SUPERSYMMETRIZATION SCHEMES OF $D = 4$ MAXWELL ALGEBRA

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Abstract

The Maxwell algebra, an enlargement of Poincaré algebra by Abelian tensorial generators, can be obtained in arbitrary dimension D by the suitable contraction of $O(D-1,1) \oplus O(D-1,2)$ (Lorentz algebra$\oplus AdS$ algebra). We recall that in $D = 4$ the Lorentz algebra $O(3,1)$ is described by the realification $Sp_R(2|C)$ of complex algebra $Sp(2|C) \simeq Sl(2|C)$ and $O(3,2) \simeq Sp(4)$. We study various $D = 4$ N-extended Maxwell superalgebras obtained by the contractions of real superalgebras $OSp_R(2N-k;2|C) \oplus OSp(k;4)$, $(k = 1,2,\ldots,2N)$ (extended Lorentz superalgebra $\oplus$ extended AdS superalgebra).

If $N = 1$ ($k = 1,2$) one arrives at two different versions of simple Maxwell superalgebra. For any fixed N we get $2N$ different superextensions of Maxwell algebra with n-extended Poincaré superalgebras ($1 \leq n \leq N$) and the internal symmetry sectors obtained by suitable contractions of the real algebra $O_R(2N-k|C) \oplus O(k)$. Finally the comments on possible applications of Maxwell superalgebras are presented.

1 Introduction

The Maxwell algebra as the enlargement of Poincaré algebra $(P_\mu, M_{\mu\nu})$ by antisymmetric Abelian tensorial charges $Z_{\mu\nu} = -Z_{\nu\mu}$ was firstly obtained for $D = 4$ more than forty years ago [1, 2] by supplementing the relations

$$\begin{aligned}
[P_\mu, P_\nu] &= i\,M^2\,Z_{\mu\nu}\,, \\
[P_\rho, Z_{\mu\nu}] &= 0, \qquad [M_{\rho\tau}, Z_{\mu\nu}] = -i(\eta_{\mu[\tau}\,Z_{\rho]\nu} - \eta_{\nu[\tau}\,Z_{\rho]\mu})\,,
\end{aligned} \qquad (1.1)$$

where M is a geometric mass parameter ($[M] = 1$; we use in $D = 4$ the metric $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$). In presenting the Maxwell algebra we introduce mass-like fundamental parameter M (in [3] denoted by $\Lambda = M^2$) which implies vanishing mass dimensionality of the tensorial charges ($[Z_{\mu\nu}] = 0$). The relations (1.1) can be however rewritten for

generators $Z_{\mu\nu}$ with any mass dimensionality. In particular if M^2 is replaced by electromagnetic dimensionless coupling constant e ($[e] = 0$; see e.g. [1, 2, 4, 5]) we obtain $[Z_{\mu\nu}] = 2$. Simple dynamical realization of such Maxwell algebra is obtained in the model of relativistic free particle moving in constant EM field backgrounds [4, 5]

We recall that Maxwell algebra can be obtained by a contraction of $O(3,1) \oplus O(3,2)$ [6, 5][1] or $O(3,1) \oplus O(4,1)$ [5] (if we replace $M^2 \to -M^2$ in (1.1)). Recently in [8] there were as well introduced the simple Maxwell superalgebras, with two Weyl supercharges $\mathbf{Q}_\alpha, \mathbf{\Sigma}_\alpha$, where $\mathbf{Q}_\alpha$ are the standard $N = 1$ Poincaré supercharges and new Maxwell supercharges $\mathbf{\Sigma}_\alpha$ are required for the supersymmetrization of the generators $Z_{\mu\nu}$. One should point out that recently appeared proposals to use the Maxwell algebra [3, 9, 7, 10] and Maxwell superalgebra [11] to the geometric extension of (super)gravity theories.

At present it appears interesting to study the problem how the Maxwell algebra can be supersymmetrized in various ways. In order to obtain important class of $D = 4$ Maxwell superalgebras we observe that $D = 4$ Lorentz algebra can be obtained as the realification[2] of complex algebra $Sp(2|C)$ ($O(3,1) \simeq Sp_R(2|C)$) with the supersymmetrization described by the realification of complex superalgebra $OSp(m; 2|C)$, (for $m = 1$ see e.g. [13]); further it is well known that $D = 4$ AdS symmetries are supersymmetrized by $D = 4$ AdS superalgebras $OSp(k; 4)$. We shall consider therefore contractions[3] of the following choices of the sum of real semisimple superalgebras

$$O(3,1) \oplus O(3,2) \xrightarrow{SUSY} OSp_R(2N - k; 2|C) \oplus OSp(k; 4) \tag{1.2}$$

where $k = 0, 1, 2 \dots 2N$ and $N = 0, 1, 2 \dots$.

We assume that the derived extended Maxwell superalgebras should satisfy the following two properties:

i) They should contain at least one standard bilinear fermionic SUSY relation characterizing Poincaré supercharges with the following algebraic structure

$$\{\mathbf{Q}_\alpha, \mathbf{Q}_\beta\} = (C\gamma^\mu P_\mu)_{\alpha\beta} + \text{ central charges.} \tag{1.3}$$

ii) Their bosonic sector should contain the Maxwell algebra.

Let us observe that in the contraction of (1.2) the Poincaré supercharges satisfying the relations (1.3) can be only obtained from the contraction of $OSp(k; 4)$. If we put $k = 0$ in (1.2) the generators $Sp(4) \simeq O(3,2)$ remain not supersymmetrized and we can get by contraction only a purely exotic version of Maxwell superalgebra [17, 15] with the standard generic SUSY relations (1.3) replaced by the following one

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[1]Recently deformation $O(3,1) \oplus O(3,2)$ of Maxwell algebra was called AdS-Maxwell algebra [7].

[2]By realification of n-dimensional complex Lie (super)algebra $\widehat{g}$ with the generators $g_i = g_i^{(1)} + ig_i^{(2)}, (i = 1, ..., n)$ we call the 2n-dimensional real Lie (super)algebra $\widehat{g}_R$ on real space with real generators $g_i^{(1)}, g_i^{(2)}$ (see e.g. [12]). If besides the complex generators g_i we introduce $\bar{g}_i = g_i^{(1)} - ig_i^{(2)}$, the real generators of $\hat{g}_R$ are linear combinations of g_i and $\bar{g}_i$, ($g_i^{(1)} = \frac{1}{2}(g_i + \hat{g}_i)$, $g_i^{(2)} = \frac{1}{2i}(g_i - \hat{g}_i)$).

[3]We need to consider the contractions of direct sum $\hat{g}_1 \oplus \hat{g}_2$ of Lie (super)algebras in which the contracted generators appear as suitable sums of the generators belonging to $\hat{g}_1$ and $\hat{g}_2$. For earlier discussion of contractions which use the linear superposition of generators from the sum of Lie (super)algebras, see [14, 15, 16].

$$\{\mathbf{S}_\alpha, \mathbf{S}_\beta\} = (C\gamma^{\mu\nu}Z_{\mu\nu})_{\alpha\beta} + \text{ central charges.} \tag{1.4}$$

The paper is organized as follows: In Sect. 2 we shall derive the simple $N = 1$ Maxwell superalgebras. If we put $N = 1$ in (1.2), for $k = 2$ and $k = 1$ we shall obtain by contractions two versions of simple $(N = 1)$ $D = 4$ Maxwell superalgebras, differing by the presence of Abelian chiral generator (see also [8, 16]). In Sect. 3 we consider the nonstandard contraction of (1.2) in general case (arbitrary N with $k = 1, 2, \ldots, 2N$); we treat separately the cases $0 \le k < N$ and $N \le k \le 2N$ requiring different explicit contractions. In Sect 4 we provide short discussion of our results.

2 $N = 1$ Maxwell superalgebras by contraction

2.1 Simple Maxwell superalgebras

In this section we consider the contractions of the superalgebras (1.2) for $N = 1$ and $k = 1, 2$ providing the simple Maxwell superalgebras with two Weyl supercharges which were proposed in [8]. The Maxwell algebra (relations (1.1)+Lorentz algebra) is extended supersymmetrically as follows[4]

$$\begin{aligned}
\{\mathbf{Q}_\alpha, \mathbf{Q}_\beta\} &= (C\gamma^\mu)_{\alpha\beta}P_\mu, & \{\boldsymbol{\Sigma}_\alpha, \boldsymbol{\Sigma}_\beta\} &= 0, \\
\{\mathbf{Q}_\alpha, \boldsymbol{\Sigma}_\beta\} &= \frac{M^2}{2}(C\gamma^{\mu\nu})_{\alpha\beta}Z_{\mu\nu} + (C\gamma_5)_{\alpha\beta}B, & & \tag{2.1} \\
[P_\mu, \mathbf{Q}_\alpha] &= -\frac{i}{2}\boldsymbol{\Sigma}_\beta(\gamma_\mu)^\beta{}_\alpha, & [P_\mu, \boldsymbol{\Sigma}_\alpha] &= 0, \\
[M_{\rho\sigma}, \mathbf{Q}_\alpha] &= -\frac{i}{2}(\mathbf{Q}\gamma_{\rho\sigma})_\alpha, & [M_{\rho\sigma}, \boldsymbol{\Sigma}_\alpha] &= -\frac{i}{2}(\boldsymbol{\Sigma}\gamma_{\rho\sigma})_\alpha, & \tag{2.2} \\
[B_C, \mathbf{Q}_\alpha] &= i\,(\mathbf{Q}\gamma_5)_\alpha, & [B_C, \boldsymbol{\Sigma}_\alpha] &= -i\,(\boldsymbol{\Sigma}\gamma_5)_\alpha, & \tag{2.3}
\end{aligned}$$

where real Dirac-Majorana matrices γ_μ $(\gamma_i^T = \gamma_i, \gamma_0^T = -\gamma_0)$ verify the $O(3,1)$ Clifford algebra $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$ and $\gamma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu] = -\gamma_{\nu\mu}$ is the 4×4 matrix realization of $O(3,1)$. The charge conjugation matrix $C = \gamma_0$ and $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3 = -\gamma_5^T$ satisfy the relations $(C\gamma_\mu)^T = (C\gamma_\mu)$, $(C\gamma_{\mu\nu})^T = (C\gamma_{\mu\nu})$, $(C\gamma_5)^T = -(C\gamma_5)$. The new supercharges $\boldsymbol{\Sigma}_\alpha$ are needed for the supersymmetrization of the generators $Z_{\mu\nu}$; the B_C describes a chiral generator and B is the central charge. In the minimal $N = 1$ Maxwell superalgebra we put $B = B_C = 0$.

2.2 Contraction of $O(3,1) \oplus OSp(2;4)$ ($k = 2$ case)

The $OSp(2,4)$ superalgebra is

$$\begin{aligned}
[\mathcal{M}_{\hat\mu\hat\nu}, \mathcal{M}_{\hat\rho\hat\sigma}] &= -i\,\eta_{\hat\rho[\hat\nu}\mathcal{M}_{\hat\mu]\hat\sigma} + i\,\eta_{\hat\sigma[\hat\nu}\mathcal{M}_{\hat\mu]\hat\rho}, & \tag{2.4} \\
\{\mathcal{Q}^i_\alpha, \mathcal{Q}^j_\beta\} &= -\delta^{ij}(C\Gamma^{\hat\mu\hat\nu})_{\alpha\beta}\mathcal{M}_{\hat\mu\hat\nu} + 2(C)_{\alpha\beta}\mathcal{B}^{ij}, \\
[\mathcal{M}_{\hat\mu\hat\nu}, \mathcal{Q}^i_\alpha] &= -\frac{i}{2}(\mathcal{Q}^i\Gamma_{\hat\mu\hat\nu})_\alpha, & [\mathcal{B}^{ij}, \mathcal{Q}^k_\alpha] &= -i\,\delta^{k[j}\mathcal{Q}^{i]}_\alpha. & \tag{2.5}
\end{aligned}$$

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[4]The mass dimension of the generators are $[P_\mu] = 1$, $[Z_{\mu\nu}] = 0$, $[\mathbf{Q}_\alpha] = \frac{1}{2}$, $[\mathbf{S}_\alpha] = \frac{3}{2}$, $[B] = 2$, $[B_5] = 0$.

Here $\mathcal{M}_{\hat{\mu}\hat{\nu}}$, ($\hat{\mu} = 0,1,2,3,4$) are $SO(3,2)$ generators with $\eta_{\hat{\mu}\hat{\nu}} = (-1,1,1,1,-1)$ and $O(3,1)$ real Dirac-Majorana matrices $\Gamma_\mu = \gamma_\mu\gamma_5$, $\Gamma_4 = -\gamma_5$ satisfy the $O(3,2)$ Clifford algebra $\{\Gamma_{\hat{\mu}},\Gamma_{\hat{\nu}}\} = 2\eta_{\hat{\mu}\hat{\nu}}$. The 4×4 matrix realization $\Gamma_{\hat{\mu}\hat{\nu}} = \frac{1}{2}[\Gamma_{\hat{\mu}},\Gamma_{\hat{\nu}}]$ of $O(3,2)$ algebra is expressed by $O(3,1)$ γ_μ-matrices $as\,\Gamma_{\mu\nu} = \frac{1}{2}[\Gamma_\mu,\Gamma_\nu] = \gamma_{\mu\nu}$, $\Gamma_{\mu4} = \Gamma_\mu\Gamma_4 = \gamma_\mu$. The charge conjugation $C = \gamma_0 = \Gamma_0\Gamma_4$ is common for $O(3,2)$ and $O(3,1)$. Supercharges $\mathcal{Q}^i_\alpha$, ($\alpha = 1,2,3,4$, $i = 1,2$) are real $O(3,2)$ spinors and $\mathcal{B}^{ij} = -\epsilon^{ij}\mathcal{B}_C$ is the $SO(2)$ generator.

The real algebra $O(3,1)$ is provided by the algebra $Sp_R(2|C) = O_R(2,1|C) \simeq O(3,1)$. The algebra $O(2,1|C) \oplus \overline{O(2,1|C)}$ is described by the generators $\mathbf{J}_{\bar{\mu}}$ and $\mathbf{J}^\dagger_{\bar{\mu}}$, ($\bar{\mu} = 0,1,2$) with the metric $\eta_{\bar{\mu}\bar{\nu}} = (-1,1,1)$,

$$[\mathbf{J}_{\bar{\mu}}, \mathbf{J}_{\bar{\nu}}] = i\,\epsilon_{\bar{\mu}\bar{\nu}}{}^{\bar{\rho}}\mathbf{J}_{\bar{\rho}}, \qquad \left[\mathbf{J}^\dagger_{\bar{\mu}}, \mathbf{J}^\dagger_{\bar{\nu}}\right] = i\,\epsilon_{\bar{\mu}\bar{\nu}}{}^{\bar{\rho}}\mathbf{J}^\dagger_{\bar{\rho}}, \qquad [\mathbf{J}_{\bar{\mu}}, \mathbf{J}^\dagger_{\bar{\nu}}] = 0, \qquad (2.6)$$

where $\epsilon^{\bar{\mu}\bar{\nu}\bar{\rho}}$ is the Levi-Civita symbol with $\epsilon^{012} = -\epsilon_{012} = 1$. If we introduce the real $O(3,1)$ generators $\mathcal{J}_{\mu\nu}$, ($\mu = 0,1,2,3$) by the relation

$$\mathbf{J}_{\bar{\mu}} = \frac{1}{4}\epsilon_{\bar{\mu}}{}^{\bar{\rho}\bar{\nu}}J_{\bar{\rho}\bar{\nu}} + \frac{i}{2}\,J_{\bar{\mu}3}\,, \qquad (2.7)$$

they satisfy the $D = 4$ Lorentz algebra with $\eta_{\mu\nu} = (-1,1,1,1)$,

$$[\mathcal{J}_{\mu\nu}, \mathcal{J}_{\rho\sigma}] = -i\,\eta_{\rho[\nu}\mathcal{J}_{\mu]\sigma} + i\,\eta_{\sigma[\nu}\mathcal{J}_{\mu]\rho}\,. \qquad (2.8)$$

We propose the following redefinitions of the generators of $O(3,1) \oplus O(3,2)$ (we recall that $\Gamma_4 = -\gamma_5$ and we put $\alpha + \beta = 1$)

$$M_{\mu\nu} = \mathcal{J}_{\mu\nu} + \mathcal{M}_{\mu\nu}, \quad P_\mu = \frac{1}{R}\mathcal{M}_{\mu4}, \quad Z_{\mu\nu} = \frac{1}{R^2M^2}(\alpha\mathcal{J}_{\mu\nu} - \beta\mathcal{M}_{\mu\nu}) \qquad (2.9)$$

and the rescaled internal symmetry generator and supercharges as follows

$$B_C = \frac{1}{R^\gamma}\mathcal{B}_C\,, \qquad (2.10)$$

$$\mathbf{Q}_\alpha = \frac{1}{2R^{1/2}}(Q^1_\alpha + (Q^2\gamma_5)_\alpha), \qquad d\mathbf{\Sigma}_\alpha = \frac{1}{2R^{3/2}}(Q^1_\alpha - (Q^2\gamma_5)_\alpha)\,, \qquad (2.11)$$

where R in the formulas (2.9)-(2.11) is a contraction parameter with dimension of length ($[R] = -1$). The superalgebra $O(3,1) \oplus OSp(2;4)$ in terms of new rescaled real generators takes the form

$$
\begin{aligned}
[P_\mu, P_\nu] &= i\,M^2\,Z_{\mu\nu} - i\frac{\beta}{R^2}\,M_{\mu\nu}, & [P_\mu, M_{\rho\sigma}] &= -i\,\eta_{\mu[\rho}P_{\sigma]}, \\
[P_\mu, Z_{\rho\sigma}] &= i\,\frac{\alpha}{R^2}\,\eta_{\mu[\rho}P_{\sigma]}, & [M_{\mu\nu}, Z_{\rho\sigma}] &= -i\,\eta_{\rho[\nu}Z_{\mu]\sigma} + i\,\eta_{\sigma[\nu}Z_{\mu]\rho}, \\
[M_{\mu\nu}, M_{\rho\sigma}] &= -i\,\eta_{\rho[\nu}M_{\mu]\sigma} + i\,\eta_{\sigma[\nu}M_{\mu]\rho}, \\
[Z_{\mu\nu}, Z_{\rho\sigma}] &= -i\frac{\beta-\alpha}{M^2\,R^2}(\eta_{\rho[\nu}Z_{\mu]\sigma} - \eta_{\sigma[\nu}Z_{\mu]\rho}) - i\frac{\beta\alpha}{M^4\,R^4}(\eta_{\rho[\nu}M_{\mu]\sigma} - \eta_{\sigma[\nu}M_{\mu]\rho})(2.12)
\end{aligned}
$$

$$\{\mathbf{Q}_\alpha, \mathbf{Q}_\beta\} = (C\gamma^\mu)_{\alpha\beta} P_\mu, \qquad \{\mathbf{\Sigma}_\alpha, \mathbf{\Sigma}_\beta\} = \frac{1}{R^2}(C\gamma^\mu)_{\alpha\beta} P_\mu,$$

$$\{\mathbf{Q}_\alpha, \mathbf{\Sigma}_\beta\} = \frac{M^2}{2}(C\gamma^{\mu\nu})_{\alpha\beta} Z_{\mu\nu} + \frac{1}{R^{2-\gamma}}(C\gamma_5)_{\alpha\beta} B_C, \qquad (2.13)$$

$$[P_\mu, \mathbf{Q}_\alpha] = -\frac{i}{2}\mathbf{\Sigma}_\beta(\gamma_\mu)^\beta{}_\alpha, \qquad [P_\mu, \mathbf{\Sigma}_\alpha] = -\frac{i}{4R^2}\mathbf{Q}_\beta(\gamma_\mu)^\beta{}_\alpha,$$

$$[Z_{\mu\nu}, \mathbf{Q}_\alpha] = \frac{i}{2R^2 M^2}(\mathbf{Q}\gamma_{\mu\nu})_\alpha, \qquad [Z_{\mu\nu}, \mathbf{\Sigma}_\alpha] = \frac{i}{2R^2 M^2}(\mathbf{\Sigma}\Gamma_{\mu\nu})_\alpha,$$

$$[M_{\rho\sigma}, \mathbf{Q}_\alpha] = -\frac{i}{2}(\mathbf{Q}\gamma_{\rho\sigma})_\alpha, \qquad [M_{\rho\sigma}, \mathbf{\Sigma}_\alpha] = -\frac{i}{2}(\mathbf{\Sigma}\gamma_{\rho\sigma})_\alpha.$$

$$[B_C, \mathbf{Q}_\alpha] = \frac{i}{R^\gamma}(\mathbf{Q}\gamma_5)_\alpha, \qquad [B_C, \mathbf{\Sigma}_\alpha] = -\frac{i}{R^\gamma}(\mathbf{\Sigma}\gamma_5)_\alpha. \qquad (2.14)$$

If we put $\alpha = 1, \beta = 0, \ 2 > \gamma > 0$ and $M = 1$ the formulas (2.12)-(2.14) describe the k-deformation of the Maxwell superalgebra given in [16]($\frac{1}{R^2}$ =k), which reproduces in the k$\to$ 0 limit the Maxwell superalgebra (2.1-2.3) with $B_C = 0$ and $B = 0$. In the case $\gamma = 0$, B_C is nonvanishing and becomes a chiral charge generating chiral transformation of $\mathbf{Q}_\alpha$ and $\mathbf{\Sigma}_\alpha$, (see (2.3)). The specific feature of our contraction is that two factors from the defining algebra $OSp(2|4) \oplus O(3,1)$ are suitably mixed (see (2.9)–(2.11)). As a result, this type of contractions will not respect the direct sum structure of the uncontracted algebras (see also [18],[19] (Sect. 8)).

2.3 Contraction of $OSp_R(1;2|C) \oplus OSp(1;4)$ ($k = 1$ case)

The $OSp(1;4)$ superalgebra is

$$[\mathcal{M}_{\hat\mu\hat\nu}, \mathcal{M}_{\hat\rho\hat\sigma}] = -i\,\eta_{\hat\rho[\hat\nu}\mathcal{M}_{\hat\mu]\hat\sigma} + i\,\eta_{\hat\sigma[\hat\nu}\mathcal{M}_{\hat\mu]\hat\rho},$$

$$\{\mathcal{Q}_\alpha, \mathcal{Q}_\beta\} = -\frac{1}{2}(C\Gamma^{\hat\mu\hat\nu})_{\alpha\beta}\mathcal{M}_{\hat\mu\hat\nu}, \qquad [\mathcal{M}_{\hat\mu\hat\nu}, \mathcal{Q}_\alpha] = -\frac{i}{2}(\mathcal{Q}\Gamma_{\hat\mu\hat\nu})_\alpha. \qquad (2.15)$$

The complex superalgebra $OSp(1;2|C) = (\mathbf{J}_{\bar\mu}, S_{+\alpha})$ contains the bosonic subalgebra $Sp(2|C) = O(2,1|C)$ with its realification given by $O(3,1)$. The complex supercharges $S_{+\alpha}$ define the fermionic sector of $OSp(1;2|C)$,

$$\{S_{+\alpha}, S_{+\beta}\} = 4i\,(C\Gamma^{\bar\rho}_+)_{\alpha\beta}\,\mathbf{J}_{\bar\rho}, \qquad [\mathbf{J}_{\bar\mu}, S_{+\alpha}] = \frac{1}{2}S_{+\beta}(\Gamma_{+\bar\mu})^\beta{}_\alpha, \qquad (2.16)$$

where the matrices $\Gamma_{+\bar\mu}$ are defined by O(3,1) gamma matrices γ_μ as

$$\Gamma_{+\bar\mu} = \gamma_{\bar\mu}\gamma_3 P_+, \quad P_\pm = \frac{1}{2}(1 \pm i\,\gamma_5), \quad [\Gamma_{+\bar\mu}, \Gamma_{+\bar\nu}] = 2i\,\epsilon_{\bar\mu\bar\nu}{}^{\bar\rho}\Gamma_{+\bar\rho} \qquad (2.17)$$

and $S_{+\alpha} = S_{+\alpha}P_+$. The complex conjugated superalgebra $\overline{OSp(1;2|C)} = (S_{-\beta} = S^\dagger_{+\beta}, \mathbf{J}^\dagger_{\bar\mu})$ has the following fermionic sector

$$\{S_{-\alpha}, S_{-\beta}\} = -4i\,(C\Gamma^{\bar\rho}_-)_{\alpha\beta}\,\mathbf{J}^\dagger_{\bar\rho}, \qquad \left[\mathbf{J}^\dagger_{\bar\mu}, S_{-\alpha}\right] = -\frac{1}{2}\mathbf{S}_{-\beta}(\Gamma_{-\bar\mu})^\beta{}_\alpha. \qquad (2.18)$$

5

Using (2.7) and introducing real supercharges $\mathcal{S}_\alpha = S_{+\alpha} + S_{-\alpha}$ one gets the following real superalgebra describing the realification $OSp_R(1; 2|C)$ of $OSp(1; 2|C)$ which extends supersymmetrically the Lorentz algebra (2.8) by the relation

$$\{\mathcal{S}_\alpha, \mathcal{S}_\beta\} = -(C\gamma^{\mu\nu})_{\alpha\beta}\mathcal{J}_{\mu\nu}, \qquad [\mathcal{J}_{\mu\nu}, \mathcal{S}_\alpha] = -\frac{i}{2}(\mathcal{S}\gamma_{\mu\nu})_\alpha. \qquad (2.19)$$

The superalgebra $OSp_R(1; 2|C) \oplus OSp(1; 4)$ before the contraction $R \to \infty$ is described by the real generators $(\mathcal{M}_{\hat\mu\hat\nu}, \mathcal{Q}_\alpha, \mathcal{J}_{\mu\nu}, \mathcal{S}_\alpha)$ and does not contain any internal symmetry generators. Using the rescalings (2.9) of $O(3,1) \oplus O(3,2)$ generators and new rescaled supercharges

$$\mathbf{Q}_\alpha = R^{-1/2}(\mathcal{Q}_\alpha + \mathcal{S}_\alpha), \quad \boldsymbol{\Sigma}_\alpha = R^{-3/2}\mathcal{S}_\alpha, \qquad (2.20)$$

we describe the real superalgebra $OSp_R(1; 2|C) \oplus OSp(1|4)$. The bosonic part is given by the relations (2.12) and the part of superalgebra which contains the supercharges $\mathbf{Q}_\alpha$ and $\boldsymbol{\Sigma}_\alpha$ is

$$
\begin{aligned}
[P_\mu, \mathbf{Q}_\alpha] &= \frac{i}{2}(\boldsymbol{\Sigma}\gamma_\mu)_\alpha - \frac{i}{2R}(\mathbf{Q}\gamma_\mu)_\alpha, & [P_\mu, \boldsymbol{\Sigma}_\alpha] &= 0, \\
[Z_{\mu\nu}, \mathbf{Q}_\alpha] &= -\frac{i}{2}\frac{1}{RM^2}(\boldsymbol{\Sigma}\gamma_{\mu\nu})_\alpha, & [Z_{\mu\nu}, \boldsymbol{\Sigma}_\alpha] &= -\frac{i}{2}\frac{1}{R^2M^2}(\boldsymbol{\Sigma}\gamma_{\mu\nu})_\alpha, \\
[M_{\rho\sigma}, \mathbf{Q}_\alpha] &= -\frac{i}{2}(\mathbf{Q}\gamma_{\rho\sigma})_\alpha, & [M_{\rho\sigma}, \boldsymbol{\Sigma}_\alpha] &= -\frac{i}{2}(\boldsymbol{\Sigma}\gamma_{\rho\sigma})_\alpha, \\
\{\mathbf{Q}_\alpha, \mathbf{Q}_\beta\} &= -\frac{1}{R}(C\gamma^{\mu\nu})_{\alpha\beta}M_{\mu\nu} + 2(C\gamma^\mu)_{\alpha\beta}P_\mu, \\
\{\mathbf{Q}_\alpha, \boldsymbol{\Sigma}_\beta\} &= -M^2(C\gamma^{\mu\nu})_{\alpha\beta}Z_{\mu\nu}, & \{\boldsymbol{\Sigma}_\alpha, \boldsymbol{\Sigma}_\beta\} &= -\frac{M^2}{R}(C\gamma^{\mu\nu})_{\alpha\beta}Z_{\mu\nu}. \quad (2.22)
\end{aligned}
$$

(2.21)

In the limit $R \to \infty$ the superalgebra $OSp_R(1; 2|C) \oplus OSp(1; 4)$ is contracted into the minimal simple Maxwell superalgebra (see [8] and (2.1-2.3)) with $B = 0$ and removed generator B_C. It should be added that if we choose $\alpha = 0, \beta = 1$ the similar contraction formulae were considered recently in [11].

2.4 Contraction of $OSp_R(2; 2|C) \oplus Sp(4)$ ($k = 0$ case)

For completeness we present the exotic version of Maxwell superalgebra obtained by the contraction of (1.2) for $k = 0$. The bosonic algebra $Sp(4) = O(3, 2)$ is given by relation (2.4) and $OSp(2; 2|C)$ is the complex superalgebra with bosonic subalgebra $Sp(2|C) \oplus O(2|C)$ where $O_R(2|C) = O(2) \oplus O(1, 1)$. Fermionic sector of $OSp_R(2; 2|C)$ is described by the complex $O(2)$ doublet of supercharges $S^i_{+\alpha}$ as follows

$$
\begin{aligned}
\{S^i_{+\alpha}, S^j_{+\beta}\} &= 4i\,\delta^{ij}(C\Gamma^{\bar\rho}_+)_{\alpha\beta}\,\mathbf{J}_{\bar\rho} + 2(CP_+)_{\alpha\beta}\,\epsilon^{ij}\mathbf{T}_+, \\
[\mathbf{J}_{\bar\mu}, S^i_{+\alpha}] &= \frac{1}{2}S^i_{+\beta}(\Gamma_{+\bar\mu})^\beta{}_\alpha, & [\mathbf{T}_+, S^i_{+\alpha}] &= i\,\epsilon^{ij}S^j_{+\alpha}, \quad (2.23)
\end{aligned}
$$

where $\mathbf{T}_+$ is the complex $O(2|C)$ generator. The fermionic sector of complex conjugate generators $(\mathbf{J}^\dagger, S^{i\dagger}_+, \mathbf{T}^\dagger_+) = (\mathbf{J}^\dagger, S^i_-, \mathbf{T}_-)$ of $\overline{OSp(2; 2|C)}$ satisfy the conjugate relations

$$
\begin{aligned}
\{S^i_{-\alpha}, S^j_{-\beta}\} &= -4i\,\delta^{ij}(C\Gamma^{\bar\rho}_-)_{\alpha\beta}\,\mathbf{J}^\dagger_{\bar\rho} + 2(CP_-)_{\alpha\beta}\,\epsilon^{ij}\mathbf{T}_-, \\
[\mathbf{J}^\dagger_{\bar\mu}, S^i_{-\alpha}] &= -\frac{1}{2}S^i_{-\beta}(\Gamma_{-\bar\mu})^\beta{}_\alpha, & [\mathbf{T}_-, S^i_{-\alpha}] &= i\,\epsilon^{ij}S^j_{-\alpha}. \quad (2.24)
\end{aligned}
$$

The generators $(\mathbf{J}, S^i_+, \mathbf{T}_+)$ and $(\mathbf{J}^\dagger, S^i_-, \mathbf{T}_-)$ are commuting. The realification of $OSp(2;2|C)$ is the real superalgebra $OSp_R(2;2|C)$ extending the Lorentz algebra (2.8) as follows

$$
\begin{aligned}
\{\mathcal{S}^i_\alpha, \mathcal{S}^j_\beta\} &= -\delta^{ij}(C\gamma^{\mu\nu})_{\alpha\beta}\mathcal{J}_{\mu\nu} + \epsilon^{ij}(C(\mathcal{T}_0 + \gamma_5\mathcal{T}_5))_{\alpha\beta}, \\
[\mathcal{J}_{\mu\nu}, S^i_\alpha] &= -\frac{i}{2}(S^i\gamma_{\mu\nu})_\alpha, \quad [\mathcal{T}_0, \mathcal{S}^i_\alpha] = i\epsilon^{ij}\mathcal{S}^j_\alpha, \quad [\mathcal{T}_5, \mathcal{S}^i_\alpha] = i\epsilon^{ij}(\mathcal{S}^i\gamma_5)_\alpha, \quad (2.25)
\end{aligned}
$$

where $\mathcal{S}^i_\alpha = S^i_{+\alpha} + S^i_{-\alpha}$ and $\mathbf{T}_+ = \mathcal{T}_0 + i\mathcal{T}_5$. Let us introduce the rescaled generators by the redefinitions (2.9) and

$$
\mathbf{S}^i_\alpha = \frac{1}{R}S^i_\alpha, \tag{2.26}
$$

$$
\mathbf{T}_0 = \frac{1}{R^{c_0}}\mathcal{T}_0, \qquad \mathbf{T}_5 = \frac{1}{R^{c_5}}\mathcal{T}_5. \tag{2.27}
$$

In terms of the generators defined by (2.9) and (2.26-2.27) the superalgebra $OSp_R(2;2|C)\oplus Sp(4)$ is described by the formulae (2.12) supplemented by the following relations;

$$
\begin{aligned}
\{\mathbf{S}^i_\alpha, \mathbf{S}^j_\beta\} &= -\delta^{ij}M^2(C\gamma^{\mu\nu})_{\alpha\beta}Z_{\mu\nu} - \frac{\alpha}{R^2}\delta^{ij}(C\gamma^{\mu\nu})_{\alpha\beta}M_{\mu\nu} \\
&\quad + \epsilon^{ij}\left(\frac{1}{R^{2-c_0}}C_{\alpha\beta}\mathbf{T}_0 - \frac{1}{R^{2-c_5}}(C\gamma_5)_{\alpha\beta}\mathbf{T}_5\right), \tag{2.28}
\end{aligned}
$$

$$
\begin{aligned}
[M_{\mu\nu}, \mathbf{S}^i_\alpha] &= -\frac{i}{2}(\mathbf{S}^i\gamma_{\mu\nu})_\alpha, \qquad [Z_{\mu\nu}, \mathbf{S}^i_\alpha] = -\frac{i\beta}{2M^2R^2}(\mathbf{S}^i\gamma_{\mu\nu})_\alpha, \\
[\mathbf{T}_0, \mathbf{S}^i_\alpha] &= \frac{i}{R^{c_0}}\epsilon^{ij}\mathbf{S}^j_\alpha, \qquad [\mathbf{T}_5, \mathbf{S}^i_\alpha] = \frac{i}{R^{c_5}}\epsilon^{ij}(\mathbf{S}^j\gamma_5)_\alpha. \tag{2.29}
\end{aligned}
$$

If $c_0 = c_5 = 2$ one gets in the contraction limit $R \to \infty$ the N=1 exotic version of Maxwell superalgebra with the basic superalgebra relation having the form (1.4) obtained from (2.28), and includes nonvanishing central charges $\mathbf{T}_0$ and $\mathbf{T}_5$. By choosing $2 > c_0, c_5 > 0$ these generators are decoupled in the contraction limit, and for $c_0 = c_5 = 0$ we obtain $\mathbf{T}_0$ and $\mathbf{T}_5$ as describing the Abelian $O(2)$ symmetry generators.

3 N-extended Maxwell superalgebras

In previous section we did see that the fermionic sector $(\mathbf{Q}_\alpha, \mathbf{\Sigma}_\alpha)$ of simple Maxwell algebra can be obtained by the contractions from the pair of $OSp(2;4)$ supercharges Q^i_α ($i = 1, 2$) or from the pair of supercharges $Q_\alpha \in OSp(1;4)$ and $S_\alpha \in OSp_R(1,2|C)$. It is interesting to see what variety of contractions can be obtained if $N > 1$, i.e. for the superalgebras (1.2) with $2N$ Weyl supercharges.

The extension to $N > 1$ of two types of contraction presented in Sect.2 is based on the observation that the standard Poincaré supercharges satisfying relation (1.3) and belonging to Maxwell superalgebra can be obtained by contraction in two ways

i) from n pairs of $OSp(2n;4)$ supercharges (see (2.11)) one gets $\mathbf{Q}^i_\alpha$ with $i = 1\ldots n$,

ii) from m pairs of supercharges of the superalgebra $OSp_R(m;2|C) \oplus OSp(m;4)$ (see (2.20)) providing m Poincaré supercharges $\mathbf{Q}^i_\alpha$, ($i = 1\ldots m$).

In general case we consider the contractions of $OSp_R(r; 2|C) \oplus OSp(k; 4)$ with $k + r = 2N$. The first type **i)** of contraction describes all Poincaré superalgebra generators only if $k = 2N$ - one gets $n = N$. The second way **ii)** of contracting describes the whole Poincaré supercharges sector only if $k = N$ - again we obtain N Poincaré supercharges. If $k \neq 0, N, 2N$ one should apply for supercharges both types of contractions **i)** and **ii)**.

We shall distinguish the following two separate cases;

a) $2N \geq k \geq N \geq r$ where $k + r = 2N$.

The supercharges in the superalgebra (1.2) can be described by $(Q_\alpha^i, Q_\alpha^{i'}, S_\alpha^i, (i = 1, ..., r, i' = r + 1, ..., k)$. From the r pairs (Q_α^i, S_α^i) one obtains r Poincaré supercharges via the second mechanism **ii)**; remaining even number $k - r = 2(k - N)$ of supercharges $Q^{i'}$ produce $k - N$ Poincaré supercharges as in **i)**. Concluding, we obtain in the contraction limit $r + (k - N) = N$ Poincaré supercharges satisfying the relation (1.3).

b) $2N \geq r \geq N \geq k$ where $k + r = 2N$.

We get the supercharges $(Q_\alpha^i, S_\alpha^i, S_\alpha^{i''})$, $(i = 1, ..., k, i'' = k + 1, ..., r)$ and we can obtain from the k pairs $(Q_\alpha^i, S_\alpha^i), (i = 1, ..., k)$, only k Poincaré supercharges by the second mechanism (see **ii)**). The remaining $r - k = 2(N - k)$ supercharges lead after contraction to the fermionic sector with exotic supercharges, satisfying the relation (1.4).

The case $k = N$ is described equally well by both cases **a)** and/or **b)**.

3.1   The superalgebras $OSp(k; 4)$ and $OSp_R(r; 2|C)$

The $OSp(k, 4)$ superalgebra extends supersymmetrically the $O(3, 2)$ algebra (2.4) as follows

$$
\begin{aligned}
\{\mathcal{Q}_\alpha^i, \mathcal{Q}_\beta^j\} &= -\delta^{ij}(C\Gamma^{\hat\mu\hat\nu})_{\alpha\beta}\mathcal{M}_{\hat\mu\hat\nu} + 2(C)_{\alpha\beta}\mathcal{B}^{ij}, \\
[\mathcal{M}_{\hat\mu\hat\nu}, \mathcal{Q}_\alpha^i] &= -\frac{i}{2}(\mathcal{Q}^i\Gamma_{\hat\mu\hat\nu})_\alpha, \qquad [\mathcal{B}^{ij}, \mathcal{Q}_\alpha^\ell] = -i\,\delta^{\ell[j}\,\mathcal{Q}_\alpha^{i]}, \\
[\mathcal{B}^{ij}, \mathcal{B}^{\ell m}] &= -i\,\delta^{\ell[j}\,\mathcal{B}^{i]m} + i\,\delta^{m[j}\,\mathcal{B}^{i]\ell}.
\end{aligned}
\tag{3.1}
$$

The supercharges $\mathcal{Q}_\alpha^i$, $(\alpha = 1, 2, 3, 4, i = 1, 2, ..., k)$ are real $SO(3, 2)$ spinors and $\mathcal{B}^{ij} = -\mathcal{B}^{ji}$ $(i, j = 1, 2, ..., k)$ are $O(k)$ generators. After 4+1 decomposition of the Lorentz indices $\hat\mu = (\mu, 4)$ the superalgebra (3.1) can be written as $D = 4$ super-AdS algebra, where $\mathcal{P}_\mu = \mathcal{M}_{\mu 4}$

$$
\begin{aligned}
[\mathcal{M}_{\mu\nu}, \mathcal{M}_{\rho\sigma}] &= -i\,\eta_{\rho[\nu}\mathcal{M}_{\mu]\sigma} + i\,\eta_{\sigma[\nu}\mathcal{M}_{\mu]\rho]}, \\
[\mathcal{P}_\mu, \mathcal{M}_{\rho\sigma}] &= -i\,\eta_{\mu[\rho}\mathcal{P}_{\sigma]}, \qquad [\mathcal{P}_\mu, \mathcal{P}_\nu] = -i\,\mathcal{M}_{\mu\nu}, \\
\{\mathcal{Q}_\alpha^i, \mathcal{Q}_\beta^j\} &= 2\,\delta^{ij}(C\gamma^\mu)_{\alpha\beta}\mathcal{P}_\mu - \delta^{ij}(C\gamma^{\mu\nu})_{\alpha\beta}\mathcal{M}_{\mu\nu} + 2(C)_{\alpha\beta}\mathcal{B}^{ij}, \\
[\mathcal{M}_{\mu\nu}, \mathcal{Q}_\alpha^i] &= -\frac{i}{2}(\mathcal{Q}^i\gamma_{\mu\nu})_\alpha, \qquad [\mathcal{P}_\mu, \mathcal{Q}_\alpha^i] = -\frac{i}{2}(\mathcal{Q}^i\gamma_\mu)_\alpha, \\
[\mathcal{B}^{ij}, \mathcal{Q}_\alpha^\ell] &= -i\,\delta^{\ell[j}\,\mathcal{Q}_\alpha^{i]}, \qquad [\mathcal{B}^{ij}, \mathcal{B}^{\ell m}] = -i\,\delta^{\ell[j}\,\mathcal{B}^{i]m} + i\,\delta^{m[j}\,\mathcal{B}^{i]\ell}.
\end{aligned}
\tag{3.2}
$$

The $OSp(r;2|C)$ superalgebra has the following form

$$
\begin{aligned}
[\mathbf{J}_{\bar\mu}, \mathbf{J}_{\bar\nu}] &= i\,\epsilon_{\bar\mu\bar\nu}{}^{\bar\rho}\mathbf{J}_{\bar\rho}, & [\mathbf{T}_+^{ij}, \mathbf{T}_+^{\ell m}] &= -i\,(\delta^{\ell[j}\,\mathbf{T}_+^{i]m} - \delta^{m[j}\,\mathbf{T}_+^{i]\ell}), \\
\{S_{+\alpha}^i, S_{+\beta}^j\} &= 2i\left(2\,\delta^{ij}(C\Gamma_+^{\bar\rho})_{\alpha\beta}\,\mathbf{J}_{\bar\rho} - i\,(CP_+)_{\alpha\beta}\,\mathbf{T}_+^{ij}\right), \\
[\mathbf{J}_{\bar\mu}, S_{+\alpha}^i] &= \frac{1}{2}\,S_{+\beta}^i\,(\Gamma_{+\bar\mu})^\beta{}_\alpha, & [\mathbf{T}_+^{ij}, S_{+\alpha}^\ell] &= -i\,\delta^{\ell[j}\,S_{+\alpha}^{i]},
\end{aligned} \tag{3.3}
$$

where $\mathbf{J}_{\bar\mu}, (\bar\mu = 0,1,2)$, $\mathbf{T}_+^{ij}, (i = 1,...,r)$ and $S_{+\alpha}^i$ are complex $SO(2,1)$, $O(r)$ and supersymmetry generators respectively. The complex conjugate generators $(\mathbf{J}, S_+^i, \mathbf{T}_+)^\dagger = (\mathbf{J}^\dagger, S_-^i, \mathbf{T}_-)$ describing the superalgebra $\overline{OSp(r,2|C)}$ satisfying the conjugate relations (for $r = 2$ see also (2.24)). One obtain finally the following real $OSp_R(r;2|C)$ superalgebra

$$
\begin{aligned}
[\mathcal{J}_{\mu\nu}, \mathcal{J}_{\rho\sigma}] &= -i\,\eta_{\rho[\nu}\mathcal{J}_{\mu]\sigma} + i\,\eta_{\sigma[\nu}\mathcal{J}_{\mu]\rho}, & [\mathcal{T}_0^{ij}, \mathcal{T}_0^{k\ell}] &= -i(\delta^{k[j}\mathcal{T}_0^{i]\ell} - \delta^{\ell[j}\mathcal{T}_0^{i]k}), \\
[\mathcal{T}_0^{ij}, \mathcal{T}_5^{k\ell}] &= -i(\delta^{k[j}\mathcal{T}_5^{i]\ell} - \delta^{\ell[j}\mathcal{T}_5^{i]k}), & [\mathcal{T}_5^{ij}, \mathcal{T}_5^{k\ell}] &= i(\delta^{k[j}\mathcal{T}_0^{i]\ell} - \delta^{\ell[j}\mathcal{T}_0^{i]k}), \\
\{\mathcal{S}_\alpha^i, \mathcal{S}_\beta^j\} &= -\delta^{ij}\,(C\Gamma^{\mu\nu})_{\alpha\beta}\mathcal{J}_{\mu\nu} + (C(\mathcal{T}_0^{ij} - \gamma_5\mathcal{T}_5^{ij}))_{\alpha\beta},
\end{aligned} \tag{3.4}
$$

$$
[\mathcal{J}_{\mu\nu}, \mathcal{S}_\alpha^i] = -\frac{i}{2}(\mathcal{S}^i\gamma_{\mu\nu})_\alpha, \quad [\mathcal{T}_0^{ij}, \mathcal{S}_\alpha^k] = -i\delta^{k[j}\,\mathcal{S}_\alpha^{i]}, \quad [\mathcal{T}_5^{ij}, \mathcal{S}_\alpha^k] = -i\delta^{k[j}\,(\mathcal{S}^{i]}\gamma_5)_\alpha, \tag{3.5}
$$

where the real generators $\mathcal{J}_{\mu\nu}, \mathcal{S}_\alpha^i, \mathcal{T}_0^{ij}, \mathcal{T}_5^{ij}$ describe respectively $SO(3,1)$, supersymmetry and the pair of internal $SO(r)$ generators that are related to the complex ones by the formula (2.7) and the relations $S_{+\alpha}^i + S_{-\alpha}^i = \mathcal{S}_\alpha^i$, $\mathbf{T}_+^{ij} = \frac{1}{2}(\mathcal{T}_0^{ij} + i\,\mathcal{T}_5^{ij})$.

3.2   Contraction of $OSp_R(r;2|C) \oplus OSp(k;4)$, $(k \geq r \geq 0)$

In this subsection we consider the case **a)** $(k \geq r)$. In order to obtain after contraction $R \to \infty$ the Maxwell algebra as bosonic subalgebra we rescale the generators $(\mathcal{M}_{\mu\nu}, \mathcal{P}_\mu, \mathcal{J}_{\mu\nu}) \in O(3,2) \oplus O(3,1)$ in accordance with the formulae (2.9) . In the contraction limit $(R \to \infty)$ we obtain the bosonic Maxwell algebra (1.1) for any value of $(\alpha, \beta = 1 - \alpha)$.

The generators of the internal symmetries $O(r|C) \oplus O(k)$, $(k \geq r)$ are split into three families,

$$
\begin{aligned}
&1) & (\mathcal{B}^{ij}, \mathcal{T}_0^{ij}, \mathcal{T}_5^{ij}) &\in O(r) \oplus O_R(r|C), & (i,j = 1,...,r), \\
&2) & \mathcal{B}^{i'j'} &\in O(k-r) = O(2(k-N)), & (i', j' = r+1,...,k), \\
&3) & \mathcal{B}^{ij'} &\in O(k)/(O(r) \oplus O(k-r)), & (i = 1,...,r,\ j' = r+1,...,k).
\end{aligned} \tag{3.6}
$$

In the first group of the generators (3.6) the diagonal subalgebra $O(r)$ remain unscaled

$$
\mathbf{B}_D^{ij} = \mathcal{B}^{ij} + \mathcal{T}_0^{ij} \tag{3.7}
$$

while the remaining ones are rescaled as [5]

$$
\mathbf{T}_0^{ij} = \frac{1}{R}\,\mathcal{T}_0^{ij}, \qquad \mathbf{T}_5^{ij} = \frac{1}{R}\,\mathcal{T}_5^{ij}. \tag{3.8}
$$

---

[5]In general case one can consider the rescaling $\mathbf{T}_0^{ij} = \frac{1}{R}((1-\alpha')\,\mathcal{T}_0^{ij} - \alpha'\,\mathcal{B}^{ij})$. It appears however that the contraction limit $R \to \infty$ will not depend on α', so we choose for simplicity $\alpha' = 0$.

The generators $\mathcal{B}^{i'j'} = \mathcal{B}_+^{i'j'} + \mathcal{B}_-^{i'j'} \in O(k-r)$ we decompose as follows

$$\mathcal{B}_-^{i'j'} \in U(k-N), \quad \mathcal{B}_+^{i'j'} \in O(2(k-N))/U(k-N), \tag{3.9}$$

where $2(k-r) \times 2(k-r)$ matrix of generators $\mathcal{B} = (\mathcal{B}^{i'j'})$ is decomposed as follows

$$\mathcal{B} = \begin{pmatrix} A_0 & S \\ -S & A_0 \end{pmatrix} + \begin{pmatrix} A_3 & A_1 \\ A_1 & -A_3 \end{pmatrix} = \mathcal{B}_- + \mathcal{B}_+, \tag{3.10}$$

where the matrices S and A_ℓ $(\ell = 1, 2, 3)$ satisfy $S^T = S$ and $A_\ell^T = -A_\ell$. If we introduce the anti-symmetric matrix $\Omega = \begin{pmatrix} 0 & 1_{k-N} \\ -1_{k-N} & 0 \end{pmatrix}$, the matrices $\mathcal{B}_\mp$ satisfy the relations $\Omega\,\mathcal{B}_\pm \pm \mathcal{B}_\pm\,\Omega = 0$. The generators $\mathcal{B}_\mp^{i'j'}$ we rescale as follows

$$\mathbf{B}_-^{i'j'} = \mathcal{B}_-^{i'j'}, \qquad \mathbf{B}_+^{i'j'} = \frac{1}{R}\,\mathcal{B}_+^{i'j'}. \tag{3.11}$$

The remaining internal symmetry generators $\mathcal{B}^{ij'}$ which occur in the anti-commutators of $\{\mathbf{Q}^i, \mathbf{Q}^{j'}\}$ and $\{\mathbf{Q}^i, \mathbf{\Sigma}^{j'}\}$ are rescaled by

$$\mathbf{B}^{ij'} = \frac{1}{R}\,\mathcal{B}^{ij'}. \tag{3.12}$$

If we perform the contraction $R \to \infty$ of the internal symmetry generators listed in (3.6) we obtain the pair of non-Abelian Lie algebras $O(r)$ (generators $\mathbf{B}_D^{ij}$) and $U(N-r)$ (generators $\mathbf{B}_-^{i'j'}$); remaining contracted generators $\mathbf{T}_0^{ij}, \mathbf{T}_5^{ij}, \mathbf{B}_+^{i'j'}, \mathbf{B}^{ij'}$ are becoming Abelian.

The k supercharges $\mathcal{Q}^i, (i = 1, ..., k)$ are split into r supercharges $\mathcal{Q}^i, (i = 1, ..., r)$ and remaining $k - r$ supercharges $\mathcal{Q}^{i'}, (i' = r+1, ..., k)$. The first ones are combined with r supercharges $\mathcal{S}^i, (i = 1, ..., r)$ to define

$$\mathbf{Q}_\alpha^i = \frac{1}{R^{1/2}}\,(\mathcal{Q}_\alpha^i + \mathcal{S}_\alpha^i), \qquad \mathbf{\Sigma}_\alpha^i = \frac{1}{R^{3/2}}\,\mathcal{S}_\alpha^i. \tag{3.13}$$

The remaining $\mathcal{Q}^{i'}, (i' = r+1, ..., k)$ are used in order to define

$$\mathbf{Q}_\alpha^{i'} = \frac{1}{R^{1/2}}\,\mathcal{Q}_\beta^{j'}\,\Pi^{+j'i'\beta}{}_\alpha, \qquad \mathbf{\Sigma}_\alpha^{i'} = \frac{1}{R^{3/2}}\,\mathcal{Q}_\beta^{j'}\,\Pi^{-j'i'\beta}{}_\alpha, \tag{3.14}$$

where $\Pi^\pm$ are the projection operators (we recall that $\gamma_5^2 = \Omega^2 = -1$),

$$\Pi^{\pm j'i'\beta}{}_\alpha = \frac{1}{2}(\delta^{j'i'}\delta^\beta{}_\alpha \pm \gamma_5{}^\beta{}_\alpha\Omega^{j'i'}), \qquad \Pi^\pm\Pi^\pm = \Pi^\pm, \quad \Pi^\pm\Pi^\mp = 0. \tag{3.15}$$

Supercharges $\mathbf{Q}^{i'}, \mathbf{\Sigma}^{i'}, (i' = r+1, ..., k)$ satisfy the identities

$$\mathbf{Q}^{i'} = \mathbf{Q}^{j'}\,\Pi^{+j'i'}, \qquad \mathbf{\Sigma}^{i'} = \mathbf{\Sigma}^{j'}\,\Pi^{-j'i'}, \tag{3.16}$$

but because $k - r = 2(k - N)$ the number of independent supercharge components is reduced to $k - N$ for both $\mathbf{Q}^{i'}$ and $\mathbf{\Sigma}^{i'}$.

10

One can write down the superalgebra $OSp_R(r; 2|C) \oplus OSp(k; 4)$ in terms of rescaled generators $(P_\mu, M_{\mu\nu}, Z_{\mu\nu}, \mathbf{Q}^i, \mathbf{Q}^{i'}, \mathbf{\Sigma}^i, \mathbf{\Sigma}^{i'}, \mathbf{B}_D^{ij}, \mathbf{T}_0^{ij}, \mathbf{T}_5^{ij}, \mathbf{B}_-^{i'j'}, \mathbf{B}_+^{i'j'}, \mathbf{B}^{ij'})$, (see (2.9), (3.7-3.8), (3.11-3.14)). If we perform the contraction limit $R \to \infty$ we obtain the following relations describing N-extended $(N > 1)$ Maxwell superalgebras,

$$
\begin{aligned}
[P_\mu, P_\nu] &= i\, M^2\, Z_{\mu\nu}, & [M_{\mu\nu}, M_{\rho\sigma}] &= -i\, \eta_{\rho[\nu} M_{\mu]\sigma} + i\, \eta_{\sigma[\nu} M_{\mu]\rho}, \\
[P_\mu, M_{\rho\sigma}] &= -i\, \eta_{\mu[\rho} P_{\sigma]}, & [M_{\mu\nu}, Z_{\rho\sigma}] &= -i\, \eta_{\rho[\nu} Z_{\mu]\sigma} + i\, \eta_{\sigma[\nu} Z_{\mu]\rho},
\end{aligned}
\tag{3.17}
$$

$$
\begin{aligned}
\left[P_\mu, \mathbf{Q}^i\right] &= \frac{i}{2} \mathbf{\Sigma}^i \gamma_\mu, & \left[M_{\mu\nu}, \mathbf{Q}^i\right] &= -\frac{i}{2} \mathbf{Q}^i \gamma_{\mu\nu}, & \left[M_{\mu\nu}, \mathbf{\Sigma}^i\right] &= -\frac{i}{2} \mathbf{\Sigma}^i \gamma_{\mu\nu}, \\
\left[P_\mu, \mathbf{Q}^{i'}\right] &= -\frac{i}{2} \mathbf{\Sigma}^{i'} \gamma_\mu, & \left[M_{\mu\nu}, \mathbf{Q}^{i'}\right] &= -\frac{i}{2} \mathbf{Q}^{i'} \gamma_{\mu\nu}, & \left[M_{\mu\nu}, \mathbf{\Sigma}^{i'}\right] &= -\frac{i}{2} \mathbf{\Sigma}^{i'} \gamma_{\mu\nu}, \\
\left[Z_{\mu\nu}, \mathbf{Q}^i\right] &= \left[Z_{\mu\nu}, \mathbf{Q}^{i'}\right] = \left[Z_{\mu\nu}, \mathbf{\Sigma}^i\right] = \left[Z_{\mu\nu}, \mathbf{\Sigma}^{i'}\right] = 0,
\end{aligned}
\tag{3.18}
$$

$$
\begin{aligned}
\{\mathbf{Q}^i{}_\alpha, \mathbf{Q}^j{}_\beta\} &= 2\, \delta^{ij} (C\gamma^\mu)_{\alpha\beta} P_\mu - C_{\alpha\beta} \mathbf{T}_0^{ij} - (C\gamma_5)_{\alpha\beta} \mathbf{T}_5^{ij}, \\
\{\mathbf{Q}^i{}_\alpha, \mathbf{\Sigma}^j{}_\beta\} &= -\delta^{ij} (C\gamma^{\mu\nu})_{\alpha\beta} M^2\, Z_{\mu\nu}, \\
\{\mathbf{Q}^i{}_\alpha, \mathbf{Q}^{j'}{}_\beta\} &= 2\, (C\mathbf{B}^{i,\ell'} \Pi^{+\ell'j'})_{\alpha\beta}. \\
\{\mathbf{Q}^{i'}{}_\alpha, \mathbf{Q}^{j'}{}_\beta\} &= 2(C\gamma^\mu \Pi^+)^{i'j'}_{\alpha\beta} P_\mu + 2\, (C\mathbf{B}_+ \Pi^+)^{i'j'}_{\alpha\beta}, \\
\{\mathbf{Q}^{i'}{}_\alpha, \mathbf{\Sigma}^{j'}{}_\beta\} &= (C\gamma^{\mu\nu} \Pi^-)^{i'j'}_{\alpha\beta} M^2\, Z_{\mu\nu}.
\end{aligned}
\tag{3.19}
$$

The internal symmetry sector containing the generators $\mathbf{B}_D^{ij} \in O(k)$ and $\mathbf{B}_-^{i'j'} \in U(N-k)$ is described by the following non-vanishing commutators,

$$
\begin{aligned}
\left[\mathbf{B}_D^{ij}, \mathbf{Q}_\alpha^\ell\right] &= -i\delta^{\ell[j} \mathbf{Q}_\alpha^{i]}, & \left[\mathbf{B}_D^{ij}, \mathbf{\Sigma}_\alpha^\ell\right] &= -i\delta^{\ell[j} \mathbf{\Sigma}_\alpha^{i]}, \\
\left[\mathbf{B}_D^{ij}, \mathbf{B}_D^{\ell m}\right] &= -i(\delta^{\ell[j} \mathbf{B}_D^{i]m} - \delta^{m[j} \mathbf{B}_D^{i]\ell}), & \left[\mathbf{B}_D^{ij}, \mathbf{T}_0^{\ell m}\right] &= -i(\delta^{\ell[j} \mathbf{T}_0^{i]m} - \delta^{m[j} \mathbf{T}_0^{i]\ell}), \\
\left[\mathbf{B}_D^{ij}, \mathbf{T}_5^{\ell m}\right] &= -i(\delta^{\ell[j} \mathbf{T}_5^{i],m} - \delta^{m[j} \mathbf{T}_5^{i]\ell}), & \left[\mathbf{B}_D^{ij}, \mathbf{B}^{\ell m'}\right] &= -i\delta^{\ell[j} \mathbf{B}^{i]m'},
\end{aligned}
\tag{3.20}
$$

$$
\begin{aligned}
\left[\mathbf{T}_0^{ij}, \mathbf{Q}_\alpha^\ell\right] &= -i\delta^{\ell[j} \mathbf{\Sigma}_\alpha^{i]}, & \left[\mathbf{T}_5^{ij}, \mathbf{Q}_\alpha^\ell\right] &= -i\delta^{\ell[j} \mathbf{\Sigma}_\alpha^{i]} \gamma_5, & \left[\mathbf{B}^{ij'}, \mathbf{Q}_\alpha^\ell\right] &= i\delta^{i\ell} \mathbf{\Sigma}_\alpha^{j'}, \\
\left[\mathbf{B}^{ij'}, \mathbf{Q}_\alpha^{\ell'}\right] &= i(\mathbf{\Sigma}^i \Pi^{+j'\ell'})_\alpha, & \left[\mathbf{B}_-^{i'j'}, \mathbf{Q}_\alpha^\ell\right] &= -i\mathbf{Q}^{[i'} \Pi^{+j']\ell'}, & \left[\mathbf{B}_+^{i'j'}, \mathbf{Q}_\alpha^{\ell'}\right] &= -i\mathbf{\Sigma}^{[i'} \Pi^{+j']\ell'}, \\
\left[\mathbf{B}_-^{i'j'}, \mathbf{\Sigma}_\alpha^{\ell'}\right] &= -i\mathbf{\Sigma}^{[i'} \Pi^{-j']\ell'}, & \left[\mathbf{B}_-^{i'j'}, \mathbf{B}^{\ell m'}\right] &= i\, \rho_-^{ij,mn} \mathbf{B}^{\ell n'}.
\end{aligned}
\tag{3.21}
$$

We see from (3.19) that all fermionic generators $\mathbf{Q}_\alpha^i$ and $\mathbf{Q}_\alpha^{i'}$ are the Poincaré super-charges.

3.3 Contraction of $OSp_R(r; 2|C) \oplus OSp(k; 4)$, $(r \geq k \geq 0)$

The generators of the internal symmetries $O(r|C) \oplus O(k)$, $(k \leq r)$ are split into the following three families,

1) $(\mathcal{B}^{ij}, \mathcal{T}_0^{ij}, \mathcal{T}_5^{ij}) \in O(k) \oplus O_R(k|C)$, $(i, j = 1, ..., k)$,

2) $(\mathcal{T}_0^{i''j''}, \mathcal{T}_5^{i''j''}) \in O_R(r-k|C) = O_R(2(r-N)|C)$, $(i'', j'' = k+1, ..., r)$,

3) $(\mathcal{T}_0^{ij''}, \mathcal{T}_5^{ij''}) \in O_R(r|C)/(O_R(k|C) \oplus O_R(r-k|C))$, $(i = 1, ..., k, j'' = k+1, ..., r)$.

In the first group we rescale the generators $\mathbf{B}^{ij}, \mathbf{T}_0^{ij}, \mathbf{T}_5^{ij}$ as in previous subsection (see (3.7-3.8)). The generators $(\mathcal{T}_0^{i''j''}, \mathcal{T}_5^{i''j''})$ and $(\mathcal{T}_0^{ij''}, \mathcal{T}_5^{ij''})$ are rescaled as follows

$$\mathbf{T}_0^{i''j''} = \frac{1}{R^2}\mathcal{T}_0^{i''j''}, \quad \mathbf{T}_5^{i''j''} = \frac{1}{R^2}\mathcal{T}_5^{i''j''}, \quad \mathbf{T}_0^{ij''} = \frac{1}{R^{3/2}}\mathcal{T}_0^{ij''}, \quad \mathbf{T}_5^{ij''} = \frac{1}{R^{3/2}}\mathcal{T}_5^{ij''}. \quad (3.22)$$

After the contraction limit $R \to \infty$ only the generators $\mathbf{B}_D^{ij}$ describe the non-Abelian $O(k)$ generators; remaining generators $(\mathbf{T}_0^{ij}, \mathbf{T}_5^{ij}, \mathbf{T}_0^{ij''}, \mathbf{T}_5^{ij''}, \mathbf{T}_0^{i''j''}, \mathbf{T}_5^{i''j''})$ are becoming Abelian.

The r supercharges $\mathcal{S}^i, (i = 1, ..., r)$ are split into k supercharges $\mathcal{S}^i, (i = 1, ..., k)$ and remaining $r - k$ supercharges $\mathcal{S}^{i''}, (i'' = k + 1, ..., r)$. The first ones are combined with k supercharges $\mathcal{Q}^i, (i = 1, ..., k)$ to define

$$\mathbf{Q}_\alpha^i = \frac{1}{R^{1/2}}(\mathcal{Q}_\alpha^i + \mathcal{S}_\alpha^i), \qquad \mathbf{\Sigma}_\alpha^i = \frac{1}{R^{3/2}}\mathcal{S}_\alpha^i. \quad (3.23)$$

The remaining generators $\mathcal{S}^{i''}, (i'' = k + 1, ..., r)$ are rescaled as

$$\mathbf{S}^{i''} = \frac{1}{R}\mathcal{S}^{i''}. \quad (3.24)$$

In the contraction limit $R \to \infty$ we obtain besides the Maxwell algebra (1.1) the following set of algebraic relations

$$
\begin{aligned}
\left[M_{\mu\nu}, \mathbf{Q}^i\right] &= -\frac{i}{2}\mathbf{Q}^i\gamma_{\mu\nu}, \quad \left[M_{\mu\nu}, \mathbf{\Sigma}^i\right] = -\frac{i}{2}\mathbf{\Sigma}^i\gamma_{\mu\nu}, \quad \left[M_{\mu\nu}, \mathbf{S}^{i''}\right] = -\frac{i}{2}\mathbf{S}^{i''}\gamma_{\mu\nu}, \\
\left[P_\mu, \mathbf{Q}^i\right] &= \frac{i}{2}\mathbf{\Sigma}^i\gamma_\mu, \quad \left[\mathbf{T}_0^{ij}, \mathbf{Q}_\alpha^\ell\right] = -i\delta^{\ell[j}\mathbf{\Sigma}_\alpha^{i]}, \quad \left[\mathbf{T}_5^{ij}, \mathbf{Q}_\alpha^\ell\right] = -i\delta^{\ell[j}\mathbf{\Sigma}_\alpha^{i]}\gamma_5. \quad (3.25)
\end{aligned}
$$

$$
\begin{aligned}
\{\mathbf{Q}_\alpha^i, \mathbf{Q}_\beta^j\} &= 2\,\delta^{ij}(C\gamma^\mu)_{\alpha\beta}P_\mu - C_{\alpha\beta}\mathbf{T}_0^{ij} - (C\gamma_5)_{\alpha\beta}\mathbf{T}_5^{ij}, \\
\{\mathbf{Q}_\alpha^i, \mathbf{\Sigma}_\beta^j\} &= -\delta^{ij}(C\gamma^{\mu\nu})_{\alpha\beta}M^2 Z_{\mu\nu}, \\
\{\mathbf{Q}_\alpha^i, \mathbf{S}_\beta^{j''}\} &= (C(\mathbf{T}_0^{ij''} - \gamma_5\mathbf{T}_5^{ij''}))_{\alpha\beta}, \\
\{\mathbf{S}_\alpha^{i''}, \mathbf{S}_\beta^{j''}\} &= -\delta^{i''j''}(C\gamma^{\mu\nu})_{\alpha\beta}M^2 Z_{\mu\nu} + (C_{\alpha\beta}\mathbf{T}_0^{i''j''} - (C\gamma_5)_{\alpha\beta}\mathbf{T}_5^{i''j''}). \quad (3.26)
\end{aligned}
$$

$$
\begin{aligned}
\left[\mathbf{B}_D^{ij}, \mathbf{Q}_\alpha^\ell\right] &= -i\delta^{\ell[j}\mathbf{Q}_\alpha^{i]}, \quad \left[\mathbf{B}_D^{ij}, \mathbf{\Sigma}_\alpha^\ell\right] = -i\delta^{\ell[j}\mathbf{\Sigma}_\alpha^{i]}, \quad \left[\mathbf{B}_D^{ij}, \mathbf{B}_D^{\ell m}\right] = -i(\delta^{\ell[j}\mathbf{B}_D^{i]m} - \delta^{m[j}\mathbf{B}_D^{i]\ell}), \\
\left[\mathbf{B}_D^{ij}, \mathbf{T}_0^{\ell m}\right] &= -i(\delta^{\ell[j}\mathbf{T}_0^{i],m} - \delta^{m[j}\mathbf{T}_0^{i]\ell}), \quad \left[\mathbf{B}_D^{ij}, \mathbf{T}_5^{\ell m}\right] = -i(\delta^{\ell[j}\mathbf{T}_5^{i],m} - \delta^{m[j}\mathbf{T}_5^{i]\ell}), \\
\left[\mathbf{B}_D^{ij}, \mathbf{T}_0^{\ell m''}\right] &= -i\delta^{\ell[j}\mathbf{T}_0^{i]m''}, \quad \left[\mathbf{B}_D^{ij}, \mathbf{T}_5^{\ell m''}\right] = -i\delta^{\ell[j}\mathbf{T}_5^{i]m''}. \quad (3.27)
\end{aligned}
$$

We see from relations (3.26) that the Maxwell superalgebras derived in this subsection have hybrid structure: the anti-commutators of k Weyl supercharges $\mathbf{Q}^i$ as in Poincaré SUSY algebra describe the fourmomenta generators P_μ (see (1.3)), but remaining $r - k$ supercharges $\mathbf{S}^{i''}$ supersymmetrizing the tensorial generators $Z_{\mu\nu}$ describe the exotic sector of N-extended Maxwell superalgebra with the SUSY relations (1.4) .

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4   Discussion

The aim of this paper is to provide a large class of superalgebraic structures describing N-extended Maxwell superalgebra. In Sect.2 we considered simple Maxwell superalgebras ($N = 1$); the extended case ($N > 1$) is described in Sect.3.

For arbitrary N we can considered the contractions of $2N$ different superalgebras described by (1.2) with $k = 1, 2, ..., 2N$. The case $k = 0$ as it was argued in introduction and in subsections 2.4 and 3.3 provides only the purely exotic supersymmetrization of Maxwell algebra with all supercharges satisfying the relations (1.4). If $N \leq k \leq 2N$ we obtain the Maxwell superalgebra with N "standard" SUSY relations (1.3); if $0 \leq k < N$ one obtains a hybrid Maxwell superalgebra, with k standard Poincaré supercharges and $r - k$ supercharges satisfying exotic SUSY relation (1.4). At present it is not known whether the exotic SUSY relation (1.4) can provide some dynamical models with physically interesting consequences but for completeness of our algebraic considerations we included as well the extended Maxwell superalgebras with exotic supercharges.

The contractions considered in this paper can be further applied to the description of various Maxwell supergravity models. For example in order to obtain $N = 1$ Maxwell supergravity one should consider firstly the gauge fields on $O(3, 1) \oplus OSp(2, 4)$ or $OSp_R(1, 2|C) \oplus OSp(1, 4)$ superalgebras and then introduce the gauge formulations of deformed Maxwell supergravity models. In the following step one can look for the deformed Maxwell supergravity actions with finite contraction limits defined in accordance with the contraction prescription in Sect.2.2 and 2.3. In general case one can consider the gauge theories on superalgebras (1.2) and study by their suitable contractions the possible new extended Maxwell supergravity models.

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