Zeno's Paradox – The Dichotomy

"If you buy the premise, you buy the joke" – anon

I believe this quote best sums up Zeno's paradox on motion "The Dichotomy." The premise for the dichotomy contains the absurdity, and if it is accepted the rest logically follows.

"For if you accept all of the steps in Zeno's argument then you must accept his conclusion (assuming that he has reasoned in a logically deductive way): it's not enough to show an unproblematic division, you must also show why the given *division is unproblematic."* [1]

The implicit assumption here is that division is a given, and that we need to prove or disprove in light of this fact. Instead I posit that disproof does not require division, calculus, infinite sums nor mathematics of any kind.

There is nothing to divide.

The Dicotomy

[The Dichotomy] asserts the non-existence of motion on the ground that that which is in locomotion must arrive at the half-way stage before it arrives at the goal. (Aristotle Physics, 239b11) [1]

Suppose Homer wants to catch a stationary bus. Before he can get there, he must get halfway there. Before he can get halfway there, he must get a quarter of the way there. Before traveling a quarter, he must travel one-eighth; before an eighth, one-sixteenth; and so on. [2]

Look closely at both of those statements and consider these terms: 'the goal', 'of the way there'. These are absurd (they do not exist) and do not govern laws of motion.

Proof by Contradiction

Consider the following scenario: Bob is a travelling salesman. He resides in city A. His normal business takes him through cities B, C and D, shown on the map below. The distance between any 2 adjacent cities below is equivalent:

A ------ B ------ C ------ D

Today Bob needs to go from A to C. Following the halving principle, first he arrives at B, which is halfway to C. The expection if we follow the paradox is that he'll keep halving the distance and getting closer to, but never quite reaching C.

However before he exits B he gets a phone call from headquarters and learns there is an urgent customer need in city D. Bob changes plans and sets out to D from B. First he traverses half the distance and arrives at C.

Result: In 2 motions Bob has arrived at C from A. How? A simple phone call changed the laws of physics?

The absurdity: "the distance" does not exist, thus there is nothing to divide in half. The absurd premise of the paradox is that there is a fixed, immutable distance that defines current and future locations, and once defined cannot be altered. Further it asserts that this distance governs the laws of motion – i.e. it is the numerator in the halving operation, reducing your motion to halved remainders vs your self-proclaimed destination. However this distance is completely contrived and has no bearing on motion.

Specifically velocity is defined as distance over time. In this usage, *distance* is measured either *in arrears* or *in the present* (instantaneous velocity), but is never constrained nor defined based on a future location. E.g. Bob travelled 60 miles in one hour, thus v = 60 / 1, or 60 miles per hour. Velocity is not constrained by where Bob thinks he is going *in the future*, proven by the scenario above: a change in plans renders "the distance" meaningless.

Simple deceit would accomplish the same goal – all I need to do is proclaim a destination that is twice as far as I really need to go, and therefore I will arrive at my real goal in the first motion.

Thus I do not need to compute infinite sums and discuss whether or not their consequences are reasonable, because the distance in question does not exist in the first place. The entire paradox is rejected as soon as it specifies a distance to be halved.

References

[1] From http://plato.stanford.edu/entries/paradox-zeno/
[2] From https://en.wikipedia.org/wiki/Zeno%27s_paradoxes#Dichotomy_paradox