Nambu-Goldstone Modes in Segregated Bose-Einstein Condensates

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Nambu–Goldstone modes in immiscible two-component Bose-Einstein condensates are studied theoretically. In a uniform system, a flat domain wall is stabilized and then the translational invariance normal to the wall is spontaneously broken in addition to the breaking of two U(1) symmetries in the presence of two complex order parameters. We clarify properties of the low-energy excitations and identify that there exist two Nambu–Goldstone modes: an in-phase phonon with a linear dispersion and a ripplon with a fractional dispersion. The signature of the characteristic dispersion can be verified in segregated condensates in a harmonic potential.

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Spontaneous symmetry breaking (SSB) is a universal phenomenon that can occur in almost all energy scales in nature. When continuum symmetries of a system are spontaneously broken, there exist gapless modes called Nambu–Goldstone (NG) modes [1], which determine the low-energy properties of the system. In Lorenz-invariant systems, the number of NG modes (n_{NG}) coincides with the number of broken symmetries (n_{SSB}) , and their dispersions are linear as $\epsilon \propto p$, where ϵ and p are the energy and momentum of the NG modes, respectively. However, this counting rule of n_{NG} fails in systems without Lorentz invariance or with spontaneously broken spacetime symmetry [2–5]. Recently, the NG theorem was generalized to the nonrelativistic system and the counting rule has been re-examined [6–9].

Superfluids or Bose–Einstein condensates (BECs) [10, 11 are important foundations upon which to study SSB and NG modes. In a single-component superfluid or a scalar BEC, the appearance of the order parameter is associated with the SSB of the global U(1) symmetry. The accompanying NG mode is known as a phonon with a linear dispersion. Another important example is a spinor BEC with spin-1 bosons [12], where the symmetry of spin rotation is also broken in addition to the global U(1). In the polar phase with $n_{\rm NG} = n_{\rm SSB} = 3$, the dispersions of the three NG modes are all linear. However, in the ferromagnetic phase, we have only two NG modes with $n_{\rm SSB} = 3$ and $n_{\rm NG} = 2$, and then a NG mode has a linear dispersion and the other has a quadratic one that comes from a conjugate pairing between the generators of the two broken symmetries [7].

The NG theorem is nontrivial when the spatial symmetry of the system is spontaneously broken. Such examples can be seen in a scalar BEC with vortices. When there is a straight vortex along the z axis in a uniform system, the translational symmetries in the x and y directions are explicitly broken, and thus $n_{\rm SSB}=3$. However, there are only two NG modes: the Kelvin mode with a quadratic dispersion, which causes helical deformation of a vortex line, and the varicose mode with a linear dispersion, which corresponds to a phonon propagating along the vortex core [10, 13, 14]. Another example can be seen

in Tkachenko modes of two-dimensional vortex lattices in rotating superfluids [9].

A nontrivial example can also be seen in twocomponent BECs, i.e., condensates of two distinguishable bosons [15]. The system is characterized by twocomponent order parameters Ψ_i (j = 1, 2), which allow the excitation of two independent phonons associated with the two broken U(1) symmetries. Two-component BECs are characterized by the inter-atomic coupling constants g_{ik} proportional to the s-wave scattering length a_{jk} between the jth and kth components. A homogeneous two-component system is miscible for $g_{12}^2 < g_{11}g_{22}$ with $g_{ij} > 0$ and then there are two NG modes with $\epsilon \propto p$, while for the "fine-tuned" interaction parameter $g_{12}^2 = g_{11}g_{22}$ the dispersion of a mode becomes quadratic with $\epsilon \propto p^2$. For $g_{12} > \sqrt{g_{11}g_{22}}$, the strong intercomponent repulsion leads to a phase-separation forming domain walls [16–19]. In the presence of a flat domain wall, the translational symmetry normal to the wall is spontaneously broken, and thus the transverse shift of the wall costs zero energy. There are some studies on interface modes in segregated two-component BECs [20–24]. Mazets showed that, under the Bogoliubov-de Gennes (BdG) analysis with a suitable ansatz of the domain wall profile, there are two branches of waves localized near a domain wall with fractional dispersions, i.e., $\epsilon \propto p^{3/2}$ and $\epsilon \propto p^{1/2}$ [20]. Later, the dispersion $\epsilon \propto p^{3/2}$ was derived from the hydrodynamic effective theory of ripple waves on a domain wall in segregated BECs [22]. However, the number of NG modes and their details in the low-energy limit have not been confirmed vet.

In this paper, we provide a full account of the NG modes in segregated two-component BECs. The semi-classical analysis of the NG modes in a uniform system reveals that the dispersion $\epsilon \propto p^{\gamma}$ with $\gamma < 1$ is forbidden for the localized modes in the low-energy limit, which conflicts with the prediction of $\gamma = 1/2$ in Ref. [20]. Even in the highly compressible BECs, the interface mode, referred to as a ripplon, has a dispersion $\epsilon \propto p^{3/2}$, which is similar to a standard capillary wave in classical incompressible hydrodynamics. We confirm that, despite $n_{\rm SSB} = 3$, there exist two NG modes, a ripplon and a

phonon, in the low-energy limit by carefully calculating the system size dependence of the dispersion relation through the numerical analysis of the BdG equation. We also discuss these low-energy modes in segregated condensates trapped by a harmonic potential.

We consider binary BECs described by complex order parameters $\Psi_j = \sqrt{n_j} e^{i\theta_j}$ in the mean-field approximation at zero temperature. Let us start with the Gross-Pitaevskii Lagrangian for the two-component BECs,

$$\mathcal{L} = \int d^3x \left(\mathcal{P}_1 + \mathcal{P}_2 - g_{12} |\Psi_1|^2 |\Psi_2|^2 \right), \tag{1}$$

where $\mathcal{P}_j = i\hbar \Psi_j^* \partial_t \Psi_j + (\hbar^2/2m_j) \Psi_j^* \nabla^2 \Psi_j - (V_j - \mu_j) |\Psi_j|^2 - g_{jj} |\Psi_j|^4/2$ with m_j , V_j , and μ_j being the atomic mass, the external trap potential, and the chemical potential of the jth component, respectively. The intra- and inter-component interaction parameters have the form $g_{jk} = 2\pi\hbar^2 a_{jk} (m_j^{-1} + m_k^{-1})$. We assume $a_{jk} > 0$ in the following.

We first consider a homogeneous system with $V_j = 0$. The condensates are miscible and immiscible when $g_{11}g_{22} > g_{12}^2$ and $g_{11}g_{22} < g_{12}^2$, respectively [16, 17]. For both cases, there occurs the trivial SSB related to the phases θ_1 and θ_2 ; the Lagrangian is invariant when the phases are rotated independently. For the immiscible case, the spatial symmetry is additionally broken in the presence of a domain wall due to the phase separation. This symmetry breaking may cause a ripplon, which is a quantum representation of ripple waves propagating on a domain wall, as a NG mode in this system. We confine ourselves to the immiscible case throughout this work.

Assuming a flat domain wall normal to the z axis in a stationary state $\Psi_j = \psi_j(z)$, Eq. (1) yields

$$0 = \left(-\frac{\hbar^2}{2m_j}\frac{d^2}{dz^2} - \mu_j + \sum_k g_{jk}\psi_k^2\right)\psi_j.$$
 (2)

Here, we have assumed $\text{Im}\psi_i = 0$ without loss of generality. Figure 1(a) shows the spatial profile of the stationary solution with a flat domain wall for the feasible parameters $\mu_1 = \mu_2 = \mu$, $g_{11} = g_{22} = g$, $m_1 = m_2 = m$ and $g_{12}/g = 1.2$, e.g., in Ref. [25]. The length is scaled by the healing length $\xi = \hbar/\sqrt{m\mu}$. The two components are separated by the domain wall along the z = 0 plane. There is an overlapping region, in which one component penetrates into the other component. The width ξ_w of the overlapping region diverges as $\xi_w \propto (1 - g_{12}/\sqrt{g_{11}g_{22}})^{-1/2}$ for $g_{12} \to \sqrt{g_{11}g_{22}}$ [17]. Here, we consider a well-segregated case with $\xi_w \sim \xi$. The tension of a flat domain wall is calculated as the excess energy in the presence of a domain wall. $\sigma = -\sum_{j} (\hbar^2/4m_j) \int dz \psi_j \partial_z^2 \psi_j$, under the pressure equilibrium $P_1 = P_2$ with the hydrostatic pressure $P_j = \mu_j^2/2g_{jj} = P$ of the jth component in the bulk, $|z| \to \infty$ [17].

The NG modes in the segregated BECs are fully solved by the BdG formalism. By writing a perturbation of the

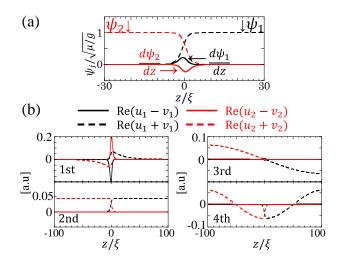


FIG. 1: (Color online) (a) Profiles of order parameters ψ_j and their spatial derivatives in a stationary state of segregated condensates with a domain wall. (b) Typical profiles of the NG modes (first: ripplon, second: in-phase phonon) and the massive modes (third, fourth) with $p\xi/\hbar=0.05$. The solid and dashed curves represent the density $[\mathrm{Re}(u_j-v_j)]$ and phase $[\mathrm{Re}(u_j+v_j)]$ fluctuations, respectively, with $\mathrm{Im}u_j\approx 0$ and $\mathrm{Im}v_j\approx 0$.

order parameters as

$$\delta\Psi_j = u_j^n(z)e^{i(\boldsymbol{p}\cdot\boldsymbol{x}-\epsilon_nt)/\hbar} - v_j^n(z)^*e^{-i(\boldsymbol{p}\cdot\boldsymbol{x}-\epsilon_nt)/\hbar}$$
 (3)

with $\mathbf{p} = (p_x, p_y, 0)^T$, one obtains the BdG equations,

$$\epsilon_n \mathbf{u}_n = \hat{h} \hat{\rho}_3 \mathbf{u}_n, \tag{4}$$

where we used $\mathbf{u}_n = (u_1^n, u_2^n, v_1^n, v_2^n)^T$, $\hat{\rho}_3 = \text{diag}(1, 1, -1, -1)$,

$$\hat{h} = \begin{pmatrix} \mathcal{H} + \mathcal{G} & \mathcal{G} \\ \mathcal{G} & \mathcal{H} + \mathcal{G} \end{pmatrix}$$
 (5)

with $[\mathcal{G}]_{jk} = g_{jk}\psi_j\psi_k$, and $\mathcal{H} = \operatorname{diag}(\mathcal{H}_1, \mathcal{H}_2)$ with $\mathcal{H}_j = \mathbf{p}^2/2m_j - \hbar^2\partial_z^2/2m_j + \sum_k g_{kj}\psi_k^2 - \mu_j$. The real quantities ϵ_n and \mathbf{p} correspond to the quantum of energy and momentum of the mode when the Bogoliubov coefficients are normalized as $\mathcal{N}_{nn} = 1$, where

$$\mathcal{N}_{nn'} \equiv \int d^3x \boldsymbol{u}_n^{\dagger} \hat{\rho}_3 \boldsymbol{u}_{n'}. \tag{6}$$

The existence of the zero modes for $p = |\mathbf{p}| = 0$ is easily confirmed because Eq. (4) is satisfied for the zero-energy perturbations

$$u_j = -v_j^* = i\frac{\Delta\Theta_j}{2}\psi_j \tag{7}$$

and

$$u_j = -v_j^* = \frac{\Delta z}{2} \frac{d\psi_j}{dz} \tag{8}$$

with small real constants $\Delta\Theta_j$ and Δz , which are connected with the independent phase rotations $[\psi_j \to e^{i\Delta\Theta_j}\psi_j]$ and the translational shift of the domain wall $[\psi_j(z) \to \psi_j(z + \Delta z)]$, respectively.

It is useful to discuss some restriction for NG modes from their asymptotic behavior for $|z| \gg \xi_w$. The asymptotic profiles of the Bogoliubov coefficients u_j and v_j are obtained by a method similar to the semi-classical approximation in quantum mechanics [26] by introducing an effective semi-classical action S in the form

$$\boldsymbol{u}(z) = \boldsymbol{U}e^{iS(z)/\hbar} \tag{9}$$

with $U = [\mathcal{U}_1, \mathcal{U}_2, \mathcal{V}_1, \mathcal{V}_2]^T$. In the semi-classical limit $\hbar \to 0$, which is well applicable in the bulk region with $d\psi_i/dz \to 0$, one obtains from Eq. (4)

$$\frac{dS}{dz} = \pm \sqrt{\frac{\epsilon^2}{c_j^2} - p^2} \tag{10}$$

with the sound velocity $c_j = \sqrt{\mu_j/m_j}$ $(j = 1 \text{ for } z \to -\infty)$ and $j = 2 \text{ for } z \to \infty)$ in the bulk [27].

Consider the NG modes $\epsilon \propto cp^{\gamma}$ for $p \to 0$ with a positive constant c and an exponent γ . In segregated BECs, the NG modes are classified into the bulk NG mode (BNG mode) and the localized NG mode (LNG mode). The former has finite coefficients u_j and/or v_j in the bulk. Since the amplitude $|\psi_1|$ ($|\psi_2|$) vanishes for $z \to \infty$ ($z \to -\infty$), a possible BNG mode is a phonon in a single-component BEC:

$$\epsilon = c_i p$$
 (BNG mode). (11)

The localized NG mode satisfies the condition that u_j and v_j vanish far from the domain wall $z \to \pm \infty$. Then, the exponent S of Eq. (9) should be imaginary and one obtains in the low-momentum limit as

$$S = \begin{cases} i(-1)^{j} \sqrt{1 - (c/c_{j})^{2}} pz & (\gamma = 1) \\ i(-1)^{j} pz & (\gamma > 1) \end{cases}$$
 (LNG mode)(12)

where we neglected the integration constants and used j=1 (2) for $z\to -\infty$ ($+\infty$) and $c< c_j$ for $\gamma=1$. Thus, $\gamma=1/2$ found in Ref. [20] is excluded for LNG modes. The localized mode with $\gamma=1/2$, an analog of surface gravity wave in hydrodynamics, can appear in the presence of external potentials [28].

To find a possible value of γ in the above semi-classical analysis, we analyze the low-energy effective theory describing the LNG mode (ripplon) by taking into account the zero mode due to a transverse shift of the wall. Here, we do not remove the condition of compressibility in the BECs, while the previous works analyzed this problem based on the assumption of incompressibility [21, 23]. Neglecting the internal structure of the domain wall and representing the position of the domain wall as $z = \eta(x, y, t)$, we may approximate Eq. (1) to the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \int dx dy \left(\int_{-\infty}^{\eta} dz \mathcal{P}_1 + \int_{\eta}^{+\infty} dz \mathcal{P}_2 - \sigma \mathcal{S} \right) \tag{13}$$

with $S = \sqrt{1 + (\partial_x \eta)^2 + (\partial_y \eta)^2}$ and the domain wall tension coefficient σ . The variation of \mathcal{L}_{eff} about small η gives

$$\mathcal{P}_1(\eta) - \mathcal{P}_2(\eta) + \sigma(\partial_x^2 + \partial_y^2)\eta = 0. \tag{14}$$

The density and phase perturbations due to a small fluctuation Eq.(3) are written respectively as $\delta n_j \approx \delta \bar{n}_j \cos(\boldsymbol{p} \cdot \boldsymbol{x}/\hbar - \epsilon t/\hbar + \alpha_j)$ and $\delta \theta_j \approx \delta \bar{\theta}_j \sin(\boldsymbol{p} \cdot \boldsymbol{x}/\hbar - \epsilon t/\hbar + \beta_j)$ with $\delta \bar{n}^2 = 4 \left[(\mathrm{Re} u_j - \mathrm{Re} v_j)^2 + (\mathrm{Im} u_j + \mathrm{Im} v_j)^2 \right] |\psi_j|^2,$ $\delta \bar{\theta}_j^2 = \left[(\mathrm{Re} u_j + \mathrm{Re} v_j)^2 + (\mathrm{Im} u_j - \mathrm{Im} v_j)^2 \right] / |\psi_j|^2,$ $\cos \alpha_j = 2\mathrm{Re}(u_j - v_j) |\psi_j| / \delta \bar{n}_j,$ and $\sin \beta_j = \mathrm{Im}(u_j - v_j) / |\psi_j| \delta \bar{\theta}_j.$ For the LNG modes, one obtains with Eq. (9) for $|z| \gg \xi_w$,

$$\delta n_j = 2\psi_j e^{a_j pz/\hbar} (\mathcal{U}_j - \mathcal{V}_j) \cos(\boldsymbol{p} \cdot \boldsymbol{x}/\hbar - \epsilon t/\hbar), (15)$$

$$\delta \theta_j = \psi_j^{-1} e^{a_j pz/\hbar} (\mathcal{U}_j + \mathcal{V}_j) \sin(\boldsymbol{p} \cdot \boldsymbol{x}/\hbar - \epsilon t/\hbar), (16)$$

with
$$\text{Im}\mathcal{U}_j = \text{Im}\mathcal{V}_j = 0$$
, $a_j = -(-1)^j$ for $\gamma > 1$, and $a_j = -(-1)^j \sqrt{1 - c^2/c_j^2}$ for $\gamma = 1$.

In the low-momentum limit $p \to 0$, δn_j vanishes because $\mathcal{U}_j/\mathcal{V}_j = \left[cp^{\gamma} + (1-a_j^2)p^2/2m_j + \mu_j\right]/\mu_j \to 1$. This justifies the incompressibility-like assumption $\delta n_j = 0$ in Refs. [21, 23] exactly in the low-energy limit even for highly compressible gaseous BECs. Additionally, we impose the kinematic boundary condition, $\frac{\hbar}{m_j}\partial_z\delta\theta_j = \partial_t\eta$, which means that a superfluid velocity in the z direction on the wall causes a shift of the wall. By combining this condition with Eq. (14), we obtain the dispersion for a ripple wave $\eta \propto \cos(\mathbf{p} \cdot \mathbf{x}/\hbar - \epsilon t/\hbar)$,

$$\epsilon = \epsilon_{\rm rip}(p) \equiv \sqrt{\frac{\sigma/\hbar}{\rho_1 + \rho_2}} p^{3/2}$$
(17)

with $\rho_j = m_j \mu_j / g_{jj}$.

To identify all NG modes in this system, we numerically diagonalized Eq. (4) for low-energy modes with $p\xi/\hbar \ll 1$, paying particular attention to the finiteness of the system size. Numerical simulations of Eq. (4) were done by imposing the boundary conditions, $du_j/dz = dv_j/dz = 0$ at the edges $z = \pm L/2$ of the system, and $\psi_j = \sqrt{\mu_j/g_{jj}}$ at $z = (-1)^j L/2$ and $\psi_j = 0$ at $z = -(-1)^j L/2$. Figure 2 shows dispersions of the several low-energy modes for the same parameters as in Fig. 1(a) with a system size $L = 204.8\xi$.

The first lowest mode for $p>p_c$ is well fitted to the analytic dispersion of Eq. (17). Here, we used the approximated form of the tension coefficient $\sigma=\sqrt{2}P\xi\sqrt{g_{12}/g-1}$ [19] for the fitting. However, the dispersion obeys $\epsilon=Ap^2$ for p smaller than a certain critical value p_c that depends on L. We found that the coefficient A is proportional to \sqrt{L} [see Fig. 2(b)], and thus the critical momentum p_c , at which the p^2 and $p^{3/2}$ branches are connected, depends on the system size L as $p_c \propto 1/L$. Therefore, the dispersion $\epsilon=Ap^2$ comes from the finite-size effect, and the first lowest mode in the infinite system

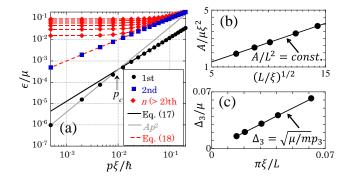


FIG. 2: (Color online) (a) The dispersion of the *n*th lowest-energy modes $(1 \le n \le 8)$ in a finite-size system $(L = 204.8\xi)$. The 1st mode has a fractional dispersion of Eq. (17) above the critical momentum p_c , which satisfies $Ap_c^2 = \epsilon_{\rm rip}(p_c)$. The dispersion of the *n*th mode $(n \ge 3)$ approaches to that of phonon (the 2nd mode) for $p \gg p_n$ but has an energy gap Δ_n for $p \to 0$. (b), (c) The system size dependence of A (b) and Δ_3 (c).

with $L=\infty$ coincides with the ripplon described by the effective theory. Actually, the spatial profiles of the density and phase fluctuations on the $p^{3/2}$ branch are well explained with the analytic forms of Eqs. (15) and (16), as shown in Fig. 1(b).

The second lowest mode is well fitted to $\epsilon = \sqrt{\mu/m} \ p$ in the whole momentum regime, being independent of L. This mode is a phonon propagating along a topological defect, which is an analog of a varicose mode propagating along a vortex [13]. A finite-size effect clearly appears for the nth lowest mode with $n \geq 3$. This effect makes the nth mode with $n \geq 3$ "massive" with an energy gap Δ_n at p=0 and the dispersion of the nth mode is asymptotic to that of the second mode with increasing p. This behavior is explained by the semi-classical theory with Eq. (10) as

$$\epsilon = \epsilon_n \equiv \sqrt{\mu/m} \sqrt{p^2 + p_n^2} \quad (n > 1),$$
 (18)

where $p_n/\hbar = (n-2)\pi/L$ is the wave number of the *n*th mode normal to the wall. The wave numbers p_n/\hbar correspond to those of standing waves emerging in the phase and density fluctuations [see Fig. 1(b)]. The standing wave is constructed with a superposition of the plane waves described by Eq. (9) with $S = \pm p_n z$, which propagate through the wall without disturbance. This transmission property of the Bogoliubov modes is an analog of so-called anomalous tunneling through a ferromagnetic domain wall in the low-energy limit [29]. Equation (18) explains very well the dispersion and the energy gap with $\Delta_n = \sqrt{\mu/m} \ p_n$ in Fig. 2(a) and (c).

The two NG modes are realized in experiments of atomic BECs. To achieve the low-energy dispersions, the domain wall width ξ_w must be much smaller than both the system sizes $L_{||}$ along a domain wall and L_{\perp} normal to the wall. For segregated condensates in a harmonic potential $V_j = (m/2)[\omega_{||}(x^2+y^2)+\omega_{\perp}^2z^2]$, $L_{||,\perp}$ may correspond to the Thomas-Fermi radii $R_{||,\perp} = \sqrt{2}\xi\mu/\hbar\omega_{||,\perp}$.

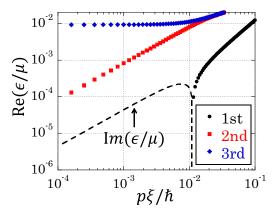


FIG. 3: (Color online) Double logarithmic plot of the dispersion of the lowest energy modes in a harmonically trapped system. The dashed curve shows the imaginary part of the dispersion of the first mode.

Figure 3 shows the dispersions of the three lowest modes in a trapped system with $R_{\perp}/\xi = 153.6$ and $\omega_{||} \rightarrow 0$ by assuming $\pi \hbar/p \ll R_{||}$, where the parameters are the same as those used in Fig. 1. Above the critical momentum p_c defined by $p_c \xi/\hbar \equiv \pi \xi/R_{\perp} = 0.02$, we find the characteristic dispersion similar to Fig. 2(a). However, the first lowest mode has imaginary dispersion $\text{Im}\epsilon > 0$ for $p \lesssim p_c$; the system may be dynamically unstable leading to amplification of ripplons. This is because the total energy decreases when the wall becomes parallel to the z axis for $R_{||} > R_{\perp}$. Therefore, a domain wall is stabilized for $R_{\perp} \gtrsim R_{||}$ and then the p and $p^{3/2}$ branches are realized [30]. The in-phase phonon (ripplon) can be excited by homogeneous (localized) perturbation potentials whose signs are in-phase (out-of-phase) between two components. Their dispersion relations can be detected directly from the time dependencies of the emergent density fluctuations on the periodicities of the perturbation potentials along the domain wall.

Finally, it should be mentioned that $n_{NG} = 2$ is smaller than $n_{\text{SSB}} = 3$ in this system. The second lowest mode, namely the in-phase phonon, is identical to the NG mode related to the breaking of a subset of the original $U(1)\times U(1)$ symmetry: the zero-energy perturbation with $\Delta\Theta_i = \Delta\Theta$ in Eq. (7). Correspondingly, the first lowest mode, the ripplon, may be understood as a result of a pairing between the out-of-phase rotation with $\Delta\Theta_i = (-1)^j \Delta\Theta$ in Eq. (7) and the domain wall shift of Eq. (8); the density and phase perturbations caused by the ripplon in the $p \to 0$ limit mimic those by the wall shift and the out-of-phase rotation [see Fig. 1(a) and (b) 1st]. According to Ref. [9], the pairing of zero modes links directly to the linear independence between the modes; we found that the two zeroenergy perturbations, namely, the out-of-phase rotation and the wall shift, are not orthogonalized to each other as $\mathcal{N}_{nn'} \neq 0$, while $\mathcal{N}_{nn'} = 0$ for other combinations

among the three zero-energy perturbations. We may expect a similar situation on the ferromagnetic domain wall in spinor BECs. To study such anomalous behaviors of zero modes for a variety of topological solitons realized in multi-component superfluid systems is an interesting direction in which to develop a universal theory of NG modes.

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- [28] In Ref. [20], the p^{1/2}-dispersion was considered to be analogous to that of surface gravity waves in hydrodynamics. However, in Ref. [22], it has been theoretically formulated and numerically confirmed that the role of gravity is played by the external forces due to the potential gradient normal to the interface in segregated condensates. We have never observed the p^{1/2}-dispersion without external potentials in our careful numerical simulations.
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- [30] Recent work [C. Ticknor, Phys. Rev. A 88, 013623 (2013)] calculated the excitation spectrum in two-dimensional two-component BECs in an isotropic trap $(R_{\parallel} = R_{\perp})$, where the rotational shift of the wall appeared as a zero mode.