An experimental review on heavy flavor v_2 in heavy-ion collision

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For over a decade now, the primary purpose of relativistic heavy–ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) has been to study the properties of QCD matter under extreme conditions – high temperature and high density. The heavy–ion experiments at both RHIC and LHC have recorded a wealth of data in p+p, p+Pb, d+Au, Cu+Cu, Cu+Au, Au+Au, Pb+Pb and U+U collisions at energies ranging from $\sqrt{s_{NN}} = 7.7$ GeV to 7 TeV. Heavy quarks are considered good probe to study the QCD matter created in relativistic collisions due to their very large mass and other unique properties. A precise measurement of various properties of heavy flavor hadrons provides an insight into the fundamental properties of the hot and dense medium created in these nucleus–nucleus collisions, such as transport coefficient and thermalization and hadronization mechanisms. The main focus of this paper is to present a review on the measurements of azimuthal anisotropy of heavy flavor hadrons and to outline the scientific opportunities in this sector due to future detector upgrade. We will mainly discuss the elliptic flow of open charmed meson (D-meson), J/ψ and leptons from heavy flavor decay at RHIC and LHC energy.

I. INTRODUCTION

In the standard model of particle physics, the strong force is described by the theory of Quantum Chromodynamics (QCD). At ordinary temperatures or densities this force just confines the quarks into hadrons. At sufficiently high temperature and/or high baryon density, Lattice QCD predicts a transition form hadronic matter to deconfined partonic matter [1–4]. Phase diagram of QCD is full of puzzles and surprises. Experimentally it is possible to create very high temperature and high dense states of nuclear matter, by colliding two heavy nuclei at ultra-relativistic speed, which would contain asymptotically free quarks and gluons. The Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) was built to study the properties of Quark-Gluon Plasma (a state of matter believed to exist just after the Big Bang) produces during collision of heavy ions at ultra-relativistic speed. The life-time of Quark-Gluon Plasma (QGP) is very short (∼5-10 fm/c), hence direct detection of QGP is not possible. Therefore, one has to rely on indirect measurement using suitable probe. Heavy quarks production in relativistic heavy-ion collisions provides unique probes of QGP.

In relativistic heavy ion collisions, heavy quarks (c, b) are produced on a short time scale (∼0.08 fm/c for $c\bar{c}$ production) in the initial hard partonic scatterings during the early stages of nucleus-nucleus collisions [5]. The probability of thermal production of heavy-quark pairs is small in the high temperature QGP. The interactions of heavy quarks are sensitive to medium dynamics. They decouple early in the evolution of QGP due to their large masses, thereby

preserving the information from the system at early stage [6–13]. While traversing the hot and dense matter produced in nucleus–nucleus collisions, hard partons (partons with high transverse momentum p_T) produced in the early stages of the collision lose energy dominantly due to multiple scatterings and radiative energy loss. Hence, become quenched. Theoretical models predict that the mechanism as well as average energy loss will be different for heavy quarks compared to light quarks [14–16]. Therefore, high p_T charmed mesons (D^0, $D^{\pm}$, D^*, $D_S^{\pm}$ etc.) will show different suppression with respect to light mesons (π, K, K_S^0, etc.). In contrast, measurements of heavy flavor decay electrons at RHIC and charm hadrons at the LHC have shown significant suppression at high transverse momentum, p_T, similar to that of light hadrons for central collisions. Therefore, a complete understanding of the energy loss mechanisms in the QGP medium requires a systematic and precise measurements of the properties of various hadrons carrying different quark flavors at RHIC and the LHC. The dependence of the partonic energy loss on the in-medium path length is expected to be different for different energy loss mechanism. It is suggested that low-momentum heavy quarks could undergo hadronization both via fragmentation in the vacuum and recombination with other quarks from the medium [17]. Azimuthal anisotropy measurements of the production of heavy–flavor hadron with respect to the reaction plane can be very useful in addressing these questions.

Elliptic flow (v_2) measured in heavy-ion collisions is believed to arise due to the pressure gradient developed when two nuclei collide at nonzero impact parameters followed by subsequent interactions among the constituents [18–23]. The elliptic flow parameter is defined as the 2nd Fourier coefficient, v_2, of the particle distributions in emission azimuthal angle (ϕ) with respect to the reaction plane angle (Ψ) [24–30]

$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos(2(\phi - \Psi)).\tag{1}$$

For a given rapidity window the second coefficient is given by

$$v_2 = \langle \cos(2(\phi - \Psi)) \rangle = \langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \rangle,\tag{2}$$

where p_x and p_y are the x and y components of the particle momenta. At small transverse momentum, p_T, a large v_2 is considered to be an evidence for the collective hydrodynamical expansion of the medium. Positive v_2, if observed at very high p_T is expected to be due to path–length dependent energy loss by hard partons. Unlike light quarks and gluons, which can be produced or annihilated during the entire evolution of the medium, heavy quarks are expected to be produced mainly in initial hard scattering processes and their annihilation rate is small. Therefore, for all p_T, the final state heavy-flavor hadrons originate from heavy quarks that have experienced each stage of the system evolution. The paper is organized in the following way. Section II describes sensitivity of heavy flavor hadron as probe of QCD medium. In Sec. III, elliptic flow of heavy-flavor decay electron are briefly discussed. Section IV and V describe elliptic flow of open charmed meson and J/ψ, respectively, measured at RHIC and LHC. Comparisons between model and data are also presented in Sec. IV and V. Finally, we summarize in Sec. VI .

II.   ELLIPTIC FLOW OF HEAVY FLAVOR AS A SENSITIVE PROBE

We have recently studied the elliptic flow of open charm mesons, using quark coalescence as a mechanism of hadronization within the framework of a multi-phase transport model (AMPT) [6]. This study includes effect of

TABLE I: Values of η_s/s for different values of α_s and μ, keeping $\sigma_{PP} = 10$ mb for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

Specific viscosity, η_s/s	QCD coupling constant, α_s	Screening mass, μ (in fm^{-1})
0.08	0.47	1.77
0.18	0.23	0.88

partonic interaction cross-section, QCD coupling constant and specific viscosity on elliptic flow of open charm mesons within the transport model approach. The AMPT model is a hybrid transport model [31]. It uses the same initial conditions as in HIJING. In the AMPT model, the value of parton parton scattering cross-section, σ_{PP}, is calculated by

$$\sigma_{PP} \approx \frac{9\pi\alpha_s^2}{2\mu^2}, \tag{3}$$

where α_s and μ are the QCD coupling constant and screening mass respectively. Using the framework of AMPT model one can study the effect of specific viscosity on elliptic flow of hadrons. For a system of massless quarks and gluons at temperature T ($T = 378$ MeV at RHIC energy in AMPT [32]), the specific viscosity is given by [32]

$$\frac{\eta_s}{s} \approx \frac{3\pi}{40\alpha_s^2} \frac{1}{\left(9 + \frac{\mu^2}{T^2}\right)\ln\left(\frac{18+\mu^2/T^2}{\mu^2/T^2}\right) - 18}. \tag{4}$$

Hadronization of heavy quarks are not implemented in AMPT model. It only gives phase space information heavy quarks at freeze-out. We have implemented quark coalescence mechanism to form open charm mesons using phase space information of quarks available from AMPT model. Within the framework of coalescence mechanism [33], the probability of producing a hadron from a soup of partons is determined by the overlap of the phase space distribution of partons at freeze-out with the parton Wigner phase space function inside the hadron. The Wigner phase space function for quarks inside a meson is obtained from its constituent quark wave function [34]

$$\begin{aligned}\rho^W(\mathbf{r}, \mathbf{k}) &= \int \psi\left(\mathbf{r} + \frac{\mathbf{R}}{2}\right)\psi^\star\left(\mathbf{r} - \frac{\mathbf{R}}{2}\right)\exp(-i\mathbf{k}\cdot\mathbf{R})d^3\mathbf{R} \\ &= 8\exp(-\frac{r^2}{\sigma^2} - \sigma^2 k^2)\end{aligned} \tag{5}$$

where the relative momentum between the two quarks is $\mathbf{k} = (\mathbf{k}_1 - \mathbf{k}_2)/2$ and the quark wave function is given by spherical harmonic oscillator described as

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{(\pi\sigma^2)^{3/4}}\exp\left[\frac{-r^2}{2\sigma^2}\right] \tag{6}$$

with $\mathbf{r} = (\mathbf{r}_1 - \mathbf{r}_2)$ being the relative coordinate and σ is the size parameter related to the root mean square radius as $\langle r^2 \rangle = (3/8)^{1/2}\sigma$. We have taken $\sigma = 0.47$ fm^2 from Ref. [34].

We have used two different value of η_s/s e.g. 0.08 and 0.18 by tuning input parameters of AMPT model keeping parton-parton interaction cross-section equal to 10 mb. The values of α_s and μ for different value of η_s/s are shown in table I. Fig. 1 shows ratio between v_2 for $\eta_s/s = 0.08$ and for $\eta_s/s = 0.18$ as function of p_T for 10-40% central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The red solid and open blue circles represents results for charged hadrons and D^0 respectively. We can see that v_2 decreases with increase in specific viscosity for both D^0 and charged hadrons .
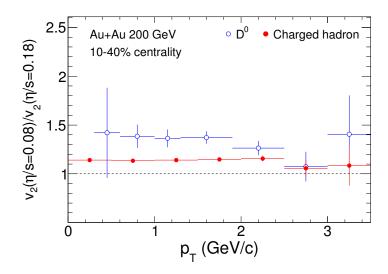
FIG. 1: (Color online) Ratio between v_2 for $\eta_s/s = 0.08$ and for $\eta_s/s = 0.18$ as a function of p_T in Au+Au collisions at $\sqrt{s_{NN}}$ = 200 GeV for charged hadrons and D^0 for 10–40% central collisions.

This is consistent with the interpretation that increased sheer viscosity reduces transverse expansion due to increased interactions and hence, reduces v_2. We can also see that the change in v_2 for charged hadrons is $\sim 15\%$, whereas for D^0 it lies between $30 - 40\%$ for $p_T < 2.0$ GeV/c. Therefore, the elliptic flow of open charmed meson is more sensitive to viscous properties of the QGP medium compared to light hadrons. Comparison between our calculation and data has been discussed in Section IV.

III.   ELLIPTIC FLOW OF HEAVY-FLAVOR DECAY ELECTRON

Measurement of electrons from semi-leptonic decay of heavy-flavor hadrons (also called non-photonic electrons, NPE) is widely used to study heavy-flavor production in high-energy collisions. These NPE give the direction of the mother D (B) mesons, especially when electron $p_T > 1.5$ (3) GeV/c. Thus, v_2 of NPE serves as a proxy for v_2 of heavy quarks. Systematic measurements of the nuclear modification factor (R_{AA} and R_{pA}) and the elliptic flow coefficient (v_2) of heavy-flavor decay electrons were performed at RHIC and LHC energies. Fig. 2 shows the azimuthal anisotropy of NPE as a function of p_T at $\sqrt{s_{NN}} = 39, 62.4$ and 200 GeV as measured by STAR and PHENIX experiments [35, 36]. A comparison between different methods for measurement of v_2 has been shown. The different methods show different sensitivity to non-flow and flow fluctuation. Non-zero positive v_2 has been observed for all methods. The increase of v_2 with p_T for $p_T > 4$ GeV/c could be due an effect of jet-like correlations as the non-flow correlation, which is estimated from p+p collision, is of the similar order with measured v_2 in Au+Au collision at high p_T. At 39 and 62.4 GeV, v_2 is consistent with zero as shown in Fig. 2(b). A very high precision measurement is required at 39 and 62.4 GeV, to understand NPE v_2 at these energies.

The nuclear modification factors (R_{AA}) and elliptic flow of NPE in Pb+Pb collisions at 2.76 TeV is shown in Fig. 3 [37]. A finite positive v_2 of NPE is observed for $p_T < 6$ GeV/c in Pb+Pb collision at 2.76 TeV, quite similar
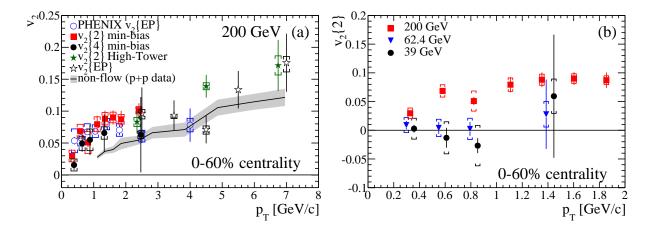
FIG. 2: (Color online) (a) Comparison of azimuthal anisotropy of NPE at $\sqrt{s_{NN}} = 200$ GeV measured by PHENIX [36] and STAR [35]. (b) NPE v_2 using two particle cumulant method at 200 and 62.4 and 39 GeV. The error bars represent the statistical uncertainty, and the brackets represent the systematic uncertainties.
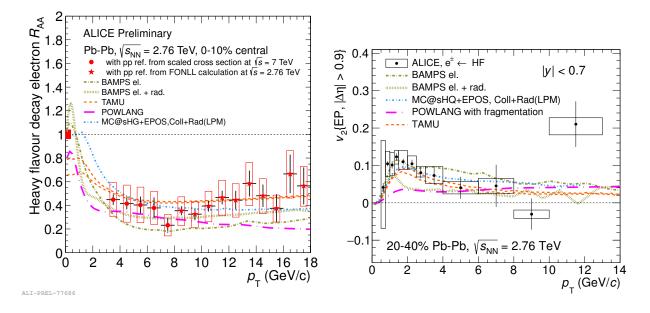

FIG. 3: (Color online) Measured v_2 and R_{AA} of NPE in Pb+Pb collision at 2.76 TeV [37–39]. Theoretical prediction for v_2 and R_{AA} of NPE are shown by lines [40–43].

to Au+Au collision at 200 GeV at RHIC. Large positive v_2 of NPE at low p_T might indicate that charm quarks participate in the collective expansion of the dense and hot QGP. Also, a strong suppression of yield of NPE is observed for $p_T > 3$ GeV/c in 0-10% most central Pb+Pb collisions.

The results from the models calculations [40–42], that include parton energy loss in the hot and dense QCD medium, are shown as lines for both v_2 and R_{AA} in Fig. 3. The simultaneous description of the measured v_2 and R_{AA} is challenging for models. BAMPS [40] gives a good description of NPE v_2 but predicts a larger in-medium suppression than measured. In BAMPS approach, heavy quarks are transported through the medium while

undergoing collisional and radiative energy loss. The prediction from POWLANG [42] describes the NPE R_{AA} but their calculation underestimates NPE v_2. In POWLANG, heavy quarks are transported following a Langevin approach and consider collisional energy loss only. The prediction from Rapp et al [41] (TAMU) and MC@sHQ+EPOS, Coll+Rad(LPM) [43] describes the NPE R_{AA} and v_2 reasonably well. Model calculation by Rapp et al includes in-medium resonance scattering and coalescence of heavy quarks in the medium. The MC@sHG+EPOS model includes radiative and collisional energy loss in an expanding medium based on the EPOS model.

IV. ELLIPTIC FLOW OF OPEN CHARMED MESON

A. Available Experimental Data

The elliptic flow of D mesons at mid-rapidity ($|y| < 0.8$) has been measured p_T differentially in Pb-Pb collisions by the ALICE at LHC [44]. The transverse momentum dependence of the v_2 of D^0, D^+ and D^{*+} mesons in the 30-50% collisions centrality at $\sqrt{s_{NN}} = 2.76$ TeV are shown in Fig. 4. Measurements were done using three different methods, namely event plane, scalar product and two-particle cumulant methods. Results from event plane method is shown in the left column of Fig. 4. The central and right panels of the Fig. 4 show the v_2 results obtained with the scalar product and two-particle cumulant methods respectively. The event plane is estimated from TPC tracks within $|\eta| < 0.8$. For the other methods, TPC tracks in $|\eta| < 0.8$ were used as reference particle. The elliptic flow of D^0, D^+ and D^{*+} mesons are consistent within uncertainties. At very high p_T ($p_T > 12$ GeV/c), v_2 is consistent with zero within the large statistical uncertainties. In the p_T range between $2 < p_T < 6$ GeV/c, the measured v_2 is found to be larger than zero with 5.7 σ significance. It suggests that low p_T charm quarks possibly participate in the collective expansion of the medium. However, the possibility that the observed D-meson v_2 is completely due to the contribution from light-quark in a scenario with hadronization via recombination cannot be ruled out. We need high precision data at low p_T and more theoretical understanding about the charm quark hadronization to understand the origin of collectivity of measured D-mesons v_2.

The p_T dependence of D^0 v_2 in the three centrality classes 0-10%, 10-30% and 30-50% are presented in Fig. 5 [44]. v_2 of charged hadrons are also shown for comparison [48]. Both the measurements are done with the event plane method. For these three centrality classes, the D^0 meson v_2 is comparable in magnitude to that of inclusive charged hadrons. These results indicate that the interactions with the medium constituents transfer information of the azimuthal anisotropy of the system to the charmed particles.

The STAR experiment at RHIC has also reported the first preliminary results of $D^0(p_T)$ v_2 at mid-rapidity ($|y| < 1.0$) in Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV using newly installed Heavy Flavor Tracker (HFT) [45, 46]. The D-meson v_2 measured by STAR in minimum bias (0-80%) Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV is shown in left panel of Fig. 6 [45]. Measurements are done at mid-rapidity ($|y| < 1.0$). The blue and black data points are the D^0 v_2 measured using two-particle correlation and event plane method respectively. Results from both the method are consistent within statistical uncertainty. The $D^{\pm}$ v_2 is shown by red symbol and calculated using event plane method.
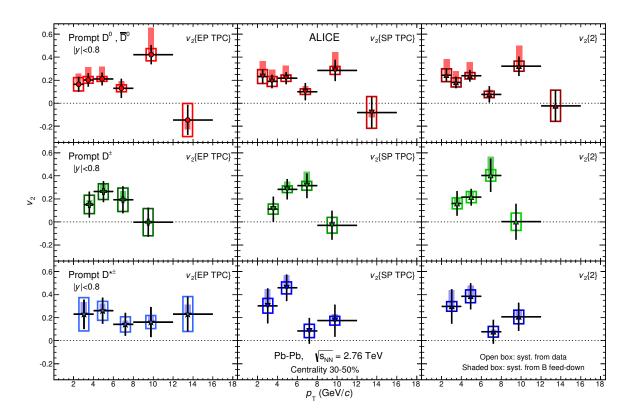
FIG. 4: (Color online) Elliptic flow as a function of p_T in the 30-50% centrality bin, for D^0, D^+ and D^{*+} mesons with the event plane, scalar product and two-particle cumulant methods [44]. The vertical error bars represent the statistical uncertainty, the open boxes are the systematic uncertainties.

D^0 azimuthal anisotropy is non-zero for $2 < p_T < 5$ GeV/c. The right panel of Fig. 6 shows the comparison of D^0 v_2 with other mesons species (K_S^0 and ϕ). It seems that D^0 v_2 for $p_T < 4$ GeV/c is systematically lower than K_S^0 [49] and ϕ [50], but one should be very careful while comparing different particle species for a wide centrality bin e.g. 0-80% centrality bin as the production of heavy open charmed meson are more biased towards central collisions than light hadrons like K_S^0 and ϕ [6].

B. Model Comparisons at LHC Energy

Various observables are compared to theoretical calculations to understand the physical mechanism behind the measurements. In this section, we will discuss the most recent theoretical baseline calculations for the elliptic flow of open charm meson. Simultaneous description of the measured v_2 and R_{AA} by theoretical model is a challenging job and is an open issue [51–59]. In Fig. 7 the v_2 (in 30-50% central collisions) and R_{AA} (in 0-20% central collisions) of D mesons (average of D^0, D^+, and D^{*+}) in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV is shown and compared to selected model predictions.
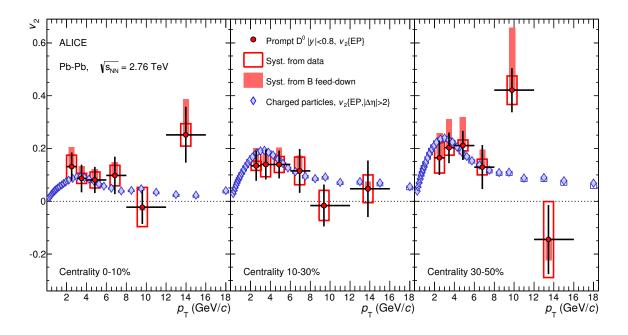
FIG. 5: (Color online) Comparison of D^0 meson [44] and charged-particle [48] v_2 in three centrality classes as a function of p_T. The vertical error bars represent the statistical uncertainty, the open boxes are the systematic uncertainties.
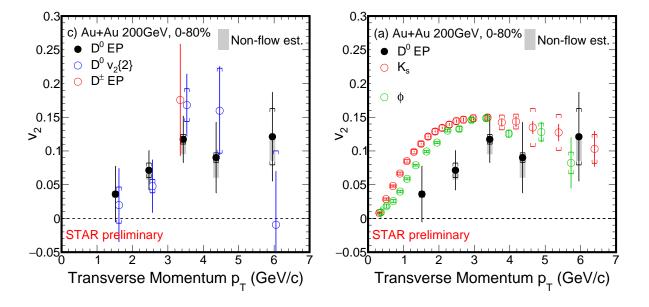

FIG. 6: (Color online) Left panel: The elliptic flow of D^0 and D^+ meson as a function of p_T in minimum bias (0–80%) Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV [45]. Right panel: v_2 for D^0 compared to that of light mesons (K_S^0 and ϕ) [49, 50]. The vertical error bars represent the statistical uncertainty, the cap symbols are the systematic uncertainties.

1. Coalescence Based Models

The model by Cao et al. is based on the Langevin approach where the space-time evolution of the medium is modeled using viscous hydrodynamic. In this model, hadronization is done using quark coalescence mechanism. This model describes R_{AA} in central collisions very well, but tend to underestimate the v_2 at low p_T. The model [57], labeled as MC@sHQ+EPOS, Coll+Rad(LPM), is a perturbative QCD (pQCD) model that includes collisional and radiative energy loss mechanisms for heavy quarks. Hadronization is performed via quark recombination in this model. It underestimates the low-p_T suppression, but yields a substantial anisotropy ($\sim$10%) which slightly underestimates observed data. It correctly describes high-p_T suppression. In the TAMU model [58], heavy-quark transport coefficient is calculated within a non-perturbative T-matrix approach. This model includes hydrodynamic medium evolution and quark coalescence as a mechanism of hadronization. This model provides a good description of the observed suppression of D mesons over the entire p_T range. However, it fails to reproduces observed anisotropy for $p_T > 4$ GeV/c. The Ultra relativistic Quantum Molecular Dynamics (UrQMD) [59] model is based on a microscopic transport theory where the phase space description of the reactions are important. The hybrid UrQMD model includes a realistic description of the medium evolution by combining hadronic transport and ideal hydrodynamics. Hadroniztion via quark recombination is implemented. The model describes the measured anisotropy and suppression in the interval 4 $< p_T < 8$ GeV/c, but fails to explain the data for very low and high p_T region.

2. Fragmentation Based Models

In WHDG model, the observed anisotropy results from path-length dependent energy loss and hadronization is performed using vacuum fragmentation function. This model describes R_{AA} in central collisions reasonably well, but tend to underestimate the v_2 at low p_T. BAMPS model is a partonic transport model which includes multi-parton scattering based on Boltzmann approach. Like WHDG model, in BAMPS, hadronization is performed using vacuum fragmentation functions. BAMPS model describes both R_{AA} and v_2 reasonably well. POWLANG [56] is also a transport model which is based on collisional processes treated within the framework of Langevin dynamics. Hadronization, in this model, is done using vacuum fragmentation functions. This model overestimates the high-p_T suppression, and significantly underestimates observed v_2 at low p_T.

In summary, models including hadronization of charm quarks from recombination with light quarks from the medium (e.g. TAMU) provide a better description of the data at low transverse momentum.

C. Model Comparisons at RHIC Energy

Figure 8 show v_2 and R_{AA} of D^0-meson for 0-80% and 0-10% centrality respectively in Au+Au collisions at $\sqrt{s_{NN}} = $ 200 GeV with selected theoretical model predictions. The physics of DUKE model by Cao et al. and TAMU is already discussed before. The SUBATECH [60] model based on pQCD calculation with the diffusion coefficient parameter $\sim$ 2-4. These three models can describe D^0 suppression reasonable well, however DUKE model underestimates measured
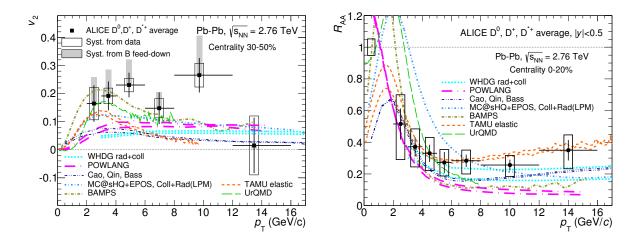
FIG. 7: (Color online) D-meson v_2 and R_{AA} in 30-50% semi-central Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV and comparison with selected theoretical models [53–59].
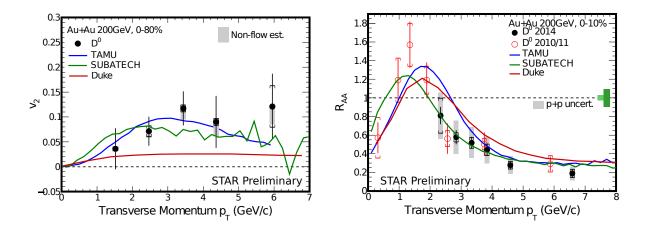

FIG. 8: (Color online) D^0-meson v_2 (0-80%) [45, 46] and R_{AA} (0-10%) [47] in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and comparison with selected theoretical models [54, 58, 60].

anisotropy. The TAMU and SUBATECH describe D^0-meson v_2 data reasonably well.

Fig. 9 (left panel) shows comparison between our AMPT model calculations [6] for D^0-meson v_2 and measured D^0-meson v_2 at 2.76 TeV for 30-50% central collisions by the ALICE experiment. Here σ_{PP} is taken to be 1.5 mb and 10mb with other parameters tuned for LHC data (charged hadron v_2 and multiplicity). Previous study shows that 1.5 mb parton parton scattering cross-section is sufficient to described charged hadron v_2 at mid-rapidity for $p_T <$ 2.0 GeV/c. However, we find that cross-sections of both both 1.5 and 10 mb underestimate LHC D^0-meson v_2 data. It would be very interesting to see how the data and model behave at low p_T (below 2 GeV/c). Therefore, the results from future ALICE upgrade [61] will be very useful to study both heavy flavor and charged hadrons v_2 at low p_T. The right panel of Fig. 9 shows the comparison between D^0 v_2 with AMPT model predictions at top RHIC energy. AMPT model calculation roughly explain data within large statistical errors.
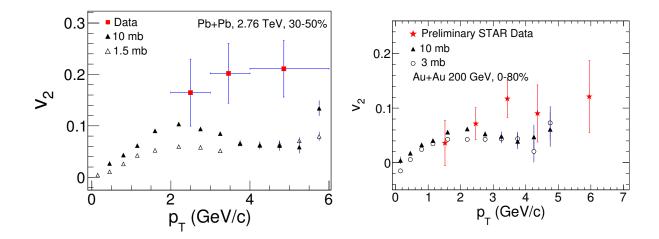
FIG. 9: (Color online) Left panel: Elliptic flow of D^0 meson at mid-rapidity in Pb+Pb collision at $\sqrt{s_{NN}} = 2.76$ TeV for 30-50% centrality and model predictions from AMPT. Only statistical error is shown for ALICE data. Right Panel: Elliptic flow of D^0 meson at mid-rapidity in 0-80% min-bias Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV and prediction from AMPT. Only statistical error is shown for STAR preliminary data.

V. ELLIPTIC FLOW OF J/ψ

The J/ψ meson is bound state of charm (c) and anti-charm ($\bar{c}$) quark. It was discovered independently by two research groups on 11 November 1974 [62, 63]. The importance of this discovery is highlighted by the fact that the subsequent rapid changes in high-energy physics around that time came to be collectively known as the "November Revolution". In relativistic heavy-ion collisions, J/ψ can be produced mainly by recombination of charm (c) and anti-charm and/or direct pQCD processes. By measuring anisotropic flow of J/ψ, one may infer the relative contribution of J/ψ particles from recombination and from direct pQCD processes. J/ψ produced from quark recombination will inherit the flow of charm quarks. On the other-hand, if J/ψ is produced from direct pQCD processes, it should have very little v_2. A detailed comparison between experimental measurements and models on J/ψ v_2 will be helpful to understand the production mechanism of J/ψ.

Figure 10 show elliptic flow of for J/ψ at mid-rapidity in 0-80% min-bias Au+Au events at $\sqrt{s_{NN}} =200$ GeV [64] compared with charged hadrons [65] and ϕ meson [66] in the upper panel and with theoretical calculations in the lower panel. J/ψ v_2 is found to be very small in comparison to that of charged hadrons and ϕ-meson. Models which include J/ψ production from coalescence of thermalized $c\bar{c}$ [68], gives the maximum of J/ψ v_2 to be almost the same in magnitude as light hadrons. v_2 of J/ψ produced from initial pQCD processes [67] is predicted to be very small compared to light hadrons. Models that include J/ψ production from both initial pQCD process and coalescence mechanism [69, 70] also give much smaller J/ψ v_2 in comparison to light hadrons.

In summary, models thats include J/ψ production from both initial pQCD process and coalescence production or entirely from initial pQCD process describe the data better at top RHIC energy. At this point, it is still unclear and we would need very high precision measurements to estimate the fraction of the total J/ψ yield that comes from
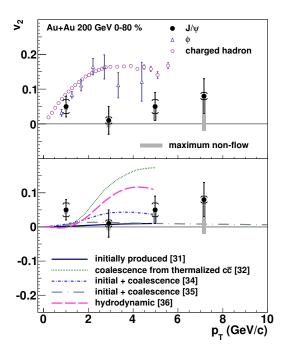
FIG. 10: (Color online) Elliptic flow of for J/ψ particles at mid-rapidity in 0-80% min-bias Au+Au events at $\sqrt{s_{NN}} =$ 200 GeV [64] compared with charged hadrons [65] and the ϕ meson [66] (upper panel) and theoretical calculations [67–70] (lower panel).

pQCD and coalescence processes.

ALICE collaboration also reported the measurement of the elliptic flow of J/ψ in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV within the rapidity range 2.5<y<4.0 [71]. J/ψ v_2 for non-central (20-60%) Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV is shown in Fig. 11. Unlike RHIC, an indication of non-zero J/ψ v_2 is observed with a maximum value of $v_2 = 0.090 \pm$ 0.041 (stat) $\pm$ 0.019 (syst) for non-central (20-60%) Pb-Pb collisions. Calculations from two transport models [72, 73] are also shown for comparison. Transport model calculations that include a J/ψ regeneration component (30%) from deconfined charm quarks in the medium describes data very well.

VI.   SUMMARY AND DISCUSSION

In summary, the measurement of elliptic flow of heavy flavor can provide valuable information about the QGP medium. This article reviewed several important results from RHIC and LHC experiments and discussed their implications. In this review article, we have focused on measurements of heavy flavor v_2 as a function of p_T, collision centrality and energy carried out in RHIC and LHC experiments. We also discussed the comparison of these experimental measurements with available theoretical model predictions.

Measurement of azimuthal anisotropy of non-photonic electron is discussed at $\sqrt{s_{NN}} = 39$, 62.4, 200 GeV and 2.76 TeV. NPE v_2 is consistent with zero at $\sqrt{s_{NN}} = 39$ and 62.4 GeV while it is non-zero v_2 at $\sqrt{s_{NN}} = 200$ GeV and 2.76 TeV. Large positive v_2 of NPE at low p_T might indicate that charm quarks participate in the collective expansion of the
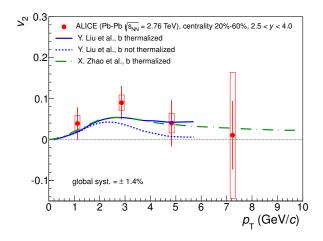
FIG. 11: (Color online) J/ψ v_2 at forward rapidity (2.5<y<4.0) for non-central (20-60%) Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [71].

dense and hot QGP. Elliptic flow of open charm D-meson measured by STAR and ALICE experiments are presented. A non-zero positive flow has been observed at $\sqrt{s_{NN}} = 200$ GeV and 2.76 TeV. Models that include recombination as mechanism of hadronization explain non-zero positive v_2 at low pT, however, a simultaneous description of v_2 and R_{AA} is still an open issue. We also discussed the elliptic flow of J/ψ measured at $\sqrt{s_{NN}} = 200$ GeV and 2.76 TeV by STAR (at mid-rapidity) and ALICE (at forward rapidity). J/ψ flow is consistent with zero at RHIC, however, the measurement is statistically limited. A positive J/ψ v_2 has been observed $\sqrt{s_{NN}} = 2.76$ TeV. Transport model calculations that include a J/ψ regeneration component from deconfined charm quarks in the medium describe data very well within current statistical uncertainties.

A precise measurement of heavy flavor hadrons will provide information on fundamental properties of the medium, such as the transport coefficient, and hadronization mechanisms. As we discussed, current heavy flavor measurements are limited by statistics. In order to circumvent this, recent upgrades have been made in STAR with the introduction of the Heavy Flavor Tracker (HFT) and Muon Telescope Detector (MTD) and dedicated high statistics run in 2016. We hope to see the results from these soon. The ALICE experiment is also upgrading its detectors to pursue high precision measurements in the heavy flavor sector [61]. ALICE is upgrading Inner Tracking System (ITS), for better position and momentum resolution with a faster readout using frontier technologies. The upgraded readout data collection rate of ALICE is expected to increase by a factor of 100. With the current set-up of ALICE, the flow analysis of D_s, Λ_c and Λ_b is not accessible in Pb-Pb due to the limited statistics. Among all open charmed meson, D_s has been considered as quantitative probe for charm quark hadronization. A comparison of D_s and D_0 R_{AA} and v_2 could be interesting to shed light on heavy quark dynamics. The new ITS is expected to allow for a precise measurement of the all D-mesons v_2 down to very low momentum [61].

VII. CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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