Circuits Theory and Electronic Fundamentals

Integrated Master in Engineering Physics, IST, University of Lisbon

Lab 5: Bandpass filter using an OP-AMP

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Contents

1 Introduction

In this laboratory assignment, a bandpass filter (BPF) with an OP-AMP was implemented and studied. Using the circuit shown in Figure 1, a theoretical analysis were made and its results were obtained by using the Octave math tool. Moreover, Ngspice scripts was made in order to simulate this circuit. In both cases, the values of the central frequency, the input and output impedances at this frequency and the gain in the passband were determined. The plots of the frequency response (gain and phase) have been obtained for both analyses. As seen below, designations have been assigned to each node in the circuit.

As opposed to the previous assignments, it was also possible to test out this circuit in the laboratory and different configurations were used. Therefore, the circuit's gain was determined for different values of the resistances and these results will be shown in this report. It is worth noting that, even though the resistance R_{2p}, in parellel with R_2, ended up not being considered for the theoretical and simulation analyses, it is still shown below, as it was used in the laboratory.

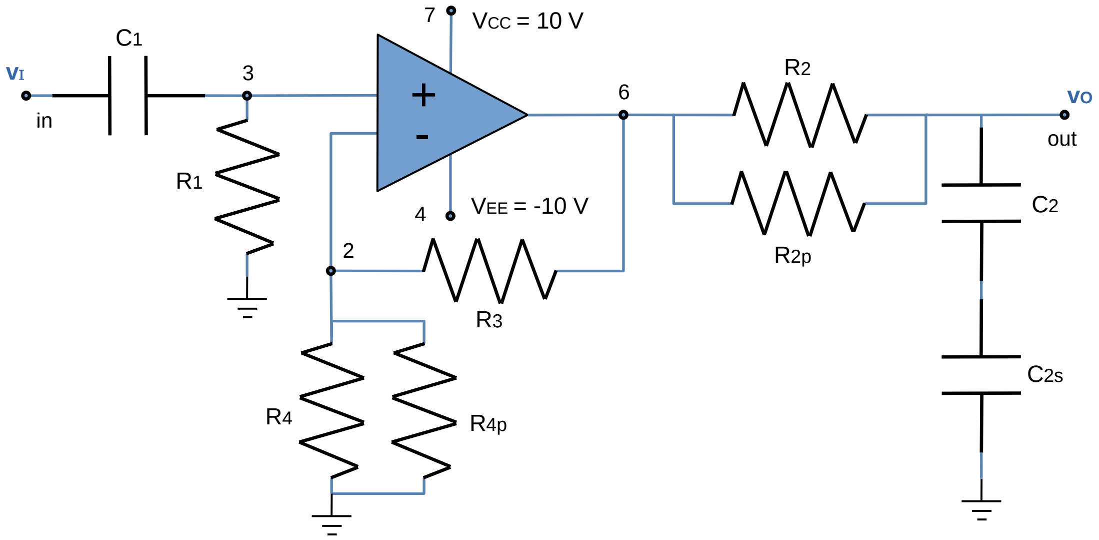


Figure 1: Circuit to be analysed in this laboratory assignment

2 Theoretical Analysis

The active bandpass filter is used to let a range of frequencies, which lie within the pass band, through the circuit without meaningful attenuation. This range of frequencies is limited between a lower cutoff frequency (f_L) and a higher cutoff frequency (f_H). The circuit shown in Figure 1 is formed by a High-Pass Filter Stage (capacitor C_1 and resistor R_1), an Amplification Stage (the OP-AMP and resistors R_3, R_4 and R_{4p}) and a Low-Pass Filter Stage (capacitors C_2 and C_{2s} and resistances R_2 and R_{2p}). It is worth noting that the reason why some resistances were put in parallel with another and two capacitors were put in series is that, in this laboratory assignment, some restrictions were imposed as to the number of components used, as well as the respective resistances and capacitances' values, as it will be further discussed in this section. The equivalent resistances and capacitances to be considered are given by:

$$C_{2,eq} = \frac{C_2 C_{2s}}{C_2 + C_{2s}} \ , \quad R_{2,eq} = \frac{R_2 R_{2p}}{R_2 + R_{2p}} \ , \quad R_{4,eq} = \frac{R_4 R_{4p}}{R_4 + R_{4p}} \tag{1}$$

As the names suggest, the High-Pass Filter Stage will significantly attenuate the signal for frequencies higher than f_L and the Low-Pass Filter Stage will "block" frequencies lower than f_H. As learnt in class, the values of these frequencies are given respectively by the following equations:

$$\omega_L = \frac{1}{R_1 C_1} = 2\pi f_L \tag{2}$$

$$\omega_H = \frac{1}{R_{2,eq} C_{2,eq}} = 2\pi f_H \tag{3}$$

On the other hand, the operation amplifier (OP-AMP) is a transistor-based device with a high gain and input impedance, a low output impedance and a differential input. In this circuit, the OP-AMP is used as a non-inverting amplifier, i.e., its gain is positive (because the resistances R_3 and $R_{4,eq}$ are positive). It is a negative feedback amplifier, meaning that the feedback is made through the inverting input.

In this theoretical model, an ideal OP-AMP is considered. Therefore, we have that the current i_+ that flows into the non-inverting input is null, because the input impedance of an ideal OP-AMP is $\infty\ \Omega$. Therefore, the currents that pass through C_1 (from node *in* to node 3) and R_1 (from node 3 to GND) have the same value, i_I. Consequently, the equation for $\frac{v_I}{i_I}$ for the High-Pass Filter sub-circuit can be easily obtained and corresponds to the **input impedance** of the circuit shown in Figure 1, thus given by

$$Z_I = R_1 + \frac{1}{j\omega C_1} = R_1 + \frac{1}{j2\pi f C_1} \tag{4}$$

In order to determine the output impedance, it is necessary to shut-off the input voltage, i.e., to consider $v_I = 0$ V. Because the current i_+ that goes into the non inverting input is zero, we have that $i_+ = -v_3\left(\frac{1}{R_1} + j\omega C_1\right) = 0 \Rightarrow v_3 = v_+ = 0$. Because of the OP-AMP's negative feedback, $v_- = v_2 = v_+ = 0$ V. Therefore, no current passes through $R_{4,eq}$; moreover, because the current that goes into the inverting input is $i_- = 0$ A (due to the $\infty\ \Omega$ input impedance), the current that flows through R_3 must be null, thus the **output impedance** is simply given by the parallel of $R_{2,eq}$ and $C_{2,eq}$'s impedances:

$$Z_O = \left.\frac{v_O}{i_O}\right|_{v_I=0} = \frac{1}{\frac{1}{R_{2,eq}} + j\omega C_{2,eq}} = \frac{1}{\frac{1}{R_{2,eq}} + j2\pi f C_{2,eq}} \tag{5}$$

In order to obtain the equation for the gain, let us consider each subsequent stage of this circuit. Firstly, by applying the Voltage Divider Law in the High-Pass Filter Stage, one easily gets the equation $v_{O,1}(s) = \frac{R_1 C_1 s}{1+R_1 C_1 s} v_I(s)$ (6) in terms of $s = j\omega$ for the output voltage of this sub-circuit. Secondly, for the Amplification Stage, we have that $v_{O,2}(s) = \left(1 + \frac{R_3}{R_{4,eq}}\right) v_{O,1}(s)$ (7). Finally, the Low-Pass Filter Stage gives us the equation $v_O(s) = \frac{1}{1+R_{2,eq}C_{2,eq}s} v_{O,2}(s)$ (8), by applying the Voltage Divider Law. By substituting equation 6 into 7 and this result into equation 8, the circuit's transfer function (thus, the output voltage **gain**) is obtained:

$$\begin{aligned} T(s) &= \frac{v_O(s)}{v_I(s)} = \frac{R_1 C_1 s}{1 + R_1 C_1 s}\left(1 + \frac{R_3}{R_{4,eq}}\right)\frac{1}{1 + R_{2,eq}C_{2,eq}s} \\ T(j\omega) &= \frac{v_O(j\omega)}{v_I(j\omega)} = \frac{R_1 C_1 j\omega}{1 + R_1 C_1 j\omega}\left(1 + \frac{R_3}{R_{4,eq}}\right)\frac{1}{1 + R_{2,eq}C_{2,eq}j\omega} \end{aligned} \tag{9}$$

The **central frequency** is given by the geometric mean of the values given by equations 2 and 3:

$$\begin{aligned} \omega_O &= \sqrt{\omega_L \omega_H} \\ f_O &= \sqrt{f_L f_H} \end{aligned} \tag{10}$$

While implementing this circuit, a few restrictions were imposed, in terms of the number of resistances and capacitors that could be used, as well as their values: at most three 1 kΩ resistors, three 10 kΩ resistors, three 100 kΩ resistors, three 220 nF capacitors and three 1 μF capacitors. By choosing the appropriate components, the main goal was to obtain a central frequency and a gain at this frequency as close as possible to 1 kHz and 40 dB, respectively. Moreover, the monetary cost of this circuit was made as low as possible. Taken this into account, the resistances and capacitances used in the theoretical and simulation analyses were the following:

Designation	Value [nF or kΩ]
C_1	220
C_2	220
C_{2s}	220
R_1	1
R_2	1
R_{2p}	∞
R_3	100
R_4	10
R_{4p}	1

Table 1: Values used for capacitances (in nF) and resistances (in kΩ) in the circuit

The values shown in Table 1 were used to obtain the theoretical results in Table 3 and Figures 2 and 3, shown below. These are the same values used to compute the Simulation Analysis in Section 4. In order to obtain the best possible results, while respecting the imposed restrictions, the resistor R_{2p} ended up not being used, thus the ∞ Ω value presented above, therefore $R_{2,eq} = R_2$. The other equivalent resistance and the equivalent capacitance are given by the values shown in Table 2.

Designation	Value [nF or kΩ]
$C_{2,eq}$	110
$R_{4,eq}$	9.090909e-01

Table 2: Values of the equivalent capacitance $C_{2,eq}$ (in nF) and the equivalent resistance $R_{4,eq}$ (in kΩ)

Finally, by using the equations which have been presented throughout Section 2, the input and output impedances (in absolute value) at the central frequency, the lower and higher cutoff frequencies, the central frequency and the gain at the central frequency have been obtained and are shown in Table 3. Moreover, by using equation 9 (with $\omega = 2\pi f$), the frequency response $\frac{v_O(f)}{v_I(f)} \equiv T(f)$ in absolute value has been plotted for a frequency vector in logarithmic scale with 10 poins per decade, from frequency $f = 10$ Hz to $f = 100$ MHz. This plot is shown in Figure 2. The last plot shows the phase, in degrees, of the output voltage, which is given by the phase of the transfer function, since the input signal is taken as a sinusoidal wave with phase equal to zero.

Designation	Value [Ω, Hz or dB]
$\lvert Z_I \rvert$	1.224745e+03
$\lvert Z_O \rvert$	8.164966e+02
f_L	7.234316e+02
f_H	1.446863e+03
f_O	1.023087e+03
Gain	3.738463e+01

Table 3: Theoretical results for the input and output impedances at the central frequency (in Ω), lower and higher cutoff frequencies and central frequency (in Hz) and gain in the passband (in dB)

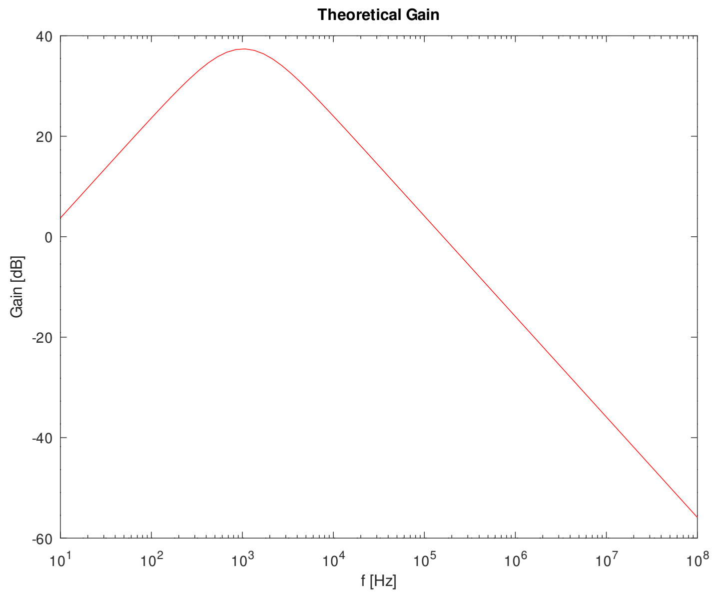


Figure 2: Plot of the gain with respect to frequency, as obtained by theoretical analysis

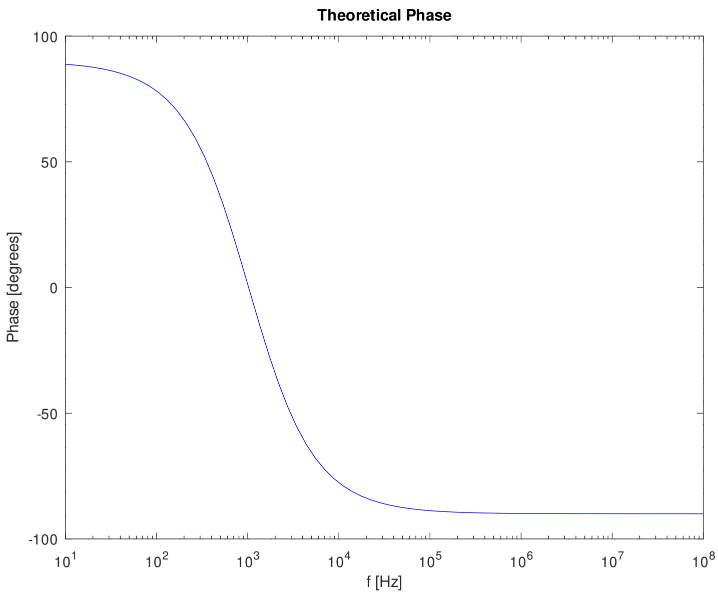


Figure 3: Plot of the output voltage's phase with respect to frequency, as obtained by theoretical analysis

Rather high values for $|Z_I|$ and $|Z_O|$ have been obtained. The value obtained for the central frequency f_O is very close to the desired result, $f = 1$ kHz. Moreover, a gain rather close to 40 dB has been obtained. Regarding the gain's plot, we can see that the theoretical model accurately depicts the functioning of a bandpass filter. For a small bandwith around f_O, the gain has its maximum value and decreases for frequencies below f_L and above f_H. This happens because, as we can see from equation 9, there is a zero at $s = 0$, which adds $\approx$ 20 dB per decade to the plot; there is a pole at f_L and another at f_H and each one adds $\approx$ -20 dB per decade to the plot. In terms of the phase, the graph looks very similar to what was expected for a bandpass filter, starting at $90°$ at tending towards $-90°$ for very high frequencies: the zero at the origin adds $\approx 45°$ to the phase slope in the decade before and in the decade after; the poles add $\approx -45°$ slopes in a similar way. These results will be further analysed in Section 4 alongside those obtained with Ngspice.

3 Experimental Results

3.1 Values obtained in the laboratory

Besides using Octave and Ngspice, it was possible to assemble the circuit shown in Figure 1 in a breadboard, for 5 different sets of values for the resistances. The values of C_1, C_2, R_3 and R_4 remained constant, but the values of the remaining resistances were changed.

Both in the laboratory and in the Ngspice simulations, the Texas Instruments $\mu A741$ OP-AMP model was used. In order to measure the input and output voltages, a signal generator was connected to the breadboard. The two channels of an oscilloscope were also connected to the breadboard. In these channels, the input and output voltage signals could be seen and both amplitudes were obtained. The oscilloscope's screen for Configuration $\#4$ is shown in Figure 4; for the other configurations, similar signals were observed, but with different amplitudes. In Figure 4, the circuit assembled in the breadboard is also shown.

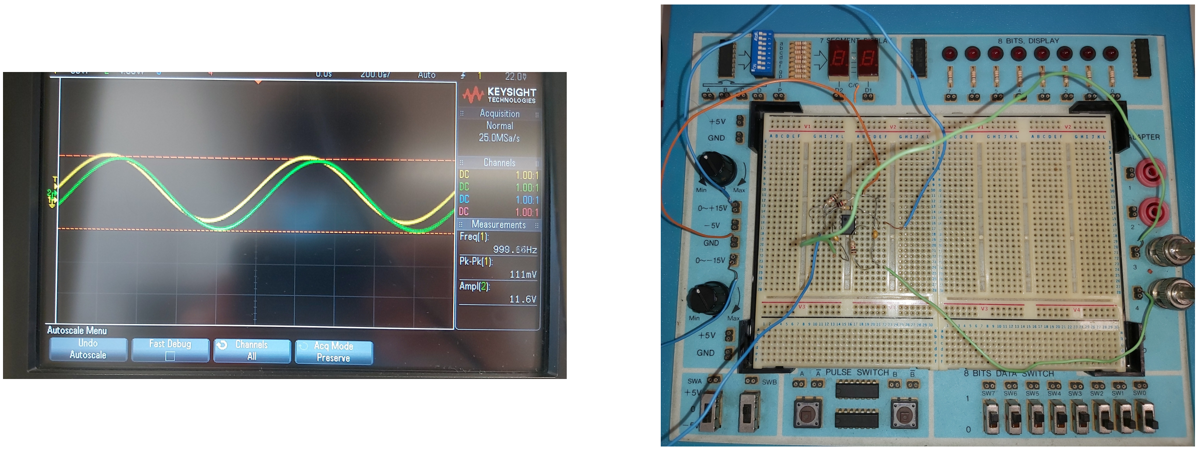


Figure 4: Input and output signals observed in the oscilloscope for configuration $\#4$ (a) and circuit implemented in the breadboard during the laboratory class (b)

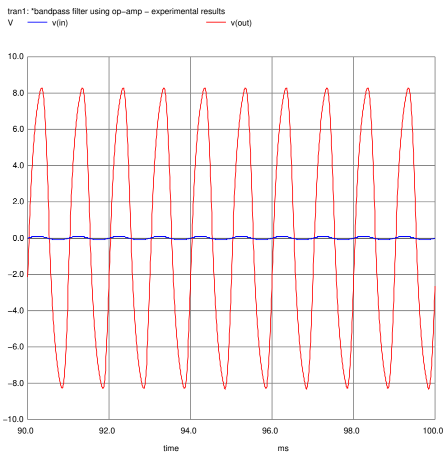
In Table 4, the results obtained for the five tested configurations are shown. A resistance of value ∞ indicates that that particular resistor wasn't used in the respective configuration. Even though the signal selected in the signal generator wasn't changed throughout the procedure, the frequency and the amplitudes measured in the oscilloscope varied slightly during each measurement, thus the values in the Amp_{v_I} column below aren't the same. The gain in linear units is given by $Gain = \frac{Amp_{v_O}}{Amp_{v_I}}$.

Configuration $\#$	R_1	R_2	R_{2p}	R_{4p}	Amp_{v_I} [mV]	Amp_{v_O} [V]	Gain [dB]
1	1	1	∞	∞	115	5.5	33.7
2	1	100	∞	∞	115	6.9	35.6
3	1	1	∞	1	113	9.5	38.5
4	1	1	2	1	111	11.6	40.4
5	1	1	1	1	113	13.0	41.2

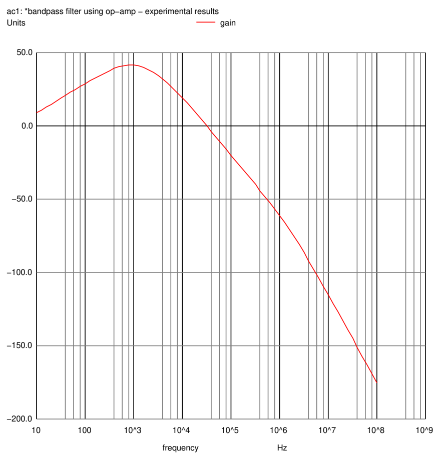
Table 4: Values obtained in the laboratory - the resistances are in kΩ, Amp_{v_I} and Amp_{v_O} are the amplitudes of the input and output signals, respectively, and the gains are approximate values with one decimal point. The following values remained constant: $C_1 = 220\ nF$, $C_2 = 220\ nF$, $R_3 = 100\ k\Omega$ and $R_4 = 1\ k\Omega$.

3.2 Simulation with the experimental values

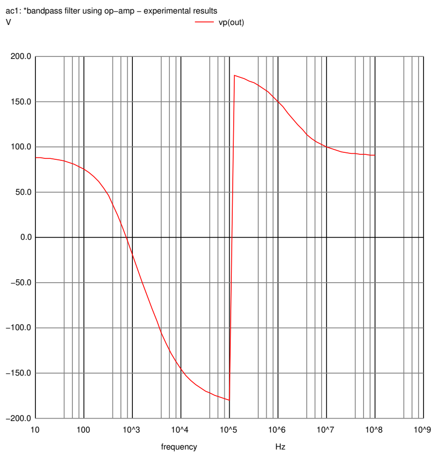
As seen in Table 4, configuration $\#4$ provided the gain closer to 4 dB. Thus, this configuration was also simulated in Ngspice, by using the same procedure which will be discussed in Section 4. The input and output signals are plotted in Figure 5a and the gain in Figure 5b. The input and output signals were represented in order to notice the discrepancies between them and the signals seen in the oscilloscope, in Figure 4: the first has a slight visible difference from a sine or cosine wave function, while the former does not. This may let us to believe that, even though the Ngspice model is very complex, it can't take into account all of the effects and phenomena that happen in an actual assembled circuit. Another possibility is that the oscilloscope might simply have a better resolution. As for the gain's plot, it's possible to verify that the assembled circuit works as a bandpass filter, with the gain being maximum around a central frequency. As for the phase, the plot is very similar to the one that was obtained and will be analysed in Section 4.



(a) Input signal (in blue) and output signal (in red) over time for f=1kHz



(b) Configuration $\#4$'s gain (in dB) with respect to frequency



(c) Configuration $\#4$'s output voltage's phase (in degrees) with respect to frequency

Finally, the same quantities computed for the theoretical analysis have been obtained for the experimental results of configuration $\#4$ and by using Ngspice. These results are shown in Table 5. The gain obtained with Ngspice is different from the gain calculated experimentally and shown in Table 4. Moreover, the central frequency is a bit off from the desired value, $f = 1$ kHz. These observations once again show that the simulated Ngspice model does not represent the assembled circuit in a totally accurate way and that some effects are difficult or unable to be predicted and/or simulated. These results will be further analysed in Section 4 and compared to those obtained in the next Ngspice simulation. In order to further compare the results of Sections 3 and 4, the monetary cost and merit of the experimental circuit (given by equations 11 and 12) are also shown.

Designation	Value [Ω, Hz or dB]
$\lvert Z_I \rvert$	9.999635e+02
$\lvert Z_O \rvert$	3.658280e+02
f_L	3.599628e+02
f_H	1.988244e+03
f_O	8.459869e+02
Gain	4.150026e+01

Designation	Value
Cost	1.342973e+04
Merit	4.312260e-07

Table 5: Experimental Results in Ngspice - absolute values of the input and output impedances at the central frequency (in Ω), lower and higher cutoff frequencies and central frequency (in Hz) and the gain in the passband (in dB), as well was the cost and merit M.

4 Simulation Analysis

In order to simulate the circuit with the values given in the Theoretical Analysis, two Ngspice scripts were made. One of the scripts was used in order to obtain the output impedance, by shutting off the input voltage, applying a test voltage source in the output and measuring the current passing through it. The quotient shown in equation 5 was then used to determine the output impedance at the central frequency. The other Ngspice script was used in order to simulate the whole circuit and to obtain the plots shown in Figures 6 and 7 and the other required values, shown in Tables 6 and 7. In the Ngspice scripts, the Texas Instruments $\mu A741$ OP-AMP model was used. It provides a quite complex OP-AMP model, which includes, for instance, two capacitors, nine resistors, two transistors and five diodes; all of these components will be considered when calculating the monetary cost.

By applying an input voltage (in node *in*) of frequency $f = 1$ kHz and conducting a frequency analysis from 10 Hz to 100 MHz, the necessary values were determined. To obtain the input impedance at the central frequency, the quotient given by equation 4 was used. The lower and higher cutoff frequencies were, respectively, considered to be the lower and higher frequencies in which (by convention) the gain was 3 dB below the maximum gain. Having obtained f_L and f_H, the central frequency was computed by equation 10. All of these values are shown in Table 6, alongside those obtained in the Theoretical Analysis.

Theoretical	
Designation	**Value [Ω, Hz or dB]**
$\lvert Z_I \rvert$	1.224745e+03
$\lvert Z_O \rvert$	8.164966e+02
f_L	7.234316e+02
f_H	1.446863e+03
f_O	1.023087e+03
Gain	3.738463e+01

Simulation	
Designation	**Value [Ω, Hz or dB]**
$\lvert Z_I \rvert$	9.999889e+02
$\lvert Z_O \rvert$	6.803833e+02
f_L	4.062898e+02
f_H	2.463678e+03
f_O	1.000484e+03
Gain	3.734444e+01

Table 6: Absolute values of the input and output impedances at the central frequency (in Ω), lower and higher cutoff frequencies and central frequency (in Hz) and gain at the central frequency (in dB), for both analyses

As we can see, very similar values were obtained with both analyses, which shows that the theoretical model considered is quite accurate. The lower and higher cutoff frequencies do differ significantly; however, the central frequencies are very similar, having the same order of magnitude and two equal digits. Moreover, they are extremely close to the desired 1 kHz value. The gains at the central frequency are also extremely close to each other, having three equal digits, even though they differ in about 2.6 dB from the desired 40 dB gain. However, it is worth noting that, because of the limitations imposed on the number and values of the components, it would be quite difficult to obtain better results. Even though the gain could have been made better, that would change the central frequency. Thus, it is safe to say that these results are quite reasonable.

By using the Ngspice script, the circuit's gain and the phase of the output voltage (i.e., the voltage at node *out*) - which, in this case, is the same as the gain's phase, because the input signal has phase zero - have been plotted and are shown below. It's in these graphs that the biggest differences from the Theoretical Analysis' results are noticeable. Firstly, although for lower and medium range frequencies the gain plots are almost indistinguishable, there is a clear difference for the higher frequencies, in which the theoretical gain is approximately -55 dB, but the simulation's gain is approximately -175 dB. Regarding the phase plots, in Section 2, where an **ideal OP-AMP model** was considered, the curve is due to the transfer function's zero at the origin and the two poles in the lower and higher cutoff frequencies, as explained in the Theoretical Analysis. The differences seen in the Simulation Analysis are due to the OP-AMP model considered. In this case, the OP-AMP model is much more complex and includes **two capacitors**. Because of them, two other poles are added; each pole adds $-45°$ to the slope in a +/- 1 decade interval. Thus, as the frequency tends to very high values, the phase will not tend to $90° - 2 \cdot 90° = -90°$ as before, but to $90° - 4 \cdot 90° = -270°$; because the Ngspice plot only considers angles in the interval [$-180°$,$180°$], at $-180°$ the curve "jumps" to $180°$, tending to $90°$ ($\equiv 270°$) for very high frequencies.

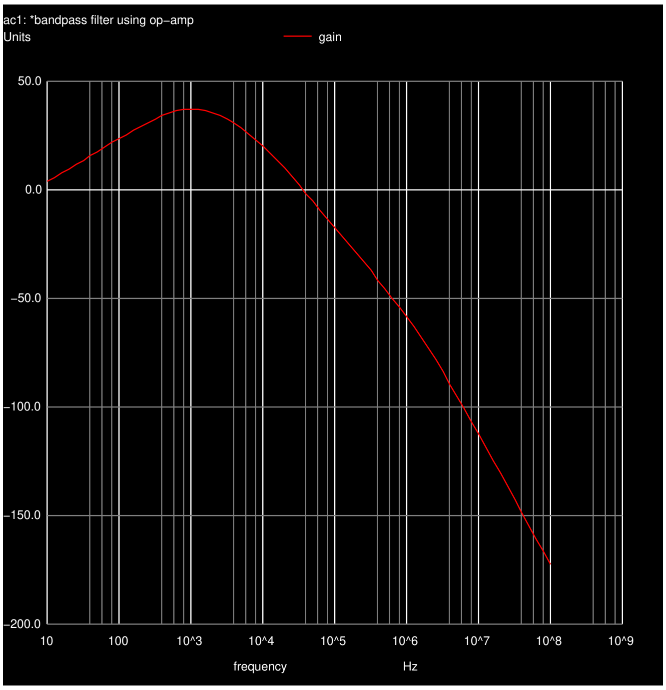


Figure 6: Plot of the gain (in dB) for frequencies (in log scale) from 10 Hz to 100 MHz

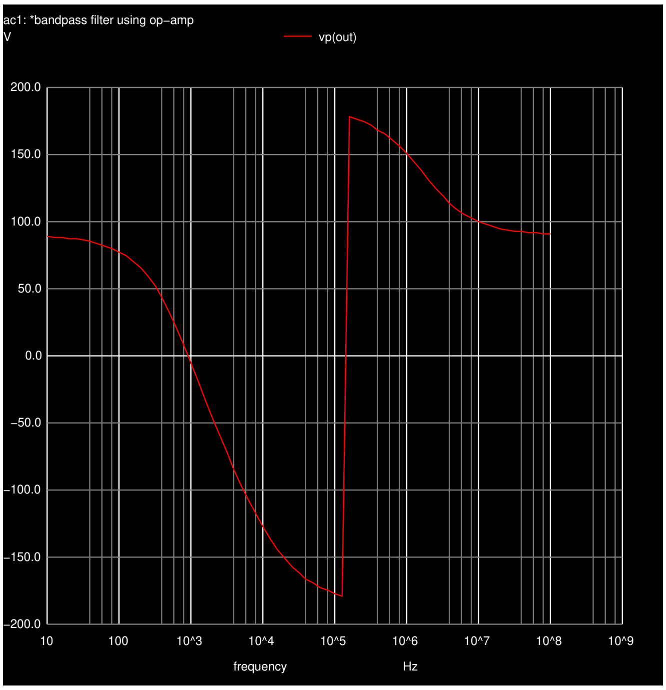


Figure 7: Plot of the output voltage's phase (in degrees) for frequencies (in log scale) from 10 Hz to 100 MHz

Finally, the monetary cost of this circuit and the merit figure, given by equations 11 and 12, were computed and are shown in Table 7. Firstly, it is important to take into account that the nine resistances (of values 100, 5.305, 5.305, 1.836, 1.836, 13190, 0.05, 0.1 and 18.16 kΩ), two capacitances (of values 8.661 and 30 pF), two transistors and five diodes which are used in the OP-AMP model were considered when calculating the monetary cost. Secondly, the monetary costs for every component were the following: 1 monetary unit (MU) per kΩ, 1 MU per μF, 0.1 MU per diode and 0.1 MU per transistor. The central frequency deviation is given by $|f_O - 1\ kHz|$ and the gain deviation by $|Gain(f_O) - 100|$ (in linear units).

$$Cost = cost\ of\ resistors\ +\ cost\ of\ capacitors\ +\ cost\ of\ transistors\ +\ cost\ of\ diodes \tag{11}$$

$$M = \frac{1}{Cost \cdot (Gain\ deviation\ +\ Central\ Frequency\ deviation\ +\ 10^{-6})} \tag{12}$$

Designation	Value
Cost	1.343695e+04
Merit	2.742830e-06

Table 7: Cost and merit obtained for this circuit

Although the monetary cost was very high and the merit was very low, it was to be expected, due to the the number of components considered, as well as their values, especially those of the OP-AMP model. Even then, this merit is still one order of magnitude bigger than the one obtained in Section 3 for the components used in the laboratory class (which couldn't have been used in the Simulation Analysis anyway, because more than three resistances of 1 kΩ were utilized in the laboratory). Thus, we may conclude that the merit is still reasonably good.

5 Conclusion

In this laboratory assignment, a bandpass filter using an operational amplifier (OP-AMP) was successfully implemented and studied in three different ways: by using a theoretical model and the Octave math tool, by implementing the circuit in the laboratory and by using Ngspice simulations. A central frequency very close to the desired 1 kHz has been obtained, as well as a gain in the passband quite close to the desired 40 dB. Taking into account the restrictions imposed on the number of components and the values of the respective resistances and capacitances, an acceptable merit was obtained. Even though the merit figure was improved as much as possible, these restrictions and the high monetary cost of the OP-AMP made it quite difficult to further increase it.

The theoretical and simulation's results were very similar to each other, except the gain and phase plots. These differences are due to the fact that an ideal OP-AMP model was considered in the Theoretical Analysis, but a quite complex model was used in the Ngspice simulations. This complex model used two capacitors, thus the simulation's results included two extra poles in the transfer function. Apart from this, the ideal OP-AMP model was able to provide good theoretical results.