



Let's Make a Deal with Monty Hall



Monty Hall Problem

- ▶ Suppose that there are three closed doors on the set of the **Let's Make a Deal**, a TV game show presented by Monty Hall.
- ▶ Behind one of these doors is a car; behind the other two are goats.
- ▶ The contestant does not know where the car is, but Monty Hall does.
- ▶ The contestant selects a door, but not the outcome is not immediately evident.

Monty Hall Problem

- ▶ Monty opens one of the remaining “wrong” doors - revealing a goat.



Figure:

Monty Hall Problem

- ▶ **Stick or Switch** After Monty has shown a goat behind the door that he opens, the contestant is always given the option to switch doors: from the one they have already to select to the other closed door.
- ▶ **Question** - What is the probability of winning the car if she stays with her first choice? What if she decides to switch?



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Game Show Problem

(This material in this article was originally published in PARADE magazine in 1990 and 1991.)

Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say #1, and the host, who knows what's behind the doors, opens another door, say #3, which has a goat. He says to you, "Do you want to pick door #2?" Is it to your advantage to switch your choice of doors?

Craig F. Whitaker
Columbia, Maryland

Yes; you should switch. The first door has a $1/3$ chance of winning, but the second door has a $2/3$ chance. Here's a good way to visualize what happened. Suppose there are a million doors, and you pick door #1. Then the host, who knows what's behind the doors and will always avoid the one with the prize, opens them all except door #777,777. You'd switch to that door pretty fast, wouldn't you?

Since you seem to enjoy coming straight to the point, I'll do the same. You blew it! Let me explain. If one door is shown to be a loser, that information changes the probability of either remaining choice, neither of which has any reason to be more likely, to $1/2$. As a professional mathematician, I'm very concerned with the general public's lack of mathematical skills. Please help by confessing your error and in the future being more careful.

Robert Sachs, Ph.D.
George Mason University

You blew it, and you blew it big! Since you seem to have difficulty grasping the basic principle at work here, I'll explain. After the host reveals a goat, you now have a one-in-two chance of being correct. Whether you change your selection or not, the odds are the same. There is



We've received thousands of letters, and of the people who performed the experiment by hand as described, the results are close to unanimous: you win twice as often when you change doors. Nearly 100% of those readers now believe it pays to switch.

The Game Show Problem

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Craig F. Whitaker

Columbia, Maryland

Is it to your advantage to switch your choice of door?

Three possible answers

- ▶ Yes - Switch Doors
- ▶ No - Stick with Your original choice
- ▶ Doesn't Matter either way

Marilyn Responds

Yes; you should switch. The first door has a $1/3$ chance of winning, but the second door has a $2/3$ chance. Here's a good way to visualize what happened. Suppose there are a million doors, and you pick door #1. Then the host, who knows what's behind the doors and will always avoid the one with the prize, opens them all except door #777,777. You'd switch to that door pretty fast, wouldn't you?

and that was the end of that....

except....

- Many readers of vos Savant's column refused to believe switching is beneficial despite her explanation.
- After the problem appeared in Parade, approximately 10,000 readers, including nearly 1,000 with PhDs, wrote to the magazine, most of them claiming vos Savant was wrong (*Tierney 1991*).

Let's read some letters!!!!



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You blew it, and you blew it big! Since you seem to have difficulty grasping the basic principle at work here, I'll explain. After the host reveals a goat, you now have a one-in-two chance of being correct. Whether you change your selection or not, the odds are the same. There is enough mathematical illiteracy in this country, and we don't need the world's highest IQ propagating more. Shame!

Scott Smith, Ph.D.

University of Florida

Your answer to the question is in error. But if it is any consolation, many of my academic colleagues have also been stumped by this problem.

Barry Pasternack, Ph.D.

California Faculty Association

You're in error, but Albert Einstein earned a dearer place in the hearts of people after he admitted his errors.

Frank Rose, Ph.D.

University of Michigan

I have been a faithful reader of your column, and I have not, until now, had any reason to doubt you. However, in this matter (for which I do have expertise), your answer is clearly at odds with the truth.

James Rauff, Ph.D.

Millikin University

May I suggest that you obtain and refer to a standard textbook on probability before you try to answer a question of this type again?

Charles Reid, Ph.D.

University of Florida

I am sure you will receive many letters on this topic from high school and college students. Perhaps you should keep a few addresses for help with future columns.

W. Robert Smith, Ph.D.

Georgia State University

You are utterly incorrect about the game show question, and I hope this controversy will call some public attention to the serious national crisis in mathematical education. If you can admit your error, you will have contributed constructively towards the solution of a deplorable situation. How many irate mathematicians are needed to get you to change your mind?

E. Ray Bobo, Ph.D.

Georgetown University

I am in shock that after being corrected by at least three mathematicians, you still do not see your mistake.

Kent Ford

Dickinson State University

Maybe women look at math problems differently than men.

Don Edwards

Sunriver, Oregon

You made a mistake, but look at the positive side. If all those Ph.D.'s were wrong, the country would be in some very serious trouble.

Everett Harman, Ph.D.

U.S. Army Research Institute

- Even when given explanations, simulations, and formal mathematical proofs, many people still do not accept that switching is the best strategy (*vos Savant 1991a*).
- ***Paul Erdős***, one of the most prolific mathematicians in history, remained unconvinced until he was shown a computer simulation confirming the predicted result (*Vazsonyi 1999*).



DOUBLE FACEPALM

FOR WHEN ONE FACEPALM DOESN'T CUT IT

Implementation with R (part 1)

- ▶ We have 3 doors to choose from, so we will define a sequence A, B and C.
- ▶ The command `sample(,n)` takes a sample of size `n` from a specified set of values.
- ▶ Here we just want to select one door to be our "correct door" and another to be "selected" door (i.e. the contestant selects)

Implementation (part 1)

These events are independent. We will perform the selection for both doors separately, but this can be implemented in one command.

```
Doors = c("A","B","C")  
Correct = sample(Doors,1)  
Choice = sample(Doors,1)
```

A wrong door must be selected to be opened. The door selected by the contestant can't be chosen. First let us select the doors that must stay closed, then find the ones we can choose from to open

```
StayClosed = union(Correct, Choice)
CanOpen = setdiff(Doors, StayClosed)
```



```
NotOpen = setdiff(Doors,Open); Stick = Choice  
#The previous statement is to aid the narrative.  
  
Switch = setdiff(NotOpen,Choice)
```

```
# Was sticking the right decision? Or was switching?
```

```
# The following logical statements will tell us.
```

```
Stick==Choice ## 1 for Yes 0 for No
```

```
Switch==Choice ## 1 for Yes 0 for No
```