Effective tensor forces and neutron rich nuclei

G. Co' and V. De Donno

Dipartimento di Fisica, Università del Salento and,
INFN Sezione di Lecce, Via Arnesano, I-73100 Lecce, ITALY

M. Anguiano and A. M. Lallena

Departamento de Física Atómica, Molecular y Nuclear,
Universidad de Granada, E-18071 Granada, SPAIN
(Dated: April 25, 2018)

We study the effects of the tensor term of the effective nucleon-nucleon interaction on nuclear excited states. Our investigation has been conducted by using a self-consistent Random Phase Approximation approach. We investigate various nuclei in different regions of the isotopes chart. Results for a set of calcium isotopes are shown.

The tensor parts of the nucleon-nucleon interaction are usually neglected in Hartree-Fock (HF) and Random Phase Approximation (RPA) calculations. In these theories, the effects of the tensor force are taken into account by modifying the values of the parameters of the other terms of the effective force. We have investigated the validity of this approach, and we present here some results of our study.

In our work we have considered only the tensor-isospin term of the nuclear interaction, which we have described as

$$v_b^{t\tau}(r) = v_{AV8'}^{t\tau}(r)(1 - e^{-br^2}) \;, \tag{1}$$

where we have indicated with $v_{AV8'}^{t\tau}(r)$ the tensor-isospin term of the Argonne V8' microscopic potential [1]. In the above expression, this term is multiplied by a function which considers the effects of the short-range correlations. The only free parameter is b, whose value rules the strength of the tensor part of the force. Small values of b increase the active range of the correlation reducing the strength of the force.

As example of the effects produced by the tensor force on the nuclear excited states, we show in Fig. 1 the evolution of the RPA eigenenergy of the first 0^- excited state in ^{16}O , obtained with a phenomenological RPA calculation. Similar sensitivity of the 0^- excitation energies to the presence of the tensor term of the force has been found in all the nuclei we have investigated, ^{12}C, ^{16}O, ^{40}Ca, ^{48}Ca and ^{208}Pb. We present here only the ^{16}O case for sake of simplicity. The results of Fig. 1 have been obtained by using single particle energies and wave functions calculated by diagonalizing a Woods-Saxon potential in a harmonic oscillator basis. The residual interaction, used in the RPA calculation, is a zero-range Landau-Migdal force whose parameters values are those given in Ref. [2]. The value obtained with this interaction is shown in the figure by the dashed line. When the full Argonne V8' tensor term is added, we obtain the value represented by the dotted line. The full line shows the experimental value of 10.96 MeV [3]. The squares indicate the values of the 0^- eigenenergies as a function of b. These values become smaller with the increase of the tensor force strength. The evolution of the 0^- eigenenergy, from that obtained with the pure Landau-Migdal force up to that obtained when the full tensor term is active, is very smooth.
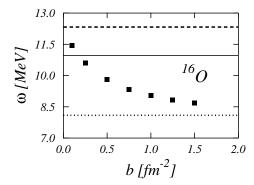

Figure 1: Energy of the first 0^- excited state in ^{16}O calculated by using a phenomenological RPA approach. The dashed line indicates the values obtained without tensor force. The dotted line shows the value obtained when the full Argonne V8' term is active. The squares indicate the values obtained for various values of the parameter b of Eq. 1. The full line shows the experimental value.

The phenomenological RPA calculations described above have been used to identify an observable very sensitive to the tensor force. We have considered this quantity in order to select the values of b.

The tensor force produces effects on the single particle basis and also on the excited states of the nucleus. For this reason we have conducted our study by doing self-consistent RPA calculations. This means that we used in the

RPA calculations the single particle wave functions and energies obtained by solving the HF equations. The effective interaction in both type of calculations is the same.

The effect of the tensor force on the single particle energies has been well clarified by Otsuka and collaborators [4]. For example, the energies of two proton spin-obit partner single particle levels are modified by the tensor-isospin interaction acting with a neutron single particle level. If the total angular momentum of the occupied neutron single particle level is $j = l + 1/2$ the energy of the $j = l - 1/2$ proton level is lowered and that of the $j = l + 1/2$ proton level is increased. This effect is reversed when the occupied neutron level has $j = l - 1/2$. For this reason, the nuclei where all the neutron spin-orbit partners single particle levels are fully occupied do not show any tensor force effect.

We have conducted our study by comparing results obtained with and without tensor force. We have considered two different parametrization of the Gogny interaction. They are the traditional D1S [5] force and the more modern D1M parametrization [6] which describes better the behaviour of the the microscopic equation of state of neutron matter at high densities.

The D1S and D1M interactions do not have tensor terms. We add to these interactions a tensor-isospin term of the form given by Eq. 1. The procedure we have adopted to construct new parametrization of the force is iterative. We first made a HF calculation without tensor force to produce single particle energies and wave functions. Then we made RPA calculations with the tensor force, selecting the value of b which reproduces the experimental energy of the 0^- in ^{16}O. With this new interaction we have recalculated the HF single particle energies and wave functions, and we changed the spin-orbit term of the interaction to reproduce the splitting between the neutron $1p_{3/2}$ and $1p_{1/2}$ levels. With this single particle basis we performed a new set of RPA calculations and the procedure has been repeated up to convergence. In this way, starting from the D1S and D1M forces, we have constructed two new parametrizations which we call respectively D1SV8 and D1MV8. The differences with the original parametrizations are only in the spin-orbit term and, obviously, in the presence of the tensor term. We did not make a complete refit of the forces.

In our studies, we have also used the GT2 interaction [7] which contains a tensor term, and we made calculations by switching off this term. Since the results obtained in this way are very similar to those obtained with the other two interactions, for sake of simplicity, we do not present them here.
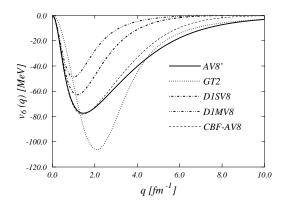

Figure 2: Tensor-isospin terms of the interactions used in our study as a function of the momentum of the interacting pair.

In Fig. 2 we show the momentum dependent part of the tensor forces, defined as

$$v_6(q)\, S_{12}(\mathbf{q}) = \int d^3 r\, e^{i\mathbf{q}\cdot\mathbf{r}}\, v^{t\tau}(r)\, S_{12}(\mathbf{r}) = -4\pi \int dr\, r^2\, j_2(qr)\, v^{t\tau}(r) S_{12}(\mathbf{r}) \ , \qquad (2)$$

where we have indicated with S_{12} the usual expression of the tensor operator [8].

In this figure, the full line shows the the tensor term of the Argonne V8' interaction used to build the D1SV8 and D1MV8 forces, indicated by the dashed-dotted and dashed-doubly dotted lines. With the dotted line we show the tensor term of the GT2 interaction [7]. The dashed line has been obtained by multiplying the bare Argonne V8' term with the scalar part of the short range correlation calculated in Correlated Basis Function theory [8]. This line indicates that the effect of the short range correlations obtained by a microscopic calculations are much smaller than those required by our procedure.

With the interactions presented above, we have studied a set of isotopes of doubly magic nuclei. These nuclei have been chosen in such a way that the single particle levels below the Fermi energy are fully occupied. In this way we avoided deformation problems and we minimized pairing effects. We have studied ground states and excitation spectra of nuclei in the oxygen, calcium, zirconium and tin regions and also ^{208}Pb, for a total number of 16 different isotopes.

We have seen that the effects of the tensor force are not important on the binding energies, as already pointed out in Ref. [9]. We have obtained variations of less than 1% of the values obtained without tensor terms.

We have also studied the protons and neutrons density distributions for all the nuclei mentioned above. Also in this case the effects of the tensor force are irrelevant. The largest differences between calculations with and without tensor force are of the order of few parts on a thousand.
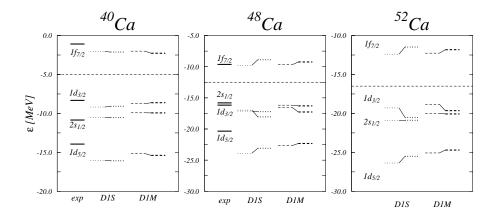

Figure 3: Single particle energies of the calcium isotopes calculated with the interactions D1S and D1M, thin lines, and with their corresponding interactions containing the tensor term, the D1SV8 and D1MV8, thick lines.

On the opposite, we have observed that the effect of the tensor force on the single particle energies in not negligible. As example we show in Fig. 3 the single particle spectrum of the three calcium isotopes considered in our study. For simplicity we present only the results obtained with D1S and D1M interactions, and their corresponding interactions containing the tensor term. In the figure, the thin lines show the results obtained without tensor force and the thick lines those obtained with the tensor force. The full lines show the experimental single particle energies taken from Ref. [10].

As expected, in ^{40}Ca the tensor term has no effect, since all the neutron spin-orbit partner levels are occupied. The small differences between the energies are due to the different spin-orbit terms between the D1S and D1SV8 forces and the D1M and D1MV8 forces.

The effect is present in the single particle spectrum of the other two isotopes since in these nuclei the neutron $j = l + 1/2$ levels are occupied, while their spin-orbit partners are empty. The figure shows that with the inclusion of the tensor term the energy of the $1d_{5/2}$ levels increases, and that of the $1d_{3/2}$ decreases. For the ^{48}Ca nucleus, this effect helps in reproducing the empirical sequence of single particle states. For the D1S interaction the tensor force lowers the energy of the $1d_{3/2}$ level below that of the $2s_{1/2}$ as it is experimentally observed. The effect in ^{52}Ca is not so dramatic. From the experimental point of view it is not yet clear if the spin and parity of the ^{51}K ground state is $3/2^+$ or $1/2^+$. All our calculations indicate $3/2^+$.

The action of the tensor force affects also the energy of the first proton level above the Fermi surface, the $1f_{7/2}$ level, by increasing its value. This means that the gap between the occupied $1d_{3/2}$ proton level and the $1f_{7/2}$ empty level increases if the tensor force is included. For this reason the energy of excited states dominated by the proton $[1f_{7/2}, 1d_{3/2}^{-1}]$ particle-hole pair should increase. This is what we observe from the results shown in Tab. I where we present the RPA eigenergies for the 4^- states in the three calcium isotopes obtained from calculations done with forces with (D1SV8 and D1MV8) and without (D1S and D1M) tensor force. We also show results of calculations where the tensor force has been switched off only in the RPA calculation. They are indicated by the D1SV8* and D1MV8* labels.

The results behave exactly as expected. The tensor force has practically no effect on the ^{40}Ca results, while there is a clear increase in ^{48}Ca and ^{52}Ca. On the other hand, the experimental values do not seem to have this trend. It would be interesting to know the experimental value of the 4^- energy in ^{52}Ca.

Table I: Energies, in MeV, of the 4^- excited state for various Ca isotopes.

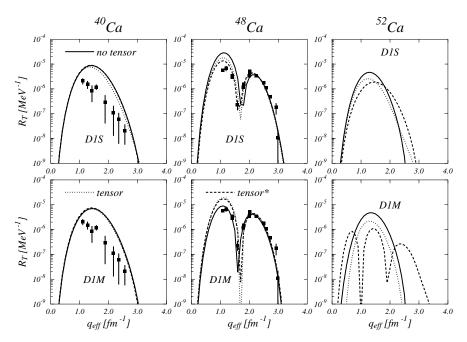|	exp	D1S	D1SV8	D1SV8*	D1M	D1MV8	D1MV8*
^{40}Ca	7.66	7.50	7.41	7.32	7.04	6.70	6.64
^{48}Ca	6.11	7.91	8.61	8.75	6.81	7.30	7.40
^{52}Ca		7.04	9.03	9.15	6.77	7.84	7.97

Figure 4: Electron scattering transverse responses calculated by using interactions without tensor, full lines, with tensor, dotted lines and by switching off the tensor force only in the RPA calculations, dashed lines. The experimental data are taken form Refs. [11] and [12].

To have information about the wave function of the excited states, we have calculated the inelastic electron scattering transverse responses. In Fig. 4, we compare our results with the experimental data from Ref. [11] and [12].

The effects of the tensor force are irrelevant in the ^{40}Ca case. We observe some sensitivity to the tensor force in the case of ^{48}Ca . In the D1S case the tensor force lowers the first peak, while the effect is reversed in the case of the D1M interaction. The main effect is that on the single particle wave functions, on the HF calculation. The situation is different for the ^{52}Ca, isotope. In this case the tensor force effects are relevant on both the HF results, and also on the results obtained by the RPA calculation. It is interesting the situation of the D1M calculation in ^{52}Ca, where the presence of the tensor force in the RPA eliminates the contamination of the neutron $[2d_{5/2}, 2p_{1/2}^{-1}]$ particle-hole pair, and it changes the shape of the response.

In general, the tensor force does not produce relevant effects on the bulk properties of the nuclear systems, such as binding energies, density distributions and excitation energies. Concerning the excited states, however, there are situations where the inclusion of the tensor form is important. Certainly the energies of the 0^- states are extremely sensitive to the presence of the tensor term of the force. Furthermore, the effects on the single particle energies can modify excitations dominated by specific particle-hole pairs. We have presented the case of the 4^- excitations in calcium isotopes which clearly shows tensor force effects when the neutrons do not occupy all the spin-orbit partners levels.

The accuracy required by modern nuclear structure calculations requires the inclusion of the tensor term in the effective nucleon-nucleon interaction, especially for the study of nuclei with neutron excess.

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