



Digital Signal Processing Systems

Problems about Phase Locked Loops (PLLs)

I — Check only one answer. The penalty for a wrong answer is $\frac{1}{4}$ point. The minimum score in this group is zero.

1) An analog multiplier used as a phase detector:

- ☐ can have positive or negative gain.
- ☐ allows a phase excursion of 2π radians.
- ☐ only works with square wave signals.
- ☐ does not generate undesirable signal components at the output.

2) In an analog PLL in which the loop filter has a pole at the origin, the hold range:

- ☐ is always limited by the VCO.
- ☐ does not depend on the saturation limits of the VCO.
- ☐ is not limited by the phase detector.
- ☐ does not depend on the characteristics of the loop filter.

3) An analog PLL (with positive return gain):

- ☐ has a lock range which may be higher than the hold range.
- ☐ is always stable if its order is less or equal to 2.
- ☐ is always unstable if its order is greater or equal to 3.
- ☐ has a lock range which is always less than the hold range.

4) A XOR phase detector:

- ☐ has a linear characteristic when operation is with sinewaves.
- ☐ allow operation with a $3\pi/2$ rad phase excursion.
- ☐ can not operate with rectangular waves.
- ☐ does not generate undesirable signal components at the output.

5) An analog phase locked loop (PLL) in which the loop filter has a pole at a finite frequency:

- ☐ responds with a nonzero but finite phase error to a phase step.
- ☐ responds with a finite nonzero phase error to a frequency step.
- ☐ responds with a finite nonzero phase error to a frequency ramp.
- ☐ responds with an infinite phase error to a phase ramp.

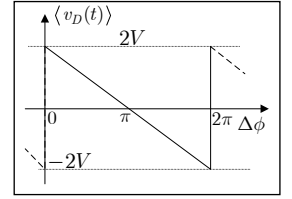
6) An analog PLL with return gain $\lambda > 0$:

- ☐ is stable only if it is of first order.
- ☐ is stable regardless of the order.
- ☐ if it is of third order it cannot be made stable.
- ☐ can be designed to be stable no matter the order.

7) A type D *flip-flop* phase detector:

- ☐ has a linear, periodic, phase characteristic.
- ☐ allows a maximum phase excursion of π radians.
- ☐ only works with square waves.
- ☐ has a phase characteristic with dead-zones which depend on the waveforms duty-cycle.

II — Consider an analog PLL in which the phase detector characteristic is shown in the box at right. The loop has a lowpass filter with $-6V$ and $+8V$ saturation voltages, DC gain $F(0)$ and a pole at f_p . The voltage-controlled oscillator (VCO) generates a wave with the characteristics in the box at right.



a) Determine the maximum filter gain $F(0)$ such that

1) the loop is stable and 2) the hold range is as large as possible.

b) Determine the static characteristics of all PLL

$$\omega_o = \begin{cases} \omega_{o1} = 10\pi \times 10^6 \text{ rad/s}, & v_E \leq -8V \\ \omega_{ol} + k_o v_E & -8V \leq v_E \leq 8V \\ \omega_{o2} = 2\pi \times 10^6 \text{ rad/s}, & v_E \geq 8V \end{cases}$$

components and sketch them interconnected. Determine k_o , ω_{ol} and the phase detector gain k_D .

If you did not answer a) consider $F(0) = 2$ from now on.

c) Determine the PLL hold range $\Delta\omega_L$ stating which components are responsible for its limitation.

Determine the filter pole frequency f_p such that the lock range is approximately $\Delta\omega_C \approx 0.8\Delta\omega_L$. Justify.

d) Determine the input frequency for which the phase shift between the input and VCO signals is $\Delta\phi = 3\pi/2$.

e) Explain how could the PLL be modified such that the phase shift between the input and VCO signals is always $\Delta\phi = 3\pi/2$, independently of the input frequency. With this modification would the hold range be different? If yes recompute it.

f) Determine the detection characteristic of a logic AND gate when operation is with rectangular waves both with duty-cycle $\delta = 0.25$ and amplitude levels 0V e 1V. Comment on its usefulness and shortcomings as a phase detector.

$$\Delta\omega_C = \sqrt{2\omega_p^2 \left(\sqrt{1 + \left(\frac{\Delta\omega_L}{\omega_p} \right)^2} - 1 \right)} \quad \Delta\omega_L \gg \omega_p \quad \approx \quad \sqrt{2\omega_p \cdot \Delta\omega_L}$$

Grades: **I** - 7 **II** - a) 2 b) 3 c) 3 d) 1 e) 2 f) 2

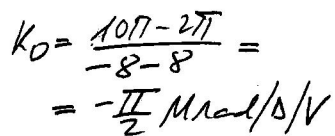
SEPS
SPDS

111

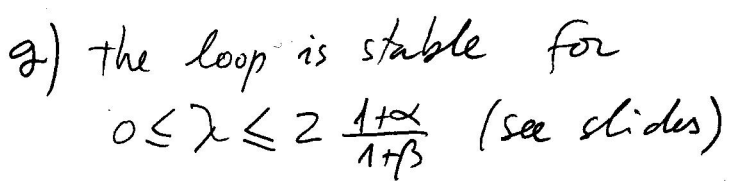
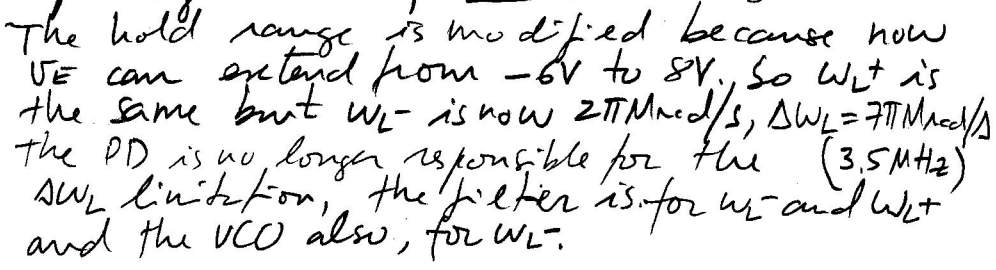
$$c) \Delta \omega_c \approx \sqrt{2\omega_p \Delta \omega_L} \rightarrow \omega_p = \frac{\Delta \omega_c^2}{2\Delta \omega_L} = \frac{0,8^2}{2} \Delta \omega_L = 0,32 \Delta \omega_L = \frac{\Delta \omega_L}{3,125}$$

$$\omega_p = \frac{\Delta \omega_L}{2} \frac{1}{\sqrt{\left(\frac{\Delta \omega_L}{\Delta \omega_C}\right)^2 - 1}} = \Delta \omega_C = 0.8 \Delta \omega_L$$

\Rightarrow approximation is not valid because $\Delta u_L \gg u_p$ is false

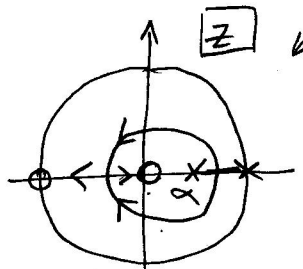


e) To have a constant phase shift $\Delta\phi$ between the input and the VCO a perfect integrator is required as loop filter so $F(s) = K \frac{1+s/\omega_z}{s}$. The zero at $-\omega_z$ is needed for stability. In this case the Sloop operates with $\langle V_D \rangle = 0$ so $\Delta\phi = \pi$. To have $\Delta\phi = 5\pi/2$ it is necessary to add 1V before the filter so $\langle V_D \rangle$ becomes -1V (the perfect integrator always operates with an input signal which, on average is zero).



So to have any $\lambda > 0$ $\frac{1+\alpha}{1+\beta}$ must be infinite, so $\beta = -1$ i.e., place a zero at $z = -1$. Works for any $0 < \alpha \leq 1$.

f) SPDS, AND gate, $\sigma = 0.25$



poles never leave
the unit circle

