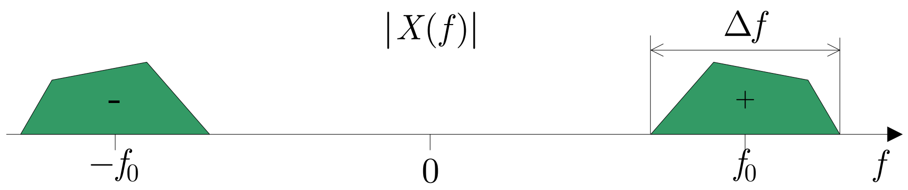
Instituto Superior Técnico

Sistemas de Processamento Digital de Sinais

Digital Signal Processing Systems

Problem: Passband signal sampling

Consider a real passband signal $x(t)$ generated by a GSM mobile phone which occupies a bandwidth of approximately $\Delta f = 300$ kHz and transmits with a carrier frequency $f_0 = 900$ MHz. **Note:** since $x(t)$ is real, its spectrum $X(f)$ has conjugate symmetry that is, $X(f) = X^*(-f)$.



a) Consider ideal impulse sampling of this signal with no loss of information. Determine the possible intervals for the sampling frequency. What is the interval for which the sampling frequency is minimum?

b) Explain why this interval (minimum sampling frequency) cannot be used in practice.

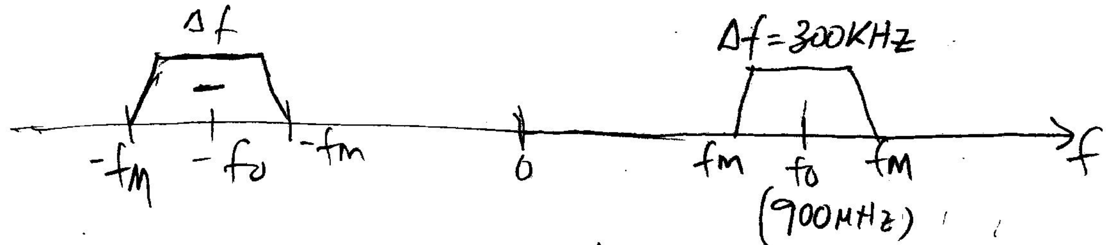
c) Determine the sampling frequency interval and the value of N which should be used if the sampling clock has a precision of ± 5 ppm (part per million).

d) Sketch the modulo of the sampled signal spectrum in the interval $|f| \leq 1$ MHz identifying the origin of each replica (positive or negative part of the original spectrum).

e) If the value of N is decreased by one what would happen relative to d)?

f) Determine the maximum RMS jitter of the sampling clock such that the SNR floor due to the clock jitter is at least 64 dB.

g) Show that due to the thermal noise from the sampling switch, folded at baseband, the output SNR increases by 3 dB for every doubling of the sampling frequency.



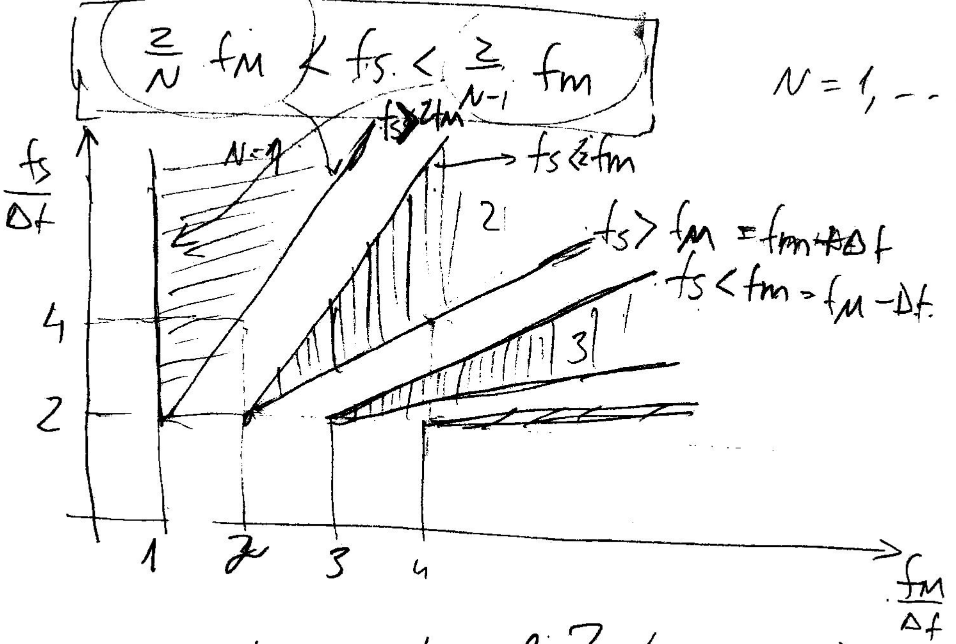
a) # intervals $= \left\lfloor \frac{f_M}{\Delta f} \right\rfloor$ because this is the number of replicas with no overlap in $[0, f_M]$ (with bandwidth Δf)

$$= \left\lfloor \frac{f_0 + \Delta f/2}{\Delta f} \right\rfloor = \left\lfloor \frac{f_0}{\Delta f} + \frac{1}{2} \right\rfloor = \left\lfloor \frac{900M}{300k} + \frac{1}{2} \right\rfloor = \lfloor 3000.5 \rfloor = 3000$$

So there are 3000 possible intervals for fs!

$$\boxed{\frac{2}{N} f_M < f_s < \frac{2}{N-1} f_m}$$

$N = 1, \ldots$ passband sampling theorem

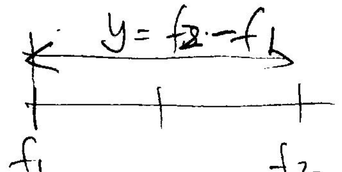


b) Cannot use this interval? (N=3000). Should use larger N ⇒ lower fs: however interval shrinks!

$N = 3000$, $f_M = 900M + 150k$, $f_m = 900M - 150k$

$$\underbrace{\frac{2}{3000}(900M + 150k)}_{f_1 = 600.1k} < f_s < \underbrace{\frac{2}{2999}(900M - 150k)}_{f_2 = 600.1000333444481k}$$

$f_2 - f_1 = 33.344\ldots$ mHz



$f_s = \frac{f_1+f_2}{2}$

Normalized tolerance $\varepsilon = \pm \frac{\frac{f_2 - f_1}{2}}{f_s} =$

$= \pm \frac{f_2 - f_1}{f_2 + f_1} =$

$\varepsilon = \pm 2.778 \times 10^{-8} = \pm 0.02778 \times 10^{-6} =$

$= \pm 0.02778$ ppm

→ Close to atomic clock! impossible!

c) $\varepsilon_{\text{desired}} = \pm 5$ ppm get N! $= \varepsilon_d$

$$|\varepsilon_{\text{real}}| = \frac{f_2 - f_1}{f_2 + f_1} \geq |\varepsilon_d| \rightarrow \frac{\frac{2}{N-1} f_m - \frac{2}{N} f_M}{\frac{2}{N-1} f_m + \frac{2}{N} f_M} = \frac{2N f_m - 2(N-1) f_M}{2N f_m + 2(N-1) f_M} \geq |\varepsilon_d|$$

$$2N f_m - 2N f_M + 2 f_M \geq (2N f_m + 2N f_M)|\varepsilon_d| - 2 f_M |\varepsilon_d|$$

$$N(2 f_m - 2 f_M - 2 f_m |\varepsilon_d| - 2 f_M |\varepsilon_d|) \geq -2 f_M - 2 f_M |\varepsilon_d|$$

$$N\left(f_m(1 - |\varepsilon_d|) - f_M(1 + |\varepsilon_d|)\right) \geq -f_M(1 + |\varepsilon_d|)$$

$$N \leq \frac{f_M(1+|\varepsilon_d|)}{-f_m(1-|\varepsilon_d|) + f_M(1+|\varepsilon_d|)} = \frac{1}{1 - \frac{f_m}{f_M}\left(\frac{1-|\varepsilon_d|}{1+|\varepsilon_d|}\right)} = 2913.12$$

$$\boxed{N \leq 2913}$$

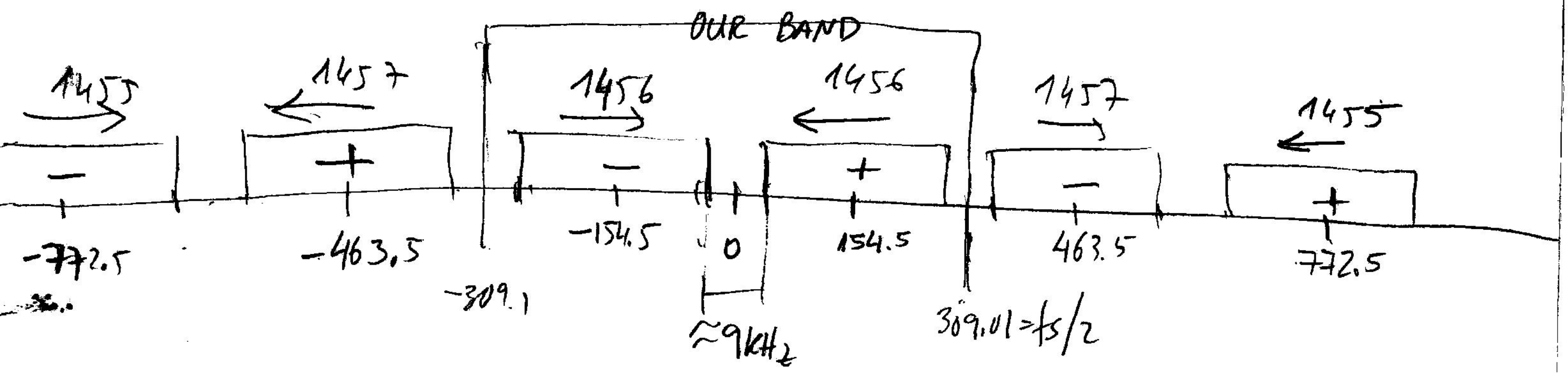
d) $N = 2913$, $\underbrace{\frac{2}{N} f_M}_{f_1 = 618.0226572} < f_s < \underbrace{\frac{2}{N-1} f_m}_{f_2 = 618.0288462\,kHz}$

$f_s = \frac{f_1 + f_2}{2} = 618.0257516\,kHz$, $\frac{f_2 - f_1}{f_2 + f_1} = 5.0071 \times 10^{-6} > 5 \times 10^{-6}$

$f_2 - f_1 = 6.189\,Hz$

$f_0 - i f_s \simeq 0 \Rightarrow i \approx \frac{f_0}{f_s} = 1456.25$, $i \approx 1456$

Positive replicas: $i = 1456$, $f' = f_0 - i f_s \simeq 154.5\,kHz$
(Equal for negative) $i = 1457$, $f' = \simeq -463.5$
1455 ≈ 772.5



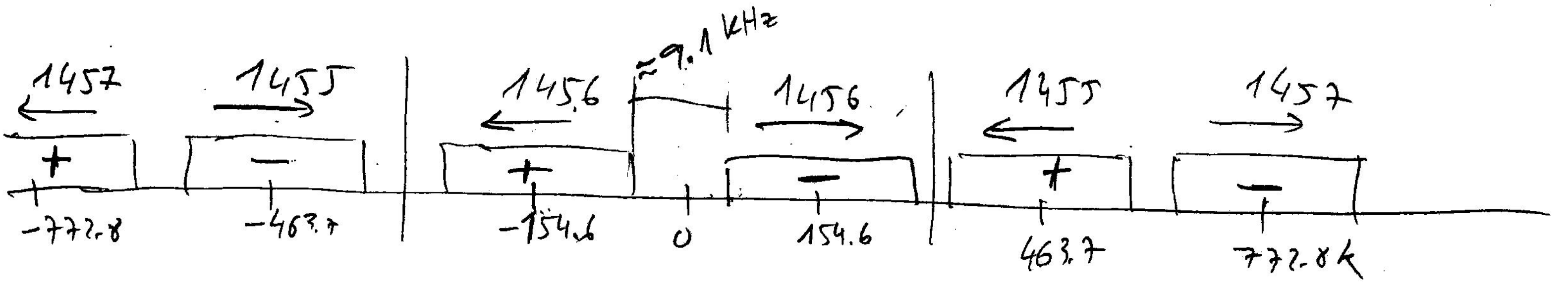
No spectrum inversion because N is odd

Suppose $N = 2912$ (even), then $f_S = 618.238\,kHz$ (slightly higher)

$i = \left\lfloor \frac{f_0}{f_S} \right\rfloor = 1455$, $f' = f_0 - i f_S = 463.7K$

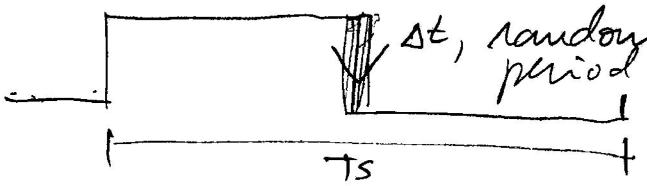
1456 $= -154.6k$

e) 1457 $= -772.8K$



Spectrum inversion because N is even.

f) SNR degradation is less than 64dB



$SNR_{jitter} \approx -16 - 20\log_{10}(f_0 \times \Delta t_{rms}) \geq 64\,dB$

$-20\log_{10}(f_0 \times \Delta t_{rms}) \geq 80$

$f_0 \times \Delta t_{rms} \leq 10^{-4}$, $\Delta t_{rms} \leq \frac{10^{-4}}{f_0} = \frac{10^{-4}}{900M} \approx 1.1\times 10^{-13} = 0.11\,ps$

Very difficult unless fs is high! What matters is the ratio $\frac{\Delta t_{rms}}{T_S} = 1.1\times10^{-13} \times \underbrace{618.238\times10^3}_{f_S} = 6.8\times10^{-8} \approx 7\times10^{-8}$

(10^{-7}) difficult!

Increase $f_S \Rightarrow$ increase $\frac{\Delta t_{rms}}{T_S} = \Delta t_{rms} f_S$

$\Downarrow$

Choose lower N

g) $SNR_{out} = SNR_{in} - 10\log_{10} N$. For every doubling of f_S, N decreases by 2, so SNR_{out} increases by $10\log_{10} 2 \approx 3dB$