The roton-assisted chiral p-wave superfluid in a quasi-two-dimensional dipolar Bose-Fermi quantum gas mixture

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The chiral p-wave ($p_x \pm ip_y$) superfluid has attracted significant attention in recent years, mainly because its vortex core supports a Majorana fermion which, due to its non-Abelian statistics, can be explored for implementing topological quantum computation. Mixing dipolar bosons with fermions in quasi-two-dimensional (2D) space offers the opportunity to use the roton minimum as a tool for engineering the phonon-induced attractive interaction between fermions. We study, within the Hartree-Fock-Bogoliubov approach, the p-wave superfluid pairings in a quasi-2D dipolar Bose-Fermi mixture. We show that enhancing the induced interaction by lowering the roton minimum can affect the stability property of the mixture as well as the effective mass of the fermions in an important way. We also show that one can tune the system to operate in stable regions where chiral p-wave superfluid pairings can be resonantly enhanced by lowering the energy cost of the phonons near the roton minimum.

I. INTRODUCTION

The chiral p-wave ($p_x \pm ip_y$) phase in both superconductors [1–6] and ultracold atomic systems [7–13] has attracted significant attention in recent years, mainly because its vortex core supports a unique bound quasiparticle (non-Abelian) excitation with zero energy known as a Majorana fermion. A two-dimensional (2D) topological system with well isolated vortices can be thought of as a quantum computing device where a unitary transformation (braiding vortices [14]), the basic operation of any quantum computer, is performed in a degenerate space [2, 15] topologically protected from other nontopological excited states by an energy gap [14]. In chiral p-wave superfluid systems, this energy gap is proportional to the p-wave superfluid gap parameter. Increasing the p-wave superfluid pairing, then, can increase the energy gap, which, in turn, makes the topologically protected computation less susceptible to finite temperature effects. However, for a wave with finite angular momentum, there is a centrifugal barrier in its scattering potential which prevents the wave from penetrating the interior and scattering effectively from the two-body potential. Thus, for typical ground state atoms, the p-wave superfluid pairing is highly suppressed.

The advent of cold-atom systems opened up the possibility of using electromagnetic fields to tune and design two-body interactions capable of enhancing chiral p-wave superfluid pairings [7–13]. The same goal may also be achieved in a Bose-Fermi mixture since bosons can induce and hence modify the Fermi-Fermi interaction in a Fermi gas [16, 17]. In a dipolar quantum gas [18, 19], the dipole-dipole interaction represents a control knob inaccessible to nondipolar bosons. Thus, mixing dipolar bosons with fermions opens up new possibilities [20–22]. Experimentally, great progress has been made in recent years in achiving dipolar quantum gases consisting of ei-

ther heteronuclear molecules with electric dipoles [23, 24] or atoms with magnetic dipoles [25–28]. In this article, we consider a quasi-two-dimensional (2D) cold-atom mixture of nondipolar fermions of mass m_F in a spin-polarized Fermi gas and bosons of mass m_B in a dipolar condensate where all the dipoles are oriented along the axial (z) direction. The collective excitation of such a system was discovered [29] to exhibit, at finite momentum, a minimum analogous to the "roton" minimum in superfluid helium [30, 31]. Dutta and Lewenstein [20] pointed out recently that the existence of dipolar bosons in such a model can significantly enhance the unconventional superfluids in the spin-polarized Fermi gas. The underlying physical mechanism [20] is the ability to enhance the induced Fermi-Fermi attraction by lowering the energy cost of the phonons of the dipolar condensate near the roton minimum [29] (see the main text for detail). This raises the questions of how the enhanced induced interaction, usually ignored in the stability analysis of the Bose-Fermi mixture, affects the stability property of the mixture, and how this same enhanced interaction affects the effective mass of the fermions near the Fermi energy and hence the p-wave superfluid critical temperature. In what follows, we apply the Hartree-Fock-Bogoliubov mean-field theory to gain insight into these questions and identify the parameter space in which the mixture is stable against density fluctuations and the p-wave superfluid pairing can be resonantly enhanced.

II. QUASI-2D DIPOLAR CONDENSATE AND ITS EXCITATION SPECTRUM

To begin with, we divide the interactions (in momentum space) into a short-range part, $U_{BF}^{3D} = 4\pi\hbar^2 a_{BF}/m_{BF}$ and $U_{BB}^{3D} = 4\pi\hbar^2 a_{BB}/m_B$, characterized with s-wave scattering lengths a_{BF}, between a boson

and a fermion, and a_{BB}, between two bosons, where $m_{BF} = 2m_B m_F/(m_B + m_F)$, and a long range part, $U_{DD}^{3D}(\mathbf{k}) = 8\pi d^2 P_2(\cos\theta_\mathbf{k})$, due to the dipole-dipole interaction, where d is the induced dipole moment, $P_2(x)$ the second-order Legendre polynomial, and $\theta_\mathbf{k}$ the polar angle of wavevector $\mathbf{k}$. Thus, $U_{BB}^{3D}(\mathbf{k}) = U_{BB}^{3D} + U_{DD}^{3D}(\mathbf{k})$ is the total Bose-Bose interaction. The particles in the mixture are confined, by sufficiently high harmonic trap potentials along the axial dimension, to a quasi-2D plane where the axial degree of freedom is "frozen" to the zero-point oscillation and can thus be factored out. Integrating out the axial degree of freedom via the relations $U_{BF} = \int n_B(k_z) U_{BF}^{3D} n_F(k_z) dk_z$ and $U_{BB}(\mathbf{k}_\perp) = \int n_B(k_z) U_{BB}^{3D}(\mathbf{k}) n_B(k_z) dk_z$, where $n_{B,F}(k_z) = \exp(-d_{B,F}^2 k_z^2/4)/\sqrt{2\pi}$, with $d_{B,F}$ the axial oscillator lengths, we arrive at a qasi-2D dipolar Bose-Fermi mixture characterized by the effective 2D interactions $U_{BF} = U_{BF}^{3D}/\sqrt{\pi(d_F^2 + d_B^2)}$, $U_{BB} = U_{BB}^{3D}/\sqrt{2\pi} d_B$, and

$$U_{BB}(k_\perp) = U_{BB}\left[1 + 2\varepsilon_{dd} - 3\varepsilon_{dd}\mathcal{G}(d_B k_\perp)\right], \quad (1)$$

where $\varepsilon_{dd} = 4\pi d^2/(3U_{BB}^{3D})$ measures the dipolar interaction relative to the s-wave contact interaction, and

$$\mathcal{G}(x) = \sqrt{\frac{\pi}{2}}x\exp\left(\frac{x^2}{2}\right)\mathrm{Erfc}\left(\frac{x}{\sqrt{2}}\right), \quad (2)$$

is a well-behaved function [32] which increases monotonically from 0 to 1, reaching 50% of its full value when $x \approx 0.61$, and $\mathrm{Erfc}(x)$ is the complementary error function. In arriving at these effective interactions, we have assumed that the axial oscillator lengths are sufficiently large compared with the relevant scattering lengths so that atoms experience essentially 3D collisions and there is no need to consider the confinement induced resonances [33–35]. Using the Bogoliubov perturbative ansatz and diagonalization procedure [36], we can describe the dipolar Bose gas near zero temperature ($T \approx 0$) as a uniform dipolar condensate of number density n_B plus a collection of phonon modes that obey the dispersion spectrum

$$E_{k,B} = \hbar k v_B \sqrt{1 + 2\varepsilon_{dd} + \chi_B^2 d_B^2 k^2 - 3\varepsilon_{dd}\mathcal{G}(d_B k)}, \quad (3)$$

where $v_B = \sqrt{n_B U_{BB}/m_B}$ is the phonon speed and $\chi_B = \xi_B/d_B$ measures the healing length, $\xi_B = \hbar/\sqrt{4m_B n_B U_{BB}}$, in terms of d_B. (From now on, $\mathbf{k}$ stands for $\mathbf{k}_\perp$, the in-plane wavevector, for notational convenience.)

In a dipolar condensate, there is a competition between the side-to-side arrangement of two dipoles where the dipole-dipole interaction is the most repulsive and the head-to-tail configuration where the dipole-dipole interaction is the most attractive. In the 3D (or large d_B) limit, the head-to-tail configuration and hence the attractive interaction can be significant, and $U_{BB}(k)$ in Eq. (1) reduces to $U_{BB}(1 - \varepsilon_{dd})$. In this limit, when $\varepsilon_{dd} > 1$, $E_{k,B}$ in Eq. (3) acquires an imaginary frequency

in the long wavelength limit, and the condensate is unstable against collapse. In the 2D (or small d_B) limit, the head-to-tail arrangement is suppressed, and $U_{BB}(k)$ in Eq. (1) asymptotes to the effective 2D contact interaction $U_{BB}(0) = U_{BB}(1 + 2\varepsilon_{dd})$, which is $1 + 2\varepsilon_{dd}$ times more repulsive than if the dipole interaction were absent. In the quasi-2D case, which is our focus here, the head-to tail configuration is not completely suppressed. This is the origin of the attractive part in Eq. (1), which depends on $\mathcal{G}(x)$ defined in Eq. (2). As a result, in quasi-2D space, $\varepsilon_{dd} > 1$ does not automatically mean condensate collapse as in the 3D limit. Instead, $\varepsilon_{dd} > 1$ signals that a roton minimum [29] at finite k can occur for sufficiently small χ_B (or equivalently sufficiently high boson density) as illustrated in Fig. 1(a), and only when the roton minimum crosses zero, becoming imaginary, does the condensate become unstable (against the formation of a density wave).

III. STABILITY OF THE QUASI-2D DIPOLAR BOSE-FERMI MIXTURE

Integrating out the phonon degrees of freedom, we obtain a total effective Hamiltonian, $\hat{H}^E = H_B^0 + \hat{H}_F^E$, for a quasi-2D mixture with an effective area A and fixed particle number densities, n_B and n_F, where $H_B^0 = AU_{BB}(0)n_B^2/2$ accounts for the dipolar condensate energy, and

$$\hat{H}_F^E = \sum_\mathbf{k}\left[\epsilon_\mathbf{k}\left(= \hbar^2 k^2/2m_F\right) + U_{BF}n_B\right]\hat{a}_\mathbf{k}^\dagger\hat{a}_\mathbf{k}$$
$$+ \frac{1}{2A}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} U(q)\hat{a}_{\mathbf{k}+\mathbf{q}}^\dagger\hat{a}_{\mathbf{k}'-\mathbf{q}}^\dagger\hat{a}_{\mathbf{k}'}\hat{a}_\mathbf{k}, \quad (4)$$

describes an effective Fermi gas in which fermions interact via the two-body interaction potential (in the static limit [16, 36]), $U(k) = -\left(U_{BF}^2/U_{BB}\right)\bar{U}(k)$, where

$$\bar{U}(k) = \frac{1}{1 + 2\varepsilon_{dd} + \chi_B^2 d_B^2 k^2 - 3\varepsilon_{dd}\mathcal{G}(d_B k)}. \quad (5)$$

Note that Fermi statistics prohibits two fermions in a spin-polarized Fermi gas to participate in s-wave scattering. In our model, two fermions of the same spin can interact indirectly by exchanging virtual phonons with the help of the intermediate states containing one phonon without violating the Pauli exclusion principle. Equation (5) describes precisely this phonon-mediated Fermi-Fermi interaction.

The Fermi pairings can benefit from this induced interaction, which is now tunable by varying the roton minimum, provided that fermions and bosons can form stable mixtures near the roton minimum. Thus, we first address the stability issue using the energy density, $\mathcal{E}(n_B, n_F) = (H_B^0 + \langle\hat{H}_F^E\rangle)/A$, where, for simplicity, the average is performed assuming that the fermions are in

the normal gas phase (at $T = 0$) and thus distribute according to $\langle \hat{a}_{\mathbf{k}}^{\dagger} \hat{a}_{\mathbf{k}} \rangle = \Theta \left(k_F^2 - k^2 \right)$. Here, $\Theta \left(x \right)$ is the step function and $k_F = \sqrt{4\pi n_F}$ the (2D) Fermi momentum. A straightforward application of the Hartree-Fock mean-field theory [37] finds,

$$\mathcal{E} \left(n_B, n_F \right) = \frac{U_{BB} \left(1 + 2\varepsilon_{dd} \right)}{2} n_B^2 + U_{BF} n_B n_F + \frac{\pi \hbar^2}{m_F} n_F^2$$
$$- \frac{\pi \hbar^2}{m_F} \frac{\lambda}{1 + 2\varepsilon_{dd}} n_F^2 + \mathcal{E}_{exc} \left(n_B, n_F \right), \quad (6)$$

where the first line, which will be denoted as $\mathcal{E}_0 \left(n_B, n_F \right)$, represents the energy of the mixture when the induced interaction is ignored, and the terms in the second line represent, respectively, the direct (Hartree) and the exchange (Fock) energy of the induced interaction Hamiltonian in Eq. (4). It can be shown that the latter, which is defined as $\mathcal{E}_{exc} = -\sum_{\mathbf{k},\mathbf{k}'} U \left(|\mathbf{k} - \mathbf{k}'| \right) \langle \hat{a}_{\mathbf{k}'}^{\dagger} \hat{a}_{\mathbf{k}'} \rangle \langle \hat{a}_{\mathbf{k}}^{\dagger} \hat{a}_{\mathbf{k}} \rangle / 2A^2$, can be reduced to the 1D integral

$$\mathcal{E}_{exc} \left(n_B, n_F \right) = \lambda \frac{\pi \hbar^2}{m_F} \frac{k_F^2}{\left(2\pi \right)^3} \int_0^{2k_F} \bar{U} \left(q \right) I \left(q \right) q dq, \quad (7)$$

where $I \left(q \right)$ is a function of q defined as

$$I \left(q \right) = 2 \sin^{-1} \sqrt{1 - \left(\frac{q}{2k_F} \right)^2} - \frac{q}{k_F} \sqrt{1 - \left(\frac{q}{2k_F} \right)^2}, \quad (8)$$

when $q < 2k_F$ and $I \left(q \right) = 0$ otherwise. The quantity λ, in both Eqs. (6) and (7), is a unitless coupling parameter defined as $\lambda = N \left(\epsilon_F \right) U_{BF}^2 / U_{BB}$, where $N \left(\epsilon_F \right) = m_F / 2\pi \hbar^2$ is the (2D) density of states independent of the Fermi energy ϵ_F. The same λ also appears in the gap equation (10) we will use to estimate the critical temperature.

The uniform mixture is unstable against small changes in densities if any of the following inequalities, (a) $\partial_{n_B}^2 \mathcal{E} < 0$, (b) $\partial_{n_F}^2 \mathcal{E} < 0$, and (c) $\left(\partial_{n_B}^2 \mathcal{E} \right) \left(\partial_{n_F}^2 \mathcal{E} \right) - \left(\partial_{n_B, n_F}^2 \mathcal{E} \right)^2 < 0$, occur. The analysis below, however, focuses only on the instability towards phase separation (c) as it always precedes collapse of the condensate (a) and that of the Fermi gas (b) for the parameters of interest. It is well-known, from stability analysis of the Bose-Fermi mixtures in which the phonon-induced interaction is insignificant and is thus neglected, that the dimensionality plays an important role. This is because the kinetic energy of the Fermi sea depends on the Fermi surface gemometry, which is strongly influenced by the dimensionality. The kinetic energy (of the Fermi sea) is $\sim \left(n_F^{3D} \right)^{5/3}$ in 3D and $\sim \left(n_F^{1D} \right)^3$ in 1D. Thus, contrary to 3D mixtures, which are stable only when n_F^{3D} is below a threshold [38], 1D mixtures are stable provided that n_F^{1D} is above a threshold [39]. 2D mixtures are distinct in the sense that the 2D kinetic energy follows the square law, $\sim n_F^2$. A simple analysis of $\mathcal{E}_0 \left(n_B, n_F \right)$
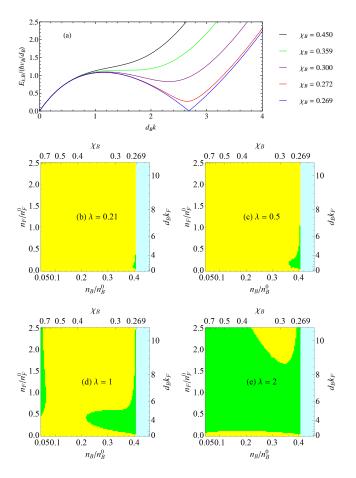

FIG. 1. For all diagrams in this article, $\varepsilon_{dd} = 2.2$, $m_F = 6u$, $m_B = 127u$, $d_F = 0.15\mu m$, $d_B = 0.9\mu m$, and $a_{BB} = 100a_0$, where u is the atomic mass and a_0 is the Bohr radius. In addition, a characteristic Fermi wavenumber k_F^0 is introduced as a basic unit, and $n_F^0 = \left(k_F^0 \right)^2$ and $n_B^0 = 2bm_B \left(k_F^0 \right)^2 / m_F$ are defined as the scaled number densities, where $b = \sqrt{\pi \hbar^2 m_F / 2m_B^2 U_{BB}}$ is a unitless parameter. Throughout, k_F^0 is fixed to $k_F^0 = 2.11\mu m^{-1}$ so that $n_F^0 = 4.45 \times 10^{12} \ m^{-2}$ and $n_B^0 = 2.99 \times 10^{14} \ m^{-2} (b = 1.344)$. (a) The excitation spectrum, $E_{\mathbf{k},B}$, is plotted as a function of $d_B k$ for $\chi_B = 0.45$ (black), 0.359 (green), 0.30 (violet), 0.272 (red), 0.269 (blue) (from top to bottom). (b)-(e) are the stability diagrams in density (n_B, n_F) space for $\lambda = 0.21$ (b), 0.5 (c), 1.5 (d), and 2.0 (e) when the Hartree-Fock energies are included. The uniform quasi-2D mixture is stable (yellow) and unstable (green) against small changes in densities, and the dipolar condensate is unstable against density waves in the light blue region where the roton minimum is imaginary.

indicates that the mixture is either stable or unstable, and the transition from the stable to unstable mixture takes place when λ exceeds the threshold value $1 + 2\varepsilon_{dd}$ irrespective of the particle number densities.

This transition is not a clear cut anymore when the Hartree-Fock energies are included because n_F and n_B enter $\bar{U} \left(k \right)$ in Eq. (5) (via the Fermi wavenumber k_F and the relative healing length χ_B) in a complicated man-

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ner. Thus, the functional dependencies of $\mathcal{E}_{exc}(n_B, n_F)$ in Eq. (7) on n_B and n_F can be highly asymmetrical and nontrivial. Figures 1(b)-(e) showcase four sets of stability diagrams in density space with different λ when the Hartree-Fock energies are included. As can be seen, the region of instability depends on densities and grows in size as λ increases. The density space is mainly occupied by the stable states (yellow) for small λ, and by unstable states (green) for sufficiently large λ. [The light blue area (in which χ_B is lower than 0.269) is the region where the roton minimum becomes imaginary.] But, the instability, particularly in the region where the roton minimum is low, develops when λ is far less than $1 + 2\varepsilon_{dd}$ (which equals 5.4 for $\varepsilon_{dd} = 2.2$) as indicated by the green areas in Fig. 1(b) and (c).

IV. CHIRAL p-WAVE SUPERFLUID AND CRITICAL TEMPERATURE

In the parameter space that hosts the single uniform mixture, the interaction induced by bosons, being attractive, provides identical fermions with the opportunity to form odd-parity superfluids. To investigate this possibility, we begin with the 2D equation for the gap parameter,

$$\Delta(\mathbf{k}) = -\frac{1}{A} \sum_{\mathbf{k}'} U(|\mathbf{k} - \mathbf{k}'|) \frac{\tanh(\beta E_{\mathbf{k}'}/2)}{2E_{\mathbf{k}'}} \Delta(\mathbf{k}'), \quad (9)$$

which is coupled to the Fock potential, $\Sigma(\mathbf{k}) = -\sum_{\mathbf{k}'} U(|\mathbf{k} - \mathbf{k}'|) \langle \hat{a}_{\mathbf{k}'}^\dagger \hat{a}_{\mathbf{k}'} \rangle / A$, via the quasiparticle energy spectrum, $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta(\mathbf{k})^2}$, where $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu_F + \Sigma(\mathbf{k})$ and $\beta = (k_B T)^{-1}$. In the space spanned by the eigenfunctions of the z-component of angular momentum, $\Delta(\mathbf{k}) = \sum_l \Delta_l(k) \exp(il\phi_{\mathbf{k}})$, where $l = 0, \pm 1, \pm 2, \cdots$ and $\phi_{\mathbf{k}}$ is the polar angle of vector $\mathbf{k}$. In this space, by virtue of the circular symmetry of the interaction, the gap parameters of different l-waves, $\Delta_l(k)$, are all decoupled, satisfying

$$\Delta_l(k) = \lambda \int_0^\Lambda d\epsilon_{k'} \bar{U}_l(k, k') \frac{\tanh(\beta \xi_{k'}/2)}{2\xi_{k'}} \Delta_l(k'), \quad (10)$$

where $\bar{U}_l(k, k') = \int_0^{2\pi} \bar{U}(|\mathbf{k} - \mathbf{k}'|) e^{il\phi}/2\pi$ with $\phi = \phi_{\mathbf{k}} - \phi_{\mathbf{k}'}$ and Λ is the energy cutoff (set to twice the Fermi energy ϵ_F [12] in our estimate below). As such, one can define independently a critical temperature

$$T_l \approx \left(T_F \equiv \frac{\epsilon_F}{k_B}\right) \times \frac{1}{\gamma_F} \exp\left[-\frac{1}{\lambda \gamma_F \bar{U}_l(k_F)}\right], \quad (11)$$

for each l-wave superfluid, where $\bar{U}_l(k_F) \equiv \bar{U}_l(k_F, k_F)$ or

$$\bar{U}_l(k_F) = \int_0^{2\pi} \bar{U}\left(2k_F \left|\sin\frac{\phi}{2}\right|\right) e^{il\phi} \frac{d\phi}{2\pi}. \quad (12)$$

In arriving at Eq. (11), we have assumed that only states near the Fermi energy contribute to the gap equation;

we have used the $T = 0$ normal Fermi gas, where the Fermi surface is well-defined, to estimate both the Fermi momentum, $k_F = \sqrt{4\pi n_F}$, and the effective mass, $m_F^* = \gamma_F m_F$, on the Fermi surface, due to the Fock exchange potential $\Sigma(k)$, where

$$\gamma_F = \left(1 + \frac{2m_F}{\hbar^2} \left.\frac{d\Sigma(k)}{dk^2}\right|_{k=k_F}\right)^{-1}. \quad (13)$$

Thus, our predictions remain quantitatively accurate only in the weak coupling regime, but, nevertheless, are expected to shed light on some qualitative aspects of superfluid pairings under more general circumstances.

In Fig. 2, we consider an example in which $a_{BF} = 200a_0$ which translates into $\lambda = 0.21$. A numerical investigation finds that the p-wave pairings dominate other odd superfluid pairings (not shown) when the product, $d_B k_F$, exceeds ~ 4. As the roton minimum is lowered [Fig. 2(a)], the p-wave interaction, $\bar{U}_1(k_F)$ (>0), increases [Fig. 2(c)], while, at the same time, the effective Fermi mass also experiences a significant increase [Fig. 2(b)]. For a particle in a lattice, its effective mass can be changed by changing the energy band structure. In our work here, fermions are submerged in a background of dipolar condensate. The (effective) mass of a fermion is different from its bare value since a fermion is now dressed with phonons of the dipolar BEC. This polaronic effect, which is typically small and negligible, is seen to increase appreciably when the roton minimum energy is lowered [Fig. 2(b)]. It is interesting to point out that while increasing the p-wave interaction surely enhances the exponential factor in Eq. (11), increasing the effective Fermi mass also serves to enhance the exponential factor as it amounts to amplifying the coupling constant, λ, by a factor γ_F according to Eq. (11). These translate into a dramatic increase in the corresponding critical temperature for the p-wave superfluid phase as clearly demonstrated in Fig. 2(d).

In our system, the induced interaction is independent of the sign of l: $\bar{U}_l(k_F) = \bar{U}_{-l}(k_F)$. Thus, the superfluid with $l = 1$ is degenerate with the superfluid with $l = -1$. A question arises: what is the nature of the p-wave superfluid in our model at $T = 0$? In principle, the gap parameter becomes the superposition of these two superfluids, $\Delta(\mathbf{k}_F) = |\Delta_F| [f(\phi_{\mathbf{k}}) \equiv c_+ \exp(i\phi_{\mathbf{k}}) + c_- \exp(-i\phi_{\mathbf{k}})]$, where $|c_+|^2 + |c_-|^2 = 1$. The ground state energy due to the superfluid pairing is then minimized when $|\Delta_F|$ is maximum [40]. A variational analysis of $|\Delta_F|$ obtained from the gap equation (9) at $T = 0$ [12] indicates that only when $c_+ = 0$ or 1 does the gap parameter reach its maximum value of $|\Delta_F| = 2k_B T_1$ so that $\Delta(\mathbf{k}_F) \approx e^{\pm i\phi_{\mathbf{k}}} 2k_B T_1$ (up to a global gauge transformation). Thus, our system realizes at zero temperature the ground state phase of chiral $p_x \pm ip_y$ waves whose gap parameters can be resonantly enhanced by tuning the roton minimum.
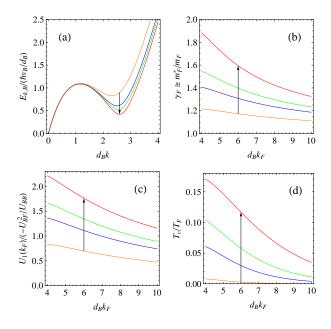
FIG. 2. As the roton minimum in (a) is lowered by decreasing χ_B along the arrow in the sequence of 0.300, 0.284, 0.280, and 0.276, the following occurs: (b) the ratio of the effective fermion mass to the bare fermion mass, γ_F, (c) the p-wave interaction strength, $U_1(k_F)$, and (d) the critical temperature for the p-wave superfluids, T_1, all increase as indicated by their respective arrows.

V. CONCLUSION

In conclusion, we have studied, within the Hartree-Fock-Bogoliubov approach, the p-wave superfluid pairings in a quasi-2D dipolar Bose-Fermi mixture. In a quasi-2D trap setting, the competition between the attractive and repulsive part of the dipole-dipole interaction can lead to the roton structure in the phonon spectrum of the dipolar condensate. Mixing dipolar bosons with fermions in quasi-2D space offers the opportunity to use the roton minimum as a tool for engineering the phonon-induced attractive interaction between fermions. The present work demonstrates that enhancing the induced interaction by lowering the roton minimum can affect the stability properties of the mixture as well as the effective mass of the fermions in an important way. It also shows that one can tune the system to operate in stable regions where chiral p-wave superfluid pairings can be resonantly enhanced by lowing the energy cost of the phonons near the roton minimum.

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