PART 2: Poisson Regression

- Poisson regression is used to model count variables.
- Poisson regression has a number of extensions useful for count models.

Conventional OLS regression

- Count outcome variables are sometimes log-transformed and analyzed using OLS regression.
- Many issues arise with this approach, including loss of data due to undefined values generated by taking the log of zero (which is undefined) and biased estimates.

Poisson Regression with R

If $\mathbf{x} \in \mathbb{R}^n$ is a vector of independent variables, then the model takes the form

$$\log_e(\mathrm{E}(Y \mid \mathbf{x})) = \beta_0 + \beta_1 x_1 + \ldots + \beta_n x_n$$

$$\mathrm{E}(Y \mid \mathbf{x}) = e^{\beta_0 + \beta_1 x_1 + \ldots + \beta_n x_n}$$

$$\mathrm{E}(Y \mid \mathbf{x}) = e^{\beta_0 + \beta_1 x_1 + \ldots + \beta_n x_n}$$

$$\mathrm{E}(Y \mid \mathbf{x}) = e^{\beta_0} \times e^{\beta_1 x_1} \times \ldots \times e^{\beta_n x_n}$$

Poisson Regression : Crabs Example

The Crabs Data Set

The crabs data set is derived from Agresti *(2007, Table 3.2, pp.76-77)*. It gives 4 variables for each of 173 female horseshoe crabs.

- **Satellites** number of male partners in addition to the female's primary partner
- **Width** width of the female in centimeters
- **Dark** a binary factor indicating whether the female has dark coloring (yes or no)
- **GoodSpine** a binary factor indicating whether the female has good spine condition (yes or no)

Let the first variable be a response variable, with the other three as predictors.

Poisson Regression : Crabs Example

The data is containted in the R package **glm2**

```
require(glm2)

data(crabs)
head(crabs)

summary(crabs[,1:4])
```

Poisson Regression : Crabs Example

```
> head(crabs)
  Satellites   Width   Dark GoodSpine  Rep1 Rep2
1          8    28.3    no         no     2    2
2          0    22.5   yes         no     4    5
3          9    26.0    no        yes     5    6
4          0    24.8   yes         no     6    6
5          4    26.0   yes         no     6    8
....
```

Poisson Regression : Crabs Example

```
> summary(crabs[,1:4])
Satellites          Width        Dark     GoodSpine
Min.   : 0.000   Min.   :21.0   no :107   no :121
1st Qu.: 0.000   1st Qu.:24.9   yes: 66   yes: 52
Median : 2.000   Median :26.1
Mean   : 2.919   Mean   :26.3
3rd Qu.: 5.000   3rd Qu.:27.7
Max.   :15.000   Max.   :33.5
```

Poisson Regression : Crabs Example

- Fit a Poisson regression model with the number of Satellites as the outcome and the width of the female as the covariate.
- What is the multiplicative change in the expected number of crabs for each additional centimeter of width?

```
crabs.pois <- glm2(Satellites ~ Width,
data=crabs, family="poisson")
summary(crabs.pois)

exp(0.164)
```

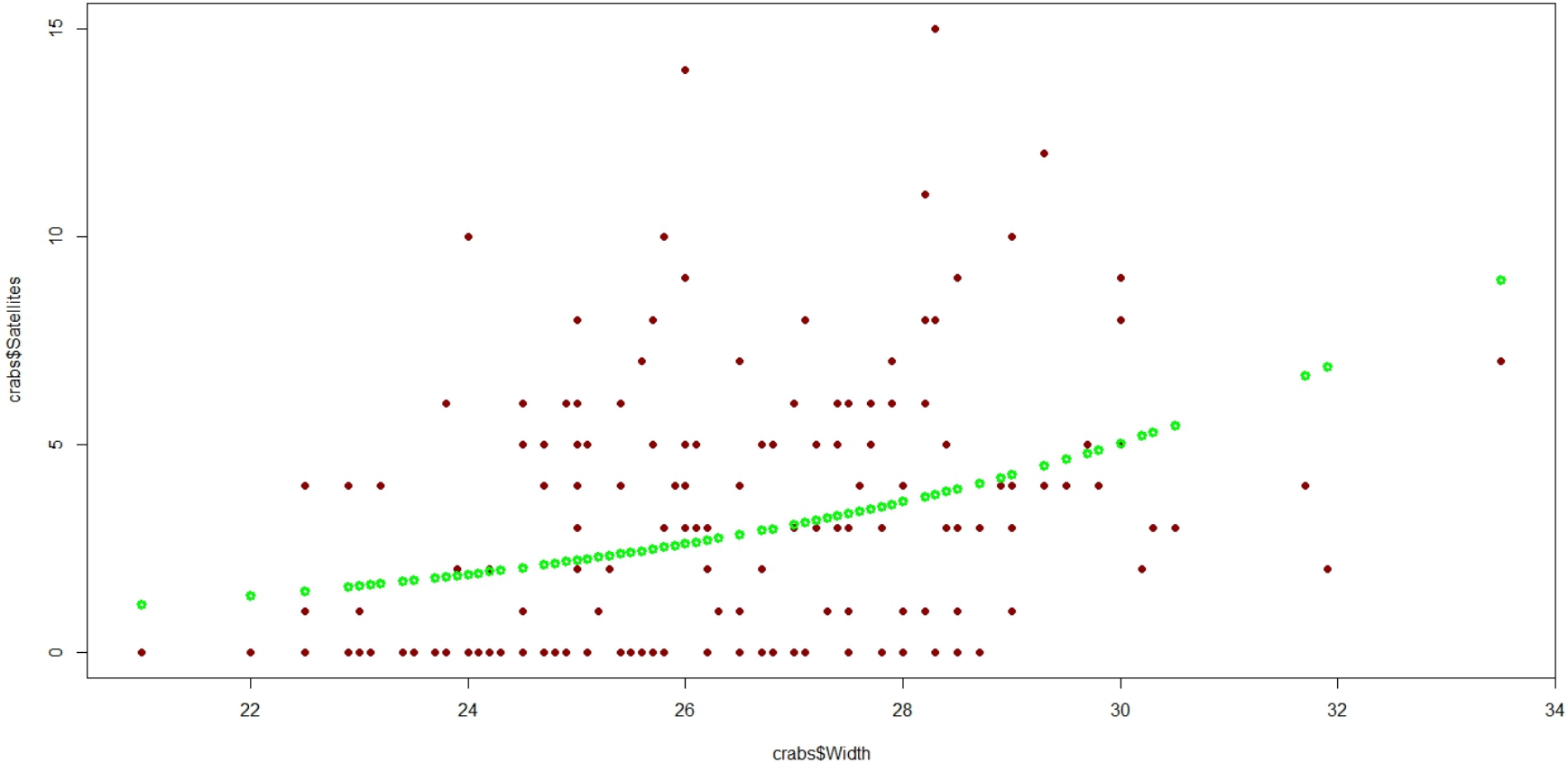
Poisson Regression : Crabs Example

```
> summary(crabs.pois)

Call:
glm2(formula = Satellites ~ Width,
family = "poisson", data = crabs)

......
......
Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -3.30476    0.54224  -6.095  1.1e-09 ***
Width        0.16405    0.01997   8.216  < 2e-16 ***
---
......
```

Poisson Regression : Crabs Example



Poisson Regression : Crabs Example

Code for Crabs Data Plot

```
plot(crabs$Width, crabs$Satellites,
   pch=16, col="darkred")
   points(crabs$Width, crabs.pois$fitted.values,
   col="green", lwd=3)
```

Other Examples of Poisson regression

- The number of awards earned by students at a secondary or high school.
- Predictors of the number of awards earned include the type of program in which the student was enrolled (e.g., vocational, general or academic) and the score on their final exam in math.

Description of the data

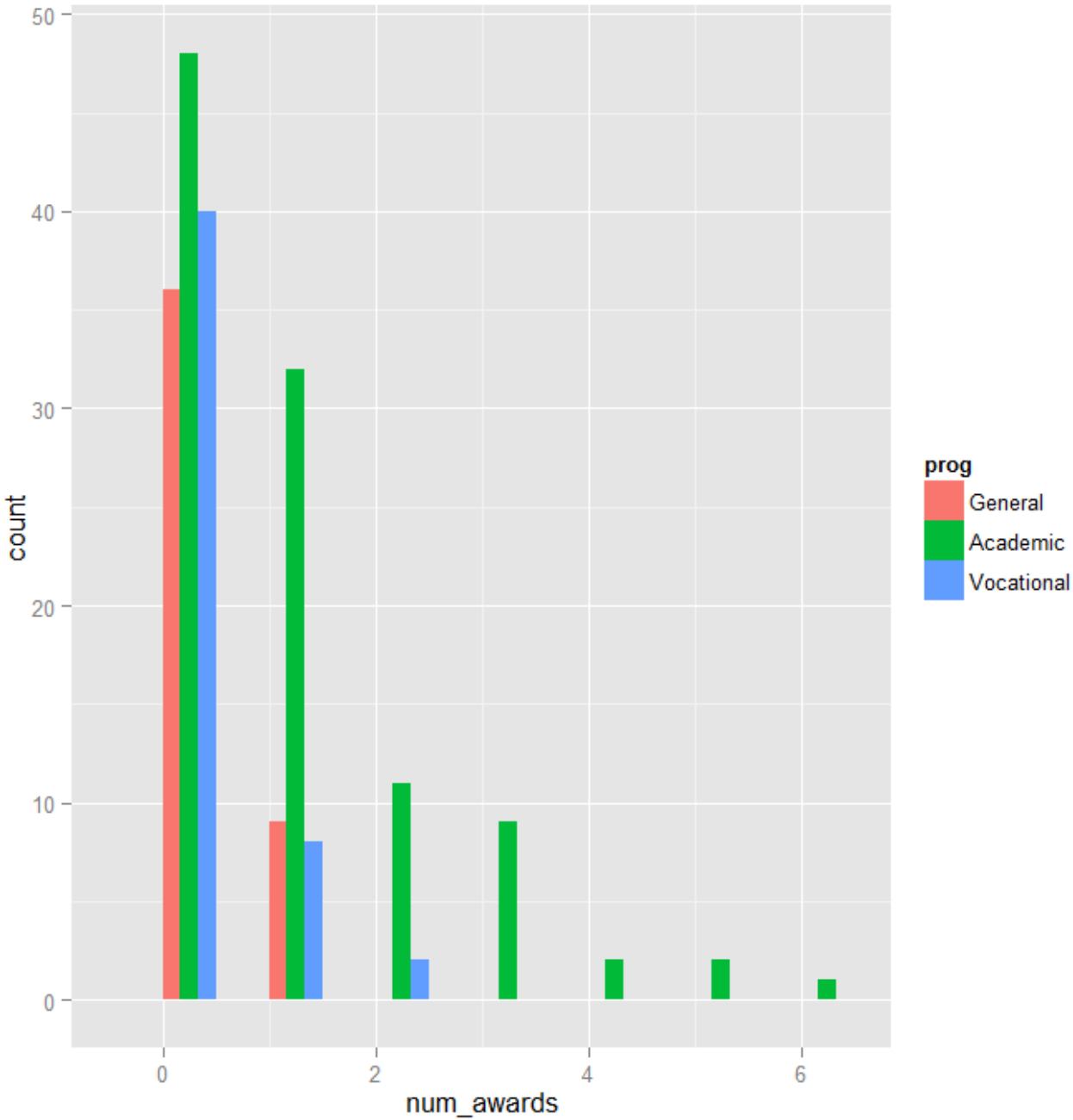
- For the purpose of illustration, we have simulated a data set for the last example.
- The data set is called *poisreg.csv*
- In this example, **num_awards** is the outcome variable and indicates the number of awards earned by students at a high school in a year.

Predictor Variables

- **math** is a continuous predictor variable and represents students' scores on their math final exam,
- **prog** is a categorical predictor variable with three levels indicating the type of program in which the students were enrolled.
- **prog** is coded as $1 =$ "*General*", $2 =$ "*Academic*" and $3 =$ "*Vocational*".

Poisson Regression with R

```
       id          num_awards          prog          math
1     :  1   Min.   :0.00   General    : 45   Min.   :33.0
2     :  1   1st Qu.:0.00   Academic   :105   1st Qu.:45.0
3     :  1   Median :0.00   Vocational: 50   Median :52.0
4     :  1   Mean   :0.63                    Mean   :52.6
5     :  1   3rd Qu.:1.00                    3rd Qu.:59.0
6     :  1   Max.   :6.00                    Max.   :75.0
  (Other):194
```


50
40
30
20
10
0
count
0
2
4
6
num_awards
prog
General
Academic
Vocational

Poisson Regression with R

- Each variable has 200 valid observations and their distributions seem quite reasonable.
- The mean and variance of our outcome variable are more or less the same.
- Our model assumes that these values, conditioned on the predictor variables, will be equal (or at least roughly so).