

# Circuits Theory and Electronic Fundamentals

Integrated Master in Engineering Physics, IST, University of Lisbon

## Lab 2: RC Circuit Analysis

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## 1 Introduction

The objective of this laboratory assignment is to analyse the RC circuit shown in Figure 1. As shown below, the nodes have been numbered, current names and directions have been assigned to all branches and potential 0V has been assigned to one of the nodes. By running the Python script `t2_datagen.py`, the values shown in Table 1 have been obtained.

In Section 2, a theoretical analysis of the circuit and the results obtained with the Octave math tool are presented. In Section 3, the results obtained using the Ngspice simulation tool are shown. The conclusions of this study are outlined in Section 4, in which the theoretical results obtained in Section 2 are compared to those presented in Section 3.

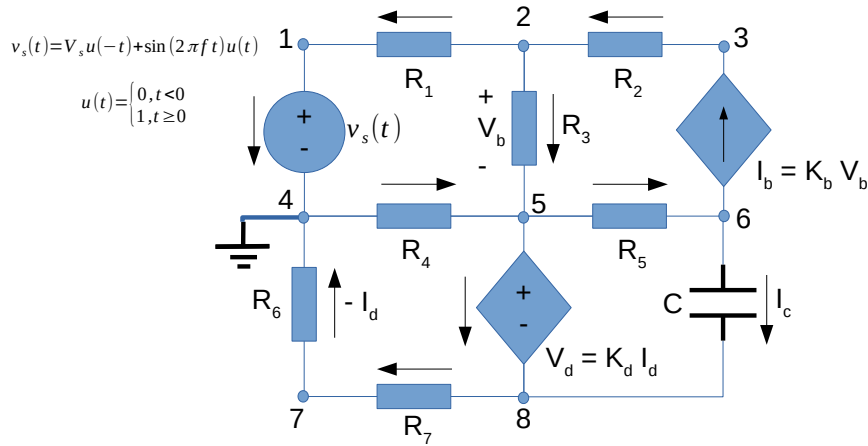


Figure 1: Circuit to be analysed in this laboratory assignment.

Designation	Value [V, k $\Omega$ , mS or $\mu$ F]
$R_1$	1.04053890347
$R_2$	2.00185929606
$R_3$	3.06593231919
$R_4$	4.15163583349
$R_5$	3.03409481751
$R_6$	2.05654586148
$R_7$	1.00587575204
$V_s$	5.16821048288
$C$	1.0127707267
$K_b$	7.29055867767
$K_d$	8.22649929708

Table 1: Values obtained by running the file `t2.datagen.py`. Resistances  $R_i$  and constant  $K_d$  are in k $\Omega$ , voltage  $V_s$  is in volts, capacitance  $C$  is in microfarads and constant  $K_b$  is in milisiemens.

## 2 Theoretical Analysis

### 2.1 Exercise 1

In this section, the circuit shown in Figure 1 is analysed theoretically for  $t < 0$ , by using the node method. The Kirchhoff Current Law (KCL) states that the sum of the currents converging or diverging in a node is null. The nodes considered for the following equations are those represented in Figure 1. Using KCL and Ohm's Law (which can also be written as  $I = VG$ ) in nodes not connected to voltage sources and additional equations for nodes related by voltage sources, it is possible to obtain a linear system from which the voltages at nodes  $V_1$  to  $V_8$  and currents in resistances  $R_1$  to  $R_7$  can be determined.

It is worth mentioning that

The following linear system is obtained:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} & 0 & -\frac{1}{R_3} & 0 & 0 & 0 \\ 0 & -K_b - \frac{1}{R_2} & \frac{1}{R_2} & 0 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_3} & 0 & -\frac{1}{R_4} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} & -\frac{1}{R_5} & -\frac{1}{R_7} & \frac{1}{R_7} \\ 0 & K_b & 0 & 0 & -\frac{1}{R_5} - K_b & \frac{1}{R_5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{R_6} + \frac{1}{R_7} & -\frac{1}{R_7} \\ 0 & 0 & 0 & -\frac{K_d}{R_6} & 1 & 0 & \frac{K_d}{R_6} & -1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{pmatrix} = \begin{pmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (1)$$

By solving the linear system 3, the values for node voltages and branch currents (calculated by using Ohm's Law) are obtained. These values, as well as the currents passing through voltage sources and currents  $I_c$  and  $I_b$ , are shown in Table 2.

Designation	Value [A or V]
$I_1$	-2.34143283E-04
$I_2$	-2.45108969E-04
$I_3$	-1.09656866E-05
$I_4$	-1.19427511E-03
$I_5$	-2.45108969E-04
$I_6$	-9.60131827E-04
$I_7$	-9.60131827E-04
$I_b$	-2.45108969E-04
$I_c$	4.33680869E-19
$I_{V_s}$	-2.34143283E-04
$I_{V_d}$	-9.60131827E-04
$V_1$	5.16821048E+00
$V_2$	4.92457529E+00
$V_3$	4.43390162E+00
$V_5$	4.95819534E+00
$V_6$	5.70187920E+00
$V_7$	-1.97455514E+00
$V_8$	-2.94032846E+00

Table 2: Values of node voltages (in volts) and currents (in amperes). Current  $I_i$  corresponds to the current passing through resistance  $R_i$ .

## 2.2 Exercise 2

In this exercise we are going to analyse the system also using the node method, as explained before. In this case our goal is to get the  $R_{eq}$ , which means, to find the value of the resistance for the Norton and Thévenin equivalents. In order to calculate the resistance one must turn off all the constant sources, so, as we want to find the equivalent seen from the capacitor, we must have  $V_s = 0$ . That's the reason, why we do that. Now we are going to use the nodal method to determine the current going to the capacitor if we put a current source with its voltage in its place and then, as we know the current and the voltage, one can simply use the Ohm's law:

$$R = \frac{V}{I} \quad (2)$$

We can use almost all the matrix in the previous exercise. The only differences are the ones stated in the previous paragraph. As we have already determined the voltages in the nodes

before, we obtain the matrix above to get the voltages in the nodes, defining  $V_x$  as the value of  $V_6 - V_8$ , and knowing that  $V_4 = 0$ , we already simplified a little in order to make the implementation easier:

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} & 0 & -\frac{1}{R_3} & 0 & 0 & 0 \\ 0 & -K_b - \frac{1}{R_2} & \frac{1}{R_2} & 0 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & K_b - \frac{1}{R_3} & 0 & -\frac{1}{R_4} & \frac{1}{R_3} + \frac{1}{R_4} - K_b & 0 & -\frac{1}{R_7} & \frac{1}{R_7} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{R_6} + \frac{1}{R_7} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\frac{K_d}{R_6} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_x \\ 0 \\ 0 \end{pmatrix} \quad (3)$$

Designation	Value [A or V]
$I_1$	-0.00000000E+00
$I_2$	0.00000000E+00
$I_3$	-0.00000000E+00
$I_4$	0.00000000E+00
$I_5$	-2.84836440E-03
$I_6$	0.00000000E+00
$I_7$	-0.00000000E+00
$I_b$	0.00000000E+00
$I_x$	-2.84836440E-03
$I_{V_s}$	-0.00000000E+00
$I_{V_d}$	2.84836440E-03
$V_1$	0.00000000E+00
$V_2$	-0.00000000E+00
$V_3$	0.00000000E+00
$V_5$	0.00000000E+00
$V_6$	8.64220765E+00
$V_7$	0.00000000E+00
$V_8$	-0.00000000E+00

Table 3: Values of node voltages (in volts) and branch currents (in amperes). Current  $I_i$  corresponds to the current passing through resistance  $R_i$ .

Now that we have all this values, we are simply going to calculate the current going throw the capacitor/voltage source as the different between the current going throw  $R_5$  and the current  $I_b$ , finally getting the result we want. We state that the current is flowing in the oposite direction of what it was suppose to flow, so, the equivalent resistance is the symetric of the result:

Designation	Value [A or V]
$I_b$	0.00000000E+00
$I_5$	-2.84836440E-03
$I_x$	-2.84836440E-03
$R_{eq}$	3.03409482E+03

Table 4: Values of node voltages (in volts) and branch currents (in amperes). Current  $I_i$  corresponds to the current passing through resistance  $R_i$ .

### 2.3 Exercise 3

Using the results from the previous exercise we understand we can simplify the circuit into a circuit with a voltage source, a resistance and and the capacitor. For this case we already have that the natural solution looks like:

$$v_n(t) = A \times e^{-\frac{t}{RC}} \quad (4)$$

Where A is the value in  $t = 0$  according to the previous exercise,  $t$  stands for time,  $R$  for the equivalent resistance and  $C$  for the capacitance of the capacitor. Plotting the result one gets:

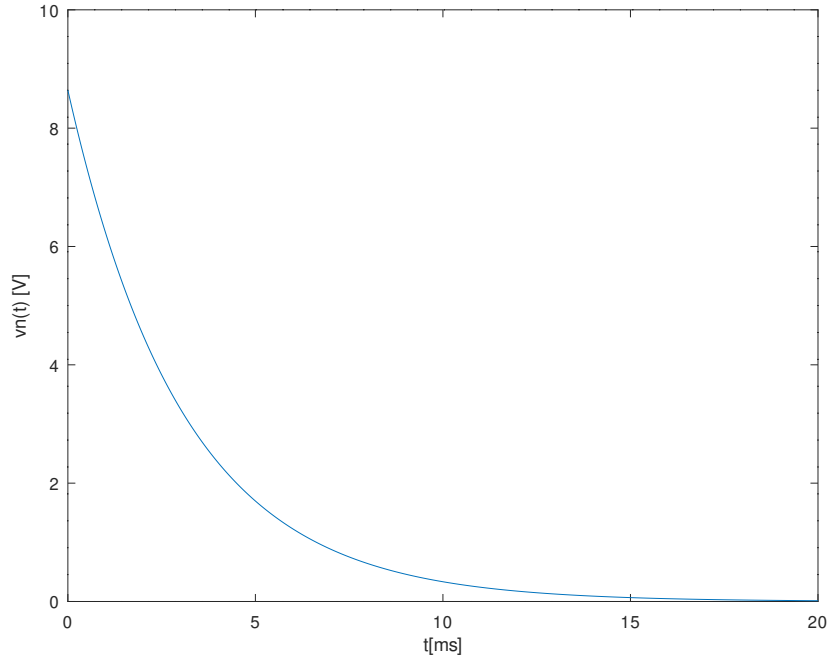


Figure 2: Natural response

### 2.4 Exercise 4

In this exercise we are going to determine the forced solution of the equation, in order to get the final solution adding the natural solution tot he forced solution. We first define  $V_s$  as 1. As the whole system is linear, so the response should be. Therefore if we aply the voltage  $V_s = 1$  we get the response of the system for one Volt, then we only have to multiply the voltages by the real  $V_s$  in order to get the real voltages. Then we get the matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} & 0 & -\frac{1}{R_3} & 0 & 0 & 0 \\ 0 & -K_b - \frac{1}{R_2} & \frac{1}{R_2} & 0 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_3} & 0 & 0 & \frac{1}{R_3} + \frac{1}{R_4} - \frac{1}{R_5} & -\frac{1}{R_5} - j\omega C & -\frac{1}{R_7} n & \frac{1}{R_7} + j\omega C \\ 0 & K_b & 0 & 0 & -K_b - \frac{1}{R_5} & \frac{1}{R_5} + j\omega C & 0 & -j\omega C \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{R_6} + \frac{1}{R_7} & -\frac{1}{R_7} \\ 0 & 0 & 0 & 0 & 1 & 0 & \frac{K_d}{R_6} & -1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{pmatrix} = \begin{pmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (5)$$

And results are:

Designation	Value [A or V]
$V_1$	1.00000000E+00+0.00000000E+00i
$V_2$	9.52858887E-015.38396699E-17i
$V_3$	8.57918158E-019.63868378E-16i
$V_4$	0.00000000E+00+0.00000000E+00i
$V_5$	9.59364050E-01-8.51381547E-18i
$V_6$	-5.64451991E-01-8.63775455E-02i
$V_7$	-3.82057802E-01+3.39054775E-18i
$V_8$	-5.68925834E-01+5.04889626E-18i

Table 5: Values of node complex amplitude (in volts).

## 2.5 Exercise 5

In this exercise we are going to plot the sum of the results of the 2 previous exercises. We obtain the following plot:

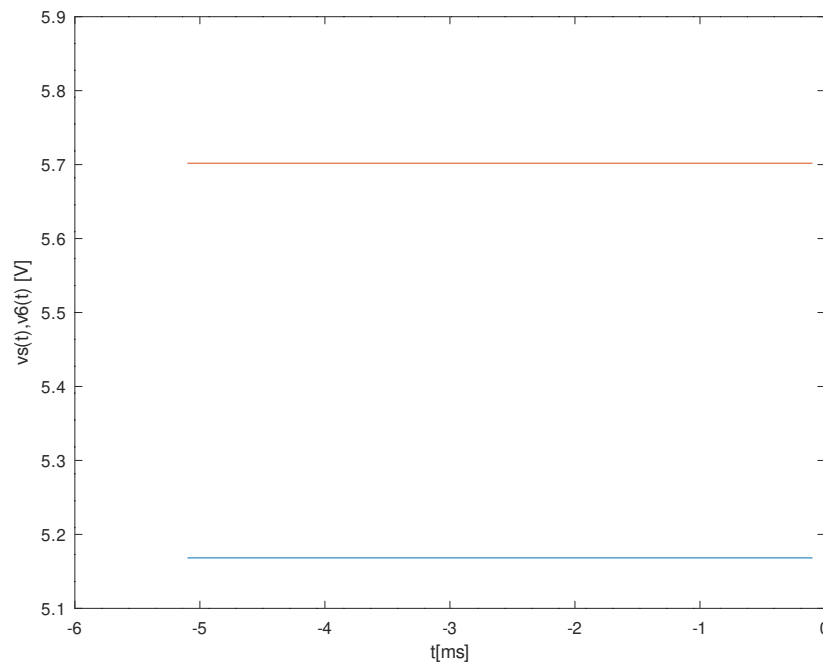


Figure 3: Final solution[-5;0]ms

We see that  $V_6$  is not a continuous function, but it should not be, as the voltage source doesn't have a continuous function for the source. Yet, we notice that the voltage in the capacitor is continuous, which is, in fact, the only thing that matters to make our system possible.

## 2.6 Exercise 6

In this exercise as we only want to compare the magnitudes and the phases of  $V_s, V_6$  and  $V_c$  for each of the different frequencies. The results are shown in the plots below:

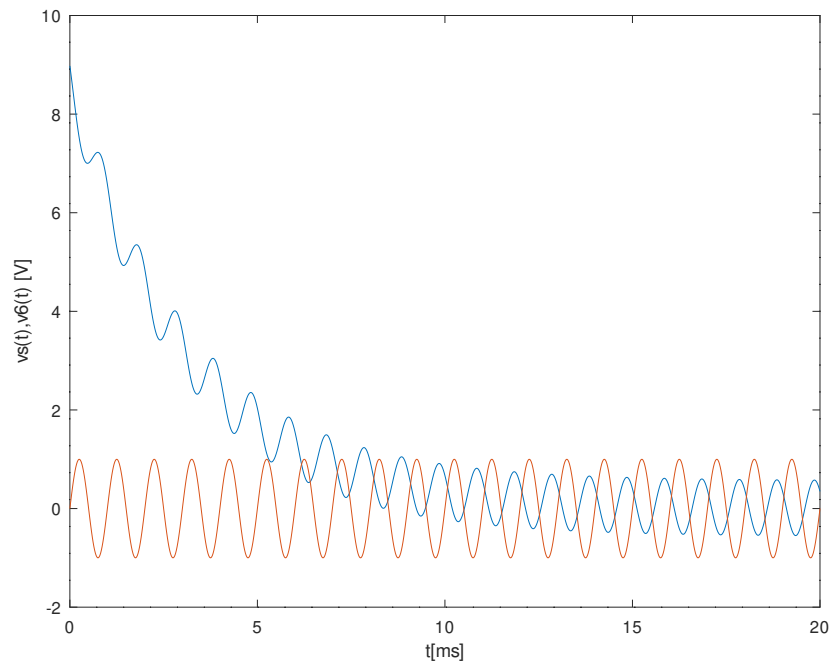


Figure 4: Final solution[0;20]ms

### 3 Simulation Analysis

#### 3.1 Exercise 1

Table 6 shows the simulated operating point results for the circuit presented in Figure 1. Again, currents designated below as  $I_i$  refer to the currents passing through the respective resistances,  $R_i$ .

Designation	Value [A or V]
$I_1$	-2.34143e-04
$I_2$	-2.45109e-04
$I_3$	-1.09657e-05
$I_4$	-1.19428e-03
$I_5$	-2.45109e-04
$I_6$	-9.60132e-04
$I_7$	-9.60132e-04
$I_b$	-2.45109e-04
$I_c$	0.000000e+00
$I_{V_s}$	-2.34143e-04
$I_{V_d}$	-9.60132e-04
$V_1$	5.168210e+00
$V_2$	4.924575e+00
$V_3$	4.433902e+00
$V_5$	4.958195e+00
$V_6$	5.701879e+00
$V_7$	-1.97456e+00
$V_8$	-2.94033e+00

Table 6: Operating point analysis table. Currents  $I_i$  are in amperes; voltages  $V_i$  are in volts.

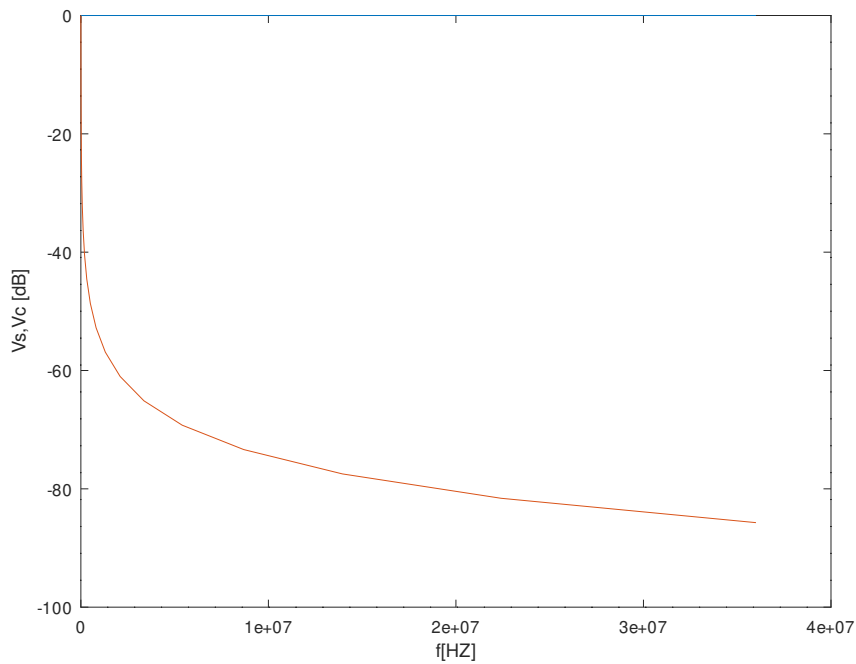


Figure 5: Magnitude of nodes

Comparing the theoretical analysis results presented in Table 2 and the results in Table 6, we can notice almost no differences. This was to be expected, since the circuit has no time dependency - meaning it's equal at any point in time. There is only a small difference between the two values of  $I_c$ , although it is negligible.

### 3.2 Exercise 2

Now we can approach the second point of our simulation where we see how the system behaves when  $v_s(0) = 0$  and the capacitor is replaced by a voltage source  $V_X = V(6) - V(8)$ , where the voltages are taken from 6.



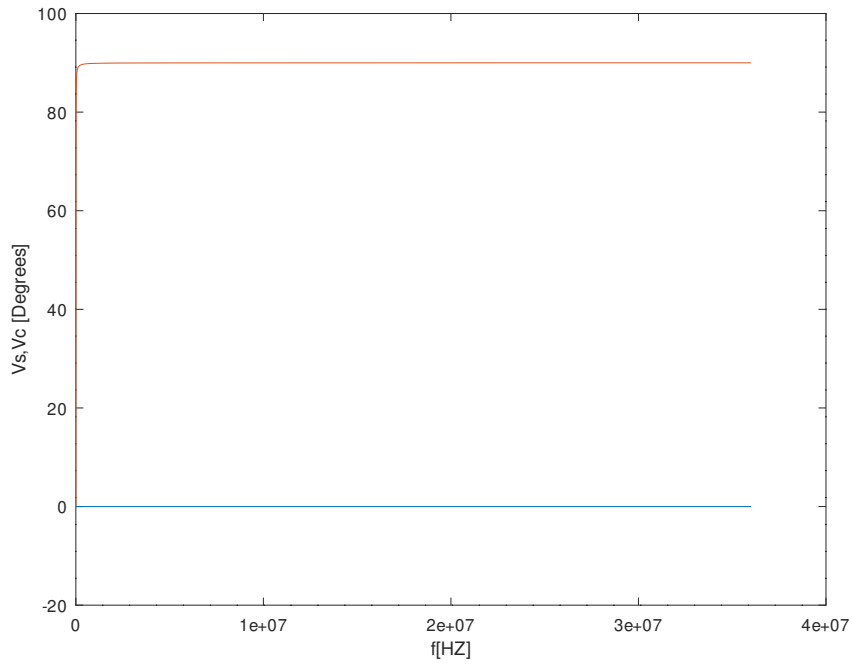


Figure 6: Phase of nodes

Designation	Value [A or V]
$I_1$	0.000000e+00
$I_2$	0.000000e+00
$I_3$	0.000000e+00
$I_4$	0.000000e+00
$I_5$	-2.84836e-03
$I_6$	0.000000e+00
$I_7$	0.000000e+00
$I_b$	0.000000e+00
$I_x$	-2.84836e-03
$I_{V_s}$	0.000000e+00
$I_{V_d}$	2.848365e-03
$V_1$	0.000000e+00
$V_2$	0.000000e+00
$V_3$	0.000000e+00
$V_5$	0.000000e+00
$V_6$	8.642208e+00
$V_7$	0.000000e+00
$V_8$	0.000000e+00

Table 7: Operating point analysis table. Currents  $I_i$  are in amperes; voltages  $V_i$  are in volts.

### 3.3 Exercise 3

Now we do a transient analysis of the values. Figure 7 shows the simulated transient analysis results obtained for the voltage in node 6.

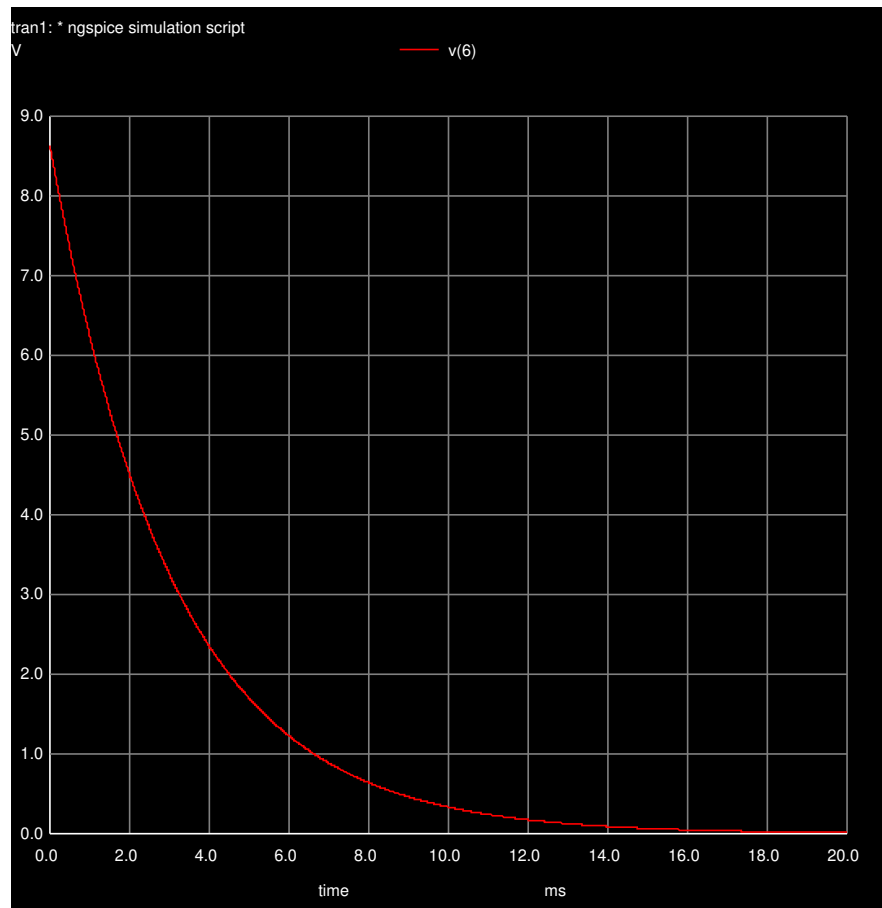


Figure 7: Natural response - value of  $v_6(t)$  in the time interval  $[0,20]$  ms.

### 3.4 Exercise 4

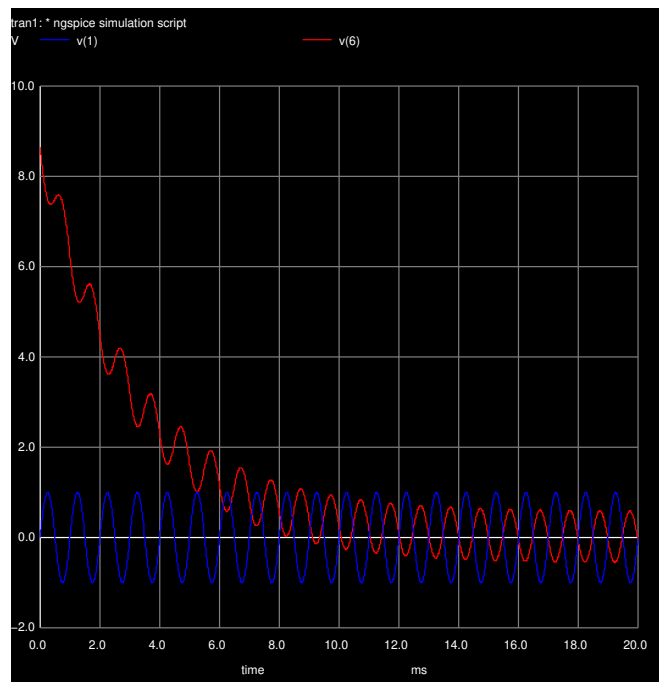


Figure 8: Forced sinusoidal response and stimulus on node 6 in time interval [0,20] ms.

### 3.5 Exercise 5

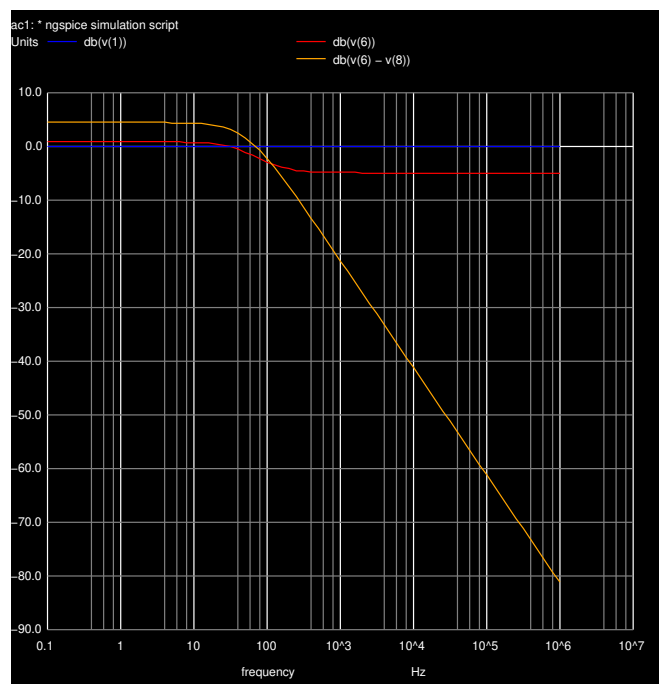


Figure 9: Frequency analysis - magnitudes of  $v_6(f)$ ,  $v_8(f)$  and  $v_c(f) = v_6(f) - v_8(f)$  in interval  $f=[0.1, 10^6]$  Hz and in dB.

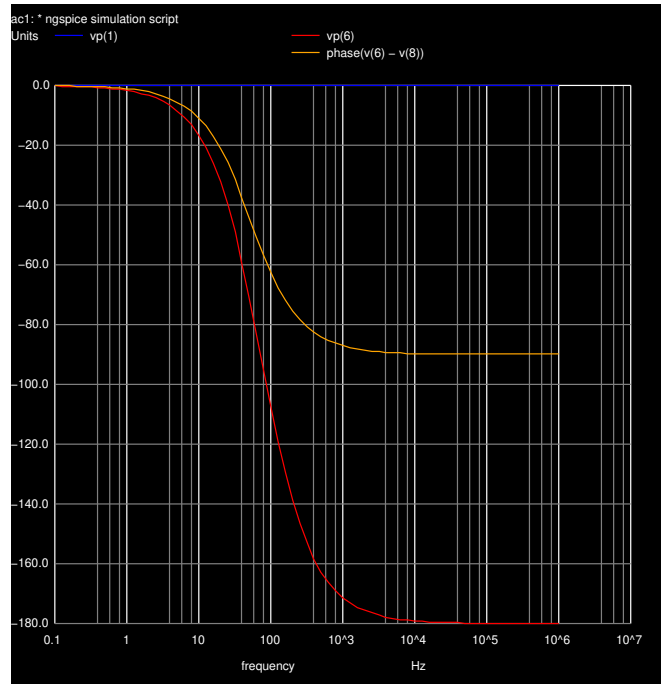


Figure 10: Frequency analysis - phases of  $v_6(f)$ ,  $v_8(f)$  and  $v_c(f) = v_6(f) - v_8(f)$  in interval  $f=[0.1, 10^6]$  Hz and in degrees.

## 4 Conclusion

Theoretical		Simulation	
Designation	Value [A or V]	Designation	Value [A or V]
$I_1$	-2.34143283E-04	$I_1$	-2.34143e-04
$I_2$	-2.45108969E-04	$I_2$	-2.45109e-04
$I_3$	-1.09656866E-05	$I_3$	-1.09657e-05
$I_4$	-1.19427511E-03	$I_4$	-1.19428e-03
$I_5$	-2.45108969E-04	$I_5$	-2.45109e-04
$I_6$	-9.60131827E-04	$I_6$	-9.60132e-04
$I_7$	-9.60131827E-04	$I_7$	-9.60132e-04
$I_b$	-2.45108969E-04	$I_b$	-2.45109e-04
$I_c$	4.33680869E-19	$I_c$	0.000000e+00
$I_{V_s}$	-2.34143283E-04	$I_{V_s}$	-2.34143e-04
$I_{V_d}$	-9.60131827E-04	$I_{V_d}$	-9.60132e-04
$V_1$	5.16821048E+00	$V_1$	5.168210e+00
$V_2$	4.92457529E+00	$V_2$	4.924575e+00
$V_3$	4.43390162E+00	$V_3$	4.433902e+00
$V_5$	4.95819534E+00	$V_5$	4.958195e+00
$V_6$	5.70187920E+00	$V_6$	5.701879e+00
$V_7$	-1.97455514E+00	$V_7$	-1.97456e+00
$V_8$	-2.94032846E+00	$V_8$	-2.94033e+00

Table 8: Exercise 1 comparison.

Theoretical		Simulation	
Designation	Value [A or V]	Designation	Value [A or V]
$I_1$	-0.00000000E+00	$I_1$	0.000000e+00
$I_2$	0.00000000E+00	$I_2$	0.000000e+00
$I_3$	-0.00000000E+00	$I_3$	0.000000e+00
$I_4$	0.00000000E+00	$I_4$	0.000000e+00
$I_5$	-2.84836440E-03	$I_5$	-2.84836e-03
$I_6$	0.00000000E+00	$I_6$	0.000000e+00
$I_7$	-0.00000000E+00	$I_7$	0.000000e+00
$I_b$	0.00000000E+00	$I_b$	0.000000e+00
$I_x$	-2.84836440E-03	$I_x$	-2.84836e-03
$I_{V_s}$	-0.00000000E+00	$I_{V_s}$	0.000000e+00
$I_{V_d}$	2.84836440E-03	$I_{V_d}$	2.848365e-03
$V_1$	0.00000000E+00	$V_1$	0.000000e+00
$V_2$	-0.00000000E+00	$V_2$	0.000000e+00
$V_3$	0.00000000E+00	$V_3$	0.000000e+00
$V_5$	0.00000000E+00	$V_5$	0.000000e+00
$V_6$	8.64220765E+00	$V_6$	8.642208e+00
$V_7$	0.00000000E+00	$V_7$	0.000000e+00
$V_8$	-0.00000000E+00	$V_8$	0.000000e+00

Table 9: Exercise 2 comparison.