Transmitter Optimization in Slow Fading MISO Wiretap Channel

Sanjay Vishwakarma and A. Chockalingam Email: sanvish1975@gmail.com, achockal@ece.iisc.ernet.in Department of ECE, Indian Institute of Science, Bangalore 560012, India

Abstract—In this paper, we consider the transmitter optimization problem in slow fading multiple-input-single-output (MISO) wiretap channel. The source transmits a secret message intended for K users in the presence of J non-colluding eavesdroppers, and operates under a total power constraint. The channels between the source and all users and eavesdroppers are assumed to be slow fading, and only statistical channel state information (CSI) is known at the source. For a given code rate and secrecy rate pair of the wiretap code, denoted by (R_D,R_s) , we define the non-outage event as the joint event of the link information rates to K users be greater than or equal to R_D and the link information rates to J eavesdroppers be less than or equal to (R_D-R_s) . We minimize the transmit power subject to the total power constraint and satisfying the probability of the non-outage event to be greater than or equal to a desired threshold $(1-\epsilon)$.

keywords: Physical layer security, MISO wiretap channel, secrecy rate, multiple eavesdroppers, slow fading.

I. INTRODUCTION

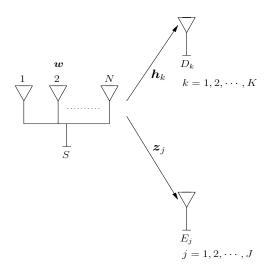
With growing applications on wireless networks, there is a need to provide security, along with reliability, from being eavesdropped, which can easily happen due to the broadcast nature of the wireless transmission. Wyner, in his work in [1], showed that a message could be transmitted at a rate called secrecy rate, at which the legitimate user could decode the message reliably whereas the eavesdropper could be kept entirely ignorant. The wiretap channel model in [1] was physically degraded and discrete memoryless. Later, the work in [1] was extended to more general broadcast channel and Gaussian channel in [2] and [3], respectively. Subsequent extension to various multi-antenna wireless wiretap channels and the corresponding achievable secrecy rates and secrecy capacities have been reported by many authors, e.g., [4]–[11].

In [12], secrecy capacity of a quasi-static single-antenna Rayleigh fading channel in terms of outage probability has been characterized. Outage probability characterization of the secrecy rate of multiple-input-single-output (MISO) wiretap channel with artificial noise has been reported in [13]-[15], and that of amplify-and-forward relay channel has been reported in [16]. Motivated by the need for outage probability characterization of secrecy rate in MISO wiretap channel, in this paper, we consider the transmitter optimization problem in slow fading MISO wiretap channel. The source transmits a secret message intended for K users in the presence of Jnon-colluding eavesdroppers, and operates under a total power constraint. The channels between the source and all users and eavesdroppers are assumed to be slow fading. Only statistical channel state information (CSI) is assumed to be known at the source. For a given code rate and secrecy rate pair of the wiretap code, denoted by (R_D,R_s) , we define the non-outage event as the joint event of the link information rates to K users be greater than or equal to R_D and the link information rates to J eavesdroppers be less than or equal to (R_D-R_s) . We minimize the transmit power subject to the total power constraint and satisfying the probability of the non-outage event to be greater than or equal to a desired threshold $(1-\epsilon)$. We obtain the achievable (R_D,R_s) region and the transmit beamforming vector. We note that we differ from the reported works in [13,14], which also consider multiple eavesdroppers scenario, in following aspects: i) number of users K can be more than one, ii) only statistical CSI of the users channels are known, and iii) channel covariance matrices of all users and eavesdroppers can be arbitrary positive semidefinite matrices.

Notations: $A \in \mathbb{C}^{N_1 \times \bar{N}_2}$ implies that A is a complex matrix of dimension $N_1 \times N_2$. $A \succeq \mathbf{0}$ and $A \succ \mathbf{0}$ imply that A is a positive semidefinite matrix and positive definite matrix, respectively. Identity matrix is denoted by I. Transpose and complex conjugate transpose operations are denoted by $[.]^T$ and $[.]^*$, respectively. $\mathbb{E}[.]$ denotes the expectation operator, and $\| . \|$ denotes the 2-norm operator. $\operatorname{diag}(a)$ denotes a diagonal matrix with elements of the vector $a \in \mathbb{C}^{N \times 1}$ on its diagonal. Trace of matrix $A \in \mathbb{C}^{N \times N}$ is denoted by $\operatorname{Tr}(A)$. $h \in \mathbb{C}^{N \times 1} \sim \mathcal{CN}(\mathbf{0}, H)$ implies that h is a circularly symmetric complex Gaussian random vector with mean vector $\mathbf{0}$ and covariance matrix H.

II. SYSTEM MODEL

Consider a MISO wiretap channel as shown in Fig. 1, which consists of a source S having N transmit antennas, Kusers $\{D_1, D_2, \cdots, D_K\}$ each having single antenna, and J non-colluding eavesdroppers $\{E_1, E_2, \cdots, E_J\}$ each having single antenna. The complex channel gain vector from S to D_k is denoted by $h_k \in \mathbb{C}^{1 \times N}$, $1 \leq k \leq K$. Likewise, the complex channel gain vector from S to E_j is denoted by $z_j \in \mathbb{C}^{1 \times N}$, $1 \leq j \leq J$. We assume that the channels between S to D_k s and those between S to E_i s fade slowly and independently with $h_k \sim \mathcal{CN}(\mathbf{0}, H_k)$, and $z_j \sim \mathcal{CN}(\mathbf{0}, Z_j)$. These channel gains are assumed to be unknown at S. We assume that the source S operates under total power constraint P_T . The communication between S and D_k s happens in n channel uses. The source S transmits secret message W which is equiprobable over $\{1, 2, \dots, 2^{nR_s}\}$. For each W drawn equiprobably from the set $\{1, 2, \cdots, 2^{nR_s}\}$, the source, using a stochastic encoder, maps W to a codeword $\{x_i\}_{i=1}^n$ of length n, where each $x_i \in \mathbb{C}$, i.i.d. $\sim \mathcal{CN}(0,1)$, and $\mathbb{E}[|x_i|^2] = 1$. Each codeword $\{x_i\}_{i=1}^n$ belongs to a collection of 2^{nR_D}



System model for MISO wiretap channel with K users and JFig. 1. eavesdroppers.

codewords (i.e., wiretap code) where $R_D \geq R_s$. The source applies the complex weight $\boldsymbol{w} = [w_1, w_2, \cdots, w_N]^T \in \mathbb{C}^{N \times 1}$ and transmits the weighted symbol which is wx_i in the ith channel use, $1 \le i \le n$. Since the source is power constrained, this implies that

$$\|\boldsymbol{w}\|^2 \le P_T. \tag{1}$$

(3)

In the following, we will use x to denote the symbols in the codeword $\{x_i\}_{i=1}^n$. Since the channel is slow fading and the CSI is unknown at the source S, we define the non-outage event for a given (R_D, R_s) pair of the wiretap code, denoted by \mathcal{E} , and impose the probability constraint on \mathcal{E} as follows:

$$\mathcal{E} = \left\{ R_{D_k} \ge R_D, \quad \forall k = 1, 2, \cdots, K, \quad \text{and} \right.$$

$$R_D - R_s \ge R_{E_j}, \quad \forall j = 1, 2, \cdots, J \right\}, \qquad (2)$$

$$\Pr(\mathcal{E}) > (1 - \epsilon), \qquad (3)$$

where $(1 - \epsilon)$ is the non-outage probability threshold, and R_{D_k} and R_{E_i} are the link information rates between S to D_k and S to E_i , respectively. In other words, when the source selects the target code rate and target secrecy rate pair of the wiretap code as (R_D, R_s) , the above constraint implies that, with probability greater than or equal to $(1 - \epsilon)$, all D_k s will be able to successfully decode the transmitted message while all E_i s will be ignorant about the transmitted message. We also note that when the CSI on all the links are known at S, the achievability of the secrecy rate R_s is shown in [11]. Let y_{D_k} and y_{E_i} denote the received signals at D_k and E_i ,

$$y_{D_k} = \mathbf{h}_k \mathbf{w} x + \eta_{D_k}, \quad \forall k = 1, 2, \cdots, K, \quad (4)$$

$$y_{E_i} = \mathbf{z}_j \mathbf{w} x + \eta_{E_i}, \quad \forall j = 1, 2, \cdots, J, \quad (5)$$

where η s are noise components, assumed to be i.i.d. \sim $\mathcal{CN}(0, N_0)$.

respectively. We have

III. TRANSMITTER OPTIMIZATION UNDER SECRECY CONSTRAINT

Using (4) and (5), and for a given h_k and z_j , the information rates at D_k and E_j are obtained, respectively, as follows:

$$R_{D_k} = I(x; y_{D_k}) = \log_2\left(1 + \frac{|\boldsymbol{h}_k \boldsymbol{w}|^2}{N_0}\right),$$
 (6)

$$R_{E_j} = I(x; y_{E_j}) = \log_2\left(1 + \frac{|z_j w|^2}{N_0}\right).$$
 (7)

Further, subject to the constraints in (1) and (3) and using (6) and (7), the optimization problem to minimize the transmit power is as follows:

$$\min \|\boldsymbol{w}\|^2 \tag{8}$$

s.t.
$$\|\boldsymbol{w}\|^{2} \leq P_{T}$$
, (9)

$$\Pr\left\{\log_{2}\left(1 + \frac{|\boldsymbol{h}_{k}\boldsymbol{w}|^{2}}{N_{0}}\right) \geq R_{D}, \quad \forall k = 1, 2, \cdots, K,$$

$$R_{D} - R_{s} \geq \log_{2}\left(1 + \frac{|\boldsymbol{z}_{j}\boldsymbol{w}|^{2}}{N_{0}}\right), \quad \forall j = 1, 2, \cdots, J\right\}$$

$$\geq (1 - \epsilon). (10)$$

Since h_k s and z_j s are independent, we rewrite the constraint (10) in the following equivalent product form:

$$\prod_{k=1}^{K} \Pr \Big\{ |\boldsymbol{h}_{k} \boldsymbol{w}|^{2} \geq (2^{R_{D}} - 1) N_{0} \Big\}
\prod_{j=1}^{J} \Pr \Big\{ |\boldsymbol{z}_{j} \boldsymbol{w}|^{2} \leq (2^{\left(R_{D} - R_{s}\right)} - 1) N_{0} \Big\} \geq (1 - \epsilon). (11)$$

We note that solving the optimization problem (8) in its original form is hard. So, in order to simplify the analysis, we replace the product probability constraint in (11) with the following K+J individual probability constraints:

$$\forall k = 1, 2, \cdots, K, \quad \text{and} \quad \forall j = 1, 2, \cdots, J,$$

$$\Pr \Big\{ |\boldsymbol{h}_k \boldsymbol{w}|^2 \ge \left(2^{R_D} - 1\right) N_0 \Big\} \ge \left(1 - \epsilon\right)^{\frac{1}{K + J}}, \quad (12)$$

$$\Pr \Big\{ |\boldsymbol{z}_j \boldsymbol{w}|^2 \le \left(2^{\left(R_D - R_s\right)} - 1\right) N_0 \Big\} \ge \left(1 - \epsilon\right)^{\frac{1}{K + J}}. \quad (13)$$

We also note that any w which satisfies all K+J constraints in (12) and (13) will also satisfy the product probability constraint in (11). However, the converse may not always be true. Further, since $h_k w$ in (12) and $z_j w$ in (13) are linear transformations of circularly symmetric complex Gaussian random vectors, $h_k w$ and $z_i w$ are also circularly symmetric complex Gaussian random variables, i.e., $h_k w \sim \mathcal{CN}(0, w^* H_k w)$, and $z_j w \sim \mathcal{CN}(0, w^* Z_j w)$. This further implies that $|\boldsymbol{h}_k \boldsymbol{w}|^2$ and $|\boldsymbol{z}_j \boldsymbol{w}|^2$ are exponential random variables, i.e.,

$$|\boldsymbol{h}_k \boldsymbol{w}|^2 \sim \frac{1}{\boldsymbol{w}^* \boldsymbol{H}_k \boldsymbol{w}} \exp^{-\frac{\lambda}{\boldsymbol{w}^* \boldsymbol{H}_k \boldsymbol{w}}}, \quad \lambda \geq 0,$$
 (14)

$$|\boldsymbol{z}_{j}\boldsymbol{w}|^{2} \sim \frac{1}{\boldsymbol{w}^{*}\boldsymbol{Z}_{j}\boldsymbol{w}} \exp^{-\frac{\lambda}{\boldsymbol{w}^{*}\boldsymbol{Z}_{j}\boldsymbol{w}}}, \quad \lambda \geq 0.$$
 (15)

Using (14) and (15), and by following standard integration steps, we get the following equivalent simplified inequalities for the probability constraints in (12) and (13):

$$\forall k = 1, 2, \cdots, K, \quad \boldsymbol{w}^* \boldsymbol{H}_k \boldsymbol{w} > a, \tag{16}$$

$$\forall j = 1, 2, \cdots, J, \quad \boldsymbol{w}^* \boldsymbol{Z}_j \boldsymbol{w} \leq b, \tag{17}$$

where $a=\frac{(2^{R_D}-1)N_0}{-\ln(1-\epsilon)^{\frac{1}{(K+J)}}}$ and $b=\frac{(2^{(R_D-R_s)}-1)N_0}{-\ln(1-(1-\epsilon)^{\frac{1}{(K+J)}})}$. Replacing the constraint in (11) with (16) and (17), we get the following upper bound for the optimization problem (8):

$$\min_{\boldsymbol{w}} \|\boldsymbol{w}\|^2 \tag{18}$$

s.t.
$$\|w\|^2 \le P_T$$
, (19)

$$\forall k = 1, 2, \cdots, K, \quad \boldsymbol{w}^* \boldsymbol{H}_k \boldsymbol{w} \geq a, \tag{20}$$

$$\forall j = 1, 2, \cdots, J, \quad \boldsymbol{w}^* \boldsymbol{Z}_i \boldsymbol{w} \leq b, \tag{21}$$

We solve the above problem for the following two cases.

A. All $H_k s$ and $Z_j s$ are diagonal matrices

When all H_k s and Z_j s are diagonal positive semidefinite matrices, the optimization problem (18) can be written as the following equivalent linear optimization problem:

$$\min_{P_1, P_2, \dots, P_N} \sum_{m=1}^{N} P_m \tag{22}$$

s.t.
$$\forall m = 1, 2, \dots, N, P_m \ge 0, \sum_{m=1}^{N} P_m \le P_T$$
, (23)

$$\forall k = 1, 2, \dots, K, \quad \sum_{m=1}^{N} P_m H_k^{mm} \geq a, (24)$$

$$\forall j = 1, 2, \dots, J, \quad \sum_{m=1}^{N} P_m Z_j^{mm} \leq b, (25)$$

where $P_m = |w_m|^2$, $\boldsymbol{H}_k = \mathrm{diag}\left([H_k^{11}, H_k^{22}, \cdots, H_k^{NN}]^T\right) \succeq \boldsymbol{0}$, and $\boldsymbol{Z}_j = \mathrm{diag}\left([Z_j^{11}, Z_j^{22}, \cdots, Z_j^{NN}]^T\right) \succeq \boldsymbol{0}$. The above problem can be easily solved using linear optimization techniques. Having obtained P_1, P_2, \cdots, P_N , the beamforming vector \boldsymbol{w} is $[\sqrt{P_1}, \sqrt{P_2}, \cdots, \sqrt{P_N}]^T$.

B. Some of $H_k s$ or $Z_j s$ are not diagonal matrices

Here, we consider the general case when H_k s and Z_j s are Hermitian positive semidefinite matrices and some of H_k s or Z_j s are not diagonal. Define $W \stackrel{\triangle}{=} ww^*$. We rewrite the optimization problem (18) into the following equivalent form:

$$\min_{\boldsymbol{W}} \operatorname{Tr}(\boldsymbol{W}) \tag{26}$$

s.t.
$$\mathbf{W} \succeq \mathbf{0}$$
, $rank(\mathbf{W}) = 1$, $Tr(\mathbf{W}) \leq P_T$, (27)

$$\forall k = 1, 2, \dots, K, \operatorname{Tr}(\boldsymbol{W}\boldsymbol{H}_k) \geq a, \quad (28)$$

$$\forall i = 1, 2, \cdots, J, \operatorname{Tr}(\boldsymbol{W}\boldsymbol{Z}_i) < b.$$
 (29)

The above optimization problem is a non-convex optimization problem. However, by relaxing the $rank(\mathbf{W}) = 1$ constraint,

the above problem can be solved using semidefinite programming techniques [17]. But the solution W of the above rank relaxed optimization problem may not have rank 1. This can be easily seen from the KKT conditions of the rank relaxed optimization problem which we discuss in the Appendix. We now take the rank-1 approximation as follows. Let w_0 be the unit-norm eigen direction corresponding to the largest eigen value of W. We substitute $W = Pw_0w_0^*$ in the above rank relaxed optimization problem and solve the resulting linear optimization problem for unknown P, i.e.,

$$\min_{P} P \tag{30}$$

s.t.
$$0 \le P \le P_T$$
, (31)

$$\forall k = 1, 2, \cdots, K, \quad P \boldsymbol{w}_0^* \boldsymbol{H}_k \boldsymbol{w}_0 \geq a, \tag{32}$$

$$\forall j = 1, 2, \cdots, J, \quad P\boldsymbol{w}_0^* \boldsymbol{Z}_j \boldsymbol{w}_0 \leq b. \tag{33}$$

Having obtained the transmit power P from (30), the beamforming vector is $\sqrt{P}w_0$.

Remark 1: We note that when the channel CSI h_k on all D_k s are perfectly known at the source S, the constraints (28) and (29) in the optimization problem (26) should be replaced with the following constraints, respectively:

$$\operatorname{Tr}(\boldsymbol{W}\boldsymbol{h}_{k}^{*}\boldsymbol{h}_{k}) \geq (2^{R_{D}} - 1)N_{0},$$
 (34)

$$\operatorname{Tr}(\boldsymbol{W}\boldsymbol{Z}_{j}) \leq \frac{(2^{(R_{D}-R_{s})}-1)N_{0}}{-\ln(1-(1-\epsilon)^{\frac{1}{J}})}.$$
 (35)

Remark 2: When the source transmits the symbol x from an equiprobable complex finite alphabet set $\mathbb{A}=\{a_1,a_2,\cdots,a_M\}$ of size M (e.g., M-ary) with $\mathbb{E}[x]=0$ and $\mathbb{E}[|x|^2]=1$, the information rates in (6) and (7) can be written in the following forms, respectively:

$$R_{D_k} = I(x; y_{D_k}) = I\left(\frac{|h_k w|^2}{N_0}\right),$$
 (36)

$$R_{E_j} = I(x; y_{E_j}) = I\left(\frac{|z_j w|^2}{N_0}\right),$$
 (37)

where

$$I(\rho) \stackrel{\triangle}{=} \frac{1}{M} \sum_{l=1}^{M} \int p_n (y - \sqrt{\rho} a_l)$$

$$\log_2 \frac{p_n (y - \sqrt{\rho} a_l)}{\frac{1}{M} \sum_{m=1}^{M} p_n (y - \sqrt{\rho} a_m)} dy,$$
(38)

and $p_n(\theta) = \frac{1}{\pi} e^{-|\theta|^2}$. Using the fact that the mutual information function, $I(\rho)$, is a strictly-increasing concave function in ρ [18,19], K+J constraints in (20) and (21) can be written in the following forms, respectively:

$$\forall k = 1, 2, \cdots, K, \quad \boldsymbol{w}^* \boldsymbol{H}_k \boldsymbol{w} > a, \tag{39}$$

$$\forall j = 1, 2, \cdots, J, \quad \boldsymbol{w}^* \boldsymbol{Z}_i \boldsymbol{w} < b, \tag{40}$$

where $a=\frac{I^{-1}(R_D)N_0}{-\ln(1-\epsilon)^{\frac{1}{(K+J)}}}$ and $b=\frac{I^{-1}(R_D-R_s)N_0}{-\ln(1-(1-\epsilon)^{\frac{1}{(K+J)}})}$. With finite alphabet input, the optimization problem (18) should be solved subject to the constraints in (39) and (40).

IV. RESULTS AND DISCUSSIONS

We have evaluated the secrecy rate through simulation with the following system parameters: $N=3,~K=2,~J=1,2,3,~N_0=1,~\epsilon=0.1$, and $P_T=12$ dB. We consider the scenarios discussed in Section III-A and Section III-B.

Scenario of Section III-B: We have used the following positive definite channel covariance matrices in the simulations:

$$H_{1} = \begin{bmatrix} 2.1670, & 0.1806 + 0.0183i, & -0.1453 - 0.3101i \\ 0.1806 - 0.0183i, & 1.9165, & 0.0696 + 0.3374i \\ -0.1453 + 0.3101i, & 0.0696 - 0.3374i, & 1.4180 \end{bmatrix} \succ \mathbf{0} \quad (41)$$

$$H_{2} = \begin{bmatrix} 1.9834, & -0.2001 + 0.0250i, & 0.0470 - 0.3424i \\ -0.2001 - 0.0250i, & 1.3867, & 0.0149 - 0.2083i \\ 0.0470 + 0.3424i, & 0.0149 + 0.2083i, & 1.4323 \end{bmatrix} \succ \mathbf{0} \quad (42)$$

$$Z_{1} = \begin{bmatrix} 0.0043, & 0.0010 - 0.0003i, & 0.0013 + 0.0009i \\ 0.0010 + 0.0003i, & 0.0074, & -0.0011 - 0.0029i \\ 0.0013 - 0.0009i, & -0.0011 + 0.0029i, & 0.0079 \end{bmatrix} \succ \mathbf{0} \quad (43)$$

$$Z_{2} = \begin{bmatrix} 0.0069, & 0.0004 - 0.0029i, & -0.0014 + 0.0014i \\ 0.0004 + 0.0029i, & 0.0070, & -0.0019 - 0.0002i \\ -0.0014 - 0.0014i, & -0.0019 + 0.0002i, & 0.0086 \end{bmatrix} \succ \mathbf{0} \quad (44)$$

$$Z_{3} = \begin{bmatrix} 0.0090, & -0.0026 + 0.0006i, & 0.0011 - 0.0009i \\ -0.0026 - 0.0006i, & 0.0064, & -0.0013 + 0.0018i \\ 0.0011 + 0.0009i, & -0.0013 - 0.0018i, & 0.0054 \end{bmatrix} \succ \mathbf{0} \quad (45)$$

For a given (R_D,R_s) pair, we solve the semidefinite rank relaxed optimization problem (26) using the tools in [20,21]. We numerically observe that, for any feasible (R_D,R_s) pair, the solution \boldsymbol{W} of the rank relaxed optimization problem (26) has rank 1. This implies that for such channel realizations, rank-1 approximation is not needed. In Fig 2(a), we plot

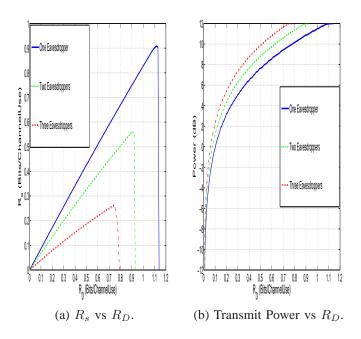


Fig. 2. R_s vs R_D and Transmit Power vs R_D in MISO wiretap channel with $N=3,~K=2,~J=1,2,3,~N_0=1,~\epsilon=0.1$ and $P_T=12$ dB, and non-diagonal covariance matrices.

the maximum achievable R_s vs R_D . In Fig 2(b), we plot the corresponding minimum transmit power vs R_D . We

observe that the maximum achievable secrecy rate R_s and the corresponding minimum transmit power increases with increase in R_D . The secrecy rate drops to zero when the entire available power, $P_T=12$ dB, is used.

Scenario of Section III-A: Here, we take H_1 , H_2 , Z_1 , Z_2 , and Z_3 as the diagonal approximation of covariance matrices in (41), (42), (43), (44), and (45), respectively. We solve the linear optimization problem (22) using the tools in [20,21], and we plot the maximum achievable R_s vs R_D and the corresponding minimum transmit power vs R_D in Fig 3(a) and Fig 3(b), respectively. As in Fig 2(a) and Fig 2(b), we

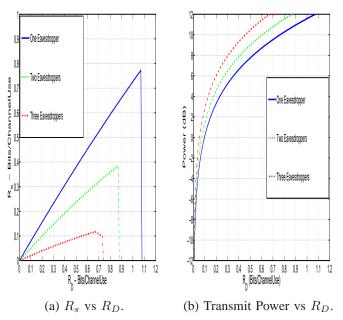


Fig. 3. R_s vs R_D and Transmit Power vs R_D in MISO wiretap channel with $N=3,~K=2,~J=1,2,3,~N_0=1,~\epsilon=0.1$ and $P_T=12$ dB, and diagonal covariance matrices.

observe that the maximum achievable secrecy rate R_s and the corresponding minimum transmit power increases with increase in R_D . The secrecy rate drops to zero when the entire available power, $P_T = 12$ dB, is used.

V. CONCLUSIONS

We considered the transmitter optimization problem in slow fading MISO wiretap channel. Secret message transmitted by the source was intended for K users in the presence of J eavesdroppers. For a given code rate and secrecy rate pair of the wiretap code, denoted by (R_D,R_s) , we defined the non-outage event and minimized the transmit power subject to the total power constraint and satisfying the probability of the non-outage event to be greater than a desired threshold $(1-\epsilon)$. We obtained the achievable (R_D,R_s) region and the transmit beamforming vector.

APPENDIX

In this appendix, we analyze the rank of the optimal solution \boldsymbol{W} of the rank relaxed optimization problem (26). We take the

Lagrangian [17] of the rank relaxed optimization problem (26) as follows:

$$\ell(\boldsymbol{W}, \boldsymbol{\Lambda}, \lambda, \mu_{k}, \nu_{j}) = \operatorname{Tr}(\boldsymbol{W}) - \operatorname{Tr}(\boldsymbol{\Lambda}\boldsymbol{W})$$

$$+ \lambda \left(\operatorname{Tr}(\boldsymbol{W}) - P_{T}\right) + \sum_{k=1}^{K} \mu_{k} \left(a - \operatorname{Tr}(\boldsymbol{W}\boldsymbol{H}_{k})\right)$$

$$+ \sum_{j=1}^{J} \nu_{j} \left(\operatorname{Tr}(\boldsymbol{W}\boldsymbol{Z}_{j}) - b\right), \quad (46)$$

where $\Lambda \succeq 0$, $\lambda \geq 0$, $\mu_k \geq 0$, and $\nu_j \geq 0$ are Lagrangian multipliers. The KKT conditions are as follows:

- K1. All the constraints in (27), (28), and (29) excluding the constraint $rank(\mathbf{W}) = 1$,
- K2. $\operatorname{Tr}(\boldsymbol{\Lambda}\boldsymbol{W})=0$. Since $\boldsymbol{\Lambda}\succeq \mathbf{0}$ and $\boldsymbol{W}\succeq \mathbf{0}$, this implies that $\boldsymbol{\Lambda}\boldsymbol{W}=\mathbf{0}$,
- K3. $\lambda (\operatorname{Tr}(\boldsymbol{W}) P_T) = 0$,
- K4. $\forall k = 1, 2, \dots, K, \quad \mu_k(a \operatorname{Tr}(\boldsymbol{W}\boldsymbol{H}_k)) = 0,$
- K5. $\forall j = 1, 2, \dots, J, \quad \nu_j (\operatorname{Tr}(\boldsymbol{W} \boldsymbol{Z}_j) b) = 0,$
- K6. $\frac{\partial \ell}{\partial \pmb{W}_j} = \pmb{0}$ implies that $\pmb{\Lambda} = (1 + \lambda) \pmb{I} \sum_{k=1}^K \mu_k \pmb{H}_k + \sum_{j=1}^J \nu_j \pmb{Z}_j \succeq \pmb{0}$,

The KKT conditions (K2), (K6), (K4), and (K5) imply that $(1+\lambda)\operatorname{Tr}(\boldsymbol{W}) - \sum_{k=1}^K \mu_k a + \sum_{j=1}^J \nu_j b = 0$. For $\boldsymbol{W} \neq \boldsymbol{0}$, this further implies that not all μ_k s can be zero simultaneously. With this, we rewrite (K6) in the following form:

$$\mathbf{\Lambda} + \sum_{k=1}^{K} \mu_k \mathbf{H}_k = (1+\lambda)\mathbf{I} + \sum_{j=1}^{J} \nu_j \mathbf{Z}_j \succ \mathbf{0}.$$
 (47)

The above equation implies that $rank(\Lambda + \sum_{k=1}^K \mu_k \boldsymbol{H}_k) = N$. This further implies that $rank(\Lambda) \geq N - rank(\sum_{k=1}^K \mu_k \boldsymbol{H}_k)$. (K2) implies that $rank(\boldsymbol{W}) \leq rank(\sum_{k=1}^K \mu_k \boldsymbol{H}_k)$ (assuming $\boldsymbol{W} \neq \boldsymbol{0}$). This means that the rank of \boldsymbol{W} may not be one.

For the special case when K=1 and \mathbf{H}_1 is a rank one positive semidefinite matrix, (47) implies that $rank(\mathbf{\Lambda}) \geq N-1$. Assuming $\mathbf{W} \neq \mathbf{0}$, (K2) further implies that $rank(\mathbf{\Lambda}) = N-1$, and $rank(\mathbf{W}) = 1$.

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