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Sistemas de Processamento Digital de Sinais

Digital Signal Processing Systems

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FIR filter design using the non-recursive frequency sampling method

a) Design a low-pass ideal FIR filter with DC gain equal to one, cut-off frequency $f_c = 1$ kHz, with $N = 8$ coefficients and linear phase using the frequency sampling method (non-recursive). Solve for a general value of N even and then generalize for any N odd.

b) Sketch approximately the frequency response of the filter and comment.

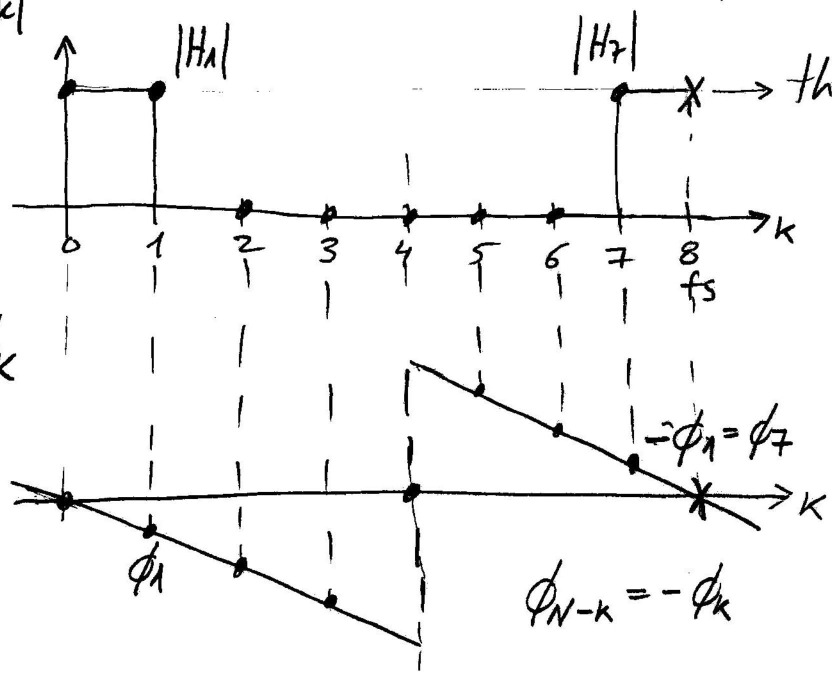
c) Determine the most accurate Q format to implement any FIR filter using fixed point arithmetic. Assume coefficients are all in the same Q format.

d) Design the same filter but using one transition sample with value ½. Sketch the frequency response of the new filter and compare with the result obtained in b).

Note: Consider the sampling frequency to be $f_s = 8$ kHz.

Frequency Sampling design: Lowpass FIR, $N=8$, $f_c = 1kHz$, $f_s = 8kHz$

a)



→ this sample dos not belong!

$$\tau_g = \frac{N-1}{2} T_s \rightarrow \phi(\omega) = -\tau_g \omega$$

$$\omega_k = \frac{2\pi}{N} k f_s$$

$$\phi(\omega_k) = -\tau_g \omega_k = -\frac{N-1}{2} T_s \frac{2\pi}{N} k f_s = -\pi \frac{N-1}{N} k = \phi_k$$

linear phase!

N even

$$e^{j\pi n} = (-1)^n$$

$$h_n = \frac{1}{N} \sum_{k=0}^{N-1} H_k e^{j\frac{2\pi}{N} nk} = \frac{1}{N}\left[H_0 + H_{\frac{N}{2}} e^{j\frac{2\pi}{N} n \frac{N}{2}} + \sum_{k=1}^{\frac{N}{2}-1}\left(H_k e^{j\frac{2\pi}{N} nk} + H_{N-k} e^{j\frac{2\pi}{N} n(N-k)}\right)\right]$$

$$= \frac{1}{N}\left[H_0 + H_{\frac{N}{2}}(-1)^n + \sum_{k=1}^{\frac{N}{2}-1}\left(|H_k| e^{j\phi_k} e^{j\frac{2\pi}{N} nk} + |H_{N-k}| e^{j\phi_{N-k}} e^{\overbrace{j\frac{2\pi}{N} n(N-k)}^{2\pi n - \frac{2\pi}{N} nk}}\right)\right]$$

$$= \frac{1}{N}\left[|H_0| + |H_{\frac{N}{2}}|(-1)^n + \sum_{k=1}^{\frac{N}{2}-1}\left(|H_k| e^{j\phi_k} e^{j\frac{2\pi}{N} nk} + |H_k| e^{-j\phi_k} e^{-j\frac{2\pi}{N} nk}\right)\right]$$

$$= \frac{1}{N}\left[|H_0| + |H_{\frac{N}{2}}|(-1)^n + \sum_{k=1}^{\frac{N}{2}-1} |H_k| \left(e^{j\phi_k} e^{j\frac{2\pi}{N} nk} + e^{-j\phi_k} e^{-j\frac{2\pi}{N} nk}\right)\right]$$

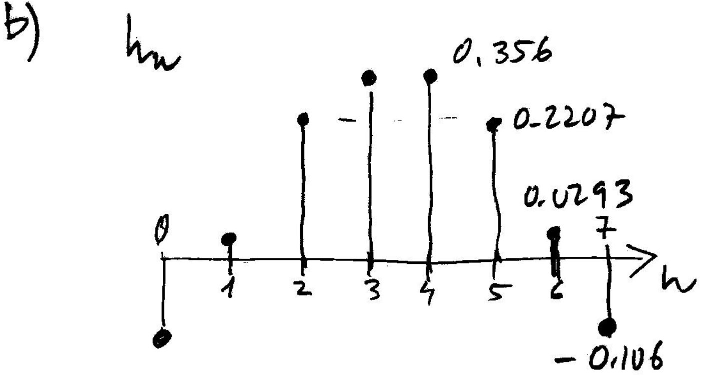
$$= \frac{1}{N}\left[|H_0| + |H_{\frac{N}{2}}|(-1)^n + 2\sum_{k=1}^{\frac{N}{2}-1} |H_k| \cos\left(\phi_k + \frac{2\pi}{N} nk\right)\right]$$ real!

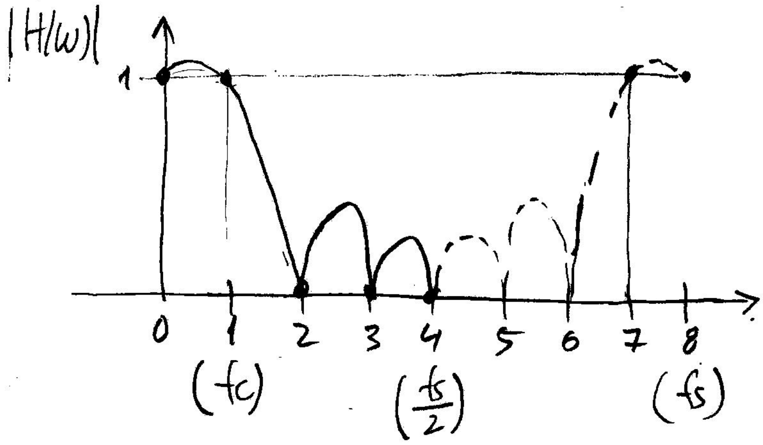
$$= \frac{1}{8}\left[1 + 2\cos\left(-\frac{7\pi}{8} + \frac{2\pi}{8} n\right)\right], \quad n = 0, \dots, 7$$

N odd (check...)

$$h_n = \frac{1}{N}\left[|H_0| + 2\sum_{k=1}^{\frac{N-1}{2}} |H_k| \cos\left(\phi_k + \frac{2\pi}{N} nk\right)\right]$$

b)





Low-pass filter ⇒ even symmetry expected

c) Q format, 16 bit fixed point

coefficients: $|h_n| < 1$ for all n ⇒ Q_{15}

Output:

$$y_n = \sum_{i=0}^{N-1} h_i x_{n-i} \rightarrow |y_n| = \left| \sum_{i=0}^{N-1} h_i x_{n-i} \right| \leq$$

$$\leq \sum_{i=0}^{N-1} |h_i| |x_{n-i}|$$

$$\leq |x_{max}| \underbrace{\sum_{i=0}^{N-1} |h_i|}_{1.42}$$

If samples x_n are in Q_m, then y_n must be stored in Q_{m-1}

d) Use one transition sample 1/2