



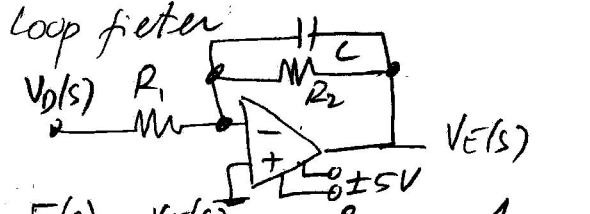
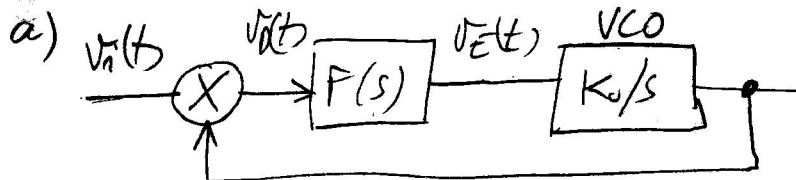
Sistemas de Processamento Digital de Sinais

Analog phase locked loop (PLL) analysis

Consider an analog PLL with a multiplier-type phase detector, a first order lowpass filter and a sinusoidal VCO with the following characteristics:

filter:	$\begin{cases} F(0) = -2 \\ \omega_p = 2\pi \times 1000 \text{ rad/s} \end{cases}$	oscillator:	$\begin{cases} \omega_o(t) = 2\pi \text{ Mrad/s}, & v_E(t) \leq -3V \\ \omega_o(t) = 10\pi \text{ Mrad/s}, & v_E(t) \geq 5V \end{cases}$
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- Sketch the PLL clock diagram and propose a circuit to implement the loop filter using an operational amplifier with $\pm 5V$ supply and saturation voltages.
- Consider $v_i(t) = V_i \sin(\omega_i t + \phi_i)$ and $v_o(t) = V_o \cos(\omega_o t + \phi_o)$ with $V_i = V_o = 2V$. Determine and sketch the static characteristics of all loop components and the hold range. Determine an approximation for the lock range.
- Consider that the loop is locked with a sinusoidal input signal with frequency $\omega_i = 4\pi \text{ Mrad/s}$. Determine the phase difference between the VCO signal and the input signal.
- Consider the linearization of the loop around the previous frequency $\omega_i = 4\pi \text{ Mrad/s}$. Sketch the loop signal flow diagram and determine the system transfer function. Compute the frequency and quality factor of the poles.
- Redesign the loop in order to operate as a phase modulator with a Butterworth characteristic for lowpass signals with bandwidth $10 \text{ kHz @ } -3\text{dB}$.



$$F(s) = \frac{v_e(s)}{v_D(s)} = -\frac{R_2}{R_1} \frac{1}{1 + \frac{s}{\omega_p}}$$

$$F(d) = -\frac{R_2}{R_1} = -2$$

$$\omega_p = \frac{1}{R_2 C} = 2\pi \times 1000 \text{ rad/s}$$

b)

$$\langle v_D(t) \rangle = \langle v_i v_o \sin(\omega_i t + \phi_i) \cos(\omega_o t + \phi_o) \rangle$$

$$= \frac{v_i v_o}{2} \langle \sin[(\omega_i + \omega_o)t + \phi_i + \phi_o] + \sin[(\omega_i - \omega_o)t + \phi_i - \phi_o] \rangle$$

eliminated by F(s)

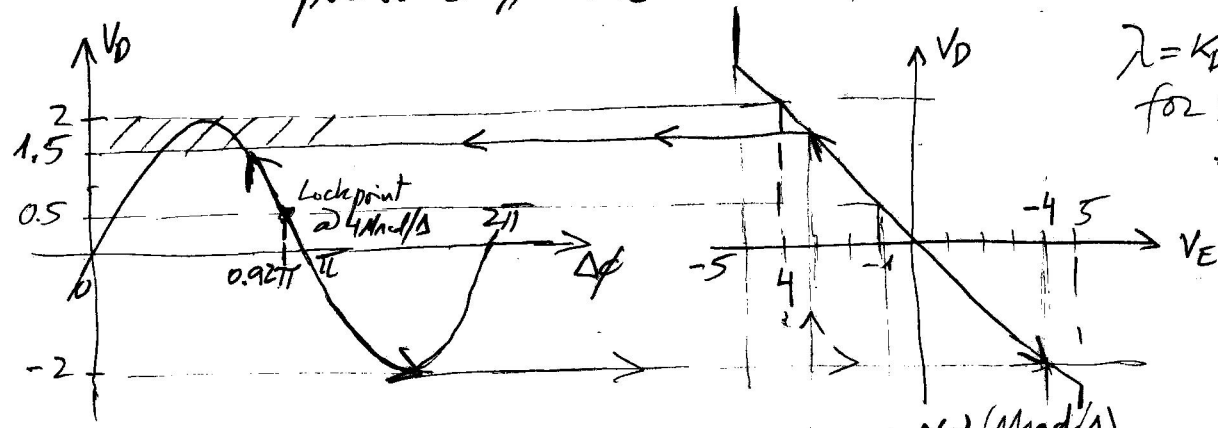
$$= \frac{v_i v_o}{2} \langle \sin[(\omega_i - \omega_o)t + \Delta\phi] \rangle$$

instantaneous, total phase difference

When Locked, $\omega_i \approx \omega_o$

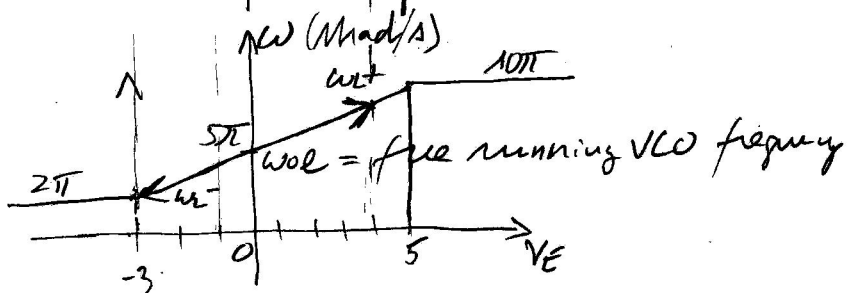
$$\langle v_D(t) \rangle = \frac{v_i v_o}{2} \sin \Delta\phi = 2 \sin \Delta\phi$$

$$\lambda = K_D F(0) K_o > 0 \text{ for stability} \Rightarrow K_D < 0$$



$$\omega(t) = \omega_{osc} + K_o v_e(t)$$

$$\begin{cases} 10\pi = \omega_L + 5K_o \\ 2\pi = \omega_L - 3K_o \end{cases} \Rightarrow \begin{cases} K_o = \pi \text{ Mrad/s/V} \\ \omega_L = 5\pi \text{ Mrad/s} \end{cases}$$



Hold range:

Upper limit = $\omega_L^+ = 9\pi \text{ Mrad/s}$ limited by the lower 'saturation' of the phase detector.

Lower limit = $\omega_L^- = 2\pi \text{ Mrad/s}$ limited by the lower saturation of the VCO

$$\Delta\omega_L = \omega_L^+ - \omega_L^- = 7\pi \text{ Mrad/s} (3.5 \text{ MHz})$$

Approximation for the lock range:

$$\Delta\omega_L \approx \sqrt{2\omega_p^2 \left(\sqrt{1 + \left(\frac{\Delta\omega_L}{\omega_p} \right)^2} - 1 \right)} \approx \sqrt{2\omega_p \Delta\omega_L} \text{ if } \Delta\omega_L \gg \omega_p \text{ which is true}$$

because $\Delta\omega_L = 7\pi \text{ Mrad/s}$ and $\omega_p = 2\pi \text{ Krad/s}$. So $\Delta\omega_L \approx 2\pi \times 83.67 \text{ Krad/s}$

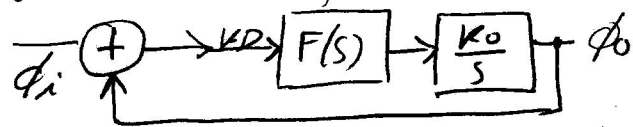
c) Loop locked with $\omega_i = 4\pi \text{ Mrad/s}$ so $v_e = -1\text{V}$, $v_D = \frac{1}{2} = 0.5\text{V}$. On the other hand $v_D = 2 \sin \Delta\phi$ so $\Delta\phi = \arcsin\left(\frac{v_D}{2}\right) = \arcsin\left(\frac{1}{4}\right) = 0.92\pi$.

Careful with your calculator! Why?

d) Gain $K_D = \left. \frac{\partial \langle v_D(\Delta\phi) \rangle}{\partial \Delta\phi} \right|_{\Delta\phi = 0.92\pi} = 2 \cos 0.92\pi = -1.935 \text{ V/rad}$

this is our 'operating' point

System transfer function:

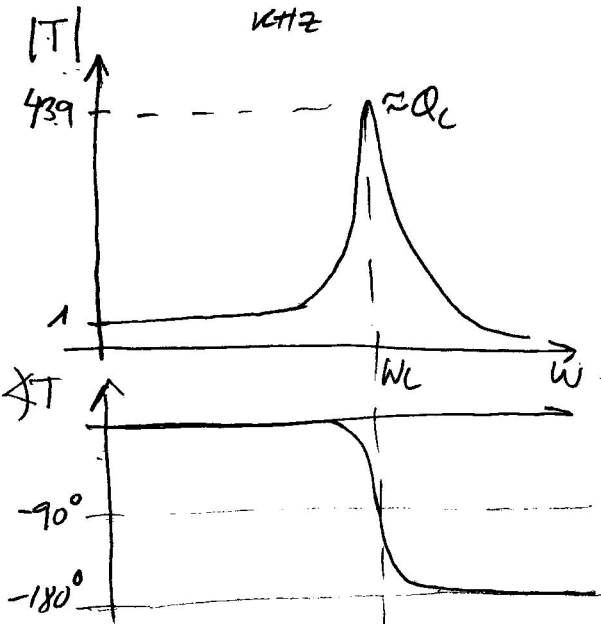


$$T(s) = \frac{\phi_o(s)}{\phi_i(s)} = \frac{K_D F(s) \cdot \frac{K_0}{s}}{1 + K_D F(s) \frac{K_0}{s}} = \frac{K_D \frac{F(0)}{1 + j\omega\tau} \frac{K_0}{s}}{1 + K_D \frac{F(0)}{1 + j\omega\tau} \frac{K_0}{s}} = \frac{\omega_p K_D K_0 F(0)}{s^2 + \omega_p s + \omega_p K_D K_0 F(0)}$$

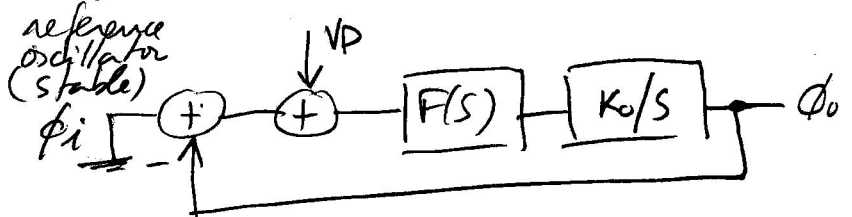
$$T(s) = \frac{\omega_p \lambda}{s^2 + \omega_p s + \omega_p \lambda} \quad \begin{cases} Q_c = \frac{\omega_c}{\omega_p} = \sqrt{\frac{2}{\omega_p \lambda}} \\ \omega_c = \sqrt{\omega_p \lambda} \end{cases}$$

$$Q_c = 43.9$$

$$\omega_c = 2\pi \times \frac{43.9 \text{ Krad/s}}{1000} = 277.8 \text{ Krad/s}$$



e) Redesign to have a phase modulator with $\omega_c = 2\pi \times 10 \text{ Krad/s}$ and $Q_c = \frac{1}{\sqrt{2}}$ (Butterworth)



$$T_{PM}(s) = \frac{\phi_o(s)}{V_P(s)} = \frac{T(s)}{K_D} \rightarrow \text{low pass}$$

$$= \frac{\omega_p \lambda / K_D}{s^2 + \frac{\omega_c}{Q_c} s + \omega_c^2}$$

$$\frac{\omega_c}{Q_c} = \omega_p = \frac{2\pi \times 10 \text{ Krad/s}}{1/\sqrt{2}} = 2\pi \times 14.1 \text{ Krad/s}$$

So, must increase filter bandwidth from 1 KHz to 14.1 KHz

$$\omega_c^2 = (2\pi \times 10^4)^2 = \omega_p \lambda \rightarrow \lambda = \frac{(2\pi \times 10^4)^2}{2\pi \times 14.1 \times 10^3} = 2\pi \times \frac{10^4}{\sqrt{2}}$$

Must change λ : choose either $F(0)$ or K_0 , K_D is difficult to change.

$$\text{Keep } F(0) \Rightarrow K_0 = \frac{2\pi \times 10^4}{\sqrt{2} F(0) K_D} = \frac{2\pi \times 10^4}{\sqrt{2} (-2) (-1.935)} = \pi \cdot 3654.2 \text{ rad/s/V}$$

(much lower)

Smaller than $\Delta\omega_L$ but much closer

Note: Now, $\Delta\omega_L = 2\pi \times 12.78 \text{ Krad/s}$ and

$\omega_p = 2\pi \times 14.1 \text{ Krad/s}$ so $\omega_p > \Delta\omega_L$ and the approximation used in b) for $\Delta\omega_L$ is not valid because $\Delta\omega_L \gg \omega_p$ is not true.

$\Delta\omega_L = K_0(V_{EMax} - V_{EMin})$ because ω_0 is the same.

$$= K_0(4 - (-3)) = 7K_0$$

\uparrow PD \uparrow VCO $= 2\pi \times 12.78 \text{ Krad/s}$
 KHz

$$\Delta\omega_c \approx \sqrt{2\omega_p^2 \left(\sqrt{1 + \left(\frac{\Delta\omega_L}{\omega_p} \right)^2} - 1 \right)}$$

$$= 2\pi \times 11.79 \text{ Krad/s}$$

KHz