TÉCNICO LISBOA

Circuits Theory and Electronic Fundamentals

Integrated Master in Engineering Physics, IST, University of Lisbon

Lab 2: RC Circuit Analysis

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April 8, 2021

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1 Introduction

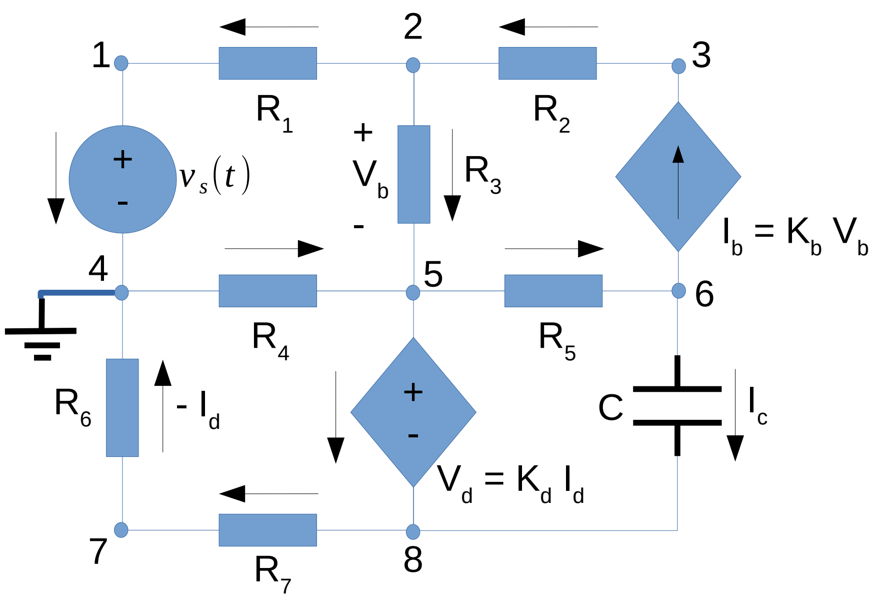

Figure 1: Circuit to be analysed in this laboratory assignment.

Designation	Value [V, kΩ, mS or μF]
R_1	1.04053890347
R_2	2.00185929606
R_3	3.06593231919
R_4	4.15163583349
R_5	3.03409481751
R_6	2.05654586148
R_7	1.00587575204
V_s	5.16821048288
C	1.0127707267
K_b	7.29055867767
K_d	8.22649929708

Table 1: Values obtained by running the file `t2_datagen.py`. Resistances R_i and constant K_d are in kΩ, voltage V_s is in volts, capacitance C is in microfarads and constant K_b is in milisiemens.

2 Theoretical Analysis

2.1 Exercise 1

In this section, the circuit shown in Figure 1 is analysed theoretically, by using the node method. The Kirchhoff Current Law (KCL) states that the sum of the currents converging or diverging in a node is null. The nodes considered for the following equations are those represented in Figure 1. Using KCL and Ohm's Law (which can also be written as $I = VG$) in nodes not connected to voltage sources and additional equations for nodes related by voltage sources, it is possible to obtain a linear system from which the voltages at nodes V_1 to V_8 and currents in resistances R_1 to R_7 can be determined.

The following linear system is obtained:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} & 0 & -\frac{1}{R_3} & 0 & 0 & 0 \\ 0 & -K_b-\frac{1}{R_2} & \frac{1}{R_2} & 0 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_3} & 0 & -\frac{1}{R_4} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} & -\frac{1}{R_5} & -\frac{1}{R_7} & \frac{1}{R_7} \\ 0 & K_b & 0 & 0 & -\frac{1}{R_5}-K_b & \frac{1}{R_5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{R_6}+\frac{1}{R_7} & -\frac{1}{R_7} \\ 0 & 0 & 0 & -\frac{K_d}{R_6} & 1 & 0 & \frac{K_d}{R_6} & -1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{pmatrix} = \begin{pmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (1)$$

By solving the linear system 1, the following values for node voltages and branch currents (calculated by using Ohm's Law) are obtained:

Designation	Value [A or V]
I_1	-2.34143283E-04
I_2	-2.45108969E-04
I_3	-1.09656866E-05
I_4	-1.19427511E-03
I_5	-2.45108969E-04
I_6	-9.60131827E-04
I_7	-9.60131827E-04
I_b	-2.45108969E-04
I_c	4.33680869E-19
I_{V_s}	-2.34143283E-04
I_{V_d}	-9.60131827E-04
V_1	5.16821048E+00
V_2	4.92457529E+00
V_3	4.43390162E+00
V_5	4.95819534E+00
V_6	5.70187920E+00
V_7	-1.97455514E+00
V_8	-2.94032846E+00

Table 2: Values of node voltages (in volts) and branch currents (in amperes). Current I_i corresponds to the current passing through resistance R_i.

3 Simulation Analysis

3.1 Exercise 1

Table 3 shows the simulated operating point results for the circuit presented in Figure 1. Again, currents designated below as I_i refer to the currents passing through the respective resistances, R_i.

Designation	Value [A or V]
I_1	-2.34143283e-04
I_2	-2.45108969e-04
I_3	-1.09656866e-05
I_4	-1.19427511e-03
I_5	-2.45108969e-04
I_6	-9.60131827e-04
I_7	-9.60131827e-04
I_b	-2.45108969e-04
I_c	0.000000000e+00
I_{V_s}	-2.34143283e-04
I_{V_d}	-9.60131827e-04
V_1	5.168210483e+00
V_2	4.924575288e+00
V_3	4.433901619e+00
V_5	4.958195341e+00
V_6	5.701879195e+00
V_7	-1.97455514e+00
V_8	-2.94032846e+00

Table 3: Operating point analysis table. Currents I_i are in amperes; voltages V_i are in volts.

Comparing the theoretical analysis results presented in Table 2 and the results in Table 3, we can notice almost no differences. This was to be expected, since the circuit has no time dependency - meaning it's equal at any point in time. There is only a small difference between the two values of I_c, although it is negligible.

3.2 Exercise 2

Designation	Value [A or V]
I_1	0.000000e+00
I_2	0.000000e+00
I_3	0.000000e+00
I_4	0.000000e+00
I_5	-2.84836e-03
I_6	0.000000e+00
I_7	0.000000e+00
I_b	0.000000e+00
I_c	-2.84836e-03
I_{V_s}	0.000000e+00
I_{V_d}	2.848364e-03
V_1	0.000000e+00
V_2	0.000000e+00
V_3	0.000000e+00
V_5	0.000000e+00
V_6	8.642208e+00
V_7	0.000000e+00
V_8	0.000000e+00

Table 4: Operating point analysis table. Currents I_i are in amperes; voltages V_i are in volts.

3.3 Exercise 3

Figure 2 shows the simulated transient analysis results obtained for the voltage in node 6.

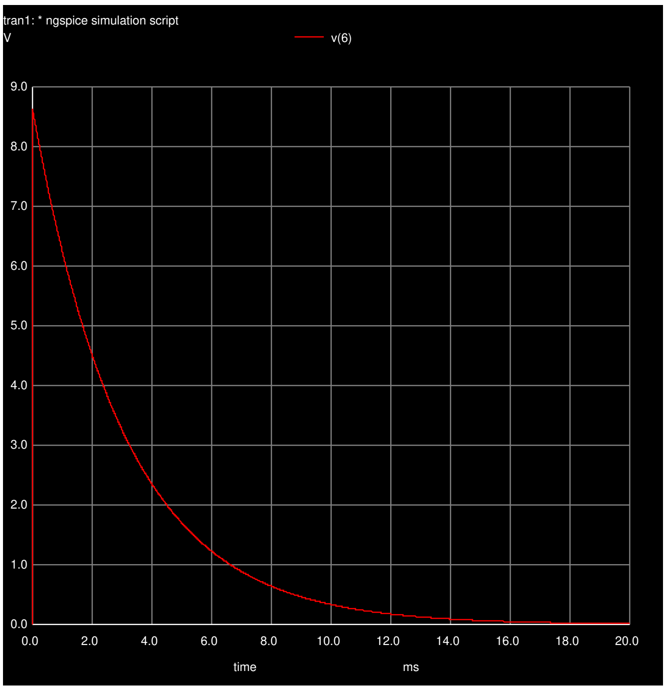

Figure 2: Value of $v_6(t)$ in the time interval [0,20] ms.

3.4 Exercise 4

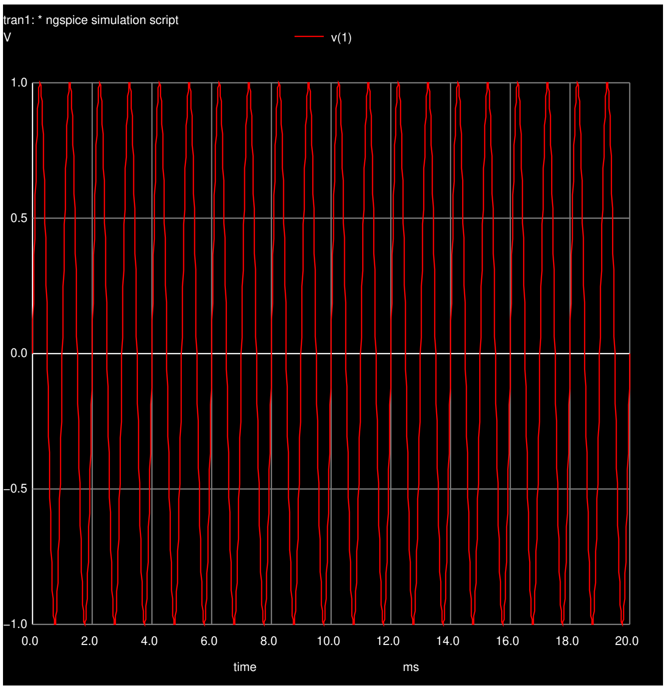

Figure 3: Stimulus voltage in time interval [0,20] ms.

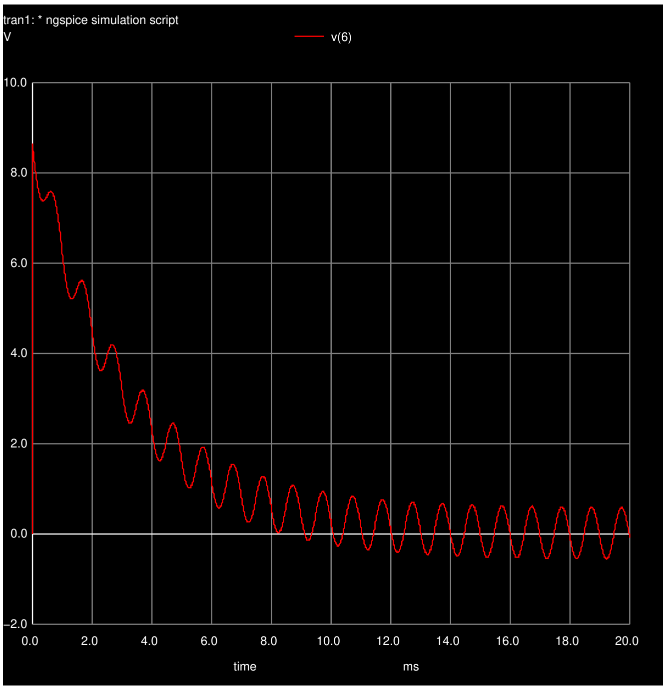

Figure 4: Forced response on node 6 in time interval [0,20] ms.

4 Conclusion

Theoretical	
Designation	**Value [A or V]**
I_1	-2.34143283E-04
I_2	-2.45108969E-04
I_3	-1.09656866E-05
I_4	-1.19427511E-03
I_5	-2.45108969E-04
I_6	-9.60131827E-04
I_7	-9.60131827E-04
I_b	-2.45108969E-04
I_c	4.33680869E-19
I_{V_s}	-2.34143283E-04
I_{V_d}	-9.60131827E-04
V_1	5.16821048E+00
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V_5	4.95819534E+00
V_6	5.70187920E+00
V_7	-1.97455514E+00
V_8	-2.94032846E+00

Simulation	
Designation	**Value [A or V]**
I_1	-2.34143283e-04
I_2	-2.45108969e-04
I_3	-1.09656866e-05
I_4	-1.19427511e-03
I_5	-2.45108969e-04
I_6	-9.60131827e-04
I_7	-9.60131827e-04
I_b	-2.45108969e-04
I_c	0.000000000e+00
I_{V_s}	-2.34143283e-04
I_{V_d}	-9.60131827e-04
V_1	5.168210483e+00
V_2	4.924575288e+00
V_3	4.433901619e+00
V_5	4.958195341e+00
V_6	5.701879195e+00
V_7	-1.97455514e+00
V_8	-2.94032846e+00

Table 5: Exercise 1 comparison.