QCD tests from pion reactions on few-nucleon systems

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Abstract. We show on two examples, namely a calculation for charge symmetry breaking in $pn \to d\pi^0$ that allows one to extract the quark mass difference induced part of the proton-neutron mass difference and a high precision calculation for pion-deuteron scattering and its implications for the value of the charged pion-nucleon coupling constant, how QCD tests can be performed from low energy hadronic observables.

Keywords: Pion-baryon interactions, Chiral Lagrangians, Electromagnetic corrections to strong-

interaction processes

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INTRODUCTION

In the last decades the effective field theory for low energy phenomena within the Standard Model, Chiral Perturbation Theory (ChPT), has developed to a mature tool to study hadronic phenomena at low energies with a clear cut connection to QCD — see Refs. [1, 2, 3] for recent reviews with emphasis on the $\pi\pi$, the single nucleon and the two–nucleon sector, respectively.

One very useful application of ChPT is its use to extract from complex reactions more fundamental quantities that can be compared to QCD predictions straight forwardly. Those QCD predictions are calculated from first principles using lattice gauge theory techniques [4]. Since those are quite involved numerically the described interplay of effective field theory and numerical methods is very rewarding. For a long time hadronic reactions were studied using models. Although very successful in providing a qualitative picture of the reaction mechanisms, it is not possible to determine the accuracy of the calculation. Here effective field theories are in a clear advantage: since in their very nature they are controlled expansions in some small parameter, they allow for uncertainty estimates. This is why the role of model calculations is decreasing in recent years. The interplay of models, ChPT and lattice QCD is illustrated in Fig. 1.

In this presentation two examples for the strategy outlined will be described. On the one hand a calculation will be sketched where from an analysis of the isospin violating forward-backward asymmetry of $pn \rightarrow d\pi^+$ the quark mass induced piece

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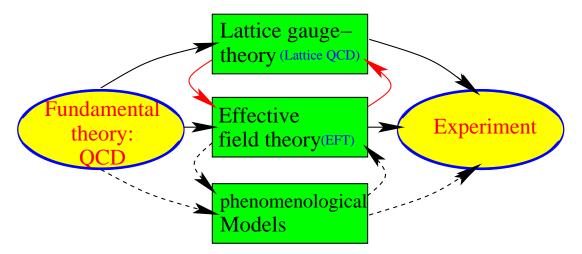


FIGURE 1. Illustration of different methods to compare properties of QCD to experiment.

of the proton-neutron mass difference was extracted [6], and on the other hand a high precision calculation is discussed that allowed for an improved extraction of the πN scattering lengths [5] from an analysis of high accuracy pionic deuterium data.

ISOSPIN VIOLATING $NN \rightarrow d\pi$

Many of the isospin violating observables are dominated by effects from the pion mass difference, since, although $m_{\pi^0} - m_{\pi^+}$ is typical for hadronic mass differences within isospin multiplets, isospin violating effects are enhanced due to the small pion mass. This is the reason for the importance of charge symmetry breaking (CSB) reactions (under charge symmetry up and down quark get interchanged), for here the pion mass difference does not contribute and therefore effects from different quark masses get enhanced.

In this section we focus on the CSB null observable $A_{fb}(pn \to d\pi^0)$ — the forward–backward asymmetry in $pn \to d\pi^0$. This is a null observable since in a charge symmetric world the final state fixes the total isospin to 1 and therefore the initial state gets projected on isospin 1 and is to behave as a proton–proton/neutron–neutron pair where forward and backward are not defined. At TRIUMF this observable was found to be [7]

$$A_{fb} = [17.2 \pm 8(\text{stat.}) \pm 5.5(\text{sys.})] \times 10^{-4}$$
 (1)

In this section we will briefly describe the first complete NLO calculation for this reaction within ChPT. Also another CSB null-observable was measured recently, namely the total cross section for $dd \to \alpha \pi^0$ [8], however, since the four-nucleon dynamics involved in the reaction is a lot more complicated, no complete theoretical analysis exists yet for this reaction — see Refs. [9, 10] for some preliminary studies.

The calculation described here became possible due to recent advances in developing a systematic power counting for reactions of the type $NN \to NN\pi$ that was complicated by the presence of the large initial momentum $p_{\text{thr}} = \sqrt{m_{\pi}M_N}$, with m_{π} (M_N) for the

pion (nucleon) mass, which calls for a different expansion parameter, namely [11, 12]

$$\chi_{
m prod} = rac{p_{
m thr}}{\Lambda_{\chi}} = \sqrt{rac{m_{\pi}}{M_N}} \; ,$$

where for the last identity the chiral symmetry breaking scale Λ_{χ} was identified with the nucleon mass. Nowadays the ChPT calculations have basically replaced the phenomenological calculations (see Ref. [13] and references therein) that dominated the field before. For a recent review see Ref. [14]. The reason why A_{fb} is linked to the proton–neutron mass difference is that the transformation properties of the quark mass term in QCD under axial rotations dictates a link between mass differences of heavy hadrons and the isospin violating pion scattering off the very same hadrons [15, 16, 17]. For the case of the $pn \to d\pi^0$ this was first studied in Refs. [18, 19], however, these calculations were not complete, for besides diagram (a) of Fig. 2, where the mentioned isospin violating πN scattering enters, also diagram (b) enters. It gives a non-vanishing contribution since the isospin conserving πN interaction is energy dependent and therefore gets sensitive to the different energy transfer in π^+ exchange, equal to $m_{\pi}/2 + M_p - M_n$, and in $\pi^$ exchange, equal to $m_{\pi}/2 + M_n - M_p$. In general CSB due to electromagnetism and due to quark mass differences enter with similar strength. Here, however, it so happens that the sum of diagram (a) and (b) are proportional only to $\delta M^{\rm qm}$ — the strong part of the proton-neutron mass difference. Using the results of Ref. [20] as input, the calculation revealed

$$A_{\rm fb}^{\rm LO} = (11.5 \pm 3.5) \times 10^{-4} \, \frac{\delta M^{\rm qm}}{\rm MeV} \,.$$
 (2)

The calculation sketched refers to a leading order calculation, however, all contributions at NLO, namely one loop diagrams with virtual photons, cancel [10] — the reason for this cancellation is now understood [21]. Thus the uncertainty was estimated to be of order $\chi^2_{prod} \sim$ 30%. Using the experimental result of Eq. (1), Eq. (2) may be converted to give

$$\delta M^{\rm qm} = (1.5 \pm 0.8 \,(\text{exp.}) \pm 0.5 \,(\text{th.})) \,\,\text{MeV} \,,$$
 (3)

where the first (second) uncertainty follows from the uncertainty of the experiment (calculation). In Fig. (3) this result is compared to previous extractions: one directly from the proton–neutron mass difference using the Cottingham sum rule to quantify the electromagnetic contribution [22] and one from lattice QCD [23]. Note, the calculation presented has a comparable accuracy to the other extractions — thus an improved measurement would be very desirable.

πd SCATTERING LENGTH AND ITS IMPLICATIONS FOR $g_{\pi NN}$

The problem of πd scattering has been studied theoretically already for many decades using phenomenological approaches, however, nowadays the high accuracy of modern experiments calls for improved tools for the analysis. Especially, a consistent treatment of strong and electromagnetic few-body effects is essential for a controlled extraction of the quite small isoscalar pion–nucleon scattering length a^+ , for especially electromagnetic effects might even outnumber its contribution to the πd scattering length [24].

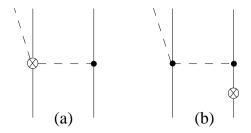


FIGURE 2. Leading order diagrams for the isospin violating *s*-wave amplitudes of $pn \to d\pi^0$. Solid (dashed) lines denote nucleons (pions). Diagram (a) corresponds to isospin violation in the πN scattering vertex explicitly whereas diagram (b) indicates an isospin-violating contribution due to the neutron-proton mass difference in conjunction with the time-dependent Weinberg-Tomozawa operator (see text for details).

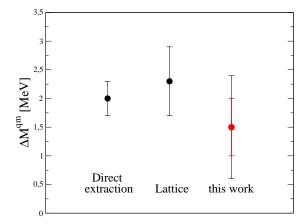


FIGURE 3. Comparison of different extractions of the quark mass induced neutron–proton mass difference. The points are from Refs. [22] (Direct extraction), [23] (Lattice), and [6] (this work). The inner (red) error bars on the last point refer to purely the theoretical uncertainty.

There were various important advances that made a high accuracy calculation for the πd scattering length, reported in Ref. [25], possible: various subleading contributions were shown to vanish [26], there exists a calculation for πN scattering of the necessary accuracy [27], the role of various few body corrections is understood [28, 29, 30], the role of the nucleon recoils is understood [31, 32], and dispersive and Delta corrections are nowadays under control quantitatively [33, 34]. In this section we will briefly sketch the results of Ref. [25], with special emphasis on isospin violating parts.

The data for hadronic scattering lengths is best deduced from high accuracy measurements of pionic atoms [35] together with properly improved Deser formulae [36, 37] — for a recent review see Ref. [38].

The theoretical limit for the accuracy of a calculation of the $\pi NN \to \pi NN$ transition operator is set by the first $(\bar{N}N)^2\pi^2$ —counter term. In the power counting of Ref. [30] it appears at $\mathcal{O}(\chi^2)$ relative to the leading two—nucleon operator shown in Fig. 4 (d_1) , with $\chi = m_\pi/M_N$. We thus aim at a calculation with up—to and including $\mathcal{O}(\chi^{3/2})$ — square root orders appear due to the connection between pion production (see previous section)

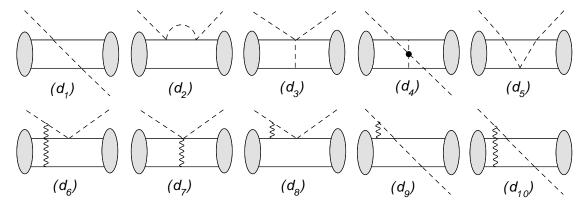


FIGURE 4. Topologies for $\pi^- d$ scattering. Solid, dashed, and wiggly lines denote nucleons, pions, and photons, respectively. The blobs indicate the deuteron wave functions.

and the dispersive corrections [33] as well as the numerical proximity of the Deltanucleon mass difference and p_{thr} introduced above [34]. The diagrams that contribute up to this order, besides those for Delta and dispersive terms, are shown in Fig. 4. Naively, one might expect the most important isospin violating contributions to πd scattering from different pion masses in the leading contributions, especially in the diagrams shown in Fig. 4 (d_1) and (d_2). However, it is the subtle interplay of one–nucleon and two– nucleon operators, driven by the Pauli principle, already discussed in Ref. [31], and the orthogonality of the nuclear wave functions [32], that strongly suppresses these effects.

More difficult is the treatment of photon loops that might get enhanced due to the masslessness of the photon together with the smallness of ε , the deuteron binding energy. For example, one finds for the leading contributions of diagrams (d_6) in an expansion in small momenta

$$M_{d_6} \sim -a^- \int \frac{d^3 p \, d^3 q \Psi^{\dagger}(\mathbf{p} - \mathbf{q}) \Psi(\mathbf{p})}{\mathbf{q}^2(\mathbf{q}^2 + 2m_{\pi}(\varepsilon + \mathbf{p}^2/M_p))} \sim -\frac{8\pi}{3\sqrt{2}} \frac{a^-}{\sqrt{m_{\pi}\varepsilon}} \left(1 + \mathcal{O}\left(\sqrt{\frac{\varepsilon}{m_{\pi}}}\right)\right)$$

and (d_8) of Fig. 4

$$M_{d_8} \sim +a^- \int \frac{d^3p \, d^3q \Psi^{\dagger}(\mathbf{p}) \Psi(\mathbf{p})}{\mathbf{q}^2(\mathbf{q}^2 + 2m_{\pi}(\varepsilon + \mathbf{p}^2/M_p))} \sim + \frac{8\pi}{3\sqrt{2}} \frac{a^-}{\sqrt{m_{\pi}\varepsilon}} \left(1 + \mathcal{O}\left(\sqrt{\frac{\varepsilon}{m_{\pi}}}\right)\right) ,$$

where Ψ denotes the deuteron wave function. Individually the amplitudes appear to be enhanced by a factor $\sqrt{m_{\pi}/\varepsilon} \sim 8$ compared to the dimension analysis estimate, however, in the sum the enhanced pieces cancel. Similar cancellations can be observed for the other potentially infrared enhanced contributions.

At the end it turns out that most of the additional contributions cancel pairwise and thus already the leading diagram — Fig. 4 (d_1) — largely exhausts the value of the πd scattering length. The numerically most important corrections are provided by an isospin violating piece to the πN scattering length and the triple scattering diagram (d_5) — from the dimensional analysis this diagram contributes at $\mathcal{O}(\chi^2)$, it is, however, enhanced by a factor π^2 due to its special topology and thus needs to be considered [30]. In addition,

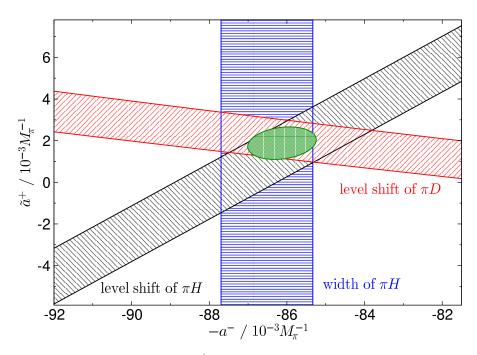


FIGURE 5. Combined constraints in the \tilde{a}^+-a^- plane from data on the width and energy shift of πH , as well as the πD energy shift.

from pionic atoms it is not possible to extract a^+ directly, but only the combination [24]

$$\tilde{a}^+ \equiv a^+ + \frac{1}{1 + m_\pi/M_N} \left\{ \frac{m_\pi^2 - m_{\pi^0}^2}{\pi F_\pi^2} c_1 - 2\alpha f_1 \right\} \,,$$

with F_{π} for the pion decay constant and the additional low energy constants c_1 and f_1 . The combined analysis for pionic hydrogen and pionic deuterium data yields from the 1σ error ellipse (c.f. Fig. 5)

$$\tilde{a}^{+} = (1.9 \pm 0.8) \cdot 10^{-3} m_{\pi}^{-1}, \ a^{-} = (86.1 \pm 0.9) \cdot 10^{-3} m_{\pi}^{-1},$$
 (4)

with a correlation coefficient $\rho_{a^-\tilde{a}^+} = -0.21$. We find that the inclusion of the πD energy shift reduces the uncertainty of \tilde{a}^+ by more than a factor of 2. Note that in the case of the πH level shift the width of the band is dominated by the theoretical uncertainty in $\Delta \tilde{a}_{\pi^-p}$, whereas for the πH width the experimental error is about 50% larger than the theoretical one. For details on the error budget see Ref. [25].

theoretical one. For details on the error budget see Ref. [25]. Taken together with $c_1 = (-1.0 \pm 0.3) \, \text{GeV}^{-1}$ [39] and the rough estimate $|f_1| \le 1.4 \, \text{GeV}^{-1}$ [40], Eq. (4) yields a non-zero a^+ at better than the 95% confidence level:

$$a^{+} = (7.6 \pm 3.1) \cdot 10^{-3} m_{\pi}^{-1}.$$
 (5)

The final result for a^+ is only a little larger than several of the contributions considered in our analysis. This emphasizes the importance of a systematic ordering scheme, and a careful treatment of isospin violation and three-body dynamics. A reduction of the theoretical uncertainty beyond that of the present analysis will be hard to achieve without

additional QCD input that helps pin down the unknown contact-term contributions in both the πN and πNN sectors.

As it was argued in the introduction, πN scattering lengths are interesting quantities by themselves, especially since they can be extracted from lattice QCD calculations relatively easily. In the last part of this section we will show that in addition they also provide an important link between pion–nucleon and nucleon–nucleon scattering and in this sense a non–trivial consistency check for our current understanding of these fundamental reactions. Extracted from a careful analysis of πN scattering data, for long the charged pion nucleon coupling constant was believed to be $g_c^2/4\pi = 14.2 \pm 0.2$ [41]. However, when extracted from NN scattering [42], the value deduced reads $g_c^2/4\pi = 13.54 \pm 0.05$, where the error includes only the fitting uncertainty and not any possible systematic uncertainties. The use of the work presented to resolve the question on the value of g_c becomes explicit, when using the Goldberger-Miyazawa-Oehme (GMO) sum rule [43] ². It reads

$$\frac{g_c^2}{4\pi} = \left(\left(\frac{M_p + M_n}{m_{\pi}} \right)^2 - 1 \right) \left[\left(1 + \frac{m_{\pi}}{M_p} \right) \frac{m_{\pi}}{4} \left(a_{\pi^- p} - a_{\pi^+ p} \right) - \frac{m_{\pi}^2}{2} J^- \right] .$$

Here the πN scattering lengths, now known to higher accuracy (c.f. Fig. 5), appear as subtraction constants for the dispersion integral

$$J^{-} = rac{1}{4\pi^2} \int dk rac{\sigma^{
m tot}_{\pi^{-}p} - \sigma^{
m tot}_{\pi^{+}p}}{\sqrt{k^2 + m_{\pi}^2}} \; ,$$

that may be expressed in terms of observable cross sections. Values for this integral can be taken from Refs. [45, 46]. Combining the findings of these works gives for the integral $(-1.073 \pm 0.034)\,\mathrm{mb}$ [39], which is consistent with previous extractions (see Ref. [47]). The GMO sum rule was used before to pin down the value of g_c , however, different analyses came to different answers. While Ref. [45] found a value as large as $g_c^2/4\pi = 14.11 \pm 0.05 \pm 0.19$, where the first uncertainty is statistical while the second is systematic, Ref. [47] found values for $g_c^2/4\pi$ between 13 and 13.3. Also other, more general analyses from this group reported lower values, namely $g_c^2/4\pi = 13.75 \pm 0.15$ [48], $g_c^2/4\pi = 13.76 \pm 0.01$ [49]. Here the uncertainties only represent the statistical uncertainty. No attempt was made to quantify the systematics. More recently Ref. [46] presented $g_c^2/4\pi = 13.56 \pm 0.36$ from a GMO analysis. There were no further developments in the last three years and the key players basically 'agreed to disagree' [50]. The basic improvements provided by the analysis discussed are that for the first time isospin violating corrections were included completely and consistently and, as the result of using a systematic effective field theory, it became possible to properly control the uncertainties of the πN scattering lengths. With the in this way improved input we find

$$g_c^2/4\pi = 13.69 \pm 0.12 \pm 0.15 = 13.7 \pm 0.2$$

² In the definition of g_c there appear subtle issues due to the Coulomb poles [44] as well as other infrared singularities. For a detailed analysis we refer to Ref. [39].

where the first error gives the uncertainty in the scattering lengths and the second in the integral. From our analysis we therefore conclude that a value for $g_c^2/4\pi$ above 14 is largely excluded.

SUMMARY

Modern lattice QCD calculations allow for first principle calculations of hadronic observables like scattering lengths [4]. However, in most cases those quantities are not directly accessible from experiments but need to be extracted from reactions with complicated few body dynamics. In this presentation on two examples, the extraction of the quark mass induced proton–neutron mass difference, $\Delta M^{\rm qm}$, from the forward–backward asymmetry in $pn \to d\pi^0$, and the extraction of the pion–nucleon scattering lengths from data on pionic hydrogen and deuterium, it is demonstrated that ChPT can be employed to extract, with controlled uncertainty, the quantities of interest from complex reactions to allow for a comparison to lattice data and thus for non–trivial test of QCD dynamics at low energies.

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REFERENCES

- 1. G. Colangelo, J. Gasser and H. Leutwyler, Nucl. Phys. B 603, 125 (2001) [arXiv:hep-ph/0103088].
- 2. V. Bernard, Prog. Part. Nucl. Phys. **60**, 82 (2008).
- 3. E. Epelbaum, H. W. Hammer and U.-G. Meißner, Rev. Mod. Phys. 81, 1773 (2009).
- 4. S. R. Beane, W. Detmold, K. Orginos and M. J. Savage, arXiv:1004.2935 [hep-lat].
- 5. V. Baru *et al.*, arXiv:1003.4444 [nucl-th].
- 6. A. Filin et al., Phys. Lett. B 681 (2009) 423
- 7. A. K. Opper *et al.*, Phys. Rev. Lett. **91** (2003) 212302.
- 8. E. J. Stephenson *et al.*, Phys. Rev. Lett. **91** (2003) 142302.
- 9. A. Gårdestig *et al.*, Phys. Rev. C **69** (2004) 044606;
- A. Nogga *et al.*, Phys. Lett. B **639** (2006) 465; T. A. Lähde and G. A. Miller, Phys. Rev. C **75** (2007) 055204 [Erratum-ibid. C **77** (2008) 019904]; A. C. Fonseca, R. Machleidt and G. A. Miller, Phys. Rev. C **80** (2009) 027001.
- 11. T. D. Cohen et al., Phys. Rev. C 53 (1996) 2661.
- 12. C. Hanhart, U. van Kolck and G.A. Miller, Phys. Rev. Lett. **85**, 2905 (2000).
- 13. C. Hanhart, J. Haidenbauer, O. Krehl and J. Speth, Phys. Rev. C 61 (2000) 064008.
- 14. C. Hanhart, Phys. Rept. **397** (2004) 155.
- 15. S. Weinberg, in *Chiral Dynamics: Theory and Experiment*, edited by A.M. Bernstein and B.R. Holstein (Springer-Verlag, 1995).

- 16. U. van Kolck, Few-Body Syst. Suppl. 9, 444 (1995); Ph.D. Dissertation, U. of Texas (1993).
- 17. U.-G. Meißner and S. Steininger, Phys. Lett. B **419** (1998) 403.
- 18. U. van Kolck, J. A. Niskanen and G. A. Miller, Phys. Lett. B 493 (2000) 65.
- 19. D. R. Bolton and G. A. Miller, Phys. Rev. C 81, 014001 (2010)
- 20. V. Baru et al., Phys. Rev. C 80 (2009) 044003
- 21. V. Lensky et al., Eur. Phys. J. A 27 (2006) 37.
- 22. J. Gasser and H. Leutwyler, Phys. Rept. 87 (1982) 77.
- 23. S. R. Beane, K. Orginos and M. J. Savage, Nucl. Phys. B 768 (2007) 38.
- 24. U.-G. Meißner et al., Phys. Lett. B 639, 478 (2006).
- 25. V. Baru et al., arXiv:1003.4444 [nucl-th].
- 26. S. R. Beane et al., Nucl. Phys. A 720, 399 (2003).
- 27. M. Hoferichter et al., Phys. Lett. B 678, 65 (2009); Nucl. Phys. A 833, 18 (2010).
- 28. A. Nogga and C. Hanhart, Phys. Lett. B 634, 210 (2006).
- 29. L. Platter and D. R. Phillips, Phys. Lett. B 641 (2006) 164.
- 30. S. Liebig et al., arXiv:1003.3826 [nucl-th].
- 31. V. Baru et al., Phys. Lett. B 589, 118 (2004).
- 32. V. Baru et al., Eur. Phys. J. A 42, 111 (2009).
- 33. V. Lensky et al., Phys. Lett. B 648, 46 (2007).
- 34. V. Baru et al., Phys. Lett. B 659, 184 (2008).
- 35. D. Gotta et al., AIP Conf. Proc. 1037, 162 (2008).
- 36. V. E. Lyubovitskij and A. Rusetsky, Phys. Lett. B **494**, 9 (2000).
- 37. U.-G. Meißner et al., Eur. Phys. J. C 41, 213 (2005).
- 38. J. Gasser et al., Phys. Rept. **456**, 167 (2008).
- 39. V. Baru et al., in preparation.
- 40. J. Gasser et al., Eur. Phys. J. C 26, 13 (2002);
 - N. Fettes and U.-G. Meißner, Phys. Rev. C **63**, 045201 (2001).
- 41. R. Koch and E. Pietarinen, Nucl. Phys. A 336 (1980) 331.
- 42. J. J. de Swart *et al.*, PiN Newslett. **13**, 96 (1997); V. G. J. Stoks, R. Timmermans and J. J. de Swart, Phys. Rev. C **47**, 512 (1993).
- 43. M. L. Goldberger, H. Miyazawa and R. Oehme, Phys. Rev. 99, 986 (1955).
- 44. D. V. Bugg and A. A. Carter, Phys. Lett. B 48 (1974) 67.
- 45. T. E. O. Ericson, B. Loiseau and A. W. Thomas, Phys. Rev. C 66, 014005 (2002).
- 46. V. V. Abaev, P. Metsä and M. E. Sainio, Eur. Phys. J. A 32, 321 (2007).
- R. L. Workman, R. A. Arndt and M. M. Pavan, Phys. Rev. Lett. 68 (1992) 1653 [Erratum-ibid. 68 (1992) 2712].
- 48. R. A. Arndt, R. L. Workman and M. M. Pavan, Phys. Rev. C 49, 2729 (1994).
- 49. R. A. Arndt, W. J. Briscoe, I. I. Strakovsky and R. L. Workman, Phys. Rev. C 74 (2006) 045205.
- 50. R. Workman, private communication.