# Nonlinear statistical effects in relativistic mean field theory

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**Abstract.** We investigate the relativistic mean field theory of nuclear matter at finite temperature and baryon density taking into account of nonlinear statistical effects, characterized by power-law quantum distributions. The analysis is performed by requiring the Gibbs conditions on the global conservation of baryon number and electric charge fraction. We show that such nonlinear statistical effects play a crucial role in the equation of state and in the formation of mixed phase also for small deviations from the standard Boltzmann-Gibbs statistics.

## 1. Introduction

Several experimental observations and theoretical calculations clearly indicate that hadrons dissociate into a plasma of their elementary constituents, quarks and gluons (QGP), at density several times the nuclear matter density and/or at temperature above few hundreds MeV. Such a QGP is expected to have occurred in the early stages of the Universe and can be found in dense and hot stars, neutron stars, nucleus-nucleus high energy collisions where heavy ions are accelerated to relativistic energies [1]. After collision, a fireball is created which may realize the conditions of the QGP. The plasma then expands, cools, freezes-out into hadrons, photons, leptons that are detected and analyzed [2].

It is a rather common opinion that, because of the extreme conditions of density and temperature in ultrarelativistic heavy ion collisions, memory effects and long-range color interactions give rise to the presence of non-Markovian processes in the kinetic equation affecting the thermalization process toward equilibrium as well as the standard equilibrium distribution [3, 4, 5]. A rigorous determination of the conditions that produce a nonextensive behavior, due to memory effects and/or long-range interactions, should be based on microscopic calculations relative to the parton plasma originated during the high energy collisions. At this stage we limit ourselves to consider the problem from a qualitative point of view on the basis of the existing theoretical calculations and experimental evidences.

On the other hand, over the last years, there has been an increasing evidence that the generalized non-extensive statistical mechanics, proposed by Tsallis [6, 7, 8, 9] and characterized by a power-law stationary particle distribution, can be considered as a basis for a theoretical framework appropriate to incorporate, at least to some extent and without going into microscopic dynamical description, long-range interactions, long-range microscopic memories and/or fractal space-time constraints. A considerable variety of physical issues show a quantitative agreement

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between experimental data and theoretical analysis based on Tsallis' thermostatistics. In particular, there is a growing interest in high energy physics applications of non-extensive statistics [10, 11, 12, 13, 14, 15, 16]. Several authors outline the possibility that experimental observations in relativistic heavy-ion collisions can reflect non-extensive statistical mechanics effects during the early stage of the collisions and the thermalization evolution of the system [17, 18, 19, 20, 21]. In this context, it is relevant to observe that the statistical origin of the nonextensive statistics lies in the deformation of the Boltzmann entropy. From the above considerations, it appears reasonable that in regime of high density and temperature both hadron and quark-gluon Equation of State (EOS) can be sensibly affected by nonextensive statistical effects [22, 23]. Furthermore, in this context it is very remarkable to observe that the relevance of these effects on the relativistic hadronic equation of state has also been recently investigated in Ref. [24].

The aim of this paper is to study the behavior of the nuclear equation of state at finite temperature and baryon density and to explore the existence of a hadron-quark mixed phase at a fixed value of the proton fraction Z/A.

# 2. Nonextensive hadronic and quark-gluon equation of state

In this Section we study the nonextensive hadronic EOS in the framework of a relativistic mean field theory in which nucleons interact through the nuclear force mediated by the exchange of virtual isoscalar-scalar ( $\sigma$ ), isoscalar-vector ( $\omega$ ) and isovector-vector ( $\rho$ ) meson fields [25, 26, 27].

The nonlinear Lagrangian density describing hadronic matter can be written as

$$\mathcal{L} = \mathcal{L}_{QHD} + \mathcal{L}_{qfm} \,, \tag{1}$$

where [27]

$$\mathcal{L}_{QHD} = \bar{\psi}[i\gamma_{\mu}\partial^{\mu} - (M - g_{\sigma}\sigma) - g_{\omega}\gamma_{\mu}\omega^{\mu} - g_{\rho}\gamma^{\mu}\vec{\tau}\cdot\vec{\rho}_{\mu}]\psi + \frac{1}{2}(\partial_{\mu}\sigma\partial^{\mu}\sigma - m_{\sigma}^{2}\sigma^{2})$$
$$-U(\sigma) + \frac{1}{2}m_{\omega}^{2}\omega_{\mu}\omega^{\mu} + \frac{1}{2}m_{\rho}^{2}\vec{\rho}_{\mu}\cdot\vec{\rho}^{\mu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\vec{G}_{\mu\nu}\vec{G}^{\mu\nu}, \qquad (2)$$

and M=939 MeV is the vacuum baryon mass. The field strength tensors for the vector mesons are given by the usual expressions  $F_{\mu\nu}\equiv\partial_{\mu}\omega_{\nu}-\partial_{\nu}\omega_{\mu}$ ,  $\vec{G}_{\mu\nu}\equiv\partial_{\mu}\vec{\rho}_{\nu}-\partial_{\nu}\vec{\rho}_{\mu}$ , and  $U(\sigma)$  is a nonlinear potential of  $\sigma$  meson

$$U(\sigma) = \frac{1}{3}a\sigma^3 + \frac{1}{4}b\sigma^4, \tag{3}$$

usually introduced to achieve a reasonable compression modulus for equilibrium nuclear matter. Following Ref.s [28, 29],  $\mathcal{L}_{qfm}$  in Eq.(1) is related to a (quasi) free gas of pions with an effective chemical potential (see below for details).

The field equations in a mean field approximation are

$$(i\gamma_{\mu}\partial^{\mu} - (M - g_{\sigma}\sigma) - g_{\omega}\gamma^{0}\omega - g_{\rho}\gamma^{0}\tau_{3}\rho)\psi = 0,$$
(4)

$$m_{\sigma}^{2}\sigma + a\sigma^{2} + b\sigma^{3} = g_{\sigma} < \bar{\psi}\psi > = g_{\sigma}\rho_{S}, \tag{5}$$

$$m_{\omega}^2 \omega = g_{\omega} \langle \bar{\psi} \gamma^0 \psi \rangle = g_{\omega} \rho_B , \qquad (6)$$

$$m_{\rho}^{2}\rho = g_{\rho} \langle \bar{\psi}\gamma^{0}\tau_{3}\psi \rangle = g_{\rho}\rho_{I}, \qquad (7)$$

where  $\sigma = \langle \sigma \rangle$ ,  $\omega = \langle \omega^0 \rangle$  and  $\rho = \langle \rho_3^0 \rangle$  are the nonvanishing expectation values of meson fields,  $\rho_I$  is the total isospin density,  $\rho_B$  and  $\rho_S$  are the baryon density and the baryon scalar density,

respectively. They are given by

$$\rho_B = 2\sum_{i=n,p} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} [n_i(k) - \overline{n}_i(k)], \qquad (8)$$

$$\rho_S = 2 \sum_{i=n,p} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{M_i^*}{E_i^*} \left[ n_i^q(k) + \overline{n}_i^q(k) \right], \tag{9}$$

where  $n_i(k)$  and  $\overline{n}_i(k)$  are the q-deformed fermion particle and antiparticle distributions:

$$n_i(k) = \frac{1}{[1 + (q-1)\beta(E_i^*(k) - \mu_i^*)]^{1/(q-1)} + 1},$$
(10)

$$\overline{n}_i(k) = \frac{1}{[1 + (q - 1)\beta(E_i^*(k) + \mu_i^*)]^{1/(q - 1)} + 1}.$$
(11)

The nucleon effective energy is defined as  $E_i^*(k) = \sqrt{k^2 + {M_i}^{*2}}$ , where  $M_i^* = M_i - g_\sigma \sigma$ . The effective chemical potentials  $\mu_i^*$  are given in terms of the meson fields as follows

$$\mu_i^* = \mu_i - g_\omega \omega - \tau_{3i} g_\rho \rho \,, \tag{12}$$

where  $\mu_i$  are the thermodynamical chemical potentials  $\mu_i = \partial \epsilon / \partial \rho_i$ . At zero temperature they reduce to the Fermi energies  $E_{Fi} \equiv \sqrt{k_{Fi}^2 + M_i^{*2}}$  and the nonextensive statistical effects disappear. The meson fields are obtained as a solution of the field equations in mean field approximation and the related meson-nucleon couplings  $(g_{\sigma}, g_{\omega} \text{ and } g_{\rho})$  are the free parameters of the model. In the following, they will be fixed to the parameters set marked as GM2 of Ref. [27].

The thermodynamical quantities can be obtained from the thermodynamic potential in the standard way. More explicitly, the baryon pressure  $P_B$  and the energy density  $\epsilon_B$  can be written as

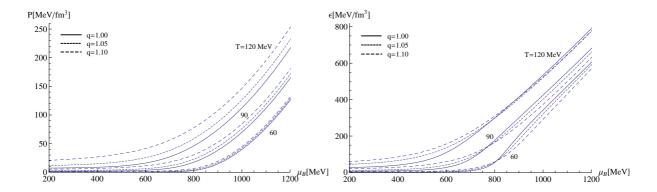
$$P_B = \frac{2}{3} \sum_{i=n,n} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{k^2}{E_i^*(k)} [n_i^q(k) + \overline{n}_i^q(k)] - \frac{1}{2} m_\sigma^2 \sigma^2 - U(\sigma) + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2 , \qquad (13)$$

$$\epsilon_B = 2\sum_{i=n,p} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} E_i^*(k) [n_i^q(k) + \overline{n}_i^q(k)] + \frac{1}{2} m_\sigma^2 \sigma^2 + U(\sigma) + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2.$$
 (14)

It is important to observe that Eq.s(9), (13) and (14) apply to  $n_i^q \equiv (n_i)^q$  rather than  $n_i$  itself, this is a direct consequence of the basic prescription related to the q-mean expectation value in nonextensive statistics [8, 15]. In addition, since all equations must be solved in a self-consistent way, the presence of nonextensive statistical effects influences the many-body interaction mediated by the meson fields.

Especially in regime of low density and high temperature the contribution of the lightest mesons to the thermodynamical potential (and, consequently, to the other thermodynamical quantities) becomes relevant. As quoted before, we have included the contribution of pions considering them as a (quasi) ideal gas of nonextensive bosons with effective chemical potentials expressed in terms of the corresponding effective baryon chemical potentials [29].

In Fig. 1, the total pressure P and energy density  $\epsilon$  are plotted as a function of  $\mu_B$  for different values of temperature and q. The different behavior from P and  $\epsilon$  reflects essentially the nonlinear combinations of the meson fields and the different functions under integration in Eq.s (13) and (14). Concerning the pressure, we have that becomes stiffer by increasing the q



**Figure 1.** Pressure (left panel) and energy density (right panel) versus baryon chemical potential for different values of temperature and q.

parameter. On the other hand, the behavior of the energy density presents features very similar to the  $\sigma$  field one. At low  $\mu_B$ , nonextensive effects make the energy density greater with respect to the standard case. At medium-high  $\mu_B$ , the standard (q=1) component of the energy density becomes dominant, this effect is essentially due to the reduction of the  $\sigma$  field for q>1. The intersection point depends, naturally, on the physical parameters of the system.

Concerning the quark-gluon EOS, we use the MIT bag model [30]. In this model, quark matter is described as a gas of free quarks with massless up and down quarks. All the non-perturbative effects are simulated by the bag constant B which represents the pressure of the vacuum. Following this line, the pressure, energy density and baryon number density for a relativistic Fermi gas of quarks in the framework of nonextensive statistics can be written, respectively, as

$$P_q = \frac{\gamma_f}{3} \sum_{f=u,d} \int_0^\infty \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{k^2}{e_f} \left[ n_f^q(k) + \overline{n}_f^q(k) \right] - B, \qquad (15)$$

$$\epsilon_q = \gamma_f \sum_{f=u,d} \int_0^\infty \frac{\mathrm{d}^3 k}{(2\pi)^3} e_f \left[ n_f^q(k) + \overline{n}_f^q(k) \right] + B, \qquad (16)$$

$$\rho_q = \frac{\gamma_f}{3} \sum_{f=u,d} \int_0^\infty \frac{\mathrm{d}^3 k}{(2\pi)^3} \left[ n_f(k) - \overline{n}_f(k) \right], \tag{17}$$

where the quark degeneracy for each flavor is  $\gamma_f = 6$ ,  $e_f = (k^2 + m_f^2)^{1/2}$ ,  $n_f(k)$  and  $\overline{n}_f(k)$  are the q-deformed particle and antiparticle quark distributions

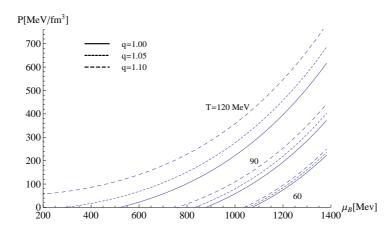
$$n_f(k) = \frac{1}{[1 + (q-1)(e_f(k) - \mu_f)/T]^{1/(q-1)} + 1},$$
(18)

$$\overline{n}_f(k) = \frac{1}{[1 + (q-1)(e_f(k) + \mu_f)/T]^{1/(q-1)} + 1}.$$
(19)

Similar expressions for the pressure and the energy density can be written for gluons treating them as a massless q-deformed Bose gas with zero chemical potential. Explicitly, we can calculate the nonextensive pressure  $P_g$  and energy density  $\epsilon_g$  for gluons as

$$P_g = \frac{\gamma_g}{3} \int_0^\infty \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{k}{[1 + (q-1)k/T]^{q/(q-1)} - 1},$$
 (20)

$$\epsilon_g = 3 P_g, \tag{21}$$



**Figure 2.** Pressure of the quark-gluon phase as a function of baryon chemical potential for different values of temperature and q.

with the gluon degeneracy factor  $\gamma_g = 16$ . In the limit  $q \to 1$ , one recovers the usual analytical expression:  $P_g = 8\pi^2/45 \, T^4$ .

Let us note that, since one has to employ the fermion (boson) nonextensive distributions, the results are not analytical, even in the massless quark approximation. Hence a numerical evaluations of the integrals in Eq.s (15)–(17) and (20) must be performed.

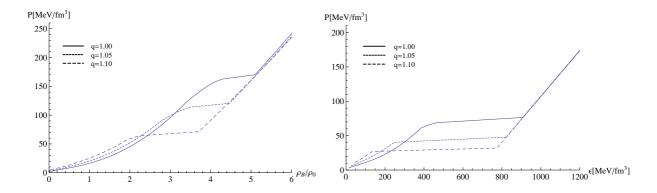
In Fig. 2, we report the total pressure as a function of the baryon chemical potential for massless quarks and gluons, for different values of q and at fixed value of Z/A = 0.4. The bag constant is set equal to  $B^{1/4}$ =190 MeV. In presence of nonextensive effects, as in the case of hadronic phase, the pressure is significantly increased even for small deviations from standard statistics.

#### 3. The hadron to quark-gluon phase transition

In this Section we investigate the hadron-quark phase transition at finite temperature and baryon chemical potential by means of the previous relativistic EOSs. Lattice calculations predict a critical phase transition temperature  $T_c$  of about 170 MeV, corresponding to a critical energy density  $\epsilon_c \approx 1~{\rm GeV/fm^3}$  [1]. In a theory with only gluons and no quarks, the transition turns out to be of first order. In nature, since the u and d quarks have a small mass, while the strange quark has a somewhat larger mass, the phase transition is predicted to be a smooth cross over. However, since it occurs over a very narrow range of temperatures, the transition, for several practical purposes, can still be considered of first order. Indeed the lattice data with 2 or 3 dynamical flavours are not precise enough to unambigously control the difference between the two situations. Thus, by considering the deconfinement transition at finite density as a the first order one, a mixed phase can be formed, which is typically described using the two separate equations of state, one for the hadronic and one for the quark phase.

The phase transition is described by using the Gibbs formalism applied to systems where more than one conserved charge is present [31]. In fact, because we are going to describe the nuclear EOS, we have to require the global conservation of two "charges": baryon number and electric charge. Each conserved charge has a conjugated chemical potential and the systems is described by two independent chemical potentials:  $\mu_B$  and  $\mu_C$ . The structure of the mixed phase is obtained by imposing the following Gibbs conditions for chemical potentials and pressure [32, 33, 34]

$$\mu_B^{(H)} = \mu_B^{(Q)}, \quad \mu_C^{(H)} = \mu_C^{(Q)},$$
(22)



**Figure 3.** Pressure as a function of baryon density (left panel) and energy density (right panel) in the mixed phase for different values of q. The temperature is fixed at T = 90 MeV.

$$P^{H}(T, \mu_B, \mu_C) = P^{Q}(T, \mu_B, \mu_C). \tag{23}$$

Therefore, at a given baryon density  $\rho_B$  and at a given net electric charge density  $\rho_C = Z/A \rho_B$ , the chemical potentials  $\mu_B$  are  $\mu_C$  are univocally determined by the following equations

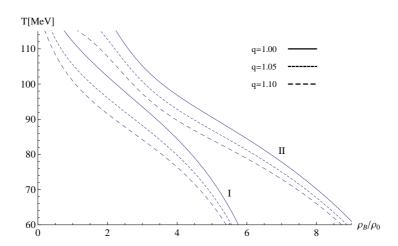
$$\rho_B = (1 - \chi) \,\rho_B^H(T, \mu_B, \mu_C) + \chi \,\rho_B^Q(T, \mu_B, \mu_C) \,, \tag{24}$$

$$\rho_C = (1 - \chi) \rho_C^H(T, \mu_B, \mu_C) + \chi \rho_C^Q(T, \mu_B, \mu_C), \qquad (25)$$

where  $\rho_B^{H(Q)}$  and  $\rho_C^{H(Q)}$  are, respectively, the net baryon and electric charge densities in the hadronic (H) and in the quark (Q) phase and  $\chi$  is the fraction volume of quark-gluon matter in the mixed phase. In this way we can find out the phase coexistence region, for example, in the  $(T, \mu_B)$  plane. We are particularly interested in the lower baryon density (baryon chemical potential) border, i.e. the first critical transition density  $\rho_{\rm cr}^{I}$  ( $\mu_{\rm cr}^{I}$ ), in order to check the possibility of reaching such conditions in a transient state during a heavy-ion collision at relativistic energies.

In Fig. 3, we report the pressure at T=90 MeV as a function of baryon density (in units of nuclear saturation density  $\rho_0=0.153~{\rm fm^{-3}}$ ) (left panel) and energy density (right panel). It is interesting to observe that pressure as a function of baryon density (or energy density) is stiffer in the pure hadronic phase for q>1 but appears a strong softening in the mixed phase. This feature results in significant changes in the incompressibility and may be particularly important in identifying the presence of nonextensive effects in high energy heavy ion collisions experiments. Related to this aspect, let us observe that possible indirect indications of a significative softening of the EOS at the energies reached at AGS have been discussed several times in the literature [34].

In Fig. 4, it is reported the phase diagram in the plane  $T - \rho_B$  for different values of q. The curves labelled with the index I and II represent, respectively, the beginning and the end of the mixed phase. For q > 1, both the first and the second critical densities are sensibly reduced, even if the shape of the mixed phase is approximately the same. Related to this aspect, let us mention that the simplest version of the MIT bag model, considered in this investigation, appears to be not fully appropriate to describe a large range of temperature and density. To overcome this shortcoming, a phenomenological approach can therefore be based on a density or temperature dependent bag constant [34, 35]. Moreover, in regime of high temperature and small baryon chemical potential the first order phase transition may end in a (second order) critical endpoint with a smooth crossover. These features cannot be incorporated in the considered



**Figure 4.** Phase diagram  $T - \rho_B$  for different values of q. The curves with index I and II indicate, respectively, the beginning and the end of the mixed phase.

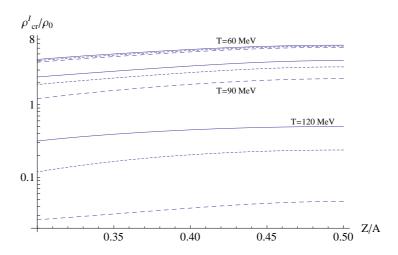
mean field approach. In our investigation, because we are focusing to nonextensive statistical effects on the nuclear EOS, instead of introducing additional parameterizations, we work with a fixed bag constant and limit our analysis to a restricted range of temperature and density, region of particular interest for high energy compressed nuclear matter experiments.

Let us now explore in more details the variation of the first transition baryon density  $\rho_{\rm cr}^I$  as a function of different physical parameters. In Fig. 5, we report the dependence of  $\rho_{\rm cr}^I$  as a function of Z/A for different values of q (y axis in logarithmic scale). It is interesting to note a significant reduction of  $\rho_{\rm cr}^I$  in presence of nonextensive statistics; as in the previous cases, this effect increases with the temperature. The dependence of the first transition baryon density as a function of Z/A is essentially a consequence of the  $\rho$  meson field behavior in the hadronic phase because it is directly connected with the isospin density of the system (as appears from Eq.(7)). In this context, let us observe that, at fixed value of q,  $\rho_{\rm cr}^I$  is significantly reduced by decreasing Z/A only at lower temperatures ( $T=60~{\rm MeV}$ ) while, as expected, at higher temperatures ( $T=120~{\rm MeV}$ ) the transition baryon density becomes very low and its isospin dependence becomes negligible, also in the framework of nonextensive statistics. This matter of fact is a consequence of fact that at low baryon chemical potentials (or baryon densities) the  $\rho$  meson field becomes almost constant and its absolute value significantly decreases.

# 4. Conclusions

Following the basic prescriptions of the Tsallis' nonlinear relativistic thermodynamics, we investigate the relevance of nonextensive statistical effects on the relativistic nuclear and subnuclear equation of state. We have focused our investigation in regime of finite temperature and baryon chemical potential, reachable in high-energy heavy-ion collisions, for which the deconfinement phase transition can be still considered of the first order.

In the first part of the work, we have investigated the hadronic equation of state and the role played by the meson fields in the framework of a relativistic mean field model which contains the basic prescriptions of nonextensive (nonlinear) statistical mechanics. We have shown that, also in presence of small deviations from standard Boltzmann-Gibbs statistics, the meson fields and, consequently, the EOS appear to be sensibly modified. In the second part, we have analyzed the QGP proprieties using the MIT Bag model and also in this case the EOS becomes stiffer in presence of nonextensive effects. Finally, we have studied the proprieties of the phase transition from hadronic matter to QGP and the formation of a relative mixed phase by requiring the



**Figure 5.** Variation of the first transition baryon density as a function of the net electric charge fraction Z/A for different temperatures and values of q (q = 1, solid lines; q = 1.05, short dashed lines; q = 1.10, long dashed lines).

Gibbs conditions on the global conservation of baryon number and electric charge fraction. We have seen that nonextensive effects play a crucial role in the deconfinement phase transition. Moreover, although pressure as a function of baryon density is stiffer in the hadronic phase, we have shown that a strong softening in the mixed phase takes place in presence of nonextensive statistics. Such a behavior implies an abruptly variation in the incompressibility and could be considered as a signal of nonextensive statistical effects in high energy heavy ion collisions.

From a phenomenological point of view, the nonextensive index q is considered here as a free parameter, even if, actually should not be treated as such because, in principle, it should depend on the physical conditions generated in the reaction, on the fluctuation of the temperature and be related to microscopic quantities (such as, for example, the mean interparticle interaction length, the screening length and the collision frequency into the parton plasma). Moreover, let us remember that, in the diffusional approximation, a value q > 1 implies the presence of a superdiffusion among the constituent particles (the mean square displacement obeys to a power law behavior  $\langle x^2 \rangle \propto t^{\alpha}$ , with  $\alpha > 1$ ) [36].

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