Production of excited double hypernuclei via Fermi breakup of excited strange systems

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Abstract

Precise spectroscopy of multi-strange hypernuclei provides a unique chance to explore the hyperon-hyperon interaction. In the present work we explore the production of excited states in double hypernuclei following the micro-canonical break-up of an initially excited double hypernucleus which is created by the absorption and conversion of a stopped Ξ^- hyperon. Rather independent on the spectrum of possible excited states in the produced double hypernuclei the formation of excited states dominates in our model. For different initial target nuclei which absorb the Ξ^- , different double hypernuclei nuclei dominate. Thus the ability to assign the various observable γ -transitions in a unique way to a specific double hypernuclei by exploring various light targets as proposed by the PANDA collaboration seems possible. We also confront our predictions with the correlated pion spectra measured by the E906 collaboration. In the present work we explore the production of excited states in double hypernuclei following the micro-canonical

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While the nucleon-nucleon scattering was extensively studied since the 50's, direct experimental investigations for the hyperon-hyperon interactions are still very sparse.

Because of their short lifetimes, hyperon targets are not available and low momentum hyperons are very difficult to produce. Since direct scattering experiments between two hyperons are impractical, the precise spectroscopy of multi-strange hypernuclei provides a unique chance to explore the hyperon-hyperon interaction. Indeed the significant progress in nuclear structure calculations [1, 2, 3, 4, 5] nurtures the hope that detailed information on excitation spectra of double hypernuclei and their structure will provide unique information on the hyperon-hyperon interactions.

The simultaneous production and implementation of two Λ particles into a nucleus is intricate. There may be a chance to produce multi-strange hypernuclei in hadron and heavy ion induced collisions [6, 7, 8], as it was im-

and heavy ion induced collisions [6, 7, 8], as it was impressively illustrated by the STAR collaboration recently [9]. However, high resolution spectroscopy of excited states will not not feasible. While in central collisions statistical approaches [10] may be applicable, the production of hypernuclei in peripheral heavy ion collisions clearly requires a detailed consideration of the collision dynamics to make definite predictions of the production probability [8]. To produce double hypernuclei in a more 'controlled' way the conversion of a captured Ξ^- and a proton into two Λ particles can be used. This process releases – ignoring binding

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energy effects – only 28 MeV. For light nuclei there exists therefore a significant probability of the order of a few percent that both Λ hyperons are trapped in the same excited nuclear fragment [11, 12, 13, 14, 15, 16].

Unfortunately Ξ^- hyperons produced in reactions with stable hadron beams have usually rather high momenta. Therefore, direct capture of the Ξ^- in the nucleus is rather unlikely. Even in case of the (K⁻, K⁺) double strangeness exchange reaction, Ξ^- hyperons are produced with typical momenta of 500 $\,\, \mathrm{MeV}/c$ at beam momenta around 1.8 GeV/c [13, 17, 18]. The advantage of this production process is that the outgoing K⁺ can be used as a tag for the reaction. A drawback is the low kaon beam intensity and hence the need for thick primary targets. Furthermore, as a consequence of the large momentum transfer, the probability to form bound Ξ^- states directly is rather small on the level of 1% [19, 20] and the production of quasi-free $\Xi^$ dominates. Still the Ξ^- hyperons in the quasi-free region may be absorbed into the target nucleus via a rescattering process on a nucleon which itself is knocked out of the primary nucleus. This two-step process is predicted to exceed the direct capture by more than a factor of 6 [12] (see also below).

On the other hand most ($\sim 80\%$) Ξ^- hyperons escape from the primary target nucleus in (K^-, K^+) reactions. However, in a second step, these Ξ^- hyperons can be slowed down in a dense, solid material (e.g. a nuclear emulsion) and form Ξ^- atoms [21]. After an atomic cascade, the Ξ -hyperon is eventually captured by a secondary target nucleus and converted via the $\Xi^- p \to \Lambda\Lambda$ reaction into two Λ hyperons. In a similar two-step process relatively

Table 1: Ground and exited states of double hypernuclei used in the

present calculation together with the relevant references. Nucleus mass g.s exc. energy J^P Ref./note											
Nucleus	mass g.s		exc. energy J^P								
	(MeV/c^2)	(MeV)									
$^4_{\Lambda\Lambda}{ m H}$	4107.47	0.0	1+	[36, 25, 46]							
				[47, 48, 49]							
$^{5}_{\Lambda\Lambda}{ m H}$	5037.54	0.0	$\frac{1}{2}^{+}$	[42, 43, 44]							
1111				[45, 46]							
$^{5}_{\Lambda\Lambda}{ m He}$	5036.98	0.0	$\frac{1}{2}^{+}$	[42, 43, 44]							
$\Lambda \Lambda^{}$			2	[45, 46]							
$^{6}_{\Lambda\Lambda}{ m He}$	5951.37	0.0	0+	[37, 41]							
	6889.82	0.0	$\frac{3}{2}^{-}$	[41]							
$\frac{^{7}_{\Lambda\Lambda}\mathrm{He}}{^{8}_{\Lambda\Lambda}\mathrm{He}}$	7825.18	0.0	0^{+}	$\Delta B_{\Lambda\Lambda} =$							
ΛΛ110	1020.10	1.80	2^{+}	$\frac{\Delta B_{KK}}{1 \mathrm{MeV}}$							
$^{9}_{\Lambda\Lambda}{ m He}$	8761.60	0.0	<u>3</u> –	$\Delta B_{\Lambda\Lambda} =$							
$\Lambda\Lambda^{11C}$	0701.00		$\frac{\overline{2}}{5}$								
7	00000	2.92	$\frac{1}{2}$	1 MeV							
$\frac{^{7}_{\Lambda\Lambda}\mathrm{Li}}{^{8}_{\Lambda\Lambda}\mathrm{Li}}$	6889.35	0.0	3- 5- 5- 3- 3- 1+	[41]							
$^{\circ}_{\Lambda\Lambda}{ m Li}$	7821.27	0.0		[41]							
		1.36	3+	[41]							
		5.63	2+	[41]							
$^9_{\Lambda\Lambda}{ m Li}$	8748.85	0.0	$\frac{3}{2}^{-}$	[41]							
		0.73	$\frac{1}{2}$	[41]							
		4.55	$\frac{\overline{7}}{2}$	[41]							
		5.96	$\frac{5}{5}$	[41]							
$^{9}_{\Lambda\Lambda}{ m Be}$	8751.04	0.0	3 2 1 2 7 2 5 2 3 2 1 2 7 2 5 2	[41]							
$\Lambda\Lambda$ DC	0101.04	0.71	$^{2}_{1}-$	[41]							
			$\frac{\overline{2}}{7}$								
		4.54	$\frac{\dot{2}}{2}$	[41]							
10 -		5.92	$\frac{3}{2}$	[41]							
$^{10}_{\Lambda\Lambda}{ m Be}$	9671.08	0.0	0+	[41]							
		2.86	2+	[41]							
$^{11}_{\Lambda\Lambda}{ m Be}$	10603.35	0.0	$\frac{3}{2}$ + $\frac{1}{2}$ +	[12]							
		1.684	$\frac{1}{2}$ +	9 excited							
				states							
		7.94	<u>5</u> -								
$^{11}_{\Lambda\Lambda}\mathrm{B}$	10603.71	0.0	5 <u>2</u> 5 <u>2</u> <u>1</u> 2 + <u>5</u> 2	[12]							
$\Lambda\Lambda^{\perp}$		2.361	$\frac{2}{5}$ -	[- -]							
			$\frac{\overline{2}}{1}$ -								
		2.75	$\frac{1}{5}$ +								
12.5	11504.05	2.788	$\frac{3}{2}$	[10]							
$^{12}_{\Lambda\Lambda}{ m Be}$	11534.05	0.0	0+	[12]							
		3.36	2+	7 excited							
		 7 5 4 9	 2 ⁺	states							
12 D	11520 50	7.542	2 '	[10]							
$^{12}_{\Lambda\Lambda}\mathrm{B}$	11532.52	0.0	3 ⁺	[12]							
		0.71835	1+	24 excited							
		 8.894	 2 ⁺	states							
13 F	1045050			[4.0]							
$^{13}_{\Lambda\Lambda}\mathrm{B}$	12456.59	0.0	$\frac{3}{2}^{-}$	[12]							

low momentum Ξ^- can also be produced in $\overline{p}p \to \Xi^-\overline{\Xi}^+$ or $\overline{p}n \to \Xi^-\overline{\Xi}^\circ$ reactions if this reactions happens in a complex nucleus where the produced Ξ^- can re-scatter [22, 23].

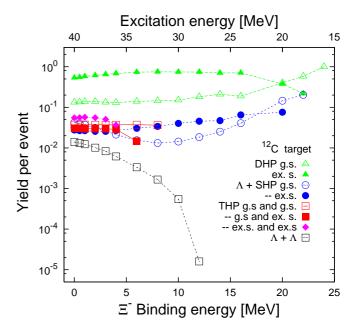


Figure 1: Predicted production probability of ground (g.s.) and excited states (ex.s.) in one single (SHP), twin (THP) and double hypernuclei (DHP) after the capture of a Ξ^- in a $^{12}\mathrm{C}$ nucleus and its conversion into two Λ hyperons . The lower and upper scale shows the binding energy of the captured Ξ^- and the excitation energy of the initial $^{13}_{\Lambda\Lambda}\mathrm{B}$ nucleus, respectively.

The advantage as compared to the kaon induced reaction is that antiprotons are stable and can be retained in a storage ring. This allows a rather high luminosity even with very thin primary targets.

Because of the two-step mechanism, spectroscopic studies, based on two-body kinematics like in single hypernucleus production, cannot be performed. Spectroscopic information on double hypernuclei can therefore only be obtained via their decay products. The kinetic energies of weak decay products are sensitive to the binding energies of the two Λ hyperons. While the double pionic decay of light double hypernuclei can be used as an effective experimental filter to reduce the background [24] the unique identification of hypernuclei groundstates only via their pionic decay is usually hampered by the limited resolution (see e.g. ref. [25] and discussion below). Instead, γ -rays emitted via the sequential decay of excited double hypernuclei may provide precise information on the level structure.

The Panda experiment [23] which is planned at the international Facility for Antiproton and Ion Research FAIR in Darmstadt aims at the high resolution γ -ray spectroscopy of double hypernuclei [22]. In this work we study the important question to what extend particle stable excited states of double hypernuclei are produced and how different secondary target nuclei by help to assign observed γ -transitions. In line with the compound nucleus model of Sano and co-workers [26, 11] we study in the present work the production of excited states in double hypernuclei fol-

lowing the break-up of an excited double hypernucleus after the absorption and conversion of a stopped Ξ^- .

For light nuclei even a relatively small excitation energy may be comparable with their binding energy. We therefore consider the explosive decay of the excited nucleus into several smaller clusters as the principal mechanism of de-excitation. Not included in the present approach are neither details of the atomic capture process nor of a possible dynamical, non-equilibrium stage prior to the Ξ^- conversion into two Λ hyperons. Unlike in case of a direct Ξ^- capture in (K^-, K^+) or other hadron induced reactions where a pre-equilibrium stage will require special attention (see e.g. Ref. [8]), pre-equilibrium processes are probably less important after an atomic capture.

To describe this break-up process we have developed a model which is similar to the famous Fermi model for particle production in nuclear reactions [27]. We assume that the nucleus with mass numbers A_0 , charge Z_0 , and the number of Λ hyperons H_0 (here $H_0=2$) break-ups simultaneously into cold and slightly excited fragments, which have a lifetime longer than the decay time, estimated as an order of 100-300 fm/c. This break-up takes place in some freeze-out volume V, where the produced fragments moves in the phase space determined by the free volume V_f . This free volume is smaller than the freeze-out volume, at least, by the proper volume of the fragments. We use the 'excluded volume' approximation for this parametrization $V = V_f + V_0$, where $V_0 = \frac{4}{3}\pi r_0^3 A_0$ denotes the initial volume of the nucleus with $r_0 = 1.3$ fm and $V_f = \kappa \cdot V_0$. In the following we assume for the parameter κ a value of 2 which is consistent with many description of fragmentation data of normal nuclei [28].

In the case of production of conventional nuclear fragments in a break-up channel, we adopt their experimental masses in ground states, and take into account their excited states, which are stable respective to emission of nucleons [29]. For hypernuclei with single Λ particle, we use the experimental masses and excited states, which were collected in various reviews [30, 31] and also some theoretical predictions[32]. One has to keep in mind that particularly for heavy hypernuclei additional excited states may be missing in the present decay channels.

For double hypernuclei the experimental information is restricted to a few cases only [33, 34, 35, 36, 37, 16]. Except for the $^{6}_{\Lambda\Lambda}$ He nucleus reported in Ref. [37] the interpretation of the observed events is however not unique [38, 39, 40, 41, 25]. Furthermore no direct experimental information on possible excited states is at hand (see e.g. discussion in Ref. [41]). Therefore, theoretical predictions of

bound and exited states of double hypernuclei predicted by Hiyama and co-workers [41] were used in the present model calculation for nuclei with mass number $6 \le A_0 \le 10$. The masses of $^8_{\Lambda\Lambda}$ He and $^9_{\Lambda\Lambda}$ He were calculated from the known single hypernuclear masses and assuming $\Delta B_{\Lambda\Lambda} = 1 \,\text{MeV}$. Here $\Delta B_{\Lambda\Lambda}$ is defined in terms of the involved nuclear masses

$$\Delta B_{\Lambda\Lambda} = 2M(^{A-1}_{\Lambda}Z) - M(^{A-2}Z) - M(^{A}_{\Lambda\Lambda}Z) \tag{1}$$

and which contains information on the binding between the two Λ hyperons. For the mirror nuclei $^5_{\Lambda\Lambda}{\rm H}$ and $^5_{\Lambda\Lambda}{\rm He}$ there seems to be a consensus that these nuclei are indeed bound [42, 43, 44, 45, 46]. In view if the theoretical uncertainties, we again assumed in our calculations a value of $\Delta B_{\Lambda\Lambda}{=}1\,{\rm MeV}$ for both nuclei. In case of $^4_{\Lambda\Lambda}{\rm H}$ the experimental situation is ambiguous [36, 25] and also various model calculations predict an unbound [47] or only slightly bound nucleus [48, 49, 46]. We checked however that our results remain unchanged for the production of heavier nuclei if we disregard this slightly bound $^4_{\Lambda\Lambda}{\rm H}$ nucleus.

For nuclei with mass number A≥11 the masses of the ground states were taken from ref. [12]. These masses correspond to $\Delta B_{\Lambda\Lambda}$ values between about 2.0 and 4.6 MeV. Also for these heavy nuclei several particle stable excited states are expected [12]. The calculations of Hiyama and co-workers [41] signal that in the mass range relevant for this work the level structure of particle stable double hypernuclei resembles the level scheme of the corresponding core nucleus. In order to explore the role of possible additional states the set of double hypernuclei with $A \ge 11$ was extended by the known excited states of the corresponding core nucleus. Only states below the lowest particle decay threshold were considered. In the present calculations $1s_{\Lambda}1p_{\Lambda}$ states have been ignored though these states may possibly contribute at high excitation energies in heavy nuclei [56]. In case of $^{11}_{\Lambda\Lambda}$ Be, $^{11}_{\Lambda\Lambda}$ B, $^{12}_{\Lambda\Lambda}$ Be and $^{12}_{\Lambda\Lambda}$ B this recipe resulted in 9, 3, 7 and 24 excited states, respectively (Tab. 1). Within the model systematic errors may arise due to missing states in single and double hyper nuclei. In order to check the sensitivity of our conclusions to the modifications of the allowed decay products, we have repeated the calculations by only considering the ground and at most two particle stable, excited states in double hyperfragments. The relative yield of some individual excited states was modified up to a factor $2^{\pm 1}$. However, the final conclusions of our paper do not not change.

In the model we consider all possible break-up channels, which satisfy the mass number, hyperon number (i.e. strangeness), charge, energy and momentum conservations, and take into account the competition between these channels. The model assumes that the probability of each break-up channel ch is proportional to the occupied phase space [28, 50, 51]. The statistical weight of the channel containing n particles with masses m_i ($i = 1, \dots, n$) can

 $^{^1}$ With respect to the number of stopped, Ξ^- hyperons, pre-equilibrium processes will decrease the yield of double hypernuclei relative to the yield for single hypernuclei (see e.g. [16]). Indeed in the simulations for the planned Panda experiment [24] a joint capture×conversion probability of 5% was assumed to mimic this pre-equilibrium stage. Taking this factor into account the probability for the various channels is compatible with present scarce experimental information [14, 15, 16].

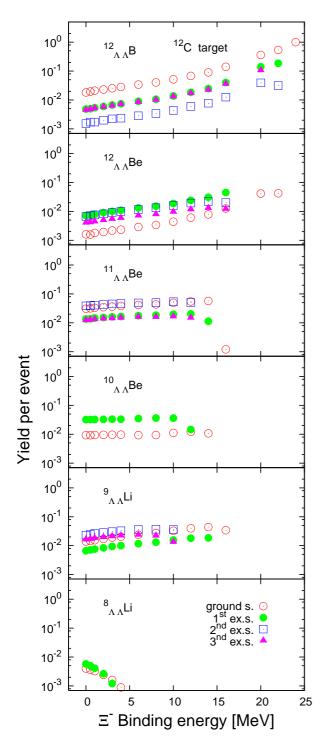


Figure 2: Individual production probability of up to four lowest lying, particle stable states in double hypernuclei after the capture of a Ξ^- by a $^{12}\mathrm{C}$ nucleus.

be calculated in microcanonical approximation:

$$W_{ch}^{mic} \propto \frac{S}{G} \left(\frac{V_f}{(2\pi\hbar)^3} \right)^{n-1} \left(\frac{\prod_{i=1}^n m_i}{m_0} \right)^{3/2}$$

$$\cdot \frac{(2\pi)^{\frac{3}{2}(n-1)}}{\Gamma(\frac{3}{2}(n-1))} \cdot \left(E_{kin} - U_{ch}^C \right)^{\frac{3}{2}n - \frac{5}{2}},$$
(2)

where $m_0 = \sum_{i=1}^n m_i$ is the summed mass of the particles, $S = \prod_{i=1}^n (2s_i + 1)$ is the spin degeneracy factor $(s_i$ is the *i*-th particle spin), $G = \prod_{j=1}^k n_j!$ is the particle identity factor $(n_j$ is the number of particles of kind j). E_{kin} is the total kinetic energy of particles at infinity which is related to the available energy via

$$E_{kin} = (M(\Xi^{-}) + M_{target})c^{2} - B_{\Xi} - \sum_{i=1}^{n} m_{i}c^{2}.$$
 (4)

Here B_{Ξ} is the binding energy of the converted Ξ^- and $E_{max} = (M(\Xi^-) + M_{target})c^2$ represents the maximum available excitation energy ignoring the binding of the Ξ^- , U_{ch}^C is the Coulomb interaction energy between fragments given in the Wigner-Seitz approximation [28]:

$$U_{ch}^{C} = \frac{3}{5} \frac{e^{2}}{r_{0}} (V/V_{0})^{-1/3} \left(\frac{Z_{0}^{2}}{A_{0}^{1/3}} - \sum_{i=1}^{n} \frac{Z_{i}^{2}}{A_{i}^{1/3}} \right), \tag{5}$$

where A_i , Z_i are mass numbers and charges of produced particles. We calculate masses of fragments in excited states by adding the corresponding excitation energy to their ground state masses.

Unfortunately, still very little is established experimentally on the interaction of Ξ hyperons with nuclei. Various data suggest a nuclear potential well depth around 20 MeV (see e.g. [52, 53]). Calculations of light Ξ atoms [21] predict that the conversion of the captured Ξ^- from excited states with correspondingly small binding energies dominates. In a nuclear emulsion experiment a Ξ^- capture at rest with two single hyperfragments has been observed [54] which was interpreted as $\Xi^- + {}^{12}\mathrm{C} \rightarrow^4_{\Lambda}\mathrm{H} + {}^9_{\Lambda}\mathrm{Be}$ reaction. The deduced binding energy of the Ξ^- varied between $0.62\,\mathrm{MeV}$ and $3.70\,\mathrm{MeV}$, depending whether only one out of the two hyperfragments or both fragments were produced in an excited particle stable state. Model calculations [19, 55, 56, 57] suggest that the width for the $\Xi^- + p \to \Lambda + \Lambda$ conversion is around 1 MeV, i.e. the conversion is rather fast and takes of the order of 200 fm/c. In order to take into account the uncertainties of the excitation energy of the converted Ξ^- -states, the calculations were performed for a range of energies $0 \le B_{\Xi} \le E_{max}$, constructing in this way the excitation functions for the production of hypernuclei.

The Fermi break-up events were generated by comparing probabilities of all possible channels with Monte–Carlo method. For example a total of 993 channels were included in case of the absorption of a Ξ^- on $^{13}\mathrm{C}$ nucleus. The Coulomb expansion stage was not considered explicitly for such light systems. The momentum distributions of the final break-up products were obtained by the random generation over the whole accessible phase space, determined by the total kinetic energy (4), taking into account exact energy and momentum conservation laws. For this purpose we applied a very effective algorithm proposed by G.I. Kopylov [58].

Previously, this model was applied rather successfully for the description of the break-up of conventional light

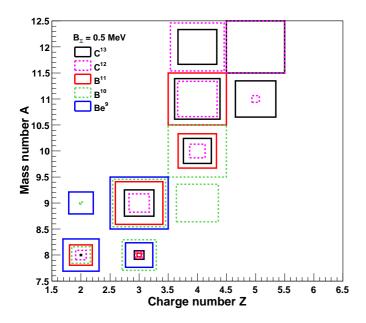


Figure 3: Relative population of the summed first + second *excited* states in all produced double hypernuclei for various Ξ -absorbing stable target nuclei $^9\mathrm{Be}$, $^{10}\mathrm{B}$, $^{11}\mathrm{B}$, $^{12}\mathrm{C}$ and $^{13}\mathrm{C}$. The area of the squares is proportional to the probability. For each secondary target the area for the most likely excited nucleus is normalized to 1.

nuclei in nuclear reactions initiated by protons, pions, antiprotons, and ions [28, 51, 59, 60]. In comparison with experimental yields of different fragments the maximum deviation was about 20–50%. We believe that this precision is sufficient for the present analysis of hypernucleus decays, since the main uncertainly is in unknown masses and energy levels of produced hyperfragments.

Fig. 1 shows as an example the production of ground (g.s.) and excited (ex.s.) states of single + one free Λ (SHP), twin (THP) and double (DHP) hypernuclei in case of a $^{12}\mathrm{C}$ target as a function of the assumed Ξ^- binding energy. With increasing Ξ^- binding energy the excitation energy of the excited primary $^{13}_{\Lambda\Lambda} B^*$ nucleus decreases from left to right from about 40 MeV to 15 MeV. For all excitation energies above 20 MeV the production of excited double hypernuclei dominates (green triangles). This can be traced back to the opening of several thresholds for various excited double hypernuclei already at moderate excitation energies (see e.g. Fig. 1 in Ref [12]). Only for small binding energies and hence large excitation energies the production of single and twin hypernuclei is significant ($\sim 10\%$). The non-monotonic behaviour for single hypernucleus + one free Λ production reflects the fact that the various lowest thresholds are relative high and widely separated, e.g. $^{12}_{\Lambda}B + \Lambda$ at B_{\(\pi\)=23.9 MeV followed by $^{11}_{\Lambda}B + n + \Lambda$} at 11.3 MeV. Twin-hypernuclei are only produced for $\Xi^$ binding energies below the threshold for ${}^{8}_{\Lambda}Li + {}^{5}_{\Lambda}He$ with $B_{\Xi}=13.6\,\mathrm{MeV}$. As discussed above the frequent observation of twin-hypernuclei [61, 62, 63, 64, 54, 65] signals a conversion from a Ξ state with only moderate binding energy. In this range of B_{\(\text{\infty}\) the production probability of dou-}

ble hypernuclei is comparable to previous estimates within a canonical statistical model [12, 13].

Fig. 2 shows the population of the four lowest lying, particle stable states in double hypernuclei. The $^{12}_{\Lambda\Lambda}$ Be+p and $^{12}_{\Lambda\Lambda}$ B+n have the lowest thresholds [12] and dominate at large binding energies and hence small excitation energies. At smaller and more realistic binding energies the lighter double hypernuclei $^{11}_{\Lambda\Lambda}$ Be, $^{10}_{\Lambda\Lambda}$ Be and $^{9}_{\Lambda\Lambda}$ Li take over. Due to the higher spin degeneracy factor the population of excited states can exceed those of ground states significantly at low binding energies. The dominant population of excited states in e.g. $^{10}_{\Lambda\Lambda}$ Be is consistent with the conjecture of Hiyama and co-workers [41] that the Demachi-Yanagi event [66, 67] can be interpreted most probably as the observation of the 2^+ excited state in $^{10}_{\Lambda\Lambda}$ Be. These calculations also indicate that by studying relative probabilities of excited states one may obtain information on the binding energy of the captured Ξ^- hyperons.

The summed population of the first and second particle stable excited states in all produced double hypernuclei are shown in Fig. 3 for various Ξ -absorbing stable target nuclei ⁹Be, ¹⁰B, ¹¹B, ¹²C and ¹³C. The probabilities are proportional to the area of the squares. For each target the yield for the most probable hypernucleus has been normalized to 1. For all five target nuclei the same Ξ^- binding energy of 0.5 MeV was adopted. Although this choice is not crucial for the main trend (see Fig. 1), the population of specific excited levels may depend somewhat on the adopted binding energy. In Fig. 3 different double hypernuclei dominate for each target. Thus combining the information of Fig. 3 with the measurement of two pion momenta from the subsequent weak decays a unique assignment of various newly observed γ -transitions to specific double hypernuclei seems possible as intended by the PANDA collaboration [22, 24].

In 2001 the BNL experiment E906 reported the observation of the $^4_{\Lambda\Lambda}{\rm H}$ hypernucleus by measuring the sequential pionic decays after a (K⁻,K⁺) reaction deposited two units of strangeness in a ⁹Be target [36]. Two structures in the correlated π^- momenta at (133,114) MeV/c and at (114,104) MeV/c were observed. The first structure was interpreted as the production of ${}^{3}_{\Lambda}H + {}^{4}_{\Lambda}H$ twins while the bump at (114,104) MeV/c was attributed to pionic decays of the double hypernucleus ${}_{\Lambda\Lambda}^4$ H. However, as it was pointed out by Kumagai-Fuse and Okabe also twin Λ -hypernuclear decays of ${}^{3}_{\Lambda}$ H and ${}^{6}_{\Lambda}$ He are a possible candidate to form this peak if excited resonance states of ⁶Li are considered [69]. More recently Randeniya and Hungerford showed that the published E906 data can be reproduced without the inclusion of $^4_{\Lambda\Lambda} H$ decay and that it is more likely that the decay of $^7_{\Lambda\Lambda} He$ was observed in the E906 experiment [25]. In their analysis this double hypernucleus was accompanied by a background of coincident decays of single hypernuclei pairs ${}_{\Lambda}^{3}H + {}_{\Lambda}^{4}H$, ${}_{\Lambda}^{3}H + {}_{\Lambda}^{3}H$, and ${}^4_{\Lambda} H + {}^4_{\Lambda} H$, respectively. Here the first twin pair dominated and the production ratio $^{7}_{\Lambda\Lambda}{\rm He}$ to coincident $^{3}_{\Lambda}{\rm H}+^{4}_{\Lambda}{\rm H}$ pairs was estimated to be in the range of 7.7 to 12.

Tab. 2 summarizes the predicted relative probabilities

Table 2: Total production probability of particle stable twin and double hypernuclei after the capture of a Ξ^- by a 9 Be target and the conversion into an excited $^{10}_{\Lambda\Lambda}$ Li* hypernucleus (third and forth [13] column). The four last columns are the results assuming the production of excited $^{8}_{\Lambda\Lambda}$ He* and $^{8}_{\Lambda\Lambda}$ H* nuclei after a knock-out process with an excitation energy of 33 MeV [12]. Here columns 5 and 6 are results of the present work, the last two columns are again from Ref. [13]. A – indicates that this particular channel cannot be reached or was not

considered.

decay channel	π pair m	nomenta	decaying system and probability					
	(MeV/c)		$^{10}_{\Lambda\Lambda} { m Li}^*$	$\int_{\Lambda\Lambda}^{10} \text{Li}^* [13]$	$^{8}_{\Lambda\Lambda}\mathrm{He^{*}}$	$^{8}_{\Lambda\Lambda}\mathrm{H}^{*}$	$^{8}_{\Lambda\Lambda}\mathrm{He}^{*}$ [13]	$^{8}_{\Lambda\Lambda} H^{*} [13]$
$^3_{\Lambda}\mathrm{H} + ^3_{\Lambda}\mathrm{H}$	114	114	_	0	0	_	0	_
$^{3}_{\Lambda}\mathrm{H}+^{4}_{\Lambda}\mathrm{H}_{gs}$	114	133	_	0	0.008	_	0.018	_
$^{3}_{\Lambda} H + ^{4}_{\Lambda} H_{1.05}$	114	134	_	_	0.014	_	_	_
$^3_{\Lambda}\mathrm{H} + ^5_{\Lambda}\mathrm{He}$	114	99	0.0001	0.011	_	_	_	_
$^3_{\Lambda}\mathrm{H}+^6_{\Lambda}\mathrm{He}$	114	108	0.004	0.012	_	_	_	_
$^3_{\Lambda}\mathrm{H}+^7_{\Lambda}\mathrm{He}_{gs}$	114	115	0.026	0.018	_	_	_	_
$^{3}_{\Lambda} H + ^{7}_{\Lambda} He_{1.66}$	114	118	0.046	0.018	_	_	_	_
$^{3}_{\Lambda}\mathrm{H}+^{7}_{\Lambda}\mathrm{He}_{1.74}$	114	118	0.068	0.018	_	_	_	_
$^4_{\Lambda}\mathrm{H}_{gs}+^4_{\Lambda}\mathrm{H}_{gs}$	133	133	_	0.005	0.017	_	0.055	_
$^4_{\Lambda}\mathrm{H}_{gs} + ^4_{\Lambda}\mathrm{H}_{1.05}$	133	134	_	_	0.096	_	_	_
${}^{4}_{\Lambda}\mathrm{H}_{1.05} + {}^{4}_{\Lambda}\mathrm{H}_{1.05}$	134	134	_	_	0.137	_	_	_
$^4_{\Lambda}\mathrm{H}_{gs}+^5_{\Lambda}\mathrm{He}$	133	99	0.022	0.045	_	_	_	_
$^4_{\Lambda}\mathrm{H}_{1.05}+^5_{\Lambda}\mathrm{He}$	134	99	0.055	_	_	_	_	_
$^4_{\Lambda}\mathrm{H}_{gs}+^6_{\Lambda}\mathrm{He}$	133	108	0.031	0.049	_	_	_	_
$^4_{\Lambda}\mathrm{H}_{1.05}+^6_{\Lambda}\mathrm{He}$	134	108	0.088	_	_	_	_	_
$^{4}_{\Lambda\Lambda}H$ $^{5}_{\Lambda\Lambda}H$	117	98	0.0006	0.026	0.003	0.0002	0.026	0
$^{5}_{\Lambda\Lambda}{ m H}$	134	99	0.007	0.139	0.069	0.635	0.108	0.877
$^{5}_{\Lambda\Lambda}\mathrm{He}$	_	_	_	0.0	0	_	0.009	_
$_{\Lambda\Lambda}^{5}$ He $_{\Lambda\Lambda}^{6}$ He	100	99 [68]	0.028	0.147	0.051	_	0.128	_
$_{\Lambda\Lambda}^{\prime}\mathrm{He}$	109	108	0.117	0.133	0.116	_	0.157	_
$^{8}_{\Lambda\Lambda}\mathrm{He}_{gs}$	116	124	0.022	small	_	_	_	_
$^{8}_{\Lambda\Lambda}\mathrm{He}_{ex}$	119	124	0.096	_	_	_	_	_
$^{9}_{\Lambda\Lambda}\mathrm{He}_{gs}$	117	121	0.021	0.025	_	_	_	_
$^{9}_{\Lambda\Lambda}\mathrm{He}_{ex}$ $^{7}_{\Lambda\Lambda}\mathrm{Li}$	122	121	0.027	_	_	_	_	_
$\int_{\Lambda\Lambda} \operatorname{Li}$	101	96	0.0001	0.008	_	_	_	_
$^{8}_{\Lambda\Lambda}{ m Li}_{gs}$	109	97	0.012	0.028	_	_	_	_
$_{\Lambda\Lambda}^{8} \text{Li}_{ex}$	111-117	97	0.028		_	_	_	_
${}^{\Lambda\Lambda}_{\substack{8\\\Lambda\Lambda}}^{g_{0}}\mathrm{Li}_{ex}$ ${}^{9}_{\substack{\Lambda\Lambda}}\mathrm{Li}_{gs}$ ${}^{9}_{\substack{\Lambda\Lambda}}\mathrm{Li}_{ex}$	123	97	0.028	0.026	_	_	_	_
$\int_{\Lambda\Lambda}^{9} \operatorname{Li}_{ex}$	124-131	97	0.098	_	_	_	_	_

for the production of particle stable twin and double hypernuclei in the E906 experiment. In the second column we list for orientation the associated pion momenta, assuming a $\Lambda\Lambda$ bond energy of $\Delta B_{\Lambda\Lambda}=1\,\text{MeV}$ and subsequent pionic two-body decays from the excited or groundstate to the corresponding groundstate. For orientation an increase of $\Delta B_{\Lambda\Lambda}$ to e.g. 4 MeV decreases the first momenta given in the second column for $\Lambda\Lambda$ hypernuclei by about 5 MeV/c. Note that lower pion momenta may also arise from multibody decays or the decay into excited intermediate nuclei. Excited states which will be deexcited prior to the weak decay will of course also result in lower pion momenta. Comparing to experimental data one also has to keep in mind that starting from pion pairs with rather similar or equal pion momenta, a finite momentum resolution will due to the sorting of the momenta result in a more asymmetric peak structure. In case of the ${}^{7}_{\Lambda\Lambda}He$ decays a momentum resolution like in the E906 experiment of σ_{rms} =4 MeV/c will shift the peak from (109,108) MeV/c

to (111,106) MeV/c.

In the third column of Tab. 2 we list the production probabilities after the capture and conversion of a stopped Ξ^- in a secondary ${}^9\mathrm{Be}$ target $\Xi^- + {}^9\mathrm{Be} \rightarrow {}^{10}_{\Lambda\Lambda}\mathrm{Li}^*$. As before a Ξ^- binding energy of 0.5 MeV was assumed corresponding to an ${}^{10}_{\Lambda\Lambda}\mathrm{Li}$ excitation of about 29 MeV. For comparison we show in column 4 of Tab. 2 the results of the canonical calculations of Ref. [13] provided in their Table 5. Those calculations use a somewhat higher excitation energy of 35 MeV. Inspecting the excitation functions for double and twin hypernuclei we have verified that this difference is not crucial.

While for most channels both calculations agree qualitatively, there are also important differences. The most striking difference is seen in the production of the $^4_{\Lambda\Lambda}{\rm H}$ and $^5_{\Lambda\Lambda}{\rm H}$ double hypernuclei where the calculations differ by a factor of 4 and more than an order of magnitude, respectively. Furthermore the total produced yield for $^9_{\Lambda\Lambda}{\rm Li}$ is about a factor of 5 higher in our model which is mainly re-

lated to the presence of excited states. Note also, that the production of ${}^4_\Lambda H + {}^4_\Lambda H$ twins is even at an excitation energy of 35 MeV energetically not possible and does therefore not occur in our micro-canonical model.

As an alternative production scheme we also consider the quasi-free/rescattering picture of Yamamoto et~al.~[13] resulting in the production of excited $^8_{\Lambda\Lambda}{\rm He}$ or $^8_{\Lambda\Lambda}{\rm H}$ nuclei. As in ref. [13] the excitation energy of the initial $^8_{\Lambda\Lambda}{\rm He}$ and $^8_{\Lambda\Lambda}{\rm H}$ nuclei was fixed to 33 MeV. The last columns of Tab.2 show the results from Ref. [13] while Columns 5 and 6 contain the values predicted by our micro-canonical model at the same excitation energy for this scenario. In both models the production of $^5_{\Lambda\Lambda}{\rm H}, ^6_{\Lambda\Lambda}{\rm He}$ and $^7_{\Lambda\Lambda}{\rm He}$ as well as the $^4_{\Lambda}{\rm H}+^4_{\Lambda}{\rm H}$ twins dominate. Due to the higher spin factor, the production of excited states of $^4_{\Lambda}{\rm H}$ is enhanced over the direct ground state production in our model.

Let us first discuss the structure at (114,104) MeV/cwhich has been attributed to double hypernuclei decays. Generally the production of double hypernuclei is energetically favored over the production of twins: all possible channels with twin-production lie energetically significantly above the thresholds for ${}^{9}_{\Lambda\Lambda}\text{Li}$, ${}^{7}_{\Lambda\Lambda}\text{He}$ and ${}^{6}_{\Lambda\Lambda}\text{He}$ production in case of the $^{10}_{\Lambda\Lambda}{\rm Li}$ compound picture. Correspondingly the production of ${}^3_{\Lambda}H$ and ${}^6_{\Lambda}He$ twins which has been suggested [69] as a possible source of the peak structure around (114,104) MeV/c is in our model by a factor of 30 and in the model of Ref. [13] by a factor of 10 lower than the $^{7}_{\Lambda\Lambda}$ He production probability. Unlike in Ref. [13] the ${}^{7}_{\Lambda\Lambda}$ He probability exceeds also the one of a ${}^{4}_{\Lambda\Lambda}$ H by more than two orders of magnitude and the production of ${}_{\Lambda\Lambda}^5H$ by a factor of about 17 in our model. Similarly, also within the quasi-free/rescattering picture the $^{7}_{\Lambda\Lambda}$ He nucleus is the most probable channel. Of course, a direct comparison with the E906 data requires a detailed consideration of the branching ratios for pionic two-body decays many of which are not known so far. Keeping that caveat in mind our microcanonical model supports independent on the assumed production scheme the interpretation of the E906 observation by Randeniya and Hungerford [25] in terms of $^{7}_{\Lambda\Lambda}$ He decays. Decays from the ground or even exited states of $^8_{\Lambda\Lambda}{\rm Li}$ or $^9_{\Lambda\Lambda}{\rm Li}$ can possibly contribute some background to the structure at (114,104) MeV/c.

In order to describe the (133,114) MeV/c structure of the E906 experiment, the production of ${}^3_\Lambda H + {}^4_\Lambda H$ twins seems mandatory [36]. However, the ${}^3_\Lambda H + {}^4_\Lambda H + t$ mass lies above the initial mass $m_0 = m(\Xi^-) + m({}^9Be)$ and can therefore not be produced in the $\Xi^- + {}^9Be$ compound production scheme. With an energy of 12.6 MeV below m_0 , the channel ${}^4_\Lambda H + {}^6_\Lambda H e$ is the most likely twin in the present scenario, followed by the ${}^4_\Lambda H + {}^5_\Lambda H e + n$ decay. Given however the experimental precision of about 1 MeV/c for the momentum calibration in E906 [36], neither the decays of ${}^4_\Lambda H + {}^5_\Lambda H e$ with (133,108) MeV/c nor decays of ${}^4_\Lambda H + {}^5_\Lambda H e$ pairs with (133,99) MeV/c seem to explain the structure around (133,114) MeV/c. Of course particularly the first one could contribute to this enhancement in the tail region.

As can be seen from the last four columns of Tab.2

also in the quasi-free/rescattering picture the production of ${}^3_\Lambda H + {}^4_\Lambda H$ twins plays only a minor role and none of the dominant decay channels can account for the observed structure at (133,114) MeV/c even though the yield ratio for ${}^7_{\Lambda\Lambda} H$ e to ${}^3_\Lambda H + {}^4_\Lambda H$ twin production of about 8.7 in ref. [13] resp. 5.2 in our model are still compatible with the estimated ratio mentioned above. More intriguing is however the fact that both statistical models predict a production of ${}^4_\Lambda H + {}^4_\Lambda H$ twins even exceeding the ${}^3_\Lambda H + {}^4_\Lambda H$ production considerably. Considering furthermore branching ratios for two-body π^- decays of $\Gamma_{\pi^-+{}^3He}/\Gamma_{total}\approx 0.26$ [70] and $\Gamma_{\pi^-+{}^4He}/\Gamma_{total}\approx 0.5$ [71] the absence of a bump which could be attributed to ${}^4_\Lambda H + {}^4_\Lambda H$ is particularly puzzling.

It seems that a process different from the ones discussed so far is required to explain the singular structure at (133,114) MeV/c in terms of ${}^3_{\Lambda}{\rm H} + {}^4_{\Lambda}{\rm H}$ twins. Indeed in a direct mechanism like $K^-+^9\mathrm{Be} \to 2\mathrm{n} + K^++^7_{\Lambda\Lambda}\mathrm{He}^*$ the production of ${}^4_{\Lambda} H + {}^4_{\Lambda} H$ would not be feasible and ${}^3_{\Lambda} H + {}^4_{\Lambda} H$ pairs would be enhanced. Clearly for such a non-equilibrium process and considering furthermore that in the analysis of the E906 experiment e.g. cuts on the 'missing mass' of the Ξ were applied [36], is difficult to predict the excitation energy distribution for the ${}^{7}_{\Lambda\Lambda}{\rm He}^*$ nuclei. This reaction scheme is therefore at present beyond the scope of our equilibrium model and needs further detailed consideration of the initial interaction. It also is clear that the present statistical decay model needs to be complemented by quantitative weak decay calculations (see e.g. [72]) to further substantiate this conjecture.

In summary we have presented a micro-canonical decay model to describe the break-up of an excited double hypernucleus after the absorption and conversion of a stopped Ξ^- hyperon. Generally the formation of excited states dominates in our model. For different Ξ^- absorbing target nuclei, different produced double hypernuclei dominate. Once combined with a weak decay model these calculations will enable more reliable estimates of the γ -ray yields for the double hypernucleus measurements envisaged by the PANDA collaboration [22] and a more consistent interpretation of the E906 data. In future studies we plan to extend the present model to the production of bound $\Xi\Lambda$ or triple Λ hypernuclei after the conversion of a Ω^- into an excited $\Xi\Lambda$ nucleus. Furthermore, the decay of excited single hyperfragments produced in electron scattering experiments will be studied with the present approach.

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