Circuit Theory and Electronics Fundamentals

Integrated Master in Engineering Physics, IST, University of Lisbon

Lab 1: Circuit analysis methods

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1 Introduction

The objective of this laboratory assignment is to study a circuit containing a sinusoidal voltage source V_I connected to a resistor R and a capacitor C in series. The circuit can be seen if Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.olaaaa

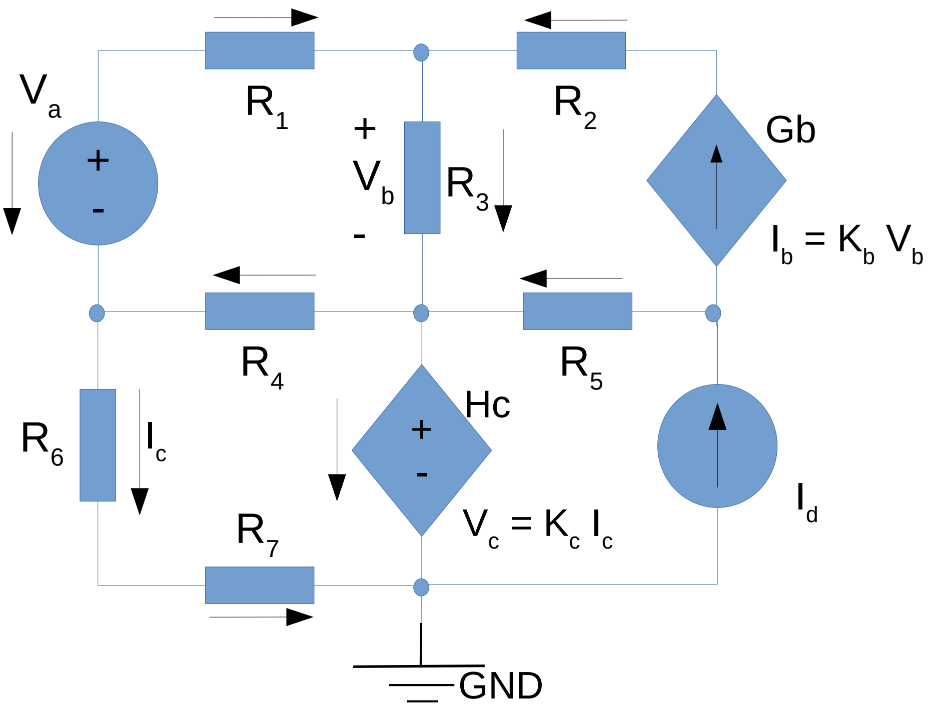

Figure 1: Voltage driven serial RC circuit.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, determinig the values of voltage and current acording to the mesh method and the node mehtod.

2.1 Mesh Method

The mesh method is based in the KVL, stating that the current in a elementary mesh is some fixed value. Furthermore, we assume that the current going throw a component is the sum of the currents of the elementary meshes, or elementary mesh, where the componet is. Then we apply the KVL law where, instead of using the Voltages, we use the current times the resistance because of the Ohm, which states that:

$$R = \frac{V}{I} \Leftrightarrow V = R \times I \tag{1}$$

Then, we write the equations acording to this format, where R is the resistance of the component, and I is the sum of the currents applied to the resistance:

$$\Sigma_i R_i I_i = 0 \tag{2}$$

It's clear that we can only write the equations for the meshes which don't have some voltage source. In order to avoid errors, one shoul previously define the direction of the current going throw each component, as well as the meshes currents directions. If this equations are not enough to solve the problem we write the KVL, or in the most unlikely of possibilities the KCL.

KVL states that the sum of the voltages in a mesh is zero, while the KCL states that the sum of the currents going to some point of the circuit is the same as the sum of the currents going out of that same point. Notice that KVL is valid both for elementary and not elementary knots.

In order to make each step clear, first the whole system will be shown,with eachof the currents marked, then the equation for each mesh, and then, finally, the matrix to solve.

Now, we are going to show the figure of the circuit with the currents in each mesh marked.

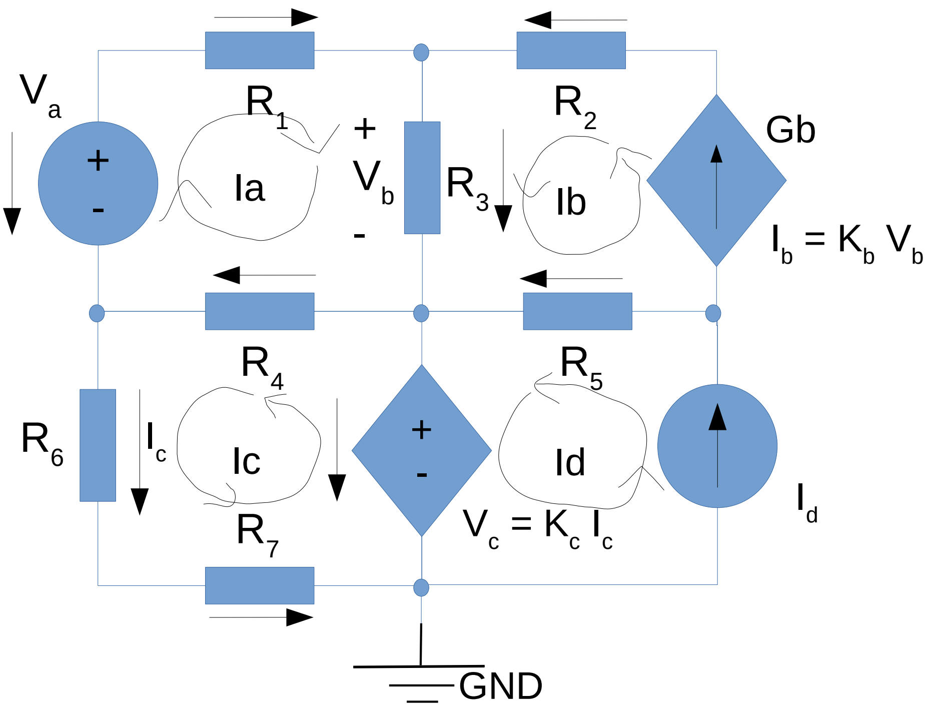

Figure 2: Circuit with the currents marked

For the mesh d it is clear that the correpondent current is the current of the current source. Therefore we have:

$$Id = I_d \tag{3}$$

For the mesh a, as it has a voltage source, we have to apply the KVL, obtaining:

$$-Va + R_1 Ia + R_3(Ia + Ib) + R4(Ia + Ic) = 0 \tag{4}$$

Then we get the final mesh a equation as:

$$Ia(R_1 + R_3 + R_4) + Ib(R_3) + Ic(R_4) = V_a \tag{5}$$

For the mesh b, we could simply write down:

$$Ib = K_b V_b = K_b(R_3(Ia + Ib)) \tag{6}$$

Then we get the final mesh equation for b as:

$$IaK_bR_3 + Ib(K_bR_3 - 1) = 0 \tag{7}$$

For the mesh c we apply, again, the KVL, obtaining:

$$-V_c + R_4(Ia + Ic) + Ic(R_6 + R_7) = 0 \tag{8}$$

One should substitute the V_C for $K_c Ic$, obtainig the final equation as:

$$Ia(R_4) + Ic(R_4 + R_6 + R_7 - K_c) = 0 \tag{9}$$

Finally, we only have to join the four final equations, obtaining the system:

$$\begin{pmatrix} R_1 + R_3 + R_4 & R_3 & R_4 \\ K_bR_3 & K_bR_3 - 1 & 0 \\ R_4 & 0 & R_4 + R_6 + R_7 - K_c \end{pmatrix} \begin{pmatrix} Ia \\ Ib \\ Ic \end{pmatrix} = \begin{pmatrix} V_a \\ 0 \\ 0 \end{pmatrix} \tag{10}$$

2.2 Node Method

The node method is based in the KCL and it is based in the fact that a node have a determined value of voltage, if we define one as the ground. Basically we say that the voltage in a component is the voltage drop in the direction of the current. Imagining we are looking at the node z. Then the node method simply states that:

$$\Sigma(Vz - Vi)Gi \tag{11}$$

where i states for the node which connects with z throw a resistance. One notices that if there are nodes which are not connect throw resistances. Therefore, we use the original form of the KCL, in order to keep the differences in mind.

In the figure showed above the ground is already marked, so one can simply name the nodes and start writing the equations. The nodes were labeled as below:

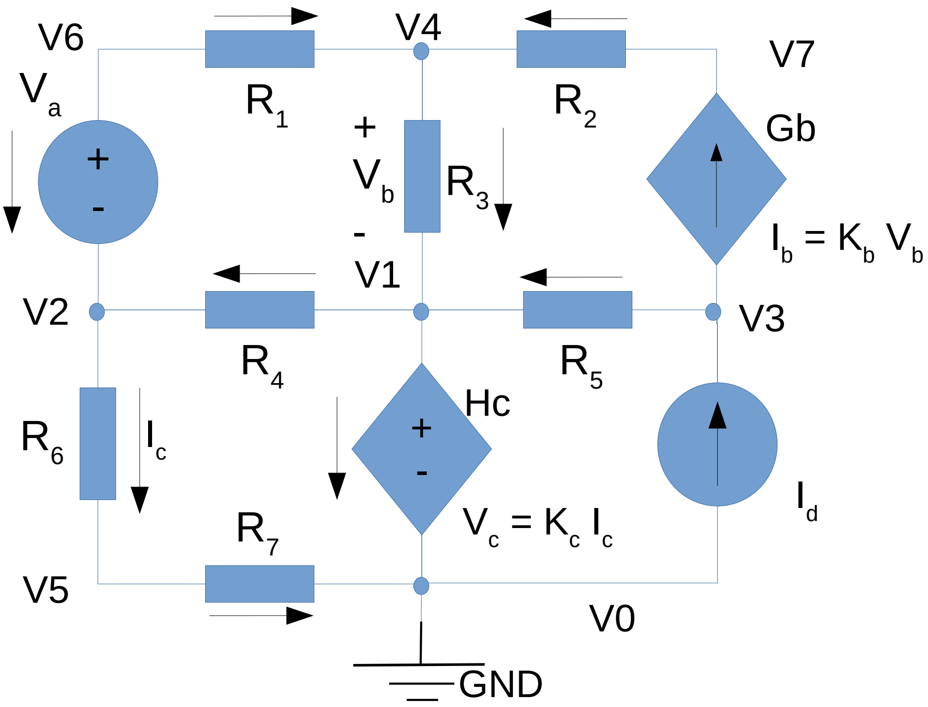

Figure 3: Circuit with the nodes marked

The value of the node V0 is 0V, because it is connected to the ground. We will simply ignore this equation.

For the node V2 we have:

$$V1(\frac{1}{R_4}) + V2(-\frac{1}{R_4} - \frac{1}{R_6}) + V4(\frac{1}{R_1}) + V5(\frac{1}{R_6}) + V6(-\frac{1}{R_1}) = 0 \tag{12}$$

For the node V3 we have:

$$V1(-\frac{1}{R_5} - K_b) + V3(\frac{1}{R_5}) + V4(K_b) = I_d \tag{13}$$

For the node V4 we have:

$$V1(-\frac{1}{R_3}) + V4(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}) + V6(-\frac{1}{R_1}) + V7(-\frac{1}{R_2}) = 0 \tag{14}$$

For the node V5 we have:

$$V2(-\frac{1}{R_6}) + V5(\frac{1}{R_6} + \frac{1}{R_7}) = 0 \tag{15}$$

For the node V7 we have:

$$V1(K_b) + V4(-\frac{1}{R_2} - K_b) + V7(\frac{1}{R_2}) = 0 \tag{16}$$

For the source V_a we have:

$$V6 - V2 = V_a \tag{17}$$

For the source V_c we have:

$$V1 + V5\frac{K_c}{R_6} + V2(-\frac{K_c}{R_6}) = 0 \tag{18}$$

Then, joining all the equations above, one gets the following matricial equation:

$$\begin{pmatrix} -\frac{1}{R_3} & 0 & 0 & \frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_2} & 0 & -\frac{1}{R1} & -\frac{1}{R_2} \\ 0 & -\frac{1}{R_6} & 0 & 0 & \frac{1}{R_6}+\frac{1}{R_7} & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 1 & -\frac{K_c}{R_6} & 0 & 0 & \frac{K_c}{R_6} & 0 & 0 \\ -\frac{1}{R_5}-K_b & 0 & \frac{1}{R_5} & K_b & 0 & 0 & 0 \\ K_b & 0 & 0 & -\frac{1}{R_2}-K_b & 0 & 0 & \frac{1}{R_2} \\ \frac{1}{R_4} & -\frac{1}{R_4}-\frac{1}{R_6} & 0 & \frac{1}{R_1} & \frac{1}{R_6} & -\frac{1}{R_1} & 0 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ V_a \\ 0 \\ I_d \\ 0 \\ 0 \end{pmatrix} \tag{19}$$

2.3 Results

Solving the matricial eqautions above we get the values below:

Name	Value [A or V]
V0	0
V1	7.898524E+00
V2	2.940328E+00
V3	1.171505E+01
V4	7.864904E+00
V5	9.657733E-01
V6	8.108539E+00
V7	7.374230E+00
Ia	2.341433E-04
Ib	-2.451090E-04
Ic	9.601318E-04
Vb	-3.362005E-02

Table 1: Values of the variables calculated, currents in A and voltages in V

3 Simulation Analysis

3.1 Operating Point Analysis

Table 2 shows the simulated operating point results for the circuit presented in Figure 1

Name	Value [A or V]
@gb[i]	-2.45109e-04
@id[current]	1.012771e-03
@r1[i]	2.341433e-04
@r2[i]	-2.45109e-04
@r3[i]	-1.09657e-05
@r4[i]	1.194275e-03
@r5[i]	1.257880e-03
@r6[i]	9.601318e-04
@r7[i]	9.601318e-04
v(1)	7.898524e+00
v(2)	2.940328e+00
v(3)	1.171505e+01
v(4)	7.864904e+00
v(5)	9.657733e-01
v(6)	8.108539e+00
v(7)	7.374230e+00

Table 2: Operating point table. A variable preceded by @ is a *current* in the unit Ampere; other variables are *voltages* in the unit Volt.

Comparing the theoretical analysis results presented in table ?To-be-determined? and the results in table 2 we can see no observable differences. This is to be expected since the circuit has no time dependency meaning its equal at any point in time and all the components are linear. For all these motives if we compare both tables all values match and there are no reasons why theys should not.

4 Conclusion

In this laboratory assignment the objective of analysing an RC circuit has been achieved. Static, time and frequency analyses have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The simulation results matched the theoretical results precisely. The reason for this perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models cannot differ. For more complex components, the theoretical and simulation models could differ but this is not the case in this work. To be continued...