The quark-hadron phase transition in
weakly isospin-asymmetric nuclear matter

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We consider the transition from quark to hadronic matter which may result during the cooling/expansion of the quark-gluon plasma formed in energetic collisions of weakly asymmetric ions. This transition involves the energy density of u and d quark matter and the one of nearly isospin-symmetric nuclear matter. Within bag models, the former entails knowledge of the bag pressure, a poorly constrained quantity. The bag pressure at high-density can be fixed imposing equality of quark and nucleonic energy densities at the (assumed known) transition point. We find this value to be very model dependent.

I. INTRODUCTION

The occurence of a transition between hadronic phase and (deconfined) quark phase in dense matter is an unsettled issue. From the experimental side, observing the formation of a quark-gluon plasma is one of the main objectives of CERN-SPS and RHIC experiments. This state of matter is also expected to take place in the interior of neutron stars, due to the high pressures typical of the stellar core. However, the phase transition in energetic heavy-ion experiments typically involves low densities and high temperatures, whereas high density cold matter constitutes the interior of stable neutron stars. Thus, there is some uncertainty as to whether laboratory observations of the quark-gluon plasma can be helpful in considerations of neutron star structure. On the other hand, there are also indications [1] that the deconfined phase occurs at a nearly constant value of the energy density, regardless the thermodynamical status of the system. A value of approximately 1 GeV fm^{-3} for the energy density has been reported from CERN-SPS experiments [2], which we will take as a reasonable estimate of the transition energy density.

Quark matter is a Fermi gas of $3A$ quarks which, together, constitute a single color-singlet baryon with baryon number A. Strange matter is quark matter where flavor equilibrium has been established by the weak interaction. Deconfined matter is typically handled within bag models, such as the MIT bag model [3]. The energy density of quark matter contains the bag parameter B, which represents the difference between the perturbative vacuum and the true vacuum. Physically, B is a pressure which maintains the quarks at finite density and chemical potential [3].

The purpose of this paper is to explore the characteristics and the model dependence of the quark-hadron phase transition in weakly isospin-asymmetric nuclear matter. The paper is organized as follows: In the next Section, we review the main steps in the calculation of the energy density of quark matter within the MIT bag model. In Section III, the phase transition is approached as resulting from energetic heavy-ion collisions of ^{208}Pb nuclei and subsequent expansion of the plasma, namely, it is a quark to hadron transition, where the hadronic phase consists of nearly symmetric nuclear matter. Assuming (approximate) knowledge of the transition density, we discuss possible values of B and a simple parametrization for its density dependence, based on the condition that the energy densities of the quark phase and the hadronic phase are equal at the transition. Often we will follow the procedure of Ref. [4] closely, with the specific intent of comparing findings. In that study, both the equation of state (EoS) based on the Brueckner-Hartree-Fock (BHF) approach implemented with three-body forces (TBF) [5] and the one based on relativistic mean field models (RMF) [6–8] are considered. On the other hand, our nucleonic equation of state is obtained within the Dirac-Brueckner-Hartree-Fock model (DBHF) [11]. A recent discussion on differences/similarities (with respect to effective or explicit inclusion of TBF) between the DBHF scheme and the BHF + TBF one can be found in Ref. [11]. It is the purpose of this note to detect differences (in the predicted features of the phase transition) which may originate from the nucleonic EoS. This is an important point, in view of the large model dependence still existing among predictions of the latter, in particular the details of its slope.

Once the bag pressure is "determined" (or, rather, constrained to be within some reasonable range of values), one can go back to the formalism we outline in Section II and calculate, for instance, the energy density of β-stable strange quark matter for the purpose of stellar structure calculations.

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Our conclusions are summarized in the last Section.

II.   COMPOSITION OF β-EQUILIBRATED STRANGE MATTER: BASIC EQUATIONS

We consider a degenerate Fermi gas of u, d, and s quarks and electrons in chemical equilibrium maintained by the weak processes:

$$u + e^- \rightarrow d + \nu_e , \tag{1}$$

$$u + e^- \rightarrow s + \nu_e , \tag{2}$$

$$d \rightarrow u + e^- + \bar{\nu}_e , \tag{3}$$

$$s \rightarrow u + e^- + \bar{\nu}_e , \tag{4}$$

$$s + u \rightarrow d + u . \tag{5}$$

The neutrinos are ignored as they are expected to have no impact on the dynamics, being an extremely diluted gas (although massive neutrinos could be found in strange matter [3]). In chemical equilibrium, we have

$$\mu_d = \mu_s = \mu , \tag{6}$$

$$\mu_u + \mu_e = \mu . \tag{7}$$

Also, charge neutrality and baryon number conservation require

$$\frac{2}{3}\rho_u = \frac{1}{3}\rho_d + \frac{1}{3}\rho_s + \rho_e , \tag{8}$$

and

$$\rho = \frac{1}{3}(\rho_d + \rho_u + \rho_s) , \tag{9}$$

where ρ is the total (fixed) baryon density. Exploiting the relation between the density of each species and the corresponding thermodynamic potential,

$$\rho_i = -\frac{\partial \Omega_i}{\partial \mu_i} , \qquad (i = u, d, s, e) \tag{10}$$

equations (6-9) can be solved for the chemical potentials of each species (and thus their densities or fractions).

To first order in a_c (the QCD coupling constant), the thermodynamic potentials are [3]

$$\Omega_u = -\frac{\mu_u^4}{4\pi^2}\left(1 - \frac{2a_c}{\pi}\right) , \tag{11}$$

$$\Omega_d = -\frac{\mu_d^4}{4\pi^2}\left(1 - \frac{2a_c}{\pi}\right) , \tag{12}$$

$$
\begin{aligned}
\Omega_s ={}& -\frac{1}{4\pi^2}\Bigg(\mu_s(\mu_s^2 - m_s^2)^{1/2}(\mu_s^2 - \frac{5}{2}m_s^2) + \frac{3}{2}m_s^4 ln\frac{\mu_s + (\mu_s^2 - m_s^2)^{1/2}}{m_s} \\
& -\frac{2a_c}{\pi}\Bigg(3\Big(\mu_s(\mu_s^2 - m_s^2)^{1/2} - m_s^2 ln\frac{\mu_s + (\mu_s^2 - m_s^2)^{1/2}}{m_s}\Big)^2 - 2(\mu_s^2 - m_s^2)^{1/2} + 3m_s^4 ln^2\frac{m_s}{\mu_s} \\
& + 6ln\frac{\rho}{\mu_s}\Big(\mu_s m_s^2(\mu_s^2 - m_s^2)^{1/2} - m_s^4 ln\frac{\mu_s + (\mu_s^2 - m_s^2)^{1/2}}{m_s}\Big)\Bigg)\Bigg) .
\end{aligned}
\tag{13}
$$

The total energy density is then

$$\epsilon = \sum_{i=u,d,s,e} (\Omega_i + \mu_i \rho_i) + B \; , \tag{14}$$

where B is a positive energy per unit volume carried by the vacuum, that is, the vacuum pressure. Its value is an open question, and will be confronted in the next section.

Quark masses are taken to be equal to their current-algebra values, which amounts to ignoring effects from chiral symmetry breaking (i. e. dynamical masses) in the quark gas. In particular, u and d quarks can be assumed to be massless, whereas a mass between 100 and 200 MeV is typically assigned to the s quark.

III. THE QUARK-HADRON PHASE TRANSITION IN NEARLY ISOSPIN-SYMMETRIC MATTER

If lead nuclei are accelerated at CERN-SPS energies, during the early stages of the collision the so-called "fireball" is formed, which is a hot and dense plasma of deconfined quarks and gluons. As the plasma cools down, it becomes more diluted until hadronization takes place. Thus, in the phase diagram one is moving from higher to lower densities, and from the quark to the hadronic phase. Weak processes are not expected to play a role due to the short expansion times, and thus strangeness, which is conserved in strong interactions, can be ignored. The bulk of nuclear matter resulting after hadronization can be described by the EoS of cold matter with an average isospin asymmetry, α, equal to 0.2 (α is defined as $(N - Z)/(N + Z)$, and 0.2 is its value in lead)). For the hadronic phase, we then have, for the energy density,

$$\epsilon_H = (e(\rho, \alpha) + x_p m_p + x_n m_n)\rho \; , \tag{15}$$

where e is the energy per particle from Ref. [11] and x_p (x_n) are the proton (neutron) fractions.

Taking the transition energy density, ϵ^{tr}, to be known, we can immediately determine the baryonic density, ρ^{tr}, which corresponds to it in our model of the nucleonic EoS. For instance, for $\epsilon^{tr} \approx 0.8 GeV fm^{-3}$, the transition baryon number density is 0.74 fm^{-3} from our EoS. Notice that the stiffness of the nucleonic EoS determines the *baryonic* density at which the appropriate transition energy density is achieved, a point worth underlining.

The bag pressure is essentially a free parameter. It has a value of 55 MeV fm^{-3} according to the original MIT bag model, whereas lattice calculations estimate it to be about 200 MeV fm^{-3} [9]. First, we consider (constant) values of B within this (rather large) range and calculate the energy density of u and d quark matter. In Fig. 1, we show the quark energy density for B equal to 55, 100, and 200 MeV fm^{-3}, and the nucleonic energy density from Eq. (15). The two panels refer to a_c equal to 0 and 0.2, respectively. Referring to the first panel, we see that, for B equal to 55 MeV fm^{-3}, the crossing density is too low to be realistic (about normal nuclear matter density), whereas with values of 100 and 200 MeV fm^{-3} transition densities equal to 0.93 and 1.2 fm^{-3} are predicted, respectively. The corresponding energy densities are not inconsistent with present experimental constraints. When a_c is non-zero, the transition density shifts to higher values, see second frame of Fig. 1, but the difference is minor.

A better way to proceed is to fit the value of B imposing equality of the quark and hadron energy densities at the appropriate baryon number density (as determined from the nucleonic EoS). That is:
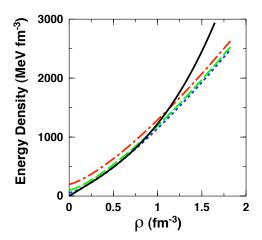
$$\epsilon_H^{tr} = \epsilon_q^{tr} \; , \tag{16}$$

where the right-hand side is the energy density of u and d quarks, which depend linearly on B, see Eq. (14). With the left-hand side known from the nucleonic EoS, it is elementary to determine B. It has been argued, though, that B should be density dependent [10], as a large reduction of its value in nuclear matter appears to be consistent with the description of phenomena such as the EMC effect [10]. In fact, allowing the MIT bag constant to depend on the local density leads to the recovery of relativistic phenomenology [10] like large cancelations between Lorentz scalar and vector potentials in nuclear matter.

Adopting an even simpler parametrization than those considered in Ref. [4], we assume that $B(\rho)$ has a constant value up to densities around the transition point and then drops sharply to its asymptotic value (basically, a very sharp Woods-Saxon distribution approximated by a step function). That is:

$$B(\rho) = \begin{cases} B_0 & \text{if } \rho < \rho^{tr} \; , \\ B_{asy} & \text{otherwise.} \end{cases} \tag{17}$$

Obviously, high-density predictions will be mostly sensitive to the asymptotic value of $B(\rho)$, thus the value of B_0 is not very relevant. We follow Ref. [4] and choose a value between 200 and 400 MeV fm^{-3}. B_{asy} can then be fixed by
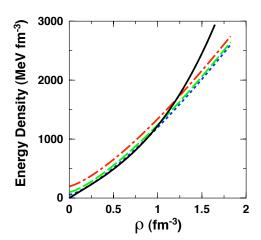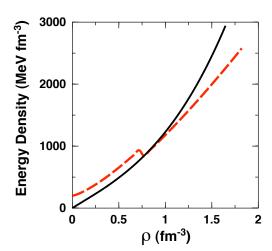
FIG. 1: (color online) The hadronic (solid line) and the quark energy densities for different values of the bag constant. Dotted line: $B=$ 55 MeV fm^{-3}; Dashed line: $B=$ 100 MeV fm^{-3}; Dash-dotted line: $B=$ 200 MeV fm^{-3}. The left and right frames refer to values of the QCD coupling constant equal to 0 and 0.2, respectively.

TABLE I: Values of B_{asy} (or B_{∞}) predicted by various hadronic models for fixed transition energy density. The "BHF + TBF" and "RMF" entries are taken from Ref. [4]. (For those cases, the given values refer to the parameter labeled as B_{∞} in Ref. [4].) The "BHF + TBF$_2$" is the EoS model labeled as "BOB" in Ref. [12], see text for.

Model	ϵ_{tr} (MeV fm^{-3})	ρ_{tr} (fm^{-3})	B_{asy} (MeV fm^{-3})
DBHF	0.8×10^3	0.74	35.7
	1.1×10^3	0.93	58.9
BHF+TBF	0.8×10^3	0.76	36.4
	1.1×10^3	0.97	51.1
RMF	0.8×10^3	0.76	37.9
	1.1×10^3	0.98	37.8
(BHF+TBF)$_2$	0.8×10^3	0.73	49.4
	1.1×10^3	0.89	122.4

imposing the condition Eq. (16). For $a_c=0$, fitted values of B_{asy} are shown in Table I for some acceptable values of the transition energy density. Our values of B_{asy} are directly comparable with those of the constant B_{∞} of Ref. [4], where a Gaussian or Woods-Saxon parametrizations are chosen for $B(\rho)$, with its asymptotic value, B_{∞}, fitted at the transition point. That is the parameter displayed in Table I for the second and third models.

First, we notice that, within a given model, B_{asy} spans a rather large range (for relatively small changes in the transition density). Furthermore, the values of B are quite sensitive to the details of the density dependence of the nucleonic EoS. Clearly, some (pre-established) value of the energy density is obtained at a lower or higher value of the number density depending on the stiffness of the original EoS. This becomes particularly clear looking at the last model considered in Table I, which is based on the "BHF + TBF" approach, but is considerably more repulsive. This EoS is the one labeled as "BOB" in Ref.[12]. It is obtained with BHF calculations together with TBF. The parameters of the TBF are chosen to be, as far as possible, consistent with those of the meson-exchange two-body potential, which is Bonn B [13].
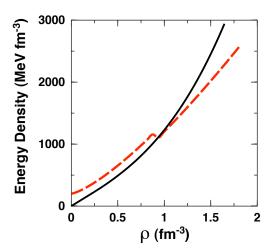
FIG. 2: (color online) The hadronic (solid line) and the quark (dashed line) energy densities with a density dependence of the bag constant as in Eq. (17). In the left (right) panel a transition energy density of 800 (1100) MeV fm^{-3} is assumed.

The quark and hadron energy densities with the sharp density dependence of $B(\rho)$ from Eq. (17) are shown in Fig. 2, for transition energy density equal to 800 and 1100 MeV fm^{-3}. A smoother density dependence of $B(\rho)$ would have impact only at the lower densities. Constructing one is straightforward but besides the point of this paper, where we want to perform a pedagogical demonstration of the uncertainty involved in determining the features of the phase transition (and the EoS thereafter), particularly as it relates to the underlying hadronic model.

IV. CONCLUSIONS

We explored the quark-hadron phase transition in weakly isospin-asymmetric matter as it could take place during the expansion phase of energetic collisions of lead nuclei. We used either a constant bag pressure, B, or the simplest parametrization of its density dependence to demonstrate the role of the nucleonic EoS in determining the features of the transition. The parameter which determines the asymptotic behavior of $B(\rho)$ is very sensitive to the details of the density dependence of the nucleonic EoS. Thus, independent information on the density dependence of B in the medium would be very helpful. Those could come, for instance, from considerations of EMC effects [10]. In turn, such information would help constrain the density dependence of the *nucleonic* EoS at the phase transition.

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