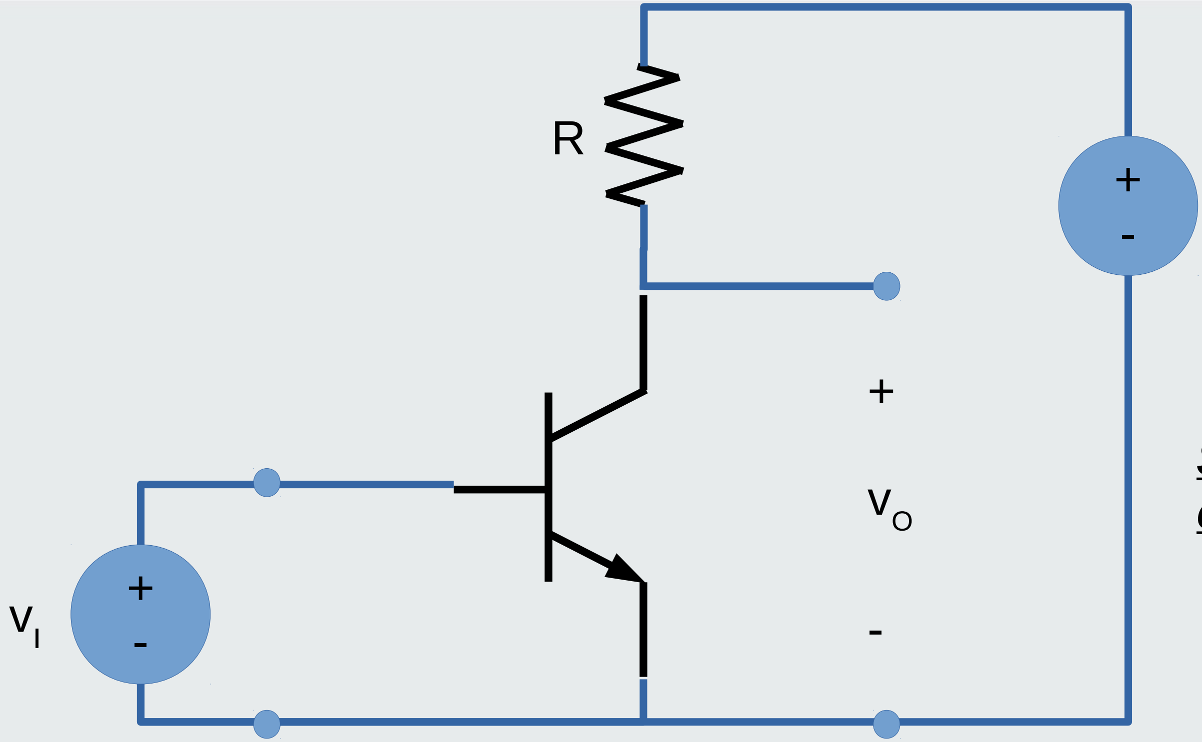
Circuit Theory and Electronics Fundamentals

Lecture 16: BJT Amplifiers

- Common emitter amplifier
  - Gain, input and output impedances
- Common collector amplifier
  - Gain, input and output impedances

The common emitter amplifier

Goal: amplify voltage



Supply voltage (active circuit)

Superposition of DC and AC components

$$v_I = V_I + v_i$$
$$v_O = V_O + v_o$$

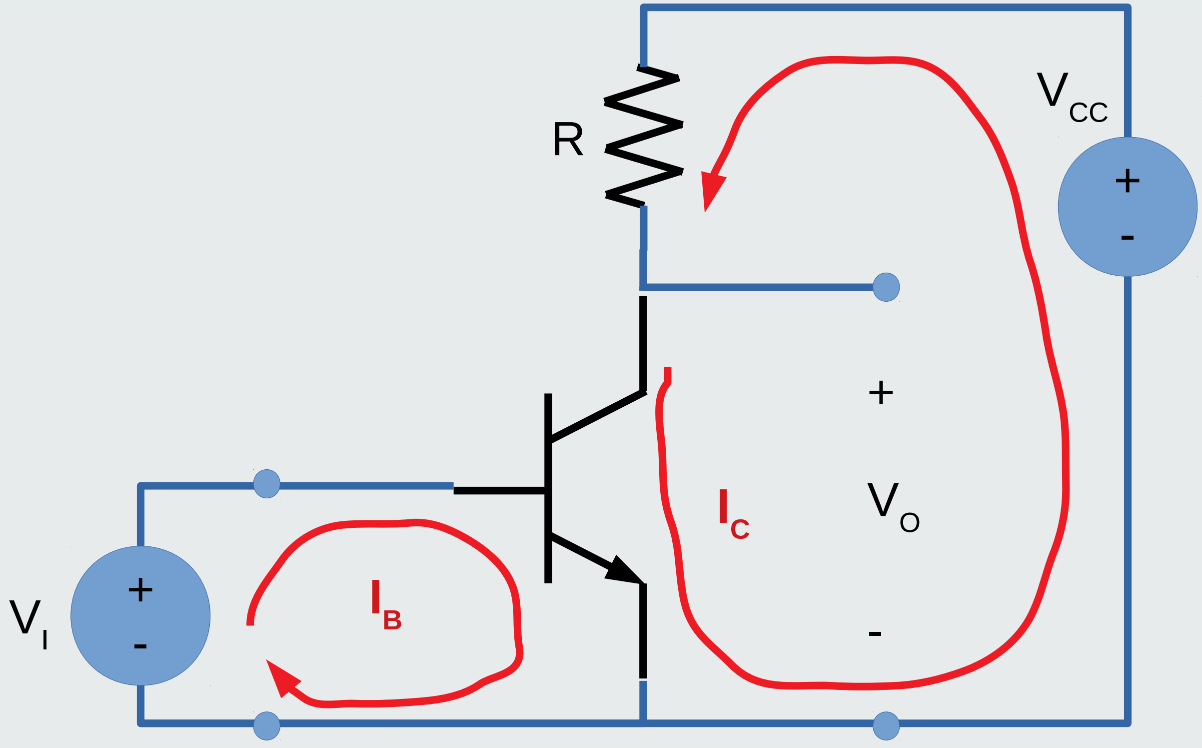
Common (to input and output) Emitter

Separate DC (operating point) and AC (incremental) analyses

Transistor must operate in Forward Active Region:

$$v_{BE} > 0 \wedge v_{BC} < 0 \Rightarrow V_{CE} > V_{BE}$$

The common emitter amplifier Operating Point (OP) analysis



Supply voltage (active circuit)

Common (to input and output) Emitter

Mesh analysis

$$\begin{cases} -V_I + V_{BE} = 0 \,(mesh\, B) \\ R I_C + V_O - V_{CC} = 0 \,(mesh\, C) \end{cases}$$

$$V_O = V_{CC} - R\, I_C$$

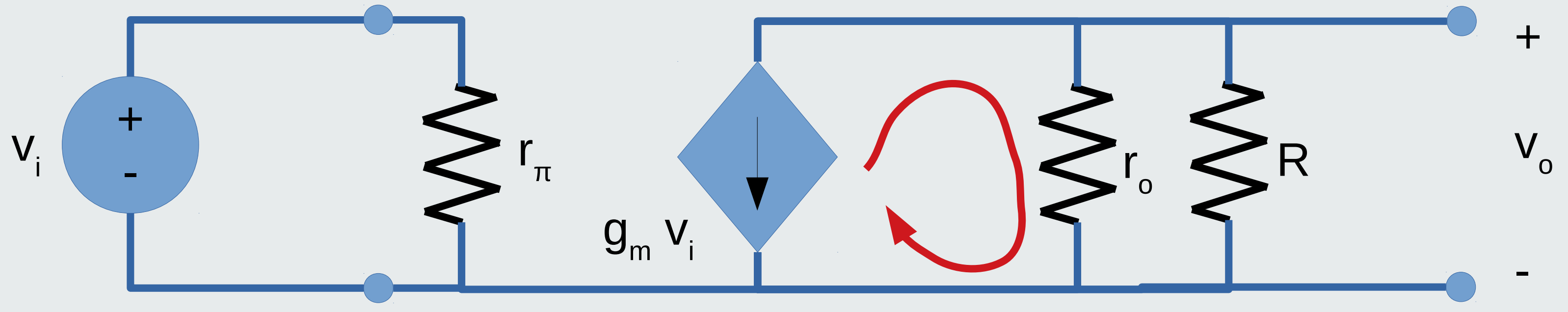
$$I_C = I_S e^{\frac{V_I}{V_T}}$$

$$V_O = V_{CC} - R\, I_S e^{\frac{V_I}{V_T}}$$

Forward Active Region condition: $v_O > v_{BE}$

Non linear model!

The common emitter amplifier
Incremental analysis: gain



Compute incremental parameters after OP

$$g_m = \frac{I_C}{V_T}$$

$$r_\pi = \frac{\beta_F}{g_m}$$

$$r_o \approx \frac{V_A}{I_C}$$

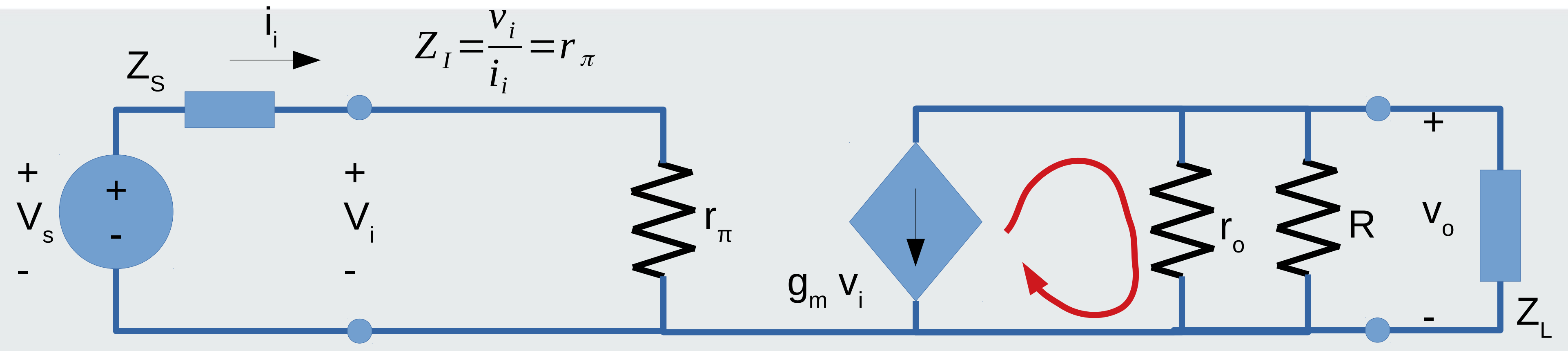
$$v_o = -g_m (r_o || R) v_i$$

$$A_V = \frac{v_o}{v_i} = -\frac{g_m}{\cancel{\frac{1}{r_o}} + \frac{1}{R}} \approx -g_m R = -\frac{R I_C}{V_T}$$

Gain is negative and High (R I_C >> v_T)

Gain is temperature Dependent!

The common emitter amplifier
Input impedance

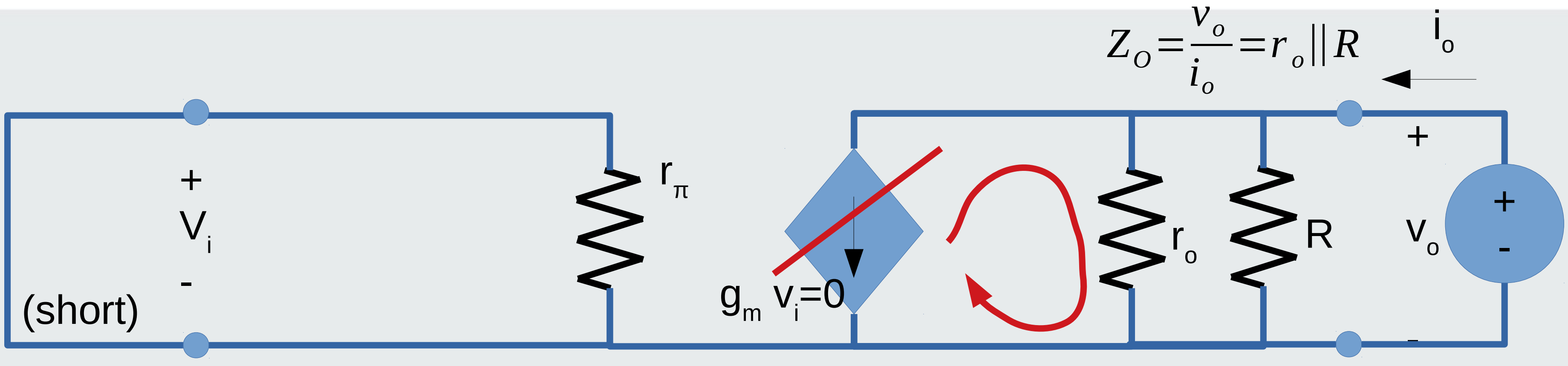


Apply Input Voltage

Measure Input Current

- If you are the source S you want to know Z_I; if Z_I does not combine with Z_S, the connection may fail and damage may occur
- In the above circuit Z_I is independent of the load , which is good!
- If it were load-dependent, the nature of the dependence must be stated; otherwise you don't know what you are connecting to.
- By convention, Z_I for when the load is absent is often given (short-circuit for current and open-circuit for voltage)

The common emitter amplifier: output impedance

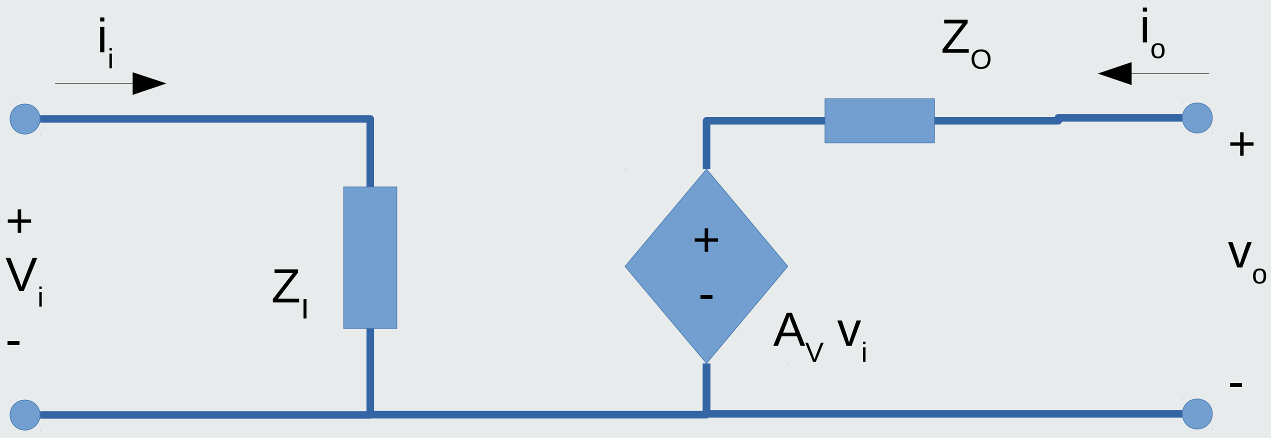


Apply Output Voltage

Measure Output Current

- If you are the load L, you want to know Z_o; if Z_O does not combine with $Z_{L,}$ the connection may fail and damage may occur!
- Z_O is depends on the source S.
- The nature of the dependence must be stated; otherwise you don’t know what you are connecting to.
- By convention, Z_O for when the source is off is given (short-circuit for voltage and open-circuit for current)

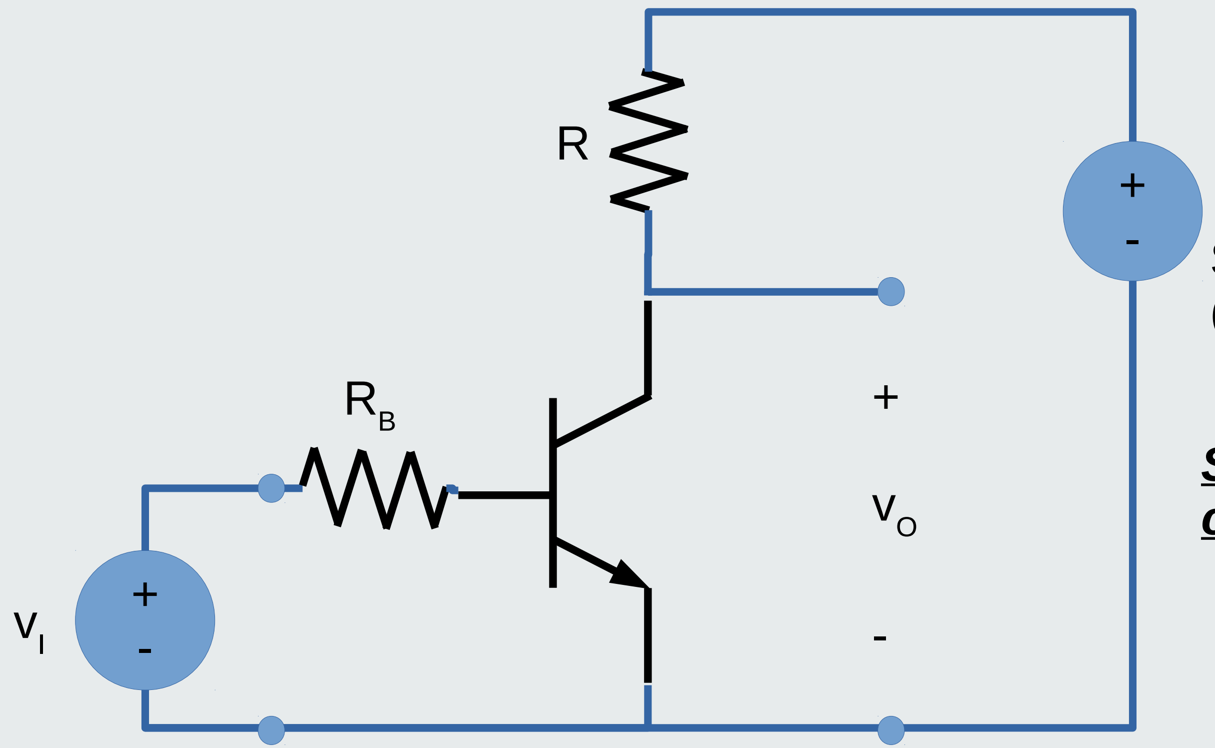
Feed-forward amplifier incremental model



- Amplifier fully characterized by 3 parameters
- Thévenin equivalent at output (A_V, Z_O)
- Equivalent impedance at input (Z_I)

The common emitter amplifier with a base resistor R_B

Goal: amplify with linear DC gain



Supply voltage (active circuit)

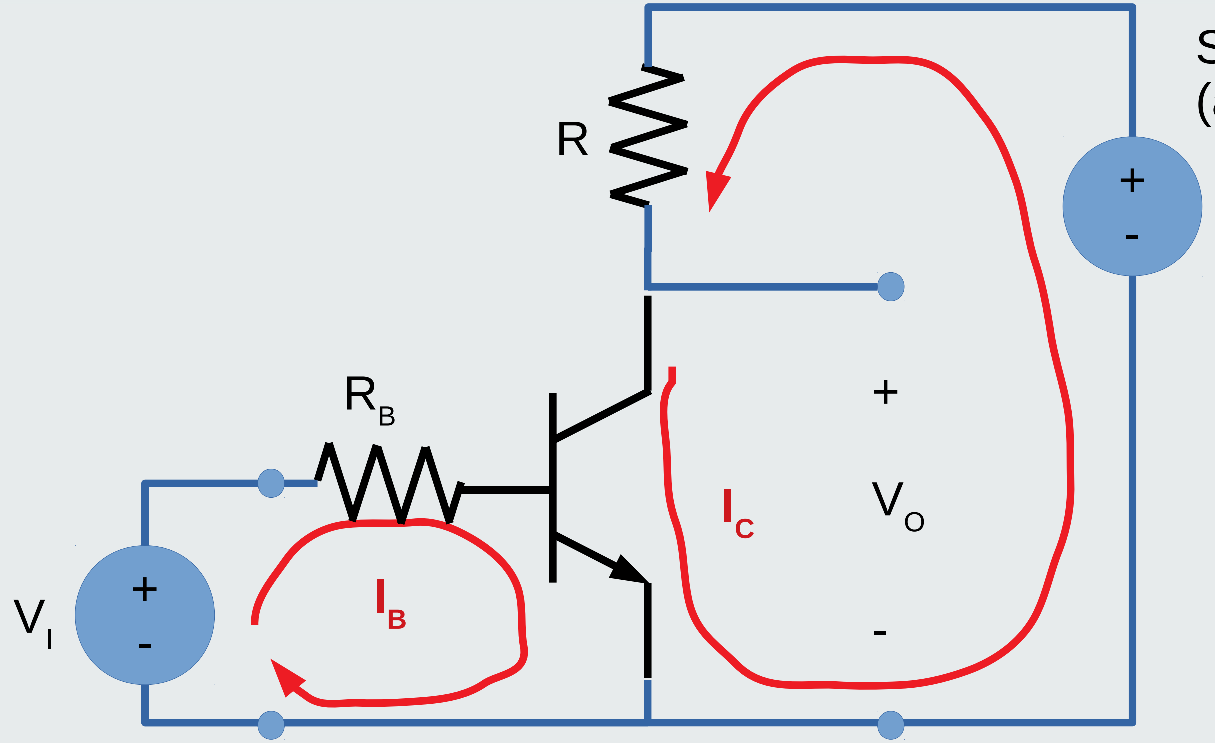
Common (to input and output) Emitter

Superposition of DC and AC components

$$v_I = V_I + v_i$$
$$v_O = V_O + v_o$$

Separate DC (operating point) and AC (incremental) analyses

The common emitter amplifier with R_B: OP analysis



Common (to input and output) Emitter

Supply voltage (active circuit)

V_{CC}

Mesh analysis

$$\begin{cases} -V_I + R_B I_B + V_{ON} = 0 \,(mesh\, B) \\ R I_C + V_O - V_{CC} = 0 \,(mesh\, C) \end{cases}$$

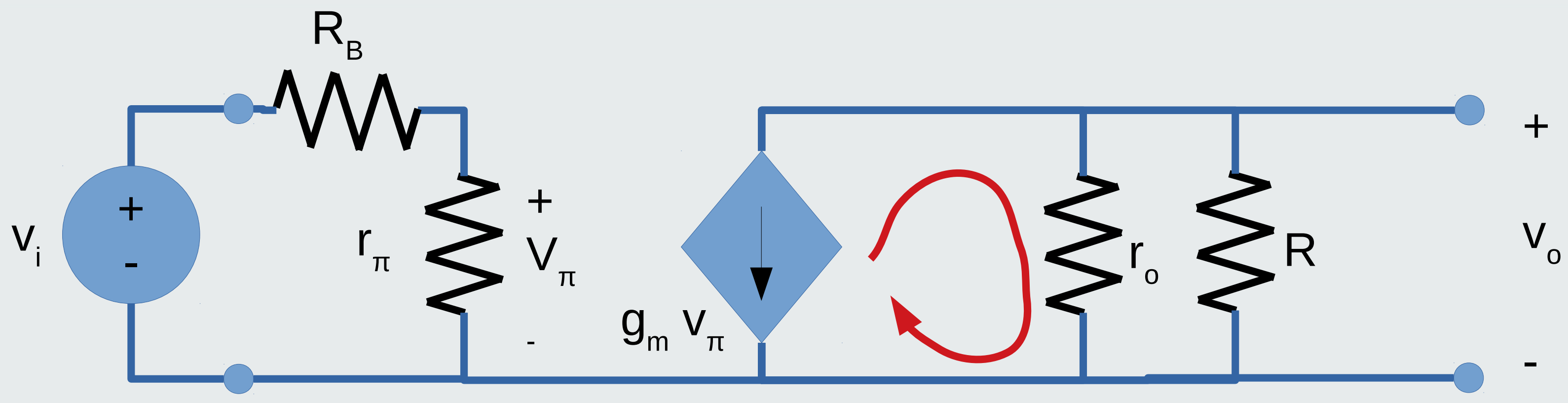
$$V_O = V_{CC} - R\, I_C$$

$$I_B = \frac{V_I - V_{ON}}{R_B} = \frac{I_C}{\beta_F}$$

$$V_O = V_{CC} - \beta_F \frac{R}{R_B}(V_I - V_{ON})$$

Can use simple V_{BEON} =0.7V linear model with good precision.

The common emitter amplifier with R_B: gain



$$A_V = \frac{v_o}{v_i} = -g_m (r_o || R) \frac{r_\pi}{r_\pi + R_B} \approx -g_m R \frac{r_\pi}{r_\pi + R_B}$$

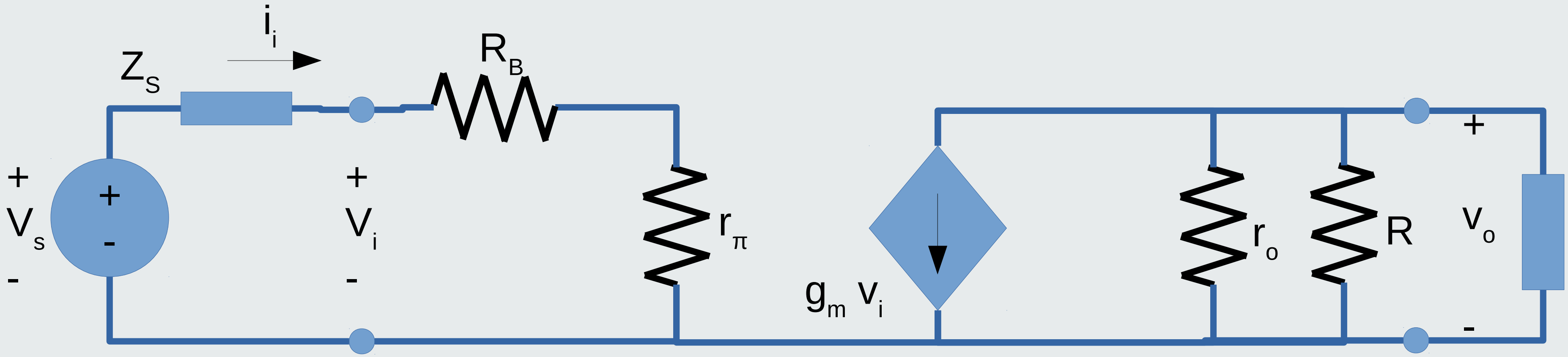
$$g_m \approx \frac{I_C}{V_T}$$
$$r_\pi = \frac{\beta_F}{g_m}$$
$$r_o \approx \frac{V_A}{I_C}$$

Gain is similar to amp without R_B, but lower due to input voltage divider

Gain is still temperature dependent

The common emitter amplifier with R_B: Input impedance

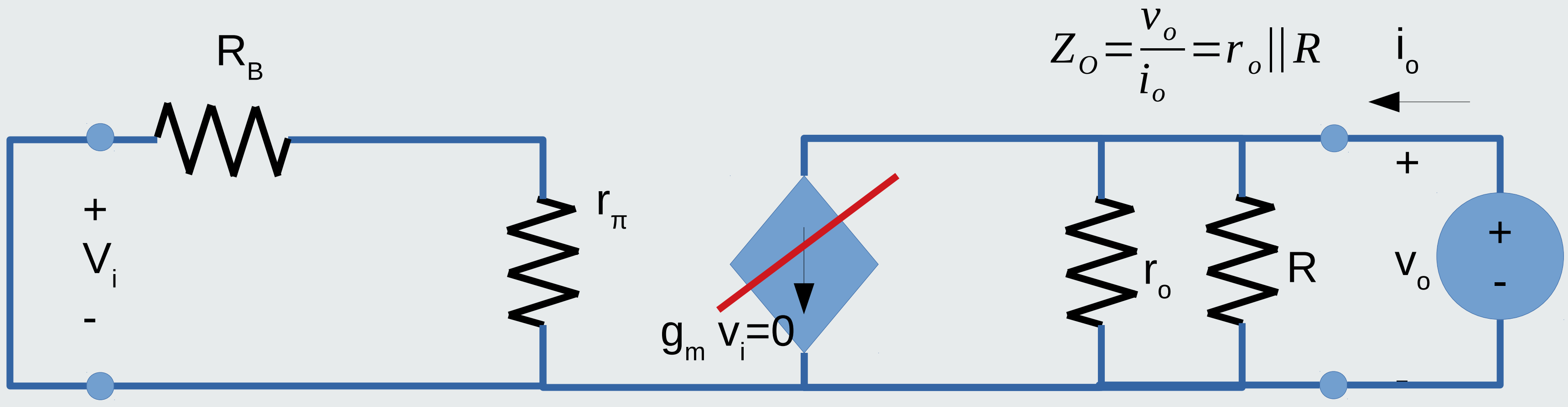
$$Z_I = \frac{v_i}{i_i} = R_B + r_\pi$$



Higher input impedance may be desirable if the source is a voltage

Prevents voltage degradation in Z_S

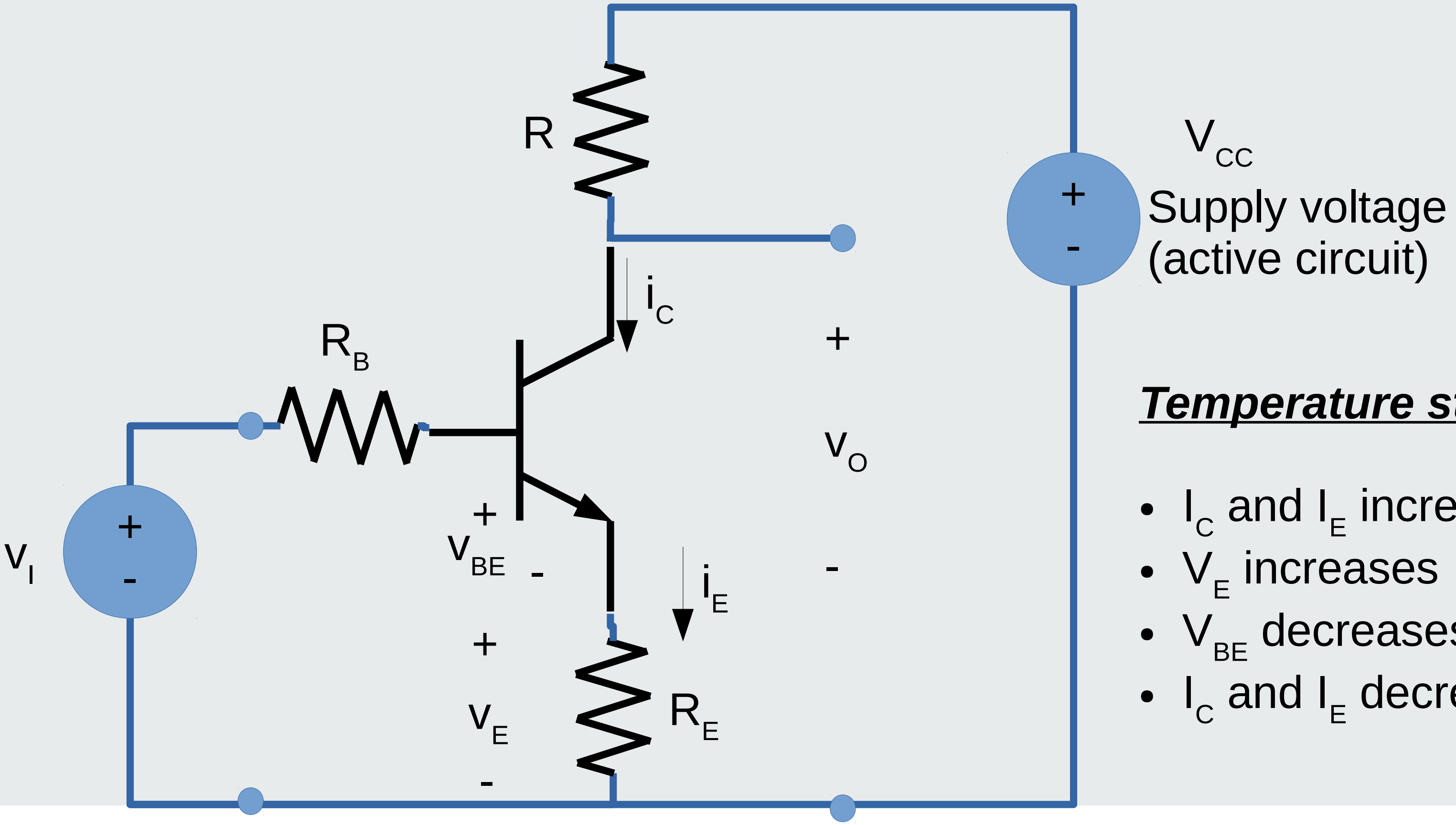
The common emitter amplifier with R_B: output impedance



- Z_o is the same with or without R_B if source is switched off (short-circuit)
- If the source is on, R_B lower the effect on Z_o

The common emitter amplifier with emitter degeneration

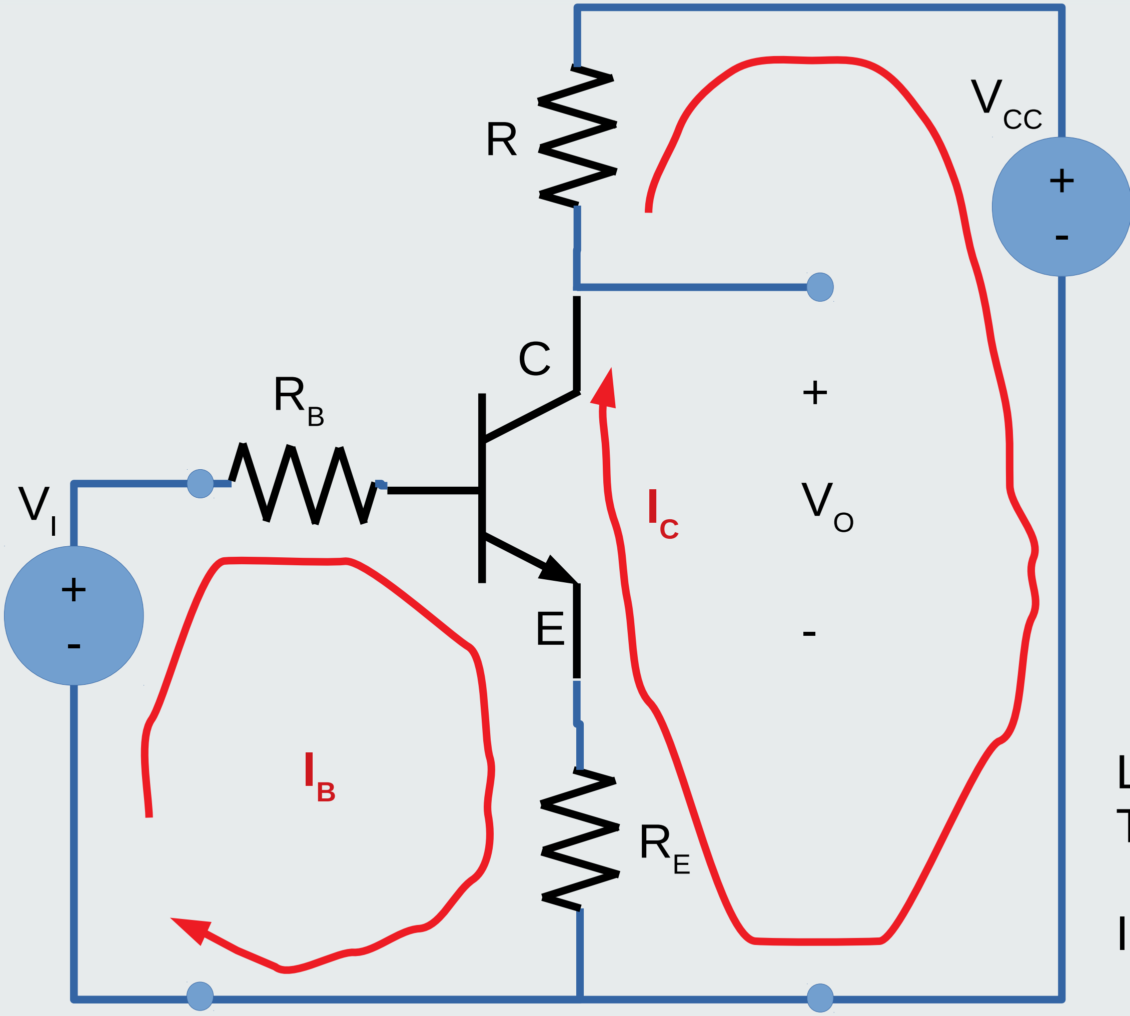
Goal: linearise DC gain, improve temperature dependency



Temperature stabilisation

- I_C and I_E increase with T
- V_E increases
- V_{BE} decreases
- I_C and I_E decrease

The common emitter amplifier with degeneration: OP



Mesh analysis

$$\begin{cases} -V_I + R_B I_B + V_{ON} + R_E I_E = 0 \ (mesh\, B) \\ -R I_C + V_{CC} - V_O = 0 \ (mesh\, C) \end{cases}$$

$$I_E = (1+\beta_F) I_B$$

$$I_B = \frac{V_I - V_{ON}}{R_B + (1+\beta_F) R_E} = \frac{I_C}{\beta_F}$$

$$V_O = V_{CC} - R I_C$$

$$V_{CE} = V_{CC} - R I_C - R_E I_E$$

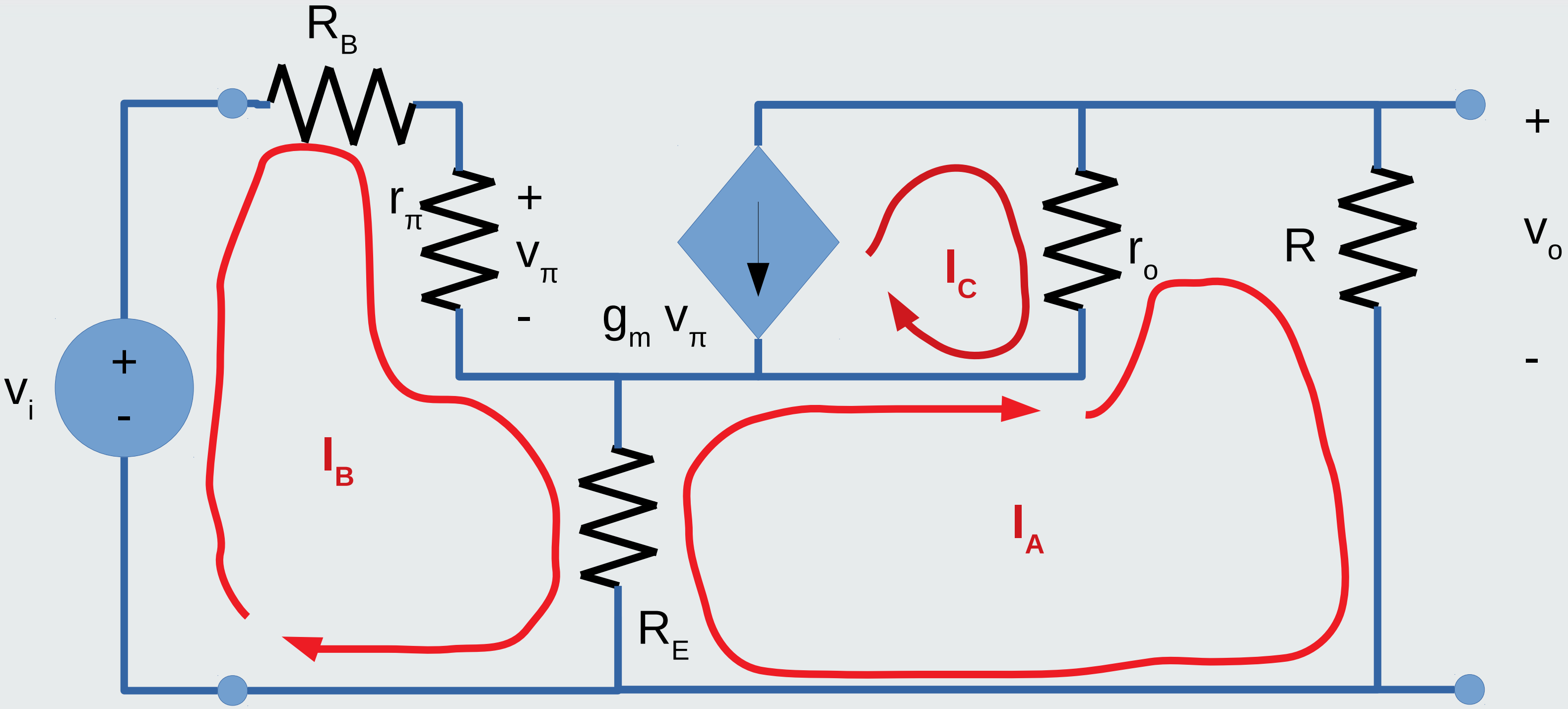
$$V_{BE} = v_I - R_B I_B - R_E I_E$$

$$V_{CE} > V_{BE}$$

Last condition must be verified for Transistor to be in F.A.R.

Imposes upper limit on v_I

The common emitter amplifier with degeneration: gain (1)



$$\begin{cases} (r_o+R+R_E)I_A-R_EI_B-r_oI_C=0 \\ -R_EI_A+(R_B+r_\pi+R_E)I_B=v_i \\ I_C=-g_mr_\pi I_B \end{cases}$$

$$\begin{bmatrix} r_o+R+R_E & -R_E & -r_o \\ -R_E & R_B+r_\pi+R_E & 0 \\ 0 & g_mr_\pi & 1 \end{bmatrix}\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}=\begin{bmatrix} 0 \\ v_i \\ 0 \end{bmatrix}$$

The common emitter amplifier with degeneration: gain (2)

$$\begin{bmatrix} r_o+R+R_E & -R_E & -r_o \\ -R_E & R_B+r_\pi+R_E & 0 \\ 0 & g_m r_\pi & 1 \end{bmatrix}\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}=\begin{bmatrix} 0 \\ v_i \\ 0 \end{bmatrix}$$

$$I_A=\frac{\begin{vmatrix} 0 & -R_E & -r_o \\ v_i & R_B+r_\pi+R_E & 0 \\ 0 & g_m r_\pi & 1 \end{vmatrix}}{\begin{vmatrix} r_o+R+R_E & -R_E & -r_o \\ -R_E & R_B+r_\pi+R_E & 0 \\ 0 & g_m r_\pi & 1 \end{vmatrix}}=\frac{R_E-g_m r_\pi r_o}{(r_o+R+R_E)(R_B+r_\pi+R_E)+g_m R_E r_o r_\pi-R_E^2} v_i$$

$$v_o=R I_A=R\frac{R_E-g_m r_\pi r_o}{(r_o+R+R_E)(R_B+r_\pi+R_E)+g_m R_E r_o r_\pi-R_E^2} v_i$$

$$r_o\rightarrow\infty, R_B=0\Rightarrow v_o=-\frac{g_m R}{1+g_m R_E} v_i$$

$$g_m R_E\gg 1\Rightarrow v_o=-\frac{R}{R_E} v_i$$

Degenerated common emitter amp. : input impedance

$$Z_I = \frac{v_i}{I_B}$$

$$\begin{bmatrix} r_o + R + R_E & -R_E & -r_o \\ -R_E & R_B + r_\pi + R_E & 0 \\ 0 & g_m r_\pi & 1 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 0 \\ v_i \\ 0 \end{bmatrix}$$

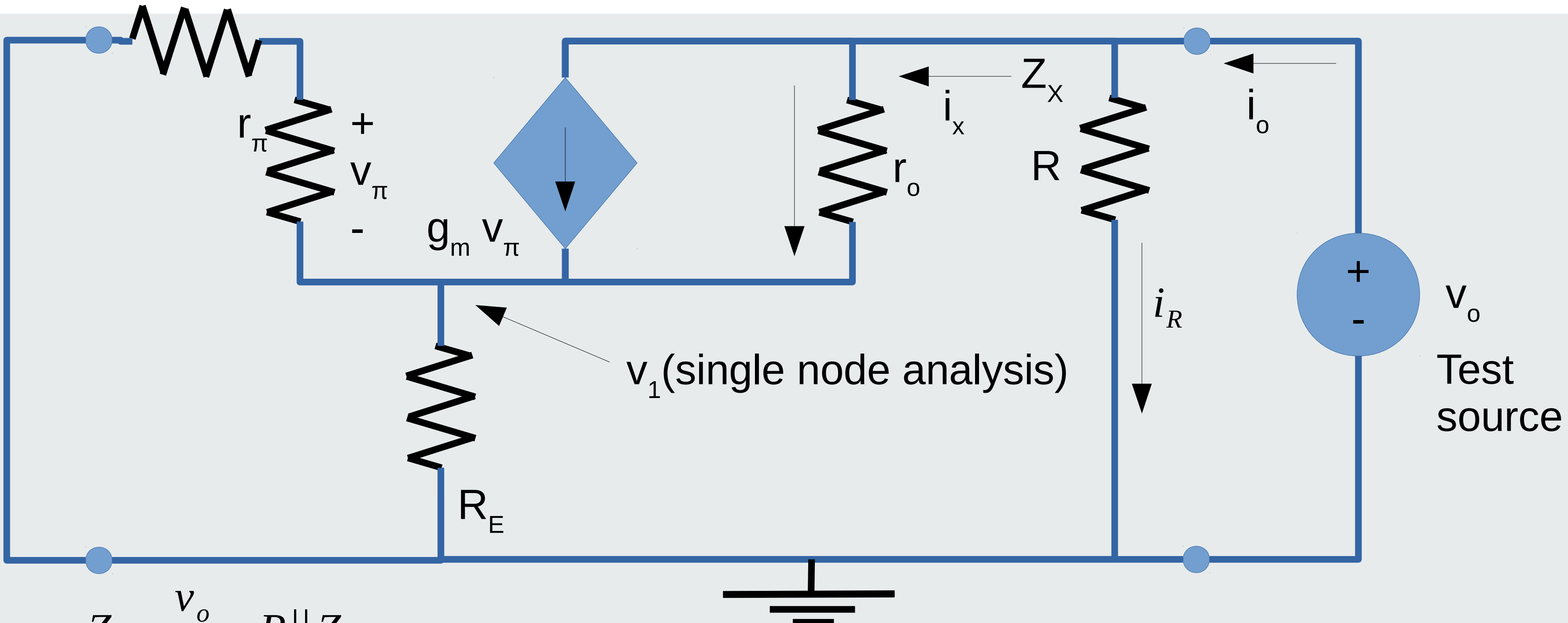
$$I_B = \frac{\begin{vmatrix} r_o + R + R_E & 0 & -r_o \\ -R_E & v_i & 0 \\ 0 & g_m & 1 \end{vmatrix}}{\begin{vmatrix} r_o + R + R_E & -R_E & -r_o \\ -R_E & R_B + r_\pi + R_E & 0 \\ 0 & g_m r_\pi & 1 \end{vmatrix}} = \frac{R + R_E + r_o}{(r_o + R + R_E)(R_B + r_\pi + R_E) + g_m R_E r_o r_\pi - R_E^2} v_i$$

$$Z_I = \frac{v_i}{I_B} = \frac{(r_o + R + R_E)(R_B + r_\pi + R_E) + g_m R_E r_o r_\pi - R_E^2}{r_o + R + R_E}$$

$$r_o \rightarrow \infty \Rightarrow Z_I = R_B + r_\pi + (1 + \beta_F) R_E$$

High!

Degenerated common emitter amplifier: output impedance



$$Z_O = \frac{v_o}{i_o} = R || Z_X$$

$$Z_X = \frac{v_o}{i_x}$$

$$i_x = \frac{v_o - v_1}{r_o} + g_m v_\pi$$

Single node equation to eliminate v_1:

$$\frac{v_1}{R_E} + \frac{v_1}{r_\pi + R_B} + \frac{v_1 - v_o}{r_o} - g_m v_\pi = 0$$

$$v_\pi = -\frac{r_\pi}{r_\pi + R_B} v_1$$

Degenerated common emitter amplifier: output impedance

$$\frac{v_1}{R_E}+\frac{v_1}{r_\pi+R_B}+\frac{v_1-v_o}{r_o}-g_m v_\pi=0$$

$$v_\pi=-\frac{r_\pi}{r_\pi+R_B}v_1$$

$$\frac{v_1}{R_E}+\frac{v_1}{r_\pi+R_B}+\frac{v_1}{r_o}+g_m\frac{r_\pi}{r_\pi+R_B}v_1=\frac{v_o}{r_o}$$

$$v_1=\frac{v_o}{r_o}\frac{1}{\frac{1}{R_E}+\frac{1}{r_\pi+R_B}+\frac{1}{r_o}+\frac{g_m r_\pi}{r_\pi+R_B}}$$

$$Z_X=\frac{v_o}{i_x}$$

$$i_x=\frac{v_o-v_1}{r_o}+g_m v_\pi=\frac{v_o}{r_o}-\frac{v_1}{r_o}-\frac{g_m r_\pi}{r_\pi+R_B}v_1$$

$$i_x=\frac{v_o}{r_o}-\left(\frac{1}{r_o}+\frac{g_m r_\pi}{r_\pi+R_B}\right)v_1$$

$$i_x=\frac{v_o}{r_o}-\left(\frac{1}{r_o}+\frac{g_m r_\pi}{r_\pi+R_B}\right)\frac{v_o}{r_o}\frac{1}{\frac{1}{R_E}+\frac{1}{r_\pi+R_B}+\frac{1}{r_o}+\frac{g_m r_\pi}{r_\pi+R_B}}$$

$$i_x=\frac{v_o}{r_o}\frac{\frac{1}{R_E}+\frac{1}{r_\pi+R_B}}{\frac{1}{R_E}+\frac{1}{r_\pi+R_B}+\frac{1}{r_o}+\frac{g_m r_\pi}{r_\pi+R_B}}$$

Degenerated common emitter amplifier: output impedance

$$i_x = \frac{v_o}{r_o}\frac{\frac{1}{R_E}+\frac{1}{r_\pi + R_B}}{\frac{1}{R_E}+\frac{1}{r_\pi + R_B}+\frac{1}{r_o}+\frac{g_m r_\pi}{r_\pi + R_B}}$$

$$Z_x = \frac{v_o}{i_x} = r_o \frac{\frac{1}{R_E}+\frac{1}{r_\pi + R_B}+\frac{1}{r_o}+\frac{g_m r_\pi}{r_\pi + R_B}}{\frac{1}{R_E}+\frac{1}{r_\pi + R_B}}$$

$$R_E \rightarrow 0 \Rightarrow Z_x = r_o$$

$$r_o \rightarrow \infty \Rightarrow Z_x = \infty$$

$$Z_o = Z_x || R$$

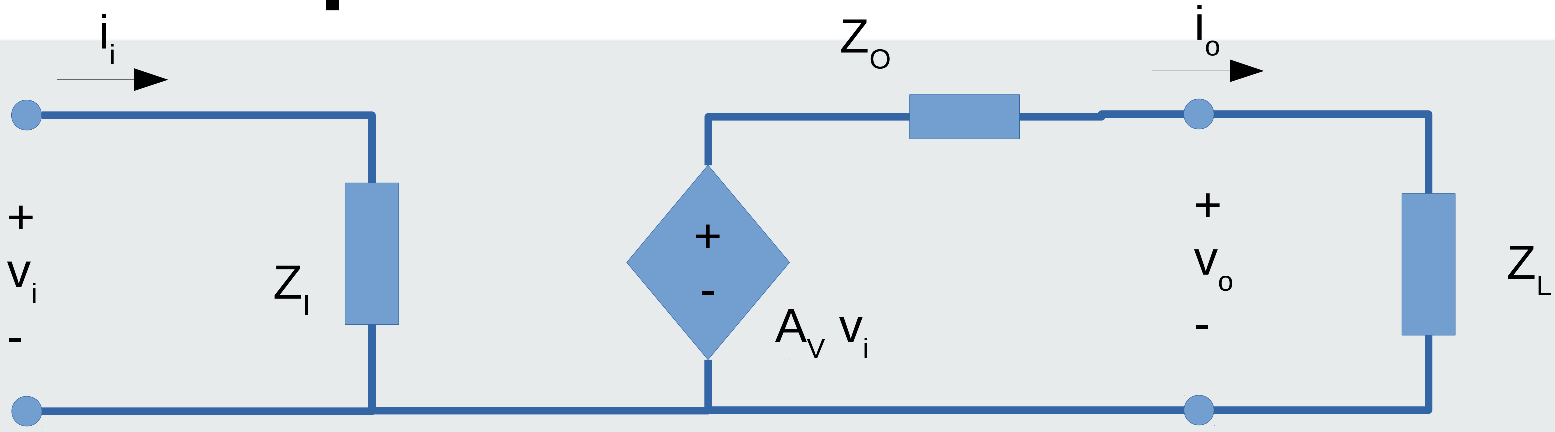
$$R_E \rightarrow 0 \Rightarrow Z_o = R || r_o$$

$$r_o \rightarrow \infty \Rightarrow Z_o = R$$

Z_o depends essentially on R!

A high R is important for high gain but makes Z_o high… :-(

Common emitter amplifier problem



Problem: because Z_o is high, a common load Z_L gets a small voltage only!

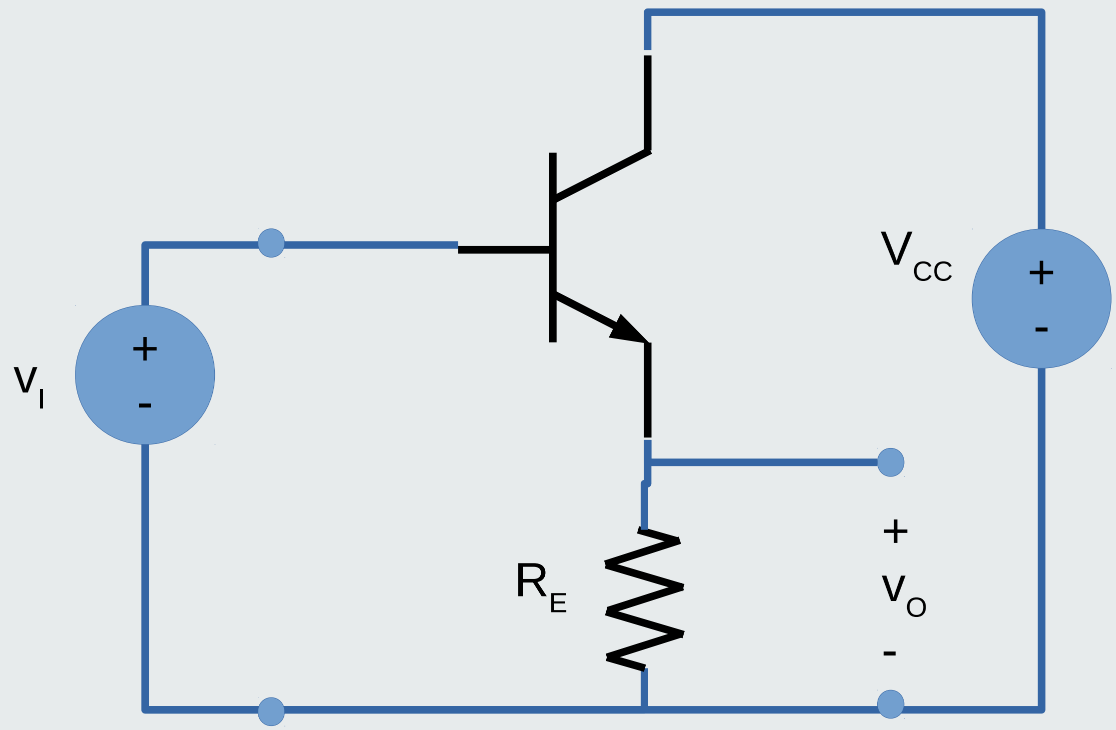
$$v_o = \frac{Z_L}{Z_L + Z_O} A_V v_i$$

This ouput voltage divider wastes the high gain A_V :-(

The common collector amplifier

Goal: supply enough current to load

Common (to input and output) collector



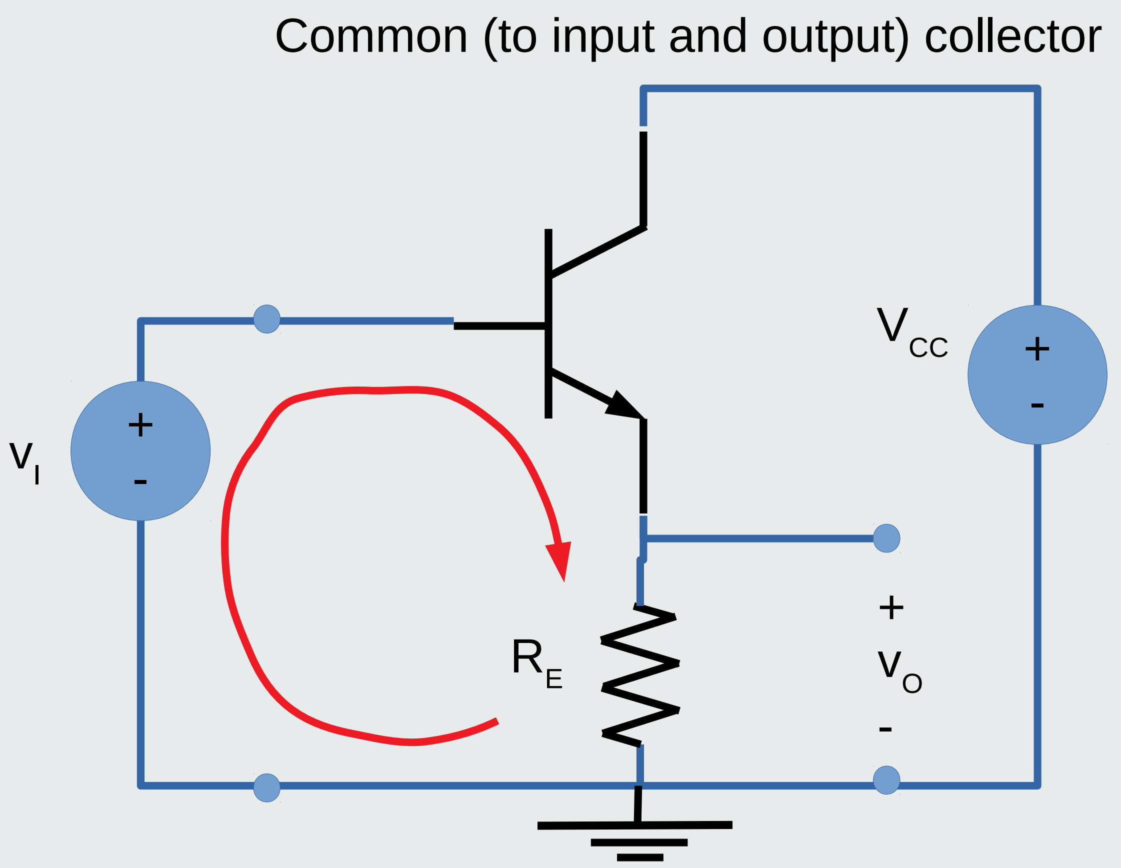
Supply voltage
(active circuit)

Superposition of DC and AC components

$$v_I = V_I + v_i$$
$$v_O = V_O + v_o$$

Separate DC (operating point) and AC (incremental) analyses

The common collector amplifier: operating point



Mesh analysis

$$-V_I + V_{ON} + R_E I_E = 0 \left(KVL \right)$$

$$V_O = V_I - V_{ON}$$

Emitter follows base voltage with constant difference V_{ON}

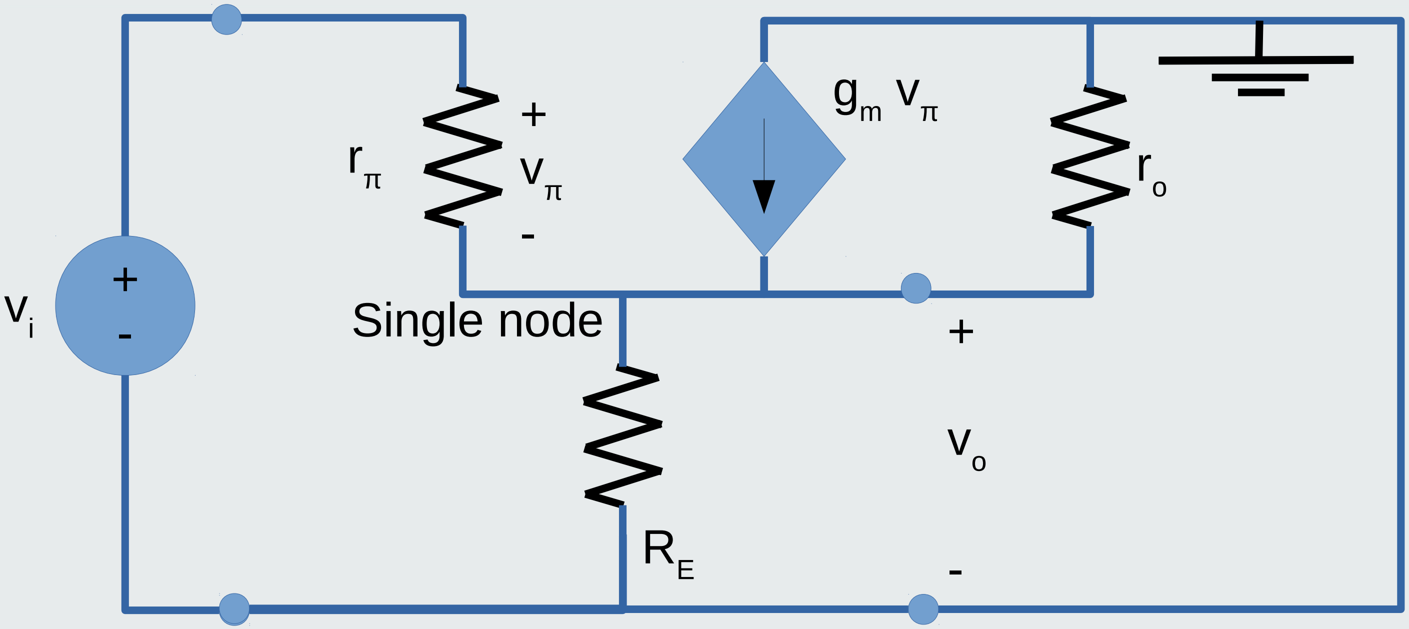
Emitter Follower Circuit

$$V_O = V_I - V_{ON} = R_E I_E$$

$$I_E = \frac{V_O}{R_E}, I_B = \frac{I_E}{1+\beta_F}, I_C = \beta_F I_B$$

$$V_{CE} = V_{CC} - R_E I_E > V_{BE}$$

The common collector amplifier: voltage gain



Convenient to work with admittances

$$g_\pi = \frac{1}{r_\pi}$$

$$g_E = \frac{1}{R_E}$$

$$g_o = \frac{1}{r_o}$$

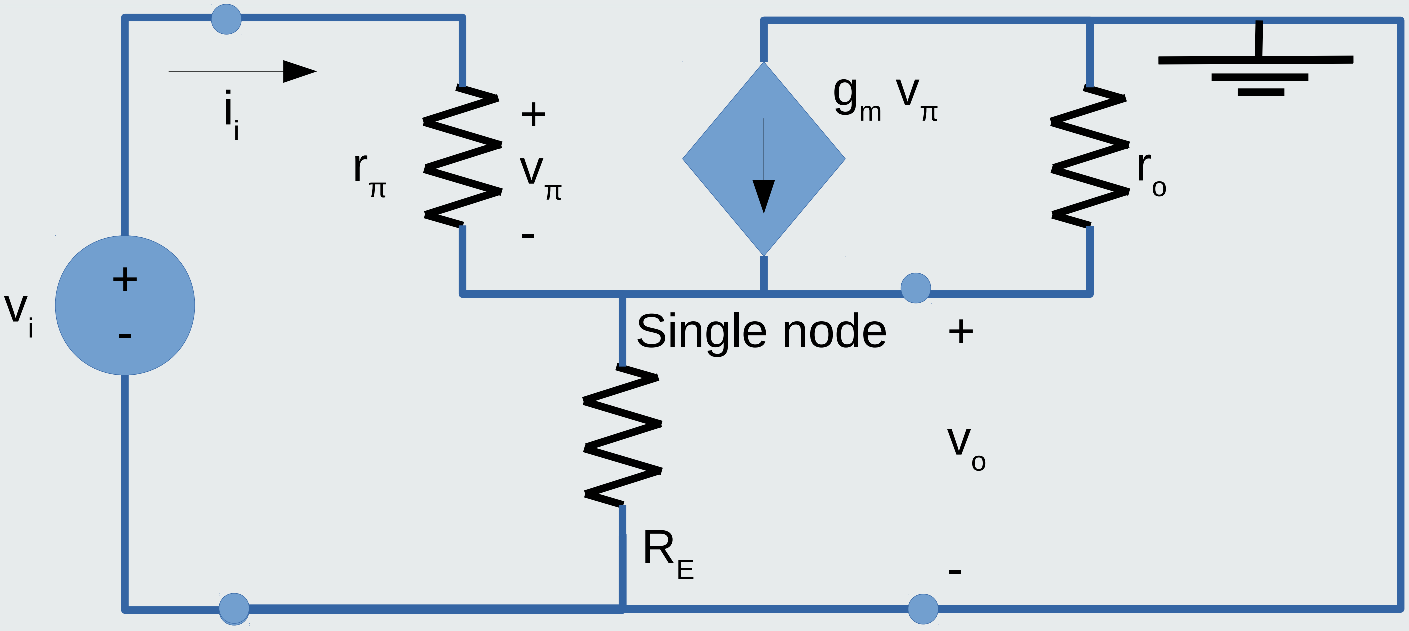
KCL: $\left(\frac{1}{R_E} + \frac{1}{r_o}\right) v_o + \frac{v_o - v_i}{r_\pi} - g_m v_\pi = 0$

$$v_\pi = v_i - v_o$$

$$\frac{v_o}{v_i} = \frac{g_m}{g_\pi + g_E + g_o + g_m} \approx 1$$

$$g_e, g_\pi, g_o \ll g_m$$

The common collector amplifier: input impedance



$$Z_I=\frac{v_i}{i_i}$$

$$i_i=g_\pi(v_i-v_o)$$

$$\frac{v_o}{v_i}=\frac{g_m}{g_\pi+g_E+g_o+g_m}$$

(From last slide)

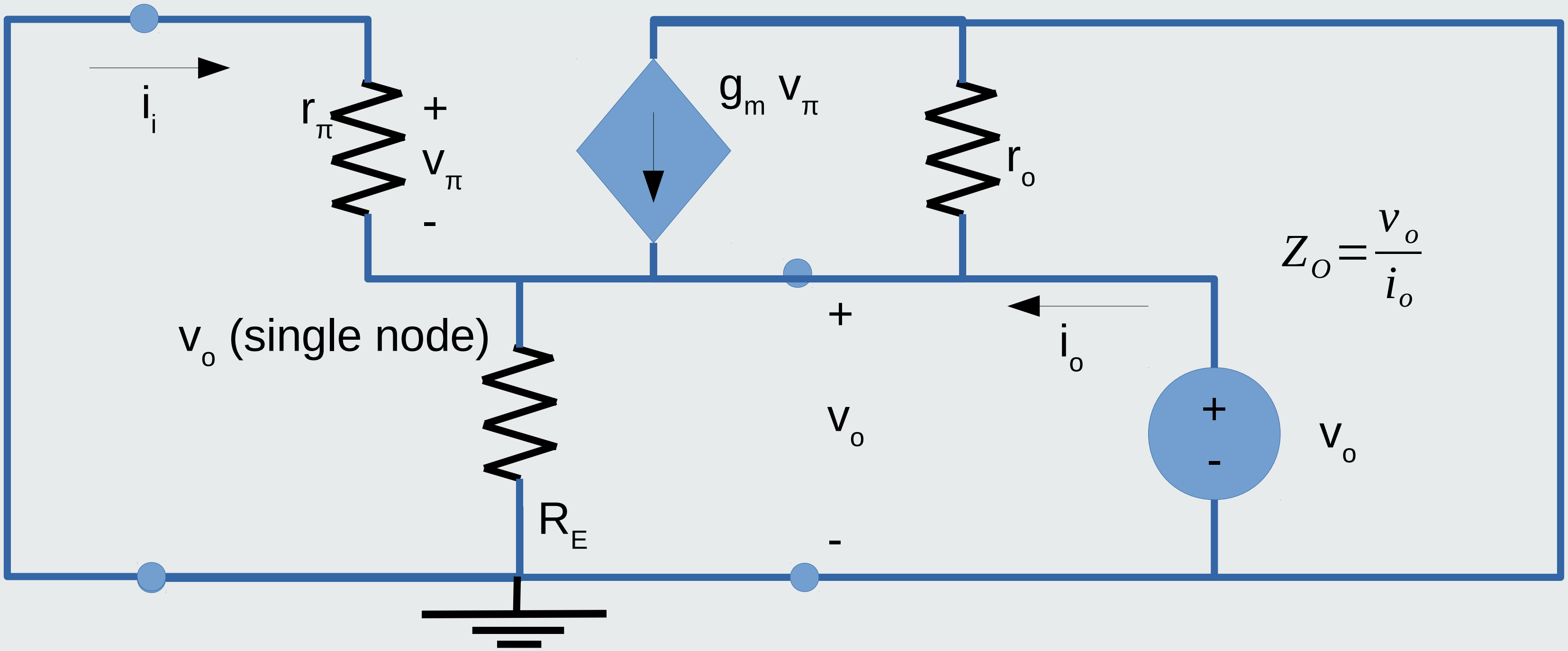
$$i_i=g_\pi\left(1-\frac{g_m}{g_\pi+g_E+g_o+g_m}\right)v_i$$

$$i_i=g_\pi\frac{g_\pi+g_E+g_o}{g_\pi+g_E+g_o+g_m}v_i$$

$$Z_I=\frac{g_\pi+g_E+g_o+g_m}{g_\pi(g_\pi+g_E+g_o)}$$

HIGH!

The common collector amplifier: output impedance



$$Z_O=\frac{v_o}{i_o}$$

$$\left(\frac{1}{R_E}+\frac{1}{r_\pi}+\frac{1}{r_o}\right)v_o-g_m v_\pi=i_o \quad \text{KCL}$$

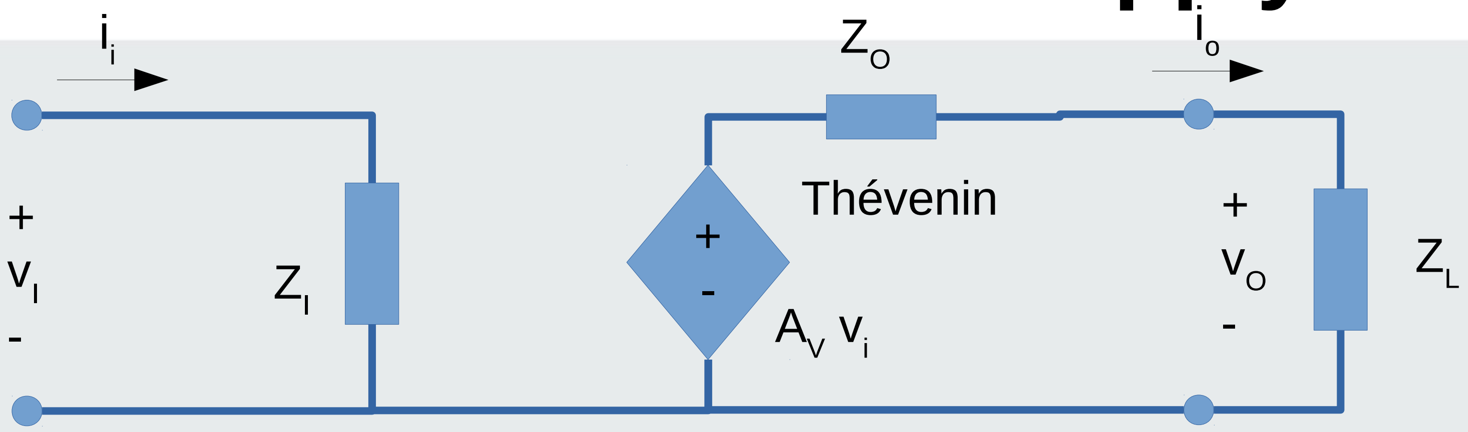
$$v_\pi=-v_o$$

$$Z_O=\frac{v_o}{i_o}=\frac{1}{g_\pi+g_E+g_o+g_m}$$

$$g_e, g_\pi, g_o \ll g_m \Rightarrow Z_O \approx \frac{1}{g_m}$$

LOW!

Common collector amplifier: solves current supply



Common Emitter

$$v_o = \frac{Z_L}{Z_L + Z_O} A_V v_i$$

Common Collector

$$A_V \approx 1$$
$$Z_O \ll Z_L$$

Depends On Z_L! $\quad A_V' = \frac{Z_L}{Z_L + Z_O} A_V$ | Effective Gain | $A_V' \approx 1$ Unaffected by Z_L!

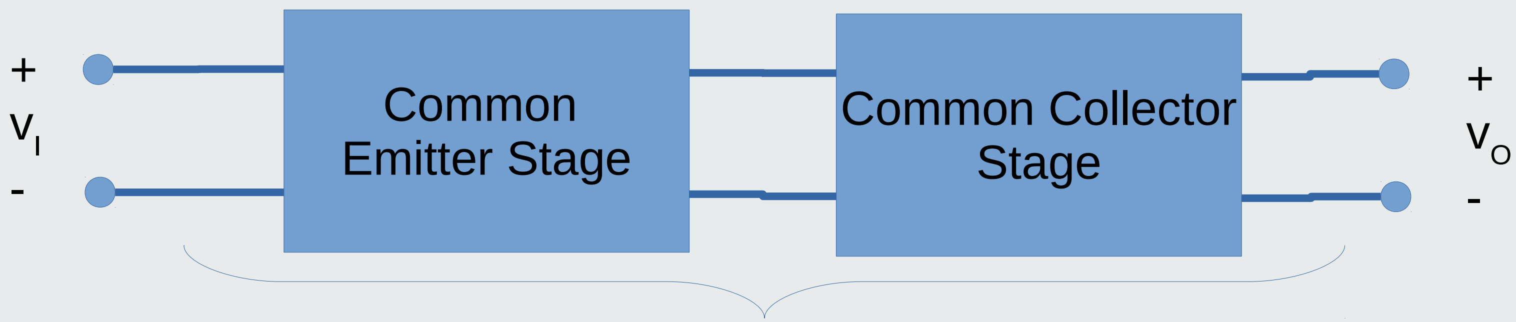
Depends On Z_O! $\quad i_o = \frac{v_o}{Z_L} = \frac{A_V}{Z_L + Z_O} v_i$ | Load Current | $i_o = \frac{v_o}{Z_L} = \frac{A_V}{Z_L} v_i$ Unaffected by Z_O!

Problem solved: enough current for load!

Good voltage amplifier

- Input is voltage
- Output is voltage
- Z_I should be high to not degrade input voltage
- Z_O should be low to not degrade output signal
- A_V should be high because we want to amplify

High Z_{I}, High A_V, High Z_O **High Z_I, Low Z_O, Low A_V**



Good Voltage Amplifier: High A_V, High Z_I, Low Z_O!

Conclusion

- BJT amplifiers presented
- Common emitter amplifier
  - High gain, input and output impedances
  - High output impedance not good for voltage amplifier
- Common collector amplifier
  - Low gain, high input impendance and low output impedances
  - Low gain is not good for voltage amplifier
- Combining common emitter and common collector stages results in a good voltage amplifier design