Heating of Coronal Loops: Weak MHD Turbulence and Scaling Laws

A.F. Rappazzo*, M. Velli*,† and G. Einaudi**

*Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109, USA †Dipartimento di Astronomia e Scienza dello Spazio, Università di Firenze, 50125 Firenze, Italy **Dipartimento di Fisica "E. Fermi", Università di Pisa, 56127 Pisa, Italy

Abstract. To understand the nonlinear dynamics of the Parker scenario for coronal heating, long-time high-resolution simulations of the dynamics of a coronal loop in cartesian geometry are carried out. A loop is modeled as a box extended along the direction of the strong magnetic field B_0 in which the system is embedded. At the top and bottom plates, which represent the photosphere, velocity fields mimicking photospheric motions are imposed.

We show that the nonlinear dynamics is described by different regimes of MHD anisotropic turbulence, with spectra characterized by intertial range power laws whose indexes range from Kolmogorov-like values ($\sim 5/3$) up to ~ 3 . We briefly describe the bearing for coronal heating rates.

Keywords: MHD — Sun:corona — Sun:magnetic fields — turbulence

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INTRODUCTION

Coronal heating is one of the outstanding problems in solar physics. Although the correlation of coronal activity with the intensity of photospheric magnetic fields seems beyond doubt, and there is large agreement that photospheric motions are the source of the energy flux that sustains an active region ($\sim 10^7\,erg\,cm^{-2}\,s^{-1}$), the debate currently focuses on the physical mechanisms responsible for the transport, storage and dissipation (i.e. conversion to heat and/or particle acceleration) of this energy from the photosphere to the corona.

A promising model is that proposed by Parker [1, 2], who was the first to suggest that coronal heating could be the necessary outcome of an energy flux associated with the tangling of coronal field lines by photospheric motions.

Over the years a number of numerical experiments have been carried out to investigate the Parker problem, with particular emphasis on exploring how the shuffling of magnetic field line footpoints leads to current sheet formation, and to estimate the heating rate.

3D cartesian simulations have been performed by Mikic et al. [3], Longcope and Sudan [4], Hendrix and Van Hoven [5], and Dmitruk and Gómez [6]. A complex coronal magnetic field results from the photospheric field line random walk, and though the field does not, strictly speaking, evolve through a sequence of static force-free equilibrium states (the original Parker hypothesis), magnetic energy nonetheless tends to dominate kinetic energy in the system. In this limit the field is structured by current sheets elongated along the axial direction. The results from these studies agreed qualitatively among themselves, in that all simulations display the development of field aligned

current sheets. However, estimates of the dissipated power and its scaling characteristics differed largely, depending on the way in which extrapolations from low to large values of the plasma conductivity of the properties such as inertial range power law indices were carried out.

The low resolution of the previous 3D studies has been partially overcome by 2D numerical simulations of incompressible MHD with magnetic forcing (Einaudi et al. [7], Georgoulis et al. [8], Dmitruk et al. [9], Einaudi and Velli [10]), which showed that turbulent current sheets dissipation is distributed intermittently, and that the statistics of dissipation events, in terms of total energy, peak energy and event duration displays power laws not unlike the distribution of observed emission events in optical, ultraviolet and x-ray wavelengths of the quiet solar corona.

More recently a first attempt to simulate full 3D sections of the solar corona with a realistic geometry has been performed by Gudiksen and Nordlund [11]. At the moment the very low resolution attainable with this kind of simulations does not allow the development of turbulence. The transfer of energy from the scale of convection cells $\sim 1000\,km$ toward smaller scales is in fact inhibited, because the smaller scales are not resolved (their linear resolution is in fact $\sim 500\,km$).

While in the future these global simulations will be able to reach the necessary high resolutions, to investigate the nonlinear dynamics of the Parker scenario at relatively high Reynolds numbers, we have recently performed high-resolution long-time simulation of the aforementioned cartesian model (Rappazzo et al. [12]).

In the next sections we describe the coronal loop model, the simulations we have carried out, and give simple scaling arguments to understand the energy spectral slopes.

PHYSICAL MODEL

A coronal loop is a closed magnetic structure threaded by a strong axial field, with the footpoints rooted in the photosphere. This makes it a strongly anisotropic system, as measured by the relative magnitude of the Alfvén velocity associated with the axial magnetic field $v_A \sim 2000~{\rm km\,s^{-1}}$ compared to the typical photospheric velocity $u_{ph} \sim 1~{\rm km\,s^{-1}}$. This means that the relative amplitude of the Alfvén waves that are launched into the corona is very small and, as an efficient energy cascade takes place [12], the relative amplitude of the fields which develop in the orthogonal planes remains small compared to the dominant axial magnetic field.

We study the loop dynamics in a simplified cartesian geometry, neglecting any curvature effect, as a "straightened out" box, with an orthogonal square cross section of size ℓ (along which the x-y directions lie), and an axial length L (along the z direction) embedded in an axial homogeneous uniform magnetic field $B_0 = B_0 \ e_z$. This simplified geometry allows us to perform simulations with both high numerical resolution and long-time duration.

The dynamics of a plasma embedded in a strong axial magnetic field are well described by the equations of reduced MHD (Kadomtsev and Pogutse [13], Strauss [14], Montgomery [15]). In this limit the velocity and magnetic fields have only per-

pendicular components, linked to the velocity and magnetic potentials φ and ψ by

$$u_{\perp} = \nabla \times (\varphi e_z), \qquad b_{\perp} = \nabla \times (\psi e_z).$$
 (1)

Although numerically we advance the equations for the potentials [see 12], in order to analyze the linear and nonlinear properties of the system it is convenient to write the equivalent equations using the Elsässer variables $z^{\pm} = u_{\perp} \pm b_{\perp}$. The more symmetric equations, which explicit the underlying physical processes at work, are given in dimensionless form by:

$$\frac{\partial z^+}{\partial t} = -\left(z^- \cdot \nabla_\perp\right) z^+ + \frac{v_A}{u_{ph}} \frac{\partial z^+}{\partial z} + \frac{(-1)^{n+1}}{R_n} \nabla_\perp^{2n} z^+ - \nabla_\perp P \tag{2}$$

$$\frac{\partial z^{-}}{\partial t} = -\left(z^{+} \cdot \nabla_{\perp}\right) z^{-} - \frac{v_{A}}{u_{nh}} \frac{\partial z^{-}}{\partial z} + \frac{(-1)^{n+1}}{R_{n}} \nabla_{\perp}^{2n} z^{-} - \nabla_{\perp} P \tag{3}$$

$$\nabla_{\perp} \cdot z^{\pm} = 0 \tag{4}$$

where $P = p + b_{\perp}^2/2$ is the total pressure, and is linked to the nonlinear terms by incompressibility (4):

$$\nabla_{\perp}^{2} P = -\sum_{i,j=1}^{2} \left(\partial_{i} z_{j}^{-} \right) \left(\partial_{j} z_{i}^{+} \right). \tag{5}$$

The gradient operator has only components in the *x*-*y* plane perpendicular to the axial direction *z*, and the dynamics in the orthogonal planes is coupled to the axial direction through the linear terms $\propto \partial_z$.

We use a computational box with an aspect ratio of 10, which then spans

$$0 \le x, y \le 1, \qquad 0 \le z \le 10.$$
 (6)

The linear terms $\propto \partial_z$ are multiplied by the dimensionless parameter v_A/u_{ph} , the ratio between the Alfvén velocity associated with the axial magnetic field $v_A = B_0/\sqrt{4\pi\rho_0}$, and the photosperic velocity u_{ph} .

Boundary conditions for our numerical simulations are specified imposing the velocity potential $\varphi(x, y)$ in the bottom (z = 0) and top (z = L) planes:

$$\varphi(x,y) = \frac{1}{\sqrt{\sum_{m,n} \alpha_{mn}^2}} \sum_{k,l} \frac{\alpha_{kl}}{2\pi\sqrt{k^2 + l^2}} \sin\left[2\pi (kx + ly) + 2\pi \xi_{kl}\right]. \tag{7}$$

These result from the linear combination of large-scale eddies with random amplitudes α_{kl} and phases ξ_{kl} (whose values are included between 0 and 1). We excite all the twelve independent modes whose wave-numbers are included in the range $3 \le \left(k^2 + l^2\right)^{1/2} \le 4$, and then normalize the result so that the velocity rms is $\sim 1 \, km \, s^{-1}$.

In terms of the Elsässer variables z^{\pm} , to impose a velocity pattern (u_{\perp}^*) at the boundary surfaces means to impose the constraint $z^+ + z^- = 2u_{\perp}^*$, and as in terms of characteristics (which in this case are simply z^{\pm} themselves) we can specify only the incoming wave

(while the outgoing wave is determined by the dynamics inside the computational box), at the top (z = L) and bottom (z = 0) planes the following "reflection" takes place:

$$z^{-} = -z^{+} + 2u_{\perp}^{0}$$
 at $z = 0$ (8)

$$z^{+} = -z^{-} + 2u_{\perp}^{L}$$
 at $z = L$ (9)

where u_{\perp}^0 and u_{\perp}^L are the forcing functions in the respective boundary surfaces. At time t=0 no perturbation is imposed inside the computational box, i.e. $b_{\perp}=u_{\perp}=$

At time t = 0 no perturbation is imposed inside the computational box, i.e. $b_{\perp} = u_{\perp} = 0$, and only the axial magnetic field B_0 is present: the subsequent dynamics are then the effect of the photospheric forcing on the system.

The linear terms ($\propto \partial_z$) in equations (2)-(3) give rise to two distinct wave equations for the z^{\pm} fields, which describe Alfvén waves propagating along the axial direction z. This wave propagation, which is present during both the linear and nonlinear stages, is responsible for the transport of energy at the large perpendicular scales from the boundaries (photosphere) into the loop. The nonlinear terms $(z^{\mp} \cdot \nabla_{\perp}) z^{\pm}$ are then responsible for the transport of this energy from the large scales toward the small scales, where energy is finally dissipated, i.e. converted to heat and/or particle acceleration.

An important feature of the nonlinear terms in equations (2)-(4) is the absence of self-coupling, i.e. they only couple counterpropagating waves, and if one of the two fields z^{\pm} were zero, there would be no nonlinear dynamics at all. This is at the basis of the so-called Alfvén effect (Iroshnikov [16], Kraichnan [17]), that ultimately renders the nonlinear timescales longer and slows down the dynamics.

From this analysis it is clear that three different timescales are present: τ_A , τ_{ph} and τ_{nl} . $\tau_A = L/v_A$ is the crossing time of the Alfvén waves along the axial direction z, i.e. the time it takes for an Alfvén wave to cover the loop length L. $\tau_{ph} \sim 5 \, m$ is the characteristic time associated with photospheric motions, while τ_{nl} is the nonlinear timescale.

For a typical coronal loop $\tau_A \ll \tau_{ph}$, and for this reason we consider a forcing which is constant in time, i.e. for which formally $\tau_{ph} = \infty$.

In the RMHD ordering the nonlinear timescale τ_{nl} is bigger than the Alfvén crossing time τ_A . This ordering is confirmed and maintained troughout our numerical simulations.

The length of a coronal section is taken as the unitary length, but as we excite all the wavenumbers between 3 and 4, and the typical convection cell scale is $\sim 1000\,km$, this implies that each side of our section is roughly $4000\,km$ long. Our grid for the cross-sections has 512x512 grid points, corresponding to $\sim 128^2$ points per convective cell, and hence a linear resolution of $\sim 8\,km$.

Between the top and bottom plate a uniform magnetic field $B = B_0 e_z$ is present. The subsequent evolution is due to the shuffling of the footpoints of the magnetic field lines by the photospheric forcing.

In this section we present the results of a simulation performed with a numerical grid with 512x512x200 points, hyper-diffusion (n=4) with $R_4=10^{19}$, and the Alfvén velocity $v_A=200\,km\,s^{-1}$ corresponding to a ratio $v_A/u_{ph}=200$. The total duration is roughly 500 axial Alfvén crossing times $(\tau_A=L/v_A)$.

Plots of the total magnetic and kinetic energies

$$E_M = \frac{1}{2} \int dV b_{\perp}^2, \qquad E_K = \frac{1}{2} \int dV u_{\perp}^2,$$
 (10)

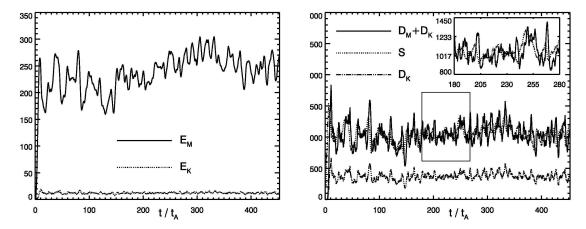


FIGURE 1. High-resolution simulation with $v_A/u_{ph}=200$, 512x512x200 grid points and $R_4=10^{19}$. Left: Magnetic (E_M) and kinetic (E_K) energies as a function of time $(\tau_A=L/v_A)$ is the axial Alfvénic crossing time). Right: The integrated Poynting flux S dynamically balances the total dissipation D. Inset shows a magnification of total dissipation and S for $180 \le t/\tau_A \le 280$.

and of the total magnetic and kinetic dissipation rates

$$D_M = -\frac{1}{R_4} \int dV \, b_\perp \cdot \nabla^8 b_\perp \qquad D_K = -\frac{1}{R_4} \int dV \, u_\perp \cdot \nabla^8 u_\perp \tag{11}$$

along with the incoming energy rate (Poynting flux) S, are shown in Figure 1. At the beginning the system has a linear behavior [see 12], characterized by a time linear growth rate for the magnetic energy, the Poynting flux and the electric current, until time $t \sim 6 \tau_A$, when nonlinearity sets in. The magnetic energy is bigger than the kinetic energy, this is the natural result of the field line bending due to the photospheric motions both in the linear and nonlinear stages. More formally this is a consequence of the fact that, while on the perpendicular magnetic field no boundary condition is imposed, the velocity field must approach the imposed boundary values at the photosphere both during the linear and nonlinear stages.

After this time, in the fully nonlinear stage, a *statistically steady state* is reached, in which the Poynting flux, i.e. the energy that is entering the system for unitary time, balances on time average the total dissipation rate $(D_M + D_K)$. As a result there is no average accumulation of energy in the box, beyond what has been accumulated during the linear stage, and a detailed examination of the dissipation time series (see inset in Figure 1) shows that the Poynting flux and total dissipations are decorrelated around dissipation peaks.

Figure 2 shows the energy spectra. The spectral index for total energy fits well the -2 value. We have shown [see 12] that this spectral index strongly depends on the ratio v_A/u_{ph} , i.e. on the relative strength of the axial magnetic field. At lower values correspond flatter spectra, with an index close to -5/3, while to higher values of the magnetic field the spectra steepens up to $\sim -5/2$ for $v_A/u_{ph} \sim 1000$.

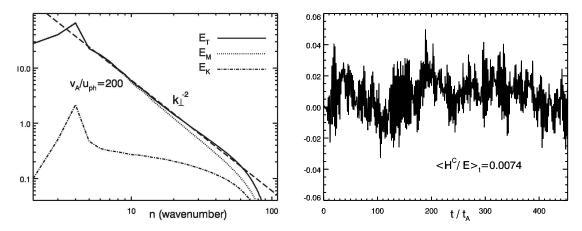


FIGURE 2. Left: Magnetic, kinetic and total energy spectra averaged in time over $\sim 500 \, \tau_A$. The total energy spectrum fits a k_{\perp}^{-2} power law. Right: Ratio between cross helicity H^C and total energy E as a function of time.

CONCLUSION AND DISCUSSION

The fact that at the large orthogonal scales the Alfvén crossing time τ_A is the fastest timescale, and in particular it is smaller than the nonlinear timescale τ_{nl} (which can be identified with the energy transfer time at the driving scale), implies that the Alfvén waves that continuously propagate and reflect from the boundaries toward the interior are basically equivalent to an anisotropic magnetic forcing function that stirs the fluid, whose orthogonal length is that of the convective cells ($\sim 1000 \, km$) and whose axial length is given by the loop length L.

Recently a lot of progress has been made in the understanding of turbulence for an MHD system embedded in a strong magnetic field (Ng and Bhattacharjee [18], Goldreich and Sridhar [19], Sridhar and Goldreich [20]). As a coronal loop is threaded by a strong magnetic field, it is no surprise that the nonlinear dynamics is described by weak MHD turbulence.

The spectra that we have found can be easily derived by order of magnitude considerations. A characteristic of anisotropic MHD turbulence is that the cascade takes place mainly in the plane orthogonal to the DC magnetic guide field (Shebalin et al. [21]). Dimensionally, and integrating over the whole box, the energy cascade rate may be written as

$$\varepsilon \sim \ell_{\perp}^2 L \rho \frac{\delta z_{\lambda}^2}{T_{\lambda}},$$
 (12)

where δz_{λ} is the rms value of the Elsässer fields $z^{\pm} = u_{\perp} \pm b_{\perp}$ at the perpendicular scale λ . Given the simmetry of the system it is expected and confirmed numerically (see Figure 2) that cross helicity is zero, hence $\delta z_{\lambda}^{+} \sim \delta z_{\lambda}^{-} \sim \delta z_{\lambda}$. ρ is the average density and T_{λ} is the energy transfer time at the scale λ , which is greater than the eddy turnover time $\tau_{\lambda} \sim \lambda/\delta z_{\lambda}$ because of the Alfvén effect [16, 17].

In the classical IK case,

$$T_{\lambda} \sim \tau_{\lambda} \frac{\tau_{\lambda}}{\tau_{A}},$$
 (13)

More generally, however, as the Alfvén speed is increased nonlinear interactions become weaker. Simply from dimensional considerations as the ratio τ_{λ}/τ_{A} is dimensionless and smaller than 1, we can suppose that the energy transfer time scales as

$$T_{\lambda} \sim \tau_{\lambda} \left(\frac{\tau_{\lambda}}{\tau_{A}}\right)^{\alpha-1}, \quad \text{with} \quad \alpha \geq 1,$$
 (14)

where α is the scaling index (note that $\alpha = 1$ corresponds to standard hydrodynamic turbulence).

The energy transfer rate (12) is then given by

$$\varepsilon \sim \ell_{\perp}^{2} L \cdot \rho \, \frac{\delta z_{\lambda}^{2}}{T_{\lambda}} \sim \ell_{\perp}^{2} L \cdot \rho \, \left(\frac{L}{v_{A}}\right)^{\alpha - 1} \, \frac{\delta z_{\lambda}^{\alpha + 2}}{\lambda^{\alpha}}. \tag{15}$$

Considering the injection scale $\lambda \sim \ell_{\perp}$, eq. (15) becomes

$$\varepsilon \sim \ell_{\perp}^{2} L \cdot \rho \, \frac{\delta z_{\ell_{\perp}}^{2}}{T_{\ell_{\perp}}} \sim \frac{\rho \ell_{\perp}^{2} L^{\alpha}}{\ell_{\perp}^{\alpha} \, \nu_{A}^{\alpha - 1}} \, \delta z_{\ell_{\perp}}^{\alpha + 2}. \tag{16}$$

On the other hand the energy injection rate is given by the Poynting flux integrated across the photospheric boundaries: $\varepsilon_{in} = \rho v_A \int da u_{ph} \cdot b_{\perp}$. Considering that this integral is dominated by energy at the large scales, due to the characteristics of the forcing function, we can approximate it with

$$\varepsilon_{in} \sim \rho \, \ell_{\perp}^2 v_A u_{ph} \delta z_{\ell_{\perp}},$$
(17)

where the large scale component of the magnetic field can be replaced with $\delta z_{\ell_{\perp}}$ because the system is magnetically dominated.

The last two equations show that the system is self-organized because both ε and ε_{in} depend on $\delta z_{\ell_{\perp}}$, the rms values of the fields z^{\pm} at the scale ℓ_{\perp} : the internal dynamics depends on the injection of energy and the injection of energy itself depends on the internal dynamics via the boundary forcing.

In a stationary cascade the injection rate (17) is equal to the transport rate (16). Equating the two yields for the amplitude at the scale ℓ_{\perp} :

$$\frac{\delta z_{\ell_{\perp}}^*}{u_{ph}} \sim \left(\frac{\ell_{\perp} v_A}{L u_{ph}}\right)^{\frac{\alpha}{\alpha+1}} \tag{18}$$

Substituting this value in (16) or (17) we obtain for the energy flux

$$\varepsilon^* \sim \ell_\perp^2 \rho \, v_A u_{ph}^2 \left(\frac{\ell_\perp v_A}{L u_{ph}} \right)^{\frac{\alpha}{\alpha+1}},$$
(19)

where $v_A = B_0/\sqrt{4\pi\rho}$. This is also the dissipation rate, and hence the *coronal heating scaling*. A dimensional analysis of eqs. (2)-(4) reveals [see 12] that the only free parameter is $f = \ell_{\perp} v_A/Lu_{ph}$, so that the scaling index α (14), upon which the strength of the stationary turbulent regime depends, must be a function of f itself, and we have determined its value computationally [12].

Identifying, as usual, the eddy energy with the band-integrated Fourier spectrum $\delta z_{\lambda}^2 \sim k_{\perp} E_{k_{\perp}}$, where $k_{\perp} \sim \ell_{\perp}/\lambda$, from eq. (15) we obtain

$$E_{k_{\perp}} \propto k_{\perp}^{-\frac{3\alpha+2}{\alpha+2}},\tag{20}$$

where for $\alpha = 1$ the -5/3 slope for the "anisotropic Kolmogorov" spectrum is recovered, and for $\alpha = 2$ the -2 slope. At higher values of α correspond steeper spectral slopes up to the asymptotic value of -3.

It has been shown computationally and analytically (Ng and Bhattacharjee [18], Galtier et al. [22]) that the scattering of Alfvén waves with random amplitudes in the weak MHD turbulence regime gives rise to the k_{\perp}^{-2} spectrum. Our MHD simulations differ from [18] as we integrate forward in time the reduced MHD equations, and the system is not three-periodic. While we confirm the presence of the k_{\perp}^{-2} spectrum, steeper spectra are found up to $\sim k_{\perp}^{-3}$, and they are clearly linked to the strength of the axial field B_0 . Future analytical and computational investigation might clarify the physical origin for the steeper spectra, their relation to boundary conditions and lying-tying, and possibly give an explicit analytical formula for $\alpha(f)$.

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