Circuits Theory and Electronic Fundamentals

Integrated Master in Engineering Physics, IST, University of Lisbon

Lab 2: RC Circuit Analysis

Alexandre Sequeira (96503), Duarte Marques (96523), João Chaves (96540)

April 8, 2021

Contents

1 Introduction

The objective of this laboratory assignment is to analyse the RC circuit shown in Figure 1. As shown below, the nodes have been numbered, current names and directions have been assigned to all branches and potential $0V$ has been assigned to one of the nodes. By running the Python script `t2_datagen.py`, the values shown in Table 1 have been obtained.

In Section 2, a theoretical analysis of the circuit and the results obtained with the Octave math tool are presented. In Section 3, the results obtained using the Ngspice simulation tool are shown. The conclusions of this study are outlined in Section 4, in which the theoretical results obtained in Section 2 are compared to those presented in Section 3.

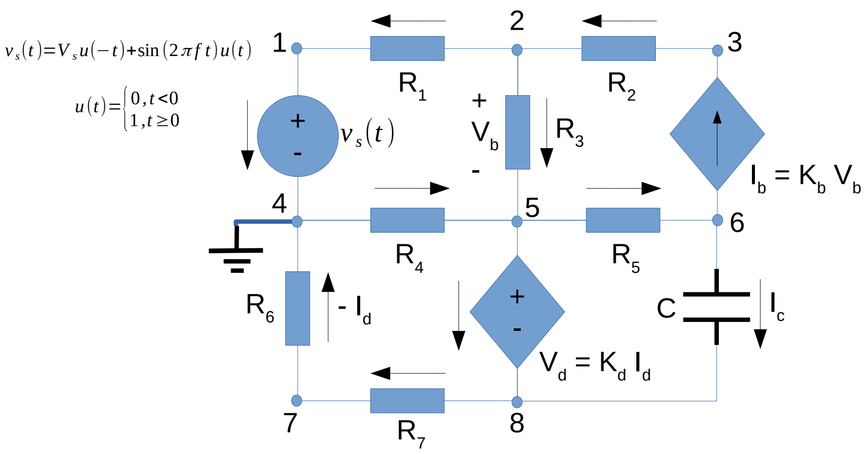

Figure 1: Circuit to be analysed in this laboratory assignment.

Designation	**Value [V, kΩ, mS or μF]**
R_1	1.04053890347
R_2	2.00185929606
R_3	3.06593231919
R_4	4.15163583349
R_5	3.03409481751
R_6	2.05654586148
R_7	1.00587575204
V_s	5.16821048288
C	1.0127707267
K_b	7.29055867767
K_d	8.22649929708

Table 1: Values obtained by running the file `t2_datagen.py`. Resistances R_i and constant K_d are in kΩ, voltage V_s is in volts, capacitance C is in microfarads and constant K_b is in milisiemens.

2 Theoretical Analysis

2.1 Exercise 1

In this section, the circuit shown in Figure 1 is analysed theoretically, by using the node method. The Kirchhoff Current Law (KCL) states that the sum of the currents converging or diverging in a node is null. The nodes considered for the following equations are those represented in Figure 1. Using KCL and Ohm's Law (which can also be written as $I = VG$) in nodes not connected to voltage sources and additional equations for nodes related by voltage sources, it is possible to obtain a linear system from which the voltages at nodes V_1 to V_8 and currents in resistances R_1 to R_7 for $t < 0$ can be determined.

It is worth mentioning that

The following linear system is obtained:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} & 0 & -\frac{1}{R_3} & 0 & 0 & 0 \\ 0 & -K_b-\frac{1}{R_2} & \frac{1}{R_2} & 0 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_3} & 0 & -\frac{1}{R_4} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} & -\frac{1}{R_5} & -\frac{1}{R_7} & \frac{1}{R_7} \\ 0 & K_b & 0 & 0 & -\frac{1}{R_5}-K_b & \frac{1}{R_5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{R_6}+\frac{1}{R_7} & -\frac{1}{R_7} \\ 0 & 0 & 0 & -\frac{K_d}{R_6} & 1 & 0 & \frac{K_d}{R_6} & -1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{pmatrix} = \begin{pmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{1}$$

By solving the linear system 3, the values for node voltages and branch currents (calculated by using Ohm's Law) are obtained. These values, as well as the currents passing through voltage sources and currents I_c and I_b, are shown in Table 2.

Designation	**Value [A or V]**
I_1	-2.34143283E-04
I_2	-2.45108969E-04
I_3	-1.09656866E-05
I_4	-1.19427511E-03
I_5	-2.45108969E-04
I_6	-9.60131827E-04
I_7	-9.60131827E-04
I_b	-2.45108969E-04
I_c	4.33680869E-19
I_{V_s}	-2.34143283E-04
I_{V_d}	-9.60131827E-04
V_1	5.16821048E+00
V_2	4.92457529E+00
V_3	4.43390162E+00
V_5	4.95819534E+00
V_6	5.70187920E+00
V_7	-1.97455514E+00
V_8	-2.94032846E+00

Table 2: Values of node voltages (in volts) and currents (in amperes). Current I_i corresponds to the current passing through resistance R_i.

2.2 Exercise 2

In this exercise we are going to analise the system also using the node method, as explained before. In this case our goal is to the get the R_{eq}, which means, to find the value of the resistance for the Norton and Thévenin equivalents. In order to calculate the resistance one must turn off all the constant sources, so, as we want to find the equivalent seen from the capacitor, we must have $V_S = 0$. That's the reason, why we do that. Now we are going to use the nodal method to determine the current going to the capacitor if we put a current source wi3th its voltage in its place and then, as we know the current and the voltage, one can simply use the Ohm's law:

$$R = \frac{V}{I} \tag{2}$$

We can use almost all the matrix in the previous exercise. The only differences are the ones stated in the previous paragraph. As we have already determined the voltages in the nodes before, we obtain the matrix above to get the voltages in the nodes,definig V_x as the value of $V_6 - V_8$,and knowing that $V_4 = 0$, we already symplified a little in order to make the implementation easier:

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} & 0 & -\frac{1}{R_3} & 0 & 0 & 0 \\ 0 & -K_b-\frac{1}{R_2} & \frac{1}{R_2} & 0 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & K_b-\frac{1}{R_3} & 0 & -\frac{1}{R_4} & \frac{1}{R_3}+\frac{1}{R_4}-K_b & 0 & -\frac{1}{R_7} & \frac{1}{R_7} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{R_6}+\frac{1}{R_7} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\frac{K_d}{R_6} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V_x \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

Designation	Value [A or V]
I_1	-0.00000000E+00
I_2	0.00000000E+00
I_3	-0.00000000E+00
I_4	0.00000000E+00
I_5	-2.84836440E-03
I_6	0.00000000E+00
I_7	-0.00000000E+00
I_b	0.00000000E+00
I_x	-2.84836440E-03
I_{V_s}	-0.00000000E+00
I_{V_d}	2.84836440E-03
V_1	0.00000000E+00
V_2	-0.00000000E+00
V_3	0.00000000E+00
V_5	0.00000000E+00
V_6	8.64220765E+00
V_7	0.00000000E+00
V_8	-0.00000000E+00

Table 3: Values of node voltages (in volts) and branch currents (in amperes). Current I_i corresponds to the current passing through resistance R_i.

Now that we have all this values, we are simply going to calculate the current going throw the capacitor/voltage source as the different between the current going throw R_5 and the current I_b, finally getting the result we want. We state that the current is flowing in the oposite direction of what it was suppose to flow, so, the equivalent resistance is the symetric of the result:

Designation	Value [A or V]
I_b	0.00000000E+00
I_5	-2.84836440E-03
I_x	-2.84836440E-03
R_{eq}	3.03409482E+03

Table 4: Values of node voltages (in volts) and branch currents (in amperes). Current I_i corresponds to the current passing through resistance R_i.

2.3 Exercise 3

Using the results from the previous exercise we understand we can simplify the circuit into a circuit with a voltage source, a resistance and and the capacitor. For this case we already have that the natural solution looks like:

$$v_n(t) = A \times e^{-\frac{t}{RC}} \tag{4}$$

Where A is the value in $t = 0$ acording to the previous exercise, t stands for time, R for the equivalent resistance and C for the capacitance of the capacitor. Plotting the result one gets:

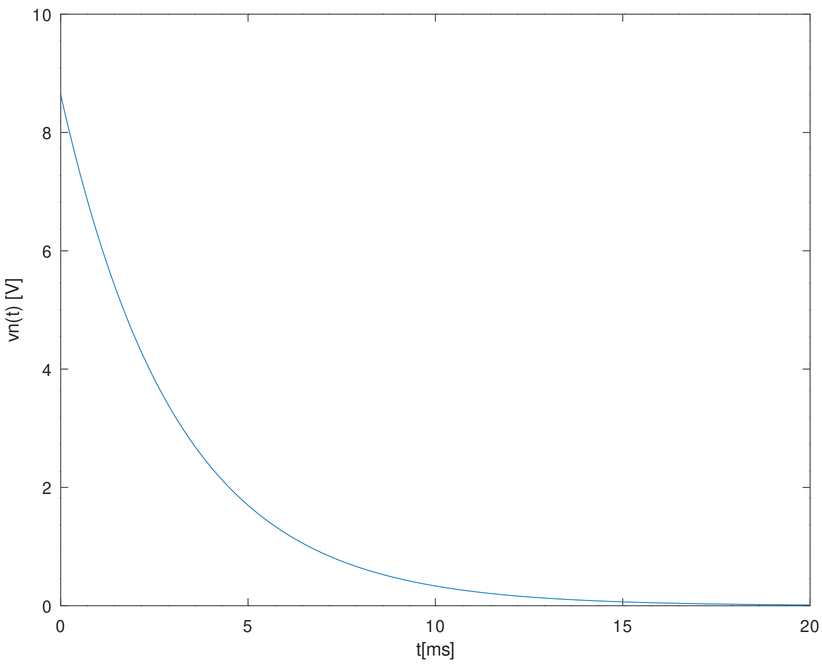

Figure 2: Natural response

2.4 Exercise 4

In this exercise we are going to determine the forced solution of the equation, in order to get the final solution ading the natural solution tot he forced solution. We first define V_s as 1. As the whole system is linear, so the response should be. Therefore if we aplly the voltage $V_s = 1$ we get the response of the system for one Volt, then we only have to multiply the voltages by the real V_s in order to get the real voltages. Then we get the matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} & 0 & -\frac{1}{R_3} & 0 & 0 & 0 \\ 0 & -K_b-\frac{1}{R_2} & \frac{1}{R_2} & 0 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_3} & 0 & 0 & \frac{1}{R_3}+\frac{1}{R_4}-\frac{1}{R_5} & -\frac{1}{R_5}-jwC & -\frac{1}{R_7}n & \frac{1}{R_7}+jwC \\ 0 & K_b & 0 & 0 & -K_b-\frac{1}{R_5} & \frac{1}{R_5}+jwC & 0 & -jwC \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{R_6}+\frac{1}{R_7} & -\frac{1}{R_7} \\ 0 & 0 & 0 & 0 & 1 & 0 & \frac{K_d}{R_6} & -1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{pmatrix} = \begin{pmatrix} Vs \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

And results are:

Designation	Value [A or V]
V_1	1.00000000E+00+0.00000000E+00i
V_2	9.52858887E-015.38396699E-17i
V_3	8.57918158E-019.63868378E-16i
V_4	0.00000000E+00+0.00000000E+00i
V_5	9.59364050E-01-8.51381547E-18i
V_6	-5.64451991E-01-8.63775455E-02i
V_7	-3.82057802E-01+3.39054775E-18i
V_8	-5.68925834E-01+5.04889626E-18i

Table 5: Values of node complex amplitude (in volts).

2.5 Exercise 5

In this exercise we are going to plot the sum of the results of the 2 previous exercises. We obtain the following plot:

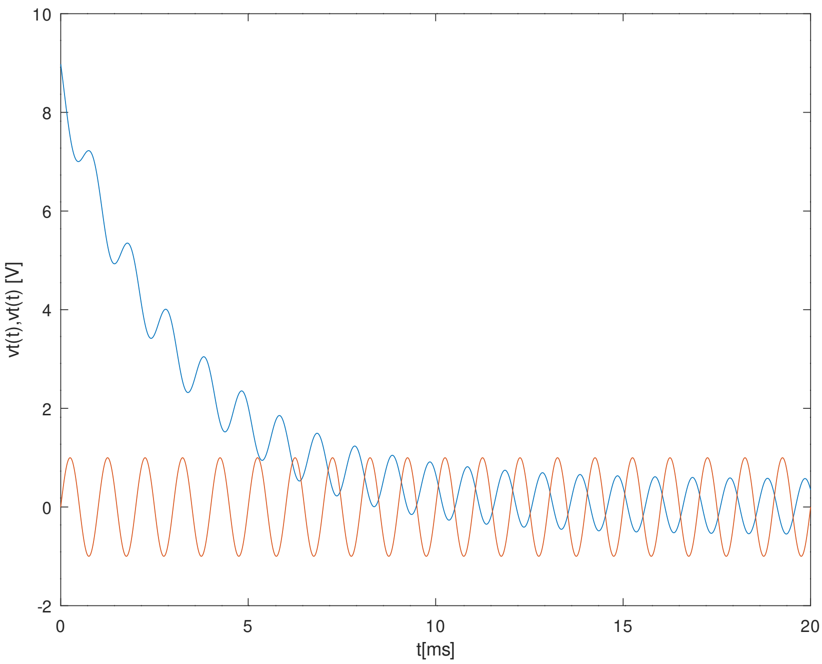

Figure 3: Final solution

2.6 Exercise 6

In this exercise as we only want to compare the magnitudes and the phases of V_s,V_6 and V_c for each of the different frequencies. The results are shown in the plots below:

3 Simulation Analysis

3.1 Exercise 1

Table 6 shows the simulated operating point results for the circuit presented in Figure 1. Again, currents designated below as I_i refer to the currents passing through the respective resistances, R_i.

Designation	Value [A or V]
I_1	-2.34143e-04
I_2	-2.45109e-04
I_3	-1.09657e-05
I_4	-1.19428e-03
I_5	-2.45109e-04
I_6	-9.60132e-04
I_7	-9.60132e-04
I_b	-2.45109e-04
I_c	0.000000e+00
I_{V_s}	-2.34143e-04
I_{V_d}	-9.60132e-04
V_1	5.168210e+00
V_2	4.924575e+00
V_3	4.433902e+00
V_5	4.958195e+00
V_6	5.701879e+00
V_7	-1.97456e+00
V_8	-2.94033e+00

Table 6: Operating point analysis table. Currents I_i are in amperes; voltages V_i are in volts.

Comparing the theoretical analysis results presented in Table 2 and the results in Table 6, we can notice almost no differences. This was to be expected, since the circuit has no time dependency - meaning it's equal at any point in time. There is only a small difference between the two values of I_c, although it is negligible.

3.2 Exercise 2

Now we can approach the second point of our simulation where we see how the system behaves when $v_s(0) = 0$ and the capacitor is replaced by a voltage souce V_X=V(6)-V(8), where the voltages are taken from 6.

Designation	Value [A or V]
I_1	0.000000e+00
I_2	0.000000e+00
I_3	0.000000e+00
I_4	0.000000e+00
I_5	-2.84836e-03
I_6	0.000000e+00
I_7	0.000000e+00
I_b	0.000000e+00
I_x	-2.84836e-03
I_{V_s}	0.000000e+00
I_{V_d}	2.848365e-03
V_1	0.000000e+00
V_2	0.000000e+00
V_3	0.000000e+00
V_5	0.000000e+00
V_6	8.642208e+00
V_7	0.000000e+00
V_8	0.000000e+00

Table 7: Operating point analysis table. Currents I_i are in amperes; voltages V_i are in volts.

3.3 Exercise 3

Now we do a transient analisis of the values. Figure 4 shows the simulated transient analysis results obtained for the voltage in node 6.

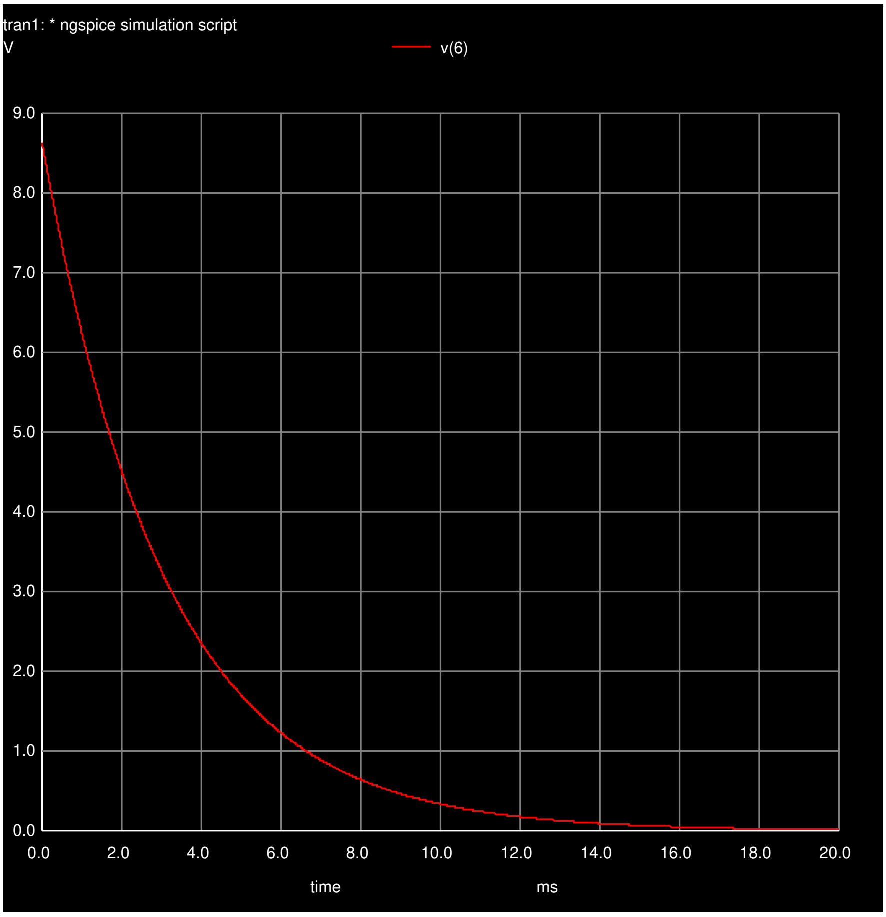

Figure 4: Natural response - value of $v_6(t$ in the time interval [0,20] ms.

3.4 Exercise 4

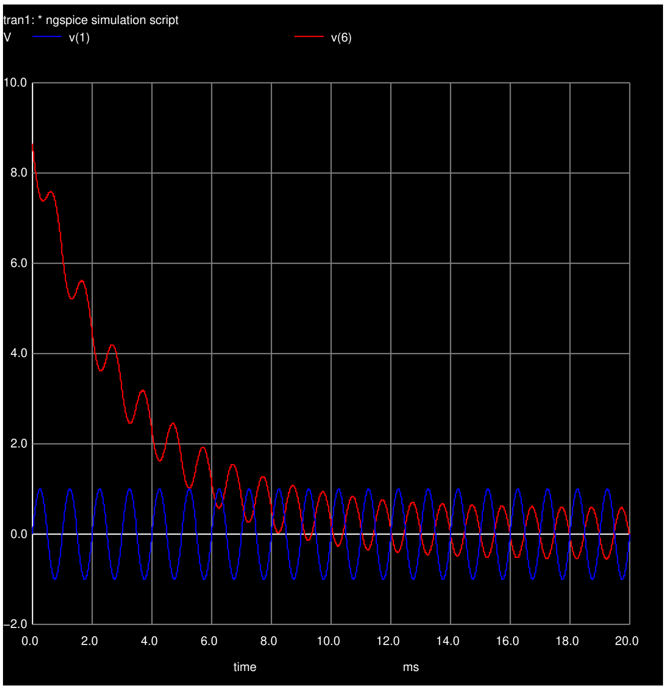

Figure 5: Forced sinusoidal response and stimulus on node 6 in time interval [0,20] ms.

3.5 Exercise 5

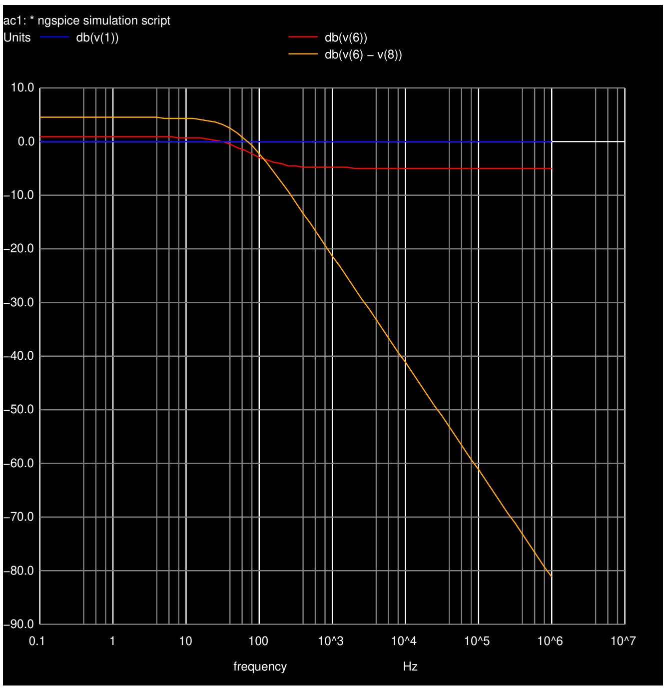

Figure 6: Frequency analysis - magnitudes of $v_6(f)$, $v_s(f)$ and $v_c(f) = v_6(f) - v_8(f)$ in interval f=[0.1,10e+6] Hz and in dB.

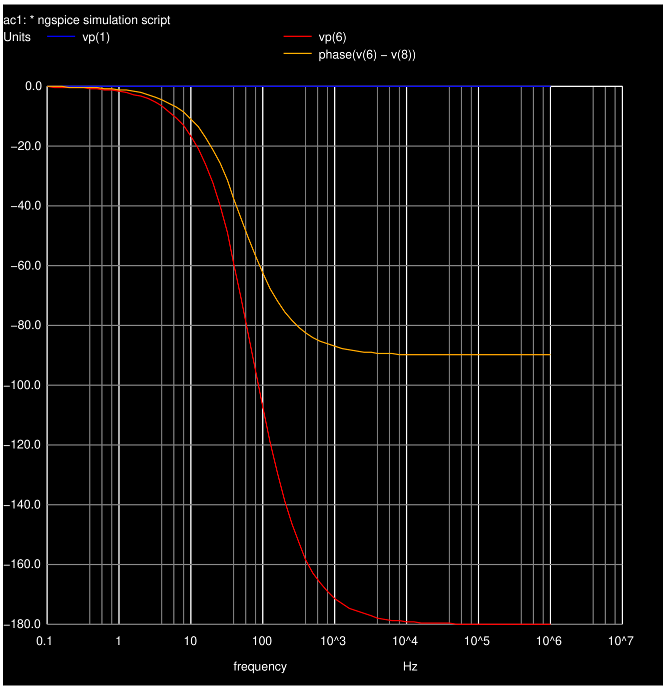

Figure 7: Frequency analysis - phases of $v_6(f)$, $v_s(f)$ and $v_c(f) = v_6(f) - v_8(f)$ in interval f=[0.1,10e+6] Hz and in degrees.

4 Conclusion

Theoretical		Simulation	
Designation	Value [A or V]	Designation	Value [A or V]
I_1	-2.34143283E-04	I_1	-2.34143e-04
I_2	-2.45108969E-04	I_2	-2.45109e-04
I_3	-1.09656866E-05	I_3	-1.09657e-05
I_4	-1.19427511E-03	I_4	-1.19428e-03
I_5	-2.45108969E-04	I_5	-2.45109e-04
I_6	-9.60131827E-04	I_6	-9.60132e-04
I_7	-9.60131827E-04	I_7	-9.60132e-04
I_b	-2.45108969E-04	I_b	-2.45109e-04
I_c	4.33680869E-19	I_c	0.000000e+00
I_{V_s}	-2.34143283E-04	I_{V_s}	-2.34143e-04
I_{V_d}	-9.60131827E-04	I_{V_d}	-9.60132e-04
V_1	5.16821048E+00	V_1	5.168210e+00
V_2	4.92457529E+00	V_2	4.924575e+00
V_3	4.43390162E+00	V_3	4.433902e+00
V_5	4.95819534E+00	V_5	4.958195e+00
V_6	5.70187920E+00	V_6	5.701879e+00
V_7	-1.97455514E+00	V_7	-1.97456e+00
V_8	-2.94032846E+00	V_8	-2.94033e+00

Table 8: Exercise 1 comparison.

Theoretical		Simulation	
Designation	Value [A or V]	Designation	Value [A or V]
I_1	-0.00000000E+00	I_1	0.000000e+00
I_2	0.00000000E+00	I_2	0.000000e+00
I_3	-0.00000000E+00	I_3	0.000000e+00
I_4	0.00000000E+00	I_4	0.000000e+00
I_5	-2.84836440E-03	I_5	-2.84836e-03
I_6	0.00000000E+00	I_6	0.000000e+00
I_7	-0.00000000E+00	I_7	0.000000e+00
I_b	0.00000000E+00	I_b	0.000000e+00
I_x	-2.84836440E-03	I_x	-2.84836e-03
I_{V_s}	-0.00000000E+00	I_{V_s}	0.000000e+00
I_{V_d}	2.84836440E-03	I_{V_d}	2.848365e-03
V_1	0.00000000E+00	V_1	0.000000e+00
V_2	-0.00000000E+00	V_2	0.000000e+00
V_3	0.00000000E+00	V_3	0.000000e+00
V_5	0.00000000E+00	V_5	0.000000e+00
V_6	8.64220765E+00	V_6	8.642208e+00
V_7	0.00000000E+00	V_7	0.000000e+00
V_8	-0.00000000E+00	V_8	0.000000e+00

Table 9: Exercise 2 comparison.