

Application of relativistic mean field and effective field theory densities to scattering observables for Ca isotopes

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In the frame work of relativistic mean field (RMF) theory, we have calculated the density distribution of protons and neutrons for $^{40,42,44,48}\text{Ca}$ with NL3 and G2 parameter sets. The microscopic proton-nucleus optical potentials for $p + ^{40,42,44,48}\text{Ca}$ systems are evaluated from the Dirac NN-scattering amplitude and the density of the target nucleus using Relativistic-Love-Franey and McNeil-Ray-Wallace parametrizations. We have estimated the scattering observables, such as elastic differential scattering cross-section, analyzing power and the spin observables with the relativistic impulse approximation (RIA). The results have been compared with the experimental data for few selective cases and find that the use of density as well as the scattering matrix parametrizations are crucial for the theoretical prediction.

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I. INTRODUCTION

Study of the nuclear reactions is a challenging subject of nuclear physics both in theory and laboratory. This is useful to explain the nuclear structure of stable as well as exotic nuclei. The Nucleon- Nucleus interaction provides a wide source of information to determine the nuclear structure including spin, isospin, momenta, densities and gives a clear path towards the formation of exotic nuclei in the laboratory. In this context the study of elastic scattering of Nucleon-Nucleus is more interesting than that of Nucleus-Nucleus at different energies. One of the theoretical method to study such type of reactions is the "Relativistic Impulse Approximation" (RIA). It is a microscopic theory where the Dirac optical potential is constructed from the Lorentz invariant Nucleon-Nucleon (NN) amplitudes obtained from relativistic meson exchange models. The basic ingredients in this approach are the NN scattering amplitude and the nuclear scalar and vector densities [1] of the target nucleus. This approach can be extended to elastic scattering of composite particles [2]. In this context proton-Nucleus (p-A) elastic scattering is of particular interest because of its relative simplicity with which it provides a satisfactory description of the reaction dynamics.

One useful application of RIA is to generate microscopic optical potential to study the elastic and inelastic scattering of nucleons for unstable proton- and/or neutron- rich nuclei. The RIA folding procedure can also be extended to calculate microscopic optical potentials for exotic nuclei using relativistic mean field formalism [3, 4].

The first theoretical introduction to elastic scattering was given by Chew [5] almost six decades ago. For a wide range of energy interval, Impulse Approximation (IA) produces the main qualitative description on quasi-elastic scattering for $A \leq 64$ nuclei [6]. During the same time, Glauber [7] studied the reaction dynamics of the composite system at low energies but this model is unable to predict extension of quasi-elastic scattering. Further the generalized Glauber formula and the unitarized impulse approximation [8, 9] were circumvented. But the development of RIA opens a path to study the above

mentioned scattering phenomenon for both the elastic and the quasi-elastic particles. This field of research was further strengthened by the experimental evidences of cross-section and analyzing power for the scattering systems $\vec{p} + ^{12}\text{C}$, $p + ^9\text{B}$ and $p + ^{16}\text{O}$ at 200 MeV, which were measured over a wide range of momentum transfer $> 6\text{ fm}^{-1}$ at IUCF [10, 11]. Recent study of proton nucleus elastic scattering within modified (Coulomb) Glauber model [12, 13] and global Dirac optical potential [14] have motivated us to study the elastic scattering phenomenon. For convenience, we consider Ca isotopes as targets and p as a projectile, because Ca satisfies the relativistic mean field nuclear structure model accurately without recoil correction to the Dirac scattering equation.

In the present paper, our aim is to calculate the nucleon-nucleus elastic differential scattering cross-section ($\frac{d\sigma}{d\Omega}$) and other related physical quantities, like optical potential (U_{opt}), analyzing power (A_y) and spin rotation parameter (Q -value) using relativistic mean field (RMF) and recently proposed effective field theory motivated relativistic mean field (E-RMF) densities. These are obtained from the successful NL3 [15] and the advanced G2 [16] parameter sets, which are given in Section II. In the Sections III and IV, the details of target densities folded with the NN-amplitude for various energetic proton projectile with Relativistic-Love-Franey (RLF) [17, 18] and McNeil-Ray-Wallace (MRW) parametrizations [19] for $^{40,42,44,48}\text{Ca}$ are given. In these sections we have outlined the expressions for the differential elastic scattering cross-section, analyzing power and spin observables. Section V describes the results obtained from our calculations. Finally a brief summary and conclusions are given in the Section VI for the present work.

II. THE RMF AND E-RMF FORMALISMS

A documentation of RMF and E-RMF formalisms are available in Refs. [20] and [16, 21] respectively for both finite and infinite nuclear matter. Here only the energy density functional and associated expressions for the densities are

presented[22, 23].

$$\begin{aligned}
\mathcal{E}(\mathbf{r}) = & \sum_{\alpha} \varphi_{\alpha}^{\dagger} \left\{ -i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + \beta(M - \Phi) + W + \frac{1}{2}\tau_3 R + \frac{1+\tau_3}{2}A - \frac{i}{2M}\beta\boldsymbol{\alpha} \cdot \left(f_v \boldsymbol{\nabla} W + \frac{1}{2}f_{\rho}\tau_3 \boldsymbol{\nabla} R + \lambda \boldsymbol{\nabla} A \right) \right. \\
& + \frac{1}{2M^2}(\beta_s + \beta_v\tau_3)\Delta A \left. \right\} \varphi_{\alpha} + \left(\frac{1}{2} + \frac{\kappa_3}{3!}\frac{\Phi}{M} + \frac{\kappa_4}{4!}\frac{\Phi^2}{M^2} \right) \frac{m_s^2}{g_s^2}\Phi^2 - \frac{\zeta_0}{4!}\frac{1}{g_v^2}W^4 + \frac{1}{2g_s^2} \left(1 + \alpha_1 \frac{\Phi}{M} \right) (\boldsymbol{\nabla}\Phi)^2 \\
& - \frac{1}{2g_v^2} \left(1 + \alpha_2 \frac{\Phi}{M} \right) (\boldsymbol{\nabla}W)^2 - \frac{1}{2} \left(1 + \eta_1 \frac{\Phi}{M} + \frac{\eta_2}{2}\frac{\Phi^2}{M^2} \right) \frac{m_v^2}{g_v^2}W^2 - \frac{1}{2g_{\rho}^2}(\boldsymbol{\nabla}R)^2 - \frac{1}{2} \left(1 + \eta_{\rho} \frac{\Phi}{M} \right) \frac{m_{\rho}^2}{g_{\rho}^2}R^2 \\
& - \frac{1}{2e^2}(\boldsymbol{\nabla}A)^2 + \frac{1}{3g_{\gamma}g_v}A\Delta W + \frac{1}{g_{\gamma}g_{\rho}}A\Delta R,
\end{aligned} \tag{1}$$

where the index α runs over all occupied states $\varphi_{\alpha}(\mathbf{r})$ of the positive energy spectrum, $\Phi \equiv g_s\phi_0(\mathbf{r})$, $W \equiv g_vV_0(\mathbf{r})$, $R \equiv g_{\rho}b_0(\mathbf{r})$ and $A \equiv eA_0(\mathbf{r})$.

The terms with g_{γ} , λ , β_s and β_v take care of the effects related with the electromagnetic structure of the pion and the nucleon (see Ref. [23]). Specifically, the constant g_{γ} concerns the coupling of the photon to the pions and the nucleons through the exchange of neutral vector mesons. The experimental value is $g_{\gamma}^2/4\pi = 2.0$. The constant λ is needed to reproduce the magnetic moments of the nucleons, defined by

$$\lambda = \frac{1}{2}\lambda_p(1 + \tau_3) + \frac{1}{2}\lambda_n(1 - \tau_3), \tag{2}$$

with $\lambda_p = 1.793$ and $\lambda_n = -1.913$, the anomalous magnetic moments of the proton and the neutron, respectively. The terms with β_s and β_v contribute to the charge radii of the nucleon [23].

The energy density contains tensor couplings, scalar-vector and vector-vector meson interactions in addition to the standard scalar self-interactions κ_3 and κ_4 . Thus, the E-RMF formalism can be interpreted as a covariant formulation of density functional theory as it contains all the higher order terms in the Lagrangian, obtained by expanding it in powers of the meson fields. The terms in the Lagrangian are kept finite by adjusting the parameters. Further insight into the concepts of the E-RMF model can be obtained from Ref. [23]. It may be noted that the standard RMF Lagrangian is obtained from that of the E-RMF by ignoring the vector-vector and scalar-vector cross interactions, and hence does not need a separate discussion.

In each of the two formalisms (E-RMF and RMF), the set of coupled equations are solved numerically by a self-consistent iteration method. The baryon, scalar, isovector, proton and tensor densities are

$$\rho(r) = \sum_{\alpha} \varphi_{\alpha}^{\dagger}(r) \varphi_{\alpha}(r), \tag{3}$$

$$\rho_s(r) = \sum_{\alpha} \varphi_{\alpha}^{\dagger}(r) \beta \varphi_{\alpha}(r), \tag{4}$$

$$\rho_3(r) = \sum_{\alpha} \varphi_{\alpha}^{\dagger}(r) \tau_3 \varphi_{\alpha}(r), \tag{5}$$

$$\rho_p(r) = \sum_{\alpha} \varphi_{\alpha}^{\dagger}(r) \left(\frac{1 + \tau_3}{2} \right) \varphi_{\alpha}(r), \tag{6}$$

$$\rho_T(r) = \sum_{\alpha} \frac{i}{M} \boldsymbol{\nabla} \cdot [\varphi_{\alpha}^{\dagger}(r) \beta \boldsymbol{\alpha} \varphi_{\alpha}(r)], \tag{7}$$

$$\rho_{T,3}(r) = \sum_{\alpha} \frac{i}{M} \boldsymbol{\nabla} \cdot [\varphi_{\alpha}^{\dagger}(r) \beta \boldsymbol{\alpha} \tau_3 \varphi_{\alpha}(r)]. \tag{8}$$

These densities are obtained from the RMF and E-RMF formalisms with NL3 [15] and G2 [16] parametrizations. We refer the readers to Refs. [20, 21] for numerical details and ground state equations for finite nuclei.

III. THE NUCLEON-NUCLEON SCATTERING AMPLITUDE

The non-linear relativistic impulse approximation (RIA) involves mainly two steps [24–26] of calculation. Basically a particular set of Lorentz covariant function [19], which multiply with the so called Fermi invariant Dirac matrix represent the Nucleon-Nucleon NN-scattering amplitudes. This functions are then folded with the target densities of proton and neutron from the relativistic Lagrangian for NL3 and G2 parameter sets to produce a first order complex optical potential. The invariant NN-scattering operator \mathcal{F} can be written in terms of five complex functions (the five terms involves in

the proton-proton pp and neutron-neutron nn scattering). In general RIA, the function \mathcal{F} can be expressed as [24–26],

$$\mathcal{F}(q, E) = \sum_{L=S}^{PS} \mathcal{F}^L(q, E) \lambda_{(0)}^L \cdot \lambda_{(1)}^L, \quad (9)$$

where λ^L stands for Dirac operator and (0) and (1) for the incident and struck nucleons respectively. The S, V, T, A and PS stands for scalar, vector, axial vector, tensor and pseudoscalar. The dot product (.) implies all Lorentz indices are contracted. The Dirac spinor is defined the initial and final two nucleons by taking the matrix elements of \mathcal{F} , which represent the NN-scattering amplitudes. The function \mathcal{F}^L are determined by equating the resultant amplitude (in center of mass frame) to the empirical amplitude, which is conventionally expressed in term of the non-relativistic Wolfenstein amplitudes A_1, A_2, \dots, A_5 [19]. Since there are five complex invariant amplitudes and A_1, A_2, \dots, A_5 are five Wolfenstein amplitudes,

the relation among them is determined by a 5×5 non-singular matrix, whose inversion is straight forward. However \mathcal{F} is an operator in the two particle Dirac space and the component cancelled out due to isospin and parity invariance and we get only 44 components [27]. From the above, it is clear, to specify that \mathcal{F} is not unique. In other words, there are infinite number of operators \mathcal{F} with same five on-shell but different negative (energy) elements. The expression of \mathcal{F} cannot predict reasonable result at lower energy region. To avoid the limitation, the pseudoscalar \mathcal{F}^{PS} is replaced by the pseudovector invariant, and is expressed as,

$$\mathcal{F}^{PS} \gamma_{(0)}^5 \gamma_{(1)}^5 = -\mathcal{F}^{PV} \frac{\gamma_{(0)}^5}{2M} \frac{\gamma_{(1)}^5}{2M}. \quad (10)$$

The meson-nucleon couplings are complex, with a real part g_i^2 and an imaginary part \bar{g}_i^2 , which can be decomposed into two parts,

$$\langle k'_0 k'_1 | \mathcal{F} | k_0 k_1 \rangle = \langle k'_0 k'_1 | t(E) | k_0 k_1 \rangle + (-1)^T \langle k'_0 k'_1 | t(E) | k_0 k_1 \rangle, \quad (11)$$

where $t(E)$ is the lowest order meson and T is the total isospin of the two nucleon. The calculation of the one-meson-exchange from Feynman diagram [17] is represented as,

$$g_i \left(\frac{\Lambda_i^2}{q^2 + \Lambda_i^2} \right) \lambda^{L(i)}(\tau)^{I_i}, \quad (12)$$

with $L(i)$ denotes spin and parity of the i^{th} meson and $I_i = (0, 1)$ is the meson's isospin. Here we neglect the energy transfer q^0 carried by the meson for different masses and cut off parameters in the real and imaginary parts of the amplitude in Eqn. (9). The contribution of i^{th} -meson to the NN-scattering amplitude by taking all kinematic is given as,

$$\begin{aligned} \overline{U_{0'}} \cdot \overline{U_{1'}} \mathcal{F}_i U_0 U_1 &\propto \frac{g_i^2}{q^2 + m_i^2} \left(\frac{\Lambda_i^2}{q^2 + \Lambda_i^2} \right)^2 \{ \tau_0 \cdot \tau_1 \}^{I_i} \overline{U_{0'}} \lambda^{L_i} U_0 \cdot \overline{U_{1'}} \lambda^{L_i} U_1 \\ &+ (-1)^T \frac{g_i^2}{Q^2 + m_i^2} \left(\frac{\Lambda_i^2}{Q^2 + \Lambda_i^2} \right)^2 \{ \tau_0 \cdot \tau_1 \}^{I_i} \overline{U_{1'}} \lambda^{L_i} U_0 \cdot \overline{U_{0'}} \lambda^{L_i} U_1, \end{aligned} \quad (13)$$

Here the direct and exchange momentum transfer are $q = k'_0 - k_0$ and $Q = k'_1 - k_1$. The first term in Eq.(13), which is already of the form of Eq.(9), can easily identify the contribution of \mathcal{F}^L . The second term is unlike to this form, so we rewrite this as,

$$\begin{aligned} \overline{U_{0'}} \cdot \overline{U_{1'}} \mathcal{F}_i U_0 U_1 &\propto \frac{g_i^2}{q^2 + m_i^2} \left(\frac{\Lambda_i^2}{q^2 + \Lambda_i^2} \right)^2 \{ \tau_0 \cdot \tau_1 \}^{I_i} \overline{U_{0'}} \lambda^{L_i} U_0 \cdot \overline{U_{1'}} \lambda^{L_i} U_1 \\ &+ (-1)^T \sum_{L'} B_{L'} \frac{g_i^2}{Q^2 + m_i^2} \left(\frac{\Lambda_i^2}{Q^2 + \Lambda_i^2} \right)^2 \{ \tau_0 \cdot \tau_1 \}^{I_i} \overline{U_{0'}} \lambda^{L'_i} U_0 \cdot \overline{U_{1'}} \lambda^{L'_i} U_1, \end{aligned} \quad (14)$$

where the transformation matrix is given as,

$$B_{L, L'} = \frac{1}{8} \begin{bmatrix} 2 & 2 & 1 & -2 & 2 \\ 8 & -4 & 0 & -4 & -8 \\ 24 & 0 & -4 & 0 & 24 \\ -8 & -4 & 0 & -4 & 8 \\ 2 & -2 & 1 & 2 & 2 \end{bmatrix}. \quad (15)$$

The row and columns are labeled in the order of S, V, T, A, PS . The contribution to the Lorentz invariants (\mathcal{F}_L) in simpler

forms are written as,

$$\mathcal{F}(q, E_c) = i \frac{M^2}{2E_c k_c} [F_D^L(q) + F_X^L(Q)], \quad (16)$$

$$F_D^L(q) = \sum_i \delta_{L,L(i)} \{\tau_0 \cdot \tau_1\}^{I_i} f^i(q), \quad (17)$$

$$F_X^L(Q) = (-1)^T \sum_i B_{L(i),L} \{\tau_0 \cdot \tau_1\}^{I_i} f^i(Q), \quad (18)$$

$$f^i(q) \equiv \frac{g_i^2}{q^2 + m_i^2} \left(\frac{\Lambda_i^2}{q^2 + \Lambda_i^2} \right)^2 - i \frac{\bar{g}_i^2}{Q^2 + m_i^2} \left(\frac{\bar{\Lambda}_i^2}{Q^2 + \bar{\Lambda}_i^2} \right)^2. \quad (19)$$

Here E_c is the total energy in the NN center of mass system. Note that f^i depends only on the magnitude of the three momentum transfer and the expressions are used to fit NN-scattering amplitude at laboratory energy. The full parametrizations are frame out in Refs. [17, 18].

IV. NUCLEON-NUCLEUS OPTICAL POTENTIAL

The Dirac optical potential $U_{opt}(q, E)$ can be written as,

$$U_{opt}(q, E) = \frac{-4\pi i p}{M} \langle \psi | \sum_{n=1}^A \exp(iq \cdot x(n)) \mathcal{F}(q, E; n) | \psi \rangle \quad (20)$$

where \mathcal{F} is the scattering operator, p is the momentum of the projectiles in the nucleon-nucleus center of mass frame, $|\psi\rangle$

is the nuclear ground state wave function for A-particle, q is the momentum transfer and E is the collision energy for a stationary target (nucleus) and incident projectile (proton). In the present calculation the nuclear recoil energy is neglected because of elastic scattering. The operator $\mathcal{F}(q, E; n)$ describe the scattering of the projectile from target nucleon 'n' without separation into direct and exchange terms. Let us define the nuclear ground state by a Dirac-Hartree wave function [28] and the incident projectile wave function as $\mathcal{U}(x)$, then the optical potential on incident wave projected to the co-ordinate space can be written as,

$$\langle x | U_{opt} | \mathcal{U}_0 \rangle = \frac{-4\pi i p}{M} \langle \psi | \sum_{\alpha}^{occ} \int d^3 y' d^3 y d^3 x' \mathcal{U}_{\alpha}(y') \times \left\{ \langle xy' | t(E) | x' y \rangle + (-1)^T \langle y' x | t(E) | x' y \rangle \right\} \mathcal{U}_0(x') \mathcal{U}_{\alpha}(y). \quad (21)$$

The antisymmetrised matrix element of $t(E)$ in coordinate space is the Fourier transforms [28] of the matrix element in the momentum space co-ordinate and is written as,

$$\begin{aligned} \langle x | U_{opt} | \mathcal{U}_0 \rangle &= \frac{-4\pi i p}{M} \sum_L \int d^3 x' \left[\rho^L(x') t_D^L(|x - x'|; E) \right] \lambda^L \mathcal{U}_0(x) \\ &\quad - \frac{-4\pi i p}{M} \sum_L \int d^3 x' \left[\rho^L(x', x) t_X^L(|x - x'|; E) \right] \lambda^L \mathcal{U}_0(x'), \end{aligned} \quad (22)$$

where

$$t_D^L(|x|; E) \equiv \int \frac{d^3 q}{(2\pi)^3} t_D^L(q, E) e^{-iqx}, \quad (23)$$

with

$$t_D^L(|q|; E) \equiv \left(\frac{iM^2}{2E_c k_c} \right) F_D^L(q), \quad (24)$$

and similarly for the exchange part $t_D^L(|Q|; E)$. The nuclear density is defined by a simple expression similar to the equation of RMF and E-RMF density,

$$\rho^L(x, x') \equiv \sum_{\alpha}^{occ'} \overline{\mathcal{U}}_{\alpha} \lambda^L \mathcal{U}_{\alpha}, \rho^L(x) \equiv \rho^L(x', x). \quad (25)$$

The prime stands for occupied states, i.e., sum over target protons (pp-amplitude) and target neutrons (pn-amplitude) used.

The first term in the Eqn. (22) defines the direct optical potential,

$$U_D^L(r, E) = \frac{-4\pi ip}{M} \int d^3x' \rho^L(x') t_D^L(|x - x'|; E). \quad (26)$$

The nonlocal second term is treated in nonlocal density approximation [29], which contains plane wave status for incident and bound nucleons. We replaced the exchange integral with local potential by,

$$U_X^L(r, E) = \frac{-4\pi ip}{M} \int d^3x' \rho^L(x', x) t_D^L(|x - x'|; E) j_0(p(|x - x'|)), \quad (27)$$

where j_0 is the spherical Bessel-function. The off diagonal one body density is approximated by the local density which result as,

$$\rho_L(x', x) \approx \rho^L(1/2(x + x')) \left(\frac{3}{sk_f}\right) j_1(sk_f), \quad (28)$$

with $s \equiv |x - x'|$ and k_f is related to the nuclear baryon density by $\rho_B(1/2(x + x')) = 2k_f^3/3\pi^2$. Now the optical potential have the form,

$$U_{opt} = U_S + \gamma^0 U^V - 2i\alpha \cdot \hat{r} U^T, \quad (29)$$

where

$$U^L \equiv U^L(r, E) = U_D^L(r, E) + U_X^L(r, E). \quad (30)$$

As the tensor contributions are small, by neglecting these, the Dirac equation for projectile has precisely the similar form as in RMF and E-RMF equation. By taking the Fourier Transform of this equation, we get the optical potential as,

$$\int \frac{d^3q}{(\pi)^3} \exp(iq \cdot x) f(q) = \frac{g^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m^2} \left\{ \frac{\Lambda^2}{\Lambda^2 - m^2} \frac{e^{-mr} - e^{-\Lambda r}}{r} - \frac{\Lambda}{2} e^{-\Lambda r} \right\}. \quad (31)$$

This equation includes all meson exchanges (except the pseudoscalar meson) with derivative coupling, which is written in the form,

$$\int \frac{d^3q}{(2\pi)^3} \exp(iq \cdot x) f(q) \frac{q^2}{4M^2} = \frac{\Lambda^2}{4M^2} \frac{g^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m^2} \left\{ \frac{m^2}{\Lambda^2 - m^2} \frac{e^{-\Lambda r} - e^{-mr}}{r} + \frac{\Lambda}{2} e^{-\Lambda r} \right\}. \quad (32)$$

The optical potential is modified by Pauli blocking factor [30–34] $a(E)$ with local density approximation as follows,

$$U^L(r, E) \longrightarrow \left[1 - a(E) \left(\frac{\rho_B(r)}{\rho_0} \right)^{2/3} \right] U^L(r, E). \quad (33)$$

Here ρ_B is the local baryon density of the target and ρ_0 is the nuclear matter density at saturation. The approximation depends on $\rho_B^{2/3}$, which agree with phase-space arguments based on isotropic scattering. The detail about the Pauli blocking factor is given in Ref. [18]. To solve the scattering state Dirac equation, the wave function is separated into two components

(upper and lower) and this equation is expressed as two coupled first order differential equations. Elimination of the lower component leads to a single second order differential equation with spin-orbit as well as both local and nonlocal potential. The nonlocal Darwin potential can be separated by rewriting the upper component of the wave function, $\mathcal{A}^{1/2}(r, E)\mathcal{U}(x)$ and

$$\mathcal{A}(r, E) \equiv 1 + \frac{U^S(r, E) - U^V(r, E)}{E + M}. \quad (34)$$

After some algebra, the equation can be written as,

$$(-\nabla^2 + V_{cent} + V_{so}\sigma.L + V_{Darwin})u(x) = (E^2 - M^2)u(x), \quad (35)$$

where the energy-dependent optical potentials are

$$V_{cent}(r, E) = 2MU^S + 2EU^V + (U^S)^2 - (U^V)^2, \quad (36)$$

$$V_{so}(r, E) = -\frac{1}{r} \frac{B'}{B}, \quad (37)$$

and

$$V_{Darwin} = \frac{3}{4} \left(\frac{B'}{B}\right)^2 - \frac{1}{r} \frac{B'}{B} - \frac{1}{2} \frac{B''}{B}. \quad (38)$$

Since the two component Dirac wave functions are eigenstate of $\sigma.L$, so by taking the second derivative of the function we can solve easily using Numerov algorithm [35, 36]. Note that $\mathcal{U}(x)$ is not equal to the upper component wave function in the region of the potential but when $A(r, E) \rightarrow 1$, as $r \rightarrow \infty$ and \mathcal{U} has the same asymptotic behavior the wave function at large r . Thus the correct boundary condition is imposed by matching \mathcal{U} to the form of Coulomb scattering solution incident in the z-direction [37].

$$\psi(r) \propto_{r \rightarrow \infty} \left\{ \exp i[pz - \eta \ln 2pr \sin^2 \theta / 2] \left[1 - \frac{\eta^2}{2ipr \sin^2 \theta / 2} \right] \right\} \chi_{inc} + \left\{ \frac{\exp i[pr - \eta \ln 2pr]}{r} [A(\theta) + B(\theta)\sigma.\hat{n}] \right\} \chi_{ref} \quad (39)$$

with $E = \sqrt{p^2 + M^2}$, χ_{inc} is a two-component Pauli spinor, θ is the scattering angle, n is the normal to the scattering plane and $\eta \equiv Ze^2/p^2$ with Z is the nuclear charge. The scattering observables like differential scattering cross-section ($\frac{d\sigma}{d\Omega}$) and other quantities, like optical potential (U_{opt}), analyzing power (A_y) and spin observables (Q -value) are easily determined from the scattering amplitude, which are written as,

$$\frac{d\sigma}{d\Omega} \equiv |A(\theta)|^2 + |B(\theta)|^2, \quad (40)$$

$$A_y \equiv \frac{2Re[A^*(\theta)B(\theta)]}{d\sigma/d\Omega}, \quad (41)$$

$$Q \equiv \frac{2Im[A(\theta)B^*(\theta)]}{d\sigma/d\Omega}. \quad (42)$$

V. DETAILS OF THE CALCULATIONS AND RESULTS

First we calculate both the scalar and vector parts of the neutrons and protons density distribution for $^{40,42,44,48}\text{Ca}$ from the RMF (NL3) and E-RMF (G2) formalisms [21]. Then evaluate the scattering observables using these densities in the RIA frame-work[38], which involves the following two steps:

(i) we generate the complex NN-interaction from the Lorentz invariant matrix $\mathcal{F}^L(q, E)$ as defined in Eq. (2). Then the interaction is folded with the ground state target nuclear density for both the RLF [17, 18] and MRW parameters [19] separately and obtained the nucleon-nucleus complex optical potential $U_{opt}(q, E)$. It is to be noted that the pairing interaction has been taken into account using the Pauli blocking approximation. Here, the Pauli blocking enters through the intermediate states of the t -matrix formalism, which has geometrical effects on the optical potential, (ii) we solve the wave function of the scattering state utilising the optical potential prepared in the first step by the well known Numerov algorithm [35]. The result is approximated with the non-relativistic Coulomb scattering for a wide range of radial component which yields the scattering amplitude and other observables [37]. By comparing our calculations with the available experimental data, we examine the validity of our RIA predictions for describing $\frac{d\sigma}{d\Omega}$, A_y and Q -values which are presented in Figures 1 – 11.

A. The neutron and proton densities

In Fig. 1, we have plotted the proton ρ_p and neutron ρ_n density distribution for $^{40,42,44,48}\text{Ca}$ using NL3 and G2 parameter sets within RMF and E-RMF formalisms. From the

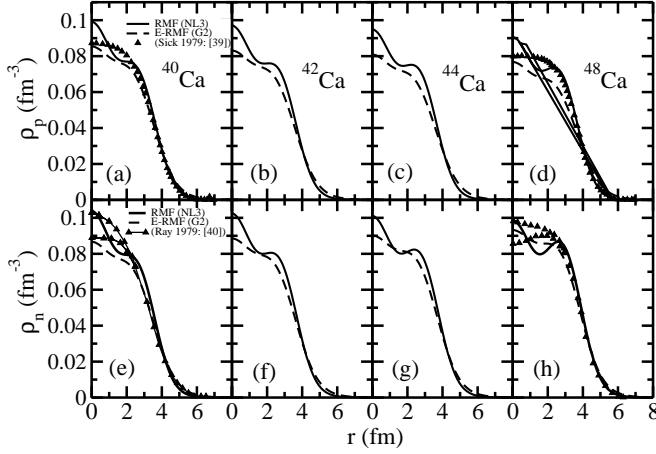


FIG. 1: The proton ρ_p (upper panel) and neutron ρ_n (lower panel) density distribution for $^{40,42,44,48}\text{Ca}$ obtained from RMF (NL3) and E-RMF (G2) parameter sets. The experimental [39] ρ_p and deduced [40] ρ_n for $^{40,48}\text{Ca}$ are also compared.

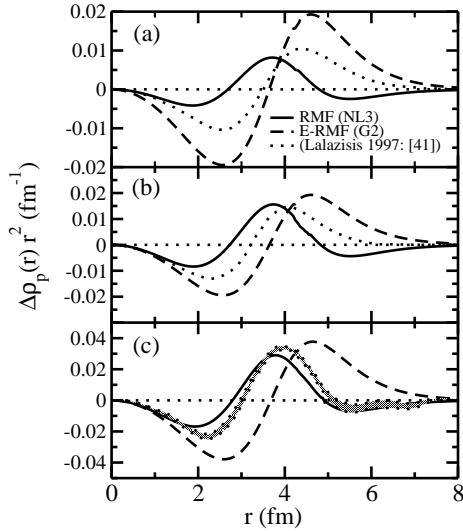


FIG. 2: The relative isotopic proton density differences $\Delta\rho_p(r)$ for $\rho_p(^{42}\text{Ca}) - \rho_p(^{40}\text{Ca})$, $\rho_p(^{44}\text{Ca}) - \rho_p(^{40}\text{Ca})$, and $\rho_p(^{48}\text{Ca}) - \rho_p(^{40}\text{Ca})$ obtained from RMF (NL3) and E-RMF (G2) are compared with the data [41] in (a), (b) and (c), respectively.

figure, we note that, there is a very small difference in the densities for NL3 and G2 parameter sets. However, a careful inspection shows a small enhancement in central density (0-1.6 fm) for NL3 set. On the other hand the densities obtained from G2 is elongated to a larger distance towards the tail region and this nominal difference has significant role to play in the scattering phenomena, which is explained later on.

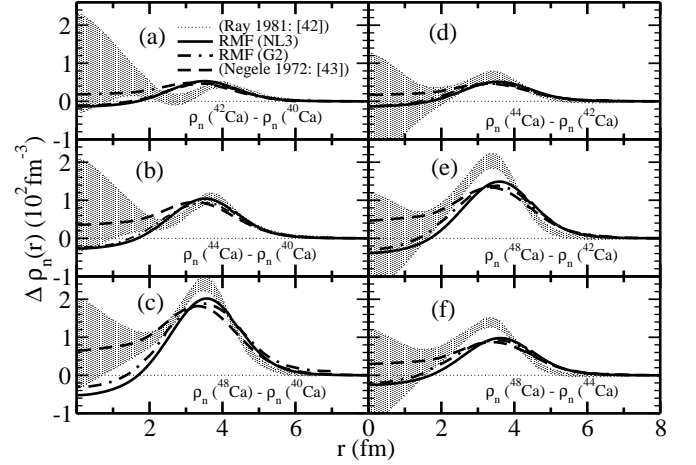


FIG. 3: The relative isotopic neutron density differences $\Delta\rho_n(r)$ for $\rho_n(^{42}\text{Ca}) - \rho_n(^{40}\text{Ca})$, $\rho_n(^{44}\text{Ca}) - \rho_n(^{40}\text{Ca})$, $\rho_n(^{48}\text{Ca}) - \rho_n(^{40}\text{Ca})$, $\rho_n(^{44}\text{Ca}) - \rho_n(^{42}\text{Ca})$, $\rho_n(^{48}\text{Ca}) - \rho_n(^{42}\text{Ca})$, and $\rho_n(^{48}\text{Ca}) - \rho_n(^{44}\text{Ca})$. The RMF (NL3) and E-RMF (G2) $\Delta\rho_n(r)$ are compared with the density-matrix-expansion (DME) data [43] and the uncertainty deduced neutron difference [42].

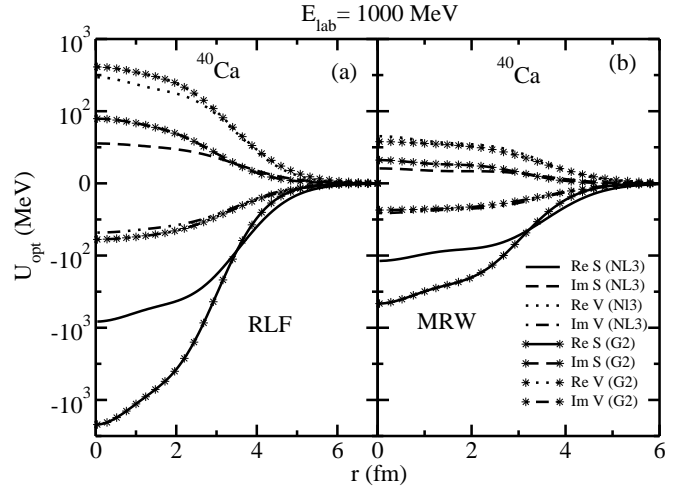


FIG. 4: The real (Re) and imaginary (Im) part of the scalar S and vector V Dirac optical potential is plotted as a function of the nuclear radius for $p + ^{40}\text{Ca}$ system using RMF (NL3) and E-RMF (G2) densities. (a) is for RLF and (b) is for MRW parametrization. The energy of the projectile proton is $E_{lab} = 1000$ MeV.

Further, the agreement of ρ_p with the experiment [39] and ρ_n with the deduced data [40] for NL3 set is slightly better than that of G2. Explicitly, it is worth mentioning that the ρ_p (NL3) matches with the data even at the central region, whereas the ρ_p of G2 under-estimates through out the density plot.

A microscopic investigation of Fig. 1 shows a change in

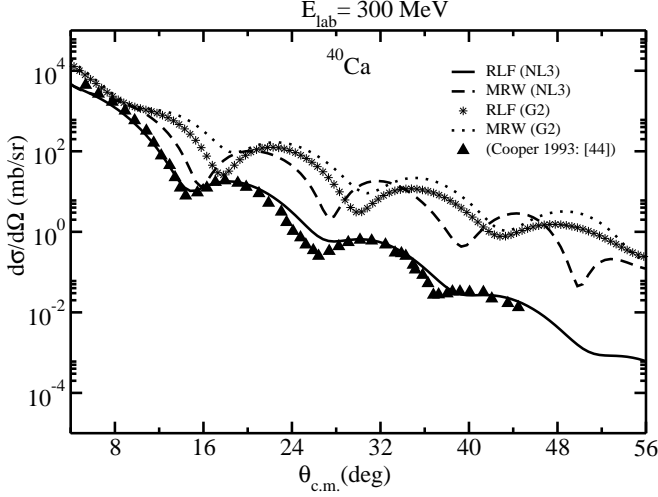


FIG. 5: The elastic differential scattering cross-section ($\frac{d\sigma}{d\Omega}$) as a function of scattering angle $\theta_{c.m.}$ (deg) for ^{40}Ca using both RLF and MRW parametrizations at $E_{lab} = 300$ MeV. Triangles are the experimental data [44].

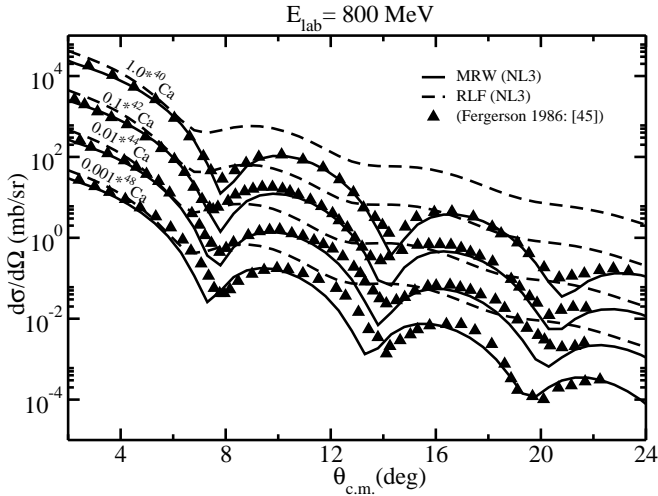


FIG. 6: Same as Fig. 5 for $^{40,42,44,48}\text{Ca}$ at $E_{lab} = 800$ MeV. The experimental data are taken from [45].

$\rho_p(r)$, $\rho_n(r)$, i.e. the area covered by the proton and neutron densities gradually increases with the mass number in an isotopic chain. From the $\rho_p(r)$ and $\rho_n(r)$, we estimate the possible relative isotopic density difference $\Delta\rho(r)$ for RMF (NL3) and E-RMF (G2) parameter sets (see Figs. 2 and 3). The calculated $\Delta\rho_p(r)$ are compared with the experimental data [41] in Fig. 2. The measured data of $\Delta\rho_p(r)$ lies in between the prediction of NL3 and G2 values as shown in Fig. 2. Comparing $\rho_p(^{42}\text{Ca}) - \rho_p(^{40}\text{Ca})$, $\rho_p(^{44}\text{Ca}) - \rho_p(^{40}\text{Ca})$,

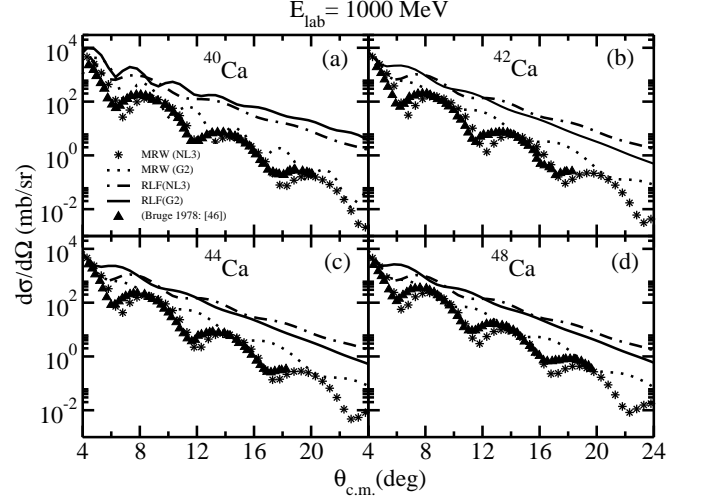


FIG. 7: Same as Fig. 6 at $E_{lab} = 1000$ MeV. The experimental data are taken from [46].

and $\rho_p(^{48}\text{Ca}) - \rho_p(^{40}\text{Ca})$ of Fig.2 [(a), (b) and (c)], we notice a better agreement of NL3 values over G2 with respect to experimental measurement in the isotopic chain which subsequently reflects in the results of scattering observables.

The relative isotopic density difference for neutron $\Delta\rho_n(r)$ is compared in Fig. 3 with the deduced neutron density difference data [42] and the density-matrix-expansion prediction [43]. The predicted results with RMF (NL3) agree well only for the double closed shell nuclei ^{40}Ca and ^{48}Ca . But in case of E-RMF (G2) we get excellent match with the deduced $\Delta\rho_n(r)$ for the considered isotopic chain. There is a peak appears in $\Delta\rho_n(r)$ at radial range $r \sim 3.4 - 3.8\text{fm}$ and this peak slightly shifted towards the center with the increase of neutron number. Although $\Delta\rho_n(r)$ for G2 set gives better agreement with the deduced values, the use of NL3 set in the RIA formalism works well for the scattering observables (shown later).

B. Optical potential

With the densities in hand, we calculate the optical potential U_{pot} for $^{40,42,44,48}\text{Ca}$ by folding the density matrix with the NN scattering amplitude of the proton projectile for 300, 800 and 1000 MeV. The U_{pot} is a complex function which constitute both real and imaginary part for both the scalar and the vector potentials. In Fig. 4, we present the U_{pot} for $\vec{p} + ^{40}\text{Ca}$ at laboratory energy $E_{lab} = 1000$ MeV as a representative case. We also examine the U_{pot} for other Ca isotopes and find similar trends with $\vec{p} + ^{40}\text{Ca}$. In other words, we do not get any significant difference in the optical potential with the increase of neutron number. Similar to the density distribution in NL3 and G2 (Fig. 1), here we find a difference in $U_{opt}(q, E)$ between the RLF and MRW parametrizations. The evaluation methods of the optical potentials using RLF or

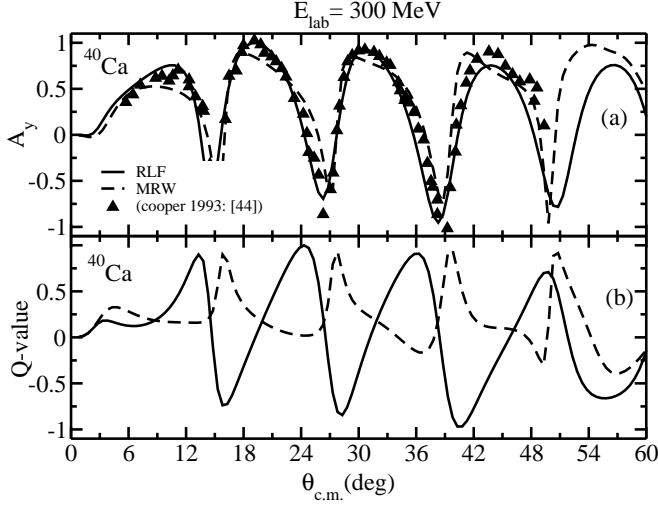


FIG. 8: Analyzing power A_y and Q-Values (spin observables) as a function of scattering angle $\theta_{c.m.}$ (deg) for ^{40}Ca at $E_{lab} = 300$ MeV. The experimental data are taken from [44].

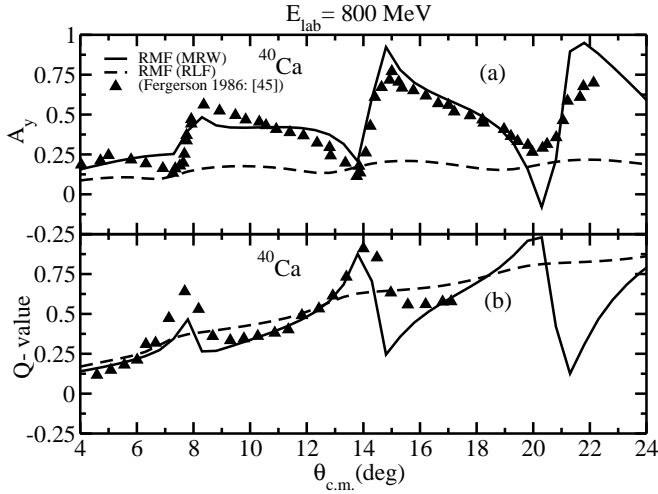


FIG. 9: Same as Fig. 8, but at $E_{lab} = 800$ MeV. The experimental data are taken from [45].

MRW (see Fig. 4) are somewhat different from each other, which is given in **Appendix A** [38], which is responsible for the use of the different parametrizations at various ranges of incident energies. For example, the RLF parameters used here are from Refs. [17, 18] which are computed for energies up to 400 MeV and are therefore suitable for lower E_{lab} whereas the MRW is better for the higher values which will be discussed in the coming sections. Further, the $U_{opt}(q, E)$ values from either RLF or MRW, differs significantly depending on the NL3 or G2 force parameters. That means, the optical po-

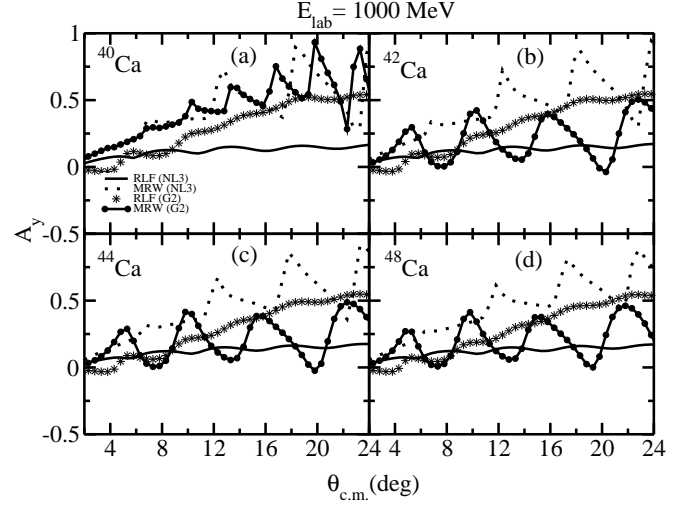


FIG. 10: Analyzing power A_y as a function of scattering angle $\theta_{c.m.}$ (deg) for $^{40,42,44,48}\text{Ca}$ at $E_{lab} = 1000$ MeV.

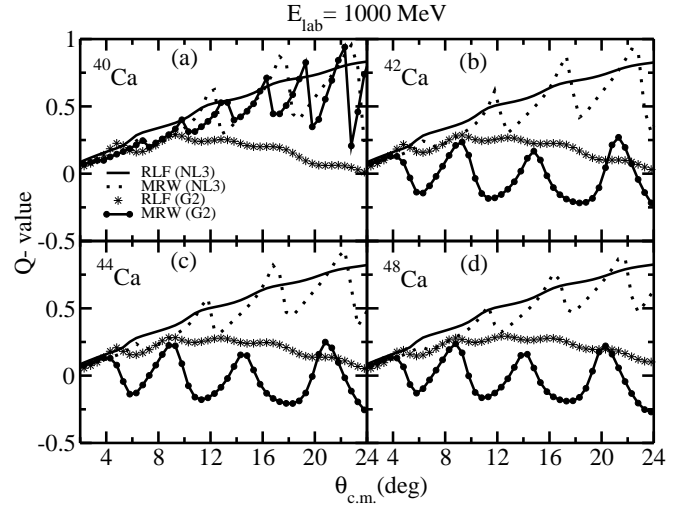


FIG. 11: The spin observable Q -value as a function of scattering angle $\theta_{c.m.}$ (deg) for $^{40,42,44,48}\text{Ca}$ at $E_{lab} = 1000$ MeV.

tential is not only sensitive to RLF or MRW but also to the use of NL3 or G2 densities. Investigating the figure, it is clear that the extreme values of the magnitude of real and imaginary part of the scalar potential are -382.9 and 110.6 MeV for RLF (NL3) and -372.4 and 177.8 MeV for RLF (G2) respectively. The same values for the MRW parametrization are -217.7 and 40.2 MeV with the NL3 and -333.8 and 61.7 MeV with the G2 sets. In case of the vector potential, the extreme values for the real and imaginary parts are 293.0 and -136.0 MeV for RLF (NL3) and 319.7 and -157.5 MeV for RLF (G2) but

with MRW parametrization these appear at 124.1 and -82.3 MeV with the NL3 and 115.5 and -77.1 MeV with the G2. From these variations in the magnitude of scalar and vector potentials, it is clear that the predicted results not only depend on the input target density, but also sensitive to the kinematics of the reaction dynamics. A further analysis of the results for the optical potential with RLF, it is noticed that the U_{opt} value extends for a larger distance than MRW. For example, with RLF the central part of U_{opt} is more expanded than MRW and ended at $r \sim 5fm$, whereas the U_{opt} persists till $r \sim 6fm$. It is important to point out that the lack of the availability of experimental data for optical potential, we are unable to justify the capability of parametrizations at different energies. We also repeat the calculations without Pauli blocking and found almost identical results for optical potential at $E_{lab} \sim 300, 800$ and 1000 MeV. The effects of RLF and MRW parametrizations are presented in the next subsections during the discussion of scattering observables.

C. Differential scattering cross-section

Evaluation of the differential elastic scattering cross-section $\frac{d\sigma}{d\Omega}$, defined in Eqn. (40) is crucial to study the scattering phenomena. The results of our calculation for $\vec{p} + {}^{40}Ca$ and $\vec{p} + {}^{40,42,44,48}Ca$ systems at incident energies 300, 800 and 1000 MeV, respectively are displayed in Figs. 5, 6 and 7 along with the available experimental data [44–46]. As it is stated earlier that the RIA prediction with the NL3 density is better to the choice of G2 for all the angular distributions, irrespective of the use of RLF or MRW parametrizations. Again considering the energy of the projectile, the RLF predictions best fit to the data for $E_{lab} \leq 400$ MeV (see Fig. 5). However, results obtained from the MRW parametrization is better for higher incident energies (Figs. 6 and 7) ($E_{lab} > 400$ MeV) [19, 38]. This result shows a fundamental difference between the RLF and MRW parametrization depending upon the incident energy ranges. Perhaps due to this reason, the explicit off shell behavior of RLF and MRW is drastically affecting the scattering predictions. Similarly for the optical potential the results are insensitive to the Pauli blocking.

D. Analyzing power and Spin Observable

The analyzing power A_y and the spin observable (Q-Value) are calculated from the general formulae given in Eqns. (41) and (42) respectively. The results of our calculations for $\vec{p} + {}^{40}Ca$ system at incident energies 300 MeV and 800 MeV are shown in Figs. 8 and 9. The RIA predictions for A_y using RLF with RMF (NL3) density show a quantitative agreement with the data [44] at 300 MeV whereas this observation is just reverse at 800 MeV [45]. That means, the prediction of A_y resemble the $\frac{d\sigma}{d\Omega}$ observations of Figs. 5 – 7. In Figs. 10 and 11, we present the A_y and Q -value for $\vec{p} + {}^{40,42,44,48}Ca$ composite system at 1000 MeV. These results are obtained for both the RLF and MRW parametrizations with NL3 and G2 densities in comparison with the experimental data [46].

The calculated A_y and Q -values obtained by these two forces differ significantly from each other for the choice of RLF and MRW parametrizations. Also, we observe small oscillations in the values of A_y and Q with the increase in scattering angle $\theta_{c.m.}$ for both RLF and MRW. This oscillatory behavior could be related with the dispersion phenomenon of the optical potential. Similar to the $\frac{d\sigma}{d\Omega}$, here also the prediction of MRW is best fitted to the data for the higher and RLF for lower incident energies. Further, investigation into the spin rotation parameter Q -value, the peak shift and diminished magnitude with the increase in neutron number (see Fig. 11) agrees with the calculation of first order Brueckner theory using Urbana V14 soft core inter-nucleon interactions [47]. It leads to the nucleon finite size correction more realistic and hence merits a structure effect for the formation of exotic nuclei in laboratory.

VI. SUMMARY AND CONCLUSION

We have calculated the density distribution of protons and neutrons for ${}^{40,42,44,48}Ca$ by using RMF (NL3) and E-RMF (G2) parameter sets. From these densities, we estimate the relative isotopic neutron density difference for both the force parameters. The comparison of $\Delta\rho_n(r)$ with the data [42] indicates the superiority of G2 over NL3. The small difference in the density at the central region significantly affect the results of scattering observables including the optical potential. A fundamental difference between RLF and MRW parametrizations as well as RMF (NL3) and E-RMF (G2) sets in the RIA predictions is noticed from the observation of $\frac{d\sigma}{d\Omega}$, A_y and Q -value. We conclude from our calculations that RLF relatively works well at lower and MRW at higher incident energies. The predicting capability of scattering observables of RMF (NL3) over E-RMF (G2) is also realised.

In conclusion, the reaction dynamics highly depends on the input density and the choice of parametrization. In addition to this, our present study indicates that the RIA is a powerful predictive model which provides a clear picture about the successful Dirac optical potentials and can be useful to study both stable and exotic nuclei.

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Appendix A

If RLF is our choice, the t^L functions in Eqns. (17-19) and (23-24) involves all the occupied states for pp and pn scattering. It is most convenient to shift variables from $x' \rightarrow$

$x + x'$ so the t^L functions are not involved in the angular integration. Now, the first order optical potential Eqn. (20)

can be written as [38],

$$U^L(r, E) = \frac{-4\pi ip}{M} \left[\int d^3 r' \rho^L(x + x') t_D^L(r'; E) \right] + \left[\int d^3 r' \rho^L(x + x', x) t_X^L(r'; E) j_0(pr') \right], \quad (A1)$$

after ϕ integration, this become

$$U^L(r, E) = \frac{-8\pi^2 ip}{M} \left[\int dr' t_D^L(r'; E) \int_{-1}^{+1} \rho^L(x + x') d\omega \right] + \left[\int dr' t_X^L(r'; E) j_0(pr') \int_{-1}^{+1} \rho^L(x + x', x) d\omega \right], \quad (A2)$$

where $\omega = \cos\theta$, $(|x + x'|^2) = (r^2 + r'^2 + 2\omega rr')$ and $(\frac{|2x+x'|^2}{4}) = \frac{1}{4}(r'^2 + 4\omega rr' + 4r^2)$. The integral evaluated by Gauss-Laguerre quadrature. At the point $(x + x')$, the radial integration must go roughly twice the nuclear radius. Note that for spherical nuclei only the scalar and vector are taken

into account, as the tensor terms are negligible.

In case of MRW, the optical potential U_{opt} is calculated somewhat differently from the RLF. Here we tranform the density $\rho^L(x)$ to momentum space, then multiply with the $\mathcal{F}^L(q, E)$, and back which leads to the equation

$$U^L(r, E) = \frac{-4\pi ip}{M} \left[\int \frac{d^3 q}{(2\pi)^3} e^{iqx} \mathcal{F}^L(q, E) \int d^3 x' e^{-iqx'} \rho^L(r') \right], \quad (A3)$$

with $\mathcal{F}^L(q, E) = \mathcal{F}_0^L(E) e^{-q^2 \beta^2(E)}$ at each proton energy E. The final equation is obtained by adding the contributions from proton and neutron states to the direct term Eqn. (A3) which is given as,

$$U^L(r, E) = \frac{-8ip}{M} \frac{1}{r} \left[\int_0^\infty dq \sin(qr) \int_0^\infty dr' r' \sin(qr') \mathcal{F}_0^L(E) e^{-q^2 \beta^2(E)} \rho^L(r') \right]. \quad (A4)$$

The above integrals is solved by double Gussian summation methods.

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