TÉCNICO LISBOA

Sistemas de Processamento Digital de Sinais (SPDSina)

IIR Digital Filter

Consider a digital signal processing system operating with $f_s = 1/T_s = 10$ kHz and the design of an IIR digital high-pass filter using the bilinear transformation applied to the first order analog filter $H_{\text{HP}}(s) = \dfrac{s}{s + \omega_c}$ with cut-off frequency ω_c.

a) Design the filter determining its transfer function $H(z)$ so that the cut-off frequency of the **digital** filter is exactly 4π krad/s (2 kHz). Write the difference equation and sketch the signal flow diagram of the filter using direct form I and direct form II.

b) Determine the gain of the digital filter for $f = 0$, $f = f_c$ and $f = f_s / 2$.

c) Sketch qualitatively the magnitude of the frequency response of the analog and the digital filters for $0 \leq f \leq f_s$ (use the values from the previous question). Explain the differences between the two frequency responses.

a) $f_s = 10\,kHz$, $f_c = 2\,kHz$, Bilinear, IIR

Pré-distorção $\omega_a = \frac{2}{T_s} tg\frac{\omega_d T_s}{2} = 2f_s tg(\pi \frac{f_c}{f_s}) = 2\pi \times \underbrace{2312.56}_{2.3125\,kHz} \; rad/s$ $\Rightarrow f_d = f_c$

Filtro passa-alto, $K = 1 = H(0)$

$$H(z) = \frac{s}{s+\omega_a}\Big|_{s=\frac{2}{T_s}\frac{1-z^{-1}}{1+z^{-1}}} = \frac{\frac{2}{T_s}\frac{z-1}{z+1}}{\frac{2}{T_s}\frac{z-1}{z+1} + \omega_a} = \frac{z-1}{z-1+\frac{T_s}{2}\omega_a(z+1)}$$

$$= \frac{z-1}{z(1+\frac{T_s}{2}\omega_a)+(1-\frac{T_s}{2}\omega_a)} = \frac{1}{1+\frac{T_s}{2}\omega_a}\,\frac{z-1}{z-\frac{1-\frac{T_s}{2}\omega_a}{1+\frac{T_s}{2}\omega_a}}$$

$$= \frac{1}{1+\frac{T_s}{2}\omega_a}\,\frac{1-z^{-1}}{1-\frac{1-\frac{T_s}{2}\omega_a}{1+\frac{T_s}{2}\omega_a}z^{-1}} = 0.5792\,\frac{1-z^{-1}}{1-0.15838z^{-1}}$$

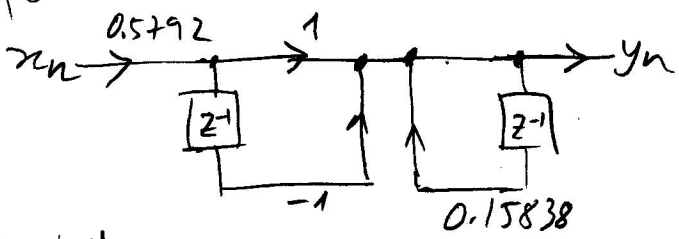
b) $= |H_a(0)|$

$f=0$, $z=1$, $|H(1)|=0$, $f=f_c \rightarrow |H(e^{j2\pi f_c T_s})| = \frac{1}{\sqrt{2}}$ pois devido à pré-distorção corresponde à freq. de corte do filtro passa-baixo.

$f=\frac{f_s}{2}$, $z=-1$, $|H(-1)| = |H_a(\infty)| = 1$

c) $y_n = 0.15838\,y_{n-1} + 0.5792(x_n - x_{n-1})$

forma directa I



forma directa II