Instituto Superior Técnico

Sistemas de Processamento Digital de Sinais

Digital Signal Processing Systems

IIR filter design

Design a low-pass IIR filter with unity DC gain and with two real poles at frequencies $f_{p_1} = 1$ kHz and $f_{p_2} = 2$ kHz using the impulse invariance method, the matched Z transform and the bilinear transformation method with pole frequency pre-distortion at the pole frequencies.

Note: Consider the sampling frequency to be $f_s = 8$ kHz.

1. <u>Impulse invariance method</u>

$$H_A(s) = \frac{\omega_{p1}\omega_{p2}}{(s+\omega_{p1})(s+\omega_{p2})} = \underbrace{\frac{\omega_{p1}\omega_{p2}}{\omega_{p2}-\omega_{p1}}}_{K} \left(\frac{1}{s+\omega_{p1}} - \frac{1}{s+\omega_{p2}} \right)$$

a) $h(t) = \mathcal{L}^{-1}\{H_A(s)\} = K\left(e^{-\omega_{p1}t} - e^{-\omega_{p2}t}\right)u(t)$

b) $h_n = h(nT_s) \times \underbrace{T_s}_{\text{keep energy!}} = K\left(e^{-\omega_{p1}T_s n} - e^{-\omega_{p2}T_s n}\right)u(n)$

c) $H(z) = Z\{h_n\} = \sum_{n=\infty}^{\infty} h_n z^{-n} = K \sum_{n=0}^{\infty} \left(e^{-\omega_{p1}T_s n} - e^{-\omega_{p2}T_s n}\right) z^{-n}$

$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, |a|<1$

$$= K\left(\frac{1}{1-e^{-\omega_{p1}T_s}z^{-1}} - \frac{1}{1-e^{-\omega_{p2}T_s}z^{-1}}\right) =$$

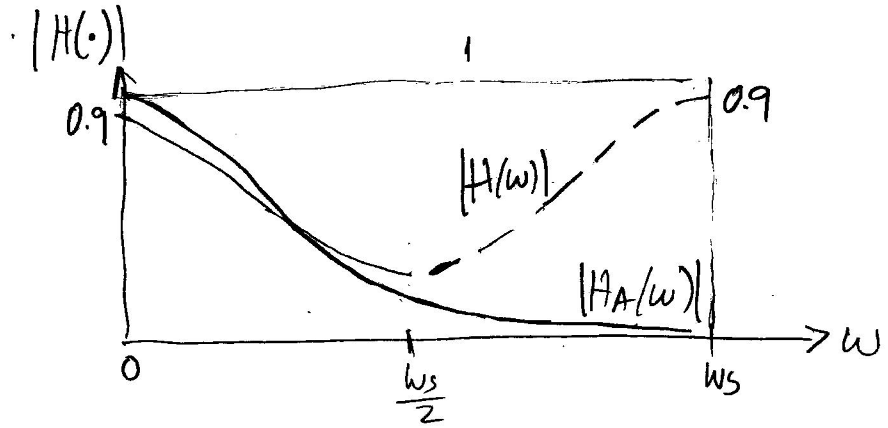
$$= K \frac{\left(-e^{-\omega_{p2}T_s} + e^{-\omega_{p1}T_s}\right) z^{-1}}{1-\left(e^{-\omega_{p1}T_s} + e^{-\omega_{p2}T_s}\right)z^{-1} + e^{-(\omega_{p1}+\omega_{p2})T_s} z^{-2}}$$

$$= 0.3891 \frac{b_0 \; z^{-1}}{1-\underbrace{0.6638}_{a_1} z^{-1} + \underbrace{0.09478}_{-a_2} z^{-2}}$$

usuallly is not implemented why?

$y_n = a_1 y_{n-1} + a_2 y_{n-2} + b_0 x_n$

$H(0) = H(z=1) \cong 0.9 < 1$



- Poor match between frequency responses
- But the target was to have a similar <u>impulse response</u>

2. Matched Z transform

$$H_A(s) = \frac{\omega_{p1}\omega_{p2}}{(s+\omega_{p1})(s+\omega_{p2})} \longrightarrow H(z) = \frac{\omega_{p1}\omega_{p2} T_s}{\left(1-e^{-\omega_{p1}T_s}z^{-1}\right)\left(1-e^{-\omega_{p2}T_s}z^{-1}\right)}$$

keep energy

$$\boxed{s+a \rightarrow 1-e^{-aT_s}z^{-1}}$$

$$= \frac{\overbrace{\omega_{p1}\omega_{p2}T_s}^{b_0}}{1-\underbrace{\left(e^{-\omega_{p1}T_s}+e^{-\omega_{p2}T_s}\right)}_{a_1=0.6638}z^{-1}+\underbrace{e^{-(\omega_{p1}+\omega_{p2})T_s}}_{-a_2=0.09478}z^{-2}}$$

$\Longrightarrow$ same poles of the previous method!

3. Bilinear transformation

$$s = \frac{2}{T_s}\frac{z-1}{z+1}$$

$$H(z) = H_A(s)\Big|_{s=\frac{2}{T_s}\frac{z-1}{z+1}} = \omega_{p1}\omega_{p2}\frac{1}{\left(\frac{2}{T_s}\frac{z-1}{z+1}+\omega_{p1}\right)\left(\frac{2}{T_s}\frac{z-1}{z+1}+\omega_{p2}\right)}$$

lots of work!

$$= K\cdot\frac{1+2z^{-1}+z^{-2}}{1-a_1z^{-1}-a_2z^{-2}}$$

$= (1+z^{-1})^2 \equiv$ double zero at $\omega = \frac{\omega_s}{2}$

$b_0 = b_2 = K, \quad b_1 = 2K$

Use predistortion at the pole frequencies:

$$s = \frac{2}{T_s}\frac{z-1}{z+1} \longrightarrow \omega_a = \frac{2}{T_s}\,tg\left(\frac{\omega_d T_s}{2}\right) > \omega_d$$

$$\left.\begin{aligned} &\underbrace{\begin{aligned}\omega_{d1} &= 2\pi\times 1\,kHz\\ \omega_{d2} &= 2\pi\times 2\,kHz\end{aligned}}_{\text{desired}} \longrightarrow \begin{aligned}\omega_{a1} &= 2\pi\times 1.0547\ kHz\\ \omega_{a2} &= 2\pi\times 2.5464\ kHz\end{aligned}\end{aligned}\right\}$$

use these predistorted frequencies in the $H_A(s)$ prototype ω_{p1}, ω_{p2}