DF DEPARTAMENTO DE FÍSICA TÉCNICO LISBOA

LIP

MATEMÁTICA para E.M.

Formulário EM/MEFT 2018/19
v0.0 (1/2/2019), Pedro Abreu

$$\int \frac{dr}{\sqrt{a^2+r^2}} = \log\left(\sqrt{a^2+r^2}+r\right)+C \qquad \int \frac{dr}{(a^2+r^2)^{3/2}} = \frac{r}{a^2\sqrt{a^2+r^2}}+C$$

$$\int \frac{rdr}{(a^2+r^2)^{3/2}} = -\frac{1}{\sqrt{a^2+r^2}}+C \qquad \int \frac{dr}{r} = \log r + C \qquad \int \frac{dr}{r^2} = -\frac{1}{r}+C \qquad \iiint_{-\infty}^{+\infty} \vec C(\vec r')\delta^3(\vec r-\vec r')\,dv' = 4\pi\vec C(\vec r)$$

Coordenadas cilíndricas

$$\begin{cases} x = R\cos\varphi \\ y = R\sin\varphi \\ z = z \end{cases} \qquad \begin{cases} \vec e_x = \cos\varphi\,\vec e_R - \sin\varphi\vec e_\varphi \\ \vec e_y = \sin\varphi\,\vec e_R + \cos\varphi\vec e_\varphi \\ \vec e_Z = \vec e_z \end{cases} \qquad \begin{cases} R = \sqrt{x^2+y^2} \\ \varphi = \arctan\frac{y}{x} \\ z = z \end{cases} \qquad \begin{cases} \vec e_R = \cos\varphi\,\vec e_x + \sin\varphi\vec e_y \\ \vec e_\varphi = -\sin\varphi\,\vec e_x + \cos\varphi\vec e_y \\ \vec e_Z = \vec e_z \end{cases}$$

$$d\vec l = dR\vec e_R + Rd\varphi\vec e_\varphi + dz\vec e_z \qquad dS = RdRd\varphi, dRdz, Rdzd\varphi \qquad dv = RdRd\varphi dz$$

$$\vec r = R\vec e_R + z\vec e_z$$

$$\vec C = C_R\vec e_R + C_\varphi\vec e_\varphi + C_z\vec e_z$$

$$\vec\nabla T = \frac{\partial T}{\partial R}\vec e_R + \frac{1}{R}\frac{\partial T}{\partial\varphi}\vec e_\varphi + \frac{\partial T}{\partial z}\vec e_z \qquad \vec\nabla\cdot\vec C = \frac{1}{R}\frac{\partial(RC_R)}{\partial R} + \frac{1}{R}\frac{\partial C_\varphi}{\partial\varphi} + \frac{\partial C_z}{\partial z}$$

$$\vec\nabla\times\vec C = \left(\frac{1}{R}\frac{\partial(C_z)}{\partial\varphi} - \frac{\partial(C_\varphi)}{\partial z}\right)\vec e_R + \left(\frac{\partial(C_R)}{\partial z} - \frac{\partial(C_z)}{\partial R}\right)\vec e_\varphi + \frac{1}{R}\left(\frac{\partial(RC_\varphi)}{\partial R} - \frac{\partial(C_R)}{\partial\varphi}\right)\vec e_z$$

$$\nabla^2\vec C = \vec e_R\left(\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial C_R}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2 C_R}{\partial\varphi^2} + \frac{\partial^2 C_R}{\partial z^2}\right) + \vec e_\varphi\,\mathrm{lap}\,C_\varphi + \vec e_z\,\mathrm{lap}\,C_z$$

Coordenadas esféricas

$$\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases} \qquad \left\{ r = \sqrt{x^2+y^2+z^2}, \quad \theta = \arccos\frac{z}{r}, \quad \varphi = \arctan\frac{y}{x} \right.$$

$$\begin{cases} \vec e_x = \sin\theta\cos\varphi\,\vec e_r + \cos\theta\cos\varphi\,\vec e_\theta - \sin\varphi\,\vec e_\varphi \\ \vec e_y = \sin\theta\sin\varphi\,\vec e_r + \cos\theta\sin\varphi\,\vec e_\theta + \cos\varphi\,\vec e_\varphi \\ \vec e_Z = \cos\theta\,\vec e_r - \sin\theta\,\vec e_\theta \end{cases} \qquad \begin{cases} \vec e_r = \sin\theta\cos\varphi\,\vec e_x + \sin\theta\sin\varphi\,\vec e_y + \cos\theta\,\vec e_z \\ \vec e_\theta = \cos\theta\cos\varphi\,\vec e_x + \cos\theta\sin\varphi\,\vec e_y - \sin\theta\,\vec e_z \\ \vec e_\varphi = -\sin\varphi\,\vec e_x + \cos\varphi\vec e_y \end{cases}$$

$$d\vec l = dr\vec e_R + rd\theta\vec e_\theta + r\sin\theta\,d\varphi\vec e_\varphi \qquad dS = rdrd\theta, r\sin\theta drd\varphi, r^2\sin\theta d\theta d\varphi \qquad dv = r^2\sin\theta drd\theta d\varphi$$

$$\vec r = r\vec e_r \qquad \vec C = C_r\vec e_r + C_\theta\vec e_\theta + C_\varphi\vec e_\varphi \qquad \vec\nabla T = \frac{\partial T}{\partial r}\vec e_r + \frac{1}{r}\frac{\partial T}{\partial\theta}\vec e_\theta + \frac{1}{r\sin\theta}\frac{\partial T}{\partial\varphi}\vec e_\varphi$$

$$\vec\nabla r = \vec e_r \qquad \vec\nabla\cdot\frac{\vec e_r}{r^2} = 4\pi\delta^3(\vec r) = \nabla^2\frac{1}{r} \qquad \vec\nabla\cdot\vec C = \frac{1}{r^2}\frac{\partial(r^2C_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial\sin\theta\,C_\theta}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial C_\varphi}{\partial\varphi}$$

$$\vec\nabla\times\vec C = \left(\frac{1}{r\sin\theta}\frac{\partial\left(\sin\theta\,C_\varphi\right)}{\partial\theta} - \frac{\partial(\sin\theta\,C_\theta)}{\partial\varphi}\right)\vec e_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial(C_r)}{\partial\varphi} - \frac{\partial\left(rC_\varphi\right)}{\partial r}\right)\vec e_\theta + \frac{1}{r}\left(\frac{\partial(rC_\theta)}{\partial r} - \frac{\partial(C_R)}{\partial\theta}\right)\vec e_\varphi$$

$$\nabla^2\vec C = \vec e_r\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C_r}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial C_r}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 C_r}{\partial\varphi^2}\right) + \vec e_\theta\,\mathrm{lap}\,C_\theta + \vec e_\varphi\,\mathrm{lap}\,C_\varphi$$

$$\vec A\cdot(\vec B\times\vec C) = \vec B\cdot(\vec C\times\vec A) = \vec C\cdot(\vec A\times\vec B) \qquad \vec\nabla\cdot(T\vec C) = T\vec\nabla\cdot\vec C + \vec C\cdot\vec\nabla T \qquad \vec A\times(\vec B\times\vec C) = \vec B(\vec A\cdot\vec C) - \vec C(\vec A\cdot\vec B)$$

$$\vec\nabla(TU) = T\vec\nabla U + U\vec\nabla T \qquad \vec\nabla\times(T\vec C) = T\vec\nabla\times\vec C - \vec C\times\vec\nabla T \qquad \vec\nabla(\vec A\cdot\vec C) = \vec A\times(\vec\nabla\times\vec C) + \vec C\times(\vec\nabla\times\vec A) + (\vec A\cdot\vec\nabla)\vec C + (\vec C\cdot\vec\nabla)\vec A$$

$$\vec\nabla\cdot(\vec A\times\vec C) = \vec C\cdot(\vec\nabla\times\vec A) - \vec A\cdot(\vec\nabla\times\vec C) \qquad \vec\nabla\cdot(\vec\nabla\times\vec C) = 0 \qquad \vec\nabla\times(\vec A\times\vec C) = (\vec C\cdot\vec\nabla)\vec A - (\vec A\cdot\vec\nabla)\vec C + (\vec\nabla\cdot\vec C)\vec A - (\vec\nabla\cdot\vec A)\vec C$$

$$\vec\nabla\times(\vec\nabla T) = 0 \qquad \vec\nabla\times(\vec\nabla\times\vec C) = \vec\nabla(\vec\nabla\cdot\vec C) - \nabla^2\vec C = \mathrm{grad}(\mathrm{div}\,\vec C) - \mathrm{lap}\,\vec C$$

$$\int_a^b \vec\nabla T\cdot d\vec l = T(b) - T(a) \qquad \iint \vec\nabla\times\vec C\cdot\vec n dS = \oint \vec C\cdot d\vec l \qquad (\vec A\cdot\vec\nabla)\vec C = A_x\frac{\partial\vec C}{\partial x} + A_y\frac{\partial\vec C}{\partial y} + A_z\frac{\partial\vec C}{\partial z}$$

$$\oint \vec\nabla T\cdot d\vec l = 0 \qquad \oiint \vec\nabla\times\vec C\cdot\vec n dS = 0 \qquad \iiint \vec\nabla\cdot\vec C dv = \oiint \vec C\cdot\vec n dS$$

$\Phi = \iint \vec{B}\cdot\vec{n}dS$ $\qquad \varepsilon_{\text{fem}} = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$ $\qquad \boxed{W_L = \frac{1}{2}LI^2}$

Formulário EM/MEFT 2018/19
v0.0 (19/6/2019), Pedro Abreu

$\varepsilon_{\text{fem}}(N \text{ espiras}) = -Nd\Phi_1(1 \text{ espira})/dt$

Circuitos RLC e corrente alterna

$\omega_0^2 = 1/(LC) \qquad \lambda = R/(2L) \qquad D = \varepsilon_f/L$

$\ddot{x} + 2\lambda\dot{x} + \omega_0^2 x = D$ tem como solução:

se $\lambda^2 > \omega_0^2$: $\frac{D}{\omega_0^2} + e^{-\lambda t}\left(A_1 e^{-\sqrt{\lambda^2-\omega_0^2}t} + A_2 e^{\sqrt{\lambda^2-\omega_0^2}t}\right)$

se $\lambda^2 = \omega_0^2$: $\frac{D}{\omega_0^2} + e^{-\lambda t}(A + Bt)$

se $\lambda^2 < \omega_0^2$: $\frac{D}{\omega_0^2} + Ae^{-\lambda t}\cos(\omega_P . t + \varphi)$ $\qquad \omega_P = \sqrt{\omega_0^2 - \lambda^2}$

$\ddot{x} + 2\lambda\dot{x} + \omega_0^2 x = \frac{\varepsilon_f}{L}\cos(\omega t) \Rightarrow \quad x(t) = A\cos(\omega t + \varphi)$

$\varphi = \tan^{-1}\frac{-2\lambda\omega}{\omega_0^2 - \omega^2}$ $\qquad \boxed{T = 2\pi/\omega}$

$A = \frac{\varepsilon_f/L}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\lambda^2\omega^2}}$ $\qquad \omega_R = \sqrt{\omega_0^2 - 2\lambda^2}$

$A_{MAX} = A(\omega_R) = \frac{\varepsilon_f}{2\lambda L\sqrt{\omega_0^2 - \lambda^2}} = \frac{\varepsilon_f}{2\lambda L\omega_P}$

impedâncias

$Z_R = R \qquad Z_C = 1/(i\omega C) \qquad Z_L = i\omega L$

Equações de Maxwell

$\vec{\nabla}\cdot\vec{D} = \rho \qquad (\vec{D}_2 - \vec{D}_1)\cdot\vec{n}_\perp = \sigma \qquad \vec{D} = \varepsilon_0\vec{E} + \vec{P} \qquad \vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

$\vec{\nabla}\cdot\vec{B} = 0 \qquad (\vec{B}_2 - \vec{B}_1)\cdot\vec{n}_\perp = 0 \qquad \vec{B} = \mu_0(\vec{H} + \vec{M})$

$\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$

$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad (\vec{E}_2 - \vec{E}_1)\cdot\vec{e}_\parallel = 0 \qquad \vec{J} = \sigma\vec{E}$

Meios LHI: $\vec{P} = \varepsilon_0\chi_e\vec{E}$, $\vec{M} = \chi_m\vec{H}$

$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} \qquad (\vec{H}_2 - \vec{H}_1)_\parallel = \vec{K}\times\vec{n} \qquad \vec{D} = \varepsilon\vec{E} \quad \vec{B} = \mu\vec{H}$

$\varepsilon = \varepsilon_0(1 + \chi_e)$

$\mu = \mu_0(1 + \chi_m)$

Densidades de Energia nos campos eletromagnéticos:

$u_E = \frac{1}{2}\vec{E}\cdot\vec{D} = \frac{\varepsilon E^2}{2}$

$u_M = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{B^2}{2\mu}$

Vetor de Poynting

$\vec{\Sigma} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \qquad \vec{\Sigma} = vu_{EM}\vec{e}_k \quad (u_{EM} = u_E + u_M) \qquad \omega = 2\pi f = 2\pi/T \qquad k = 2\pi/\lambda$

Ondas Eletromagnéticas

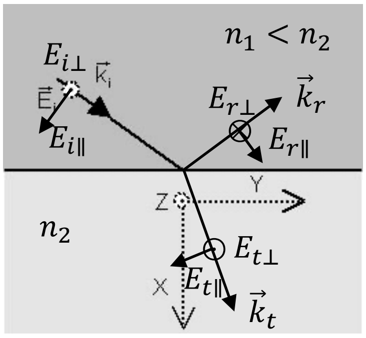
(campos físicos: $\text{Re}[\vec{E},\vec{B}]$) $\quad \vec{E} = \vec{E}_0 e^{i(\omega t - \vec{k}\cdot\vec{r} + \varphi)} \qquad \vec{B} = \vec{B}_0 e^{i(\omega t - \vec{k}\cdot\vec{r} + \varphi)} \qquad \vec{B} = \frac{1}{\omega}\vec{k}\times\vec{E} \qquad \vec{E} = -\frac{\omega}{k^2}\vec{k}\times\vec{B}$

$v = \frac{\omega}{k} = \lambda f = \frac{c}{n} = \frac{1}{\sqrt{\varepsilon\mu}} \qquad |\vec{E}_0| = v|\vec{B}_0| \qquad \mu = \mu_0 \Leftrightarrow \mu_r = 1 \Leftrightarrow n = \frac{c}{v} = \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{\varepsilon_r}$

$|\vec{\Sigma}| = \frac{1}{v\mu_0}E_0^2 = v\varepsilon E_0^2 = nc\varepsilon_0 E_0^2 \qquad I = \langle|\vec{\Sigma}|\rangle = \alpha\cdot nc\varepsilon_0 E_0^2$ ($\alpha = 0.5$ se pol.linear, $\alpha = 1.0$ se pol.circular, $\alpha =$ outros valores se pol.elíptica)

Superfície de separação

$i = \arctan\frac{k_{iy}}{k_{ix}} \qquad r = i \qquad n_1 \operatorname{sen} i = n_2 \operatorname{sen} t \qquad i_{RT} = \arcsin\frac{n_2}{n_1} \qquad i_B = \arctan\frac{n_2}{n_1}$



Refletância

$R = \frac{I_r\cos r}{I_i\cos i}$

Transmitância

$T = \frac{I_t\cos t}{I_i\cos i}$

$R = r^2 \qquad T = t^2\frac{\tan i}{\tan t} \quad (i \neq 0)$

$T = t^2 n_2/n_1 \quad (i = 0)$

Cons.Energia: $R + T = 1$

$(I_i\cos i = I_r\cos r + I_t\cos t\ )$

$R(T) = R(T)_\parallel\cos^2\gamma_i + R(T)_\perp\operatorname{sen}^2\gamma_i$

EQUAÇÕES DE FRESNEL

$i \neq 0°$:

$r_\perp = \frac{E_{0r\perp}}{E_{0i\perp}} = -\frac{\operatorname{sen}(i-t)}{\operatorname{sen}(i+t)}$

$t_\perp = \frac{E_{0t\perp}}{E_{0i\perp}} = \frac{2\operatorname{sen} t\cos i}{\operatorname{sen}(i+t)}$

$r_\parallel = \frac{E_{0r\parallel}}{E_{0i\parallel}} = \frac{\tan(i-t)}{\tan(i+t)}$

$t_\parallel = \frac{E_{0t\parallel}}{E_{0i\parallel}} = \frac{2\operatorname{sen} t\cos i}{\operatorname{sen}(i+t)\cos(i-t)}$

$\tan\gamma_i = \frac{E_{0i\perp}}{E_{0i\parallel}}$

$i = 0°$:

$r_\perp = r_\parallel = \frac{E_{0r}}{E_{0i}} = \frac{n_1 - n_2}{n_1 + n_2}$

$t_\perp = t_\parallel = \frac{E_{0t}}{E_{0i}} = \frac{2n_1}{n_1 + n_2}$

Meios dispersivos

Veloc.fase = $v = \frac{\omega}{k} = \frac{c}{n}$ $\qquad$ Veloc.grupo = $u = \frac{d\omega}{dk}$

Relações de dispersão: $u = \frac{c}{n + \omega\frac{dn}{d\omega}} = v + k\frac{dv}{dk} = v - \lambda\frac{dv}{d\lambda}$