Circuits Theory and Electronic Fundamentals

Integrated Master in Engineering Physics, IST, University of Lisbon

Lab 1: Circuit analysis methods

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1 Introduction

The objective of this laboratory assignment is to analyse the circuit shown in Figure 1, that is, to find all node voltages and branch currents. In Figure 1, the nodes have been numbered, current names and directions have been assigned to all branches and potential 0 has been assigned to one of the nodes.

By running the Python script `t1_datagen.py`, the values of the resistances R_1 to R_7, voltage V_a, current I_d and constants K_b and K_c are obtained. These are shown in Table 1, shown below. In Section 2, a theoretical analysis of the circuit and the results obtained with the Octave math tool are presented. In Section 3, the results obtained using the Ngspice simulation tool are shown. The conclusions of this study are outlined in Section 4, in which the theoretical results obtained in Section 2 are compared to those presented in Section 3.

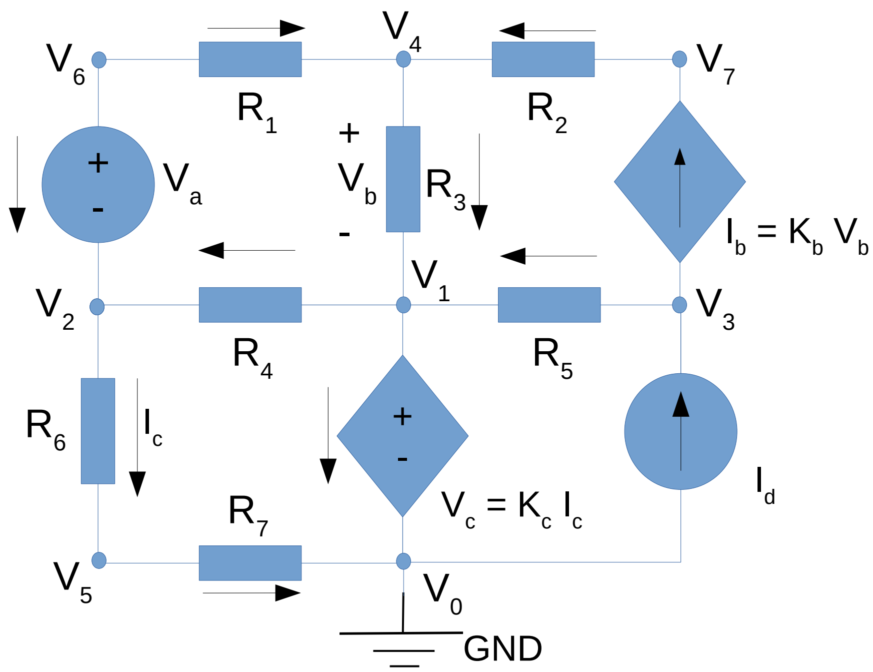

Figure 1: Circuit to be analysed in this laboratory assignment.

Designation	Value
R_1	1.04053890347 kΩ
R_2	2.00185929606 kΩ
R_3	3.06593231919 kΩ
R_4	4.15163583349 kΩ
R_5	3.03409481751 kΩ
R_6	2.05654586148 kΩ
R_7	1.00587575204 kΩ
V_a	5.16821048288 V
I_d	1.0127707267 A
K_b	7.29055867767 mS
K_c	8.22649929708 kΩ

Table 1: Values obtained by running the file `t1_datagen.py`.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, by determining the values of voltages and currents, using the mesh and node methods.

2.1 Mesh Method

The Kirchhoff Voltage Law (KVL) states that the sum of voltages in a circuit loop is zero. In Figure 2, the chosen directions for the mesh currents I_a, I_b, I_c and I_d are shown. It is also worth mentioning Ohm's Law, which states the following:

$$R = \frac{V}{I} \Leftrightarrow V = R \times I \tag{1}$$

Using this information, an equation is written for each of the four meshes represented in Figure 2. The value of current I_d is known and it was obtained by running `t1_datagen.py`. By solving the system that comes from applying KVL, the values of I_a, I_b and I_c will be obtained. Knowing all the mesh currents and using the other values given by `t1_datagen.py`, it is possible to know all node voltages abd branch currents, using Ohm's Law.

By applying KVL and Ohm's Law in mesh 'a', the following equation is obtained:

$$-V_a + R_1 I_a + R_3(I_a + I_b) + R_4(I_a + I_c) = 0 \Leftrightarrow I_a(R_1 + R_3 + R_4) + I_b(R_3) + I_c(R_4) = V_a \quad (2)$$

For mesh 'b', we can write down:

$$I_b = K_b V_b = K_b(R_3(I_a + I_b)) \Leftrightarrow I_a(K_b R_3) + I_b(K_b R_3 - 1) = 0 \quad (3)$$

The equation for mesh 'c' is:

$$-V_c + R_4(I_a + I_c) + I_c(R_6 + R_7) = 0 \Leftrightarrow I_a(R_4) + I_c(R_4 + R_6 + R_7 - K_c) = 0 \quad (4)$$

For mesh 'd', it is clear that the correpondant current is the same as the current source's. Therefore, by inspection, we simply obtain:

$$I_d = I_d \quad (5)$$

Finally, by joining equations 2, 3 and 4 togehter, we get the following system:

$$\begin{pmatrix} R_1 + R_3 + R_4 & R_3 & R_4 \\ K_b R_3 & K_b R_3 - 1 & 0 \\ R_4 & 0 & R_4 + R_6 + R_7 - K_c \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = \begin{pmatrix} V_a \\ 0 \\ 0 \end{pmatrix} \quad (6)$$

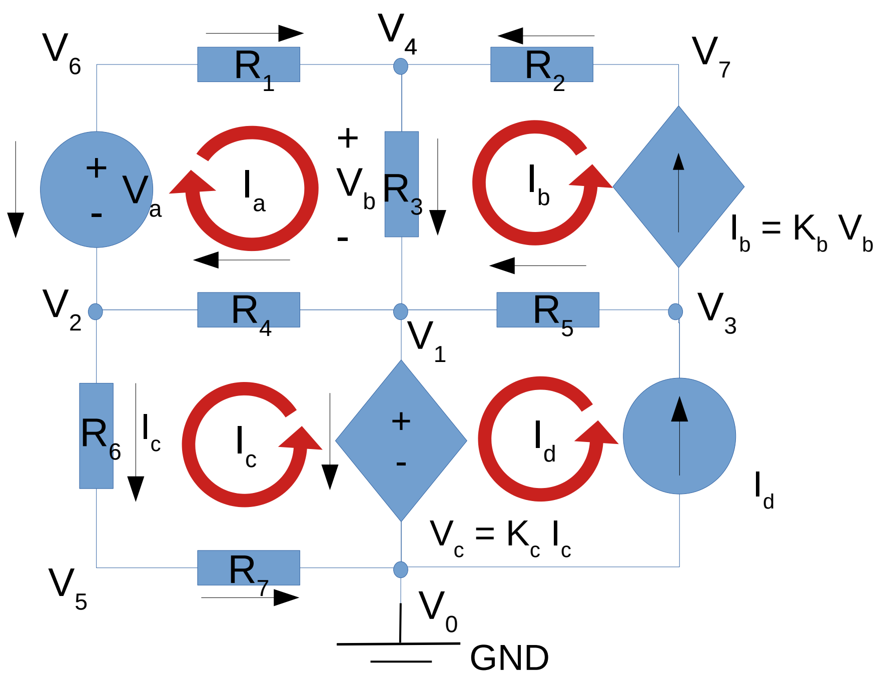

Figure 2: Circuit with the respective mesh currents.

2.2 Node Method

The Kirchhoff Current Law (KCL) states that the sum of the currents converging or diverging in a node is null. The nodes considered for the following equations are those represented in Figure 1. Using KCL and Ohm's Law (which can also be written as $I = VG$) in nodes not connected to voltage sources and additional equations for nodes related by voltage sources, it is possible to obtain a linear system from which the voltages at nodes V_1 to V_7 can be determined. Using these values and equation 1, currents I_a, I_b and I_c could be easily calculated.

Let us now consider, in the first place, nodes not connected to voltage sources.

For node V_2, we have that:

$$V_1(G_4) + V_2(-G_4 - G_6) + V_4(G_1) + V_5(G_6) + V_6(-G_1) = 0 \tag{7}$$

The equation for node V_3 is the following:

$$V_1(-G_5 - K_b) + V_3(G_5) + V_4(K_b) = I_d \tag{8}$$

For node V_4:

$$V_1(-G_3) + V_4(G_1 + G_2 + G_3) + V_6(-G_1) + V_7(-G_2) = 0 \tag{9}$$

The equation for node V_5 is:

$$V_2(-G_6) + V_5(G_6 + G_7) = 0 \tag{10}$$

And finally, for node V_7, we have that:

$$V_1(K_b) + V_4(-G_2 - K_b) + V_7(G_2) = 0 \tag{11}$$

Now, additional equations can be obtained for the nodes related by voltage sources (V_0, V_1, V_2 and V_6).

The value of the voltage in node V_0 is $0V$, because it is connected to GND. Therefore, this equation will not be used in the final matrix.

The source V_a can be used to obtain the following equation:

$$V_6 - V_2 = V_a \tag{12}$$

Finally, for node V_1, we have that V_1 - V_0 = V_1 = V_c = K_c I_c. Using Ohm's Law, we can replace I_c by $(V_2 - V_5)G_6$. Therefore, the equation obtained is:

$$V_1 + V_5(K_cG_6) + V_2(-K_cG_6) = 0 \tag{13}$$

Considering equations 7-13 above, one gets the following linear system:

$$\begin{pmatrix} 1 & -\frac{K_c}{R_6} & 0 & 0 & \frac{K_c}{R_6} & 0 & 0 \\ \frac{1}{R_4} & -\frac{1}{R_4} - \frac{1}{R_6} & 0 & \frac{1}{R_1} & \frac{1}{R_6} & -\frac{1}{R_1} & 0 \\ -\frac{1}{R_5} - K_b & 0 & \frac{1}{R_5} & K_b & 0 & 0 & 0 \\ -\frac{1}{R_3} & 0 & 0 & \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2} & 0 & -\frac{1}{R1} & -\frac{1}{R_2} \\ 0 & -\frac{1}{R_6} & 0 & 0 & \frac{1}{R_6} + \frac{1}{R_7} & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ K_b & 0 & 0 & -\frac{1}{R_2} - K_b & 0 & 0 & \frac{1}{R_2} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ I_d \\ 0 \\ 0 \\ V_a \\ 0 \end{pmatrix} \tag{14}$$

2.3 Results

By solving the linear systems 6 and 14, the following values are obtained:

Designation	Value [A or V]
V_0	0
V_1=V_c	7.898524E+00
V_2	2.940328E+00
V_3	1.171505E+01
V_4	7.864904E+00
V_5	9.657733E-01
V_6	8.108539E+00
V_7	7.374230E+00
I_a	2.341433E-04
I_b	-2.451090E-04
I_c	9.601318E-04
V_b=V_4-V_1	-3.362005E-02

Table 2: Values of voltages (in volts) and mesh currents (in amperes).

3 Simulation Analysis

3.1 Operating Point Analysis

Table 3 shows the simulated operating point results for the circuit presented in Figure 1. Voltages *v(i)* correspond to the respective voltages V_i in Table 2. Besides this, currents represented below as *@rj[i]* refer to the currents passing through the respective resistances, R_j. Therefore, @r1[i] is the value of I_a, @r2[i] is the value of I_b and @r6[i]=@r7[i] is the value of I_c. On the other hand, similarly to what is indicated in Table 2, v(1)=V_c and v(4)-v(1)=V_b.

It is also worth mentioning that @id[current] is the value of I_d and that @gb[i] is the current I_b.

Name	Value [A or V]
@gb[i]	-2.45109e-04
@id[current]	1.012771e-03
@r1[i]	2.341433e-04
@r2[i]	-2.45109e-04
@r3[i]	-1.09657e-05
@r4[i]	1.194275e-03
@r5[i]	1.257880e-03
@r6[i]	9.601318e-04
@r7[i]	9.601318e-04
v(1)	7.898524e+00
v(2)	2.940328e+00
v(3)	1.171505e+01
v(4)	7.864904e+00
v(5)	9.657733e-01
v(6)	8.108539e+00
v(7)	7.374230e+00

Table 3: Operating point analysis table. A variable preceded by @ is a current, in the unit *ampere*; other variables are voltages, in the unit *volt*.

Comparing the theoretical analysis results presented in Table 2 and the results in Table 3, we can notice no observable differences. This was to be expected, since the circuit has no time dependency - meaning it's equal at any point in time - and all the components are linear.

4 Conclusion

In this laboratory assignment, the objective of analysing the given circuit has been achieved. Static analysis has been performed both theoretically, using the Octave maths tool, and by circuit simulation, using the Ngspice tool. By comparing Tables 2 and 3, it is possible to verify that the simulation results matched the theoretical results precisely. The reason for this perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models shall not differ. For more complex components, like transistors, it should be expected that the theoretical and simulation models could differ, but this is not the case in this lab assignment.