



**DF**  
DEPARTAMENTO  
DE FÍSICA  
TÉCNICO LISBOA

## Nanotechnologies and Nanoelectronics

2021-semester 1-P2

# Homework #3

The work should be sent to: [susana.freitas@tecnico.ulisboa.pt](mailto:susana.freitas@tecnico.ulisboa.pt)

Deadline: 21 January 2022 @ 23:59<sup>1</sup>.

Please send the work in one single file<sup>2</sup> (e.g., pdf, jpeg, word)  
identified as: **NN2021\_YourName\_HW3**

---

<sup>1</sup> The documents received up to 5 days after the deadline will have a penalty of 1 point (out of 20) per day. No documents will be accepted after the 6<sup>th</sup> day.

<sup>2</sup> In case you need to have pictures of hand-written work, or multiple format digital formats for your solutions, please merge them into one single document.

Support bibliography:

Book "Theory of Magnetic Recording" by H.Neal Bertram, Cambridge University Press [in Fenix]  
Chapters 4 (to answer questions 1-4) and Chapter 8 (to answer question 5)

Magnetic data storage and positioning systems relies on highly sensitive and reliable transducers to measure encoded patterns in magnetic media. A magnetized recording media is characterized by a finite magnetized thickness  $t$  and a bit length  $B$  while its track width ( $w$ ) along the  $y$  axis is much wider than the other dimensions of interest. Consequently, the medium magnetization  $\vec{M}$  lies in the  $xOz$  plane (2D approach) and the normal  $H_n$  and longitudinal  $H_l$  magnetic field components produced at each bit surface arise by virtue of a linear charge density  $\vec{n}' \cdot \vec{M}$ , where  $\vec{n}'$  is the unit vector normal to the outward surface.

$$\vec{H} = H_n \vec{n} + H_l \vec{l}$$

$$\begin{cases} H_n = -\frac{\vec{n}' \cdot \vec{M}}{2\pi} \Delta\theta \\ H_l = -\frac{\vec{n}' \cdot \vec{M}}{2\pi} \ln \frac{r_1}{r_2} \end{cases}$$

Figure 1 shows the schematic of an in-plane recorded magnetic medium with a uniform magnetization along the  $+x$  axis comprising  $N = 2$  bits and assuming a 2D approximation. The magnetic medium is a hard-ferromagnetic material that in the absence of an external magnetic field exhibits a uniform magnetization  $M_r = 580$  kA/m along a thickness of  $t = 100$  nm and bit length of  $B$ . Consequently, the signal is measured at the position  $(x, z = t + h = \text{constant})$ , where  $h$  is equivalent to half of the bit length ( $B/2$ ).

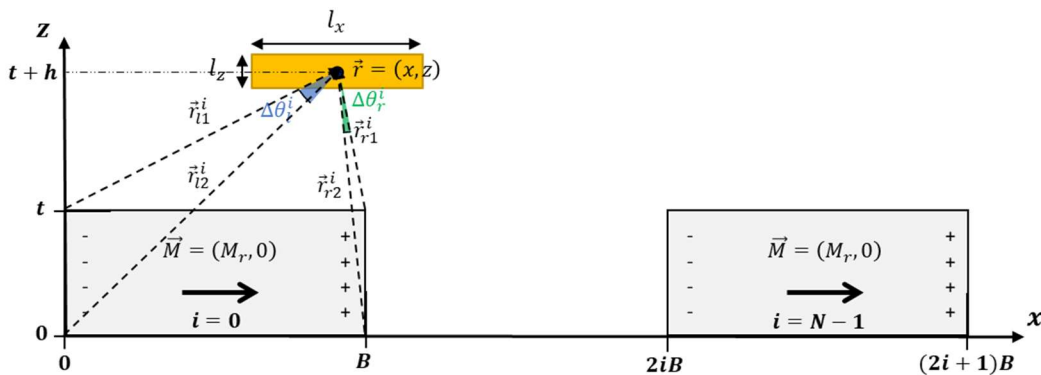


Figure 1: Schematic of an in-plane recorded magnetic medium with a uniform magnetization along the  $+x$  axis comprising  $N = 2$  bits and assuming a 2D approximation.

The magnetic signal is detected by a magnetoresistive sensor at  $z = t + h$  while it is moving along the  $x$  axis. The sensor has a length of  $l_x = 20 \mu\text{m}$  and  $l_z = 2 \mu\text{m}$  along the  $x$  and the  $z$  axis, respectively. Furthermore, it is characterized by a magnetoresistance of  $MR = 200 \%$ , a minimum resistance of  $R_{\min} = 1 \text{ k}\Omega$ , and a linear range between  $\pm H_{\text{keff}} = 2 \text{ kA/m}$ . Assuming a bias current of  $I = 0.1 \text{ mA}$  and the average magnetic field over the sensor area  $\langle H \rangle$ , the sensor output  $\Delta V$  is given by:

$$\Delta V(H) = MR \cdot \frac{R_{\min} I}{2H_{\text{keff}}} \langle H \rangle$$

## Questions

**NOTE: Explain and comment all answers, and present the proofs of all computer calculations**

1. Based in the geometric parameters described in Figure 1 write the expressions for the longitudinal and the normal magnetic field components for a single bit and for  $N$  bits magnetized along the  $+x$  axis assuming a linear superposition of bits.
2. Plot the longitudinal magnetic field component ( $H_l = H_z$ ) as a function of the position  $x$  generated by a single bit and 10 bits for a bit length of  $10 \mu\text{m}$ ,  $100 \mu\text{m}$  and  $1\text{mm}$ .
3. Plot the sensor output voltage produced by the longitudinal magnetic field component as a function of the  $x$  position taking into account the sensor area for a bit length of  $10 \mu\text{m}$ ,  $100 \mu\text{m}$  and  $1\text{mm}$  ( $l_x = 20 \mu\text{m}$  and  $l_z = 2 \mu\text{m}$ ).
4. Plot the maximum sensor output produced by the longitudinal magnetic field component as a function of  $l_x$  ( $l_z = 2 \mu\text{m}$ ) for a bit length of  $100 \mu\text{m}$ . Which is the maximum  $l_x$  to neglect the sensor dimension? Please consider the maximum sensor output around the middle of the pattern under study with enough bits ( $N \gg 1$ ) to neglect the border effects.
5. Indicate (no need to calculate) how the equations would change to include:
  - a. An inhomogeneous magnetization profile along the media thickness
  - b. An irregular bit transition, with width  $\delta$  ( $\delta \sim 10\%$  of the bit size)
6. [Optional] Check examples of product technical details from manufacturers of similar magnetic readout systems for linear encoders:
  - a. [Sensitec](#) (Germany)
  - b. [Renishaw](#) (USA)