Circuits Theory and Electronic Fundamentals

Integrated Master in Engineering Physics, IST, University of Lisbon

Lab 2: RC Circuit Analysis

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1 Introduction

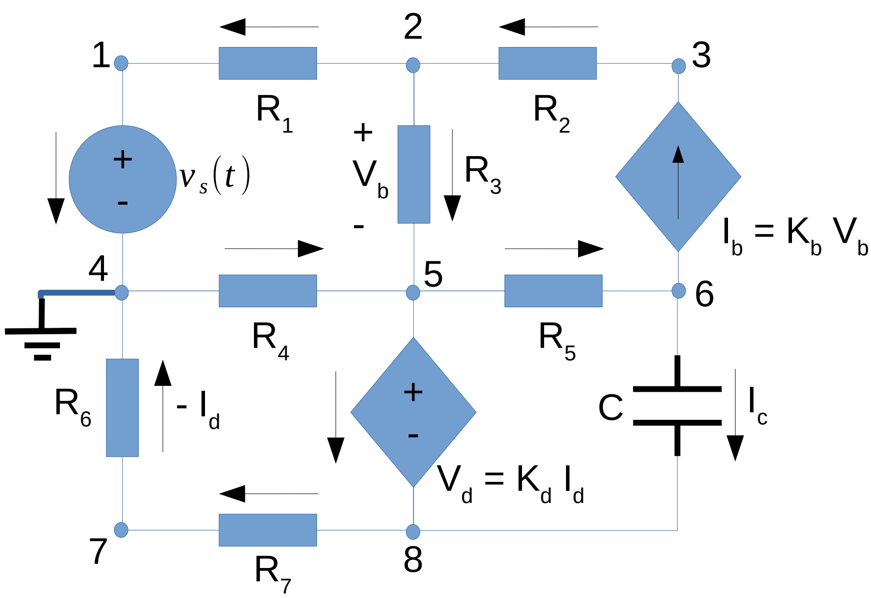

Figure 1: Circuit to be analysed in this laboratory assignment.

Designation	Value [V, kΩ, mS or μF]
R_1	1.04053890347
R_2	2.00185929606
R_3	3.06593231919
R_4	4.15163583349
R_5	3.03409481751
R_6	2.05654586148
R_7	1.00587575204
V_s	5.16821048288
C	1.0127707267
K_b	7.29055867767
K_d	8.22649929708

Table 1: Values obtained by running the file `t2_datagen.py`. Resistances R_i and constant K_d are in kΩ, voltage V_s is in volts, capacitance C is in microfarads and constant K_b is in milisiemens.

2 Theoretical Analysis

2.1 Exercise 1

In this section, the circuit shown in Figure 1 is analysed theoretically, by using the node method. The Kirchhoff Current Law (KCL) states that the sum of the currents converging or diverging in a node is null. The nodes considered for the following equations are those represented in Figure 1. Using KCL and Ohm's Law (which can also be written as $I = VG$) in nodes not connected to voltage sources and additional equations for nodes related by voltage sources, it is possible to obtain a linear system from which the voltages at nodes V_1 to V_8 and currents in resistances R_1 to R_7 can be determined.

The following linear system is obtained:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} & 0 & -\frac{1}{R_3} & 0 & 0 & 0 \\ 0 & -K_b-\frac{1}{R_2} & \frac{1}{R_2} & 0 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_3} & 0 & -\frac{1}{R_4} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} & -\frac{1}{R_5} & -\frac{1}{R_7} & \frac{1}{R_7} \\ 0 & K_b & 0 & 0 & -\frac{1}{R_5}-K_b & \frac{1}{R_5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{R_6}+\frac{1}{R_7} & -\frac{1}{R_7} \\ 0 & 0 & 0 & -\frac{K_d}{R_6} & 1 & 0 & \frac{K_d}{R_6} & -1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{pmatrix} = \begin{pmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{1}$$

By solving the linear system 3, the following values for node voltages and branch currents (calculated by using Ohm's Law) are obtained:

Designation	Value [A or V]
I_1	-2.341433E-04
I_2	-2.451090E-04
I_3	-1.096569E-05
I_4	-1.194275E-03
I_5	-2.451090E-04
I_6	-9.601318E-04
I_7	-9.601318E-04
I_b	-2.451090E-04
I_c	4.336809E-19
I_{V_s}	-2.341433E-04
I_{V_d}	-9.601318E-04
V_1	5.168210E+00
V_2	4.924575E+00
V_3	4.433902E+00
V_5	4.958195E+00
V_6	5.701879E+00
V_7	-1.974555E+00
V_8	-2.940328E+00

Table 2: Values of node voltages (in volts) and branch currents (in amperes). Current I_i corresponds to the current passing through resistance R_i.

2.2 Exercise 2

In this exercise we are going to analise the system also using the node method, as explained before. In this case our goal is to the get the R_{eq}, which means, to find the value of the resistance for the Norton and Thévenin equivalents. In order to calculate the resistance one must turn off all the constant sources, so, as we want to find the equivalent seen from the capacitor, we must have $V_S = 0$. That's the reason, why we do that. Now we are going to use the nodal method to determine the current going to the capacitor if we put a current source wi3th its voltage in its place and then, as we know the current and the voltage, one can simply use the Ohm's law:

$$R = \frac{V}{I} \tag{2}$$

We can use almost all the matrix in the previous exercise. The only differences are the ones stated in the previous paragraph. As we have already determined the voltages in the nodes before, we obtain the matrix above to get the voltages in the nodes,definig V_n as the value of $V_8 - V_6$:

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} & 0 & -\frac{1}{R_3} & 0 & 0 & 0 \\ 0 & -K_b-\frac{1}{R_2} & \frac{1}{R_2} & 0 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & K_b-\frac{1}{R_3} & 0 & -\frac{1}{R_4} & \frac{1}{R_3}+\frac{1}{R_4}-K_b & 0 & -\frac{1}{R_7} & \frac{1}{R_7} \\ 0 & 0 & 0 & -\frac{K_d}{R_6} & -1 & 1 & \frac{K_d}{R_6} & 0 \\ 0 & 0 & 0 & -\frac{1}{R_6} & 0 & 0 & \frac{1}{R_6}+\frac{1}{R_7} & -\frac{1}{R_7} \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_n \\ 0 \\ V_n \end{pmatrix} \tag{3}$$

Designation	Value [A or V]
I_1	1.181855E-03
I_2	1.237205E-03
I_3	5.535009E-05
I_4	-2.553371E-04
I_5	-1.048370E-03
I_6	-1.437192E-03
I_7	-1.437192E-03
I_b	1.237205E-03
I_c	-2.285575E-03
I_{V_s}	1.181855E-03
I_{V_d}	8.483826E-04
V_1	0.000000E+00
V_2	1.229766E+00
V_3	3.706477E+00
V_5	1.060067E+00
V_6	4.240919E+00
V_7	-2.955652E+00
V_8	-4.401288E+00

Table 3: Values of node voltages (in volts) and branch currents (in amperes). Current I_i corresponds to the current passing through resistance R_i.

Now that we have all this values, we are simply going to calculate the current going throw the capacitor/voltage source as the different between the current going throw R_5 and the current I_b, finally getting the result we want. We get:

Designation	Value [A or V]
I_b	1.237205E-03
I_5	-1.048370E-03
$I_{capacitor}$	-2.285575E-03
R_{eq}	3.781197E+03

Table 4: Values of node voltages (in volts) and branch currents (in amperes). Current I_i corresponds to the current passing through resistance R_i.

2.3 Exercise 3

Using the results from the previous exercise we understand we can simplify the circuit into a circuit with a voltage source, a resistance and and the capacitor. For this case we already have that the natural solution looks like:

$$v_n(t) = A \times e^{-\frac{t}{RC}} \tag{4}$$

Where A is the value in $t = 0$ acording to the previous exercise, t stands for time, R for the equivalent resistance and C for the capacitance of the capacitor. Plotting the result one gets:

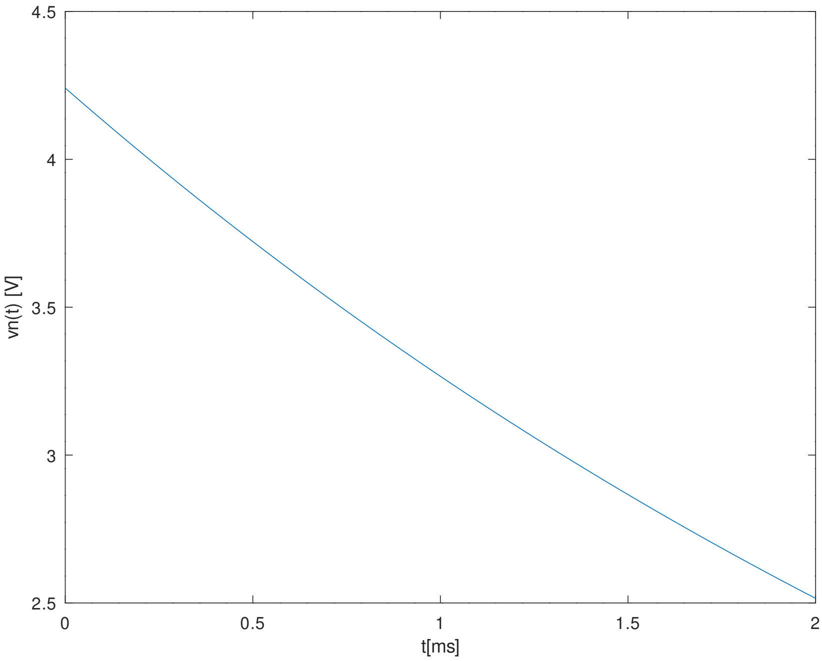

Figure 2: Forced sinusoidal response.

3 Simulation Analysis

3.1 Exercise 1

Table 5 shows the simulated operating point results for the circuit presented in Figure 1. Again, currents designated below as I_i refer to the currents passing through the respective resistances, R_i.

Designation	Value [A or V]
I_1	-2.34143e-04
I_2	-2.45109e-04
I_3	-1.09657e-05
I_4	-1.19428e-03
I_5	-2.45109e-04
I_6	-9.60132e-04
I_7	-9.60132e-04
I_b	-2.45109e-04
I_c	0.000000e+00
I_{V_s}	-2.34143e-04
I_{V_d}	-9.60132e-04
V_1	5.168210e+00
V_2	4.924575e+00
V_3	4.433902e+00
V_5	4.958195e+00
V_6	5.701879e+00
V_7	-1.97456e+00
V_8	-2.94033e+00

Table 5: Operating point analysis table. Currents I_i are in amperes; voltages V_i are in volts.

Comparing the theoretical analysis results presented in Table 2 and the results in Table 5, we can notice almost no differences. This was to be expected, since the circuit has no time dependency - meaning it's equal at any point in time. There is only a small difference between the two values of I_c, although it is negligible.

Now we ca approach the second point of our simulation where we see how the system behaves when $v_s(0) = 0$ and the capacitor is replaced by a voltage souce V_X=V(6)-V(8), where the voltages are taken from 5.

Designation	Value [A or V]
I_1	0.000000e+00
I_2	0.000000e+00
I_3	0.000000e+00
I_4	0.000000e+00
I_5	-2.84836e-03
I_6	0.000000e+00
I_7	0.000000e+00
I_b	0.000000e+00
I_{V_x}	-2.84836e-03
I_{V_s}	0.000000e+00
I_{V_d}	2.848365e-03
V_1	0.000000e+00
V_2	0.000000e+00
V_3	0.000000e+00
V_5	0.000000e+00
V_6	8.642209e+00
V_7	0.000000e+00
V_8	0.000000e+00

Table 6: Operating point analysis table. Currents I_i are in amperes; voltages V_i are in volts.

3.2 Transient analisis

Now we do a transient analisis of the values.

4 Conclusion

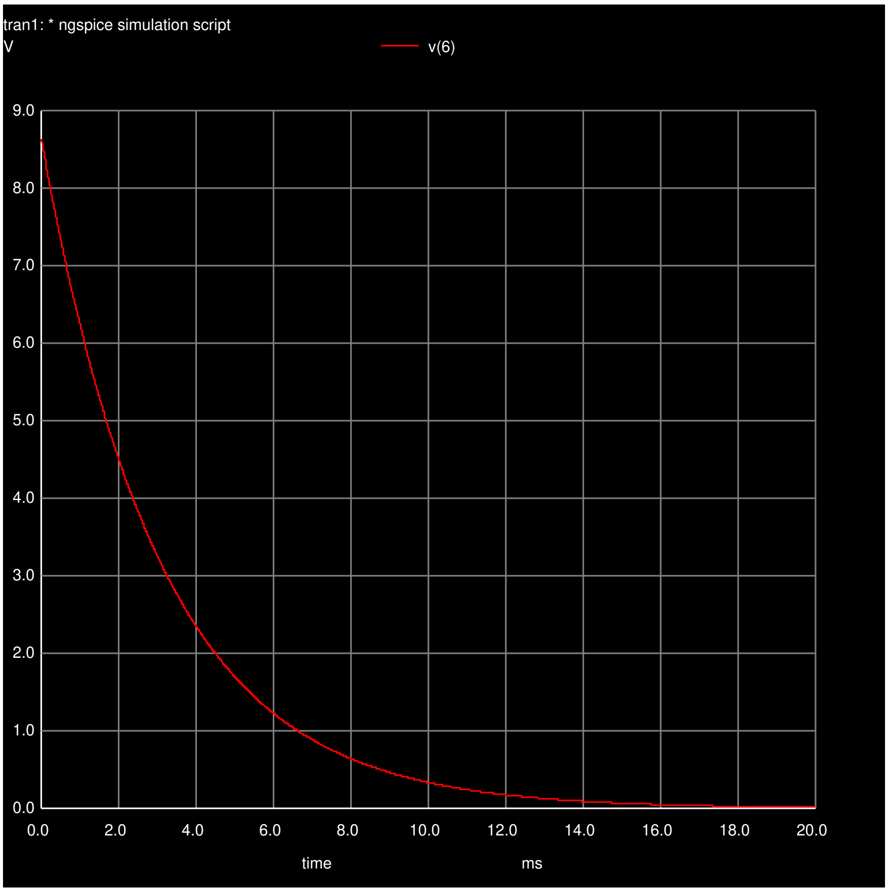

Figure 3: Natural response.

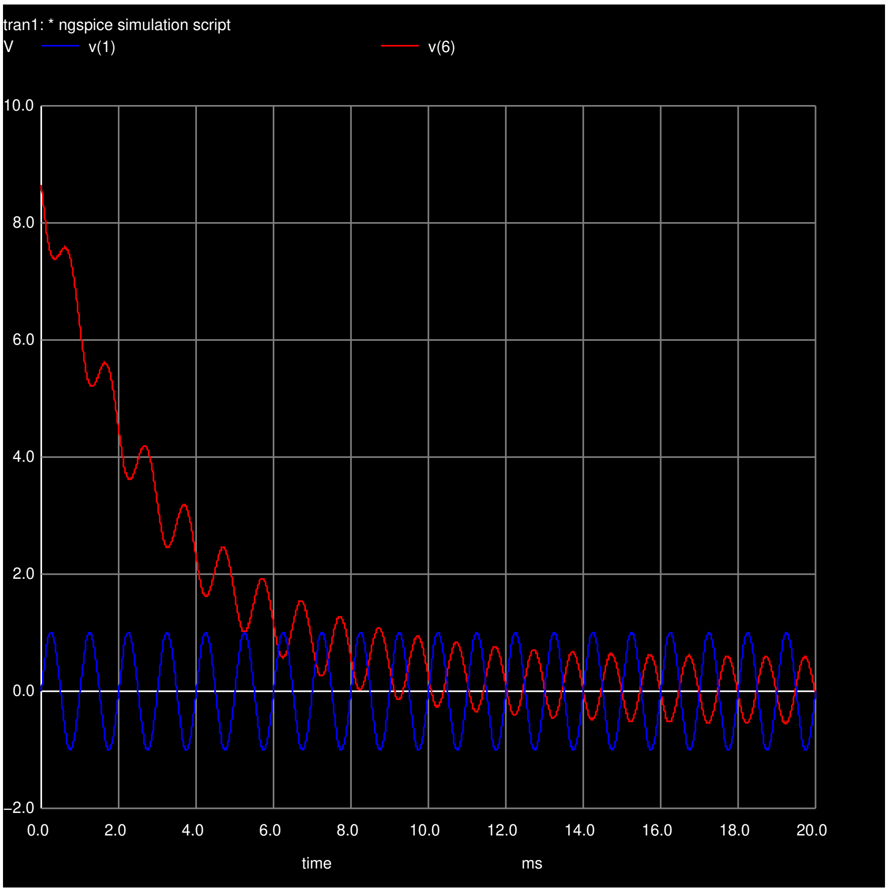

Figure 4: Forced sinusoidal response and stimulus.