



TÉCNICO
LISBOA

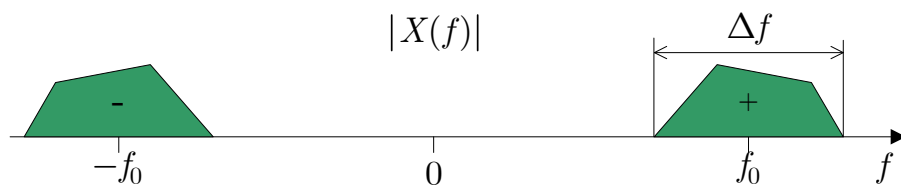
Instituto Superior Técnico

Sistemas Electrónicos de Processamento de Sinal

Problem: Passband signal sampling

Consider a real passband signal $x(t)$ generated by a 2G GSM mobile phone which occupies a bandwidth of approximately $\Delta f = 300$ kHz and transmits with a carrier frequency $f_0 = 900$ MHz.

Note: since $x(t)$ is real, its spectrum $X(f)$ has conjugate symmetry that is, $X(f) = X^*(-f)$.



- Consider ideal impulse sampling of this signal with no loss of information. Determine the possible intervals for the sampling frequency. What is the interval for which the sampling frequency is minimum?
- Explain why this interval (minimum sampling frequency) cannot be used in practice.
- Determine the sampling frequency interval and the value of N which should be used if the sampling clock has a precision of ± 5 ppm (part per million).
- Sketch the modulo of the sampled signal spectrum in the interval $|f| \leq 1$ MHz identifying the origin of each replica (positive or negative part of the original spectrum).
- If the value of N is decreased by one what would happen?
- Determine the maximum RMS jitter of the sampling clock such that the SNR due to the clock jitter is at least 64 dB.

c) $\epsilon = \pm 5 \text{ ppm}$ get $N! = \epsilon_d$

$$|\epsilon|_{\text{real}} = \frac{f_2 - f_1}{f_2 + f_1} \gg |\epsilon_d| \rightarrow \frac{\frac{2}{N-1} f_m - \frac{2}{N} f_m}{\frac{2}{N-1} f_m + \frac{2}{N} f_m} = \frac{2N(f_m - 2(N-1)f_m)}{2Nf_m + 2(N-1)f_m} \gg |\epsilon_d|$$

$$2Nf_m - 2Nf_m + 2f_m \gg (2Nf_m + 2Nf_m)|\epsilon_d| - 2f_m|\epsilon_d|$$

$$N(2f_m - 2f_m - 2f_m|\epsilon_d| - 2f_m|\epsilon_d|) \gg -2f_m - 2f_m|\epsilon_d|$$

$$N(f_m(1-|\epsilon_d|) - f_m(1+|\epsilon_d|)) \gg -f_m(1+|\epsilon_d|)$$

$$N \leq \frac{f_m(1+|\epsilon_d|)}{-f_m(1-|\epsilon_d|) + f_m(1+|\epsilon_d|)} = \frac{1}{1 - \frac{f_m}{f_m} \left(\frac{1-|\epsilon_d|}{1+|\epsilon_d|} \right)} = 2913.12$$

$$\boxed{N \leq 2913}$$

d) $N = 2913$, $\frac{2}{N} f_m < f_s < \frac{2}{N-1} f_m$

$$f_s = \frac{f_1 + f_2}{2} = 618.0257516 \text{ kHz}, \quad f_1 = 618.0226572 \text{ kHz}, \quad f_2 = 618.0288462 \text{ kHz}$$

$$f_2 - f_1 = 6.189 \text{ Hz}$$

$$\frac{f_2 - f_1}{f_2 + f_1} = 5.0071 \times 10^{-6} > 5 \times 10^{-6}$$

$$f_0 - i f_s \approx 0 \Rightarrow i \approx \frac{f_0}{f_s} = 1456.25, \quad i \approx 1456$$

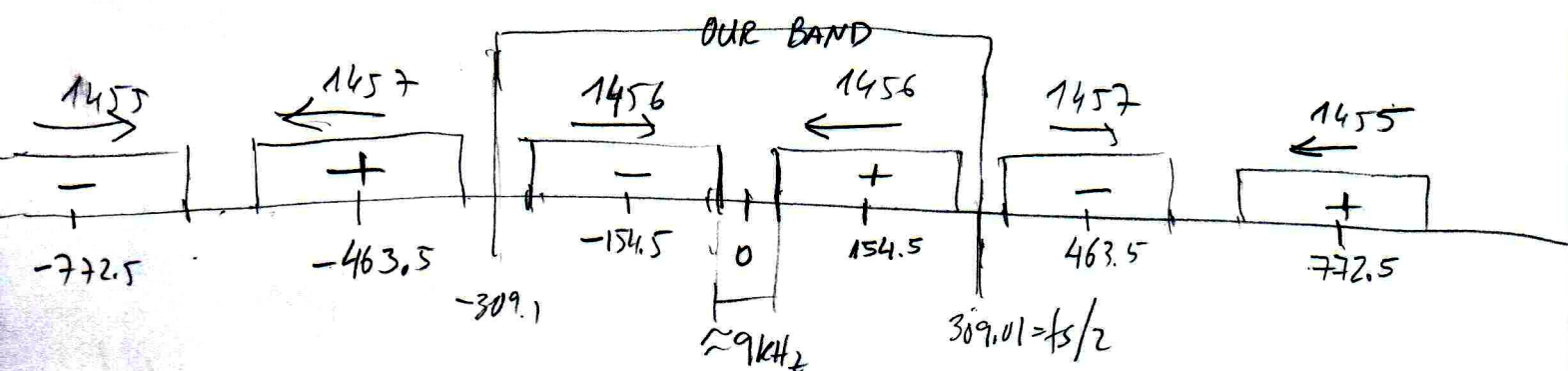
Replica positions:

$$i = 1456, \quad f' = f_0 - i f_s \approx 154.5 \text{ kHz}$$

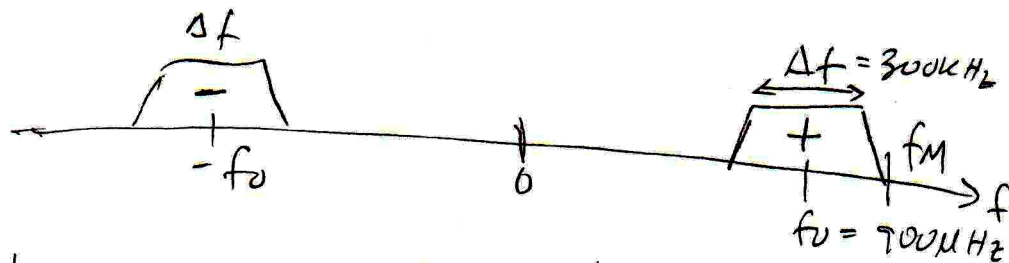
$$i = 1457, \quad f' = -463.5$$

$$1455$$

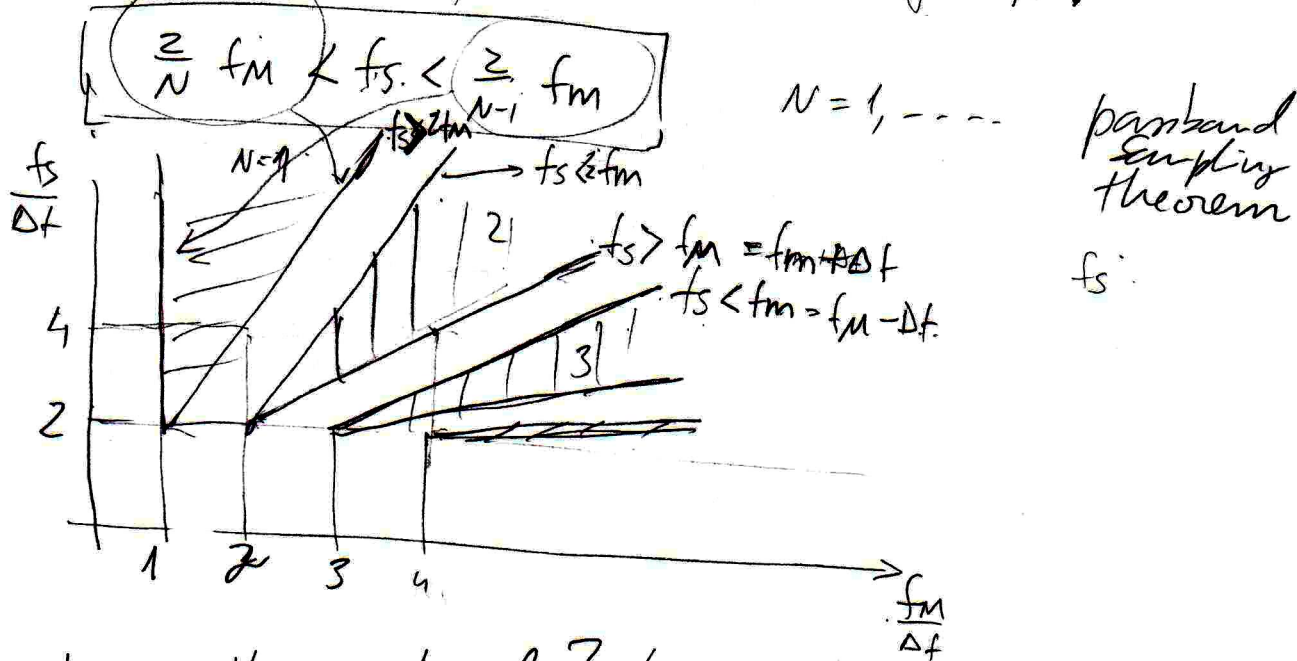
$$\approx 772.5$$



No spectrum inversion
because N is odd



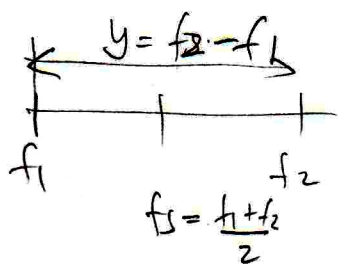
- a) # intervals = $\left\lfloor \frac{f_m}{\Delta f} \right\rfloor$ because this is the number of replicas that fit $[0, f_m]$ (with bandwidth Δf).
 $= \left\lfloor \frac{f_0 + \Delta f/2}{\Delta f} \right\rfloor = \left\lfloor \frac{f_0}{\Delta f} + \frac{1}{2} \right\rfloor = \left\lfloor \frac{900 \text{ M}}{300 \text{ k}} + \frac{1}{2} \right\rfloor = \left\lfloor 3000.5 \right\rfloor = 3000$
 So there are 3000 possible intervals for f_s !



- b) Can't use this interval? ($N=3000$). Should use layer $N \Rightarrow$ lower f_s ; however interval shrinks!
 $N=3000, f_m = 900 \text{ M} + 150 \text{ k}, f_{m'} = 900 \text{ M} - 150 \text{ k}$

$$\frac{2}{3000} (900 \text{ M} + 150 \text{ k}) < f_s < \frac{2}{2999} (900 \text{ M} - 150 \text{ k})$$

$$f_1 = 600.1 \text{ k} < f_s < f_2 = 600.1000333444481 \text{ k}$$



$$f_2 - f_1 = 33.344... \text{ MHz}$$

Normalized tolerance $\epsilon = \pm \frac{f_2 - f_1}{2 f_s} =$

$$= \pm \frac{f_2 - f_1}{f_2 + f_1}$$

$$\epsilon = \pm 2.778 \times 10^{-8} = \pm 0.02778 \times 10^{-6} =$$

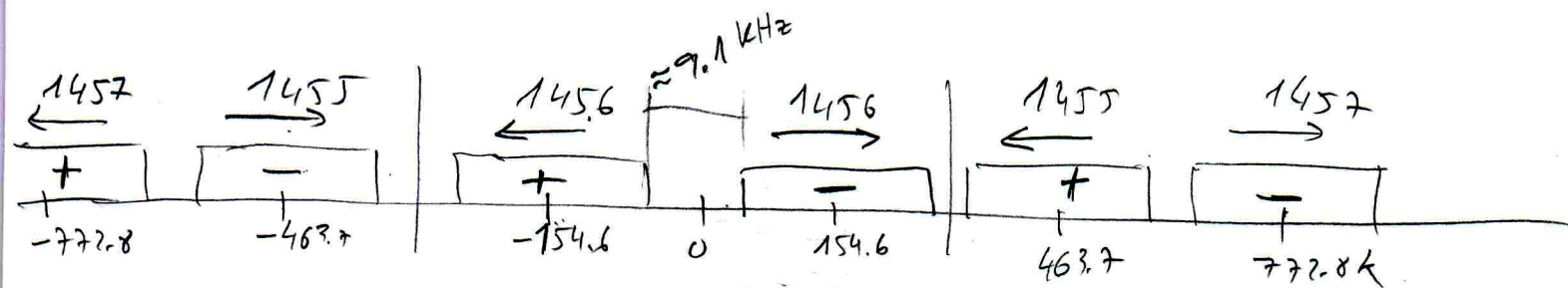
$$= \pm 0.02778 \text{ ppm}$$

\rightarrow atomic clock! impossible!

Suppose $N = 2912$ (even), then $f_s = 618.238 \text{ kHz}$ (slightly higher)⁻²⁻

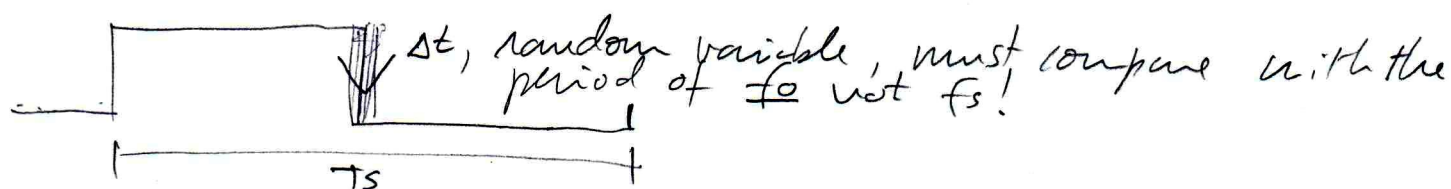
$$i = \left\lfloor \frac{f_0}{f_s} \right\rfloor = 1455, \quad f' = f_0 - i f_s = 463.7 \text{ K}$$

$$\begin{array}{rcl} & 1456 & = -154.6 \text{ K} \\ e) & 1457 & = -772.8 \text{ K} \end{array}$$



Spectrum inversion
because N is even.

f) SNR degradation is less than 40dB



$$\text{SNR}_{\text{jitter}} \approx -16 - 20 \log_{10}(f_0 \times \Delta t_{\text{rms}}) \geq 64 \text{ dB}$$

$$-20 \log_{10}(f_0 \times \Delta t_{\text{rms}}) \geq 80$$

$$f_0 \times \Delta t_{\text{rms}} \leq 10^{-4}, \quad \Delta t_{\text{rms}} \leq \frac{10^{-4}}{f_0} = \frac{10^{-4}}{900 \text{ M}} \approx 1.1 \times 10^{-13} = 1.1 \text{ ps}$$

Very difficult unless f_s is high! What matters is

the ratio $\frac{\Delta t_{\text{rms}}}{T_s} = 1.1 \times 10^{-13} \times \frac{618.238 \times 10^3}{f_s} = 6.8 \times 10^{-8} \approx 7 \times 10^{-8}$
(10^{-7})
difficult!

$$\text{Increase } f_s \Rightarrow \text{increase } \frac{\Delta t_{\text{rms}}}{T_s} = \Delta t_{\text{rms}} f_s$$



Choose lower N