Tracing a phase transition with fluctuations of the largest fragment size: Statistical multifragmentation models and the ALADIN S254 data

T. Pietrzak[1], P. Adrich[2], T. Aumann[2], C. O. Bacri[3],
T. Barczyk[1], R. Bassini[4], S. Bianchin[2], C. Boiano[4],
A. S. Botvina[2,5], A. Boudard[6], J. Brzychczyk[1], A. Chbihi[7],
J. Cibor[8], B. Czech[8], M. De Napoli[9], J.-É. Ducret[6], H. Emling[2],
J. D. Frankland[7], M. Hellström[2], D. Henzlova[2], G. Immè[9],
I. Iori[4,*], H. Johansson[2], K. Kezzar[2], A. Lafriakh[6],
A. Le Fèvre[2], E. Le Gentil[6], Y. Leifels[2], J. Lühning[2],
J. Łukasik[8], W. G. Lynch[10], U. Lynen[2], Z. Majka[1], M. Mocko[10],
W. F. J. Müller[2], A. Mykulyak[11], H. Orth[2], A. N. Otte[2],
R. Palit[2], P. Pawłowski[8], A. Pullia[4], G. Raciti[9,*], E. Rapisarda[9],
H. Sann[2,*], C. Schwarz[2], C. Sfienti[2], H. Simon[2],
K. Sümmerer[2], W. Trautmann[2], M. B. Tsang[10], G. Verde[10],
C. Volant[6], M. Wallace[10], H. Weick[2], J. Wiechula[2],
A. Wieloch[1] and B. Zwiegliński[11]

[1] Institute of Physics, Jagiellonian University, 30-059 Kraków, Poland
[2] GSI Darmstadt, D-64291 Darmstadt, Germany
[3] IPN Orsay, IN2P3-CNRS et Universite, F-91406 Orsay, France
[4] Istituto di Scienze Fisiche, Università and INFN, I-20133 Milano, Italy
[5] Institute for Nuclear Research, 117312 Moscow, Russia
[6] DAPNIA/SPhN, CEA/Saclay, F-91191 Gif-sur-Yvette, France
[7] GANIL, CEA et IN2P3-CNRS, F-14076 Caen, France
[8] Institute of Nuclear Physics PAN, 31-342 Kraków, Poland
[9] Dipartimento di Fisica and INFN-LNS, I-95123 Catania, Italy
[10] Dept. of Physics and NSCL, MSU, East Lansing, MI 48824, USA
[11] A. Sołtan Institute for Nuclear Studies, Pl-00681 Warsaw, Poland
* deceased

Abstract

A phase transition signature associated with cumulants of the largest fragment size distribution has been identified in statistical multifragmentation models and examined in analysis of the ALADIN S254 data on fragmentation of neutron-poor and neutron-rich projectiles. Characteristics of the transition point indicated by this signature are weakly dependent on the A/Z ratio of the fragmenting spectator source. In particular, chemical freeze-out temperatures are estimated within the range 5.9 to 6.5 MeV. The experimental results are well reproduced by the SMM model.

In nuclear multifragmentation studies a special attention has been paid to the largest fragment size which is expected to play the role of the order parameter, and thus provide valuable insight into the phase behavior of investigated systems [1, 2, 3, 4, 5]. In particular, percolation studies have shown that a cumulant analysis focused on the skewness, K_3, and the kurtosis excess, K_4, of the largest fragment size distribution is a valuable method to reveal the presence of a phase transition (critical behavior) in finite systems [5]. The percolation transition is indicated by $K_3 = 0$ and a K_4 minimum of about -1 for all system sizes. Events may be sorted according to various measurable quantities that are correlated with the control parameter.

In this work we show that such cumulant features are not restricted to the percolation transition but also are observed at a liquid-gas phase transition present in statistical multifragmentation models. With this justification the cumulant analysis will be applied to the ALADIN S254 experimental data obtained with radioactive beams to investigate the presence and isotopic dependence of the transition signals.

It is instructive to look first at the predictions of the thermodynamic model which is known to contain a first-order phase transition. The thermodynamic model is a simplified version of the statistical multifragmentation model (SMM), allowing to compute the partition function and thus to obtain thermodynamic properties of the system [6]. Our calculations were performed for the canonical ensemble of noninteracting one-component fragments at the constant freeze-out volume three times larger than the normal nucleus volume. Figure 1 shows the specific heat and the statistical measures of the largest fragment size distribution for three systems with 64, 216, and 1000 nucleons. The broadening of C_V peaks and reduction in peak intensities, as well as the decrease of the transition temperatures indicated by the C_V maxima, is observed with decreasing system size due to finite-size effects.
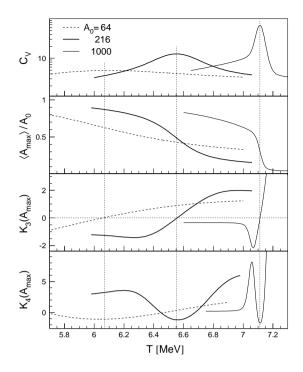
Figure 1: Canonical thermodynamic model predictions. From top to bottom: the specific heat, the mean, skewness and kurtosis of the largest fragment mass distribution as a function of the freeze-out temperature for different system masses.

At the transition point, the mean A_{max} exhibits the fastest decrease, which tends with increasing system size to a step discontinuity expected in the thermodynamic limit. It is also clearly seen that the transition is associated with $K_3 = 0$ and minimum K_4, as in the case of the percolation transition. Although these cumulant features are similar for both transition types, the shapes of the transitional A_{max} distributions are distinctly different. In the present model the distribution has a bimodal structure as expected for the order parameter at a first-order phase transition in the canonical ensemble. In percolation the transition is continuous and the distribution is single-peaked.

In the next step we have examined the cumulants of the largest fragment charge (Z_{max}) distributions within the SMM model [7]. Calculations were performed with the standard SMM code which includes secondary decays. The freeze-out density of one third of the normal nuclear density was assumed. For a given system (A,Z), events were uniformly generated over a

wide range of the excitation energy, and then sorted according to the measurable quantity Z_{bound} used as a control parameter.The simulation results have shown that, similarly as in the thermodynamic model, the cumulant signal is well observed at a point where Z_{max} rapidly decreases. The mean excitation energy and the mean microcanonical temperature at this Z_{bound} point correspond to the flattest part of the caloric curve. The temperature derived in this way, referred to as the SMM breakup temperature, T_b, is shown in Fig. 2(a) as a function of the system charge Z for the A/Z ratios of 2.17 and 2.49. These ratios are relevant to the neutron-poor and neutron-rich systems investigated in the S254 experiment. As one can see, the temperature is around 6 MeV and weakly depends on the system size and the A/Z ratio. For a comparison, Fig. 2(b) shows the Hartree-Fock limiting temperatures [8]. In this case, regarding their absolute magnitude which depends on the nuclear potential form used in the calculations [9], the mass/charge dependences are much stronger. The discrimination between the two scenarios was one of the motivations for the S254 experiment.
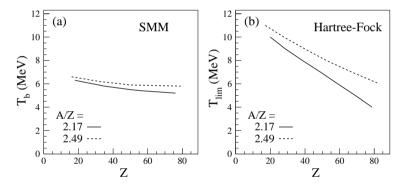

Figure 2: SMM breakup temperatures (a) and limiting temperatures from [8] (b).

The ALADIN experiment S254 was conducted at GSI Darmstadt. Beams of ^{107}Sn, ^{124}Sn and ^{124}La were used to investigate isotopic effects in projectile-spectator fragmentation at 600 AMeV. The secondary beams with neutron-poor ^{107}Sn and ^{124}La projectiles contained also some fraction of neighbouring isotopes. The mean compositions of the nominal ^{107}Sn (^{124}La) beams were $\langle Z \rangle = 49.7\,(56.8)$ and $\langle A/Z \rangle = 2.16\,(2.19)$, respectively [10].

The most prominent result of the experiment is the observation that the isotopic dependence of the projectile fragmentation is weak [11]. The mean IMF multiplicity, the mean largest fragment charge, and the double-

isotope temperatures, T_{HeLi} and T_{BeLi}, determined as a function of Z_{bound}, are nearly invariant with respect to the projectile A/Z ratio. These can be seen in the top panels of Fig. 3 for the former quantities. When comparing the Sn and La results, the resemblance is better if $\langle Z_{max} \rangle$ and Z_{bound} are normalized to the projectile charge. Figure 3 allows to examine the pattern of Z_{max} fluctuations by looking at the variance and the K_3, K_4 cumulants of the Z_{max} distribution. Also in this case the results are similar for all the projectiles.
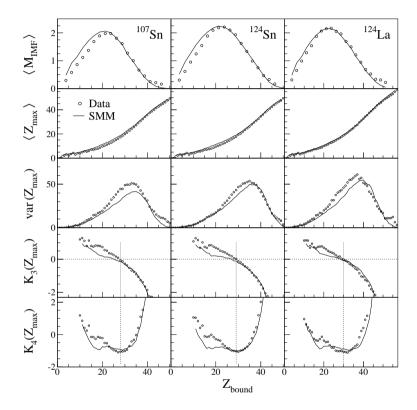

Figure 3: Experimental data versus SMM predictions. From top to bottom: the mean IMF multiplicity, the mean, variance, skewness and kurtosis of the Z_{max} probability distribution as a function of Z_{bound}.

The signals characteristic of a phase transition ($K_3 = 0$ and minimum K_4) are observed at $Z_{bound} = 28$, 29 and 30 for the ^{107}Sn, ^{124}Sn and ^{124}La projectiles, respectively. It should be noted that the shapes of the investigated Z_{max} distributions are highly irregular since the constraints imposed by fixed Z_{bound} values are strong in such small systems. Therefore, the

presence of some distribution features (*e.g.* bimodality) cannot be trustworthy concluded. Despite that, the evolution of the cumulant values is quite smooth so that the transition signals are clearly identified. Given the signal locations on the Z_{bound} axis, we could estimate parameters of the fragmenting sources at the transition as shown in Table 1. The mean charge of the fragmenting system, Z_0, was estimated on the basis of percolation and SMM simulations as 10-20% larger than the Z_{bound} value. The values of the isotope temperature T_{HeLi} were raised by 15% with a possible error of $\pm 5\%$, according to the expected range of corrections for secondary decays feeding [12]. The obtained temperatures are in an approximate agreement with the SMM predictions given in the last column. The SMM breakup temperatures were read out from Fig. 2 with the assumption that the A/Z ratio of the fragmenting system remains that of the projectile.

Projectile	Z_{bound}	Z_0	T_{HeLi} (MeV)	T_b - SMM (MeV)
^{107}Sn	28	32.2±2	5.9±0.3	5.87±0.05
^{124}Sn	29	33.5±2	6.5±0.3	6.18±0.05
^{124}La	30	34.5±2	6.2±0.3	5.82±0.05

Table 1: Freeze-out characteristics of the fragmenting systems at the transition indicated by Z_{max} fluctuations.

For more detailed comparisons with the experimental data we have performed SMM ensemble calculations following the procedure described in [13]. The ensemble specifies the probability distribution of the thermalized sources defined by A, Z and E^*. Parameters of the ensemble were adopted from earlier results for ^{197}Au fragmentation [13], and then individually adjusted to improve agreements with the experimental IMF multiplicity distributions. The quality of the obtained agreements can be seen in the top panel of Fig. 3. The next panels show the predicted characteristics of the Z_{max} distributions. The overall agreement with the data can be concluded as satisfactory, considering the fact that no optimization of the model parameters was performed.

In summary, simulations with the canonical thermodynamic model and the SMM model corroborate the percolation suggestion that cumulants such as the skewness (K_3) and the kurtosis excess (K_4) of the largest fragment size distribution are valuable observables in searching for a phase transition in multifragmentation. The location of a transition is well indicated by $K_3 = 0$ and K_4 minimum, even in very small systems. The measurable quantity Z_{bound} may be used to sort events as a control parameter. The cumulant

analysis applied to the ALADIN S254 experimental data has shown that the transition signals are observed at nearly the same Z_{bound} values of 28-30 for all the projectiles. The corresponding freeze-out temperatures are in the vicinity of 6 MeV and the temperature difference between the neutron-rich and neutron-poor sources is within a few percent, in agreement with SMM predictions. A satisfactory overall description of the experimental Z_{max} distributions can be obtained with SMM ensemble calculations. The observed agreement with the statistical model indicate that in the multifragmentation process the accessible phase-space is of dominant importance.

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