TÉCNICO LISBOA

Circuit Theory and Electronics Fundamentals

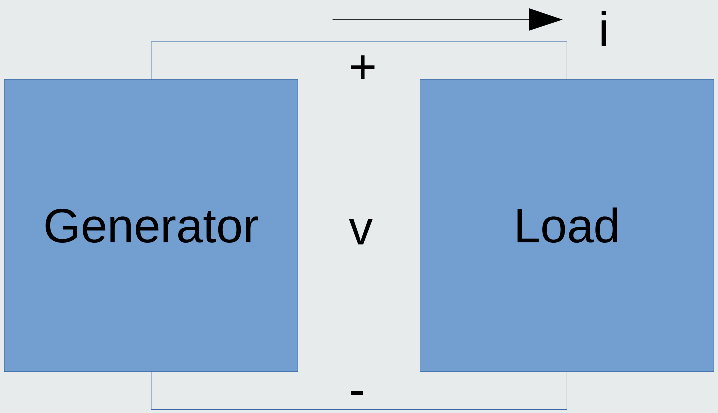
Lecture 7: Power and energy in sinusoidal steady-state analysis

- Computing power using time functions
- Effective voltage, effective current, power factor, average power
- Computing energy
- Computing power with phasors: complex, apparent, active and reactive power
- Maximum power transfer theorem
- Power factor compensation

Sinusoidal analysis: power

Power is energy per unit of time

$$p(t)=v(t)i(t)$$



In a sinusoidal steady-state analysis:

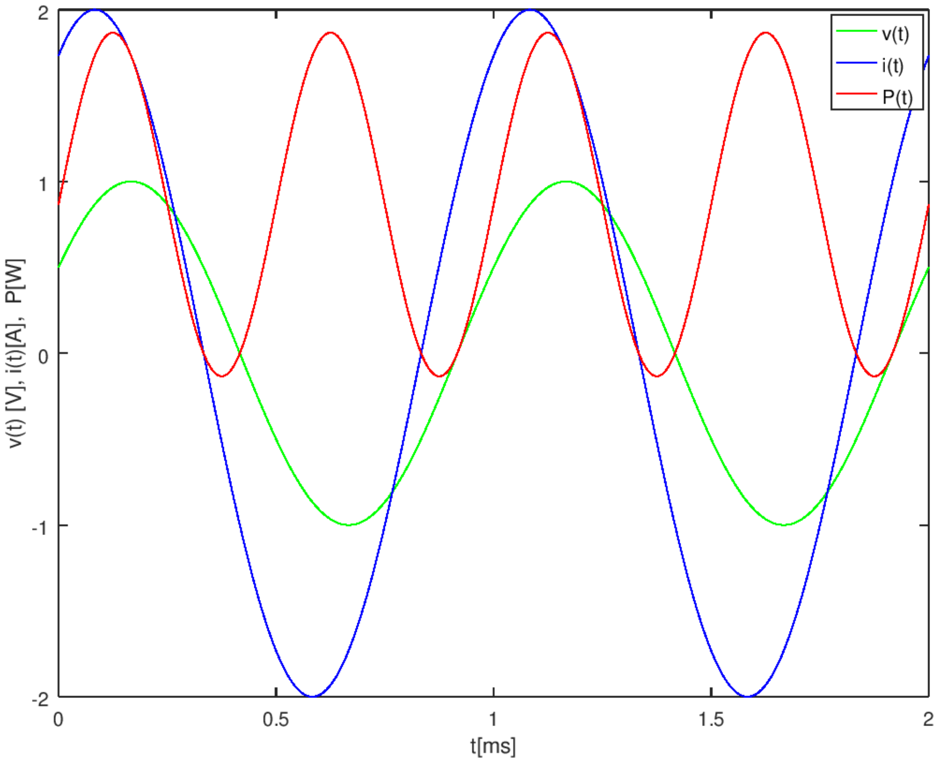
$$p(t)=V\cos(\omega t+\phi_V)\,I\cos(\omega t+\phi_I)$$

$$\cos(a)\cos(b)=\frac{1}{2}[\cos(a+b)+\cos(a-b)]$$

$$\cos(-a)=\cos(a)$$

$$p(t)=\frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}}[\cos(\phi_V-\phi_I)+\cos(2\omega t+\phi_V+\phi_I)]$$

Sinusoidal analysis: power



Voltage and current are out of phase: component has complex impedance

Power has twice the frequency: consequence of multiplying sinusoidal functions

Power is sometimes negative: component is producing energy in those times

Sinusoidal average power

$$\bar{P}=\frac{1}{T}\int_T p(t)\,dt$$

Average power during one period T

$$\bar{P}=\frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}}\left[\frac{1}{T}\int_T \cos(\phi_V-\phi_I)\,dt+\cancel{\frac{1}{T}\int_T \cos(2\omega t+\phi_V+\phi_I)}\right]$$

$$\boxed{\bar{P}=V_{RMS}\,I_{RMS}\cos(\phi_V-\phi_I),}\quad V_{RMS}=\frac{V}{\sqrt{2}},\quad I_{RMS}=\frac{I}{\sqrt{2}}$$

$$PF=\cos(\phi_v-\phi_i)$$

Power factor

Effective Voltage: Root Mean Square (RMS) Voltage

Effective Current: Root Mean Square (RMS) Current

Energy computation

$$E=\int_{t=0}^{t=t_f} p(t)\,dt$$

Energy consumed in time interval [0, t_f]

$$t_f \gg T \Rightarrow E \approx \bar{P} * t_f$$

No need to integrate

$$\bar{P}=V_{RMS} I_{RMS} PF$$

How to use effective voltage, current and power factor

$$PF=\cos(\phi)$$

Power factor expression

$$\phi=\phi_V-\phi_I$$

Voltage-current phase difference

Computing power with phasors

$$\tilde{P}=\frac{\tilde{V}\,\tilde{I}^{*}}{2}$$

Complex power definition (very convenient)

$$\tilde{P}=\frac{V\,I}{2}\,e^{j\,\phi}$$

$$P_{apparent}=\frac{V\,I}{2}=V_{RMS}\,I_{RMS}$$ [VA]

$$\tilde{P}=P_{apparent}\,e^{j\,\phi}$$

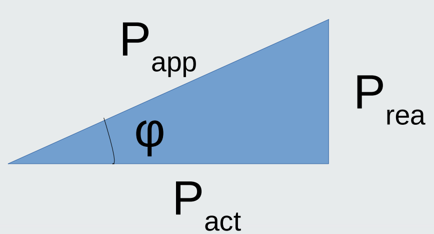
Complex power definition using apparent power

$$\Re\left\{\tilde{P}\right\}=P_{a}\cos\left(\phi\right)$$

Average real or active power! (convenience explained) [W]

$$\Im\left\{\tilde{P}\right\}=P_{a}\sin\left(\phi\right)$$

Reactive power! (power that goes back and forth without being consumed) [VAR]



Maximum power transfer theorem (DC analysis)

$$P_R = V_R I_R$$

$$V_R = \frac{R}{R_S + R} V_S \quad \text{Volt. Div.}$$

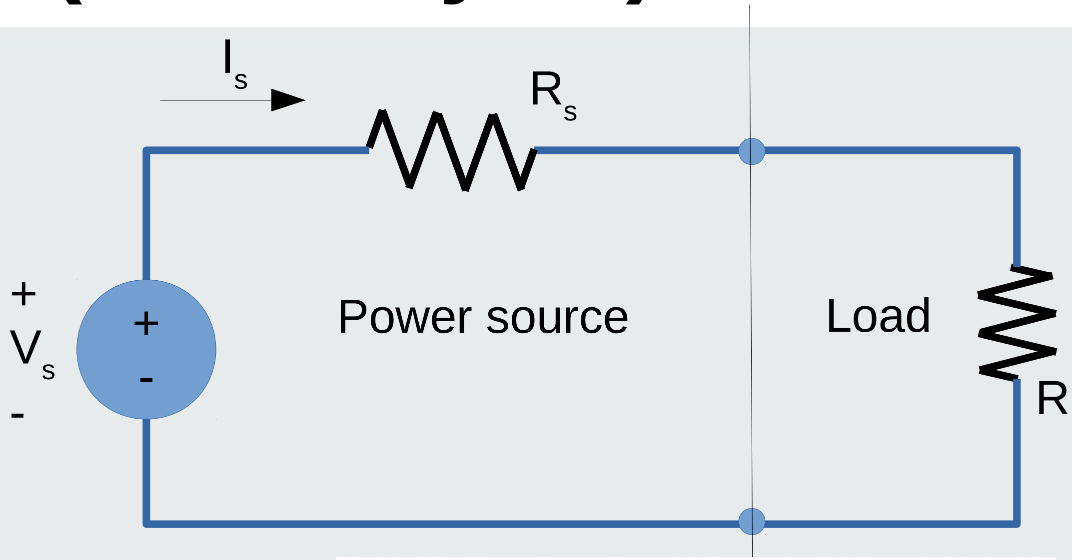
$$I_R = I_s = \frac{V_S}{R_S + R} \quad \text{Ohms'}$$

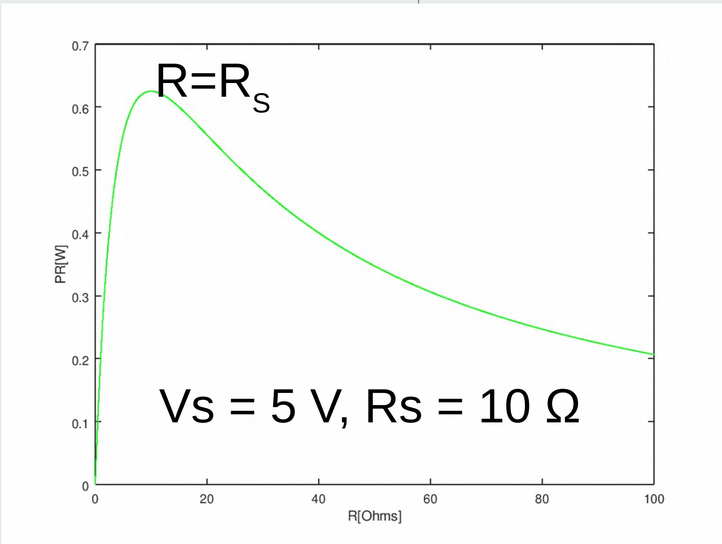
$$P_R = \frac{R}{(R_S + R)^2} V_s^2$$

$$P_R(R_s) = max(P_R)$$

Just find the maximum of function $P_R(R)$

$$R_{max\ power} = R_S$$

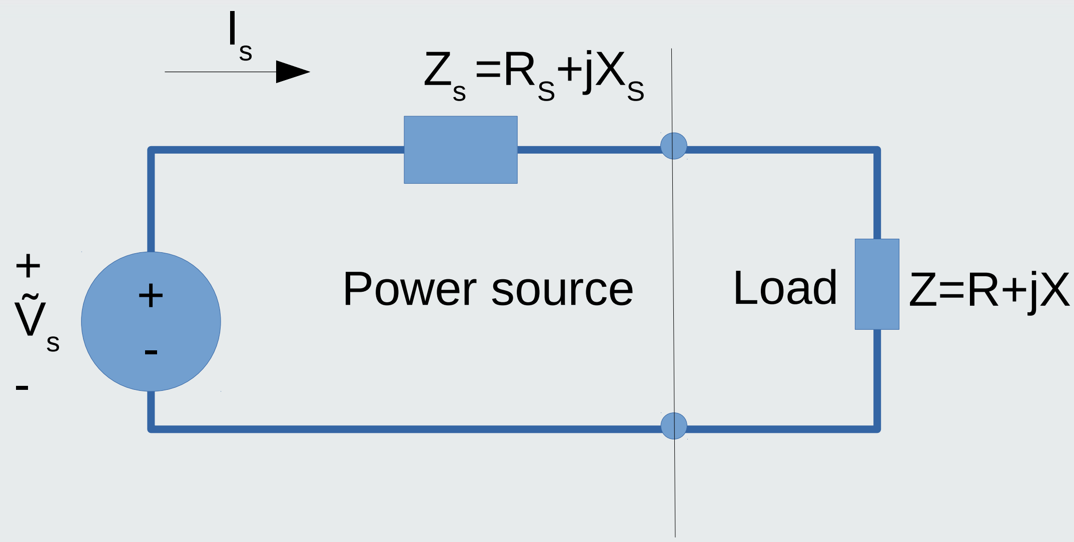




Maximum power transfer theorem in sinusoidal analysis

$$P_Z(Z_S^*) = max(P_Z)$$

$$Z_{max\ power} = Z_S^*$$
$$R = R_S$$
$$X = -X_S$$



Just find the maximum for real function of a complex variable $P_Z(Z)$

$R^2 \rightarrow R : Z \rightarrow P_Z$ active power on load

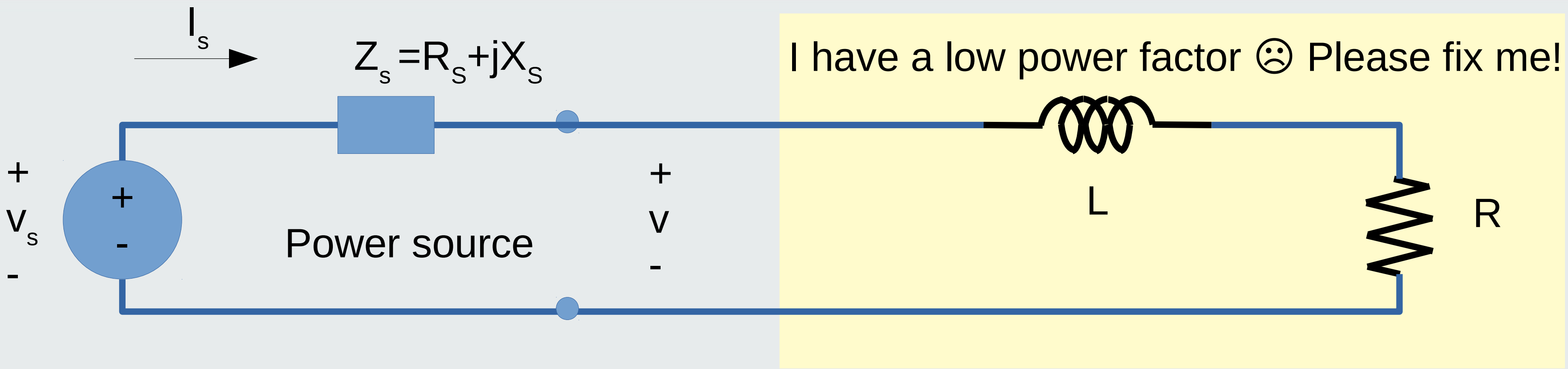
Complex loads are common due to transformers, motors, etc

Complex loads show power factor magnitudes lower than 1 and this is a problem...

Power factor compensation

- In some situations a low power factor is not desirable
- Example: industrial electrical installations
- A low power factor stresses the electrical grid with high voltage and current peaks, only to transport reactive energy back and forth, with no useful purpose
- A low power factor exacerbates power losses and compromises power distribution efficiency
- Electrical utility companies charge a penalty cost for low power factor installations
  - Simple penalty scheme: charge money for Apparent Power instead of Active Power!
- Consumers have thus an incentive to compensate their power factors, reduce reactive power, and save big money in electricity bills!

Power factor compensation example



$$PF=\cos(\phi)$$

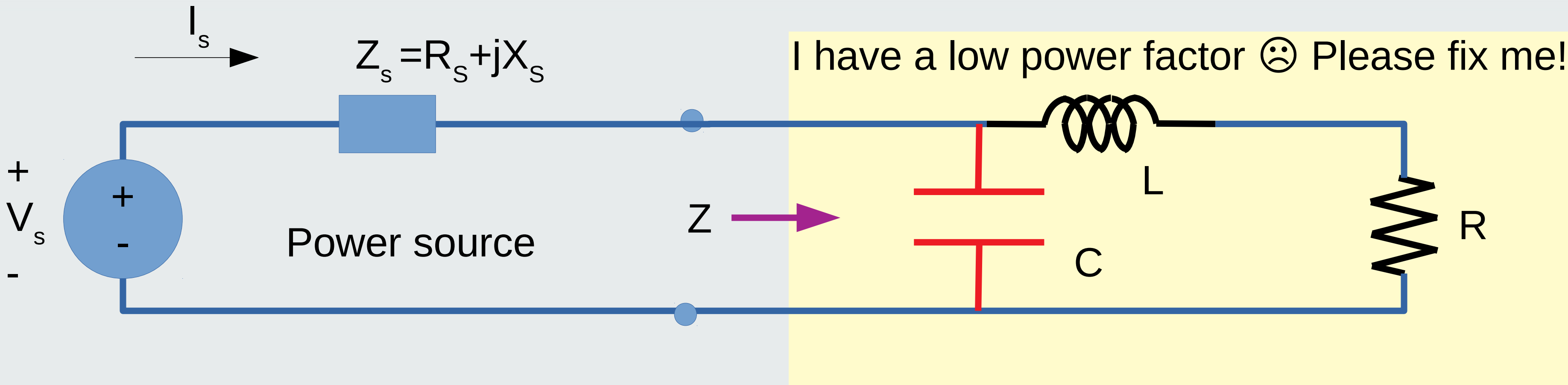
$$Z=\frac{\tilde{V}}{\tilde{I}}=\frac{V}{I}e^{j(\phi_v-\phi_i)}=\frac{V}{I}e^{j\phi}$$

$$\phi=\measuredangle Z$$

$$Z=R+j\omega L$$

$$\phi=\arctan\left(\frac{\omega L}{R}\right)$$

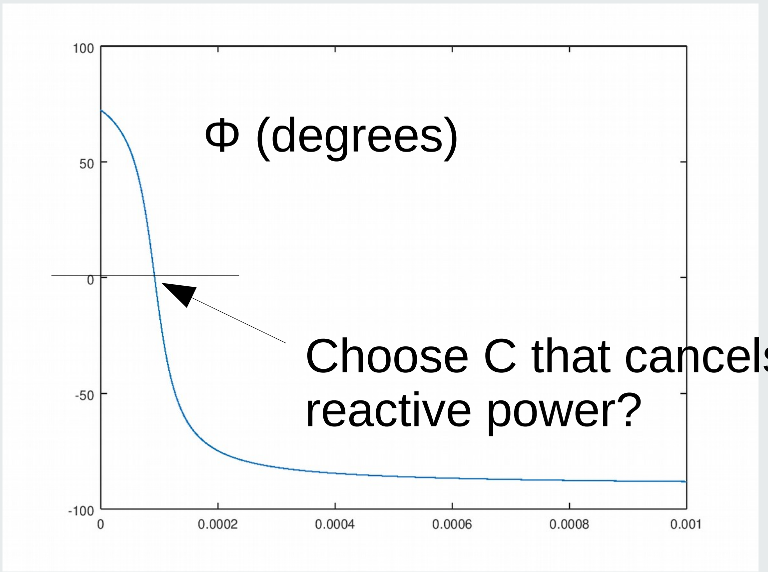
Power factor compensation using a capacitor



$$\cos(\phi) = \cos(\sphericalangle Z)$$ Power factor

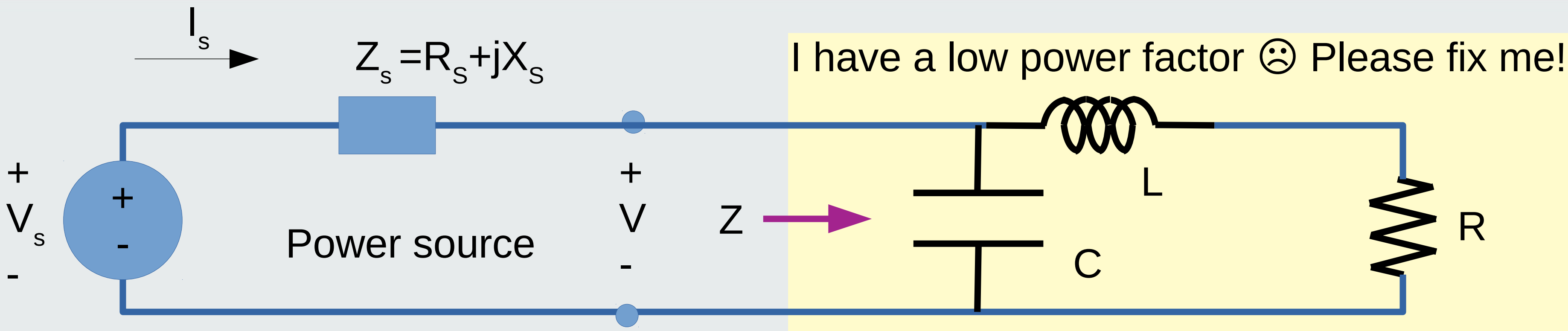
$$Z = \frac{1}{\frac{1}{R + j\omega L} + j\omega C}$$

Z = C || L-R



WARNING:
Natural solution being ignored!
Important when switching networks on and off!

Power factor compensation effect on delivered power



$$P_R = \Re\{\tilde{P}_R\}$$

Active power delivered to R

$$\tilde{P}_R = \frac{1}{2}\tilde{V}_R \tilde{I}_R^{\,*}$$

Complex power delivered to R

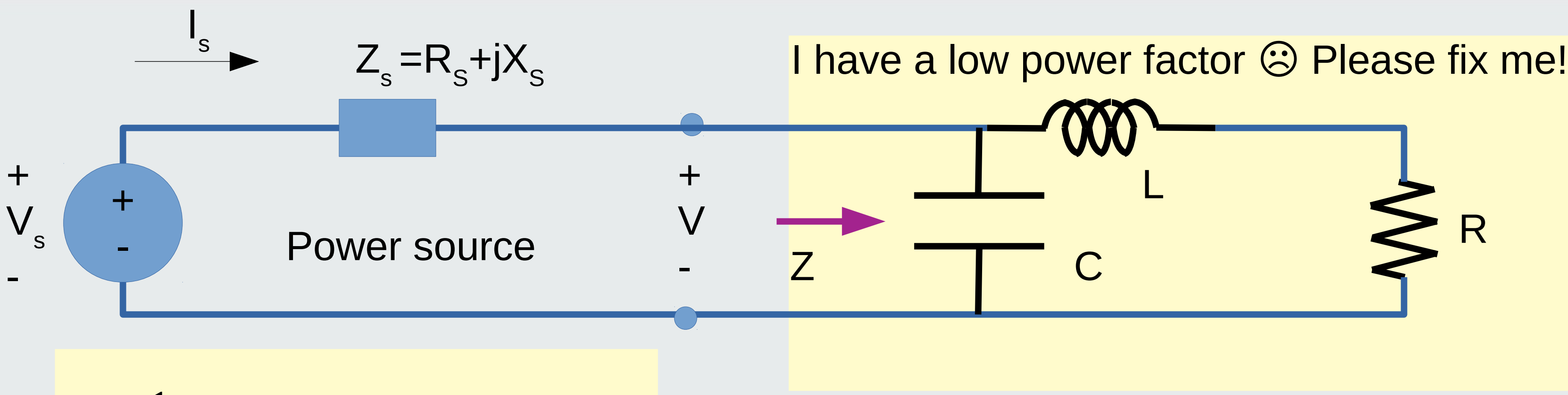
$$\tilde{V}_R = \frac{R}{R + j\omega L}\tilde{V}$$

Voltage Divider

$$\tilde{I}_R = \frac{\dfrac{1}{R + j\omega L}}{j\omega C + \dfrac{1}{R + j\omega L}}\tilde{I}_S$$

Current Divider

Power factor compensation: input voltage and current



Z=C||L-R

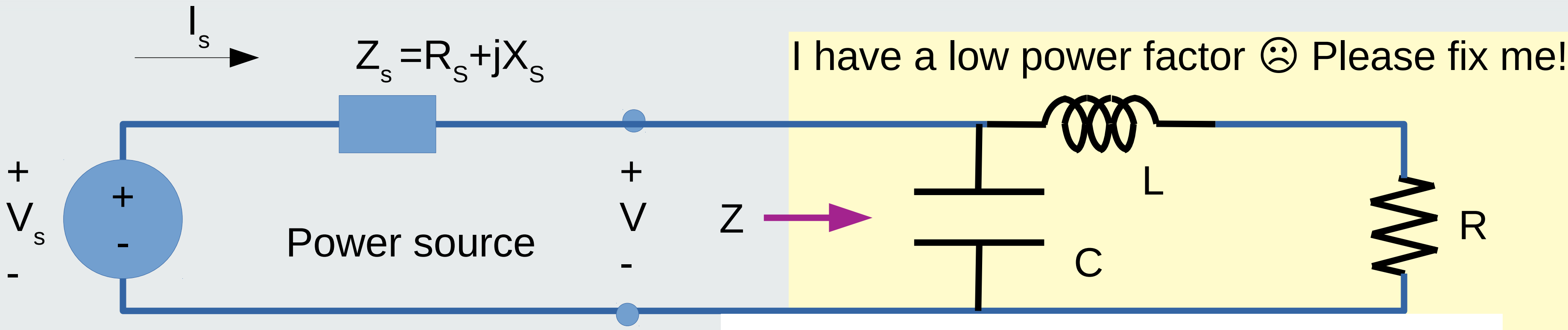
$$\tilde{V} = \frac{\dfrac{1}{j\omega C + \dfrac{1}{R + j\omega L}}}{\dfrac{1}{j\omega C + \dfrac{1}{R + j\omega L}} + Z_S} \tilde{V}_S$$

Voltage Divider

$$\tilde{I}_S = \frac{\tilde{V}_S}{\dfrac{1}{j\omega C + \dfrac{1}{R + j\omega L}} + Z_S}$$

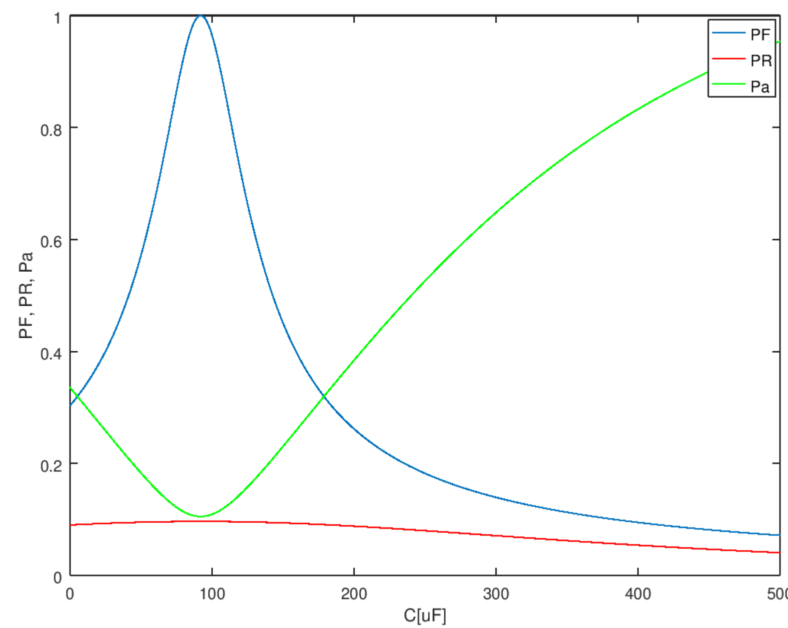
Ohm's Law

Power factor compensation effect on delivered power



$PF = \cos(\measuredangle Z)$ Power factor

$P_R = \Re\{\widetilde{P}_R\}$ Active power in R



Fixed with C = 92 μF !

Compensating power factors pays!

Electricity bill reduced 3 times!

Conclusion

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