Instituto Superior Técnico

Sistemas de Processamento de Sinal (SPDSina)

Analog phase locked loop (PLL) analysis

Consider an analog PLL with a multiplier-type phase detector, a first order lowpass filter and a sinusoidal VCO with the following characteristics:

filter: $\begin{cases} F(0) = -2 \\ \omega_p = 2\pi \times 1000 \text{ rad/s} \end{cases}$	oscillator: $\begin{cases} \omega_o(t) = 2\pi \text{ Mrad/s}, \ \ v_E(t) \leq -3V \\ \omega_o(t) = 10\pi \text{ Mrad/s}, \ \ v_E(t) \geq 5V \end{cases}$

a) Sketch the PLL clock diagram and propose a circuit to implement the loop filter using an operational amplifier with ± 5V supply and saturation voltages.

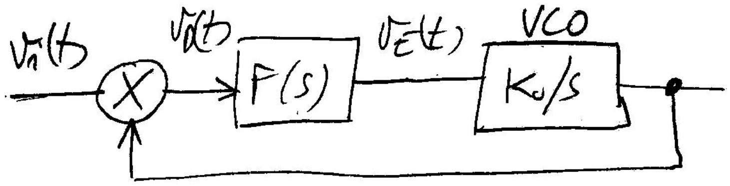
b) Consider $v_i(t) = V_i \sin(\omega_i t + \phi_i)$ and $v_o(t) = V_o \cos(\omega_o t + \phi_o)$ with $V_i = V_o = 2V$. Determine and sketch the static characteristics of all loop components and the hold range. Determine an approximation for the lock range.

c) Consider that the loop is locked with a sinusoidal input signal with frequency $\omega_i = 4\pi$ Mrad/s. Determine the phase difference between the VCO signal and the input signal.

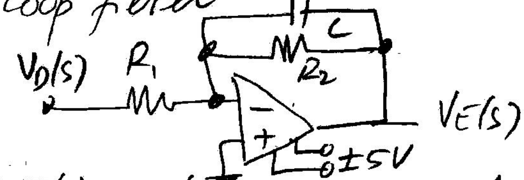
d) Consider the linearization of the loop around the previous frequency $\omega_i = 4\pi$ Mrad/s. Sketch the loop signal flow diagram and determine the system transfer function. Compute the frequency and quality factor of the poles.

e) Redesign the loop in order to operate as a phase modulator with a Butterworth characteristic for lowpass signals with bandwidth 10 kHz @ -3dB.

a)



Loop filter



$$F(s) = \frac{V_E(s)}{V_D(s)} = -\frac{R_2}{R_1}\,\frac{1}{1+\frac{s}{\omega_P}}$$

$$F(0) = -\frac{R_2}{R_1} = -2$$

$$\omega_P = \frac{1}{R_2 C} = 2\pi \times 1000 \text{ rad/s}$$

b)

$$\langle v_D(t)\rangle = \langle V_i V_0 \sin(\omega_i t + \phi_i)\cos(\omega_0 t + \phi_0)\rangle$$

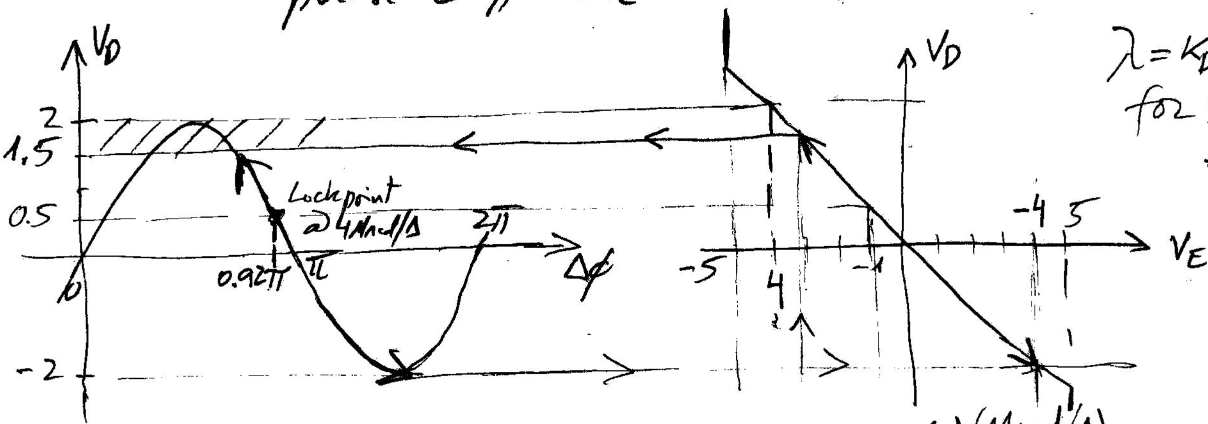
$$= \frac{V_i V_0}{2}\langle \underbrace{\sin[(\omega_i+\omega_0)t + \phi_i + \phi_0]}_{\text{eliminated by } F(s)} + \sin[(\omega_i-\omega_0)t + \underbrace{\phi_i - \phi_0}_{\Delta\phi}]\rangle$$

$$= \frac{V_i V_0}{2}\langle \sin[\underbrace{(\omega_i-\omega_0)t + \Delta\phi}_{\text{instantaneous, total phase difference}}]\rangle$$

When locked, $\omega_i \approx \omega_0$

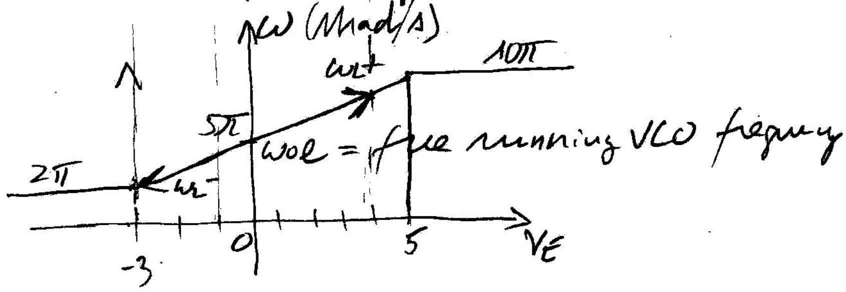
$$\langle v_D(t)\rangle = \frac{V_i V_0}{2}\sin\Delta\phi = 2\sin\Delta\phi$$

$$\lambda = \underset{<0}{K_D}\, \underset{>0}{F(0)}\, K_0 > 0$$ for stability $\Rightarrow K_D < 0$



$$\omega(t) = \omega_{0\ell} + K_0 V_E(t)$$

$$\begin{cases} 10\pi = \omega_{0\ell} + 5K_0 \\ 2\pi = \omega_{0\ell} - 3K_0 \end{cases} \rightarrow \begin{cases} K_0 = \pi \text{ Mrad/s/V} \\ \omega_{0\ell} = 5\pi \text{ Mrad/s} \end{cases}$$



Hold range:

Upper limit $= \omega_L^+ = 9\pi$ Mrad/s limited by the lower "saturation" of the phase detector.
Lower limit $= \omega_L^- = 2\pi$ Mrad/s limited by the lower saturation of the VCO

$$\Delta\omega_L = \omega_L^+ - \omega_L^- = 7\pi \text{ Mrad/s} \quad (3.5\text{ MHz})$$

Approximation for the lock range:

$$\Delta\omega_C \approx \sqrt{2\omega_p^2\left(\sqrt{1+\left(\frac{\Delta\omega_L}{\omega_p}\right)^2} - 1\right)} \approx \sqrt{2\omega_p \Delta\omega_L}$$

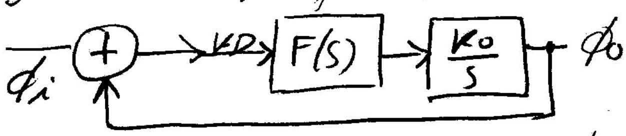
if $\Delta\omega_L \gg \omega_p$ which is true because $\Delta\omega_L = 7\pi$ Mrad/s and $\omega_p = 2\pi$ Krad/s. So $\Delta\omega_C \approx 2\pi \times \underbrace{83.67}_{\text{KHz}}$ Krad/s

c) Loop locked with $\omega_i = 4\pi$ Mrad/s so $V_E = -1V$, $V_D = \frac{-1}{-2} = 0.5V$. On the other hand $V_D = 2\sin\Delta\phi$ so $\Delta\phi = a\sin\left(\frac{V_D}{2}\right) = a\sin\frac{1}{4} = 0.92\pi$. ⚠
Careful with your calculator! Why?

d) Gain $K_D = \left.\frac{\partial\langle V_D(\Delta\phi)\rangle}{\partial\Delta\phi}\right|_{\Delta\phi = 0.92\pi} = 2\cos 0.92\pi = -1.935$ V/rad.

this is our "operating" point

System transfer function:



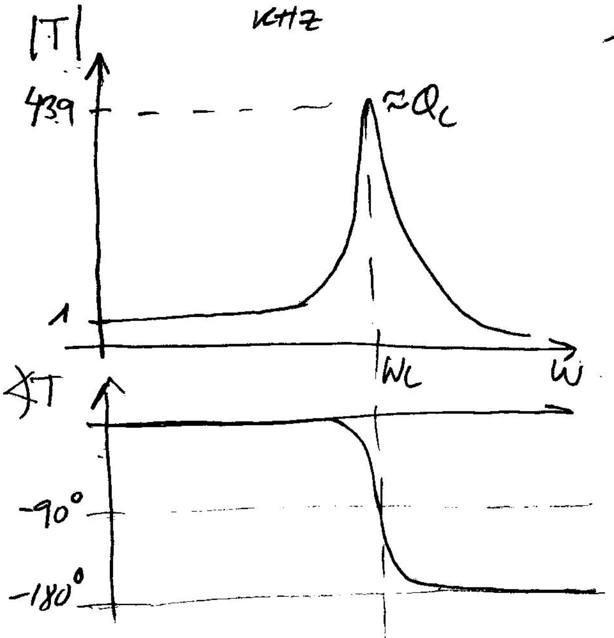
$$T(s) = \frac{\phi_o(s)}{\phi_i(s)} = \frac{K_D F(s) \frac{K_o}{s}}{1 + K_D F(s) \frac{K_o}{s}} = \frac{K_D \frac{F(0)}{1+s/\omega_p} \frac{K_o}{s}}{1 + K_D \frac{F(0)}{1+s/\omega_p} \frac{K_o}{s}} = \frac{\omega_p K_D K_o F(0)}{s^2 + \omega_p s + \omega_p K_D K_o F(0)}$$

$$T(s) = \frac{\omega_p \lambda}{s^2 + \underbrace{\omega_p}_{\frac{\omega_c}{Q_c}} s + \underbrace{\omega_p \lambda}_{\omega_c^2}}$$

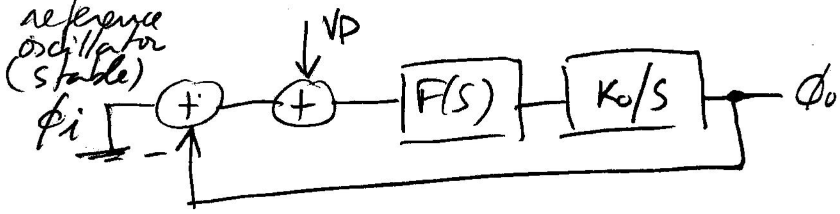
$$\begin{cases} Q_c = \frac{\omega_c}{\omega_p} = \sqrt{\frac{\lambda}{\omega_p}} \\ \omega_c = \sqrt{\omega_p \lambda} \end{cases}$$

$Q_c = 43.9$

$\omega_c = 2\pi \times \underbrace{43.9}_{kHz}$ Krad/s



e) Redesign to have a phase modulator with $\omega_c = 2\pi \times 10$ Krad/s and $Q_c = \frac{1}{\sqrt{2}}$ (Butterworth)



$$T_{PM}(s) = \frac{\phi_o(s)}{V_p(s)} = \frac{T(s)}{K_D} \rightarrow \text{lowpass}$$

$$= \frac{\omega_p \lambda / K_D}{s^2 + \frac{\omega_c}{Q_c} + \omega_c^2}$$

$$\frac{\omega_c}{Q_c} = \omega_p = \frac{2\pi \times 10 \text{ Krad/s}}{1/\sqrt{2}} = 2\pi \times \underbrace{14.1}_{kHz} \text{ Krad/s}$$

So, must increase filter bandwidth from 1kHz to 14.1kHz

$$\omega_c^2 = (2\pi \times 10^4)^2 = \omega_p \lambda \rightarrow \lambda = \frac{(2\pi \times 10^4)^2}{2\pi \times 14.1 \times 10^3} = 2\pi \times \frac{10^4}{\sqrt{2}}$$

Must change λ: choose either $F(0)$ or K_o, K_D is difficult to change.

Keep $F(0) \Rightarrow K_o = \frac{2\pi \times 10^4}{\sqrt{2} F(0) K_D} = \frac{2\pi \times 10^4}{\sqrt{2}(-2)(-1.935)}$

$= \pi \cdot 3654.2$ rad/s/V (much lower)

e) (continuation)

$\Delta\omega_L = K_o(V_{Emax} - V_{Emin})$ because ω_{ol} is the same.

$$= K_o(\underset{\underset{PD}{\uparrow}}{4} - (\underset{\underset{VCO}{\uparrow}}{-3})) = 7K_o = 2\pi \times \underbrace{12.78}_{kHz} \text{ Krad/s}$$

$$\Delta\omega_c \cong \sqrt{2\omega_p^2\left(\sqrt{1+\left(\frac{\Delta\omega_L}{\omega_p}\right)^2}-1\right)}$$

$= 2\pi \times \underbrace{11.79}_{kHz}$ Krad/s

Smaller than $\Delta\omega_L$ but much closer

Note: Now, $\Delta\omega_L = 2\pi \times 12.78$ Krad/s and $\omega_p = 2\pi \times 14.1$ krad/s so $\omega_p > \Delta\omega_L$ and the approximation used in b) for $\Delta\omega_c$ is not valid because $\Delta\omega_L \gg \omega_p$ is not true.