# Strangeness S=-3 and -4 baryon-baryon interactions in chiral EFT

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**Abstract.** I report on recent progress in the description of baryon-baryon systems within chiral effective field theory. In particular, I discuss results for the strangeness $S = -3$ to $-4$ baryon-baryon systems, obtained to leading order.

## INTRODUCTION

Chiral effective field theory (EFT) as proposed in the pioneering works of Weinberg [1] is a powerful tool for the derivation of nuclear forces. In this scheme there is an underlying power counting which allows to improve calculations systematically by going to higher orders in a perturbative expansion. In addition, it is possible to derive two- and corresponding three-nucleon forces as well as external current operators in a consistent way. For reviews we refer the reader to Refs. [2, 3, 4].

Over the last decade or so it has been demonstrated that the nucleon-nucleon (*NN*) interaction can be described to a high precision within the chiral EFT approach [5, 6]. Following the original suggestion of Steven Weinberg, in these works the power counting is applied to the *NN* potential rather than to the reaction amplitude. The latter is then obtained from solving a regularized Lippmann-Schwinger equation for the derived interaction potential. The *NN* potential contains pion-exchanges and a series of contact interactions with an increasing number of derivatives to parameterize the shorter ranged part of the *NN* force.

Recently, also hadronic systems involving the strange baryons $\Lambda$ and $\Sigma$, and the $S = -2$ baryon $\Xi$ were investigated within EFT by the group in Jülich [7, 8, 9, 10, 11]. Specifically, the interactions in the $\Lambda N$ and $\Sigma N$ channels [7] as well as those in the $S = -2$ sector ($\Lambda\Lambda$, $\Sigma\Sigma$, $\Lambda\Sigma$, $\Xi N$) [8] were considered. In these works the same scheme as applied in Ref. [6] to the *NN* interaction is adopted. In the present contribution I focus on a recent extension of that study to systems with $S = -3$ and $-4$ [10].

## FORMALISM

To leading order (LO) in the power counting, as considered in the aforementioned investigations [7, 8, 10], the baryon-baryon potentials involving strange baryons consist of four-baryon contact terms without derivatives and of one-pseudoscalar-meson exchanges, analogous to the *NN* potential of [6]. The potentials are derived using constraints from SU(3) flavor symmetry. Details on the derivation of the chiral potentials for the $S = -1$ to $S = -4$ sectors at LO using the Weinberg power counting can be found in Ref. [7]. The contributions of one-pseudoscalar-meson exchanges are identical to those already discussed extensively in the literature, see, e.g., [7]. The LO $SU(3)_f$ invariant contact terms for the octet baryon-baryon interactions that are Hermitian and invariant under Lorentz transformations follow from the Lagrangians

$$
\begin{aligned}
\mathscr{L}^1 &= C_i^1 \langle \bar{B}_a \bar{B}_b (\Gamma_i B)_b (\Gamma_i B)_a \rangle , \quad \mathscr{L}^2 = C_i^2 \langle \bar{B}_a (\Gamma_i B)_a \bar{B}_b (\Gamma_i B)_b \rangle , \\
\mathscr{L}^3 &= C_i^3 \langle \bar{B}_a (\Gamma_i B)_a \rangle \langle \bar{B}_b (\Gamma_i B)_b \rangle .
\end{aligned}
\tag{1}
$$

As described in [7], in LO the Lagrangians give rise to six independent low-energy coefficients (LECs) – $C_S^1$, $C_T^1$, $C_S^2$, $C_T^2$, $C_S^3$ and $C_T^3$ – that need to be determined by a fit to experimental data. Here $S$ and $T$ refer to the central and spin-spin
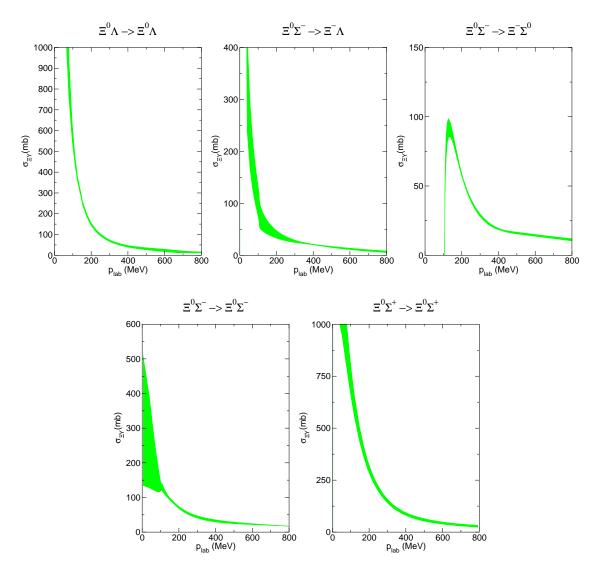
**FIGURE 1.** Total cross sections for various reactions in the strangeness $S = -3$ sector as a function of $p_{lab}$. The shaded band shows the chiral EFT results for variations of the cut-off in the range $\Lambda = 550\ldots700$ MeV.

parts of the potential, respectively. The scattering amplitude is obtained by solving a Lippmann-Schwinger equation for the LO potential. Thereby, the possible coupling between different baryon-baryon channels, $\Lambda N - \Sigma N$ or $\Xi\Lambda - \Xi\Sigma$, say, is taken into account. The potentials in the LS equation are cut off with a regulator function, $\exp\left[-\left(p'^4 + p^4\right)/\Lambda^4\right]$, in order to remove high-energy components of the baryon and pseudoscalar meson fields [6].

## RESULTS AND DISCUSSION

The LO chiral EFT interaction for the $S = -3$ and $-4$ baryon-baryon sector depends only on those five contact terms that enter also in the $YN$ interaction, cf. Table 1 in [10]. Thus, based on the values which were fixed in our study of the $YN$ sector [7] we can make genuine predictions for the interaction in the $S = -3$ and $-4$ channels that follow from the imposed $SU(3)_f$ symmetry.

Corresponding results for the $\Xi^0\Lambda \to \Xi^0\Lambda$, $\Xi^0\Sigma^- \to \Xi^-\Lambda$, $\Xi^0\Sigma^- \to \Xi^-\Sigma^0$, $\Xi^0\Sigma^- \to \Xi^0\Sigma^-$, and $\Xi^0\Sigma^+ \to \Xi^0\Sigma^+$ scattering cross sections are presented in Fig. 1. Partial waves with total angular momentum up-to-and-including $J = 2$ are taken into account. The shaded bands show the cut-off dependence. From that figure one observes that the $\Xi^0\Lambda \to \Xi^0\Lambda$ and $\Xi^0\Sigma^+ \to \Xi^0\Sigma^+$ cross sections are rather large near threshold. Though the cross section for
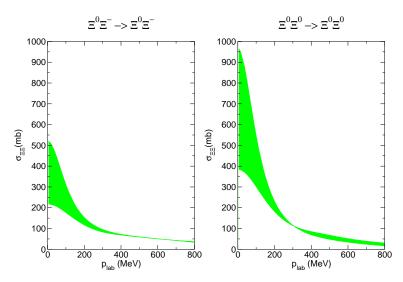
**FIGURE 2.** Total cross sections for the reactions $\Xi^0\Xi^- \rightarrow \Xi^0\Xi^-$ and $\Xi^0\Xi^0 \rightarrow \Xi^0\Xi^0$ as a function of $p_{lab}$. The shaded band shows the chiral EFT results for variations of the cut-off in the range $\Lambda = 550\ldots700$ MeV.

**TABLE 1.** Selected $\Xi Y$ and $\Xi\Xi$ singlet and triplet scattering lengths $a$ and effective ranges $r$ (in fm) for various cut-off values $\Lambda$. The last columns show results for the Nijmegen potential (NSC97a, NSC97f) [15] and the model by Fujiwara et al. (fss2) [16].

| | EFT | | | | NSC97a | NSC97f | fss2 |
|---|---|---|---|---|---|---|---|
| $\Lambda$ (MeV) | 550 | 600 | 650 | 700 | | | |
| $a_s^{\Xi\Lambda}$ | −33.5 | 35.4 | 12.7 | 9.07 | −0.80 | −2.11 | −1.08 |
| $r_s^{\Xi\Lambda}$ | 1.00 | 0.93 | 0.87 | 0.84 | 4.71 | 3.21 | 3.55 |
| $a_t^{\Xi\Lambda}$ | 0.33 | 0.33 | 0.32 | 0.31 | 0.54 | 0.33 | 0.26 |
| $r_t^{\Xi\Lambda}$ | −0.36 | −0.30 | −0.29 | −0.27 | −0.47 | 2.79 | 2.15 |
| $a_s^{\Xi^0\Sigma^+}$ | 4.28 | 3.45 | 2.97 | 2.74 | 4.13 | 2.32 | −4.63 |
| $r_s^{\Xi^0\Sigma^+}$ | 0.96 | 0.90 | 0.84 | 0.81 | 1.46 | 1.17 | 2.39 |
| $a_t^{\Xi^0\Sigma^+}$ | −2.45 | −3.11 | −3.57 | −3.89 | 3.21 | 1.71 | −3.48 |
| $r_t^{\Xi^0\Sigma^+}$ | 1.84 | 1.72 | 1.70 | 1.70 | 1.28 | 0.96 | 2.52 |
| $a_s^{\Xi\Xi}$ | 3.92 | 3.16 | 2.71 | 2.47 | 17.28 | 2.38 | −1.43 |
| $r_s^{\Xi\Xi}$ | 0.92 | 0.85 | 0.79 | 0.75 | 1.85 | 1.29 | 3.20 |
| $a_t^{\Xi\Xi}$ | 0.63 | 0.59 | 0.55 | 0.52 | 0.40 | 0.48 | 3.20 |
| $r_t^{\Xi\Xi}$ | 1.04 | 1.05 | 1.08 | 1.11 | 3.45 | 2.80 | 0.22 |

$\Xi^0\Sigma^- \rightarrow \Xi^-\Lambda$ rises too, in this case it is only due to the phase space factor $p_{\Xi^-\Lambda}/p_{\Xi^0\Sigma^-}$. There is a clear cusp effect visible in the $\Xi^0\Sigma^-$ cross section at $p_{lab} \approx 106$ MeV/c, i.e. at the opening of the $\Xi^-\Sigma^0$ channel. On the other hand, we do not observe any sizeable cusp effects in the $\Xi^0\Lambda$ cross section around $p_{lab} = 690$ MeV/c, i.e. at the opening of the $\Xi\Sigma$ channels. The latter is in line with the results reported by the Nijmegen group for their interactions [15], where a cusp effect in that channel is absent too. In this context I would like to remind the reader that the cusp seen in the corresponding strangeness $S = -1$ case, namely in the $\Lambda N$ cross section at the $\Sigma N$ threshold, is rather pronounced in our chiral EFT interaction [7] but also in conventional meson-exchange potential models [12, 13, 14].

Predicted cross sections for the $\Xi^0\Xi^0$ and $\Xi^0\Xi^-$ channels are shown in Fig. 2, again as a function of $p_{lab}$ and with shaded bands that indicate the cut-off dependence.

Results for the $\Xi^0\Lambda$, $\Xi^0\Sigma^+$, and $\Xi\Xi$ scattering lengths and effective ranges are listed in Table 1. Here we also include predictions by other models [15, 16] for channels where pertinent results are available in the literature. This Table reveals the reason for the sizeable $\Xi^0\Lambda$ cross section predicted by the chiral EFT interactions, namely a rather large scattering length in the corresponding $^1S_0$ partial wave. It is obvious that its value is strongly sensitive to cut-off

variations. It even changes sign (in other words, it becomes infinite) within the considered cut-off range. This means that a virtual bound state transforms into a real bound state, where the strongest binding occurs for the cut-off $\Lambda = 700$ MeV and leads to a binding energy of $-0.43\,\text{MeV}$. While this behaviour is interesting per se, one certainly has to stress that in such a case the predictive power of our LO calculation is rather limited. One has to wait for at least an NLO calculation, where we expect that the cut-off dependence will become much weaker so that more reliable conclusions on the possible existence of a virtual or a real bound state should be possible. The $^1S_0$ scattering lengths of the other potentials suggest also an overall attractive interaction in this partial wave though only a very moderate one.

The results for the $^3S_1$ state of the $\Xi^0\Lambda$ channel are fairly similar for all considered interactions. Moreover, with regard to the chiral EFT interaction there is very little cut-off dependence. The $S$-waves in the $\Xi\Sigma$ $I = 3/2$ channel belong to the same ($10^*$ and 27, respectively) irreducible representations where in the $NN$ case real ($^3S_1 - {}^3D_1$) or virtual ($^1S_0$) bound states exist, cf. Table 1 in Ref. [10]. Therefore, one expects that such states can also occur for $\Xi\Sigma$. Indeed, bound states are present for both partial waves in the Nijmegen model, cf. the discussion in Sect. III.B in Ref. [15]. Their presence is reflected in the positive and fairly large singlet and triplet scattering lengths for $\Xi^0\Sigma^+$, cf. Table 1. The chiral EFT interaction has positive scattering lengths of comparable magnitude for $^1S_0$, for all cut-off values, and therefore bound states, too. These binding energies lie in the range of $-2.23\,\text{MeV}$ ($\Lambda = 550\,\text{MeV}$) to $-6.15\,\text{MeV}$ (700 MeV). In the $^3S_1 - {}^3D_1$ partial wave the attraction is obviously not strong enough to form a bound state. The same is the case (but for both $S$ waves) for the quark model fss2 of Fujiwara et al. [16].

The $^1S_0$ state of the $\Xi\Xi$ channel belongs also to the 27plet irreducible representation and also here the Nijmegen as well as the chiral EFT interactions produce bound states. In our case the binding energies lie in the range of $-2.56\,\text{MeV}$ ($\Lambda = 550\,\text{MeV}$) to $-7.28\,\text{MeV}$ (700 MeV). The predictions of both approaches for the $^3S_1$ scattering length are comparable. The quark model of Fujiwara et al. exhibits a different behavior for the $\Xi\Xi$ channel, see the last column in Table 1. The small and negative $^1S_0$ scattering length signals an interaction that is only moderately attractive. The large and positive scattering length in the $^3S_1 - {}^3D_1$ partial wave, produced by that potential model, is usually a sign for the presence of a bound state, though according to the authors this is not the case for this specific interaction. Further results, and specifically $\Xi\Lambda$, $\Xi\Sigma$, and $\Xi\Xi$ phase shifts, can be found in [17].

## SUMMARY

Our investigations show that the chiral EFT scheme, successfully applied in Ref. [6] to the $NN$ interaction, also works well for the $\Lambda N$, $\Sigma N$ [7, 11] and $\Lambda\Lambda$ [8] interactions. Moreover, as reported here, it can be used o make predictions for the $S = -3$ and $-4$ baryon-baryon interactions invoking constraints from SU(3) flavor symmetry. It will be interesting to see whether the new facilities J-PARC (Tokai, Japan) and FAIR (Darmstadt, Germany) allow access to empirical information about the interaction in the $S = -3$ and $-4$ sectors. Such information could come from formation experiments of corresponding hypernuclei or from proton-proton and antiproton-proton collisions at such high energies that pairs of baryons with strangeness $S = -3$ or $S = -4$ can be produced.

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