# GeoGebra Tools with Proof Capabilities

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#### Abstract

We report about significant enhancements of the complex algebraic geometry theorem proving subsystem in GeoGebra for automated proofs in Euclidean geometry, concerning the extension of numerous GeoGebra tools with proof capabilities. As a result, a number of elementary theorems can be proven by using GeoGebra's intuitive user interface on various computer architectures including native Java and web based systems with JavaScript. We also provide a test suite for benchmarking our results with 200 test cases.

# 1 Introduction

GeoGebra is an educational mathematics software tool, with millions of users. In 2005, its founder Markus Hohenwarter broadened its software development into an open source project. GeoGebra's features (including dynamic geometry, computer algebra, spreadsheets and function investigation) primarily focus on facilitating student experiments in Euclidean geometry, and not on formal reasoning. Including automated deduction tools in GeoGebra's dynamic geometry system (DGS) could introduce a whole new range of learning and teaching scenarios.

Since automated theorem proving (ATP) in geometry has reached a rather mature stage, in 2010 some ATP experts agreed on starting a project of incorporating and testing a number of different automated provers for geometry in GeoGebra. This collaboration was initiated by Tomás Recio. Since the initial kickstart this project reached the following milestones:

1. A workshop for **theoretical planning** took place in Santiago de Compostela, Spain, February 2011.

- 2. A second workshop for **implementation planning** took place in Alcalá de Henares, Spain, January 2012.
- 3. A **prototype** implementation was presented in Alcalá de Henares in June 2012 by demonstrating 44 test cases using 5 different theorem prover methods [2, 12].
- 4. **First public release** in GeoGebra 5.0 in October 2014 with 60 test cases [4].
- 5. Full **documentation** and fixing several issues according to users' feedback in July 2015 in [15].
- 6. Extension of the set of the translated dynamic geometry construction tools to cover 200 test cases.

In this paper we report about the last milestone. In section 2 we give a comprehensive overview about the first milestones. Section 3.1 summarizes our results by focusing on the general improvements in GeoGebra. Section 3.2 shows some tables concerning our test results. Section 4 sketches up our next planned steps for another milestone in the implementation.

# 2 Overview

An interactive prover system designed mainly for secondary school students can differ from expert prover systems in some aspects. For example, GCLC [9] and OpenGeoProver [19] process a program code written in its special language and print the output as a precise report about the computation details. By contrast, a DGS tool should collect all pieces of information about the relationships of the objects purely by analyzing the construction being created by point-and-click edits and possibly some other input parameters for the prover commands; finally the output is typically a yes/no answer and eventually some extra prescribed conditions to avoid degeneracy cases.

There is a plenty of literature on reports on successful applications of DGS by extending one with an ATP subsystem. Among them, here we mention GeoProof [18] and LADucation [1] which are open-sourced, and thus it is possible to continue their efforts by external researchers also. A publicly available variant of LADucation was already able to import a GeoGebra construction and set up an equation system which was solved by an external computer algebra system (CAS). Other systems including JGEX [24] and Cinderella [11] are not open-sourced, but built upon a similar approach: visualizations in the DGS must be supported by ATP computations.

Our project harnessed GeoGebra's success in the classrooms and tried to address some problems of the existing DGS/ATP prototypes including small distribution, being unmaintained or incomplete operation. In our solution in GeoGebra a user creates a dynamic geometry construction which contains free

points and dependent points as usual. All dependent points are already determined by the free points, however, all free points can be dragged by the user as desired. When a free point is dragged, some dependent points will also be changed by following the definitions in the construction steps. In such a way geometric theorems can be visualized by experiment.

This technique is well known in the world of DGS. Going one step forward, an ATP subsystem can give a more sound answer whether the visually obvious facts (for example, if three dependent points in a given construction are always collinear) generally hold. GeoGebra's command line interface with its **Prove** and **ProveDetails** commands and the graphical *Relation Tool* [16] introduce a higher level interface to investigate the problem setting by using an ATP subsystem.

Proving Euclidean elementary geometry theorems was introduced in GeoGebra with its version 5 in September 2014. A report [4] shows a benchmark about 60 theorems which can be directly checked with the **Prove** and **ProveDetails** commands in GeoGebra. More details are shown in [14] about how the prover subsystem is embedded to GeoGebra's user interface intuitively by using and extending the Relation Tool.

There are several approaches to compute a proof internally by using GeoGebra's portfolio prover [13], including

- Wu's method [23] by using OpenGeoProver externally, and also
- the area method [6] (via OpenGeoProver), moreover
- Recio's exact check method [12] and
- the Gröbner basis method [10, 17].

In our present work we focused on the internally implemented *Gröbner basis* method which translates the geometric objects to algebraic equations directly and manipulates on the algebraic equation system by eliminating the dependent variables. Our work could be however used for Wu's method also, since we just defined a set of equations to translate geometric construction tools into an algebraic approach. We used complex algebraic geometry in our computations which is a standard way to set up a Euclidean geometry question (see [7, chapter 6]).

# 3 Our enhancements

We report about our contributions to GeoGebra in two major areas:

- 1. Implementation of symbolic equations for various geometric tools (section 3.1).
- 2. Creating a number of tests to extend the benchmarks (section 3.2).

# 3.1 Symbolic equations

GeoGebra's geometry tools have been classified by [20, p. 104] as "easy to use", "middle" and "difficult to use". Preiner defines two criteria for a tool to be easy (p. 121):

- The tool does not depend on already existing objects, or just requires existing points which can also be created 'on the fly' by clicking on the drawing pad. The order of actions is irrelevant and no additional keyboard input is required.
- 2. The tool directly affects only one type of existing object or all existing objects at the same time and requires just one action. Again, the order of actions is irrelevant and no additional keyboard input is required.

The basic concept in our work was to implement theorem proving features for the easier tools in GeoGebra. Also it was important that the usually discussed classroom theorems can be quickly constructed by using the easier tools. The classroom theorems usually require points, segments, rays, lines and circles, and angles. For some more advanced topics tangents, parabolas, ellipses and hyperbolas may be needed.

### 3.1.1 Translating geometry to algebra

Implementing angles and conics may have theoretical difficulties in our approach. For angles, we refer to the fact that it is not possible to define only the interior bisector of an angle: we always need to work together with internal and external angles at the same time (cf. [5, p. 40]). This is a consequence of handling angles: there is no way to check equality unless one computes the tangent of them, that is, instead of checking  $\alpha = \beta$  one verifies  $\tan(\alpha) = \tan(\beta)$  and these formulas are equivalent only if we set up some restrictions, say  $0 \le \alpha, \beta < \pi$ . In this sense we cannot distinguish  $0^{\circ}$  and  $180^{\circ}$ .

For conics, ellipses and hyperbolas must also be handled as non-distinguishable objects, because using the synthetic approach we need to define them with their foci, and the defining relations are the same. More precisely, given foci A and B and conic point C, another point P is an element of the conic if and only if AC + CB = AP + PB in the case of an ellipse and |AC - CB| = |AP - PB| (that is,  $(AC - CB)^2 = (AP - PB)^2$ ) in the case of a hyperbola. Since the lengths in these equations are non-negative quantities, we either need to add constraints  $AC \geq 0$ ,  $CB \geq 0$ ,  $AP \geq 0$  and  $PB \geq 0$  (which are not possible in complex algebraic geometry due to lack of inequalites), or we need to use the squared quantities  $AC^2$ ,  $CB^2$ ,  $AP^2$  and  $PB^2$  and express these equations exclusively by them. In this second case we need to eliminate the non-squared quantities from the equation. With the help of the following computer algebra command we learn that for both the ellipse and the hyperbola we get the same product of 8th degree (here we used Giac [14] for computations):

returns

```
[(ac-cb-ap-pb)*(ac-cb-ap+pb)*(ac-cb+ap-pb)*(ac-cb+ap+pb)*(ac+cb-ap-pb)*(ac+cb-ap+pb)*(ac+cb+ap-pb)*(ac+cb+ap-pb)]
```

which has the same result as for the input

Interpreting the result, it is only possible to define the set  $AC\pm CB = \pm AP\pm PB$  in the complex algebraic geometry sense which consists of 8 theoretical curves:

- 1. AC + CB = AP + PB, the ellipse,
- 2.  $AC + CB = AP PB \Leftrightarrow AC + CB + BP = AP$ , which—according to the triangle inequality—is possible only in a degenerate case when A, B and C (and also P) are collinear,
- 3.  $AC + CB = -AP + PB \Leftrightarrow PA + AC + CB = PB$ , similar to the previous collinear case,
- 4.  $AC + CB = -AP PB \Leftrightarrow AC + CB + AP + PB = 0$  which is possible only in a degenerate case when A = B = C = P.
- 5.  $AC CB = AP + PB \Leftrightarrow CA + AP + PB = CB$ , similar to the former collinear cases,
- 6. AC CB = -AP + PB, one branch of the hyperbola,
- 7. AC CB = AP PB, the other branch of the hyperbola,
- 8.  $AC CB = -AP PB \Leftrightarrow CA + AP + PB = CB$ , again similar to the former collinear cases.

That is, we indeed obtained that an ellipse and a hyperbola cannot be distinguished in this model (but all other non-degenerate curves can be distinguished from them). This issue will give some limitations to investigate special features of conics, but still enable investigating some common features of them. For example, the following generalization of Pascal's hexagon theorem for conics holds (see Fig. 1):

**Theorem 1.** Let c be the union of an ellipse and a hyperbola, both defined with foci A, B and circumpoint C. Let a denote line AB and let the perpendicular bisector of a be b. Let C' be the reflection of C to the line a and C'' to b. Also let us take an arbitrary point D on c and by reflection to a and b, respectively, obtain points D' and D''. Now the intersections of CD and C''D'', CD' and C''C'', moreover C'D and D'D'' will be collinear.

A consequence of this example that some *formulas* can also be difficult to distinguish, and may require further investigation by using elimination and factorization with the help of a CAS.

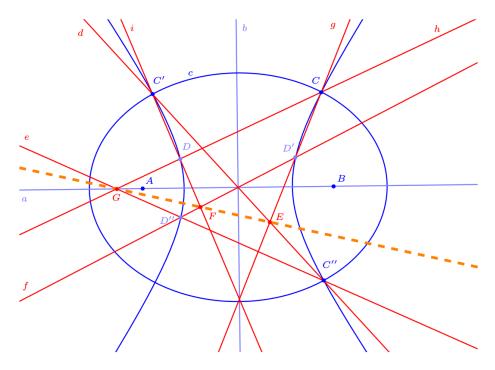


Figure 1: A generalization of Pascal's hexagon theorem

In general, when a construction is given, it is important to identify geometrical hypotheses which are non-distinguishable from other geometrical hypotheses because they are translated with the same algebraic formula. When the prover disproves the respective statement in the algebraic translation, it should not be interpreted that the geometry statement was false. This is the case when attempting to prove that the internal bisectors of a triangle are concurrent: the algebraic translation actually disproves that the union of the internal and external angle bisectors are concurrent.

Also it is important to identify geometrical theses which are non-distinguishable from other geometrical theses because they are translated with the same algebraic formula. When the prover proves the respective statement in the algebraic translation, it should not be interpreted that the geometry statement was true.

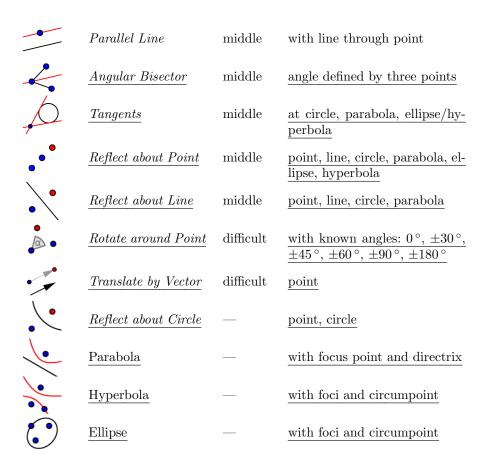
### 3.1.2 The implemented tools

Apart from considering these issues, we managed to handle many typical class-room situations, and we report that most "easy" tools are implemented, and also some other tools from the "middle" and "difficult to use" toolset.

The following basic geometrical shapes are now implemented: segment, line, ray and vector, each defined by two points, circle defined with center and through

point or through three points, angle, parabola with focus point and directrix, ellipse and hyperbola defined with two focus points. This table summarizes them, and also those tools which can operate on the basic geometrical shapes (the latter ones printed in italicized description, underlined objects are new enhancements compared to [15]):

Tool	Description	Difficulty	Implementation remarks
<ul><li>A</li></ul>	Point	easy	
No. of	Line	easy	
	Segment	easy	
	Circle through 3 Points	easy	
•	Midpoint or Center	easy	points and segments
	Perpendicular Bi- sector	easy	at line and segment
cm <sup>2</sup>	<u>Area</u>	easy	polygons of
	Ray	middle	
	Vector	middle	
α	Angle	middle	
•	Circle with Center through Point	middle	
	Polygon	middle	
<b>X</b>	Intersect	middle	line with line (cannot decide properly for segments), with circle, with parabola, with ellipse, with hyperbola; circle with circle (for other conics we cannot decide properly)
	Perpendicular Line	middle	at line through point



The remaining, yet unimplemented "easy" tools in GeoGebra are: *Conic through 5 Points* and *Slope*. The former one is actually not widely used in the classroom, and the latter is a non-synthetic tool, that is, it is related to *analytic geometry*. Some other missing, but planned features are listed in section 4.

### 3.1.3 An example

**Theorem 2.** Let c be a circle with center A and circumpoint B. Let a be a line through B and C. Now—not considering some degenerate cases—reflecting line a about c the image is a circle, that is, for arbitrary point  $D \in a$  its reflection D' about c always lies on the same circle (which is the circumcircle of points A, B'(=B) and C', where B' and C' are the mirror images of B and C about c, respectively).

In other words, an inversion translates lines to circles in general. To use GeoGebra's Relation Tool (see Fig. 2) one needs to set up the construction as described in the Algebra View on the left (either by selecting tools from the top, or by using commands in the Input Bar on the bottom). Finally one has to

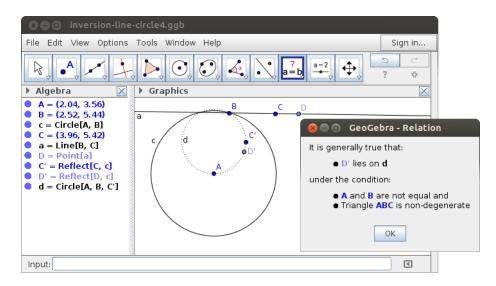


Figure 2: Inversion translates lines to circles

select the Relation Tool from the top and choose point D' and line d (or enter the command  $\mathbf{Relation}[\mathbf{D},\mathbf{d}]$  in the Input Bar). GeoGebra now numerically checks if  $D \in d$ , the answer is yes, and the user can request a symbolical check by clicking on "More...". Finally GeoGebra concludes that—under some non-degeneracy conditions—the statement is generally true.

From the computational point of view, GeoGebra here uses the Gröbner basis method. Thus it sets up the following 6 equations in 13 variables, but point A will be fixed to the origin (so there are only 11 variables remaining). The following log information is printed only in debug mode in GeoGebra, including the timestamp in the first column:

```
19:48:26.550 // Free point A(v1,v2)
19:48:26.550 // Free point B(v3,v4)
19:48:26.550 c = Circle[A, B] /* Circle through B with center A */
19:48:26.551 // Free point C(v5,v6)
19:48:26.551 a = Line[B, C] /* Line through B, C */
19:48:26.551 D = Point[a] /* Point on a */
19:48:26.555 // Constrained point D(v7,v8)
19:48:26.555 Hypotheses:
19:48:26.555 1. -1*v7*v6+v8*v5+v7*v4+-1*v5*v4+-1*v8*v3+v6*v3
19:48:26.556 C' = Mirror[C, c] /* C mirrored at c */
19:48:26.560 // Constrained point C'(v9,v10)
v2^2 + v6^2 * v1 + 2 * v9 * v5 * v1 + v5^2 * v1 + -1 * v4^2 * v1 + -2 * v5 * v3 * v1 + -1 * v3^2 * v1 + -2 * v6 * v2 * v1 + 2 * v4 * v2 * v1 + 2 * v5 * v1
             v1+-1*v9*v1^2+-1*v5*v1^2+2*v3*v1^2
19:48:26.562 3. -1*v10*v6^2+-1*v10*v5^2+v6*v4^2+v6*v3^2+2*v10*v6*v2+v6^2*v2+v5^2*v2+-2*v6*v4
             v5*v2*v1+2*v3*v2*v1+-1*v10*v1^2+v6*v1^2
19:48:26.562 D' = Mirror[D, c] /* D mirrored at c */
19:48:26.566 // Constrained point D'(v11,v12)
v7*v2^2+v8^2*v1+2*v11*v7*v1+v7^2*v1+-1*v4^2*v1+-2*v7*v3*v1+-1*v3^2*v1+-2*v8*v2*v1+2*v4*
             v2*v1+-1*v11*v1^2+-1*v7*v1^2+2*v3*v1^2
```

```
v7*v2*v1+2*v3*v2*v1+-1*v12*v1^2+v8*v1^2
19:48:26.568 Hypotheses have been processed.
19:48:26.574 substitutions: {v1=0, v2=0}
19:48:26.574 Thesis reductio ad absurdum (denied statement)...
v13*v11*v10*v4^2+-1*v13*v12*v9*v4^2+-1*v13*v12^2*v10*v3+-1*v13*v11^2*v10*v3+v13*v11
     ^2*v3+v13*v12*v9^2*v3+v13*v11*v10*v3^2+-1*v13*v12*v9*v3^2+v13*v11*v10^2*v2+-1*v13*v12
     ^2*v9*v2+-1*v13*v11^2*v9*v2+v13*v11*v9^2*v2+-1*v13*v11*v4^2*v2+v13*v9*v4^2*v2+v13*v1
    ^2*v3*v2+v13*v11^2*v3*v2+-1*v13*v10^2*v3*v2+-1*v13*v9^2*v3*v2+-1*v13*v11*v3^2*v2+v13*v9
    *v3^2*v2+-1*v13*v11*v10*v2^2+v13*v12*v9*v2^2+v13*v11*v4*v2^2+-1*v13*v9*v4*v2^2+-1*v13*
    v12*v3*v2^2+v13*v10*v3*v2^2+v13*v12^2*v10*v1+v13*v11^2*v10*v1+-1*v13*v12*v10^2*v1+-1*
    v13*v12*v9^2*v1+-1*v13*v12^2*v4*v1+-1*v13*v11^2*v4*v1+v13*v10^2*v4*v1+v13*v9^2*v4*v1+
    v13*v12*v4^2*v1+-1*v13*v10*v4^2*v1+v13*v12*v3^2*v1+-1*v13*v10*v3^2*v1+-1*v13*v111*v10*v1
     ^2+v13*v12*v9*v1^2+v13*v11*v4*v1^2+-1*v13*v9*v4*v1^2+-1*v13*v12*v3*v1^2+v13*v10*v3*v1^2
19:48:26.592 Eliminating system in 11 variables (6 dependent)
```

Then the underlying CAS (here Giac) eliminates variables v8, v9, v10, v11, v12 and v13 to describe non-degeneracy conditions between the coordinates of the free points. The obtained equation system in factorized form is produced in the following output (which is compatible with Singular's arrays, cf. [15, p. 146]):

```
[1]:
      [1]:
          _[1]=1
            _[2]=-v6^2-v5^2
                  [3]=v7
      [2]: 1,1,1
[2]:
      [1]:
          _[1]=1
            _{2}=v4^2+v3^2
          _[3]=v5
               _[4]=v7
      [2]: 1,1,1,1
[12]:
      [1]:
            _[1]=1
             \  \  \, \_[2] = v4 * v6 * v7^3 - v4 * v6 * v7^2 * v3 - v4 * v6 * v7 * v5^2 + v4 * v6 * v5^2 * v3 - v6^2 * v5 * v3^2 + v7^3 * v5 * v3 - v7^2 * v5 * v3 + v6 * v7^2 * v5 * v5 + v7^2 * v
                                  v3^2-v7*v5^3*v3
      [2]: 1,1
[13]:
      [1]:
            _[1]=1
            _[2]=v4^2+v3^2
            _{1}[3]=-v5*v4+v6*v3
                   [4]=-1
      [2]: 2,2,1,1
```

This is interpreted by GeoGebra as 13 possible sets of degeneracy conditions. Here—because of its geometrical meaning, simplicity and being fully synthetic—the 13th set will be selected, which means: "if  $(v4^2+v3^2)^2*(-v5*v4+v6*v3)$  differs from 0, then the thesis will be true on all possible values of the coordinates of the free points". Since A=(0,0), B=(v3,v4) and C=(v5,v6), this clearly means that the two non-degeneracy conditions being shown are "A differs from B" (that is, circle c is non-degenerate) and "A, B and C are not collinear" (that is, such a line must be chosen for a which is not going through the center of c).

```
Finally, GeoGebra concludes that
```

```
19:48:26.714 Statement is GENERALLY TRUE
19:48:26.714 Benchmarking: 487 ms
```

This computation is done faster than half of a second.<sup>1</sup>

#### 3.1.4 Technical notes

Technically speaking, GeoGebra is a Java application. From the developer's point of view, the Java public interface SymbolicParametersBotanaAlgo has to be implemented in GeoGebra's Algo\* classes by creating suitable algebraic equations (and corresponding new variables) to describe the symbolic background of a newly used tool.<sup>2</sup>

To check the validity of a thesis, the public interface SymbolicParametersB-otanaAlgoAre must be implemented.<sup>3</sup> Currently the following checks are implemented: collinearity, concurrency, concyclicity, congruency, equality, parallelism, perpendicularity, incidence, and formula checking (to prove equations).

### 3.2 Benchmarks

In our improvements the benchmark suite was extended by additional 140 theorems. 57 of these extra tests were chosen from [5]—these tests were computed in Chou's book by using Wu's [23] characteristic method.

Here we summarize our results by sharing a list of the recent benchmarking outputs. GeoGebra's prover benchmarking system is available as a command line tool in its source folder test/scripts/benchmark/prover/.

### 3.2.1 Desktop version

GeoGebra's desktop version runs as a Java native application on the mostly used operating system platforms including Windows, Mac OS X and Linux. Due to the internally used native Giac CAS each platform requires its own compiled version of the embedded computer algebra system.

The following table is the output of the "jar-paper" scenario, launched by the command line xvfb-run ./runtests -S jar-paper -r in this folder. This scenario tests the **Prove** command exclusively. See also [4].

- The first column abbreviates the name of the test cases.
- Column E1 ("Engine 1") refers to Recio's exact check method programmed by Simon Weitzhofer.

<sup>&</sup>lt;sup>1</sup>The steps and the output for computing this example have been simplified to fit this paper. See also [15] for the detailed algorithm of symbolical checking in the Relation Tool.

<sup>&</sup>lt;sup>2</sup>See https://dev.geogebra.org/javadoc/common/org/geogebra/common/kernel/algos/SymbolicParametersBotanaAlgo.html for a recent list of the implemented classes.

<sup>&</sup>lt;sup>3</sup>See https://dev.geogebra.org/javadoc/common/org/geogebra/common/kernel/algos/SymbolicParametersBotanaAlgoAre.html for a recent list.

- Column E2 ("Engine 2") refers to the Gröbner basis method via SingularWS (also known as Botana's method) programmed by the authors of this paper. (See [3] for more on SingularWS.)
- Column E2/Giac refers to Gröbner basis method via the Giac computer algebra tool (instead of SingularWS) programmed by Bernard Parisse and the authors of this paper.
- Column E3a ("Engine 3a") refers to OpenGeoProver's Wu's method implementation programmed by Ivan Petrović and Predrag Janičić.
- Column E3b ("Engine 3b") refers to OpenGeoProver's Area method programmed by Damien Desfontaines.
- The Auto approach refers to the automatic selection of methods which is already implemented in GeoGebra and it usually starts with "Engine 1" and then it continues with "Engine 2" (either via SingularWS or Giac: if SingularWS is available, then in SingularWS, otherwise in Giac). If the Gröbner basis method is not conclusive, then "Engine 3a" is tried. If it is not conclusive either, then OpenGeoProver's Area method (Engine 3b) is used.

See [4] for more details about the used methods. Explanation of the used colors:

- Green means that the test returns a correct yes/no answer. Intensity of green means speed (the lighter the slower). Numbers are in milliseconds.
- Pink means that GeoGebra returns the wrong answer.
- Yellow means the output is not conclusive, thus using this method GeooGebra shows "undefined", i.e. there is no error here.
- The R. ("Result") column provides some extra information about the result, such as f ("false") when the statement was false on purpose.

The S. ("Speed") column shows the timing. Highlighted entries are the best results, italicized entries are the slowest (but working) results in a row. The test cases are also available for download in GeoGebra's .ggb format from the GeoGebra online source code directly.

For testing we used a PC with 16 GB RAM,  $8 \times \text{Intel}(R)$  Core(TM) i7 CPU 860 @ 2.80GHz, and Linux Mint 17.2.

Test		E1		E2	E2/	Giac	I	E3a	E	СЗЪ	P	uto
Test	R.	S.	R.	S.	R.	S.	R.	S.	R.	S.	R.	S.
lines-parallel	f	3	f	35		185	f	65	f	69	f	7
midpoint-third		3	f	217		201		58		72		311
orthocentear		4	f	48		202		75		85		312
points-colar	f	3	f	34		196	f	62	f	64	f	3
points-equal	f	4	f	33		190	f	57	f	68	f	8
Pythagorasn4		1	f	214		196		61	t	85	t	310
tangents-cns		5		1767		1080		210		84		1118
translater3		2	f	42		t/o		61		69		t/o
translatear	f	4	f	37		196		61		67	f	9

					400		0.00		0.0		200
altitudes82	2	t	45	t	198		37		38	t	200
altitudes05	5	t	46	t	200	t	356		t/o	t	199
angle-bisee2	3	t	379	t	323		56		67	t	320
angle-bisele	2	t	377	t	319		63		69	t	322
area-polygon-2	1	t	103	t	242		69		66	t	241
area-polygon	1	t	118	t	252		73		80	t	272
bisector-i20	2	t	231		342		38		34		452
bisector-i19	2	t	180	t	371	t	747		70	t	327
bisector-m22	2	t	224	t	350		2150		70	t	310
bisector-m21	2	t	170	t	305	t	431		69	t	299
bisector-mnt	1	t	40	t	192	t	69	t	73	t	194
	2	L .		· ·	739	t t		U		L L	
bisector-o37			590				88		89		873
bisector-p28	1	t	787		2153	t	517		69	t	2630
bisector-p19	2	t	214		374	t	652		69	t	980
Brianchonla	1		t/o		t/o		1135		71		t/o
cathetus	1	t	224	t	221		67		74	t	221
centroid-mo1	2	t	45	t	197	t	590	t.	174	t	199
centroid-mo2	2	t	215	t	214		58	t	80	t	214
centroid-mo3	2	t	221	t	220		39		39	t	207
centroid-mo4	1		214	t	207		33		38	t	214
		t		t							
Ceva1	1		20		181		57	t	82	t	285
Ceva2	2	t	251	t	798		66	t	91	t	814
Ceva3	3	t	260	t	694		61	t	91	t	748
Ceva4	1	t	239	t	759		68		t/o	t	678
Ceva5	2	t	244	t	758		63		80	t	672
circle-are71	2	t	72		224	t	334		96	t	556
circle-bis16	2		136		289	-	38		32	-	388
circle-bis94	2				421		900		86		474
circle-bis94			t/o			t					
circle-mid11	2	t	71	t	233		1911	t	95	t	243
circle-onent	2	t	39	t	194		30		35	t	186
circle-per75	2	t	45		202	t	492	t	92		243
circle-per95	1	t	46		195		39		35		240
circle-per72	5	t	55		199		191	t	105		363
circle-per67	2	t	54		11175	t	1169		t/o		11324
circle-per68	2	t	53		t/o	t	906	t	1608		t/o
	5	t	54		228	t	860	t	1673	t	933
circle-qua65						t		· ·		L	
circle-ray06	1	t	64		8312		38		38		8289
circles-ar90	4	t	54		t/o		34		34		t/o
circles-ch98	3	t	54		200		33		34		235
circumcenter1	t 5	t	41	t	187	t	236	t	102	t	5
circumcenter2	t 6	t	37	t	198	t	106	t	97	t	7
circumcenter3	t 9	t	36	t	197	t	141	t	98	t	4
circumcenter4	t 4	t	44	t	196	t	141	t	102	t	5
circumcenter5	5	t	46	t	195	t	108	t	96	t	190
			47	t	196		128	t	106		12
circumcenter6		t				t				t	
circumcent97	4	t	49	t	203	t	682	t	156	t	199
circumcircnt	1	t	805		t/o		t/o		68		t/o
constructne	1		61		252		78		84		349
constructnt	1		45		238		77		80		340
def-line-pne	t 4	t	40	t	186	t	70	t	65	+	6
def-pointse1		_		t				t			209
	4		60		215	l t				t	
	4	t	60 68		215	t	210		84	t	
def-pointse2	3	t	68	t	209	t	210 145	t	84 82	t	212
def-pointse2 def-pointsne	3 t 6	t	<b>68</b> 36		209 197	t t	210 145 57	t t	84 82 65	t	212 6
def-pointse2 def-pointsne Desargues	3 t <b>6</b> 5	t t	68 36 49	t	209 197 1108	t t	210 145 57 1068	t	84 82 65 80	t t	212 6 2014
def-pointse2 def-pointsne Desargues diameters74	3 t 6 5 2	t t t	68 36 49 50	t	209 197 1108 203	t t	210 145 57 1068 554	t t	84 82 65 80 85	t t t	212 6 2014 202
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2	3 t 6 5 2 5	t t	68 36 49 50 43	t	209 197 1108 203 196	t t	210 145 57 1068 554 53	t t	84 82 65 80 85 63	t t	212 6 2014 202 185
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3	3 t 6 5 2 5 5 5	t t t	68 36 49 50 43 t/o	t t t	209 197 1108 203 196 216	t t	210 145 57 1068 554 53	t t	84 82 65 80 85 63	t t t t t	212 6 2014 202 185 255
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2	3 t 6 5 2 5	t t t	68 36 49 50 43	t	209 197 1108 203 196	t t	210 145 57 1068 554 53	t t	84 82 65 80 85 63	t t t	212 6 2014 202 185
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle	3 t 6 5 2 5 5	t t t t	68 36 49 50 43 t/o	t t t	209 197 1108 203 196 216	t t	210 145 57 1068 554 53	t t	84 82 65 80 85 63	t t t t t	212 6 2014 202 185 255
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-yy2	3 t 6 5 2 5 5 5	t t t t	68 36 49 50 43 t/o 28	t t t	209 197 1108 203 196 216 189	t t	210 145 57 1068 554 53 59	t t	84 82 65 80 85 63 63	t t t t t	212 6 2014 202 185 255 181
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syry	3 t 6 5 2 2 5 5 4 4	t t t t t t	68 36 49 50 43 t/o 28 46 40	t t t t	209 197 1108 203 196 216 189 206	t t t	210 145 57 1068 554 53 59 59 62 61	t t	84 82 65 80 85 63 63 62 64	t t t t t	212 6 2014 202 185 255 181 236
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle ellipse-circle ellipse-syy2 ellipse-syry EulerLine	3 t 6 5 2 2 5 5 5 4 t 13	t t t t t t t	68 36 49 50 43 t/o 28 46 40 50	t t t t t t t	209 197 1108 203 196 216 189 206 192	t t	210 145 57 1068 554 53 59 59 62 61	t t	84 82 65 80 85 63 63 62 64 56	t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syry EulerLine expressiond1	3 t 6 5 2 5 5 4 4 t 13	t t t t t t t t t	68 36 49 50 43 t/o 28 46 40 50 237	t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213	t t t	210 145 57 1068 554 53 59 59 62 61 729 64	t t	84 82 65 80 85 63 63 62 64 56 238	t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syry EulerLine expressiond1 expressiond2	3 t 6 5 2 5 5 5 4 t 13 1	t t t t t t t t t t t	68 36 49 50 43 t/o 28 46 40 50 237 222	t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213 209	t t t	210 145 57 1068 554 53 59 62 61 729 64	t t	84 82 65 80 85 63 63 62 64 56 238 75	t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syry EulerLine expressiond1 expressiond2 expressiond2	3 t 6 5 2 5 5 5 4 t 13 1 1 2	t t t t t t t t t t t t	68 36 49 50 43 t/o 28 46 40 50 237 222 220	t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213 209 219	t t t	210 145 57 1068 554 53 59 59 62 61 729 64 73	t t	84 82 65 80 85 63 63 62 64 56 238 75 78	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syd1 expressiond1 expressiond2 foot-exists	3 t 6 5 2 5 5 4 4 t 13 1 1 2 3	t t t t t t t t t t t t	68 36 49 50 43 t/o 28 46 40 50 237 222 220 40	t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213 209 219	t t t	210 145 57 1068 554 53 59 62 61 729 64 73 65	t t	84 82 65 80 85 63 62 64 56 238 75 78	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syry EulerLine expressiond1 expressiond2 expression-ex31 foot-exists geometric-mean	3 t 6 5 2 2 5 5 5 4 t 13 1 1 2 3 1	t t t t t t t t t t t t	68 36 49 50 43 t/o 28 46 40 50 237 222 220 40 227	t t t t t t t t t t t t	209 197 1108 203 196 216 206 192 198 213 209 219 219	t t t t	210 145 57 1068 554 53 59 59 62 61 729 64 73 65 314	t t	84 82 65 80 85 63 63 62 64 56 238 75 78 74 70	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syd1 expressiond1 expressiond2 foot-exists	3 t 6 5 2 5 5 5 4 4 t 13 1 2 3 3 1 3	t t t t t t t t t t t t	68 36 49 50 43 t/o 28 46 40 50 237 222 220 40 227 402	t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213 209 219	t t t	210 145 57 1068 554 53 59 62 61 729 64 73 65	t t	84 82 65 80 85 63 63 62 64 56 238 75 78 70 70	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 198 209
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syry EulerLine expressiond1 expressiond2 expression-ex31 foot-exists geometric-mean	3 t 6 5 2 2 5 5 5 4 t 13 1 1 2 3 1	t t t t t t t t t t t t	68 36 49 50 43 t/o 28 46 40 50 237 222 220 40 227	t t t t t t t t t t	209 197 1108 203 196 216 206 192 198 213 209 219 219	t t t t	210 145 57 1068 554 53 59 59 62 61 729 64 73 65 314	t t	84 82 65 80 85 63 63 62 64 56 238 75 78 74 70	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syd1 expressiond1 expressiond2 expression.ex31 foot-exists geometric-mean incenter1 incenter2	3 t 6 5 2 5 5 5 4 4 t 13 1 2 3 3 1 3	t t t t t t t t t t t t t t	68 36 49 50 43 t/o 28 46 40 50 237 222 220 40 227 402	t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213 209 219 190 213 1205	t t t t	210 145 57 1068 554 53 59 59 62 61 729 64 73 65 314 63	t t	84 82 65 80 85 63 63 62 64 56 238 75 78 70 70	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 198 209
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syy2 ellipse-syy2 expressiond1 expressiond2 expressiond2 expression-ex31 foot-exists geometric-mean incenter1 incenter2 inversionnt	3 t 6 5 2 2 5 5 5 4 t 13 1 1 2 3 1 3 3 2 3 3	t t t t t t t t t t t t t t t t t t t	68 36 49 50 43 4/o 28 46 40 50 237 222 220 220 40 227 402 363 37	t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 199 219 198 213 209 219 190 213 1205 1348 201	t t t t	210 145 57 1068 554 53 59 62 61 729 64 73 65 314 63 462 643 80	t t	84 82 65 80 85 63 62 64 56 238 75 78 74 70 76 58	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 198 209 1601 1478
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syty EulerLine expressiond1 expressiond2 expression.ed3 foot-exists geometric-mean incenter1 incenter2 inversionnt inversione2	3 t 6 5 2 5 5 5 4 t 13 1 1 2 3 1 3 2 3 2 3 2	t t t t t t t t t t t t t t t t t t t	68 36 49 50 43 46 40 50 237 222 220 40 227 402 363 37 41	t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 219 198 219 190 211 190 213 209 1348 201 195	t t t t	210 145 57 1068 554 53 59 59 62 61 729 64 73 65 55 314 63 462 643 80 75	t t	84 82 65 80 85 63 63 62 64 56 238 75 78 70 76 58 85	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 198 209 1601 1478 185
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syy EulerLine expressiond1 expressiond2 expressiond2 expressiond3 foot-exists geometric-mean incenter1 incenter2 inversionnt inversionne	3 t 6 5 2 5 5 5 4 t 13 1 1 2 3 3 2 2 2 2	t t t t t t t t t t t t t t t t t t t	68 36 49 50 43 46 40 50 237 222 220 40 227 402 363 37 41	t t t t t t t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 219 209 219 219 190 213 1205 1348 201 195	t t t t	210 145 57 1068 554 53 59 62 61 1729 64 73 65 314 63 462 643 80 75	t t	84 82 65 80 85 63 63 62 64 56 238 75 78 74 70 76 58 89 87	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 1601 1478 185 203 209 1601 218 229 249 25 25 209 209 209 209 209 209 209 209 209 209
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syy1 expressiond1 expressiond2 expressiond3 foot-exists geometric-mean incenter1 incenter2 inversionnt inversione2 inversionne	3 t 6 5 2 5 5 5 4 4 t 13 1 1 2 3 3 1 1 3 2 3 2 5 5 5	t t t t t t t t t t t t t t t t t t t	68 36 49 50 28 46 40 237 222 220 40 227 402 363 37 41 40	t t t t t t t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213 209 219 190 213 1205 1348 201 195 188	t t t t	210 145 57 1068 554 59 62 61 729 64 73 314 63 462 643 80 755 81	t t t	84 82 65 80 85 63 62 64 56 238 75 78 74 70 76 58 65 89	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 198 209 1601 1478 185 203 203
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syd1 expressiond1 expressiond2 expression-ex31 foot-exists geometric-mean incenter1 incenter2 inversionnt inversionne inversionne	3 t 6 5 2 5 5 5 4 4 t 13 1 1 2 3 1 1 2 3 2 3 2 5 5 3	t t t t t t t t t t t t t t t t t t t	68 36 49 50 43 43 46 40 50 28 46 40 237 222 220 40 227 402 363 37 41 40 32 370	t t t t t t t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213 209 219 190 213 1205 1348 201 195 188 179 778	t t t t	210 145 57 1068 554 53 59 59 62 61 729 64 73 65 314 63 462 643 80 75 81	t t t	84 82 65 80 85 63 63 62 64 56 238 75 78 76 76 58 65 89 87	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 198 209 1478 1478 185 203 281 180 773
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syy2 ellipse-syd1 expressiond2 expressiond2 expressiond2 expressiond2 expressionnt inversionnt inversionnt inversionne inversionne inversionne	3 t 6 5 2 5 5 5 4 t 13 1 1 2 3 3 2 2 5 5 3 3 2	t t t t t t t t t t t t t t t t t t t	68 36 49 50 43 46 40 50 237 222 220 40 227 402 363 37 41 40 32 37 40 96	t t t t t t t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 219 198 219 219 190 213 1205 1348 201 195 188 179 778	t t t t	210 145 57 1068 554 53 59 62 61 729 64 73 65 314 63 80 75 81 83 99	t	84 82 65 80 85 63 63 62 64 56 238 75 78 74 70 65 89 87 89	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 1601 1478 185 203 281 180 773
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syd1 expressiond1 expressiond2 expression-ex31 foot-exists geometric-mean incenter1 incenter2 inversionnt inversionne inversionne	3 t 6 5 2 5 5 5 4 4 t 13 1 1 2 3 1 1 2 3 2 3 2 5 5 3	t t t t t t t t t t t t t t t t t t t	68 36 49 50 43 43 46 40 50 28 46 40 237 222 220 40 227 402 363 37 41 40 32 370	t t t t t t t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213 209 219 190 2118 201 190 218 179 778 254	t t t t	210 145 57 1068 554 53 59 59 62 61 729 64 73 65 314 63 462 643 80 75 81	t t t	84 82 65 80 85 63 63 62 64 56 238 75 78 76 76 58 65 89 87	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 198 209 1478 1478 185 203 281 180 773
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syy2 ellipse-syd1 expressiond1 expressiond2 expressiond2 expressiond2 expressiond2 incenter1 incenter1 incenter2 inversionnt inversionnt inversionne inversionne inversione2 inversione3 inversione3 inversione3 inversione3	3 t 6 5 2 5 5 5 4 t 13 1 1 2 3 3 2 2 5 5 3 3 2	t t t t t t t t t t t t t t t t t t t	68 36 49 50 43 46 40 50 237 222 220 40 227 402 363 37 41 40 32 37 40 96	t t t t t t t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 219 198 219 219 190 213 1205 1348 201 195 188 179 778	t t t t t	210 145 57 1068 554 53 59 62 61 729 64 73 65 314 63 80 75 81 83 99	t	84 82 65 80 85 63 63 62 64 56 238 75 78 74 70 65 89 87 89	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 1601 1478 185 203 281 180 773
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syy1 expressiond1 expressiond1 expressiond2 expression.ex31 foot-exists geometric-mean incenter2 inversionnt inversionnt inversionnt inversionnt inversionnt inversione2 inversione3 inversione3 inversione3	3 t 6 5 2 5 5 5 4 4 t 13 1 1 2 3 3 1 3 2 2 2 2 5 5 3 3 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	t t t t t t t t t t t t t t t t t t t	68 36 49 50 48 48 46 40 50 28 40 50 50 60 28 40 20 40 227 402 363 37 41 40 32 370 96	t t t t t t t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213 209 219 190 2118 201 190 218 179 778 254	t t t t t	210 145 57 1068 554 53 59 62 61 729 64 73 314 63 3462 643 80 75 81 83 99	t	84 82 65 80 85 63 62 64 56 238 75 78 70 76 58 65 89 87 88 90	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 198 209 1601 1478 185 203 281 180 773 250
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syy3 ellipse-syy3 foot-exists geometric-mean incenter1 incenter2 inversionnt inversionne inversionne inversionne inversionne inversione2 inversione3 inversione3 inversione3 inversione3 inversione3 inversione3 inversione1 isogonals1	3 t 6 5 2 5 5 5 4 4 t 13 1 1 2 3 3 2 2 2 2 5 5 3 2 1 1 5	t t t t t t t t t t t t t t t t t t t	68 36 49 50 28 40 40 227 222 220 40 227 402 363 37 41 40 32 370 96 91 288	t t t t t t t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213 209 190 211 1205 1348 201 195 188 179 778 254 259 430 358	t t t t t	210 145 57 1068 554 559 62 61 729 64 73 65 314 63 80 75 81 83 99 90 86 66 66	t	84 82 65 80 85 63 62 64 56 238 75 78 74 70 65 89 87 89 89 90 89 92	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 218 205 209 1601 1478 185 203 281 180 773 250 345 528
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syy1 expressiond1 expressiond1 expressiond2 expressiond2 expressiond2 inversionne inversionne inversionne inversionne inversionne inversione2 inversionne inversione2 inversionle isogonals1 isogonals2	3 t 6 5 2 5 5 5 4 4 t 13 1 1 2 3 3 1 3 2 2 5 5 5 3 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	t t t t t t t t t t t t t t t t t t t	68 36 49 50 28 46 40 50 28 46 40 237 222 220 40 227 402 363 37 41 40 32 370 96 91 286 228 877	t t t t t t t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 219 219 219 219 190 2113 209 218 201 195 1348 201 195 188 277 188 254 430 358	t t t t t	210 145 57 1068 554 53 59 62 61 729 64 73 314 63 462 643 80 75 81 83 99 90 66 66	t	84 82 65 80 85 63 62 64 56 238 75 78 70 76 58 65 89 87 89 90 92 95 64	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 198 209 1601 1478 185 203 281 180 773 250 345 528 426
def-pointse2 def-pointsne Desargues diameters	3 t 6 5 2 2 5 5 5 5 4 4 t 13 1 1 2 2 3 3 3 2 2 2 5 5 3 3 2 2 1 1 5 5 1 1 1 1 1 1 1 1 1 1 1 1 1 1	t t t t t t t t t t t t t t t t t t t	68 36 49 50 43 1/o 28 46 40 50 227 220 227 400 227 400 363 37 41 286 96 91 288 877 201	t t t t t t t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 219 209 213 209 219 190 213 1205 1348 201 195 188 179 778 254 259 430 358 t/o	t t t t t	210 145 57 1068 554 53 59 62 61 1729 64 73 65 314 63 80 75 81 83 99 66 66 61	t	84 82 65 80 85 63 63 62 64 56 238 75 78 74 70 76 58 89 87 87 88 99 90 92	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 198 209 1601 1478 185 203 281 180 345 528 426 t/o 468
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syy3 ellipse-syy3 foot-exists geometric-mean incenter1 incenter2 inversionnt inversionnt inversionne inversionnt inversione2 inversionnt inversione3 inversionle isogonals1 isogonals2 isogonals3 isoscelesor	3 t 6 5 2 5 5 5 4 4 t 13 1 1 2 3 3 1 1 2 2 2 1 5 5 1 1 1 1 2 2 2 2 2 2 2 2	t t t t t t t t t t t t t t t t t t t	68 36 49 50 28 46 40 40 227 222 220 227 40 227 402 363 37 41 40 96 91 288 877 201	t t t t t t t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213 209 219 190 213 1205 1348 201 195 1348 241 179 778 254 430 358 t/o 347	t t t t t	210 145 57 1068 554 59 62 61 729 64 73 65 314 63 80 75 81 83 99 90 66 61 1100	t	84 82 65 80 85 63 62 64 56 238 75 78 76 58 65 89 87 89 90 89 92 95 64 72 82 82	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 198 209 1601 1478 281 180 773 250 345 528 426 t/o 468 370
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syy2 ellipse-syd1 expressiond1 expressiond2 expression-ex31 foot-exists geometric-mean incenter1 incenter2 inversionnt inversionne inversionne inversionne inversione2 inversionle isogonals1 isogonals2 isoscelesr2 isoscelesor	3 t 6 5 2 5 5 5 4 4 t 13 1 1 2 3 3 2 2 5 5 5 1 1 1 1 2 5 1 1 2 5 5	t t t t t t t t t t t t t t t t t t t	68 36 49 50 43 1t/o 28 46 40 50 237 222 220 40 227 402 363 37 41 40 320 368 228 887 201 163	t t t t t t t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213 209 219 219 190 213 1205 1348 201 195 188 201 195 430 358 430 358 t/o 347 307	t t t t t t t t t t t t t	210 145 57 1068 554 53 59 62 61 1729 64 73 65 314 80 75 81 81 83 99 90 86 62 66 61 1000 111 63	t	84 82 65 80 85 63 63 62 64 56 238 75 78 74 70 76 58 89 90 90 92 64 72 82 82 82 82	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 198 209 1601 1478 185 203 281 180 773 250 345 528 426 468 370
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syy3 ellipse-syy3 foot-exists geometric-mean incenter1 incenter2 inversionnt inversionnt inversionnt inversionnt inversione2 inversionnt inversione3 inversione3 inversione3 inversionle isogonals1 isogonals2 isoscelesr2 isoscelesr2 isoscelesor isoscel-ex91 line-circlon	3 t 6 5 2 5 5 5 5 4 4 t 13 1 1 2 3 3 2 2 3 2 1 5 5 4 4 4 4 1 1 1 1 2 3 3 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4	t t t t t t t t t t t t t t t t t t t	68 36 49 50 48 46 40 40 237 222 200 40 227 402 363 37 41 20 32 37 41 40 32 370 96 91 286 877 201 163	t t t t t t t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213 209 190 211 1205 1348 201 195 188 179 778 254 259 430 358 t/o 347 307	t t t t t	210 145 57 1068 554 53 59 62 61 729 64 73 65 314 63 80 75 81 83 99 66 61 100 1114 63 84	t	84 82 65 80 85 63 63 62 64 56 238 75 78 74 70 65 89 87 88 90 89 92 95 64 72 62 82 82 82	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 218 205 209 1601 1478 185 203 281 180 773 250 345 528 426 t/o 468 370 216
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syy1 expressiond1 expressiond2 expressiond3 foot-exists geometric-mean incenter1 incenter2 inversionne inversionne inversionne inversione2 inversione1 inversione2 inversione2 inversione1 isogonals1 isogonals1 isogonals3 isoscelesr2 isoscelesr2 isoscelesor isoscel-ex91 line-circlon Menelaus	3 t 6 5 2 5 5 5 4 4 t 13 1 1 2 3 3 1 3 2 2 2 2 1 1 1 1 1 1 2 1 1 1 1	t t t t t t t t t t t t t t t t t t t	68 36 49 50 28 46 40 50 50 28 46 40 237 222 220 40 227 402 363 37 41 40 32 370 96 228 877 201 163 15	t t t t t t t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213 209 219 190 213 1205 1348 201 195 188 254 430 358 1/o 347 307 174 208	t t t t t t t t t t t t t	210 145 57 1068 554 559 62 61 729 64 73 314 63 80 75 81 83 99 90 100 114 63 83 84 63 86 66 61 100	t	84 82 65 80 85 63 62 64 56 238 75 78 70 76 58 65 89 87 88 90 89 92 95 64 72 62 82 82 82 82 82 82 83 84 85 86 86 87 88 88 88 88 88 88 88 88 88	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 205 209 198 209 1601 1478 185 203 281 180 773 250 426 426 426 427 428 429 429 429 429 429 429 429 429 429 429
def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 ellipse-syy3 ellipse-syy3 foot-exists geometric-mean incenter1 incenter2 inversionnt inversionnt inversionnt inversionnt inversione2 inversionnt inversione3 inversione3 inversione3 inversionle isogonals1 isogonals2 isoscelesr2 isoscelesr2 isoscelesor isoscel-ex91 line-circlon	3 t 6 5 2 5 5 5 5 4 4 t 13 1 1 2 3 3 2 2 3 2 1 5 5 4 4 4 4 1 1 1 1 2 3 3 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4	t t t t t t t t t t t t t t t t t t t	68 36 49 50 48 46 40 40 237 222 200 40 227 402 363 37 41 20 32 37 41 40 32 370 96 91 286 877 201 163	t t t t t t t t t t t t t t t t t t t	209 197 1108 203 196 216 189 206 192 198 213 209 190 211 1205 1348 201 195 188 179 778 254 259 430 358 t/o 347 307	t t t t t t t t t t t t t	210 145 57 1068 554 53 59 62 61 729 64 73 65 314 63 80 75 81 83 99 66 61 100 1114 63 84	t	84 82 65 80 85 63 63 62 64 56 238 75 78 74 70 65 89 87 88 90 89 92 95 64 72 62 82 82 82	t t t t t t t t t t t t t t t t t t t	212 6 2014 202 185 255 181 236 189 21 218 218 205 209 1601 1478 185 203 281 180 773 250 345 528 426 t/o 468 370 216

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midpoints60	2	t	46		213	t	977	f	1539		258
midpoints66	4	t	55		235	t	1150	t	321	t	1187
midpoint-w96	2	t	73	t	230	t	506	t	88	t	218
mirrored-rel	3	t	45	t	193		63		62	t	206
nine-pointle	t 35	t	69	t	225	t	430	t	79	t	34
orthocenter1	t <b>7</b>	t	39	t	193	t	225	t	69	t	7
orthocenter2	t <b>5</b>	t	33	t	189	t	100	f	625	t	6
orthocenter3	t 5	t	35	t	184	t	128	t	70	t	5
orthocenter4	t 6	t	33	t	187	t	126	t	67	t	5
orthocenter5	t 15	t	42	t	196	t	113	t	170	t	17
orthocenter6	t 7	t	36	t	186	t	105	f	635	t	9
orthocenter7	1	t	25	t	189	t	119	t	164	t	201
orthocente70	1	t	107	t	319	t	34	t		· ·	350
									35		
orthocente30	1	t	40	t	191	t	168		95	t	186
orthocentec2	2	t	65	t	222		86		87	t	213
orthocenteic	2	t	75	t	212		81		91	t	222
Pappus	3	t	40	t	235	t	957	t	80	t	243
parabola-dn2	5	t	51	t	208		459		62	t	201
parabola-dn3	2	t	42	t	195		460		60	t	200
parabola-don	2	t	46	t	199	f	160		63	t	200
parabola-ds2	3	t	48	t	205		37		36	t	204
parabola-ds3	5	t	41	t	191		57		59	t	194
parabola-dus	2	t	47	t	198		34		33	t	192
parallel-chords	t 88	t	44		195	t	667		t/o	t	85
parallel-ex121	4	t	48	t	201		38		33	t	203
parallel-l51	2	t	53	t	203	t	1163	t	145	t	198
parallelog84	2		24	U	191	·	69		79		231
parallelogls	t 10	t	42	t	191	t	400		t/o	t	13
	5		25	t	183	t	400		33		223
parallelog79				4				4		_	
parallelog 69	2	t	50	t	199		175	t	95	t	205
parallelog85	5	t	57		205	t	741	f	11860	t	1064
Pascal-ellipse	2		t/o		t/o		72		63		t/o
Pascal-ellec	3		t/o		t/o		69		68		t/o
Pascal-ellec	1	t	93		977		65		62		1073
Pascal-hypla	2		t/o		t/o		66		63		t/o
Pascal-parabola	2		t/o		t/o	t	1074		63		t/o
perpendcul81	2	t	45	t	204		39		37	t	199
perpendicu88	3	t	45	t	209	t	678		t/o	t	202
perpendicu87	3	t	67	t	234	t	319	t	70	t	230
perpendicu86	5	t	67	-	254		536	t	84		298
point-equal2	4	t	34	t	177		47	-	59	t	185
point-equal	t 9	t	12	t	182	t	55	t	70	t	8
	3				204	_		t			196
powerlinear		t	48	t		t	131		79	t	
Pythagorasn2	1	t	230	t	217		67	t	91	t	211
Pythagorasn3	2	t	226	t	216		66	t	91	t	212
Pythagorason	2	t	219	t	206		61	t	79	t	202
Pythagoras	2	t	216	t	202		61	t	80	t	201
quadrangle61	2	t	49		222	t	1772	t	89	t	1898
rational-nn2	2	t	215	t	203		55		69	t	211
rational-non	2	t	217	t	205		62		76	t	216
reflection1	2	t	34	t	188		55		59	t	189
reflection2	2	t	41	t	189		55		69	t	186
reflection11	2	t	45		208		60		61		283
reflection2a	2	t	42	t	187		54		61	t	194
reflectionG4	2	t	47	t	217		61		64	t	209
reflectionnt	2	t	28	t	188	t	65		71	t	194
regular-trle	5	t	40	t	193	t	115		84	t	191
rotate1	2		23	-	183		65		64	-	251
rotate-byes	3	t	33	t	191		48		61	t	184
rotate-byes	1	t	32	t	189		32		35	t	185
rotate-byes	1	t	220	t	207		60		67	t	203
rotate-byes	1	t	236	t	206		55		66	t	216
	1		36	t			34				183
rotate-byes		t			188				30	t	
rotate-byes	2	t	36	t	191		57		61	t	183
rotate-byes	1	t	34	t	194		31		29	t	185
Simson1	4	t	53		247	t	311	t	190	t	543
Simson2	2	t	51		239	t	277	t	206	t	496
simsons-li93	1	t	114		2999		44		41		3041
simsons-li97	2	t	58	t	213	t	658		93	t	205
simsons-th88	1	t	44		235	t	337	t	184	t	548
square1	1	t	42	t	197		38		41	t	196
square2	1	t	39	t	192	t	112		86	t	190
square3	2	t	56	t	204	t	100		83	t	202
symmedians	3	t	441		t/o		68		63		t/o
tangent-arar	2	t	42		193		79		81		272
	2	t	60		203		59		70		245
		t	48	t	193	f	210		61	t	201
tangent-paa2		L.		t		1				ı	
tangent-paa2 tangent-pala	5	- 4			190		165	1	77		234
tangent-pala tangent-pole	2	t	41				01.5				100
tangent-paa2 tangent-pala tangent-pole Thales1	2 5	t	42	t	200	t	215		t/o	t	186
tangent-paa2 tangent-pala tangent-pole Thales1 Thales2	2 5 2	t t	42 41	t	200 194	t	101		78	t	195
tangent-pala tangent-pole tangent-pole Thales1 Thales2 Thales3	2 5 2 2	t	42 41 37		200 194 195		101 122		78 92		195 202
tangent-paa2 tangent-pala tangent-pole Thales1 Thales2 Thales3 theorem-3se	2 5 2 2 2	t t	42 41 37 t/o	t	200 194 195 t/o	t t	101 122 71		78 92 69	t	195 202 t/o
tangent-pala tangent-pole tangent-pole Thales1 Thales2 Thales3	2 5 2 2	t t	42 41 37	t	200 194 195	t	101 122	t	78 92	t	195 202

translater2		3	t	53	t	259		68		75	t	273
translateic		4	t	68	t	237		58		70	t	228
trapezoid49		2	t	45		204	t	997		t/o		239
trapezoid03		2	t	49	t	206		33		33	t	197
triangle-areas		1	t	65	t	216		52	t	208	t	210
triangle-c74		2		30		184		80		91		239
triangle-mns	t	5	t	43	t	190	t	523	t	82	t	5
triangle-m53		5		63		566	t	4042		98		621
triangle-m79		5	t	48	t	193	t	236		t/o	t	196
triangle-m80		2	t	50	t	201		41		43	t	201
triangle-mt1	t	10	t	31	t	190	t	72	t	74	t	8
triangle-mt2	t	4	t	37	t	184	t	68	t	72	t	5
triangle-mt3	t	7	t	39	t	189	t	78	t	75	t	4
triangle-mt4	t	6	t	26	t	186	t	75	t	67	t	9
triangle-mt5	t	6	t	40	t	189	t	73	t	76	t	7
triangle-p34		3	t	56	t	210		179	t	151	t	216
triangle-p23	t	67	t	49	t	232		171	t	90	t	72
triangle-p55		2	t	46	t	207	t	225	t	73	t	208
triangle-p48		2	t	57	t	211	t	327		t/o	t	203
triangles31	t l	5173	t	52	t	211	t	818		t/o	t	5330
true	t	1	t	1	t	1	t	1	t	1	t	1
two-circle01		2		94		239		263		85		486
Varignon	t	9	t	40	t	203	t	82	t	73	t	13
Total (of 200)	33		1	.75		127		84		67		145

### We highlight that:

- Our theorem corpus has a significant number of test cases. Cf. [22].
- The best performing theorem prover—when using our corpus—is the complex algebraic geometry prover via Singular [8]. Here the [7, chapter 6, §4] algorithm was used. Timing is remarkably under one second in most test cases.
- The table can be misleading when investigating other columns. Actually, there is no implementation for intersections with conics in GeoGebra for Recio's method. Also E2/Giac can use a different algorithm with better (but slightly slower) results. Some GeoGebra commands are not yet implemented in the communication layer between GeoGebra and Open-GeoProver, that is, columns E3a and E3b show only a limited amount of positive test cases.
- For the end user the significant case is the last column, since Singular is disabled by default to ensure the same behavior on offline and online runs.

### 3.2.2 Web version

The web version runs in a web browser. All major browsers including Google Chrome, Mozilla Firefox and Internet Explorer are supported.

The following table was generated by using the command line xvfb-run ./runtest -p "Auto Web" -r in this folder. It compares the outputs of the **Prove** command in the desktop version ("Auto") and the web version ("Web").

Test	A	uto	Web		
Test	R.	S.	R.	S.	
lines-parallel	f	7	f	11	
midpoint-third		297		779	
orthocentear		303		626	
points-colar	f	5	f	9	
points-equal	f	4	f	11	

Pythagorasn4 tangents-cns translater3 translatear	t	311		
translater3		011		612
translater3		1153		t/o
translate		t/o		7190
	f	9	f	
				12
altitudes82	t	200	t	715
altitudes05	t	202	t	564
angle-bisee2	t	391	t	1206
angle-bisele	t	340	t	1188
area-polygon-2	t	275	t	620
	t	277	t	661
area-polygon	· ·		L L	
bisector-i20		442		1585
bisector-i19	t	348	t	1020
bisector-m22	t	323	t	1069
			_	
bisector-m21	t	318	t	932
bisector-mnt	t	188	t	427
bisector-o37		726		5511
Discetor-oor				
bisector-p28	t	2668		t/o
bisector-p19	t	1146		1569
Brianchonla		t/o		4344
		0/0		
cathetus	t	214	t	870
centroid-mo1	t	190	t	580
centroid-mo2	t	210	t	864
centroid-mo3	t	211	t	848
centroid-mo4	t	214	t	845
Ceva1	t	298		265
Ceva2	t	874		841
Ceva3	t	852		859
Ceva4	t	715		t/o
		721		+/-
Ceva5	t			t/o
circle-are71	t	527		887
circle-bis16		353		438
		437		1650
circle-bis94				
circle-mid11	t	252	t	817
circle-onent	t	186	t	446
circle-per75	_	278	_	896
circle-per95		255		569
circle-per72		369		740
circle-per67		11148		t/o
circle-per68		t/o		t/o
circle-qua65	t	909		2383
circle-ray06		8343		t/o
circles-ar90		t/o		t/o
circles-ch98		260		823
circumcenter1	t	12	t	36
circumcenter2	t	6	t	36
circumcenter3	t	6	t	21
	t			t/o
circumcenter4				
circumcenter4		106		477
circumcenter5	t	196	t	477
			t	477 51
circumcenter5 circumcenter6	t	196 14	t	477 51
circumcenter5 circumcenter6 circumcent97	t	196 14 210		477 51 603
circumcenter5 circumcenter6 circumcent97 circumcircnt	t	196 14 210 t/o	t	477 51 603 9376
circumcenter5 circumcenter6 circumcent97 circumcircnt constructne	t	196 14 210 t/o 358	t	477 51 603 9376 271
circumcenter5 circumcenter6 circumcent97 circumcircnt constructne	t	196 14 210 t/o 358	t	477 51 603 9376
circumcenter5 circumcenter6 circumcent97 circumcircnt constructne	t t	196 14 210 t/o 358 353	t	477 51 603 9376 271 201
circumcenter5 circumcenter97 circumcircnt constructne constructnt def-line-pne	t t t	196 14 210 t/o 358 353 8	t	477 51 603 9376 271 201 12
circumcenter5 circumcenter6 circumcent97 circumcircnt constructne def-line-pne def-pointse1	t t	196 14 210 t/o 358 353 8 212	t	477 51 603 9376 271 201 12
circumcenter5 circumcenter6 circumcent97 circumcircnt constructne def-line-pne def-pointse1	t t t	196 14 210 t/o 358 353 8	t	477 51 603 9376 271 201 12
circumcenter5 circumcenter6 circumcent97 circumcircnt constructne constructnt def-line-pne def-pointse1 def-pointse2	t t t t	196 14 210 t/o 358 353 8 212 212	t t t	477 51 603 9376 271 201 12 509
circumcenter5 circumcenter6 circumcent97 circumcircnt constructne constructnt def-line-pne def-pointse1 def-pointse2 def-pointse2	t t t t t t t t t t t t t t t t t t t	196 14 210 t/o 358 353 8 212 212	t t	477 51 603 9376 271 201 12 509 525
circumcenter5 circumcenter6 circumcent97 circumcircnt constructnt def-line-pne def-pointse1 def-pointse2 def-pointsne	t t t t	196 14 210 t/o 358 353 8 212 212 7 1928	t t t t	477 51 603 9376 271 201 12 509 525 10
circumcenter5 circumcenter6 circumcent97 circumcircnt constructnt def-line-pne def-pointse1 def-pointse2 def-pointsne	t t t t t t t t t t t t t t t t t t t	196 14 210 t/o 358 353 8 212 212	t t t	477 51 603 9376 271 201 12 509 525
circumcenter5 circumcenter6 circumcent97 circumcircnt constructne constructne def-pointse1 def-pointse2 def-pointsne Desargues diameters74	t t t t t t t t t t t t t t t t t t t	196 14 210 t/o 358 353 8 212 212 7 1928	t t t t t t t	477 51 603 9376 271 201 12 509 525 10 13654 496
circumcenter5 circumcenter6 circumcent97 circumcircnt constructne constructnt def-line-pne def-pointse1 def-pointse2 def-pointse2 def-upcintse2 def-pointsne	t t t t t t t t t t t t t t t t t t t	196 14 210 t/o 358 353 8 212 212 7 1928 200 189	t t t t	477 51 603 9376 271 201 12 509 525 10 13654 496
circumcenter5 circumcenter6 circumcent97 circumcircnt constructnt def-line-pne def-pointse2 def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3	t t t t t t t t t t t t t t t t t t t	196 14 210 t/o 358 353 8 212 212 7 1928 200 189 273	t t t t t t t	477 51 603 9376 271 201 12 509 525 10 13654 496 400
circumcenter5 circumcenter6 circumcent97 circumcircnt constructne constructnt def-line-pne def-pointse1 def-pointse2 def-pointse2 def-upcintse2 def-pointsne	t t t t t t t t t t t t t t t t t t t	196 14 210 t/o 358 353 8 212 212 7 1928 200 189	t t t t t t t	477 51 603 9376 271 201 12 509 525 10 13654 496 400
circumcenter5 circumcenter6 circumcent97 circumcircnt constructne constructnt def-line-pne def-pointse1 def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3	t t t t t t t t t t	196 14 210 t/o 358 353 8 212 212 7 1928 200 189 273 183	t t t t t t t t t	477 51 603 9376 271 201 12 509 525 10 13654 496 400 1271 393
circumcenter5 circumcenter6 circumcent97 circumcircnt constructnt def-line-pne def-pointse1 def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle3 ellipse-circle	t t t t t t t t t t t t t t t t t t t	196 14 210 t/o 358 353 8 212 212 7 1928 200 189 273 183	t t t t t t t t t t t t t	477 51 603 9376 271 201 12 509 525 10 13654 496 400 1271 393
circumcenter5 circumcenter6 circumcent97 circumcircnt constructnt constructnt def-line-pne def-pointse2 def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2	t t t t t t t t t t t t t t t t t t t	196 14 210 t/o 358 353 8 212 212 7 1928 200 189 273 183 260 190	t t t t t t t t t t t t	477 51 603 9376 271 201 12 509 525 10 13654 496 400 1271 393 699 503
circumcenter5 circumcenter6 circumcent97 circumcircnt constructnt def-line-pne def-pointse1 def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle3 ellipse-circle	t t t t t t t t t t t t t t t t t t t	196 14 210 t/o 358 353 8 212 212 7 1928 200 189 273 183	t t t t t t t t t t t t t	477 51 603 9376 271 201 12 509 525 10 13654 496 400 1271 393
circumcenter5 circumcenter6 circumcent97 circumciernt constructnt constructnt def-line-pne def-pointse1 def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syy2 EulerLine	t t t t t t t t t t t t t t t t t t t	196 14 210 t/o 358 8 212 7 1928 200 189 273 183 260 190	t t t t t t t t t t t t t t t t t t t	477 51 603 9376 271 122 509 525 100 13654 496 400 1271 393 699 503
circumcenter5 circumcenter6 circumcent97 circumcircnt constructnt def-line-pne def-pointse1 def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-syy2 ellipse-syry EulerLine expressiond1	t t t t t t t t t t t t t t t t t t t	196 14 210 t/o 358 353 8 212 7 1928 200 189 273 189 260 190 13 213	t t t t t t t t t t t t t t t t t t t	477 51 603 9376 271 201 122 509 525 10 13654 496 400 1271 393 699 503 611 751
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circumcenter5 circumcenter6 circumcent97 circumcircnt constructnt constructnt def-line-pne def-pointse2 def-pointse2 def-pointsne Desargues diameters74 ellipse-circle2 ellipse-circle3 ellipse-circle ellipse-circle ellipse-circle ellipse-circle dellipse-syry EulerLine expressiond1 expressiond2 expressiond2 expression-ex31 foot-exists geometric-mean incenter1 incenter2 inversionnt inversionne	t t t t t t t t t t t t t t t t t t t	196 14 210 210 4/o 358 8 353 8 212 212 212 212 219 1928 200 189 273 183 260 190 190 13 211 217 196 209 1696 1391 198	t t t t t t t t t t t t t t t t t t t	477 511 603 9376 2711 201 12 12 10 13534 496 400 1271 393 61 751 716 784 503 871 t/o 412 467
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circumcenter5 circumcenter6 circumcent97 circumcircnt constructnt constructnt def-line-pne def-pointse2 def-pointse2 def-pointse2 def-pointsr4 ellipse-circle2 ellipse-circle3 ellipse-circle6 ellipse-syy2 ellipse-circle delipse-syy3 foot-exists geometric-mean incenter1 incenter2 inversionnt inversionne inversionne inversionne inversionne inversionne		196 14 14 210 1/o 358 8 353 8 1212 212 212 200 189 273 183 260 190 13 211 217 196 209 1696 1391 198 192 289	t t t t t t t t t t t t t t t t t t t	477 511 603 9376 2711 201 12 519 525 10 13654 496 400 1271 393 611 751 751 751 404 407 447 487 487
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powerlinear Pythagorasn2 Pythagorasn3 Pythagorason Pythagoras quadrangle61 rational-nn2 rational-non	t t t t t t t t	203 213 211 206 203 1902 209 207	t t t t t t t t	496 691 684 577 589 1042 807 770
powerlinear Pythagorasn2 Pythagorason Pythagorason Pythagoras quadrangle61 rational-nn2 rational-non reflection1	t t t t t t t t t t t t	203 213 211 206 203 1902 209 207 185	t t t t t t t t t	496 691 684 577 589 1042 807 770 449
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powerlinear Pythagorasn2 Pythagorasn3 Pythagorason Pythagoras quadrangle61 rational-nn2 rational-non reflection1 reflection2 reflection11 reflection2a reflection74 reflectionnt regular-trle rotate1	t t t t t t t t t t t t t t t t t t t	203 213 211 206 203 1902 207 185 191 286 193 205 185 186 276	t t t t t t t t t t t t t t t t t t t	496 691 684 577 589 1042 807 770 449 492 772 474 867 387 437
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Thales3	t	209	t	550
theorem-3se		t/o		2805
theorem-of42	t	190	t	454
translater1	t	9	t	33
translater2	t	273	t	852
translateic	t	215	t	543
trapezoid49		251		642
trapezoid03	t	204	t	592
triangle-areas	t	212	t	524
triangle-c74		261		259
triangle-mns	t	8	t	31
triangle-m53		640		t/o
triangle-m79	t	189	t	480
triangle-m80	t	196	t	558
triangle-mt1	t	10	t	23
triangle-mt2	t	4	t	23
triangle-mt3	t	9	t	15
triangle-mt4	t	4	t	16
triangle-mt5	t	8	t	33
triangle-p34	t	211	t	879
triangle-p23	t	79	t	166
triangle-p55	t	202	t	766
triangle-p48	t	214	t	623
triangles31	t	5263		t/o
true	t	1	t	0
two-circle01		486		1131
Varignon	t	12	t	37
Total (of 200)		145		125

# We highlight that:

- The web version does not return any incorrect output in any cases.
- It is properly working in 125/200 cases (62.5%) which is 86.2% of the performance ratio of the desktop version.
- The web version is definitely slower than the desktop version by a ratio between 2 and 6.
- Despite its limited availability and speed, the web version is already applicable in many classroom situations. The users only need a web browser which should be accessed not only on desktop computers and laptops, but also on tablets and mobile phones.

### 3.2.3 Theorems in the classrooms

To sum up, we list some important theorems which are usually discussed in secondary schools. Now they can be proven with GeoGebra's help, that is, at least a yes/no answer is provided for many theorems, including:

- The Pythagorean theorem. The intercept theorem. The geometric mean and cathetus theorems. Thales' theorem.
- Concurrency of medians, bisectors, altitudes. Euler line. The midline theorem, Varignon's theorem. The nine points circle. Simson's theorem.
- The angle bisector theorem.
- Basic properties of translations and rotations.
- Basic properties of reflections about a point, a line or a circle.

- <u>Ceva's theorem</u>, <u>Menelaus' theorem</u>. Desargues's theorem, Pappus' theorem
- Basic properties of conic sections (including tangents).

The underlined theorems can be proven with the internal complex algebraic geometry prover in GeoGebra by using the enhancements implemented in the last milestone in our work.

### 4 Future work

Finally, we summarize the currently planned new features in the forthcoming versions of GeoGebra.

GeoGebra tool	Description	To implement
cm <sup>2</sup>	Area	of conics
**	Translate by Vector	line, segment, ray, circle, parabola, ellipse, hyperbola, polygons
	Reflect about Line	ellipse, hyperbola
•	Reflect about Circle	line
<b>.</b>	Rotate around Point	general angles

There is still room for further enhancements:

- Improve formula handling by eliminating non-squared quantities automatically and identifying formulas for a correct decision about the truth of the statement.
- Currently it is not possible to mirror a line about a circle directly: in this case the implementation should handle that the object type is changing from line to circle in general.
- The **ShowProof** command [13] might be implemented in cases when a readable proof can be produced automatically.
- Allow proofs for 3D Euclidean geometry (cf. [21]).
- Improve Gröbner bases computations in Giac to implement transcendent coefficients (see [7, chapter 6, §4]). This would speed up computations in a number of cases which are currently infeasible: an indirect reduction of variables would be achieved in this way.

• GeoGebra's **LocusEquation** command is capable of computing algebraic loci [3]. It would be possible to unify the code base for the locus and the prover subsystem, and the unified system could be maintained and improved easier.

Also implementing conic sections for Recio's exact check method would speed up GeoGebra's proofs significantly.

# Acknowledgments

The theorem proving subsystem in GeoGebra is a joint work with contributions from several researchers, programmers and teachers. We are especially thankful to Bernard Parisse for improving the Giac CAS to be competitive to Singular and some commercial systems, making possible that GeoGebra has a robust embedded theorem prover also.

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