TÉCNICO
LISBOA

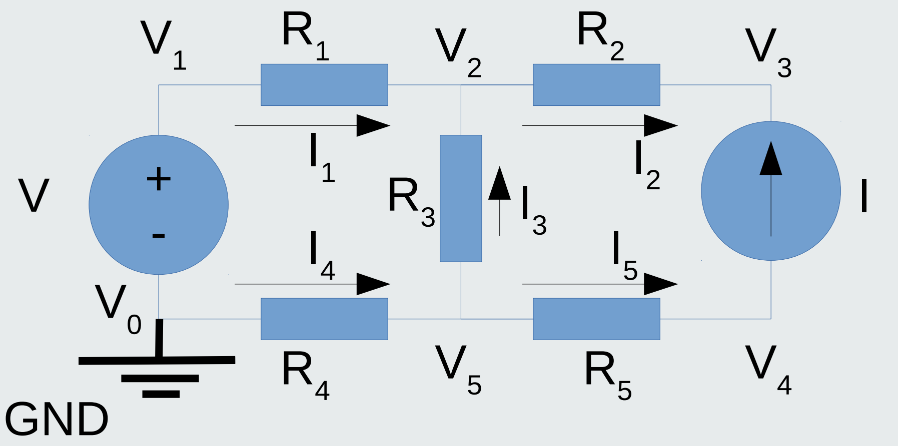
Circuit Theory and Electronics Fundamentals

Lecture 2: Circuit analysis methods

- Analysis for circuits with resistors and independent I/V sources
- Superposition Theorem
- Thévenin and Norton Theorems
- Dependent or controlled sources
- Circuit mesh and nodal analysis methods

Analysis of circuits with resistors and independent V/I sources

Analysis: find all node voltages and branch currents

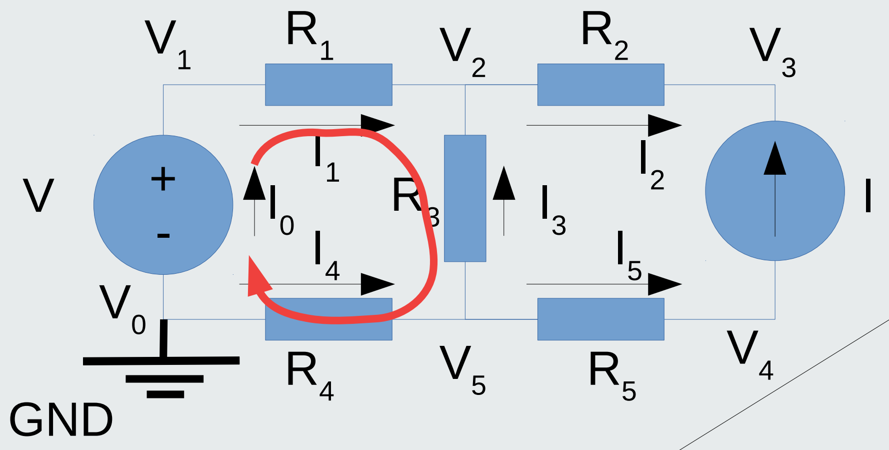


SETUP

1) Number nodes arbitrarily (e.g. 0 to n-1)
2) Assign current names and directions to all branches arbitrarily
3) Assign potential 0 to one of the nodes

Solving circuits

Find all node voltages V_0-V_5 and branch currents



$$I_0 = I_1$$
$$I_2 = -I$$
$$I_4 = -I_1$$
$$I_5 = I$$

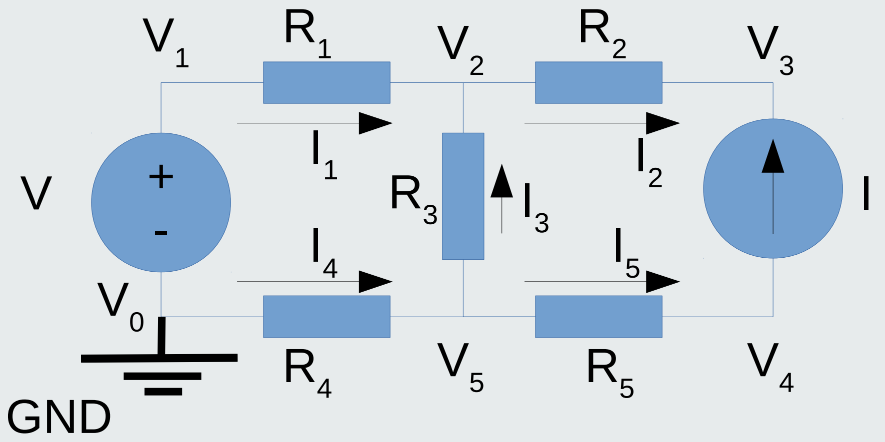
$$I_1 + I_3 - I_2 = I_1 + I_3 + I = 0 \,(KCL\, node\, 2)$$
$$I_3 + I_5 - I_4 = I_3 + I + I_1 = 0 \,(KCL\, node\, 5)$$
$$V = R_1 I_1 - R_3 I_3 + R_4 I_1 \,(KVL\, loop\, V, R_{1,} R_{3,} R_4)$$

$$I_1 = \frac{1}{R_1 + R_3 + R_4} V - \frac{R_3}{R_1 + R_3 + R_4} I$$
$$I_3 = -\frac{1}{R_1 + R_3 + R_4} V - \frac{R_3 + R_4}{R_1 + R_3 + R_4} I$$

KCL2 = KCL5 !
This may happen if we are not following a structured method!

Computing node voltages from branch currents

Use Ohm's Law



$$V_0 = 0$$

$$V_1 = V$$

$$V_2 = V_1 - R_1 I_1 = \frac{R_3 + R_4}{R_1 + R_3 + R_4} V + \frac{R_1 R_3}{R_1 + R_3 + R_4} I$$

$$V_3 = V_2 - R_2 I_2 = V_1 - R_1 I_1 = \left(1 - \frac{R_1}{R_1 + R_3 + R_4}\right) V + \left(\frac{R_1 R_3}{R_1 + R_3 + R_4} + R_2\right) I$$

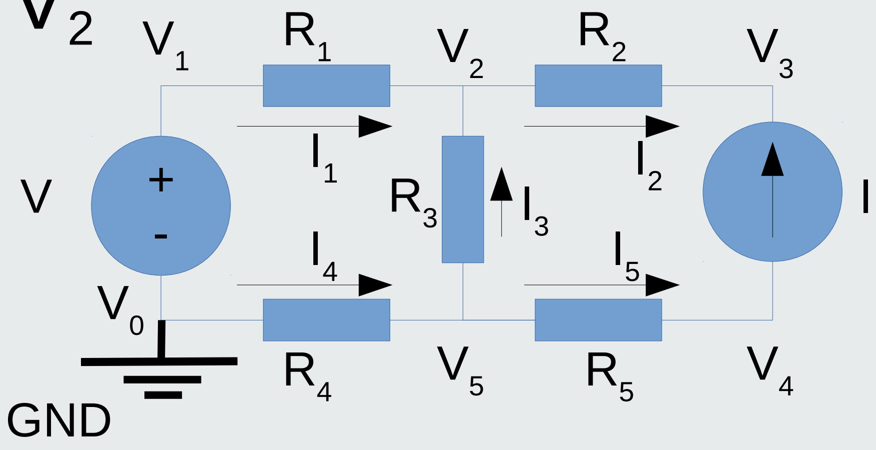
$$V_4 = V_5 - R_5 I_5 = \frac{R_4}{R_1 + R_3 + R_4} V - \left(\frac{R_3 R_4}{R_1 + R_3 + R_4} + R_5\right) I$$

$$V_5 = V_0 - R_4 I_4 = \frac{R_4}{R_1 + R_3 + R_4} V - \frac{R_3 R_4}{R_1 + R_3 + R_4} I$$

Superposition Theorem

Note that the solution for V_2

$$V_2 = \frac{R_3 + R_4}{R_1 + R_3 + R_4} V + \frac{R_1 R_3}{R_1 + R_3 + R_4} I$$



can be expressed as

$$V_2 = V_2\big|_{V=0} + V_2\big|_{I=0}$$

V_2 is given by a <u>Linear Superposition</u> of the effects of the independent sources V and I

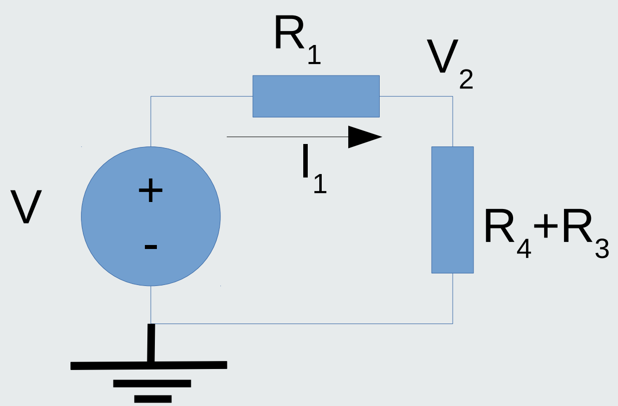
This is a consequence of all-linear relationships

Computing the Effect of V on V_2

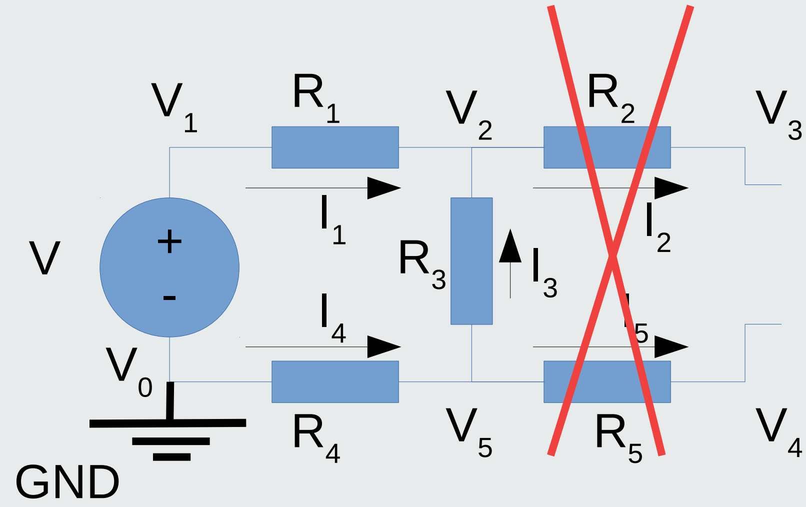
Switch off current source I

$$I=0 \Rightarrow I_2=I_5=0 \Rightarrow V_2$$

Then use Voltage Divider



$$V_2=\frac{R_3+R_4}{R_1+R_3+R_4}V$$

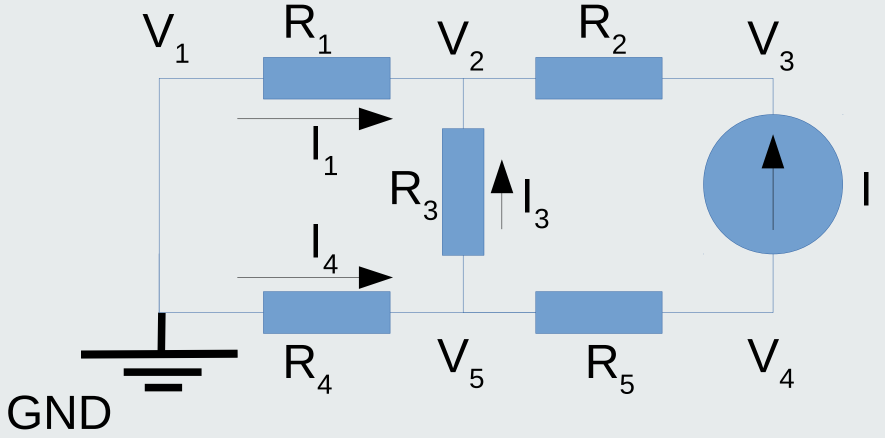


Switching off current source I leaves an open-circuit between nodes 3 and 4 (internal resistance is infinite). R_2 and R_5 become dead-end branches and may be removed.

Computing the Effect of I on V_2

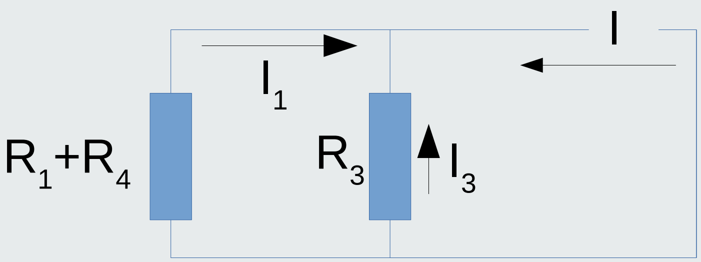
Switch off voltage source V

$$V=0 \Rightarrow V_1=0 \Rightarrow V_2=-R_1 I_1$$



Then use Current Divider

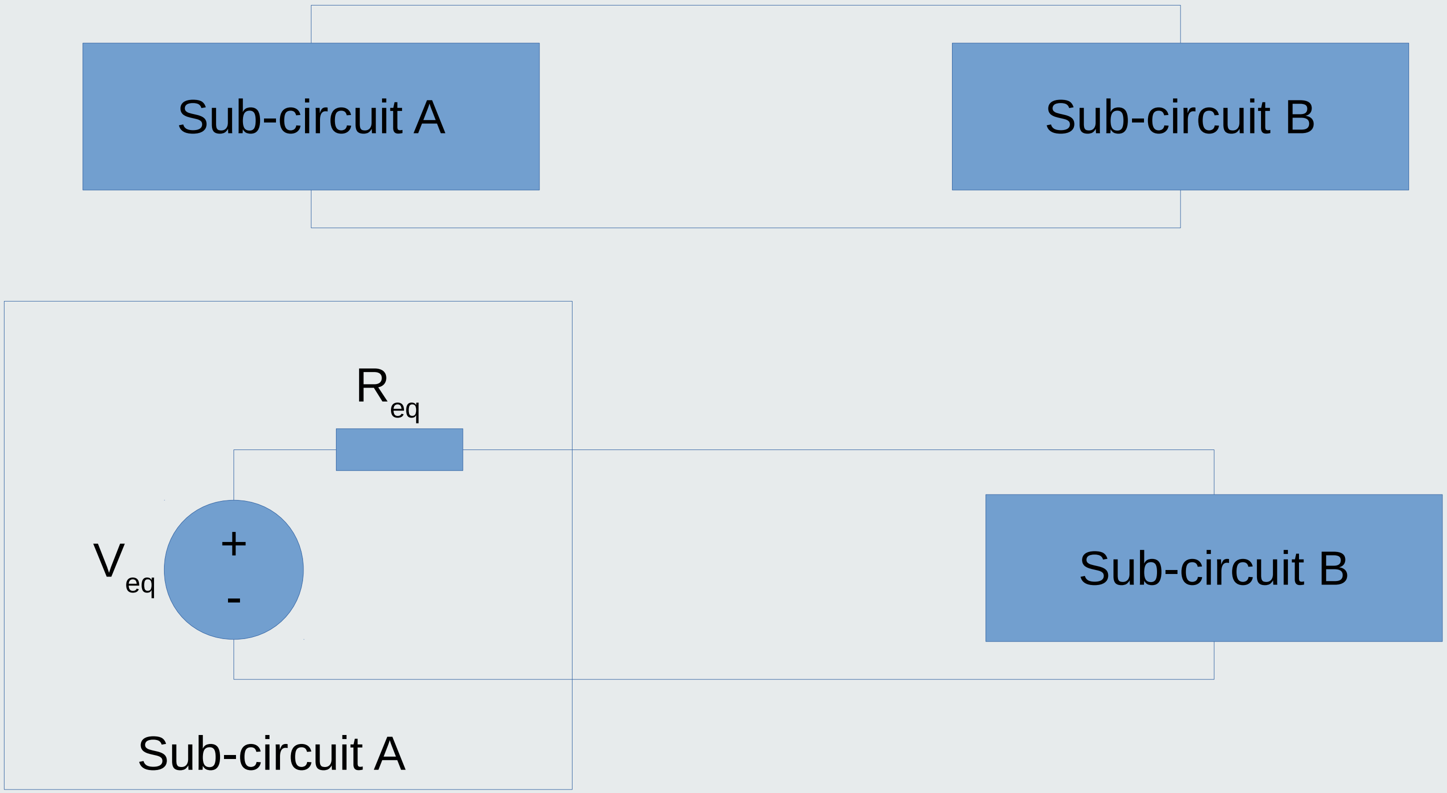
$$-I_1=\frac{\frac{1}{R_1+R_4}}{\frac{1}{R_1+R_4}+\frac{1}{R_3}} I \Rightarrow V_2=\frac{R_1 R_3}{R_1+R_3+R_4} I$$



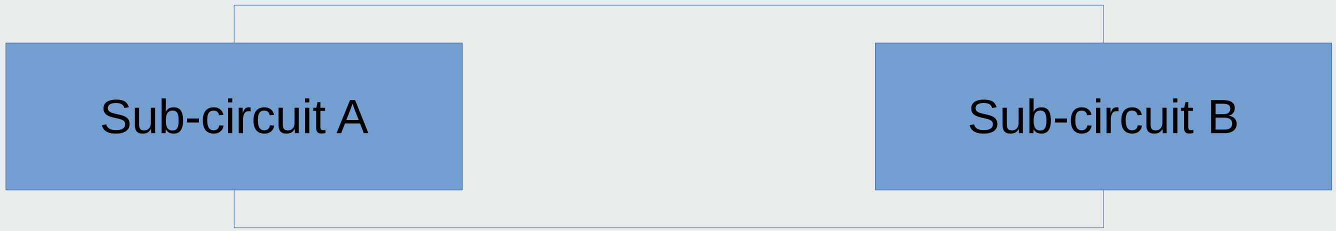
Switching off voltage source V leaves a short-circuit between nodes 0 and 1 (internal resistance is zero). R_1 and R_4 are now directly connected in a resistor series.

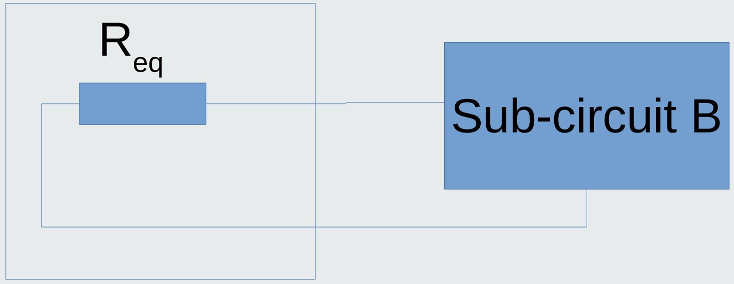
Thévenin Theorem

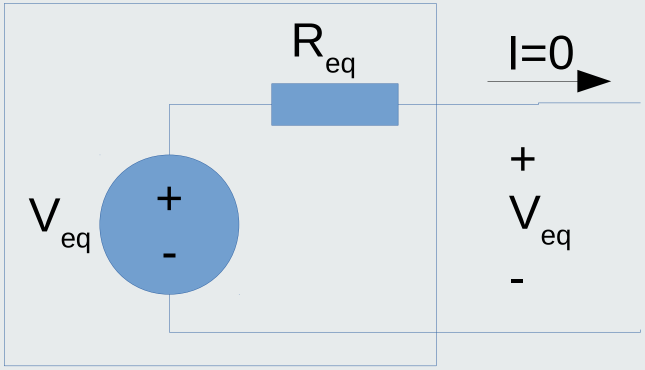
Replaces a linear complex 2-port circuit with a simpler 2-port circuit having a voltage source V_{eq} in series with a resistor R_{eq}



Thévenin V_{eq} and R_{eq}





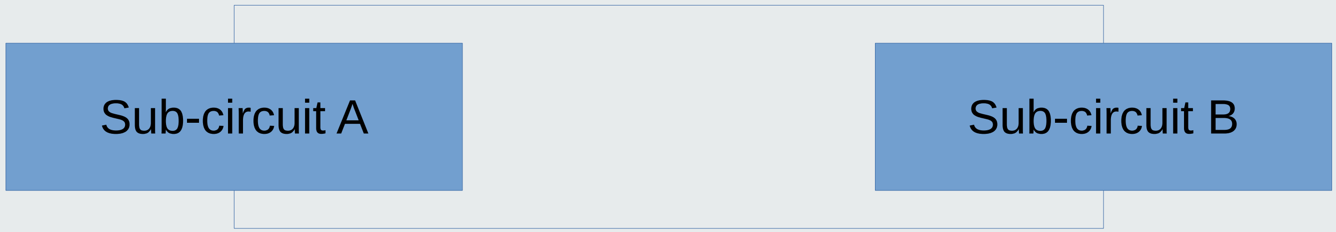


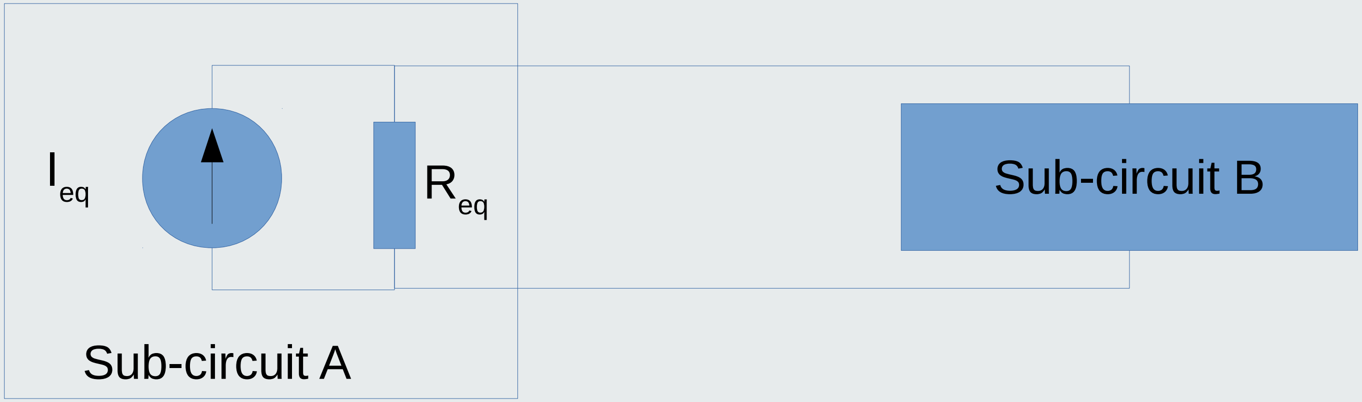
R_{eq} is the equivalent resistor when all sources of Sub-circuit A are switched off

V_{eq} is the voltage drop between the 2 ports if sub-circuit B is removed, leaving an open-circuit between the ports

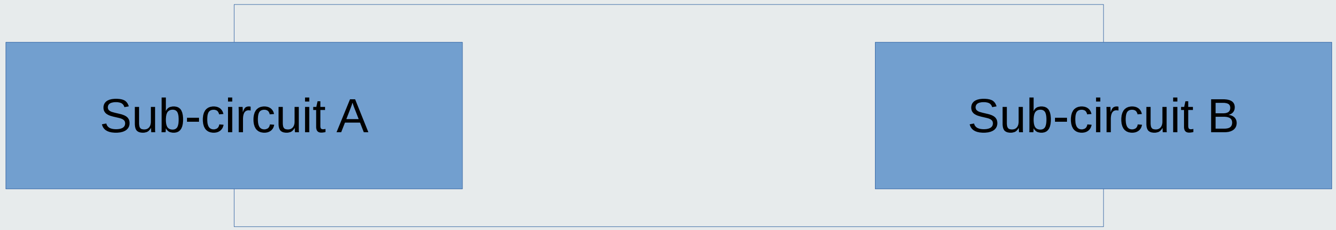
Norton Theorem

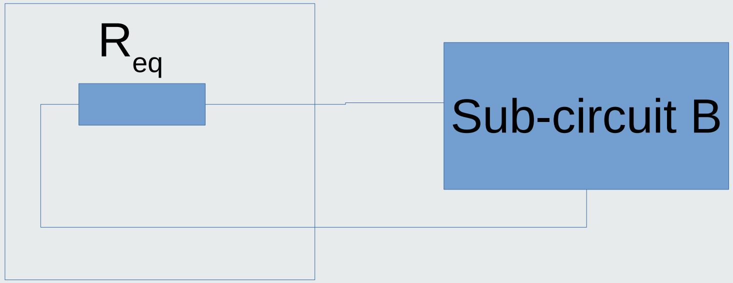
Replaces a linear complex 2-port circuit with a simpler 2-port circuit having a current source I_{eq} in parallel with a resistor R_{eq}

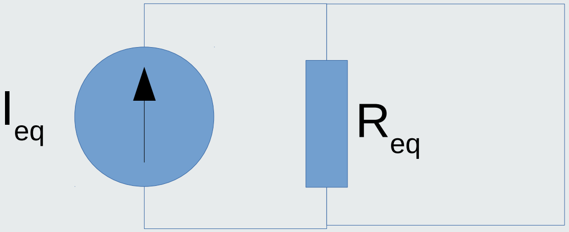




Norton I_{eq} and R_{eq}







R_{eq} is the equivalent resistor when all sources of Sub-circuit A are switched-off

I_{eq} is the current between the 2 ports when Sub-circuit B is replaced with a shunt (short-circuit)

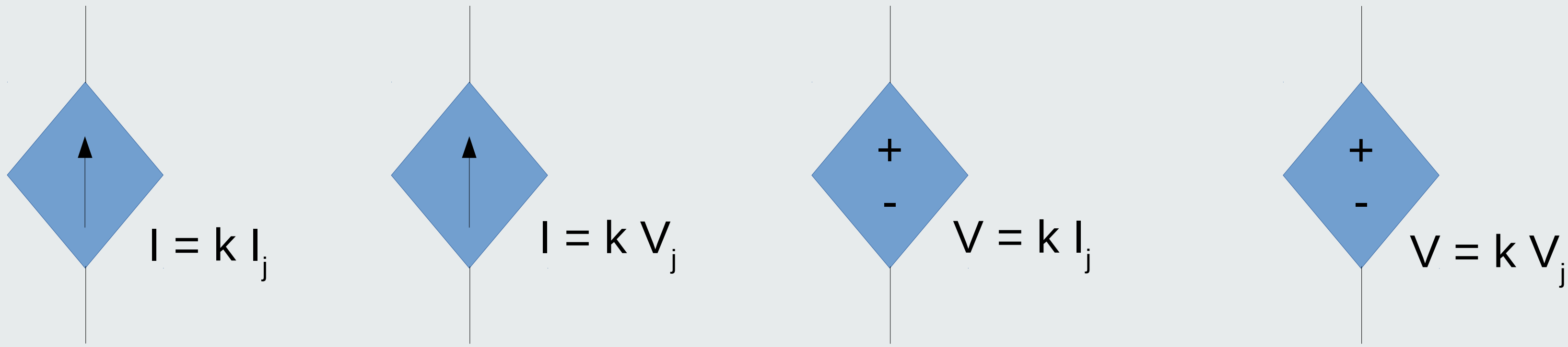
Relationship between I_{eq} and V_{eq}

Applying the Thévenin Theorem to the Norton equivalent (do it as exercise) or

Applying the Norton Theorem to the Thévenin equivalent (do it as exercise) we get the useful relationship:

$$V_{eq} = R_{eq} I_{eq}$$

Linearly Dependent Sources



Current controlled Current source

($Ri = \infty$)

Voltage controlled Current source

($Ri = \infty$)

Current controlled Voltage source

($Ri = 0$)

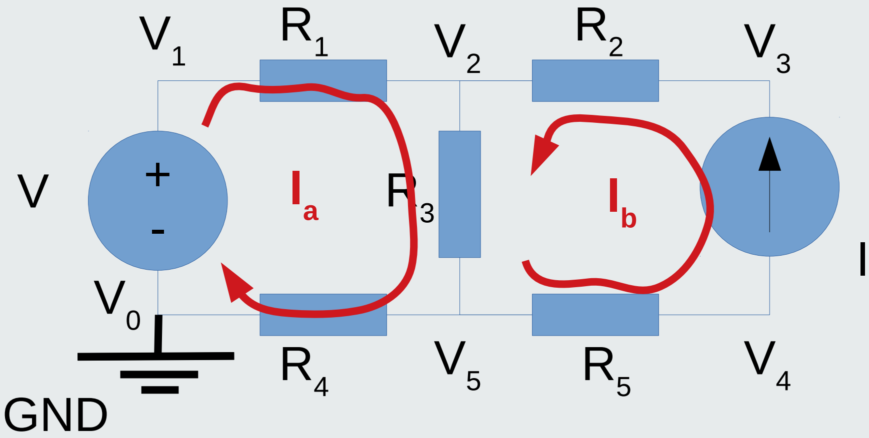
Voltage controlled Voltage source

($Ri = 0$)

Mesh Analysis

A mesh is a loop that contains no other loops

Consider 2 meshes ***a*** and ***b***



$$V = R_1 I_a + R_3 (I_a + I_b) + R_4 I_a \,(KVL\, mesh\, 'a')$$

$$(R_1 + R_3 + R_4) I_a + R_3 I_b = V$$

$$I_b = I \,(by\ inspection)$$

$$\begin{bmatrix} R_1 + R_3 + R_4 & R_3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} V \\ I \end{bmatrix}$$

$$I_a = \frac{V - R_3 I}{R_1 + R_3 + R_4}$$

Knowing mesh currents, we can know any node voltages or branch currents, using Ohm’s law. For example for V2:

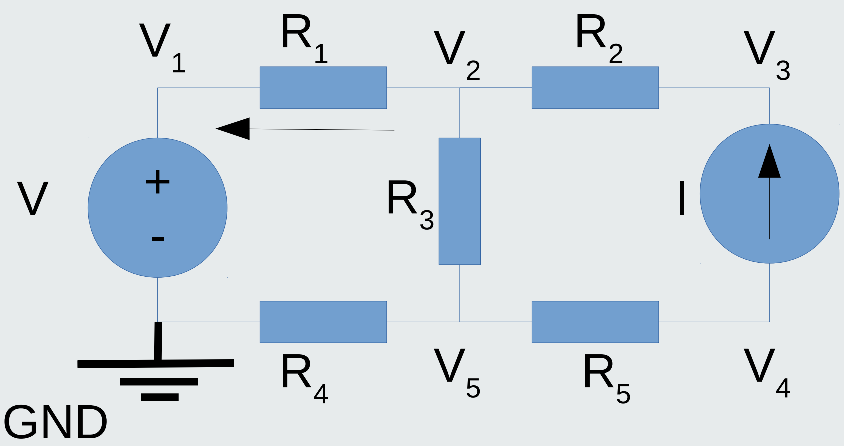
$$V_2 = V - R_1 I_a = V - R_1 \frac{V - R_3 I}{R_1 + R_3 + R_4} = \frac{R_3 + R_4}{R_1 + R_3 + R_4} V + \frac{R_1 R_3}{R_1 + R_3 + R_4} I$$

Same result as before: check

Node Analysis

Nodes connect components in a circuit. Nodal equations comprise

- KCL in nodes *not connected to voltage sources*
- Additional equations for *nodes related by voltage sources*



$KCL\ equations:$

$$node\ 2 \rightarrow (V_2 - V_1)G_1 + (V_2 - V_5)G_3 + (V_2 - V_3)G_2 = 0$$

$$node\ 3 \rightarrow (V_3 - V_2)G_2 - I = 0$$

$$node\ 4 \rightarrow (V_4 - V_5)G_5 + I = 0$$

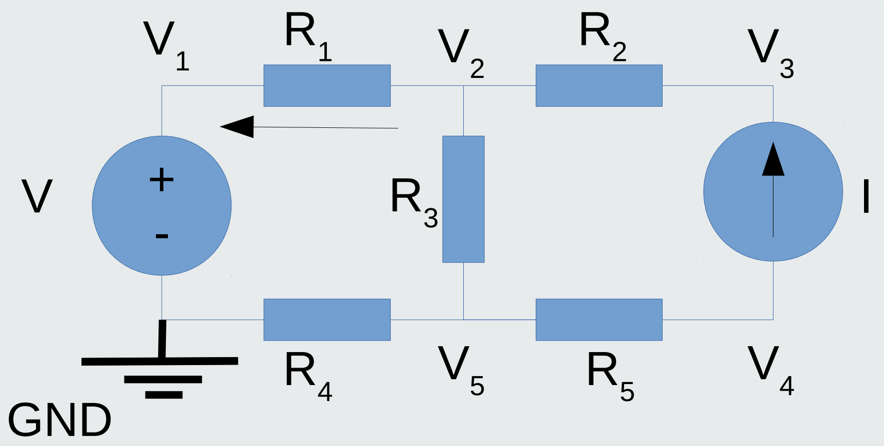
$$node\ 5 \rightarrow (V_5 - V_4)G_4 + (V_5 - V_2)G_3 + (V_5 - V_4)G5 = 0$$

$Additional\ equations:$

$$V_1 - V_0 = V$$

$$V_0 = 0$$

Node Analysis Matrix Form



- The nodal method is systematic
- Automatic extraction of nodal equations from circuit description
- Preferred method for circuit simulation
- Solve in Octave!

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -G_1 & G_1+G_3+G_3 & -G_2 & 0 & -G_3 \\ 0 & -G_2 & G_2 & 0 & 0 \\ 0 & 0 & 0 & G_5 & -G_5 \\ 0 & -G_3 & 0 & -G_5 & G_3+G_4+G_5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} V \\ 0 \\ I \\ -I \\ 0 \end{bmatrix}$$

Conclusion

- Analysis for circuits with resistors and independent I/V sources
- Superposition Theorem
- Thévenin and Norton Theorems
- Dependent or controlled sources
- Mesh analysis method
- Nodal Analysis method