Collective and fractal properties of pion jets in the four-velocity space at intermediate energies

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Experimental results are presented for study of collective and fractal properties of soft pion jets in the space of relative four-dimensional velocities. Significant decreasing is obtained for mean square of second particle distances from jet axis for pion-proton interactions at initial energies ~ 3 GeV in comparison with hadron-nuclear collisions at close energies. The decreasing results in power dependence of distance variable on collision energy for range $\sim 2-4$ GeV. The observation allows us to estimate the low boundary of manifestation of color degree of freedom in pion jet production. Cluster dimension values were deduced for pion jets in various reactions. Fractional values of this dimension indicate on the manifestation of fractal-like properties by pion jets. Changing of mean kinetic energy of jet particles and fractal dimension with initial energy increasing is consistent with suggestion for presence of color degrees of freedom in pion jet production at intermediate energies.

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I. INTRODUCTION

Investigation of the mixed phase, transition from meson-nucleon to quark-gluon degrees of freedom is now one of the most actual and important tasks of the world program of researches in the field of strong interactions. Experimental study of hadron-hadron and hadron-nuclear interactions can give the important information concerning manifestation of new (color) degrees of freedom. In various fields of physics the beginning of manifestation of new degrees of freedom and transition processes are accompanied by presence of self-affine, fractal features for collective effects. Thus, study of collective and geometrical (fractal) properties, in particular, soft hadron jets at intermediate energies can give the new important information concerning hadronization mechanisms, non-perturbative physics and transition to manifestation of color degrees of freedom in collective phenomena.

A relativistic invariant method for investigation of collective effects in multiparticle production processes of the type I + II \rightarrow 1 + 2 + ... was proposed in [1]. Features of this method for events with two jets were in detail considered in [2, 3]. In this case the secondary particles emitted in the regions of target and beam fragmentation can be separated by means of a relativistic invariant variable $X_{\rm l} = 0.1 - 0.2$. One of the main variables of the method is $b_k = -(V - U_k)^2$ – a square of distance k-th particle from an axis of jet V in the space of four-dimensional velocities $U_i = P_i/m_i, i = I, II, 1, 2...$ [1]. It should be stressed that in the framework of this approach the range of values $10^{-2} \le b_k \sim 1$ corresponds to transition from domination of meson-nucleon degrees of freedom to manifestation of internal structure of initial particles and, thus, to quark-gluon degrees of freedom in processes of production of secondary particles.

In [4] it was suggested to study the geometrical properties of pion jets in four-velocity space by means of cluster dimension, D, defined by a relation between number of particles in jet, $N(b_k)$, and its radius $N(b_k) \propto b_k^{D/2}$.

In this paper, collective and geometrical properties of soft pion jets have been studied using the space of relative four-dimensional velocities for following reactions:

$$\pi^- + p \to N + k\pi^- + l\pi^+ + j\pi^0, \quad k = 2, l = 1, 2, j \ge 0, \quad P_0 = (3.93 \pm 0.01) \text{ GeV/c};$$
 (1)

$$\pi^{+} + p \rightarrow \begin{cases} N + K^{+} (K^{0}) + \bar{K}^{0} (K^{-}) + k\pi^{-} + l\pi^{+} + j\pi^{0}, & P_{0} = (3.90 \pm 0.30) \text{ GeV/c} \\ B_{s} + K^{0,+} + k\pi^{-} + l\pi^{+} + j\pi^{0}, & k, j = 0, 1, l \ge 0; \end{cases}$$
(2)

$$\pi^+ + p \rightarrow N + k\pi^- + l\pi^+ + j\pi^0$$
, $k = 1, 2, l = 2 - 4, j = 0, 1$, $P_0 = (4.23 \pm 0.08) \text{ GeV/c}$; (3)

$$\pi^- + (C_2F_5Cl_3) \rightarrow \Lambda^0 + K^0 + k\pi^- + l\pi^+ + mp + X, \quad k, l, m > 0, \quad P_0 = (3.86 \pm 0.04) \text{ GeV/c};$$
 (4)

$$\pi^- + \text{Ne} \to k\pi^- + l\pi^+ + mp + X, \quad k, l, m > 0, \quad P_0 = (6.20 \pm 0.10) \text{ GeV/c};$$
 (5)

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Here P_0 – beam momentum, N=p,n, $B_s = \Lambda^0, \Sigma^{0,\pm}$ are final state strange baryon, moreover l = 1, 2 at $B_s = \Lambda^0$ and l = 0 - 3 at $B_s = \Sigma^{0,\pm}$ in the reaction (2). The technique of experiments and a selection criteria are described in detail in [3, 5]. The general statistics for reactions (1) – (5) is more than $1.9 \cdot 10^5$ events [6].

II. COLLECTIVE PROPERTIES OF PION JETS

Fig.1 shows dependencies of mean square of distance from axis of pion jet in the four-velocity space – $\langle b_k \rangle$ – on initial energy \sqrt{s} (mean total energy of final hadron state in the c.m. frame, W, for $\bar{\nu}$ N) for secondary π^- mesons at cutoff $X_1 = 0.1$ (a,b) and 0.2 (c,d) for various interactions.

At initial energies $\sqrt{s} > 8$ the dependence $\langle b_k \rangle (\sqrt{s})$ has been fitted by logarithmic function

$$\langle b_k \rangle = a_1 + a_2 \ln \left(s/s_0 \right), \tag{6}$$

where $s_0=1~{\rm GeV^2}$. Results of above fit are presented on Fig.1b,d (dashed line) and in Table I. Only experimental data are fitted at $X_1=0.1$ for energy range under considered. As seen the Lund model calculations for $\pi^- p$ interactions at 40 GeV/c are in good agreement with experimental data, unlike results obtained by the model of homogeneously filled phase volume (Fig.1b). The fit with taking into account results of Lund model calculations for $\pi^- p$ at 40 and 360 GeV/c gives parameter values which coincide within errors with the values indicated in Table I at some improvement of fit quality ($\chi^2/{\rm n.d.f.}=4.53$). The value of a_2 parameter in (6) coincides with zero within errors at $X_1=0.2$ in the energy domain $\sqrt{s}>8$ GeV. Therefore data were fitted at the fixed value $a_2=0$, that allows to obtain better quality of approximation. However, it is impossible to exclude unambiguously the weak logarithmic growth for $\langle b_k \rangle$ in accordance with (6) taking into account insignificant ensemble of accessible experimental data. Earlier in [3] statistically reasonable description by function (6) had been obtained for experimental data for hadron-hadron and $\bar{\nu}N$ reactions, and also for hadron-nuclear interactions, at $3.5 < \sqrt{s} < 9$ GeV (solid lines on Fig.1b,d). Thus, dependence $\langle b_k \rangle (\sqrt{s})$ supposes universal approximation (6) at $\sqrt{s}>3.5$ GeV for a wide class of interactions at any values X_1 under study.

The hypothesis has been suggested in [3] about change of dynamic regime at $\sqrt{s} < 3-4$ GeV, reflected in the behaviour of underlying dependence. This energy domain has been investigated in more details in the present paper. Dependencies $\langle b_k \rangle (\sqrt{s})$ are shown on Fig.1a,b in collision energy range $\sqrt{s} < 4$ GeV with taking into account of uncertainties of initial momenta for various types of interactions with exception of hadron-nuclear ones at $X_1 = 0.1$ and 0.2, respectively. The results obtained here for a pion-proton reactions (1) and (2) indicate clearly on the change of behaviour of dependence $\langle b_k \rangle$ at $\sqrt{s} \sim 3$ GeV (Fig.1), confirming the our earlier suggestion [3]. Results for symmetric nuclear collisions agree well with the general tendency (Fig.1a,b) at $X_1 = 0.1$. Dependence of $\langle b_k \rangle$ values on $\sqrt{s}(W)$ has been fitted by following power function at $\sqrt{s} < 4$ GeV:

$$\langle b_k \rangle = a_1 \left(\sqrt{s/s_0} - a_2 \right)^{a_3}, \quad \sqrt{s/s_0} \ge a_2. \tag{7}$$

The results of the above fit are shown on Fig.1 (dotted lines) and in Table I. In [3, 6] it has been shown, that properties of soft pion jets depend on region of fragmentation in the energy domain under study. This effect leads to significant distinction of corresponding $\langle b_k \rangle$ values. Therefore sharp behaviour of dependence $\langle b_k \rangle$ (\sqrt{s}) and disorder of experimental points result in statistically unacceptable fit quality. However, as seen from Fig.1, power function agrees well enough with experimental data at all values of X_1 at qualitative level.

Experimental data samples were fitted for target fragmentation region (curves 1 on Fig.1a,c) and for beam fragmentation one (curves 2 on Fig.1a,c) separately. The values of fit parameters are listed in Table II. Substantial improvement of fit quality is observed for any fragmentation region and values of X_1 . The data samples at $X_1 = 0.2$ were fitted at the fixed value of parameter $a_3 = 0.5$ taking into account the results obtained at soft cutoff and volumes of accessible data samples at $X_1 = 0.2$. Thus, dependence $\langle b_k \rangle (\sqrt{s})$ is described by power function (7) at $\sqrt{s} < 4$ GeV for studied types of interactions (hh, $\bar{\nu}$ N, AA) both for separate samples for various fragmentation regions and for total data ensemble. It is important to note, that in general the power behaviour is one of the characteristic features of transition domain in which new degrees of freedom of investigated system begin to be manifest. Value of parameter a_2 can be put in correspondence with the energy (in GeV), $\sqrt{s_c}$, at which the internal structure of interacting particles, i.e. new degrees of freedom, begins to manifest in the pion jet production. Thus, the estimation is obtained, that new (color) degrees of freedom begin to be manifest experimentally at $\sqrt{s_c} \simeq 2.5 - 2.8$ GeV in jet production of pions. Dependencies $\langle b_k \rangle (\sqrt{s})$ are presented at Fig.2 for secondary neutral strange particles, K^0 mesons (a,c) and Λ^0

Dependencies $\langle b_k \rangle (\sqrt{s})$ are presented at Fig.2 for secondary neutral strange particles, K^0 mesons (a,c) and Λ^0 hyperons (b,d), at various values of X_1 . As seen, usually, $\langle b_k \rangle$ value is smaller for target fragmentation region, than corresponding $\langle b_k \rangle$ for beam fragmentation one. The reactions (2) and (4) investigated here with absence of strange particles in an initial state and with its production in a final state can be considered as additional argument in favour

TABLE I: Fit parameters for dependence $\langle b_k \rangle (\sqrt{s})$.

Fit parameter	$X_{l} = 0.1$	$X_{l} = 0.2$			
hh-, hA-interactions ($\sqrt{s} > 8 \text{ GeV}$)					
a_1	3.7 ± 0.2	4.88 ± 0.05			
a_2	0.12 ± 0.05	0.0 (fixed)			
$\chi^2/\mathrm{n.d.f.}$	7.20	1.71			
hh-, AA-interactions (\sqrt{s} < 4 GeV)					
a_1	3.69 ± 0.02	3.63 ± 0.05			
a_2	2.76 ± 0.01	2.51 ± 0.03			
a_3	0.49 ± 0.01	0.40 ± 0.01			
$\chi^2/\mathrm{n.d.f.}$	133	9.25			

TABLE II: Parameter values for $\langle b_k \rangle (\sqrt{s})$ at $\sqrt{s} < 4$ GeV. Separated fitting for target and beam fragmentation regions.

X_{l}	Fit parameters				
	a_1	a_2	a_3	$\chi^2/\text{n.d.f.}$	
target fragmentation					
0.1	3.79 ± 0.03	2.82 ± 0.02	0.52 ± 0.05	5.70	
0.2	3.52 ± 0.07	2.46 ± 0.04	0.5 (fixed)	0.09	
beam fragmentation					
0.1	2.65 ± 0.13	2.43 ± 0.04	0.50 ± 0.35	5.19	
0.2	4.6 ± 1.9	2.5 ± 0.3	0.5 (fixed)	2.29	

of manifestation of quark degrees of freedom at $\langle b_k \rangle \sim 1$ at intermediate energies. Accessible samples of results is much less in case of neutral strange particles, than for π^- mesons. Therefore it is possible to study qualitative behaviour of $\langle b_k \rangle$ (\sqrt{s}) at Fig.2 only. For K^0 mesons as well as for secondary pions, the increasing of $\langle b_k \rangle$ at growth of initial energy is observed at any X_1 under considered (Fig.2a,c). One can see that behaviour of $\langle b_k \rangle$ (\sqrt{s}) for Λ^0 hyperons at $X_1 = 0.1$ (Fig.2b) is similar to corresponding dependence for K^0 mesons (Fig.2a), however $\langle b_k \rangle$ values for Λ^0 do not depend on collision energy practically at more hard cutoff $X_1 = 0.2$ (Fig.2d).

The dependence of $\langle b_k \rangle$ value on particle mass has been investigated for secondary $\pi^{0,\pm}$, $K^{0,\pm}$, Λ^0 and $\Sigma^{0,\pm}$ particles from reaction (2). The relativistic invariant variable X_1 was used for separation of various fragmentation region for all particle types with exception of Σ hyperons. The Feynman variable x_F was used for separation of more heavy Σ particles on fragmentation region. Significant decreasing of mean square of distance from jet axis is observed for kaons in comparison with pions with the weaker decreasing at further increasing of particle mass that agrees with the behaviour of similar dependence at higher energies $\sqrt{s} \simeq 8.7$ GeV qualitatively [6].

The important characteristic of applied approach is the mean kinetic energy of particles in jet in the its rest frame (this parameter is called "temperature" often), $\langle T_k \rangle$, which is calculated on the basis of fitting of invariant $F(b_k)$ -distributions [7]. Fig.3 shows dependencies $\langle T_k \rangle$ (\sqrt{s}) for pion jets in the target fragmentation region (a,c) and for beam fragmentation one (b,d). The published distributions $1/NdN/db_k$ [10] have been used for an estimation of $\langle T_k \rangle$ in case of $\bar{p}p, pp, \pi^-C$ interactions, results for π^-p reaction at 40 GeV/c are taken from [11]. Accessible experimental data allow to study behaviour of $\langle T_k \rangle$ (\sqrt{s}) at a qualitative level. The increasing of $\langle T_k \rangle$ is observed with growth of initial energy both for hadron-hadron and for hadron-nuclear collisions. It is important to note significant increasing of $\langle T_k \rangle$ in a narrow range of \sqrt{s} for reactions (1), (3), and much smoother growth at the further increasing of initial energy. Additional experimental investigations are necessary for interactions of various types at $\sqrt{s} \sim 3-20$ GeV, however, the obtained results do not contradict the hypothesis above concerning manifestation of new degrees of freedom in processes of soft pion jet production at $\sqrt{s} \sim 3$ GeV.

The results obtained for π^- mesons from jets (clusters) of secondary particles of various types in proton-nuclear and in nuclear-nuclear interactions with using of normalized relative four-velocity space for central region and for beam fragmentation [9] are agreed with behaviour of $\langle T_k \rangle$ (\sqrt{s}) at $X_l = 0.1$ for beam fragmentation (Fig.3b) qualitatively.

III. CLUSTER DIMENSIONS FOR JETS

Fig.4 shows cluster dimension D, calculated for secondary π^- -mesons, versus collision energy \sqrt{s} for hadron-hadron and hadron-nuclear interactions at $X_1 = 0.1$ (a,b) and at $X_1 = 0.2$ (c,d) for region of target fragmentation (a,c) and for beam fragmentation (b,d). Experimental data used for calculation of cluster dimension are taken from [10] for $\bar{p}p$ and pp interactions, from [10, 11] – for π^-p and π^-C at 40 GeV/c. Cluster dimension shows fractional value for all reactions under study with exception of hadron-nuclear interactions (4) and (5), that allows to suggest the presence of indication on manifestation of fractal-like properties of pion jets in collision energy range under considered. The parameter D have values close to the integer ones within error bars for reactions (4) and (5), excepting for π^-Ne reaction for beam fragmentation at more hard cutoff $X_1 = 0.2$. Accessible experimental data allow to study the behaviour of $D(\sqrt{s})$ and some features of the dependence only at a qualitative level. At any used X_1 cluster dimension for reactions (4) and (5) is close to the integer value within larger errors as indicated above, results obtained for hadron-nuclear reactions show increasing of D with growth of initial energy. The hadron-hadron interactions show a different behaviour of $D(\sqrt{s})$ in various range of initial energies for region of target fragmentation. Significant growth of cluster dimension

is observed at small increasing of collision energy for reactions (1) and (3), the further increasing of \sqrt{s} leads to much weaker growth of D with the subsequent reaching of a constant at $X_1 = 0.1$ (Fig.4a) or to absence of changes at all for $X_1 = 0.2$ (Fig.4c). The dependencies $D(\sqrt{s})$ for beam fragmentation region (Fig.4b,d) do not contradict with behaviour of corresponding dependencies described above for a target fragmentation. The observed behaviour of cluster dimension versus collision energy and sharp amplification of manifestation of fractal-like properties, which is one of the important and characteristic features for occurrence of new degrees of freedom, does not contradict a hypothesis about presence of transition energy range to experimentally noticeable manifestation of quark degrees of freedom in hadron jet production at $\sqrt{s} \sim 3$ GeV.

There is an intensive production of meson resonances in the energy range under study. Influence of resonances on cluster dimension of pion jets has been investigated for following exclusive channels $\pi^- + p \to p + \pi^+ + 2\pi^-$, $\pi^+ + p \to p + 2\pi^+ + \pi^-$ of reactions (1) and (3), respectively. For jets (clusters) of particles with identical masses there is the unambiguous relation between $\langle b_k \rangle$ and jet effective mass [6, 12]. In consequence of this relation meson resonances, in particular, clear identified $\rho(700)$ and $a_1(1320)$, give rise to the normalized dependence of number of particles in jet on its radius, $N(b_k)/N_{\rm tot}$, in value range $3 \le b_k < 4$ for beam fragmentation region. The influence of the specified resonances on cluster dimension is negligible for $\pi^- p$ exclusive channel under study at soft condition of selection $X_1 = 0.1$ and significantly larger at hard cutoff. But the influence of these resonances is observable for corresponding $\pi^+ p$ channel at any X_1 . The contribution from decay of meson resonances leads to significant growth of fractal cluster dimension of pion jets. Influence of resonances has been investigated in details for exclusive channels with one π^0 in reactions (1) and (3) also. However because of narrowness of η and ω mesons it is possible to speak only about presence of indication on the influence of these resonances on cluster dimension.

IV. SUMMARY

In conclusion, we summarize the main results of this study.

Values of $\langle b_k \rangle$ are significantly smaller for reactions (1) - (3) at $\sqrt{s} \sim 3$ GeV than those in other interactions at slightly higher energies, that leads to changing of dependence $\langle b_k \rangle (\sqrt{s})$ from logarithmic behaviour to power one. It is possible to assume, that this effect is caused by occurrence of experimentally noticeable manifestation of quark degrees of freedom in soft pion jet production and corresponding transition from the description of the such process at language of nucleon-meson degrees of freedom to using of color (quark-gluon) degrees of freedom. The behaviour of dependence $\langle T_k \rangle (\sqrt{s})$ for hadron-hadron interactions qualitatively confirms this hypothesis.

The cluster dimension of soft pion jets has been obtained for various reactions at $\sqrt{s} \sim 3-20$ GeV for the first time. The dimension shows fractional value for main part of reactions under considered that allows to suggest the presence of fractal-like properties for pion jets. Features of behaviour of dependence $D\left(\sqrt{s}\right)$ obtained for hadron-hadron reactions do not contradict the above suggestion of manifestation of new degrees of freedom at $\sqrt{s} \sim 3$ GeV. Influence of meson resonances is observed on fractal dimensions of pion jets in beam fragmentation region, leading to significant increasing of D.

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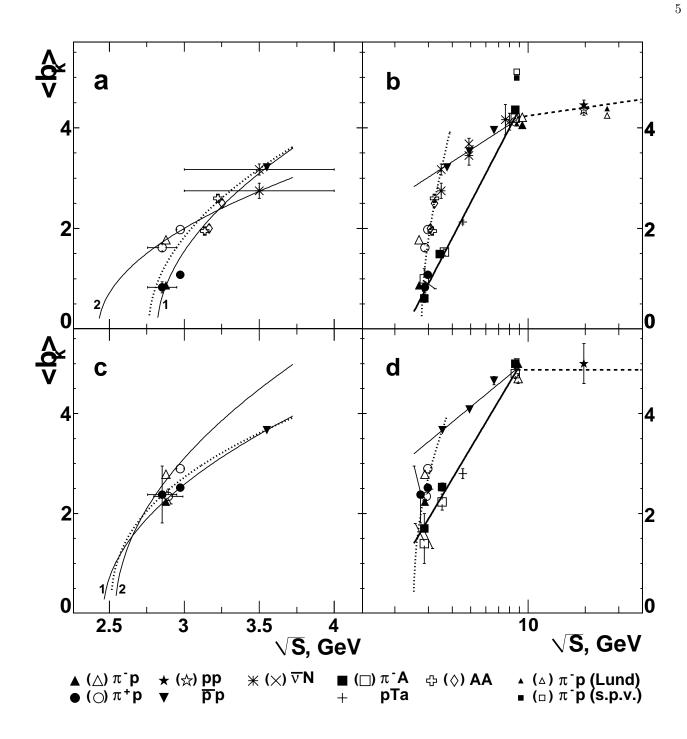


FIG. 1: Dependence of $\langle b_k^\pi \rangle$ on $\sqrt{s}(W)$ at $X_l = 0.1$ (a,b) and $X_l = 0.2$ (c,d). Experimental data for reactions (3) – (5) are from [2, 3, 6], for $\bar{p}p$ at 5.7, 12 and 22.4 GeV/c, $\bar{\nu}N$ at W=3-4, 4-6 and \geq 6 GeV, pp at 205 GeV/c, π^-p at 40 GeV/c, pTa + p (C₃H₈) at 10 GeV/c, π^-C at 40 GeV/c, CC at 4.2 GeV/c/A, MgMg at 4.5 GeV/c/A, model calculations – from [7]. The curves are described in the main body of the text.

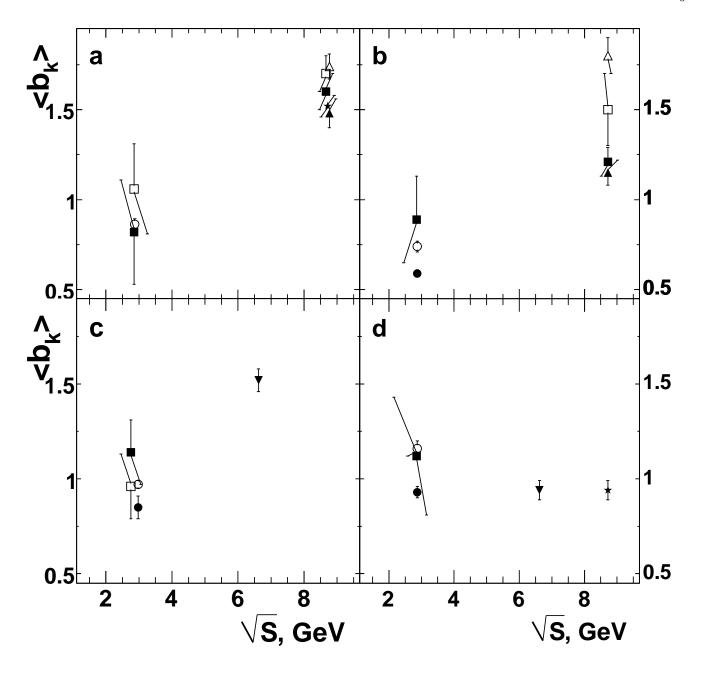


FIG. 2: Values of $\langle b_k \rangle$ versus collision energy for strange particles at $X_1 = 0.1$ (a,b) and $X_1 = 0.2$ (c,d). Left column corresponds to K^0 -mesons (a,c), right column – to Λ^0 -hyperons (b,d). Experimental points for target (beam) fragmentation region are marked as follows: \bullet (\circ) – reaction (2), ∇ – $\bar{p}p$ at 22.4 GeV/c [7, 8] \star – pp at 40 GeV/c [8], \blacktriangle (\triangle) – π^- p at 40 GeV/c [8], \blacksquare (\square) – π^- (C₂F₅Cl₃), π^- C at 3.9 and 40 GeV/c [8], respectively.

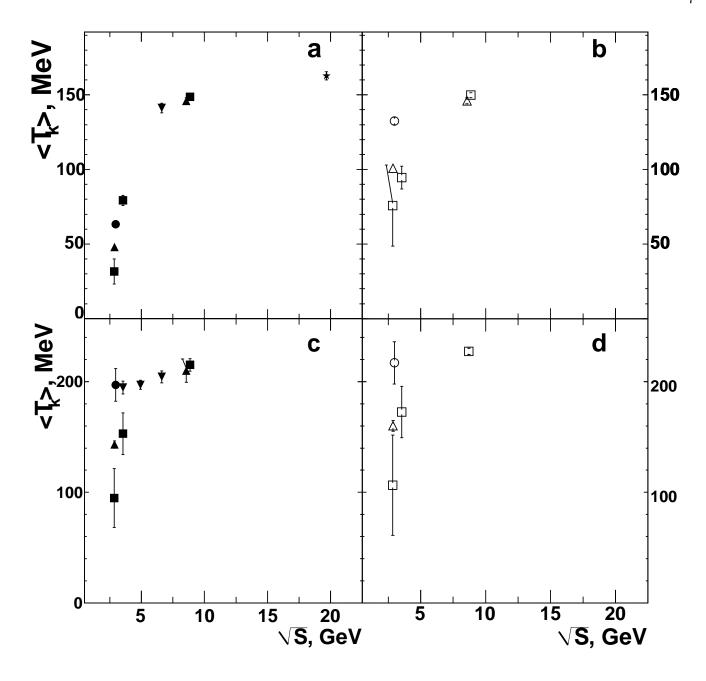


FIG. 3: Dependence $\langle T_k \rangle$ (\sqrt{s}) for π^- -mesons at $X_l = 0.1$ (a,b) and $X_l = 0.2$ (c,d). Left column corresponds to the target fragmentation region (a,c), right column – to the beam fragmentation (b,d). Experimental points for target (beam) fragmentation region are marked as follows: \bullet (\circ) – π^+ p at 4.2 GeV/c, \blacktriangledown – $\bar{p}p$ at 5.7, 12 and 22.4 GeV/c, \star – pp at 205 GeV/c, \blacktriangle (\triangle) – π^- p at 3.9 and 40 GeV/c, \blacksquare (\square) – π^- (\square) – π^- C at 3.9, 6.2 and 40 GeV/c, respectively.

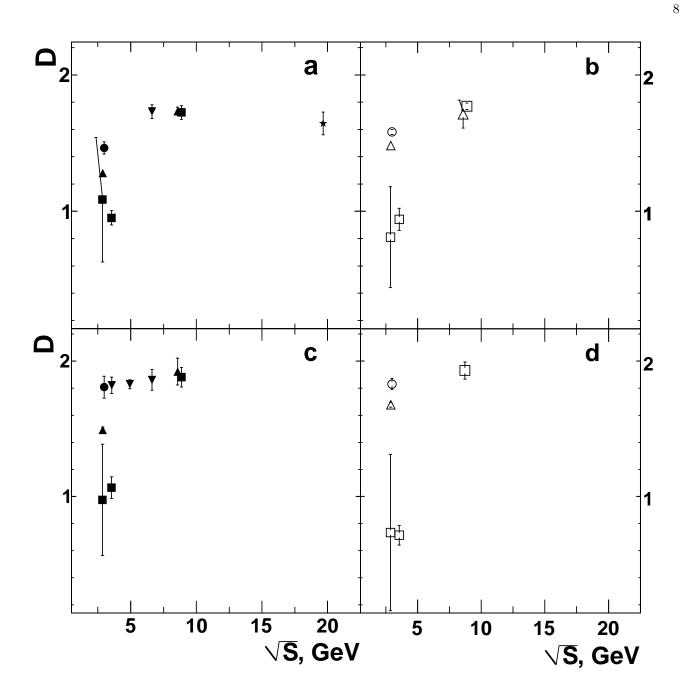


FIG. 4: Dependence of the cluster dimension D, calculated for π^- -mesons, on collision energy at $X_{\rm l}=0.1$ (a,b) and $X_{\rm l}=0.1$ 0.2 (c,d). Left hand corresponds to the target fragmentation region (a,b), right hand – to the beam fragmentation (b,d). Experimental points for target (beam) fragmentation region are marked as follows: \bullet (\circ) – $\pi^+ p$ at 4.2 GeV/c, \blacktriangledown – $\bar{p}p$ at 5.7, 12 and 22.4 GeV/c, \star – pp at 205 GeV/c, \blacktriangle (\triangle) – $\pi^- p$ at 3.9 and 40 GeV/c, \blacksquare (\square) – π^- (C₂F₅Cl₃), π^- Ne, π^- C at 3.9, 6.2 and 40 GeV/c, respectively.