TÉCNICO LISBOA

Sistemas de Processamento Digital de Sinais

Problems about FIR & IIR filters

I — Check only one answer. The penalty for a wrong answer is ¼ point.

1) When designing FIR filters with the non-recursive frequency sampling method, transition samples are used to:

☐ precisely control the filter cutoff frequencies.

☐ decrease the attenuation in the stopband.

☐ increase the transition bandwidth.

☐ decrease the ripple in the passband.

2) A FIR digital filter designed with the non-recursive frequency sampling method:

☐ cannot be lowpass.

☐ cannot be highpass.

☐ cannot have linear phase.

☐ is always stable.

3) A FIR filter with an even number of coefficients:

☐ cannot be lowpass.

☐ cannot be highpass.

☐ cannot have linear phase.

☐ can be passband.

4) In the impulse response invariance method (design of IIR filters):

☐ a stable analog prototype filter always originates a stable digital filter.

☐ the frequency response of the digital filter is equal to the frequency response of the analog filter.

☐ the DC gain of the digital filter is always the same of the analog filter.

☐ the analog frequency band $(0,\infty)$ is mapped into the digital frequency band $(0,\omega_s/2)$.

5) When designing FIR digital filters, coefficient windows are used to:

☐ precisely control the filter cutoff frequencies.

☐ increase the attenuation in the stopband relative to the passband.

☐ increase the transition band.

☐ increase the ripple in the passband.

6) When designing IIR digital filters using the bilinear transformation method:

☐ high-pass filters cannot be designed.

☐ there is no frequency warping effect.

☐ an unstable analogue prototype filter may originate a stable digital filter.

☐ the analog frequency band $(0,\infty)$ is mapped into the digital frequency band $(0,\omega_s/2)$.

II — Consider the design of a FIR linear phase, high-pass ideal unit-gain filter using the Fourier series development method with $N = 9$ coefficients. The cutoff frequency is $f_c = 2$ kHz and the sampling frequency is $f_s = \frac{1}{T_s} = 10$ kHz .

a) Explain what kind of symmetry is expected for the filter coefficients. Compute the filter coefficients.

b) Determine the fixed-point arithmetic format that should be used to represent the filter output if the input samples are represented in Q_{11}. Justify.

c) Sketch approximately and qualitatively (for $0 \leq f \leq f_s$) the amplitude frequency response of the filter and of the same filter if a Hanning window was applied to the coefficients. Justify and explain the advantages and disadvantages of using windows with progressive decay.

Window	**Transition width**	**Passband ripple**	**Relative attenuation**
Rectangular	$0.9\, f_s/N$	0.74 dB	13 dB
Hanning	$3.1\, f_s/N$	0.05 dB	31 dB

III — Consider a DSP system operating with a sampling frequency $f_s = \frac{1}{T_s} = 10$ kHz .

a) Design an IIR filter from an analog filter with transfer function $H(s) = 40\dfrac{s+10}{s^2 + 40s + 400}$ using the matched Z transform method (MZT) and keeping the DC gain.

b) Write the difference equation and the signal flow diagram of the filter in direct form II.

c) Consider the bilinear method of IIR filter design. Explain the differences, advantages and disadvantages of this method with respect to the MZT method.

IV – Consider a digital signal processing system operating with $f_s = \frac{1}{T_s} = 10$ kHz and the design of a IIR digital filter with the same impulse response of an analog, first-order lowpass filter $H_{LP}(s) = K\dfrac{\omega_c}{s + \omega_c}$ with cutoff frequency $\omega_c = 4\pi$ krad/s .

a) Design the filter determining its frequency response $H(z)$.

b) Determine the gain of the digital filter for $f = 0$ and $f = f_s / 2$.

c) Sketch the magnitude of the frequency response of the analog and the digital filters for $0 \leq f \leq f_s$ (make use of the values from previous question). Explain the differences between the two responses.

Points: **I -** 6 **II -** a) 3 b) 1 c) 1.5 **III -** a) 2 b) 1 c) 1 **IV -** a) 1.5 b) 1 c) 2

Problems about FIR & IIR filters -I- 4/4/4/1/2/4 1

II- $N=9$, high-pass $f_c = 2KHz$, $f_s = 10KHz$, FIR linear phase

a) Because N is odd, coefficients must be symmetrical. If N were even then coefficients would be anti-symmetric. No other cases are possible because $H(\frac{w_s}{2})=0$.

$$h_n = \frac{1}{w_s}\int_{-w_s/2}^{w_s/2} |H(jw)|\, e^{jwnT_s}\, dw =$$

$$= \frac{1}{w_s}\left[\int_{-w_s/2}^{-w_c} e^{jwnT_s}\, dw + \int_{w_c}^{\frac{w_s}{2}} e^{jwnT_s}\, dw\right]$$

$$= \frac{1}{w_s}\left[\frac{e^{-jw_cnT_s} - e^{-j\frac{w_s}{2}nT_s}}{jnT_s} + \frac{e^{j\frac{w_s}{2}nT_s} - e^{jw_cnT_s}}{jnT_s}\right]$$

$$= \frac{1}{2\pi n}\left[\frac{e^{-jw_cnT_s} - e^{jw_cnT_s} + e^{j\pi n} - e^{-j\pi n}}{j}\right]$$

$$= -\frac{\sin w_cnT_s}{\pi n} + \frac{\sin \pi n}{\pi n}$$

$$= \begin{cases} 1 - \frac{w_cT_s}{\pi}, & n=0 \\ -\frac{\sin w_cnT_s}{\pi n}, & n\neq 0 \end{cases}$$

* see page #3

$w_cT_s = 2\pi \times \frac{2}{10} = \frac{2\pi}{5}$

$h_0 = 1 - \frac{2}{5} = \frac{3}{5} \rightarrow h_4$

$h_{-1} = h_1 = -0.3027 \rightarrow h_3 = h_5$

$h_{-2} = h_2 = -0.0935 \rightarrow h_2 = h_6$

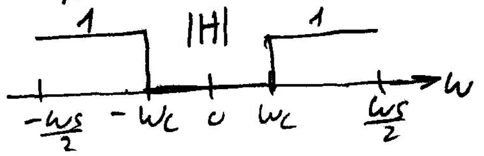
$h_{-3} = h_3 = 0.0624 \rightarrow h_1 = h_7$

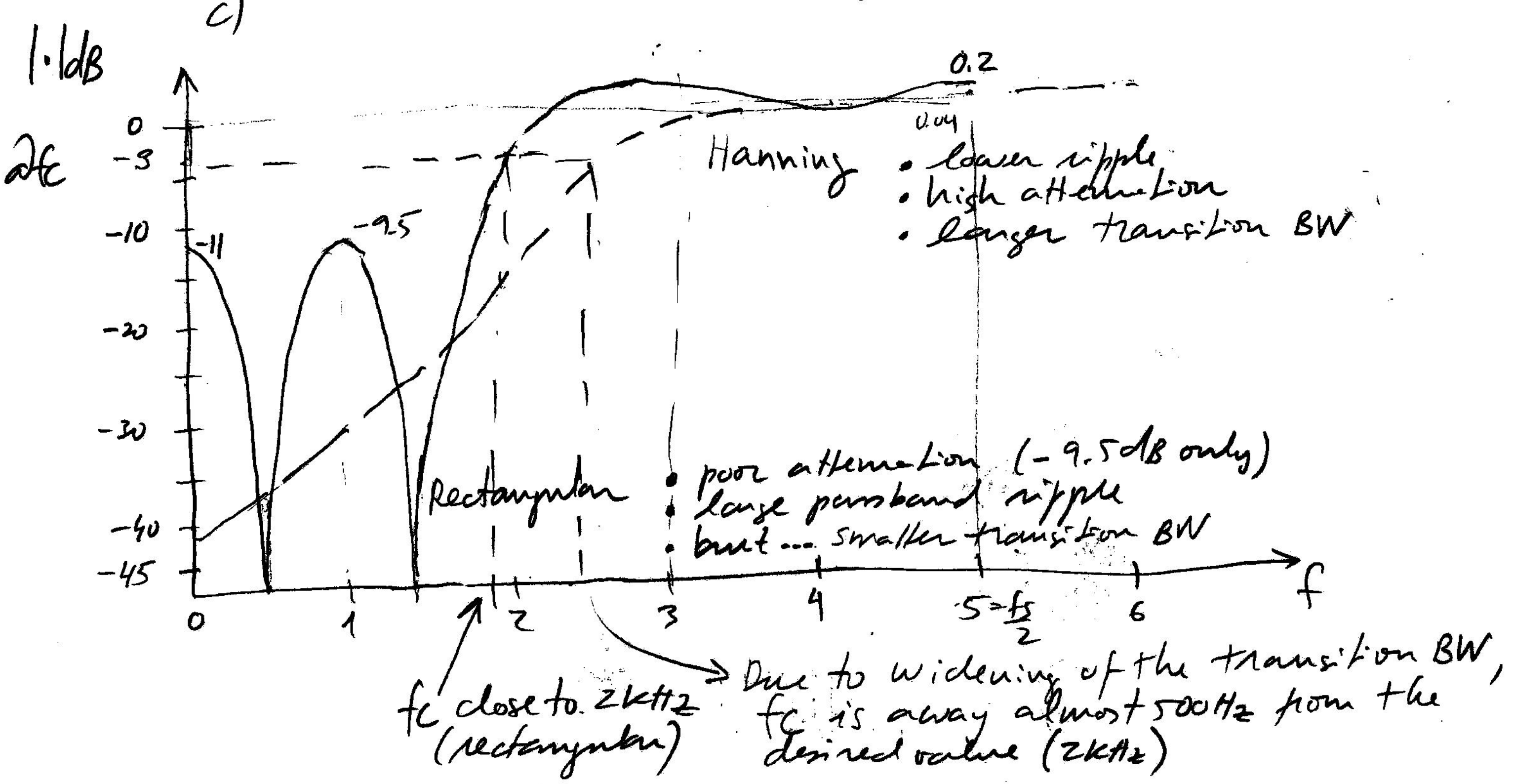
$h_{-4} = h_4 = 0.0757 \rightarrow h_0 = h_8$

b) Assuming signed 16 bit words

$\sum_{n=0}^{8} |h_n| = 1.667 \rightarrow Q_{10}$ for output

c)





III- $H(s) = 40\dfrac{s+10}{s^2+40s+400}$, MZT, $f_s = 10KHz$; $H(0) = 40\frac{10}{400} = 1$.

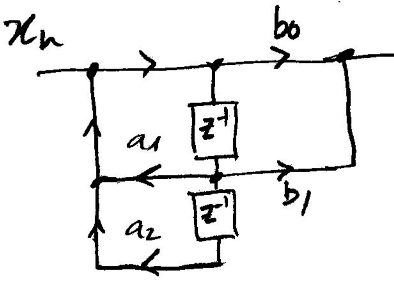
$s_{1,2} = \frac{-40 \pm \sqrt{40^2 - 4\times 400}}{2} = -20 \pm j0$ $\quad H(s) = 40\frac{s+10}{(s+20)^2} \rightarrow H(z) = KT_s\, 40\, \frac{1 - e^{-10T_s}z^{-1}}{(1-e^{-20T_s}z^{-1})^2}$

$$= KT_s 40 \frac{1 - e^{-10T_s}z^{-1}}{1 \underbrace{-2e^{-20T_s}}_{a_1} z^{-1} + \underbrace{e^{-40T_s}}_{-a_2} z^{-2}}\ ;\ H(z=1) = 1 \Rightarrow K = \frac{(1-e^{-20T_s})^2}{1-e^{-10T_s}} \times \frac{f_s}{40} = 0.9985$$

$b_0 = KT_s 40$, $b_1 = -KT_s 40\, e^{-10T_s}$

b) $b_0 = K T_s \cdot 40 = 0.003994$, $b_1 = -K T_s 40 \cdot e^{-10T_s} = -0.003990$

$a_1 = 2\times e^{-20T_s} = 1.996$, $a_2 = -e^{-40T_s} = -0.996008$



$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1} - a_2 z^{-2}}$$

$$\boxed{y_n = a_1 y_{n-1} + a_2 y_{n-2} + b_0 x_n + b_1 x_{n-1}}$$

c) **Bilinear**

- ⊕ no aliasing
- ⊕ can do highpass & band-reject
- ⊕ simple
- ⊕ stable ⇒ stable
- ⊖ frequency-warping

MZT

- ⊕ simple
- ⊕ stable ⇒ stable
- ⊖ can not do hippass & band reject
- ⊖ aliasing
- ⊕ matches poles and zeros

IV- $f_s = 10KHz$, impulse invariance, IIR

$$H_{lp}(s) = K\frac{w_c}{s+w_c} \xrightarrow{\mathcal{L}^{-1}} h(t) = K w_c e^{-w_c t} u(t)$$

$$h_n = K w_c T_s e^{-w_c n T_s} u(n), \quad H(z) = \sum_{n=-\infty}^{\infty} h_n z^{-n} = \frac{K w_c T_s}{1 - e^{-w_c T_s} z^{-1}}$$

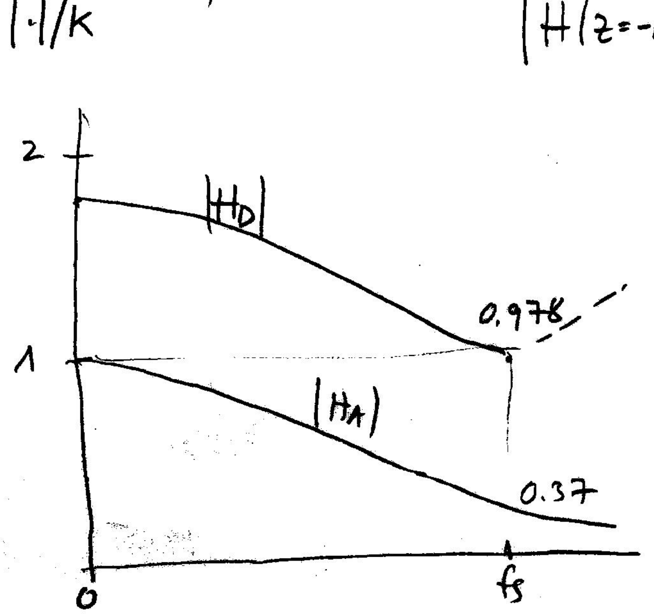
$$= \frac{1.256637\,K}{1 - 0.2846\, z^{-1}}$$

b) $H_{Lp}(0) = K$, $H(z=1) = 1.75657K$ much different!

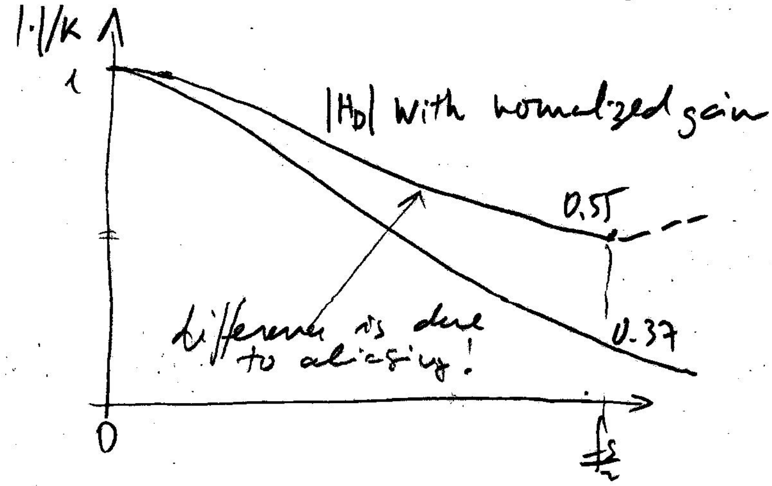
$$\left|H_{Lp}\left(\frac{w_s}{2}\right)\right| = K\frac{w_c}{\sqrt{\left(\frac{w_s}{2}\right)^2 + w_c^2}} = 0.37139K$$

$$|H(z=-1)| = 0.9782249\,K$$

c)

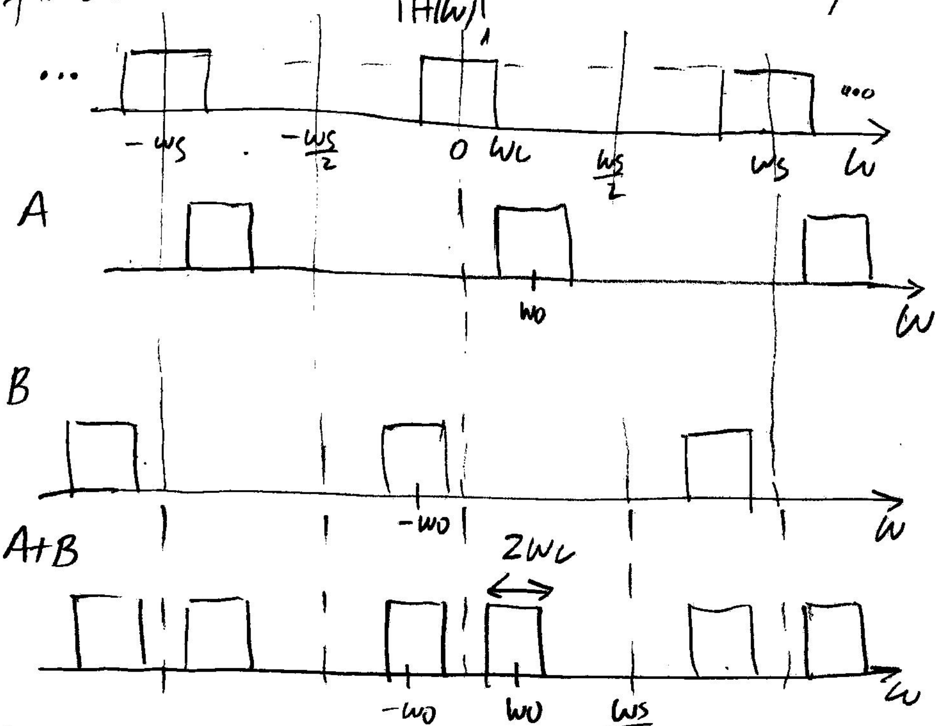


This shows that the filter gain should have been normalized.



Another way to solve II a)

Obtain a bandpass filter or a high-pass filter from a low-pass filter



$$h_n = \frac{1}{w_s}\int_{-w_s}^{w_s} e^{jw n T_s} = \frac{\sin w_c n T_s}{n\pi}$$

Low-pass

This is $|H(w-w_0)|$, i.e, $|H(w)|$ shifted w_0 to the right

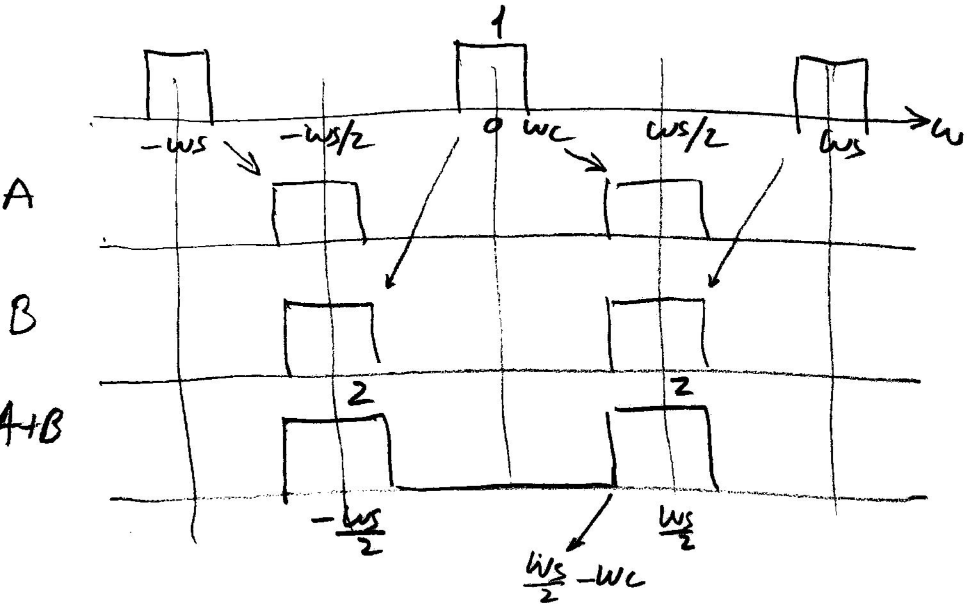
This is $|H(w+w_0)|$, i.e, $|H(w)|$ shifted w_0 to the left

this is the result from adding the two magnitudes a bandpass filter centered at w_0 with $BW = 2w_c$

So, the coefficients are

$$h_{BP}(n) = h_n e^{jw_0 n T_s} + h_n e^{-jw_0 n T_s} = 2h_n\cos(w_0 n T_s)$$

What about the high-pass case? We must set $w_0 = \frac{w_s}{2}$.



In fact you get a high pass filter but with twice the gain and a cutoff frequency which is $\frac{w_s}{2}-w_c$, not w_c! this means that you should start with a low pass filter with a cutoff of $\frac{w_s}{2}-w_c$.

To obtain the correct coefficients then:

$$h_{HP}(n) = h'_n \cdot \cos\left(n\frac{w_s}{2}T_s\right) = \frac{\sin\left(\frac{w_s}{2}-w_c\right)nT_s}{\pi n}\cdot\underbrace{\cos(n\pi)}_{(-1)^n} =$$

$$= \frac{\sin(n\pi - w_c n T_s)}{n\pi}\cos n\pi = \underbrace{\frac{\sin(n\pi)\cos(w_c n T_s)}{n\pi}}_{=1 \text{ if } n=0,\ 0 \text{ otherwise}}\cos n\pi - \frac{\cos(n\pi)\sin(w_c n T_s)}{n\pi}\cos n\pi$$

$$= \begin{cases} 1-\frac{w_c T_s}{\pi}, & n=0 \\ -\frac{\sin(w_c n T_s)}{n\pi}, & n\neq 0 \end{cases}$$

which coincides with the previous result on page #1 obtained using direct integration