Hybrid neutron stars based on a modified PNJL model

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We discuss a three-flavor Nambu–Jona-Lasinio (NJL) type quantum field theoretical approach to the quark matter equation of state (EoS) with scalar diquark condensate, isoscalar vector mean field and Kobayashi-Maskawa-'t Hooft (KMT) determinant interaction. While often the diquark and vector meson couplings are considered as free parameters, we will fix them here to their values according to the Fierz transformation of a one-gluon exchange interaction. In order to estimate the effect of a possible change in the vacuum pressure of the gluon sector at finite baryon density we exploit a recent modification of the Polyakov-loop NJL (mPNJL) model which introduces a parametric density dependence of the Polyakov-loop potential also at $T = 0$, thus being relevant for compact star physics. We use a Dirac-Brueckner-Hartree-Fock (DBHF) EoS for hadronic matter phase and discuss results for mass-radius relationships following from a solution of the TOV equations for such a hybrid EoS in the context of observational constraints from selected objects.

§1. Introduction

In the present contribution we will introduce a model for the EoS of hybrid stars with deconfined quark matter cores and recent observational constraints[1,2] for their masses (M) and radii (R).

The status of the theoretical approach to the neutron star matter equation of state is very different from that for the high-temperature case at low or vanishing net baryon densities, where *ab initio* lattice QCD simulations provide EoS with almost physical quark masses systematically approaching the continuum limit.[3] This guidance is yet absent at zero temperature and high baryon number densities, where a variety of models exists on different levels of the microphysical description which make different predictions for the state of matter. It is the current hope that the situation might change in a not too far future when very accurate measurements of the $M - R$ relationship for compact stars become possible, e.g., with the *International X-Ray Observatory (IXO)* project.[4] Once we are in possession of such data which allow upon inversion of the Tolman-Oppenheimer-Volkoff (TOV) equations the extraction of the cold dense compact star EoS,[5] we will face the problem of interpreting the physical content of this numerical result. Then, a broad basis of alternative EoS models like the one presented here may become very useful.

A common feature of present hybrid star models is that the transition from hadronic to quark matter cannot yet be described on a unique footing where hadrons would appear as bound states (clusters) of quarks and their possible dissociation at high densities as a kind of Mott transition[6] like in nonideal plasmas[7] or in nuclear matter.[8,9] Early nonrelativistic potential model approaches[10,11] are insufficient since they cannot accomodate the chiral symmetry restoration transition in a

proper way. Therefore, at present the discussion is restricted to so-called two-phase aproaches where the hadronic and the quark matter EoS are modeled separately followed by a subsequent phase transition construction to obtain a hybrid EoS.

Widely employed for a description of quark matter in compact stars are thermodynamical bag models of three-flavor quark matter with eventually even density-dependent bag pressure $B(n)$, as in Ref.[12] A qualitatively new feature of the phase structure appears in chiral quark models of the Nambu–Jona-Lasinio type which describe the dynamical chiral symmetry breaking of the QCD vacuum and its partial restoration in hot and dense matter, see Ref.[13] for a review. In contrast to bag models, in these approaches at low temperatures the light and strange quark degrees of freedom may appear sequentially with increasing density,[14]–[16] so that strangeness may even not appear in the quark matter cores of hybrid stars, before their maximum mass is reached. Once chiral symmetry is restored, a rich spectrum of color superconducting quark matter phases may be realized in dense quark matter, depending on it's flavor composition and isospin asymmetry[17] with far-reaching consequences for hybrid star phenomenology, e.g., $M - R$ relationships and cooling behavior.

We will consider here a color superconducting three-flavor NJL model with self-consistently determined density dependences of quark masses and scalar diquark gaps, developed in Refs.,[18]–[20] including the flavor-mixing KMT determinant interaction.[21],[22] Only recently it became clear[23],[24] that this flavor mixing is crucial for the possible stability of strange quark matter phases in hybrid stars. As a new aspect, we will couple the chiral quark sector to a Polyakov-loop potential which is a modification[25] of the one used in standard PNJL models.[26],[27]

§2. Modified PNJL model with color superconductivity

We start from the path integral representation of the partition function for the modified color superconducting three-flavor PNJL model

$$Z[T, V, \{\mu\}] = \int \mathcal{D}\bar{q}\mathcal{D}q \; e^{-\int^{\beta} d^4 x \{\bar{q}[i\gamma^{\mu}\partial_{\mu} - \hat{m} - \gamma^0(\hat{\mu} + i\lambda_3\phi_3)]q - \mathcal{L}_{\text{int}} + \mathcal{U}(\Phi)\}} \qquad (2\cdot1)$$

with the interaction Lagrangian $\mathcal{L}_{\text{int}} = \mathcal{L}_{\bar{q}q} + \mathcal{L}_{qq}$, where

$$\mathcal{L}_{\bar{q}q} = G_S \sum_{a=0}^{8} \left[(\bar{q}\tau_a q)^2 + (\bar{q}i\gamma_5\tau_a q)^2 \right] + G_V (\bar{q}i\gamma_0 q)^2$$
$$- K \left[\det_f(\bar{q}(1 + \gamma_5)q) + \det_f(\bar{q}(1 - \gamma_5)q) \right], \qquad (2\cdot2)$$

$$\mathcal{L}_{qq} = G_D \sum_{a,b=2,5,7} (\bar{q}i\gamma_5\tau_a\lambda_b C\bar{q}^T)(q^T C i\gamma_5\tau_a\lambda_b q), \qquad (2\cdot3)$$

with τ_a and λ_b being the antisymmetric Gell-Mann matrices acting in flavor and color space, respectively. We have suppressed the flavor index for the current quark mass matrix $\hat{m}$ and the chemical potential matrix $\hat{\mu}$. The scalar (G_S), diquark (G_D), vector (G_V) and KMT (K) couplings are to be determined by hadron phenomenology, see.[28] The Polyakov-loop $\Phi = \text{Tr}_c[\exp(i\beta\lambda_3\phi_3)]/N_c$ is an order parameter for

confinement, weighted with the phenomenological potential

$$\mathcal{U}(\Phi) = (aT^4 + b\mu^2 T^2 + c\mu^4)\Phi^2 + a_2 T_0^4 \ln(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4) , \qquad (2\cdot4)$$

which is a modification[25] of the standard PNJL model, now accounting for an explicit chemical potential dependence of $\mathcal{U}(\Phi)$, even at $T = 0$, which is not present in the traditional parametrizations by, e.g., Refs.[26],[27]

Our description of quark matter is based on the grand canonical thermodynamic potential [13],[18]–[20],[29] which in the meanfield (stationary phase) approximation to the partition function (2·1) is given by

$$\Omega_{\mathrm{MF}}(T, \{\mu\}) = \frac{\phi_u^2 + \phi_d^2 + \phi_s^2}{8G_S} + \frac{K\phi_u\phi_d\phi_s}{16G_S^3} - \frac{\omega_u^2 + \omega_d^2 + \omega_s^2}{8G_V} + \frac{\Delta_{ud}^2 + \Delta_{us}^2 + \Delta_{ds}^2}{4G_D}$$

$$- \int \frac{d^3p}{(2\pi)^3} \sum_{n=1}^{18} \left[E_n + 2T \ln \left(1 + e^{-E_n/T} \right) \right] - \Omega_0 + \mathcal{U}(\Phi_0) , \qquad (2\cdot5)$$

where $E_n = E_n(p, \mu; \mu_Q, \mu_3, \mu_8, \phi_u, \phi_d, \phi_s, \omega_u, \omega_d, \omega_s, \Delta_{ud}, \Delta_{us}, \Delta_{ds}, \Phi_0)$ are the quasiparticle dispersion relations, obtained by numerical diagonalization of the quark propagator matrix in color-, flavor-, Dirac- and Nambu-Gorkov spaces. The values of the meanfields (order parameters) are obtained from a minimization of $\Omega(T, \{\mu\})$, which is equivalent to the selfconsistent solution of the set of corresponding gap equations. The subtraction of Ω_0 garantees that in the vacuum $\Omega(0, 0) = 0$. For applications to compact stars, we will have to add the contribution from leptons (*e.g.*, electrons, muons and the corresponding neutrino flavors) and obey the constraints of color and electric neutrality as well as β-equilibrium of weak interactions between quark flavors and leptons.

The mean-field contribution of the Polyakov-loop potential could be viewed as a T, μ-dependent modification of the bag function $\Delta B(T, \mu) = \mathcal{U}(\Phi_0)$ which accounts for possible changes in the pressure of the gluon sector related to a partial melting of the gluon condensate at finite T and μ. Therefore, it takes negative values. In the present contribution, we will discuss exploratory calculations of those effects for compact star structure, we will compare results with the above modified PNJL (mPNJL) model with those of approximating the influence of these effects by a simple (negative) bag constant ΔB. Note that the mPNJL model provides us with a more micoscopic picture on the origin of density (and temperature) dependence of nonperturbative QCD thermodynamics which in some phenomenological approaches are subsumed in a density dependent bag constant as, e.g., in Ref.[12]

As pointed out in,[13] due to the mixing of the light and strange flavor sectors by the KMT interaction, the difference in the critical chemical potentials for the chiral phase transitions in these sectors (which coincide with the onset of 2SC and CFL phases, respectively) gets diminished. This entails that the phase transition between hadronic matter (described by a realistic nuclear EoS, e.g., the DBHF one, see[29]) and superconducting quark matter may eventually proceed directly into the CFL phase, provided the diquark coupling is sufficiently strong, see Ref.[24]

In the present study, we will not investigate the dependence of the thermodynamics and phase structure on the strengths of the coupling constants which was

done in a previous work,[24] but rather adopt for their ratios with the scalar coupling strength the values given by the Fierz transformation of the one-gluon exchange interaction, i.e., $G_V = 0.5\ G_S$ and $G_D = 0.75\ G_S$. The values of G_S, cutoff Λ and current quark mases $m_{u,d}$ and m_s are chosen such as to describe the pion decay constant, pion and kaon masses as well as a light quark constituent mass of 350 MeV. The KMT coupling is fixed such that the $\eta - \eta'$ mass splitting is obtained.

In Fig. 1 we show the $T = 0$ EoS as $P(\mu)$ for the *ab initio* DBHF approach to nuclear matter together with the results of evaluating the mPNJL model for quark matter in meanfield approximation (2·5) for specific choices of the coefficient c in $\mathcal{U}(\Phi_0)$. It is interesting to note that those results compare well with those of the simplifying ansatz $\mathcal{U}(\Phi_0) = \mathcal{U}(\Phi_0; \mu = 0) + \Delta B$, for the region close to the hadron-to-quark matter transition as relevant for applications to compact star structure. Note, that in the present model the range of ΔB-values is limited to $|\Delta B| < 50$ MeV/fm^3. For the mPNJL model, there is no such limitation of the values for the parameter c.
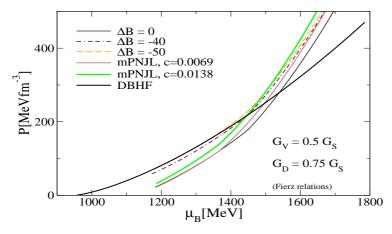

Fig. 1.   Equations of state for neutron star matter in beta-equilibrium. The hadronic phase is described by the DBHF EoS (solid line) and the quark matter EoS correspond to the mPNJL model (2·5) and its abridged version, for details see text.

In Fig. 2 we show the $M - R$ and $M - n_c$ relationships for nonrotating compact star sequences obtained as solutions of the Tolman-Oppenheimer-Volkoff equations for the hybrid EoS shown in Fig. 1 where the c-parameter of the mPNJL model (2·4) and the bag constant ΔB of the abridged mPNJL model are varied as respective free parameters. The diquark and the vector couplings are set to the values of the Fierz relation for one-gluon exchange, $\eta_V = G_V/G_S = 0.5$ and $\eta_D = 0.75$, resp. For $\Delta B[\text{MeV/fm}^3] = -50$ and -40 we obtain sequences of stable hybrid stars with 2SC quark core, in the mass range $1.74 < M/M_\odot < 2.04$ and $2.10 < M/M_\odot < 2.12$, respectively. For the other parameter choices, there is a direct transition from hadronic to CFL quark matter with a very small sequence of marginally stable CFL quark core hybrid stars at the respective maximum mass. A more detailed investigation is subject to ongoing work.

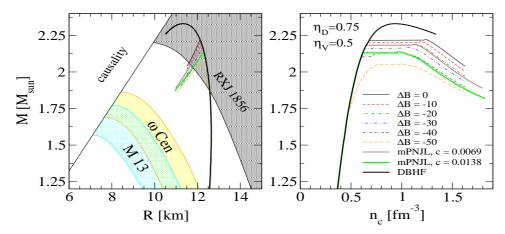The question arises whether flavor-mixing interactions other than the KMT one

Fig. 2. Compact star sequences for different values of the c parameter (bag constant ΔB in MeV/fm^3) in the (abridged) mPNJL model (2·4). For a discussion, see text.

have to be invoked. It seems necessary in a next step to include channels which appear after Fierz transformation of the KMT interaction and couple chiral condensates with diquark condensates.[30] This may lead to a new critical point in the QCD phase diagram[31] and, depending on the sign of the coupling, to a further reduction of the strange chiral condensate which enforces the flavor mixing effect studied here.

The observational constraints for masses and radii are not yet settled. There is a lower limit for the $M - R$ relation from RXJ 1856.5-3754[32] which requires either a large radius $R > 14$ km for a star with $M = 1.4\ M_\odot$ or a mass larger than $\sim 2\ M_\odot$ for a star with 12 km radius. The latter is accomodated with the present EoS. $M - R$ relations for the quiescent binary neutron stars in globular clusters M13[33] and ω Cen[34] point to rather light and compact stars as described by those sequences obtained here, but without quark matter cores for masses $M < 1.9\ M_\odot$.

In order to test any of the predictions for quark matter phases in compact stars and their possible consequences, we look forward to future observational campaigns devoted to determine the masses[35] and $M - R$ relationship for compact stars[36] with high precision and thus to constrain the dense matter EoS.[5]

§3. Conclusions

The effect of density-dependent modifications of the gluon condensate on the sequential occurrence of superconducting two- and three-flavor quark matter phase transitions in the EoS for cold dense matter has been studied in a modified PNJL model with color superconductivity and flavor-mixing KMT determinant interaction. Different from previous work, here we fix the coupling constants from hadron phenomenology and Fierz relations which leaves the unknown chemical potential dependence of the modified Polyakov-loop potential as a free parameter of the present study. It is found that for sufficiently large values of the parameter c there can be a stable quark matter core in massive hybrid stars.

Acknowledgements

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