Breathing-mode measurements in Sn isotopes and isospin dependence of nuclear incompressibility

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Abstract

T. Li et al. [Phys. Rev. C 81, 034309 (2010)] have analyzed their measured breathing-mode energies of some tin isotopes in terms of a first-order leptodermous expansion, and find for the symmetry-incompressibility coefficient K_{τ} the value of -550 \pm 100 MeV. Removing an approximation that they made, we find that the first-order estimate of K_{τ} shifts to -661 \pm 144 MeV. However, taking into account higher-order terms in the leptodermous expansion shows that the data are compatible with the significantly lower magnitudes indicated by both another experiment and some theoretical estimates.

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Li et al. [1] have measured the energies E_{GMR} of the giant isoscalar monopole resonance in the seven even-even isotopes $^{112-124}$ Sn, expressing their results in terms of the finite-nucleus incompressibility

$$K(Z,A) = \frac{M}{\hbar^2} R^2 E_{GMR}^2 \quad , \tag{1}$$

where R is the rms matter radius (see also Ref. [2]). Now adopting the leptodermous picture of the nucleus [3], K(Z, A) can be expanded about K_v , the incompressibility of symmetric nuclear matter, in powers of the small quantities $A^{-1/3}$ and I^2 , where I = (N - Z)/A; if we retain only the lowest-order terms beyond K_v we have [4]

$$K(Z,A) = K^{(1)}(Z,A) \equiv K_v + K_{sf}A^{-1/3} + K_{\tau}I^2 + K_{coul}\frac{Z^2}{A^{4/3}}$$
 (2)

However, as Refs. [1, 2] recall, it was shown long ago [5] that fitting Eq. (2) to the measured values of K(Z,A) cannot determine a unique value of K_v , essentially because even though there are only three other coefficients, the data are insufficiently accurate and too few in number. Nevertheless, Refs. [1, 2] argue that it is still possible to extract a value for K_{τ} from their data on the Sn isotopes. They do this by making three assumptions: i) the variation of $A^{-1/3}$ over the measured chain of Sn isotopes is so small that the first two terms on the right-hand side of Eq. (2) can be lumped together as a constant; ii) all higher-order terms in the expansion (2) can be neglected; iii) K_{coul} lies within the range -5.2 \pm 0.7 MeV. They then find that K_{τ} falls in the range -550 \pm 100 MeV.

This result is to be compared with the value of -370 ± 120 MeV extracted from measurements of isospin diffusion in heavy-ion collisions [6]. The two results are not inconsistent, but there is some theoretical support for a lower magnitude of K_{τ} . For example, referring to Table 1 of Ref. [6] (where our K_{τ} is denoted by $K_{sat,2}$), we see that most Skyrme forces and various forms of Gogny forces predict values of K_{τ} lying in the range -325 ± 55 MeV; the only exceptions in this table are forces for which K_v is excessively large. A more recently developed Skyrme force, BSk18 [7], which gives a precision fit to essentially all the mass data, and satisfies several realistic constraints, especially relating to neutron matter, has for K_{τ} the value of -344 MeV. Actually, it is on the basis of this disagreement between the value of K_{τ} that they extract from experiment and the Skyrme values that the authors of Refs. [1, 2] conclude that their data "rule out a vast majority of Skyrme forces". However, further theoretical support for a lower magnitude of K_{τ} comes in the form of three different

Brueckner-Hartree-Fock calculations [8] which yield values of K_{τ} lying in the range -344 to -335 MeV. Moreover, and most significantly, it has been argued [9, 10] that a value of -550 MeV is too strongly negative to be compatible with the behavior of low-density neutron matter, which is determined unambiguously by low-energy neutron-neutron scattering.

Accordingly, in this note we revisit the analysis of Refs. [1, 2] in an attempt to see whether their quoted error bars could be widened to accommodate the lower magnitude of K_{τ} for which there appears to be significant evidence. We find their assumption (iii) (see above) to be reasonable but contest assumptions (i) and (ii), as follows.

Non-constancy of $A^{-1/3}$. If we take account of the variation of $A^{-1/3}$ over the measured chain of Sn isotopes then fitting Eq. (2) to the data will require assumptions about K_v and K_{sf} . For the former we take the well established range 240 ± 10 MeV [11], while K_{sf} is used along with K_{τ} to fit the data (the fits of Refs. [1, 2] had only one free parameter, K_{τ}). Taking into account the range of uncertainty on K_{coul} assumed by Ref. [2], we find that fitting Eq. (2) to the data yields $K_{\tau} = -661 \pm 144$ MeV. These lower and upper limits of K_{τ} correspond to Sets 1 and 2 of Table I and curves 1 and 2, respectively, in Fig. 1.

Comparison of this new value for K_{τ} with the value of -550 \pm 100 MeV given by Refs. [1, 2] shows that the variation of $A^{-1/3}$ over the isotope chain is significant, even if it amounts to only 3%. More seriously, our new analysis of the data has aggravated the conflict with both theory and the only other recent measurement of K_{τ} [6]. Nevertheless, we now show that it is possible to reconcile the data with a significantly smaller magnitude of K_{τ} : bearing in mind the values indicated by the heavy-ion experiment [6] and the various theoretical calculations mentioned above, we shall, to be specific, consider just the value of -350 MeV.

Higher-order terms. Going to the next order in powers of $A^{-1/3}$ and I^2 , Eq. (2) is replaced by

$$K(Z,A) = K^{(1)}(Z,A) + K_{ss}I^2A^{-1/3} + K_{cv}A^{-2/3} + K_{\tau 4}I^4 \quad . \tag{3}$$

Of the three new terms, the last one relates to infinite nuclear matter, whence the coefficient $K_{\tau 4}$ can be reliably calculated for Skyrme forces. However, Table I of Ref. [6] (where our $K_{\tau 4}$ is denoted by $K_{sat,4}$) makes it clear that this term will make only a negligible contribution to K(Z,A), so we study the effect only of the K_{ss} (surface-symmetry) and K_{cv} (curvature) terms (a possible role for the surface-symmetry term was suggested by Colò [12]). Setting $K_v = 240 \text{ MeV}$, $K_{coul} = -5.2 \text{ MeV}$ and $K_{\tau} = -350 \text{ MeV}$, we leave K_{sf} , K_{ss} and K_{cv} as fitting

parameters. Since only two parameters are required for a good fit to the data, an infinite number of fits are possible, one for each value of K_{cv} . We took just two values for K_{cv} , simply to assess the role of the curvature term, and defined thereby Sets 3 and 4 of Table I; the corresponding curves in Fig. I are indistinguishable from the curve 2, corresponding to Set 2.

Conclusions. We see that by invoking the surface-symmetry term it is possible to reconcile the breathing-mode data of Refs. [1, 2] with a value of -350 MeV for K_{τ} ; this value is consistent with the heavy-ion experiment of Ref. [6] and the various theoretical estimates that we have mentioned above. The required value of K_{ss} can be reduced if we make use of the curvature term as well. Of course, one might ask whether the required values of K_{sf} , K_{ss} and K_{cv} are physically plausible. In principle, we could answer this question by making the appropriate calculation of semi-infinite nuclear matter with various forces, but such calculations tend to be somewhat unreliable because of stability problems. In any case, our required values of K_{sf} , K_{ss} and K_{cv} must be regarded as "effective" values, since they absorb the effect of all the terms of still higher order, such as a term in $I^2A^{-2/3}$, that we have neglected in fitting the data. In fact, it has been suggested [13] that the leptodermous expansion may not converge at all, no matter how many terms are taken. The only reliable approach to the problem would be to calculate the breathing-mode energies of each nucleus directly through self-consistent quasiparticle random-phase approximation (QRPA) (or constrained Hartree-Fock-Bogoliubov (HFB)) calculations with a succession of different forces until agreement with the data is reached [14]. This is quite beyond the scope of the present note.

This state of affairs is somewhat unsatisfactory. However, we remark that the situation is little better with the analysis of Refs. [1, 2]: the tacit assumption of zero values for K_{ss} and K_{cv} is just as much in need of justification as are our required non-zero values. Thus it was altogether premature of Refs. [1, 2] to conclude that their data "rule out a vast majority of Skyrme forces".

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TABLE I: Parameter sets (in MeV) for calculating K(Z,A).

	K_v	K_{coul}	$K_{ au}$	K_{sf}	K_{ss}	K_{cv}
Set 1	250.0	-5.9	-805.0	-386.0	0	0
Set 2	230.0	-4.5	-517.0	-352.0	0	0
Set 3	240.0	-5.2	-350.0	-382.5	-980.0	0
Set 4	240.0	-5.2	-350.0	-488.0	-835.0	500.0

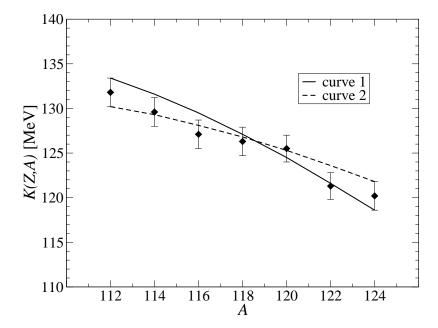


FIG. 1: Fits of parameter sets of Table I to experimental values of K(Z, A). Curve 1 corresponds to Set 1, curve 2 to Sets 2, 3 and 4.