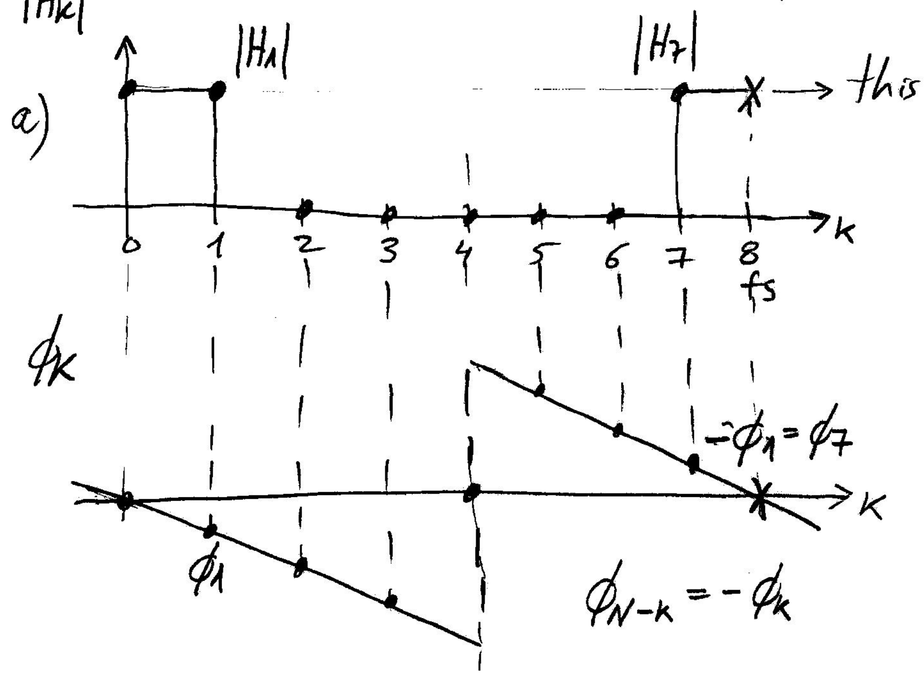
Frequency Sampling design: Lowpass FIR, $N=8$, $f_c = 1kHz$, $f_s = 8kHz$

a)



$$\tau_g = \frac{N-1}{2} T_S \rightarrow \phi(\omega) = -\tau_g \omega$$

$$\omega_k = \frac{2\pi}{N} k f_S$$

$$\phi(\omega_k) = -\tau_g \omega_k = -\frac{N-1}{2} T_S \frac{2\pi}{N} k f_S = -\pi \frac{N-1}{N} k = \phi_k$$

linear phase!

N even

$$e^{j\pi n} = (-1)^n$$

$$h_n = \frac{1}{N} \sum_{k=0}^{N-1} H_k e^{j\frac{2\pi}{N} nk} = \frac{1}{N}\Big[H_0 + H_{\frac{N}{2}} e^{j\frac{2\pi}{N} n \frac{N}{2}} + \sum_{k=1}^{\frac{N}{2}-1} \Big(H_k e^{j\frac{2\pi}{N} nk} + H_{N-k} e^{j\frac{2\pi}{N} n(N-k)} \Big)\Big]$$

$$= \frac{1}{N}\Big[H_0 + H_{\frac{N}{2}} (-1)^n + \sum_{k=1}^{\frac{N}{2}-1} \Big(|H_k| e^{j\phi_k} e^{j\frac{2\pi}{N} nk} + |H_{N-k}| e^{j\phi_{N-k}} e^{j\overbrace{\frac{2\pi}{N} n(N-k)}^{2\pi n - \frac{2\pi}{N} nk}} \Big)\Big]$$

$$= \frac{1}{N}\Big[|H_0| + |H_{\frac{N}{2}}| (-1)^n + \sum_{k=1}^{\frac{N}{2}-1} \Big(|H_k| e^{j\phi_k} e^{j\frac{2\pi}{N} nk} + |H_k| e^{-j\phi_k} e^{-j\frac{2\pi}{N} nk} \Big)\Big]$$

$$= \frac{1}{N}\Big[|H_0| + |H_{\frac{N}{2}}| (-1)^n + \sum_{k=1}^{\frac{N}{2}-1} |H_k| \Big(e^{j\phi_k} e^{j\frac{2\pi}{N} nk} + e^{-j\phi_k} e^{-j\frac{2\pi}{N} nk} \Big)\Big]$$

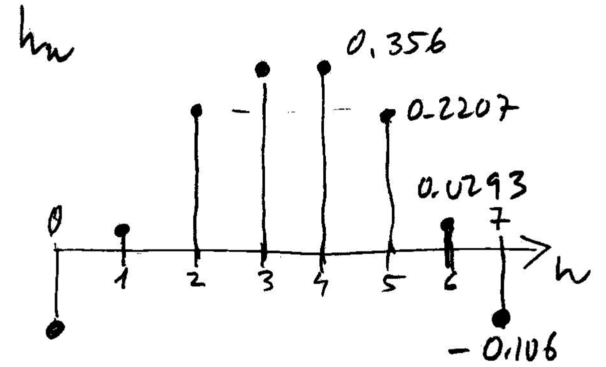
$$= \frac{1}{N}\Big[|H_0| + |H_{\frac{N}{2}}| (-1)^n + 2 \sum_{k=1}^{\frac{N}{2}-1} |H_k| \cos\Big(\phi_k + \frac{2\pi}{N} nk \Big)\Big] \quad \text{real!}$$

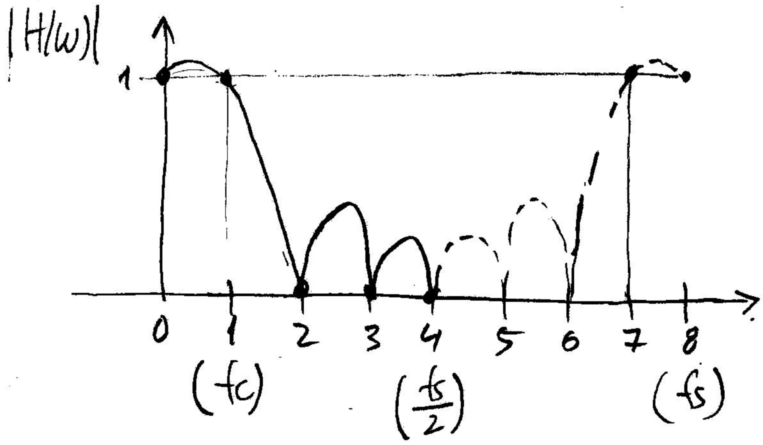
$$= \frac{1}{8}\Big[1 + 2\cos\Big(-\frac{7\pi}{8} + \frac{2\pi}{8} n \Big)\Big], \quad n = 0, \dots, 7$$

N odd (check...)

$$h_n = \frac{1}{N}\Big[|H_0| + 2 \sum_{k=1}^{\frac{N-1}{2}} |H_k| \cos\Big(\phi_k + \frac{2\pi}{N} nk \Big)\Big]$$

b)





Low-pass filter $\Rightarrow$ even symmetry expected

c) Q format, 16 bit fixed point

coefficients: $|h_n| < 1$ for all n $\Rightarrow Q_{15}$

Output:

$$y_n = \sum_{i=0}^{N-1} h_i x_{n-i} \rightarrow |y_n| = \left| \sum_{i=0}^{N-1} h_i x_{n-i} \right| \leq$$

$$\leq \sum_{i=0}^{N-1} |h_i| |x_{n-i}|$$

$$\leq |x_{max}| \underbrace{\sum_{i=0}^{N-1} |h_i|}_{1.42}$$

If samples x_n are in Q_m, then y_n must be stored in Q_{m-1}

d) Use one transition sample 1/2