Effective interactions and long distance symmetries in the Nucleon-Nucleon system

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Abstract. Effective interactions, when defined in a coarse grained sense, such as V_{lowk} at the scale $\Lambda = 450\mathrm{MeV}$, display a remarkable symmetry pattern. Serber symmetry works with high accuracy for spin triplet states. Wigner SU(4) spin-isospin symmetry with nucleons in the fundamental representation works only for even partial waves exactly as predicted by large N_c limit of QCD with accuracy $\mathcal{O}(1/N_c^2)$. This suggests tailoring the very definition of effective interactions to provide a best possible fulfillment of long distance symmetries. With the V_{lowk} definition Wigner symmetry requires that chiral potentials have low cut-offs $\Lambda_\chi \sim 450\mathrm{MeV}$. Perturbative saturation of exchanged heavy mesonic resonances does not faithfully display the Serber symmetry pattern.

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INTRODUCTION

The NN interaction is too hard to be treated directly in *ab initio* nuclear structure calculations for medium and heavy nuclei mainly because the strong short range repulsion generates short distance correlations. While much work has been devoted to handle this problem, one should recognize that calculations become involved precisely in the region where the interaction is least known since NN elastic scattering experiments probe distances larger than the minimal de Broglie wavelength $1/\sqrt{m_\pi M_N} \sim 0.5\mathrm{fm}$ corresponding to pion production threshold. The momentum space V_{lowk} approach [1] (see e.g. Ref. [2] for a review) takes a Wilsonian point of view of integrating out high energy components and working within an effective Hilbert space. The out-coming potentials are smooth and more amenable to mean field and perturbative treatments at the expense of introducing scale dependent three- and higher-body forces. For the two-body problem one replaces [1] the Lippmann-Schwinger (half-off shell) equation for the *bare* potential

$$T(k',k;k^2) = V(k',k) + \frac{2}{\pi} \int_0^\infty \frac{dq\,q^2}{k^2 - q^2} V(k',q) T(q,k;k^2) \tag{1}$$

by the equivalent V_{lowk} potential defined by the equation restricted to $(k,k') \leq \Lambda$,

$$T(k',k;k^2) = V_{\mathrm{lowk}}(k',k) + \frac{2}{\pi} \int_0^\Lambda \frac{dq\,q^2}{k^2 - q^2} V_{\mathrm{lowk}}(k',q) T(q,k;k^2) \tag{2}$$

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For instance, in perturbation theory the V_{lowk} potential becomes

$$V_{\text{lowk}}(k',k) = V(k',k) + \frac{2}{\pi} \int_\Lambda^\infty \frac{dq\,q^2}{k^2 - q^2} V(k',q)V(q,k) + \ldots \tag{3}$$

The amazing finding [1] was that *all* high precision interactions, i.e. fitting the NN elastic scattering data with $\chi^2/\text{DOF} \sim 1$ and including One Pion Exchange (OPE), have quite different momentum space $V(k,k')$ behaviour, but reduced to an *universal* V_{lowk} potential when $\Lambda \leq 2.1\text{fm}^{-1}$. The coordinate space equivalent (alias V_{HighR}) corresponds to the universality of boundary conditions obtained from integrating in from large distances taking the experimental phase shifts *and* the OPE potential down to a shortest possible distance cut-off r_c where *all other* components of the potential are negligible [3, 4, 5].

LONG DISTANCE SYMMETRIES

In this contribution we point out that such an interpretation unveils important symmetries of the effective NN interaction relevant to Nuclear Structure. Given a symmetry group with a generic element G, a standard symmetry means that $[V,G] = 0$ implies $[V_{\text{lowk}}, G] = 0$. The reverse, however, is not true. We define a *long distance symmetry* as a symmetry of the effective interaction, i.e. $[V_{\text{lowk}}, G] = 0$ but $[V,G] \neq 0$. From a renormalization viewpoint that corresponds to a symmetry of the potential broken only by counter-terms. In coordinate space the symmetry is broken by the boundary condition at the cut-off radius, r_c [3, 4, 5]. Important features of the NN interaction are both the spin-orbit interaction and the tensor force. However, it was recognized in early partial wave studies (see e.g. [6]) that there appear strong statistical correlations among different channels which could be better handled by defining suitable linear combinations for phase shifts at fixed orbital angular momentum. This may be understood by separating the NN potential as the sum of central components and (small) non-central components,
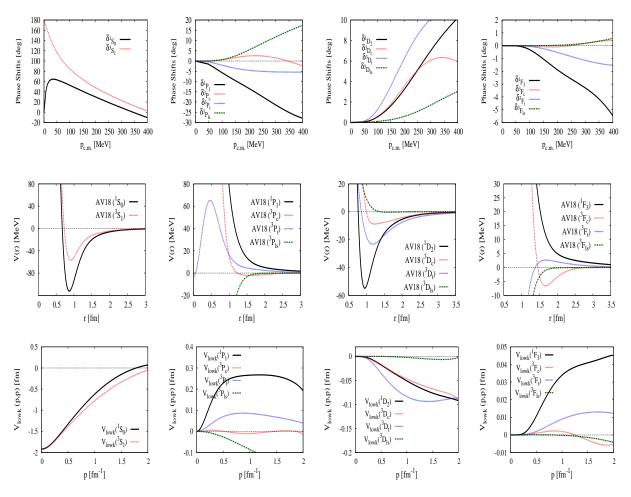
$$\mathcal{V}_{NN} = V_0 + V_1\,, \tag{4}$$

where $[\vec{L}, V_0] = 0$ whereas $[\vec{J}, V_1] = 0$ and $[\vec{L}, V_1] \neq 0$. The zeroth order potential commutes with L, S, T and so the corresponding phase shift is denoted as δ_L^{ST}. The total potential commutes with the total angular momentum $J = L + S$ and the phase-shift is δ_{LJ}^{ST}. Using first order perturbation theory around the central potential one obtains

$$\delta_{LJ}^{ST} = \delta_C^{LST} + \delta_{S,1}\,\delta_T^{LST}(S_{12}^J)_{LL} + \delta_{LS}^{LST}\frac{1}{2}\left[J(J+1) - L(L+1) - S(S+1)\right]\,, \tag{5}$$

where $(S_{12}^J)_{J-1,J-1} = -2(J-1)/(2J+1)$, $(S_{12}^J)_{J,J} = 2$, $(S_{12}^J)_{J+1,J+1} = -2(J+2)/(2J+1)$. The phases δ_C^{LST}, δ_T^{LST} and δ_{LS}^{LST} represent the center of the multiplet, the splitting due to spin-orbit and the tensor force and can be obtained by inverting Eq. (5).

In Fig. 1 we show the low partial waves with orbital angular momentum $L = 0, 1, 2, 3$ using the average values of the phase shifts analyzed by the Nijmegen group [7]. Likewise we have depicted also similar combinations for the Argonne-V18 potential [8] and the corresponding V_{lowk} potential [1] obtained from it. Spin singlet ($^1S_0$) and spin

FIGURE 1. Top panel:Average values of the phase shifts [7] (in degrees) as a function of the CM momentum (in MeV). Middle panel: Argonne V-18 potentials [8] (in MeV) as a function of distance (in fm). Bottom panel: Diagonal $V_{\text{lowk}}(p,p)$ potentials (in fm) as a function of the momentum p (in fm^{-1}) [1].

triplet ($^3S_1$) S-waves have very different phase shifts. However, both the low energy interaction as well as the potential at long distances are very similar. This property also holds for D- and G-waves, i.e., $^1S_0 =^3 S_1$, $^1D_2 =^3 D_c$ and $^1G_3 =^3 G_c$ and corresponds to spin independence of the forces in even-L channels (Wigner symmetry). On the contrary, triplet P-waves and F-waves average to null, i.e. $^3P_c =^3 F_c = 0$ in agreement with the Serber symmetry $|f_{pn}(\theta)|^2 = |f_{pn}(\pi - \theta)|^2$ in the pn differential cross section in the CM scattering angle, θ. All this could be observed in old analyses [6]. The new feature unveiled in Refs. [3, 4, 5] has been the recognition that this is a property of the effective interaction. This amazing fact suggests that the V_{lowk} approach is a good filter for an otherwise unforeseen symmetry. Note also that LS couplings in D- and F-waves are rather small within this framework, i.e. $^3D_{LS} =^3 F_{LS} = 0$. In passing we note that Wigner symmetry requires that V_{lowk} Chiral forces to N^{3}LO [9] have their cut-off $\Lambda_\chi \sim \Lambda_{\text{lowk}}$ [4] hindering larger $\Lambda_\chi \sim 600$MeV values where increasing violations are found.

LARGE N_c

While QCD is entitled to eventually generate all features of Nuclear Physics, one may profit from a semiquantitative analysis based on distinct properties of the fundamental theory. Some time ago [10], it was found that within a large N_c expansion the leading piece of the NN potential is $\mathcal{O}(N_c)$ and has the tensorial structure

$$V_{NN}(\vec{x}) = V_c(r) + \tau_1 \cdot \tau_2 \left[\sigma_1 \cdot \sigma_2 W_S(r) + S_{12}(\hat{x}) W_T(r) \right] + \mathcal{O}(1/N_c) \tag{6}$$

with corrections (comprising relativistic corrections, spin orbit, meson widths, etc.), suppressed by a relative $1/N_c^2$. This potential *only* complies to the Wigner symmetry for *even* partial waves. These large-N_c counting rules are based on quark and gluon dynamics, but for large distances quark-hadron duality allows to saturate them by the standard (multi-)meson exchange picture [11]. The striking thing [3, 4, 5] is that this symmetry pattern emerges in the effective interaction !!. This important point prevents us from using large N_c literally, but rather as a long distance symmetry. This way, the ubiquitous fine tunings, triggered by unknown short distance physics, are efficiently disentangled from long distance physics with the help of renormalization. We are pursuing this large-N_c framework for Nuclear Physics with rather encouraging results [12] for the low partial waves and the deuteron in the case of One Boson Exchange (OBE) and its Meson Exchange Currents [13] where only $\pi, \sigma, \rho, \omega, A_1$ contribute to Eq. (6) [11].

RESONANCE SATURATION VS SCALE SATURATION

The momentum space V_{lowk} approach [1] makes clear that the long distance behaviour is not determined by the low momentum components of the original potential *only*; one has to add virtual high energy states which also contribute to the interaction at low energies. Actually, in the limit $\Lambda \to 0$ one may Taylor expand the effective S-wave interaction

$$V_{\text{lowk}}(k, k') = (M_N/16\pi^2) \left[C_0 + C_2(k^2 + k'^2) + \ldots \right] \tag{7}$$

Plugging this ansatz into Eq. (2) and fixing the T matrix at low energies to the scattering length α_0, effective range r_0, one gets (for $C_2 = 0$) a result depending on the scale Λ,

$$C_0(\Lambda) = (16\pi^2 \alpha_0/M_N)/(1 - 2\alpha_0\Lambda/\pi) \tag{8}$$

which for $\Lambda \sim 150(200)$MeV reproduces the exact V_{lowk} value in the $^1S_0(^3S_1)$ channel. The inclusion of tensor force mixing and range corrections saturates to almost 100% smoothly the V_{lowk} for the same $\Lambda \sim 250$MeV and complies to the Wigner symmetry pattern, $C_{^1S_0} = C_{^3S_1}$ as can be seen in Fig. 1 at $p = 0$. Higher partial waves and the relation to Skyrme forces are analyzed further in Ref. [14]. Note that since $M_N \sim N_c$ for $C_0 \sim N_c$ one needs $\alpha_0 \sim N_c^2$ and $\Lambda \sim 1/N_c^2$ in Eq. (8).

Inspired by the success of the resonance saturation hypothesis of the exchange forces in $\pi\pi$ scattering (see e.g. [15]), the OBE picture [16] low momentum contributions from the exchange of heavier mesons have been identified as generating the counter-terms for chiral potentials [17]. Taking e.g. a heavy scalar meson with mass $m \gg \Lambda$ and coupling

g one can use Eq. (3) and the potential reads

$$\frac{g^2}{(\mathbf{p}'-\mathbf{p})^2+m^2} = \frac{g^2}{m^2} - \frac{g^2(\mathbf{p}'-\mathbf{p})^2}{m^4} + \ldots = C_0 + C_2\left(\mathbf{p}^2+\mathbf{p}'^2\right) + C_1\mathbf{p}\cdot\mathbf{p}' + \ldots, \quad (9)$$

Note that here $C_0 = g^2/m^2$ does not depend on Λ, unlike Eq. (8). The OBE resonance matching to chiral potentials besides identifying terms scaling differently in N_c, $C_0^{\text{OBE}} \sim N_c$ vs $C_0^{\text{Chiral}} \sim g_A^4/f_\pi^2 \sim N_c^2$ do not comply to Serber symmetry for P-waves as it happens for the V_{lowk} determination [4]. This does not mean, however, that the effective interaction cannot be represented in the polynomial form of Eq. (9), but rather that the coefficients cannot be computed *directly* and generically as the Fourier components of the potential, since the corrections are not necessarily small (unlike the $\pi\pi$ case).

CONCLUSIONS

The non-trivial fact that the Wigner and Serber symmetries occur unequivocally for V_{lowk} and not so much for the bare V reinforces the V_{lowk}-approach as an efficient and symmetry-based coarse grained interaction. Once the symmetries emerge, simplifications are expected, and this affects the very definition of effective interactions.

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