The current evaluation of V_{ud}

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The V_{ud} element of the Cabibbo-Kobayashi-Maskawa matrix can be determined from several different experimental approaches: either $0^+ \rightarrow 0^+$ superallowed nuclear β decays, neutron decay, nuclear mirror decays, or pion β decay. Currently all give consistent results but, because the nuclear superallowed value has an uncertainty at least a factor of seven less than all other results, it dominates the result. A new survey of world superallowed-decay data establishes the $\mathcal{F}t$ values of 14 separate superallowed transitions to a precision of order 0.1% or better; and all 14 are statistically consistent with one another. This very robust data set yields the result $V_{\mathrm{ud}} = 0.97417(21)$, the value we recommend.

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1 Superallowed nuclear beta decay

Beta decay between nuclear analog states of spin-parity, $J^\pi = 0^+$, and isospin, $T = 1$, has a unique simplicity: It is a pure vector transition and is nearly independent of the nuclear structure of the parent and daughter states. Such transitions are called "superallowed." Their measured strength – expressed as an "ft value" – can be related directly to the vector coupling constant for semi-leptonic decays, G_V, with the intervention of only a few small ($\sim$1%) calculated terms to account for radiative and nuclear-structure-dependent effects. Once G_V has been determined in this way, it is only another short step to obtain a value for V_{ud}, the up-down mixing element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

The ft value of any β transition is simply the product of the phase-space factor, f, and the partial half-life of the transition, t. It depends on three measured quantities: the total transition energy, Q_{EC}, the half-life, $t_{1/2}$, of the parent state, and the branching ratio, R, for the particular transition of interest. The Q_{EC} value is required to determine f, while the half-life and branching ratio combine to yield the partial half-life.

In dealing with superallowed decays, it is convenient to combine some of the small correction terms with the measured ft-value and define a corrected $\mathcal{F}t$-value. Thus, we write [1]

$$\mathcal{F}t \equiv ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{2G_V^2(1 + \Delta_R^V)} \, , \tag{1}$$

where $K = 8120.2776(9) \times 10^{-10}$ GeV^{-4}s, δ_C is the isospin-symmetry-breaking correction and Δ_R^V is the transition-independent part of the radiative correction. The terms δ'_R and δ_{NS} constitute the transition-dependent part of the radiative correction, the former being a function only of the electron's energy and the Z of the daughter nucleus, while the latter, like δ_C, depends in its evaluation on nuclear structure. From this equation, it can be seen that a measurement of any one superallowed transition establishes a value for G_V. The measurement of several tests the Conserved Vector Current (CVC) hypothesis that G_V is not renormalized in the nuclear medium. If indeed G_V is constant – i.e. all the $\mathcal{F}t$-values are the same – then an average value for G_V can be determined and V_{ud} obtained from the relation $V_{ud} = G_V/G_F$, where G_F is the well known [2, 3] weak-interaction constant for purely leptonic muon decay.

It is important to note that if, instead, the $\mathcal{F}t$ values show a significantly non-statistical inconsistency, one to the other, then the remaining steps cannot be taken since inconsistency would demonstrate that the correction terms were not correct or, less likely, that CVC had been violated. Without consistency, there is no coupling "constant" and there can be no justification for extracting a value for V_{ud}.

Early in 2015, we published [1] a new critical survey of all half-life, decay-energy and branching-ratio measurements related to 20 superallowed $0^+ \rightarrow 0^+$ β decays. Included were 222 individual measurements of comparable precision obtained from
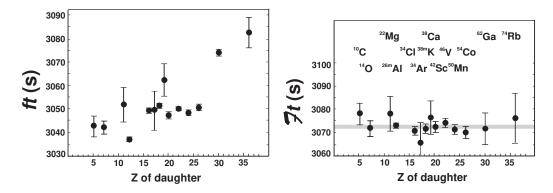
Figure 1: Results from the 2015 survey [1]: uncorrected ft values for the 14 best known superallowed decays on the left; the same results but incorporating the δ'_{R}, δ_{C} and δ_{NS} correction terms on the right. The grey band in the right panel is the average $\mathcal{F}t$ value and its uncertainty.

177 published references. We obtained world-average ft values for each of the 18 transitions that had a complete set of data, and then applied radiative and isospin-symmetry-breaking corrections to extract corrected $\mathcal{F}t$ values. A total of 14 of these $\mathcal{F}t$ values have a precision of order 0.1% or better; their uncorrected ft values and corrected $\mathcal{F}t$ values are shown in Fig. 1.

It is immediately evident from the figure that the $\mathcal{F}t$ values are all consistent with one another from $A=10$ to $A=74$. This simultaneously confirms the CVC expectation of a constant value for G_{v} and demonstrates the absence of any significant scalar current, which would introduce an upward or downward curve into the $\mathcal{F}t$-value locus at low Z [1]. It also goes a long way towards validating the particular set of calculated transition-dependent corrections that were used in the analysis. These calculations of δ_{C} and δ_{NS} were an updated version of those presented in Ref. [4] and employed the best available shell-model wave functions, which in each case had been based on a wide range of spectroscopic data for nuclei in the same mass region. They were further tuned to agree with measured binding energies, charge radii and coefficients of the isobaric multiplet mass equation for the specific states involved. This means that the origins of these correction terms are completely independent of the superallowed decay data, so consistency in the corrected $\mathcal{F}t$ values gives powerful support to the calculated corrections used in the derivation of those $\mathcal{F}t$ values. We will return later to the question of alternative calculations for these correction terms.

With a mutually consistent set of $\mathcal{F}t$ values, one is then justified in proceeding to determine the value of G_{v} and, from it, V_{ud}. The result we obtained from the new survey is

$$|V_{\mathrm{ud}}| = 0.97417(21) \qquad \text{[nuclear superallowed]}.$$

2

2 Other methods for determining V_{ud}

Neutron β decay is the simplest β decay to involve both the vector and axial-vector weak interactions. It is an attractive option for determining V_{ud} since its analysis does not require the application of corrections for isospin-symmetry-breaking, δ_{C}, or for nuclear-structure-dependent radiative effects, δ_{NS}. However, it has the distinct disadvantage that it requires a difficult correlation measurement in order to separate the vector-current contribution to its decay from the axial-vector one. Not only that, but neutrons are inherently more difficult to handle and contain than nuclei.

Since the Q_{EC} value and the branching ratio for neutron β decay are very well known, the crucial measurements required to determine V_{ud} are its mean-life and a decay correlation – usually selected to be the β asymmetry from the decay of polarized neutrons. World data for both these quantities are not statistically consistent among themselves, the normalized chi-squared (χ^2/N) for the mean-life average being 3.4 and that for the β assymmetry being 3.8. More alarming still is the fact that the mean-life results from two different measurement techniques appear to be systematically different from one another. The average mean-life obtained when the decay products are recorded from a beam of neutrons is 888.1(20)s; while it is 879.5(7)s when neutrons are confined in a "bottle" and the survivors are counted a known time later. It is difficult to know how to deal with such conflicts so we employ two different methods. With the first, we follow exactly the same procedures as we do for the superallowed decays, averaging all world data for each parameter and increasing its uncertainty by the square root of the normalized chi-squared. For the second we simply assign a range to the mean-life, which encompasses both the conflicting sets of results. The results for V_{ud} are

$$|V_{\mathrm{ud}}| = 0.9754(14) \qquad \text{[neutron average]},$$
$$0.9707 \leq V_{\mathrm{ud}} \leq 0.9761 \qquad \text{[neutron range]}.$$

Neutron β decay is just a special case of decay between $T = 1/2$ mirror nuclei. Like neutron decay, these nuclear mirror decays are mixed vector and axial-vector decays; so, in addition to Q_{EC} values, half-lives and branching ratios, they also require a β-asymmetry measurement. Of course, unlike the neutron, these decays as well require the corrections δ_{C} and δ_{NS} for small nuclear-structure-dependent effects. There are five mirror decays, ^{19}Ne, ^{21}Na, ^{29}P, ^{35}Ar and ^{37}K, for which sufficient data are known. The relevant world data were first surveyed in 2008 [5], from which a value of $|V_{\mathrm{ud}}|$ was obtained [6]. More data have appeared since and been incorporated [7] although there has been very little change in the $|V_{\mathrm{ud}}|$ value obtained. The current result is

$$|V_{\mathrm{ud}}| = 0.9718(17) \qquad \text{[mirror nuclei]}.$$

Finally, the rare pion beta decay, $\pi^+ \to \pi^0 e^+ \nu_e$, which has a branching ratio of $\sim 10^{-8}$, is one of the most basic semi-leptonic electroweak processes. It is a pure
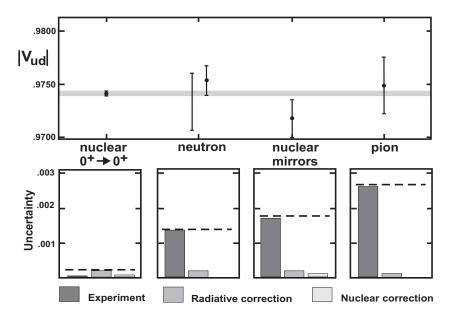
Figure 2: The five values of $|V_{ud}|$ given in the text are shown in the top panel, the grey band being the average value. The four panels at the bottom show the error budgets for the corresponding results with points and error bars at the top.

vector transition between two spin-zero members of an isospin triplet and is therefore analogous to the superallowed $0^+ \to 0^+$ decays. In principle, it can yield a value of V_{ud} unaffected by nuclear-structure uncertainties. In practice, the branching ratio is very small and has proved difficult to measure with sufficient precision. The most recent, and by far the most precise, measurement of the branching ratio is by the PIBETA group [8]. This leads to the result [9]

$$|V_{ud}| = 0.9749(26) \qquad \qquad \text{[pion]}.$$

3   Recommended value for V_{ud}

The five results we have quoted for $|V_{ud}|$ are plotted in Fig. 2. Obviously they are consistent with one another but, because the nuclear superallowed value has an uncertainty a factor of 7 to 13 smaller than the other results, it dominates the average. Furthermore, the more precise of the two neutron results can hardly be considered definitive since it ignores a serious systematic uncertainty in the data. Consequently we recommend using the nuclear superallowed result as the best value for $|V_{ud}|$: i.e.

$$|V_{ud}| = 0.97417(21). \qquad \qquad (2)$$

4   Potential for improvement

The uncertainty budgets plotted in the bottom panels of Fig. 2 reveal three important facts. First, experimental uncertainties dominate in the cases of the less-precisely known neutron, nuclear-mirror and pion β-decays; while theory makes the largest contribution to the overall uncertainty for the key $0^+ \to 0^+$ decays. Second, by far the most important theoretical contribution to the latter is from the radiative correction, principally it turns out [1] from Δ_R^V, the transition-independent part of the radiative correction. Finally, the size of the Δ_R^V contribution is the same for all measurement methods; thus we can conclude that no major improvement in the value of $|V_{ud}|$ can be achieved in future without improved calculations of Δ_R^V.

Unfortunately, experiment can play no role in reducing the Δ_R^V uncertainty. That must remain a purely theoretical challenge. The impact of any improvement would be immediate though: If the Δ_R^V uncertainty were to be cut in half, the $|V_{ud}|$ uncertainty would be reduced by 30%.

In the meantime, some small improvement in the $|V_{ud}|$ uncertainty can still be made with the help of nuclear experiments. These experiments can contribute to improving the nuclear-structure-dependent corrections (δ_C - δ_{NS}), which produce the second largest component of the $|V_{ud}|$ uncertainty budget for the $0^+ \to 0^+$ decays (see Fig. 2). In the past few years, a number of different groups have published δ_C values from calculations based upon a variety of different model approaches. Typically each calculation covers only a subset of the measured transitions but the subsets are not the same from calculation to calculation and, where overlap does exist, the results are not notably consistent with one another. This diversity of results has prompted us to develop a test [10] to assess the quality of each calculated set of corrections and determine its relative merit. The test is based on the premise that the CVC hypothesis is valid and thus the corrected $\mathcal{F}t$ values for all measured transitions should be statistically consistent with one another (*i.e.* with $\chi^2/N \sim 1$).

This test has already contributed to reducing the uncertainty in δ_C. As part of our recent survey [1], we applied the test to all sets of calculations that cover at least half the number of well-measured superallowed transitions. The resultant χ^2/N values for the various calculations spanned a wide range, with only a single set [4] yielding a value near one. In this way, we identified that set as the one to use in our ultimate analysis of the experimental data (see Fig. 1). No allowance for systematic differences between otherwise acceptable calculations was required since no other set passed the acceptability test.

There is a second test that can be expected to refine the selection process for δ_C calculations even further. It involves the measurement of mirror pairs of superallowed transitions, which has only just become possible, with the first case — $^{38}Ca \to {}^{38m}K$ and $^{38m}K \to {}^{38}Ar$ — having appeared very recently [11, 12]. This test also depends on the expected constancy of $\mathcal{F}t$ values, but in this instance it applies to the two

members of a mirror pair of $0^+ \to 0^+$ transitions. Considering current capabilities for producing superallowed $T_Z = -1$ parent nuclei in sufficient quantity for a high-statistics measurement, we conclude that there are three mirror pairs in addition to the one at $A=38$ that can be completed in the immediate future. These are ^{26}Si $\to^{26m}$Al and ^{26m}Al $\to^{26}$Mg; ^{34}Ar $\to^{34}$Cl and ^{34}Cl $\to^{34}$S; and ^{42}Ti $\to^{42}$Sc and ^{42}Sc $\to^{42}$Ca.

These tests have already played a role in reducing the $|V_{ud}|$ uncertainty. With improved measurement precision on the already known ft values, together with the addition of new mirror pairs of transitions, some modest further improvement can be expected. However, ultimately these will only have a significant impact on the $|V_{ud}|$ result after meaningful improvements have been made in the calculation of Δ_R^V.

5   Note on $|V_{us}|$ and the CKM unitarity test

The standard model does not prescribe the individual elements of the CKM matrix – they must be determined experimentally – but absolutely fundamental to the model is the requirement that the matrix be unitary. To date, the most demanding test of CKM unitarity comes from the sum of squares of the top-row elements, $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$, which should equal exactly one. Combining our value for $|V_{ud}|$ in Eq. (2) with the values of $|V_{us}|$ and $|V_{ub}|$ recommended by the Particle Data Group (PDG) [2], the top-row sum yields the result 0.99978(55), in excellent agreement with unitarity.

Unfortunately this cannot be the last word since the PDG evaluation does not include recent results from the most recent lattice calculations, which are used to extract $|V_{us}|$ from semileptonic kaon decays ($K \to \pi\ell\nu_\ell$), and $|V_{us}|/|V_{ud}|$ from the ratio of the pure leptonic decay of the kaon ($K^\pm \to \mu^\pm\nu$) to to that of the pion ($\pi^\pm \to \mu^\pm\nu$). In the past, the results for $|V_{us}|$ and $|V_{us}|/|V_{ud}|$ have formed a consistent set with the result for $|V_{ud}|$. As the quoted uncertainties on the lattice calculations have been reduced, however, some tension has appeared, with the combination of results for $|V_{ud}|$ and $|V_{us}|/|V_{ud}|$ continuing to yield excellent agreement with unitarity but the combination of $|V_{ud}|$ and $|V_{us}|$ being low by two standard deviations. This is not a cause for serious concern, but the inconsistency between the two kaon-decay approaches will need to be resolved in future.

This subject is discussed in more detail in section VB of Ref. [1].

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