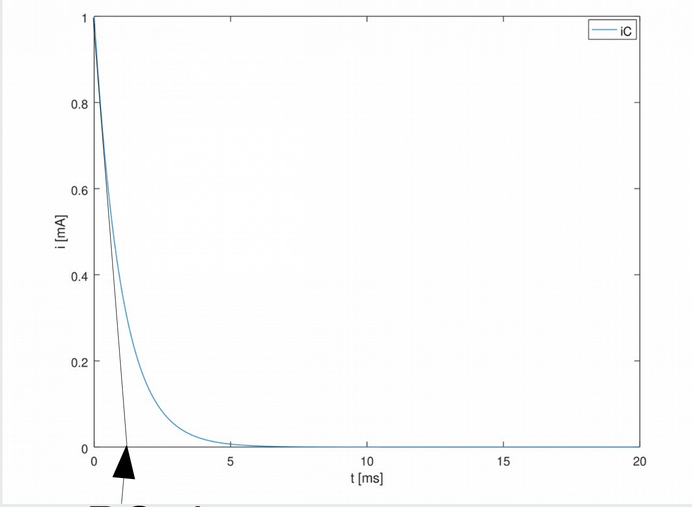
Final solutions voltage and current are now computed

RC series: final solution plots



V=1V, R=1kΩ, C=1μF



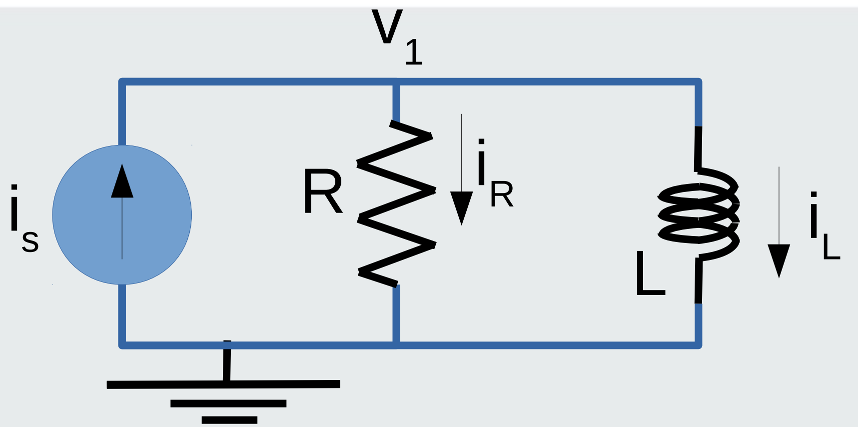


RC=1ms

t=0: no voltage across C (electric field build up starts), v_1 is null, all voltage across R, and *i* is max.

t=∞: all voltage across C (open-circuit behaviour), v_1 is max, no voltage across R and *i* is null.

RL parallel: circuit analysis



$$i_s = i_R + i_L$$ KCL

$$v_1 = L\frac{di_L}{dt}$$

$$i_R = \frac{v_1}{R}$$

$$\frac{L}{R}\frac{di_L}{dt} + i_L = i_s$$

Solution for constant current source

$$i_s(t) = I$$

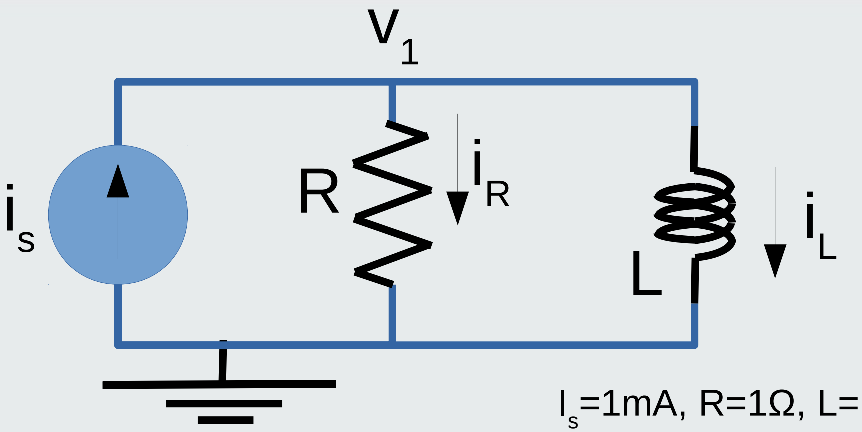
$$i_L(t) = I\left(1 - e^{-\frac{R}{L}t}\right)$$

$$i_R = i_s - i_L$$

$$i_R(t) = I\,e^{-\frac{R}{L}t}$$

$$v_1(t) = RI\,e^{-\frac{R}{L}t}$$

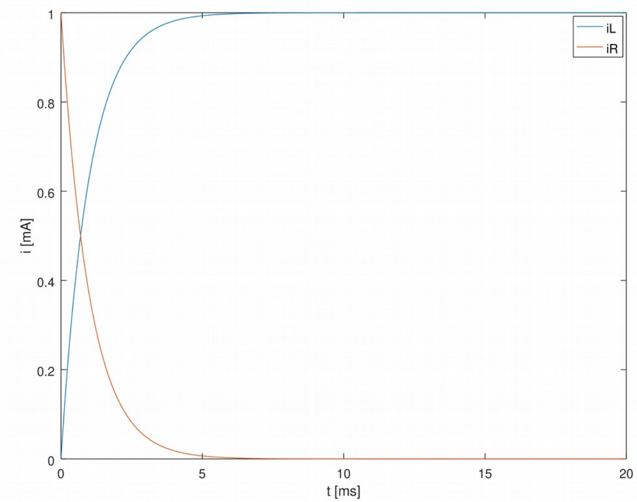
RL parallel: final solution plots

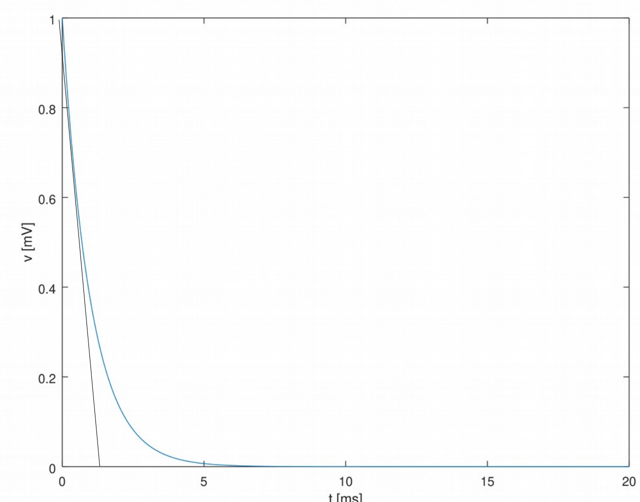


I_s=1mA, R=1Ω, L=1mH

t=0: no current through L
(magnetic field build up starts)
All current through R
v_1 is max

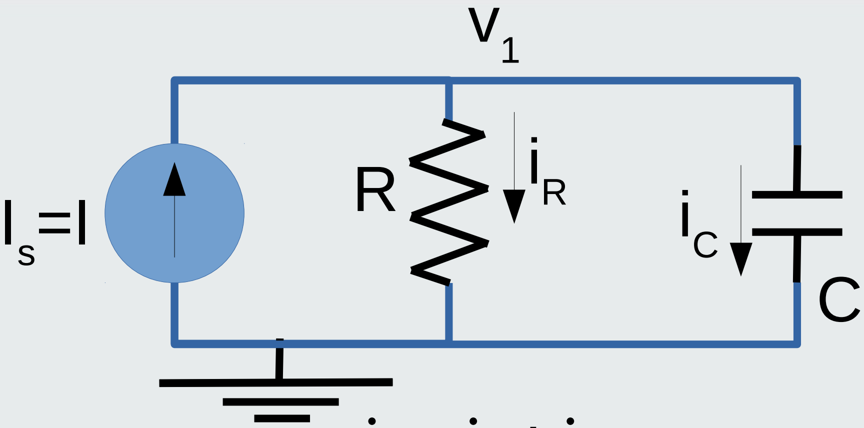
t=∞: all current through L
(short-circuit behavior)
No current through R
v_1 is null





L/R = 1ms

RC parallel: circuit analysis



$$i_s = i_R + i_C$$ KCL

$$i_C = C\frac{dv_1}{dt}$$

$$i_R = \frac{v_1}{R}$$

$$C\frac{dv_1}{dt} + \frac{v_1}{R} = i_s$$

Solution for constant i_s

$$i_s(t) = I$$

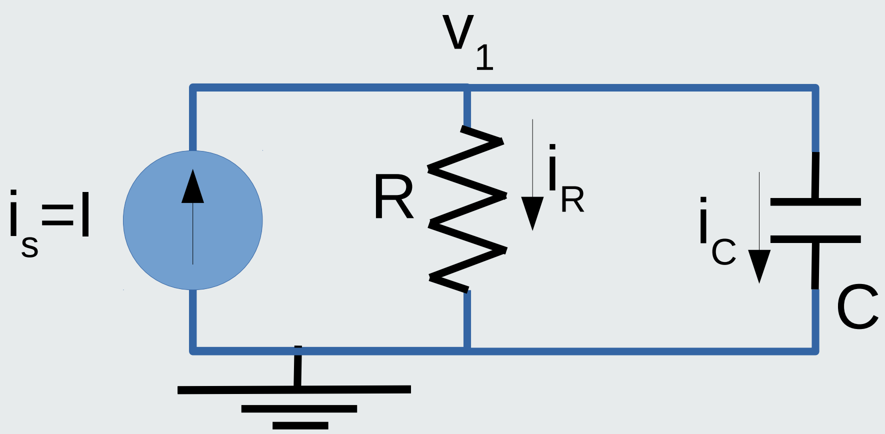
$$v_1(t) = RI\left(1 - e^{-\frac{t}{RC}}\right)$$

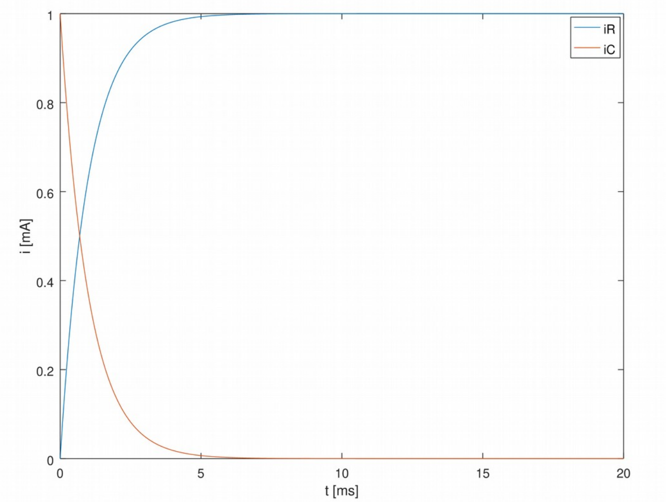
$$i_C(t) = I\,e^{-\frac{t}{RC}}$$

$$i_R = i_s - i_C$$

$$i_R(t) = I\left(1 - e^{-\frac{t}{RC}}\right)$$

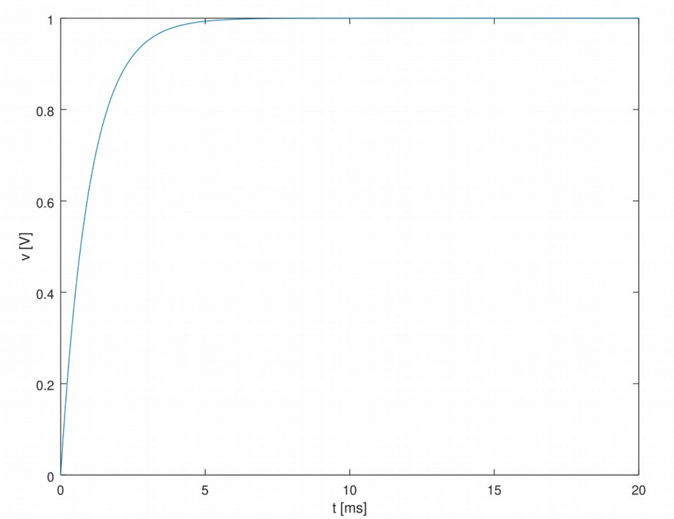
RC parallel circuit analysis





t=0: no current through R
(electric field build starts)
All current through C
v_1 is nul

t=∞: all current through R
No current through C
(open-circuit behavior)
v_1 is max (v_1=RI)



Conclusion

- Images of components R, L and C shown
- Capacitor and inductor laws
- Series and parallel of capacitors and inductors
- Analysis of circuits containing a single capacitor or single inductor, using 1st order linear differential equations
  - RC series
  - RC parallel
  - RL parallel