Answers

Chapter 1

Exercise 1A

1 **a** numerical **b** numerical **c** categorical
d categorical **e** numerical
f numerical **g** categorical
h categorical

2 **a** nominal **b** nominal **c** ordinal
d ordinal **e** ordinal **f** nominal

3 **a** discrete **b** discrete **c** continuous
d continuous **e** discrete
f continuous

4 B **5** D **6** B

Exercise 1B

1 **a**

Grades	Frequency	
	Count	%
A	3	27.3
B	5	45.5
C	3	27.3
Total	11	100.1

b

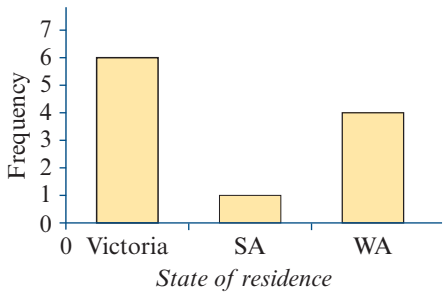
Shoe size	Frequency	
	Count	%
8	6	50.0
9	3	25.0
10	2	16.7
11	0	0
12	1	8.3
Total	12	100.0

2 **a** categorical

b

State of residence	Frequency	
	Count	%
Victoria	6	54.5
SA	1	9.1
WA	4	36.4
Total	11	100.0

c



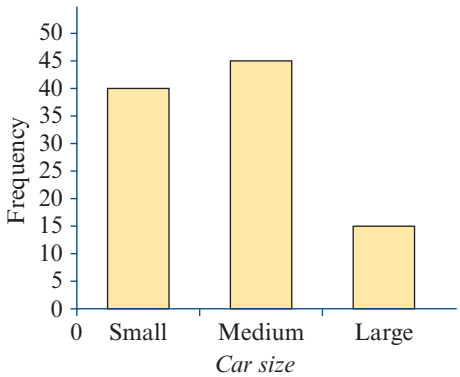
ISBN 978-1-009-11041-9

3 a categorical

b

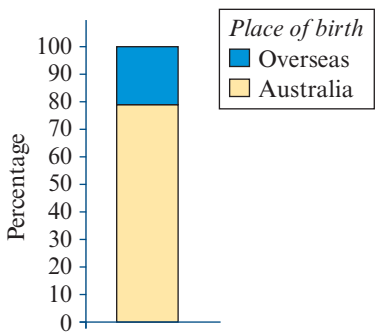
Car size	Frequency	
	Count	%
Small	8	40
Medium	9	45
Large	3	15
Total	20	100

c



4 a nominal

b

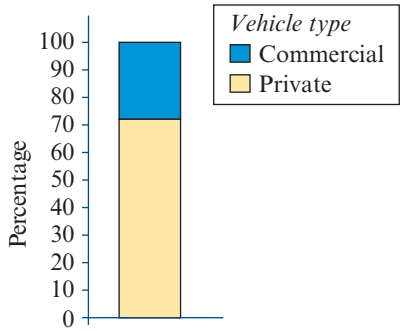


5 a nominal

b

Type of vehicle	Frequency	
	Count	%
Private	132 736	73
Commercial	49 109	27
Total	181 845	100

c



6 a 20, 55 **b** 5 **c** 20 **d** 55%

e Report: 20 schools were classified according to school type. The majority of these schools, 55%, were found to be government schools. Of the remaining schools, 25% were independent while 20% were Catholic schools.

7 a 7, 45.5, 100.0

b Report: When 22 students were asked the question, 'How often do you play sport', the most frequent response was 'sometimes', given by 45.5% of the students. Of the remaining students, 31.8% of the students responded that they played sport 'rarely' while 22.7% said that they played sport 'regularly'.

8 Report: The eye colours of 11 children were recorded. The majority, 54.5%, had brown eyes. Of the remaining children, 27.3% had blue eyes and 18.2% had hazel eyes.

9 B

ISBN 978-1-009-11041-9

Cambridge University Press

Exercise 1C

1 a

Number	Frequency	
	Count	%
0	6	30
1	4	20
2	3	15
3	3	15
4	2	10
5	2	10
Total	20	100

b 20%

c 0

2 a

Number	Frequency	
	Count	%
2	1	2.5
3	0	0
4	17	42.5
5	13	32.5
6	9	22.5
Total	40	100.0

b 2.5%

c 4

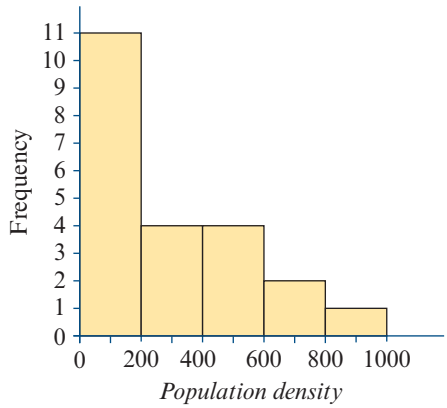
3 a

Height (cm)	Frequency
160–164	5
165–169	5
170–174	5
175–179	6
180–184	3
185–189	1
Total	25

b 175–179

c 16%

4



5 a **i** 17% **ii** 13% **iii** 46% **iv** 33%

b **i** 6 **ii** 4

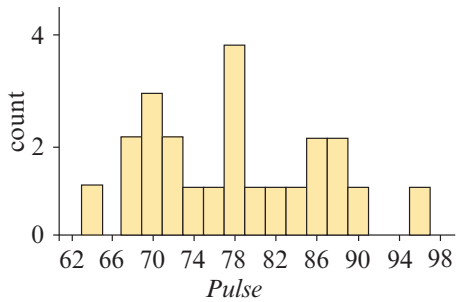
c 15–19 words/sentence

6 a 21

b **i** 13 **ii** 8 **iii** 5 **iv** 0

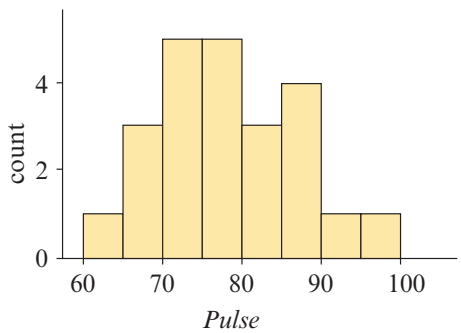
c **i** 4.8% **ii** 57.1%

7 a



b **i** 69 **ii** 3; 69, 70, 70

c

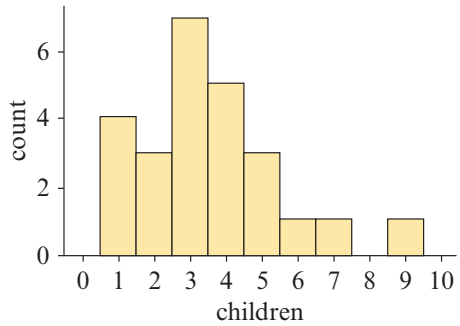


d 3

ISBN 978-1-009-11041-9

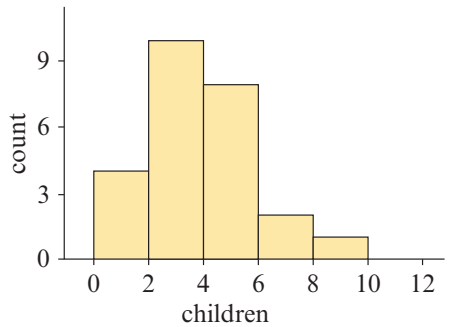
Cambridge University Press

8 a



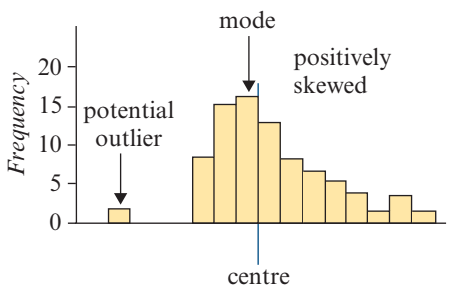
b 3.5, 5

c



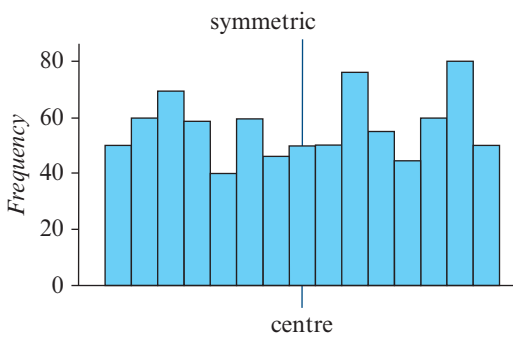
d **i** 2 **ii** 6 and 7

9 a



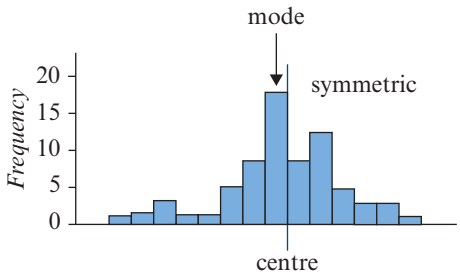
Histogram A

b



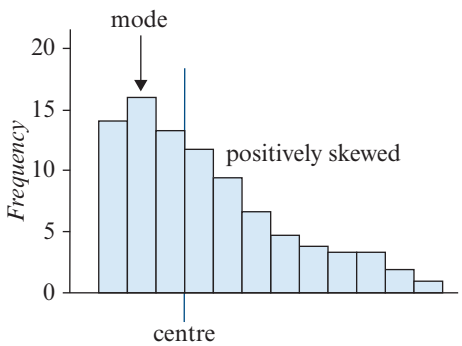
Histogram B

c



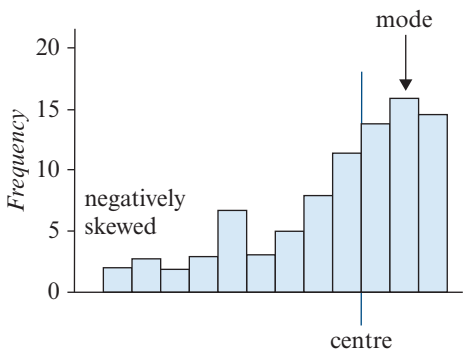
Histogram C

d



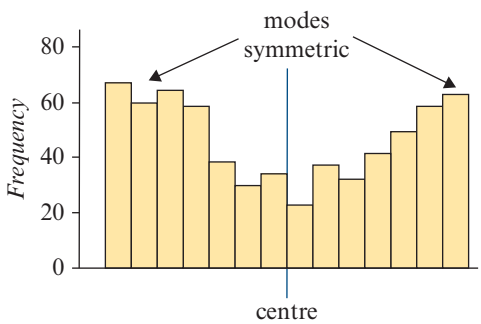
Histogram D

e



Histogram E

f



Histogram F

ISBN 978-1-009-11041-9

Cambridge University Press

10 a All of the distributions are approximately symmetric about their respective centres.

b There are no clear outliers in any of the distributions.

c In A the centre lies in the interval 8-10, in B it lies in the interval 24-26, and in C it lies in the interval 40-42.

d The spread is the lowest in B, since the range is only 8, compared to 14 for A, and 18 for C.

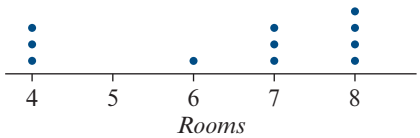
11 B

12 A

Exercise 1D

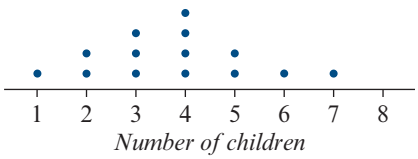
1 a discrete

b



2 a discrete

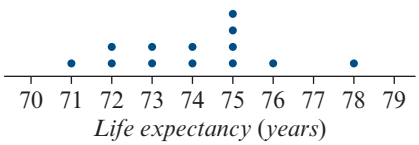
b



c 4; mode is the most frequently occurring number of children for these families

3 a continuous

b



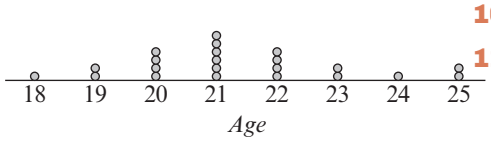
c 75; mode is the most common life expectancy for these countries

4 a negatively skewed

b positively skewed

5

a



b 21 years

c approximately symmetric **d** 14%

6 a continuous

b

Key: 1|6 = 16

0	3 3 6 9 9
1	2 2 6 7
2	0 2 2 5 7 8 9
3	1 5
4	
5	4 6
6	
7	
8	
9	9 9
10	0

7 a continuous

b **i**

Key:16|5 = 16.5

16	5 7 9
17	0 1 2 3 6 6 7
18	2 4 5
19	3 9

ii

Key:16|5 = 16.5

16	5 7 9
17	0 1 2 3
17	6 6 7
18	2 4
18	5
19	3
19	9

8 a positively skewed

b negatively skewed

9 a 40 **b** approx symmetric

c 11

10 C

11 B

ISBN 978-1-009-11041-9

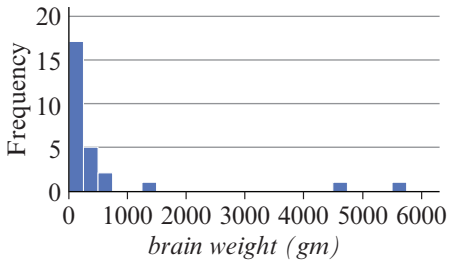
Cambridge University Press

Exercise 1E

1 **a** 0.4 **b** 1.4 **c** 2.4 **d** 3.4
e −0.3 **f** −1.3 **g** −2.3 **h** −3.3

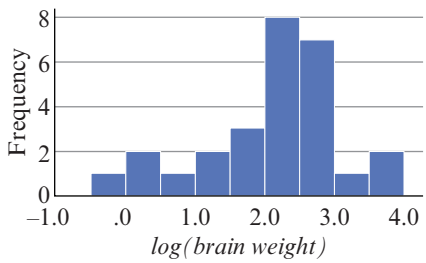
2 **a** 0.0032 **b** 0.032 **c** 0.32 **d** 1.0

3 **a**



The shape is positively skewed with outliers.

b



The shape is slightly negatively skewed but closer to symmetric.

4 **a** −0.4 **b** 3.8 **c** 100 g **d** 0.1 g
e **i** 5 **ii** 12 **iii** 24

5 B **6** D

Exercise 1F

1 **a** 5 **b** 12

2 $850

3 M=1

4 **a** M=7.3 **b** R=6.4

5 **a** $M = 2$ **b** $Q_1 = 1,\ Q_3 = 3$ **c** $IQR = 2$
d $R = 7$

6 **a** $M = 11$ **b** $Q_1 = 10,\ Q_3 = 15$
c $IQR = 5,\ R = 18$

7 **a** approximately symmetric with no outliers.
b $M = 26$
c $Q_1 = 17.5, Q_3 = 30.5$
d $IQR = 13,\ R = 29$

8 **a** positively skewed with a possible outlier at 6.
b $M = 0$
c $IQR = 1$ **d** $R = 6$

9 **a** $M = 21$ **b** $Q_1 = 10.5,\ Q_3 = 28$
c $IQR = 17.5,\ R = 54$

10 Median from 65 to less than 70, Q_1 from 60 to less than 65, Q_3 from 75 to less than 80.

11 **a** Median in the interval 5.0-9.9.
b max $IQR = 19.9$

12 **a** $n = 4,\ \Sigma x = 12,\ \bar{x} = 3$
b $n = 5,\ \Sigma x = 104,\ \bar{x} = 20.8$
c $n = 7,\ \Sigma x = 21,\ \bar{x} = 3$

13 **a** $\bar{x} = 3,\ M = 3,$ Mode = 2
b $\bar{x} = 5,\ M = 5,$ Mode = 5

14 **a** **i** mean = 36.1 **ii** median = 36.0
b The mean and median almost co-incide because the distribution is approximately symmetric.

15 **a** **i** mean = $3.65 **ii** median = $1.70
b The median. The mean is inflated because of the one large sale and not representative of the sales in general.

16 **a** strongly positively skewed distribution
b positively skewed distribution with outliers

17 **a** symmetric; either
b mean = 82.55 median= 82.5

18 **a** *IQR* **b** range **c** standard deviation

19 7.1, 0 **20** b, d, f

21 **a** 20.1, 1.8 **b** symmetric

ISBN 978-1-009-11041-9

Cambridge University Press

22

	TVs	*Cars*	*Alcohol*
mean	450	376	14.9
SD	100	107	5.1

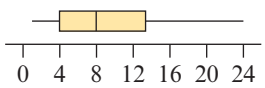
23 C **24** C **25** C **26** B

Exercise 1G

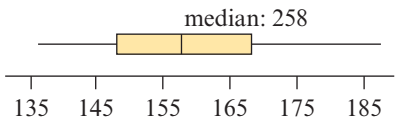
1 4, 5, 6, 7, 9

2 136, 148, 158, 169, 189

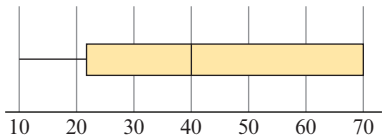
3 a



b



4 a

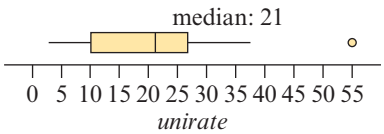


b Q_3 and the maximum are equal.

5 b Upper fence = $26.5 + 1.5 \times 16 = 50.5$

c 55 is larger than the upper fence.

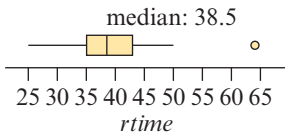
d



6 a lower fence = 7, upper fence = 95.

b 6 and 99

7 a



b 25, 35, 38.5, 43, 64; 64 is a possible outlier.

8 a **i** 10 **ii** 5, 21 **iii** 16 **iv** 0, 45 **v** none

b **i** 27 **ii** 12, 42 **iii** 30 **iv** 5, 50 **v** none

c **i** 38 **ii** 32, 42 **iii** 10 **iv** 5, 50 **v** 5

d **i** 16 **ii** 14, 21 **iii** 7 **iv** 1.5, 50 **v** 1.5, 3, 36, 40, 50

e **i** 34 **ii** 30, 39 **iii** 9 **iv** 30, 55 **v** 55

9 a **i** 55 **ii** 15

b **i** 100 **ii** 20

10 a 30;

b no, 31 > 30, so inside the lower fence

11 a 25% **b** 75% **c** 25% **d** 50% **e** 75%

12 a 25% **b** 25% **c** 50% **d** 25% **e** 75% **f** 50%

13 Boxplot 1 matches histogram B, Boxplot 2 matches histogram D, Boxplot 3 matches histogram C, Boxplot 4 matches histogram A.

14 a The distribution is negatively skewed with no outliers. The distribution is centred at about 42, the median value. The spread of the distribution, as measured by the IQR, is 15 and, as measured by the range, 47.

b The distribution is postively skewed with no outliers. The distribution is centred at 800, the median value. The spread of the distribution, as measured by the IQR, is 1200 and, as measured by the range, 3200.

15 a The distribution is negatively skewed with an outlier. The distribution is centred at 39, the median value. The spread of the distribution, as measured by the *IQR*, is 10 and, as measured by the range, 45. There is an outlier at 5.

b The distribution is positively skewed with outliers. The distribution is centred at 16, the median value. The spread of the distribution, as measured by the *IQR*, is 6 and, as measured by the range, 35. The outliers are at 5, 8, 36 and 40.

ISBN 978-1-009-11041-9

Cambridge University Press

c The distribution is approximately symmetric but with outliers. The distribution is centred at 41, the median value. The spread of the distribution, as measured by the *IQR*, is 7 and, as measured by the range, 36. The outliers are at 10, 15, 20 and 25.

16 The median time it takes Taj to travel to university is 70 minutes. The range is of the distribution of travel time is 60 minutes, but the interquartile range is only 15 minutes. The distribution of travel times is positively skewed with two outliers, unusually long travel times of 110 minutes and 120 minute respectively.

17 B **18** A **19** B **20** D

Exercise 1H

1 **a** 114 and 154 **b** 94 and 174 **c** 74 and 194 **d** 154 **e** 94 **f** 74 **g** 134

2 **a** 68% **b** 99.7% **c** 16% **d** 2.5% **e** 0.15% **f** 50%

3 **a** **i** 84% **ii** 50% **iii** 47.5% **b** 25

4 **a** **i** 99.7% **ii** 2.5% **iii** 81.5% **b** 800

5 **a** **i** 50% **ii** 34% **iii** 81.5% **b** 1994

6 **a** $z = 1$ **b** $z = 2$ **c** $z = -1$ **d** $z = 0$ **e** $z = -3$ **f** $z = 0.5$

7 **a** 1.0 **b** −1.0 **c** 1.4 **d** −1.4

8

Subject	*z-score*	*Rating*
English	2.25	Top 2.5%
Biology	3	Top 0.15%
Chemistry	0	Exactly average
Further Maths	1.1	Top 16%
Psychology	−2.25	Bottom 2.5%

9 **a** 2.5% **b** 15.85%

10 **a** 2.5% **b** 13.5%

11 **a** 120 **b** 116 **c** 142 **d** 100 **e** 72 **f** 50

12 $1.50

13 101 g

14 mean = 3.5 kg, st dev = 0.5 kg

15 mean = 66.0 marks, st dev = 7.7 marks

16 **a** 0.2 **b** 46.5 **c** 2.5% **d** 34% **e** 16% **f** 97.5%

17 **a** **i** 16% **ii** 2.5%
b 130
c 133

18 **a** **i** 84% **ii** 97.5%
b 184 cm
c 144 cm
d 150.4 cm

19 A **20** D **21** C **22** C **23** C

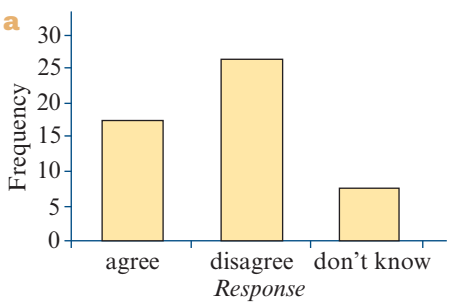
Chapter 1 review

Multiple-choice questions

1 A	**2** B	**3** D	**4** C
5 D	**6** B	**7** C	**8** C
9 C	**10** D	**11** E	**12** B
13 C	**14** B	**15** A	**16** B
17 A	**18** D	**19** D	**20** A
21 B	**22** B	**23** A	**24** D
25 B	**26** D	**27** C	**28** B
29 C	**30** E	**31** A	**32** A
33 E	**34** B	**35** E	

Written-response questions

1 **a**



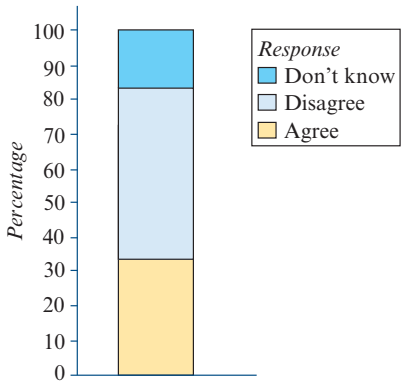
ISBN 978-1-009-11041-9

Cambridge University Press

b

	Frequency	
Legalised?	Count	%
Agree	18	34.6
Disagree	26	50.0
Don't know	8	15.4
Total	52	100.0

c



d Report: In response to the question, 'Do you agree that the use of marijuana should be legalised?', 50% of the 52 students disagreed. Of the remaining students, 34.6% agreed, while 15.4% said that they didn't know.

2 a i 50 **ii** 5

b \$105 – < \$110 **c** 28 **d** 16%

e i approximately symmetric

ii \$110 – < \$115 **iii** \$120 – < \$125

3 a positively skewed.

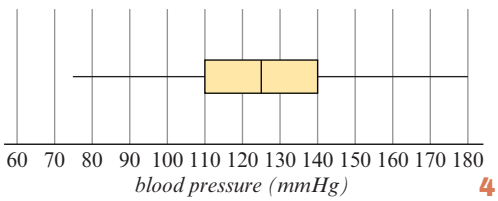
b $M = 2.65$ kg

c $IQR = 1.25$ kg

d 15.6%

e No, it is less than the upper fence (5.33 kg).

4 a



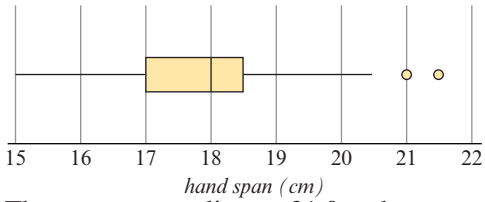
b lower fence = 65 , upper fence = 185, no outliers.

c i 68% **ii** 68 mmHg **iii** 3 **iv** 0

5 a 18 cm

b 5.5%

c



There are two outliers at 21.0 and one at 21.5.

Chapter 2

Exercise 2A

1 a Two categorical

b One categorical and one numerical

c Two numerical **d** Two categorical

2 a EV: *colour*; RV: *toxicity*

b EV: *diet*; RV: *weight loss*

c EV: *age*; RV: *price*

d EV: *fuel*; RV: *cost*

e EV: *location*; RV: *house price*

3 a EV: *age* (numerical), RV: *exercise level* (categorical)

b EV: *years of education* (numerical), RV: *salary level* (numerical)

c EV: *temperature* (numerical), RV: *comfort level* (categorical)

d EV: *time of year* (categorical), RV: *incidence of hay fever* (categorical)

e EV: *age group* (categorical), RV: *musical taste* (categorical)

f EV: *state of residence* (categorical), RV: *AFL team* (categorical)

4 B **5** B **6** C

ISBN 978-1-009-11041-9

Cambridge University Press

Exercise 2B

1 a EV: *gender*, RV: *intends to go to university*

b

Intends to go to university	*Gender*	
	Male	Female
Yes	4	8
No	4	4
Total	8	12

2 a EV: *age group*, RV: *reduce university fees?*

b

Reduce university fees?	*Age group*		
	17-18	19-25	26 or more
Yes	8	6	6
No	3	3	4
Total	11	9	10

c

Reduce university fees?	*Age group (%)*		
	17-18	19-25	26 or more
Yes	72.7	66.7	60.0
No	27.3	33.3	40.0
Total	100.0	100.0	100.0

3 a *enrolment status*

b No. The percentage of full-time and part-time students who drank alcohol is similar: 80.5% to 81.8%. This indicates that drinking behaviour is not related to enrolment status.

4 a *handedness*

b

Handedness	*Gender (%)*	
	Male	Female
Left	9.0	9.8
Right	91.0	90.2
Total	100.0	100.0

c No, there is little difference in the percentage of males and females who are left handed, 9.0% compared to 9.8%.

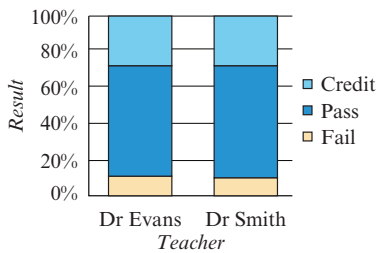
5 a *course* **b** ordinal **c** 54.9%

d Yes; the percentage of Business students who exercise regularly (18.6%) was much higher that the percentage of Arts who exercise regularly (5.9%).

6 a

Result	*Teacher (%)*	
	Dr Evans	Dr Smith
Fail	11.1	9.4
Pass	61.1	62.5
Credit	27.8	28.1
Total	100.0	100.0

b



c There is no evidence students of Dr Evans receive higher grades than students of Dr Smith. The percentage of students achieving each grade level is almost the same for both classes (eg. 61.1% compared 62.5% for students who received a Pass).

7 The data supports the contention that people who are satisfied with their job are more likely to be satisfied with their life, with 70% of people who are satisfied with their job reporting that they are satisfied with their life, compared to only 50% of people who are dissatisfied with their job.

8 a EV: *type of treatment*, RV: *treatment outcome*

b The data supports the contention that the special pillow is more effective at treating snoring than the drug treatment, with just over 30% of

ISBN 978-1-009-11041-9

people who used the special pillow reporting they were cured, compared to only 10% of people who used the drug reported they were cured.

9 a 11.9% **b** 52.3% **c** marital status **d** ordinal

e Yes. There are several ways that this can be seen. For example, by comparing the married and widowed groups, we can see that a smaller percentage of those widowed found life exciting (33.8%) compared to those who were married (47.6%).
Or: a bigger percentage of widowed people found life pretty routine (54.3% to 48.7%) and dull (11.9% to 3.7%) compared to those who were married.

10 A **11** B **12** C

Exercise 2C

1 a *country of origin*: categorical, *number of days away*: numerical

b The number of days these tourists spend away from home was associated with their country of origin. The median number of days spent away from home for Japanese tourists (M = 17 days) is considerably higher than for Australian tourists (M = 7 days). The variability for the number of days away is also higher for Japanese tourists (IQR = 16.5) compared to that for Australian tourists (IQR = 10.5).

2 a *age*: numerical, *gender*: categorical

b From this information it can be concluded that the median age of the people admitted to the hospital during this week was associated with their age. The median age of the females (34 years) admitted to the hospital was considerably higher than the median age of males (25.5 years). The variability of the ages was also higher for the females (IQR = 28 years) compared that of the males (IQR = 13 years).

3 a *hours online*: numerical, *year level*: categorical

b From this information it can be concluded that the median number of hours spent online was associated with year level. The median time spent online by the Year 10 students (20 hours) was higher than the median number of hours by the Year 11 students (16.5 hours). The variability of the hours spent online was lower for the Year 10 students (IQR = 9.5 hours) compared that of the Year 11 students (IQR = 13 hours).

4 a *age at marriage*: numerical, *gender*: categorical

b For this data there is an association between age at marriage and gender. The age at marriage is higher for men (M = 23 years) than for women (M = 20.5 years). The variability is also greater for the men (IQR = 12 years) than for the women (IQR = 8.5 years). The distributions of age at marriage are positively skewed for both men and women. There are no outliers.

5 a *pulse rate*: numerical, *gender*: categorical

b For this data there is an association between pulse rate and gender. The pulse rates for males (M = 73 beats/min) are lower than the pulse

ISBN 978-1-009-11041-9

Cambridge University Press

rates for women (M = 76 beats/min). The variability is also lower for the males (IQR = 8 beats/min) than for the women (IQR = 14 beats/min). Both distributions are approximately symmetric, with no outliers.

6 a *lifetime*: numerical; *price*: categorical

b For this data there is an association between the lifetime of a battery and its price. The lifetime of the high price batteries (M = 51 hours) is longer than that of the medium price batteries (M = 35 hours), which is in turn slightly longer than that of the low price batteries (M = 32 hours). The variability in lifetime increased as price decreased, from IQR = 7 hours for the high price batteries, to IQR = 12 hours for the medium price batteries, and IQR = 17 hours for the low price batteries. All three distributions are approximately symmetric, with no outliers.

7 A **8** D

Exercise 2D

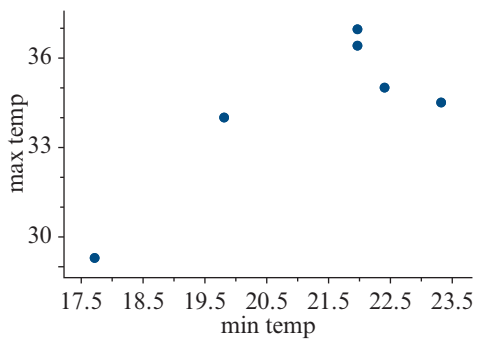
1 a number of seats

b numerical

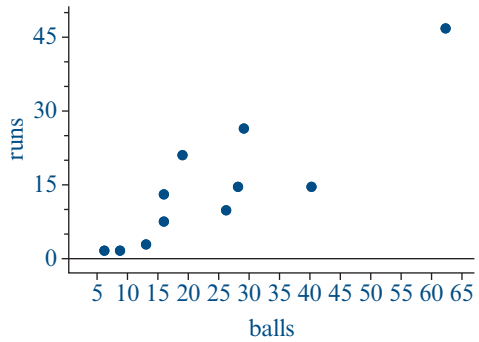
c 8 aircraft

d around 800 km/h

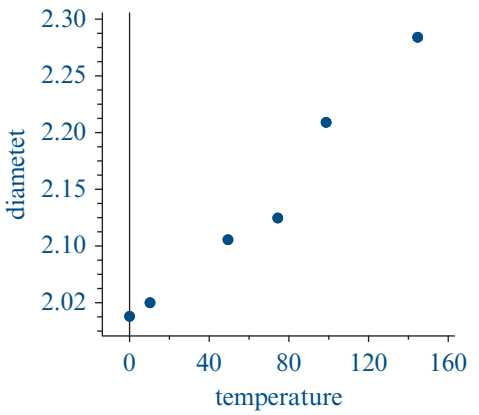
2



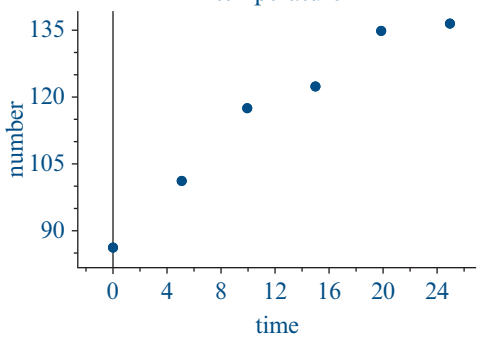
3



4



5



6 D

Exercise 2E

1 Note: There are no absolute right or wrong answers to these questions as answering them requires a degree of personal judgment.

i no association **ii** yes, positive

iii yes, positive **iv** yes, positive

v yes, negative **vi** yes, negative

2 a i moderate, positive, linear association

ii weak, negative, linear association

iii strong, positive, linear association

ISBN 978-1-009-11041-9

Cambridge University Press

iv no association

b **i** Those people who smoke more tend to have a higher lung cancer mortality rate.

ii Older children tended to score lower on the aptitude test.

iii Intersections with higher levels of traffic volume also tended to have higher CO levels.

Exercise 2F

1 **a** A: strong, positive, non-linear relationship with no outliers

B: strong, negative, linear relationship with an outlier

C: weak, negative, linear relationship with no outliers

b A: non-linear

2 $r = 0.73$

3 **a–c** Answers given in question.

4 **a** strong positive

b strong positive

c moderate negative

Exercise 2G

1 **a** 45.6% **b** 11.9% **c** 32.1% **d** 45.3% **e** 1.5%

2 **a** The coefficient of determination is $r^2 = (-0.611)^2 = 0.373$ or 37.3%; that is, 37.3% of the variation observed in hearing test scores can be explained by variation in age.

b The coefficient of determination is $r^2 = (0.716)^2 = 0.513$ or 51.3%; that is, 51.3% of the variation observed in mortality rates can be explained by variation in smoking rates.

c The coefficient of determination is $r^2 = (-0.807)^2 = 0.651$ or 65.1%; that is, 65.1% of the variation observed in life expectancies can be explained by variation in birth rates.

d The coefficient of determination is $r^2 = (0.818)^2 = 0.669$ or 66.9%; that is, 66.9% of the variation observed in daily maximum temperature is explained by the variability in daily minimum temperatures.

e The coefficient of determination is $r^2 = (0.8782)^2 = 0.771$ or 77.1%; that is, 77.1% of the variation in the runs scored by a batsman is explained by the variability in the number of balls they face.

3 **a** $r = 0.906$ **b** $r = -0.353$

4 E **5** A **6** E **7** D

8 D

Exercise 2H

Note: These answers are for guidance only. Alternative explanations for the source of an association may be equally acceptable as the variables suggested.

1 Not necessarily. In general, older children are taller and have been learning mathematics longer. Therefore they tend to do better on mathematics tests. Age is the probable common cause for this association.

2 Not necessarily. While one possible explanation is that religion is encouraging people to drink, a better explanation might be that towns with large numbers of churches also have large populations, thus explaining the larger amount of alcohol consumed. Town size is the probable common cause for this association.

3 Probably not. The amount of ice-cream consumed and the number of drownings would both be affected by weather

ISBN 978-1-009-11041-9

Cambridge University Press

conditions. Weather conditions are the probable common cause.

4 Maybe but not necessarily. Bigger hospitals tend to treat more people with serious illnesses and these require longer hospital stays. A common cause could be the type of patients treated at the hospital.

5 Not necessarily. Possible confounding variables include age and diet.

6 There is no logical link between eating cheese and becoming tangled in bed sheets and dying. The correlation is probably spurious and the result of coincidence.

7 Not necessarily. For example, the more serious the fire, the more fire trucks in attendance and the greater the fire damage. A possible common cause is the severity of the fire.

8 E

Exercise 2I

1 a segmented bar chart
b scatterplot **c** parallel box plots
d scatterplot **e** scatterplot
f segmented bar chart
g segmented bar chart
h parallel box plots or back-to-back stem plots

2 E

3 D

Chapter 2 review

Multiple-choice questions

1 A	**2** D	**3** B	**4** D
5 E	**6** B	**7** D	**8** A
9 C	**10** E	**11** D	**12** E
13 C	**14** C	**15** C	**16** D
17 A	**18** E	**19** B	**20** C
21 E	**22** C		

Written-response questions

1 a *Number of accidents* and *age*; both categorical variables

b RV: *Number of accidents*; EV: *age*

c 470

d

No of accidents	< 30	≥ 30
At most one	21.7%	42.5%
More than one	78.3%	57.5%

e The statement is correct. Of drivers aged less than 30, 78.3% had more than one accident compared to only 57.5% of drivers in the older category.

2 a Numerical: *conversation test score*. Categorical: *completed weeks of course*

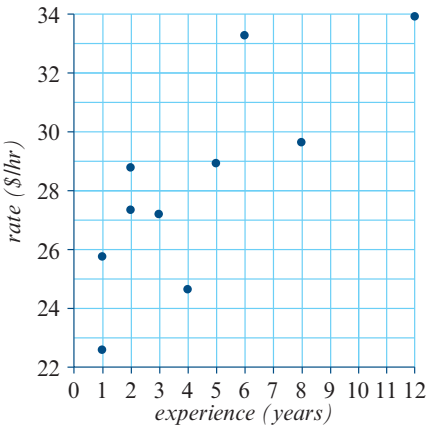
b There is an association between the students' scores on the conversation test, and the number of weeks of the course they have completed. The median score at the beginning of the course (M=38) showed a little improvement after six weeks (M =42), followed by a very large improvement by the end of the 12 week course (M =72). The variability of the scores changed little over the course (IQR=12 at the beginning, IQR = 12 at 6 weeks, IQR = 14 at 12 weeks). The distributions of scores at 0 weeks is approximately symmetric with an outlier at 66, positively skewed with an outlier at 76 at 6 weeks, and approximately symmetric with no outliers at 12 weeks.

3 a *rate* is the response variable, *experience* is the explanatory variable.

ISBN 978-1-009-11041-9

Cambridge University Press

b



c There is a moderate positive linear relationship; that is, people with more experience are generally being paid a higher hourly pay rate.

d 0.786. Note that this value is on the borderline between a moderate and a strong linear relationship, but with such a small data set it is difficult to assess the strength from the scatterplot.

e Coefficient of determination = 0.618; that is, 61.8% of variation in pay rate is explained by the variation in experience.

4 a 60%

b For these people there is an association between the person's quality of sleep and their participation in the course, with 85% of people rating their sleep quality as good after the course, compared to only 60% of people rating their sleep quality as good before the course.

Chapter 3

Exercise 3A

1 A residual is the difference between a data value and its value predicted by a regression line.

2 C

3 The data is numerical; the association is linear; there are no clear outliers.

4 a x

b $y = 9.23 + 1.00x$

5 a RV: *pollution level*; EV: *traffic volume*

b *pollution level* $= -330 + 49 \times$ *traffic volume*

6 a RV: *life expectancy*; EV: *birth rate*

b *life expectancy* $= 110 - 1.5 \times$ *birth rate*

7 a y **b** $r = -0.450$

8 a RV: *distance travelled*; EV: *age of car*

b $r = 0.947$

9 a r is also negative.

b Slope is zero: regression line is horizontal.

c intercept $= \bar{y}$ (mean of RV)

10–11 Answers given in question.

12 a Answers given in question.

b *runs* $= -2.6 + 0.73 \times$ *balls faced*

13 a RV: *number of TVs*

b Answer given in question.

c *number of TVs* $=$ $61.2 + 0.930 \times$ *number of cars*

14 C **15** A **16** C

Exercise 3B

1 *mark* $= 80 - 4.3 \times$ *days absent*

2 a 2.9: On average a person who is 0 cm in height has a hand span of 2.9 cm - not sensible!

b 0.33: On average a person's hand span increases by 0.33 cm for each 1 cm increase in height.

3 a 575: On average, the company will have $575 in sales when their online advertising expenditure is $0.

b 4.85: On average sales will increase by $4.85 for each additional $1.00 spent on online advertising.

ISBN 978-1-009-11041-9

Cambridge University Press

4 **a** 80 cm, extrapolating
b 92 cm, interpolating
c 98 cm, extrapolating

5 **a** $487.50, extrapolating
b $1023.50, interpolating
c $1224.50, extrapolating

6 **a** 173 cm, reliable, interpolating
b 189 cm, unreliable, extrapolating
c 165 cm, reliable, interpolating

7 **a** 20.3%
b 42.3%
c The number of *hours* is more important as it explains 42.3% of the variation in *exam score*, much more than *IQ* which explains only 20.3% of the variation in *exam score*.

8 Answers given in question.

9 **a** 9.7 **b** −0.8

10 **a** 2 **b** −1 **c** 2

11 A: clear curved pattern in the residuals (not random), C: curved pattern in the residuals (not random).

12 **a** 27.8: On average a packet of chips with 0 gm of fat contains 27.8 calories.
b 14.7: On average, the calorie content increases by 14.7 for each one additional gram of fat included.
c 75.7% of the variation in calorie content of the chips is explained by the variation in fat content.
d 145.4 **e** −13.4

13 **a** −0.278: On average, for each additional one metre the golfer is from the hole the success rate decreases by 27.8%.
b 73.5
c 3.54 m
d −0.705
e 49.7%: 49.7% of the variation in success rate in putting is explained by the variation in the distance the golfer is from the hole.

14 **a** yes, linear relationship
b 0.9351 or 93.5%
c 93.5%
d *pay rate* = 18.56 + 0.289 × *experience*
e Intercept = $18.56. On average, a worker with no experience will earn $18.56 per hour.
f On average, the pay rate increases by 29 cents per hour for each additional one year of experience.
g **i** $20.87 **ii** $0.33
h yes; no clear pattern in the residual plot

15 **a** $r = -0.608$
b 37% of the variation in the hearing test score is explained by the variation in age.
c *score* = 4.9 − 0.043 × *age*
d −0.043; the hearing test score, on average, decreases by 0.043 for each one additional year of age.
e **i** 4.04 **ii** −2.04
f **i** 0.3 **ii** −0.4
g yes; no clear pattern in the residual plot

16 negative, drug dose, −0.9492; 55.9; −9.31; decreases, 9.31; 55.9; 90.1, response time, drug dose; clear pattern

17 The scatterplot shows that there is a strong positive linear relationship between radial length and femur length: $r = 0.9876$. There are no outliers. The equation of the least squares regression line is: *radial length* = −7.2 + 0.74 × *femur length*

ISBN 978-1-009-11041-9

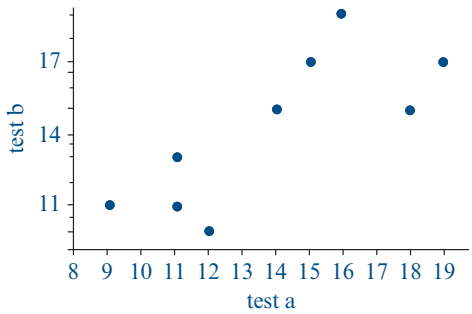
Cambridge University Press

The slope of the regression line predicts that, on average, radial length increased by 0.74 cm for each one centimetre increase in femur length.
The coefficient of determination indicates that 97.5% of the variation in radial lengths can be explained by the variation in femur lengths.
The residual plot shows no clear pattern, supporting the assumption that the relationship between radial and femur length is linear.

18 E **19** B **20** A

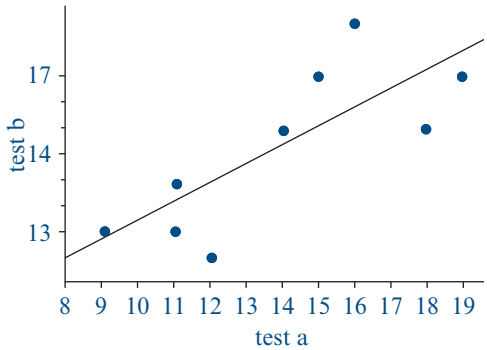
Exercise 3C

1 a

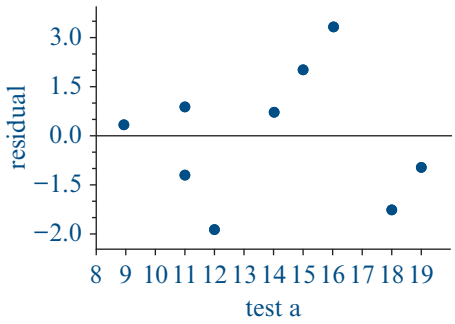


b *test B score* = 4.2 + 0.72 × *test A score*, $r = 0.78$, $r^2 = 0.61$

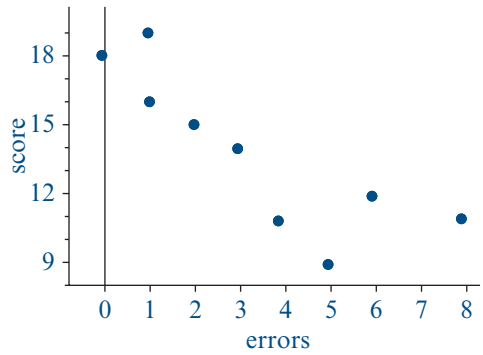
c



d

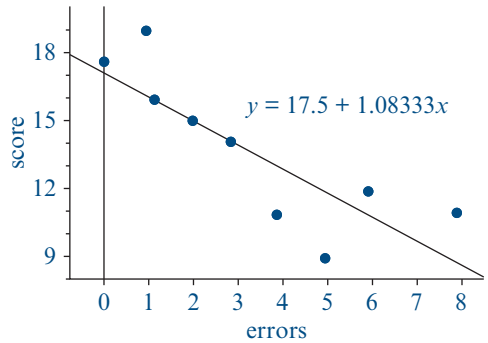


2 a

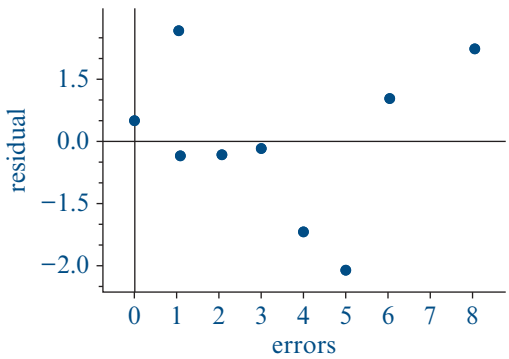


b *score* = 17.5 − 1.08 × *error*, $r = -0.841$, $r^2 = 0.707$

c



d

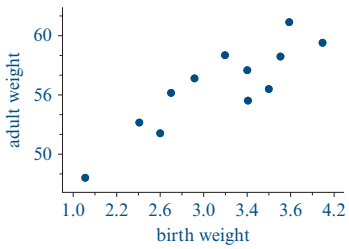


ISBN 978-1-009-11041-9

Cambridge University Press

3 a RV: *adult weight*; EV: *birth weight*

b



c i strong positive linear association with no outliers

ii approximately 0.9

d *adult weight* = $38.4 + 5.87 \times$ *birth weight*, $r^2 = 0.765$, $r = 0.875$

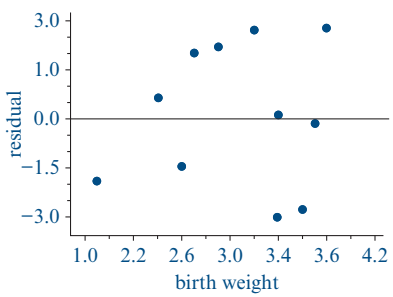
e 76.5% of the variation in the adult weight is explained by the variation in birth weight.

f On average, adult weight increases by 5.9 kg for each additional kilogram of birth weight.

g i 56.0 **ii** 53.1 **iii** 61.3

h Yes. 76.5% of the variation in the adult weight is explained by the variation in birth weight.

i



The lack of a clear pattern in the residual plot supports the assumption that the association between adult weight and birth weight is linear.

Chapter 3 review

Multiple-choice questions

1 C	**2** D	**3** A	**4** C
5 E	**6** C	**7** B	**8** B
9 D	**10** A	**11** A	**12** A
13 D	**14** E	**15** A	**16** C
17 C	**18** A	**19** A	**20** D
21 C	**22** C	**23** D	

Written-response questions

1 a i 5 years

ii mean= 767, st dev = 35

b *airspeed* = $673 + 0.372 \times$ *number of seats*

c 74.1%

2 a days of rain **b** −6.88, 2850 **c** 2024

d decrease, 6.88 **e** −0.696

f 48.4, days of rain **g i** 1873 **ii** −483

h interpolation

3 a *cost*

b There is a strong, positive, linear association between the cost of the meals and the number of meals prepared.

c i \$307.30 **ii** extrapolating

d i 222.48: the fixed costs of preparing meals is \$222.48.

ii \$4.039: The slope of the regression line predicts that, on average, meal preparation costs increase by \$4.039 for each additional meal produced.

e Answer given in question.

4 a RV: *height*; EV: *femur length*

b *height* = $36.3 + 5.35 \times$ *femur length*

c On average, height increases by 5.35 cm for each cm increase in femur length.

d $r^2 = 0.988$; that is, 98.8% of the variation in height is explained by the variation in femur length.

e 97.6%

5 a RV: *height*; EV: *age*

b strong positive association with no outliers.

c Answer given in question.

d i Answer given in question

ISBN 978-1-009-11041-9

Cambridge University Press

ii Extrapolating

e On average, for each additional one year of age, height increases by 6.4 cm.

f $r^2 = 0.995$; that is, 99.5% of the variation in height is explained by the variation in age.

g i 140.3 cm ii −0.7 cm

h i Answer given in question.

ii Residuals show a clear curved pattern.

6 a moderate, positive linear association with no outliers

b i 142

ii extrapolating

c −6.3

d i linearity

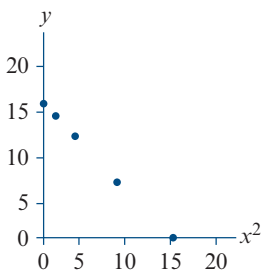
ii the lack of a clear pattern in the residual plot supports the linearity assumption.

Chapter 4

Exercise 4A

1 a 19.5 b 11.7 c 23.8 d 126.7

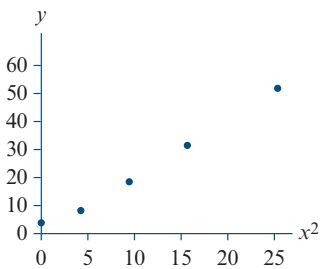
2 a



b $y = 16 - x^2$

c when $x = -2, y = 12$

3 a

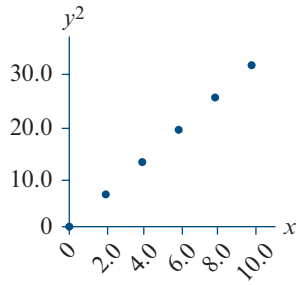


b $y = 1 + 2x^2$

c when $x = 6$, $y = 73$

4 a ±4.7 b ±1.3 c 6 d −8

5 a



b $y^2 = 1.5 + 3.1x$

c $y = \pm 5.4$, but only the positive solution applies here because the model is only defined for $y > 0$.

6 a *number of people* = 0.0 + 4.1 × *diameter*²

b 7

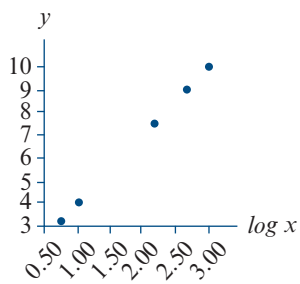
7 a *time*² = 18 − 9.3 × *amount* b 3.8 min

8 D 9 A 10 B

Exercise 4B

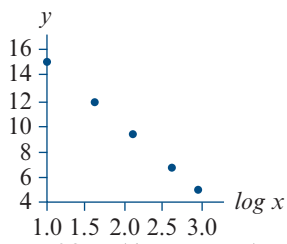
1 a 6.6 b 1.1 c −3.1 d 138.5

2 a



b $y = 1 + 3 \log x$ c 7

3 a



b $y = 20 - 5 \log x$ c 5

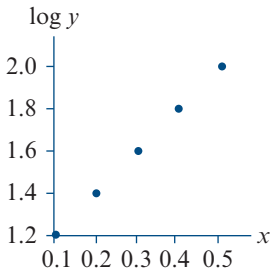
4 a 100 b 218.8 c 1 000 000

d 0.8

ISBN 978-1-009-11041-9

Cambridge University Press

5 a



b $\log y = 1 + 2x$ **c** 158.5

6 a $level = 1.8 + 2.6\log(time)$ to 2 sig. figs

b 2.8 to 1 d.p.

7 a $\log(number) = 1.314 + 0.08301 \times month$ to 4 sig. figs

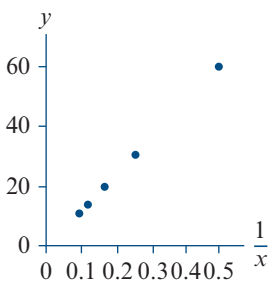
b 139 to nearest whole number

8 C **9** E **10** A

Exercise 4C

1 a 13.3 **b** 2.8 **c** 4.9 **d** 309.5

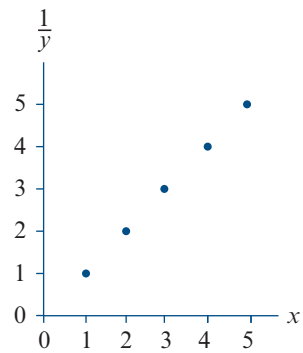
2 a



b $y = \frac{120}{x}$ **c** 24

3 a 0.17 **b** 0.07 **c** 0.16 **d** 0.06

4 a



linear

b $\frac{1}{y} = x$

c 4

5 a $horsepower = 22.1 + \frac{690}{consumption}$

b 99 to nearest whole number

6 a $\frac{1}{errors} = -0.00024 + 0.050 \times times$ to 2 sig. figs

b 3 to nearest whole number

7 A **8** E **9** C

Exercise 4D

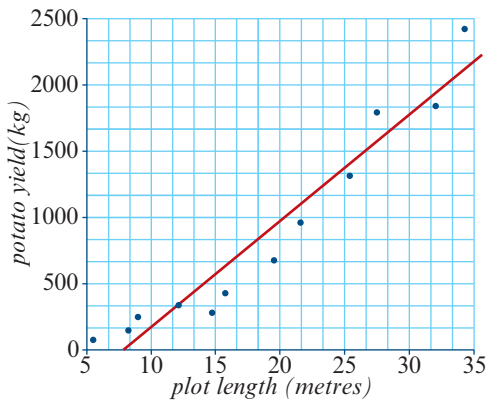
1 a $\log y$, $\frac{1}{y}$, $\log x$, $\frac{1}{x}$

b None; trend needs to be consistently increasing or decreasing.

c $\log y$, $\frac{1}{y}$, x^2

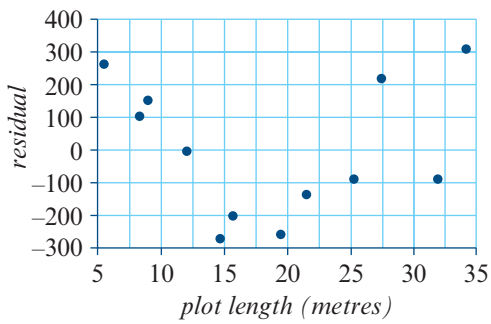
d x^2, y^2

2 a



b $yield = -620.0 + 80.23 \times length$

c



No, the residuals show a clear curved pattern.

d $\log y$, $\frac{1}{y}$, x^2

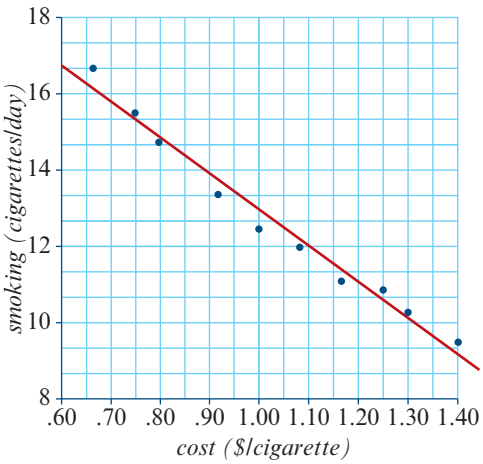
e $yield = 3.983 + 2.030 \times (length)^2$

ISBN 978-1-009-11041-9

Cambridge University Press

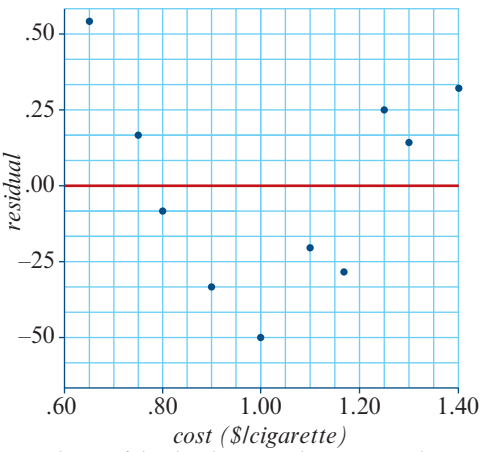
f $r^2 = 97.5\%$

3 a



b $smoking = 22.49 - 9.501 \times cost$

c



No, the residuals show a clear curved pattern.

d $\log x$, $\log y$, $\frac{1}{y}$, $\frac{1}{x}$

e Either the $\log x$ and $\frac{1}{x}$ could be recommended as both transformations give very good results. That is

$smoking = 3.420 + \frac{9.045}{cost}$ or

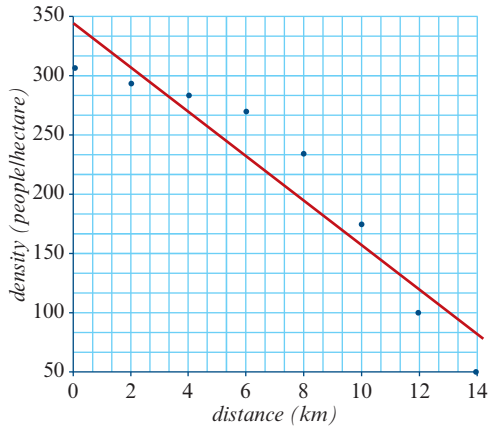
$smoking = 12.73 - 21.90 \times \log(cost)$

The $\frac{1}{x}$ transformation is more intuitive and easier to interpret.

f $\frac{1}{x}$: $r^2 = 99.3\%$

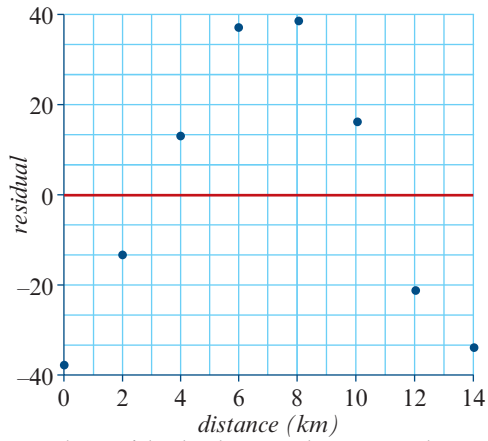
$\log x$: $r^2 = 99.6\%$

4 a



b $density = 345.3 - 18.65 \times distance$

c



No, the residuals show a clear curved pattern.

d x^2, y^2

e $density = 308.9 - 1.345 \times (distance)^2$

f $r^2 = 99.1\%$

Chapter 4 review

Multiple-choice questions

1 A	**2** D	**3** D	**4** B
5 A	**6** B	**7** E	**8** D
9 D	**10** D	**11** D	

Written-response questions

1 a $\frac{1}{age} = 2.606 - 1.053 \times length$

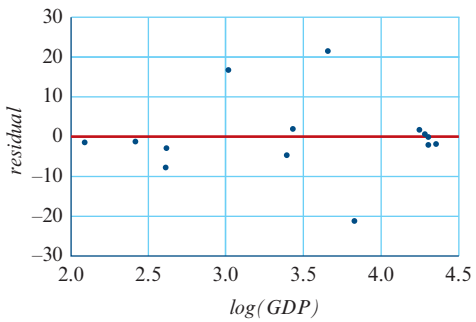
b 2.6 years

2 a $literacy\ rate = -44.2 + 33.3 \log(GDP)$

ISBN 978-1-009-11041-9

Cambridge University Press

b

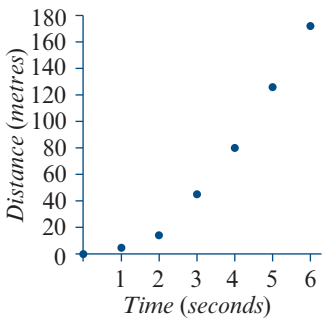


Residual plot shows no clear pattern

c 89%

d 0.077

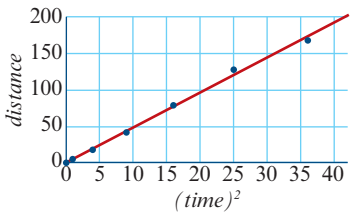
3 a



b

Time	0	1	2	3	4	5	6
Distance	0	5.2	18	42	79	128	168
$Time^2$	0	1	4	9	16	25	36

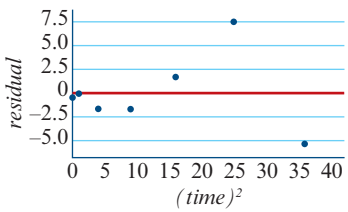
c



d $distance = 0.45 + 4.8 \times time^2$

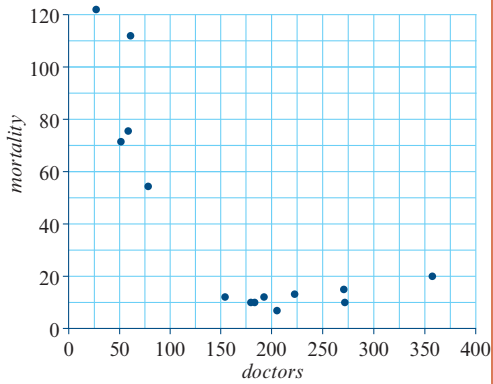
e 236 metres

f

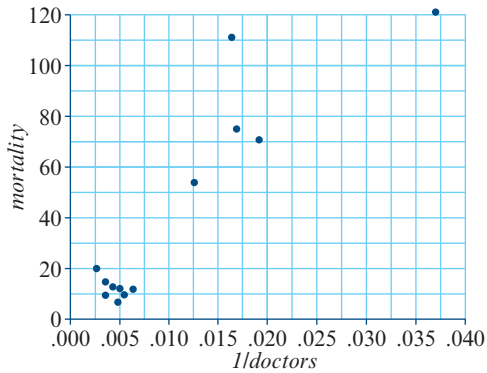


The residual plot shows no clear structure indicating that the assumption of linearity is justified.

4 a

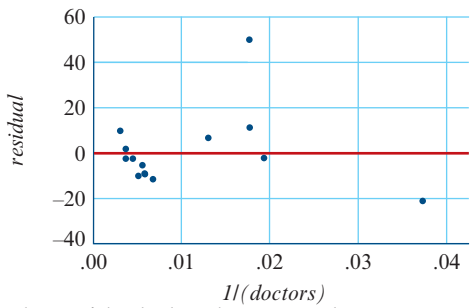


b



c $mortality = -1.194 + 3856 \times \dfrac{1}{doctors}$

d



The residual plot shows no clear structure indicating that the assumption of linearity is justified.

e r^2=82.8%

f 37

g Since 100 is within the range of the data we are interpolating, and the prediction is reliable.

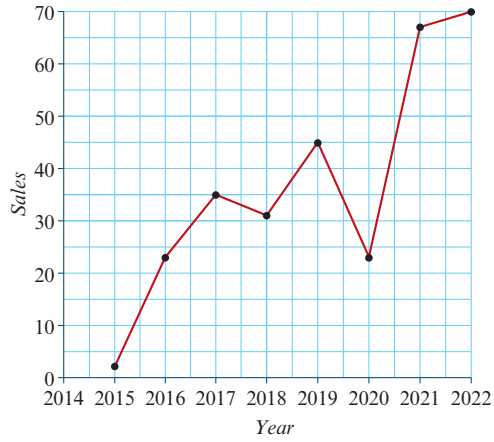
ISBN 978-1-009-11041-9

Cambridge University Press

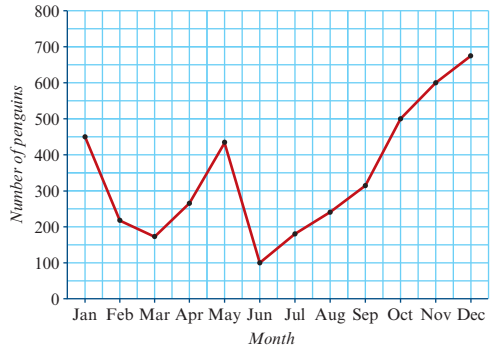
Chapter 5

Exercise 5A

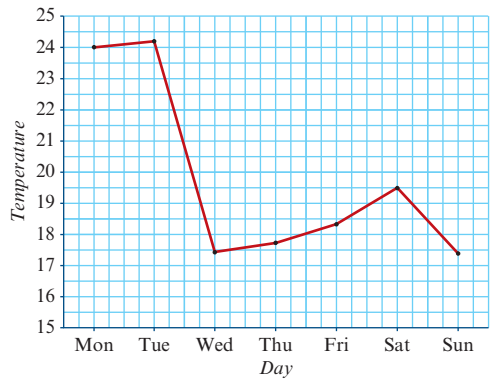
1



2



3



4

Feature	A	B	C
Irreg fluct	✓	✓	✓
Inc			✓
Dec	✓		

5

Feature	A	B	C
Irreg fluct	✓	✓	✓
Inc trend			✓
Dec trend	✓		
Cycles		✓	
Seasonality	✓		✓

6

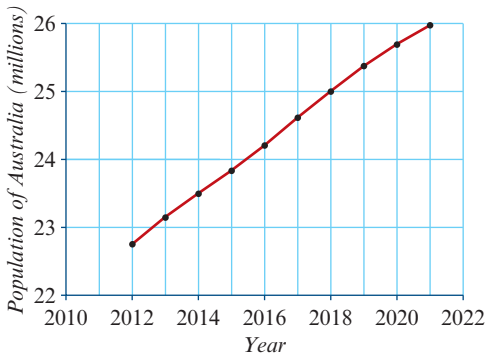
Feature	A	B	C
Irreg fluct	✓		✓
Struct change	✓		
Inc trend	✓		
Dec trend	✓		
Seasonality			✓

7

Feature	A	B	C
Irreg fluct	✓	✓	✓
Struct change		✓	
Inc trend	✓		
Dec trend			
Outlier			✓

8 The number of mobile phones per 100 people increases rapidly over the years 2000-2008. The number continues to increase from 2009 until 2019, but the increase in the number of phones is at much lower rate than in the preceding years.

9 a

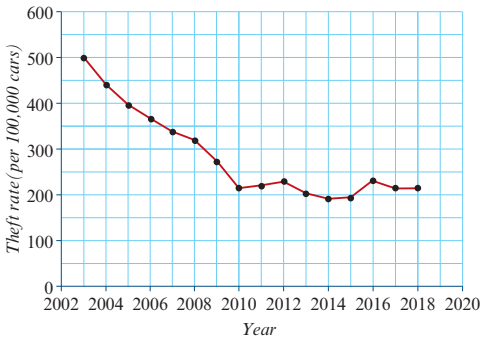


ISBN 978-1-009-11041-9

Cambridge University Press

b The plot shows a steady increase in the population of Australia over the years 2012 - 2021.

10 a



b The plot shows a steady decline in the number of vehicle thefts over the years 2003 -2010, after which the number of vehicle thefts has remained reasonably steady, showing only irregular variation.

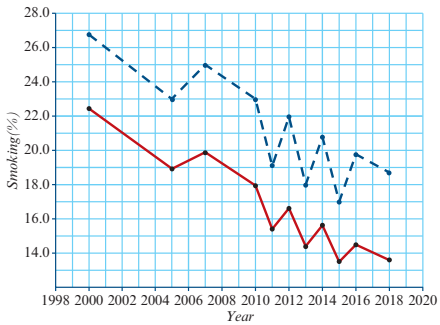
11 The number of cases of measles show an increasing trend between 1989 and 1992. In 1993-1994 there is a rapid increase in the number of measles cases, followed be a rapid decrease in 1994-1995. The number of cases continued to decrease until 2000, since then have remained low, showing only irregular variation over the years 2001-2019.

12 The number of overseas arrivals (millions people per month) in Australia increased steadily from November 2011 until April 2020. The number of arrivals is clearly seasonal, with the peak time for arrivals in the January quarter each year. The number of arrivals dropped suddenly to almost zero in April 2020, and remained at this level until October 2021.

13 a i The percentage of males who smoke has consistently decreased since 1945, while the percentage of females who smoke increased from 1945 to 1975 but then decreased at a similar rate to males over the period 1975–1992.

ii The difference in smoking rates between males and females has decreased over these years.

b i



ii Whilst both plots show irregular fluctuation, overall the percentages of male and females who smoke have declined substantially over the years 2000-2018.

iii The difference in smoking rates between males and females has remained almost the same over these years.

14 E

15 D

Exercise 5B

1 a i 3 **ii** 1 **iii** 4

b i 3.2 **ii** 1.2 **iii** 2.2

c i 2.6 **ii** 2.0

d 2.3

2 a 24.4 **b** 20.0 **c** 23.2

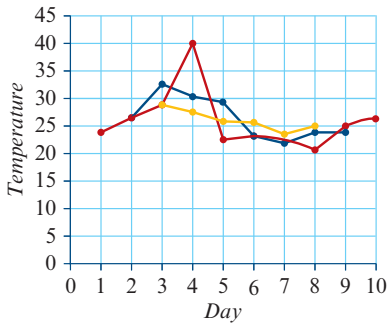
3

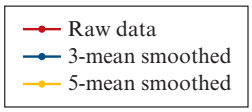
t	1	2	3	4	5	6	7	8	9
y	10	12	8	4	12	8	10	18	2
3-mean	–	10	8	8	8	10	12	10	–
5-mean	–	–	9.2	8.8	8.4	10.4	10	–	–

ISBN 978-1-009-11041-9

Cambridge University Press

4 a, c



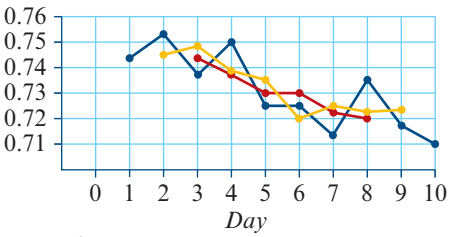


The smoothed plots show that the 'average' maximum temperature changes relatively slowly over the 10-day period (the 5-day average varies by only 5°) when compared to the daily maximum, which can vary quite widely (for example, nearly 20° between the fourth and fifth day) over the same period of time.

b

Day	*Temp. (°C)*	*3-moving mean*	*5-moving mean*
1	24	–	–
2	27	26.3	–
3	28	31.7	28.2
4	40	30.0	28.0
5	22	28.3	27.0
6	23	22.3	25.6
7	22	22.0	22.6
8	21	22.7	23.4
9	25	24.0	–
10	26	–	–

5 a, c



- exchange rate
- 3-moving mean exchange rate
- 5-moving mean exchange rate

The exchange rate has a downward trend over the 10-day period. This is most obvious from the smoothed plots, particularly the 5-moving mean plot.

b

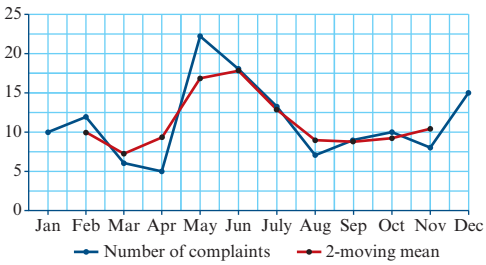
Day	*Exchange rate*	*3-moving mean*	*5-moving mean*
1	0.743	–	–
2	0.754	0.745	–
3	0.737	0.747	0.742
4	0.751	0.737	0.738
5	0.724	0.733	0.730
6	0.724	0.720	0.729
7	0.712	0.724	0.722
8	0.735	0.721	0.720
9	0.716	0.721	–
10	0.711	–	–

6 a 3.8 **b** 2.0

7 a 3.3 **b** 1.5 **c** 2.4 **d** 1.9

8 a 13.1 **b** 12.2 **c** 10.7

9 a, c

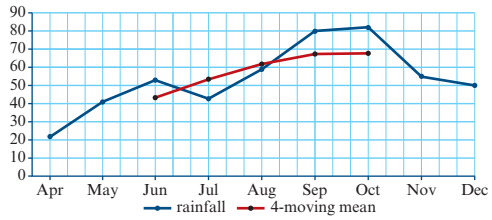


Two-mean smoothing of the plot has not had a big effect, with both plots showing that the that the number of complaints between April and July is considerably higher than the number in the rest of the year.

ISBN 978-1-009-11041-9

Cambridge University Press

10 a, c



Four-mean smoothing of the plot shows a steady increase in rainfall from June to October.

11 A **12** C **13** E **14** D

Exercise 5C

1 a (3, 3) **b** (2, 2) **c** (3, 2) **d** (3, 3)

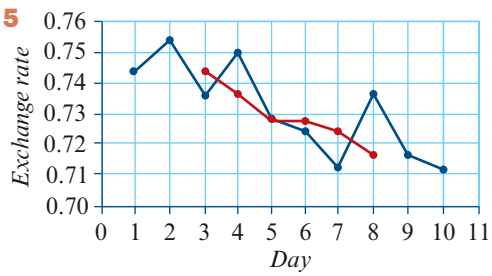
2 a 30°C **b** 25°C

3

The smoothed plot shows that sales were quite consistent up to 2017 when they dropped, and have remained at this lower level from 2017-2022.

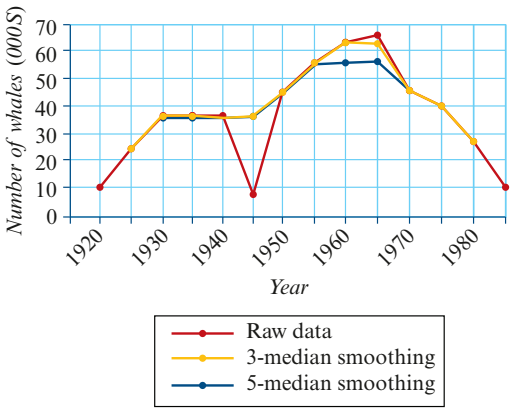
4 a 25°C **b** 25°C

5

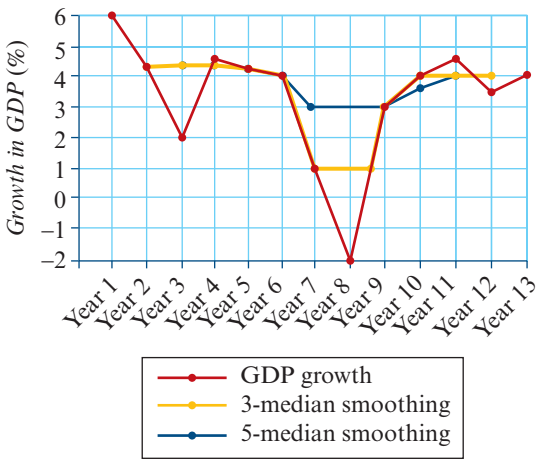


The smoothed plot shows that there was a general decreasing trend in the exchange rate over this period.

6



7 a 4 **b i, ii**



c The plot of GDP growth over 1 year, shows a great deal of variability, with no clear trend apparent. When smoothed over a 3-year period, GDP growth is still variable but to a lesser extent. No clear trend is apparent, but GDP appears to be going through a period of below average growth during the time period from Year 7 to Year 9. When smoothed over a 5-year period, GDP growth is much less variable but clearly shows the period of below average growth during the from period from Year 7 to Year 9.

8 D **9** C **10** D

ISBN 978-1-009-11041-9

Cambridge University Press

Exercise 5D

1 a 1.0

b Sales in February are typically 30% higher than sales in the average month.

c Sales in September are typically 10% lower than sales in the average month.

2 a 7.8 **b** 6.7

3 a 3.9 **b** 6.9

4 a Increase by 42.9%.

b Decrease by 23.1%

5 a 1.2 **b** 1514 **c** 1437 **d** 1005

6

	Sum	Aut	Win	Spr
Number of students:	56	125	126	96
Deseasonalised:	112	125	97	80
Seasonal index	0.5	1.0	1.3	1.2

7 a, c

	Q1	Q2	Q3	Q4
Deseasonalised:	152	142	148	153
Seasonal index	1.30	1.02	0.58	1.10

b In quarter 1 the restaurant chain employs 30% more waiters than the number employed in an average quarter.

8

Q1	Q2	Q3	Q4
0.89	0.83	1.12	1.16

9

Jan	Feb	Mar	April	May	June
0.89	0.96	1.04	1.26	1.33	1.11

July	Aug	Sept	Oct	Nov	Dec
0.67	0.74	0.59	0.81	1.11	1.48

10

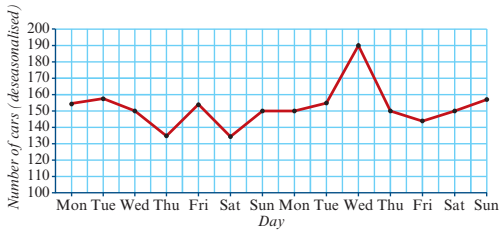
Jan	Feb	Mar	April	May	June
1.06	0.96	1.18	1.18	1.03	0.86

July	Aug	Sept	Oct	Nov	Dec
0.96	0.79	0.74	0.54	1.18	1.50

11 a

Mon	Tues	Wed	Thur	Fri	Sat	Sun
155	157	150	134	153	134	150
150	154	190	148	143	150	157

11 b



12 a, d

b

Q1	Q2	Q3	Q4
1.03	0.93	0.93	1.11

c

Year	Q1	Q2	Q3	Q4
1	206	209	211	205
2	214	212	211	215
3	224	220	218	221

13 E **14** D **15** E **16** B

17 B

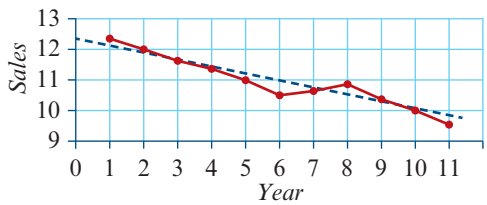
Exercise 5E

1 a There was a general increasing trend in the number of university students in Australia during the period 2010–2019

b $students(000s) = -37563 + 18.927 \times year$. On average, the number of university students in Australia has increased by 18 927 per year.

c 859 000 (to nearest thousand)

2 a



ISBN 978-1-009-11041-9

Cambridge University Press

b There was a general decreasing trend in the percentage of retail sales made in department stores.

c $sales = 12.5 - 0.258 \times year$
The percentage of total retail sales that are made in department stores is decreasing by 0.258% per year.

d 8.6%

3 a $age = -147 + 0.0882 \times year$; On average, the average age of mothers increased by 0.0882 years (equivalent to 1 month) each year between 2010 and 2020.

b 32.0 years; Unreliable as we are extrapolating 10 years beyond the period in which the data were collected.

4 a $earnings = -83\,280 + 42.07 \times year$; On average, average weekly earnings increased by \$42.07 each year between 2014 and 2021.

b \$2122.10; Unreliable as we are extrapolating 9 years beyond the period in which the data were collected.

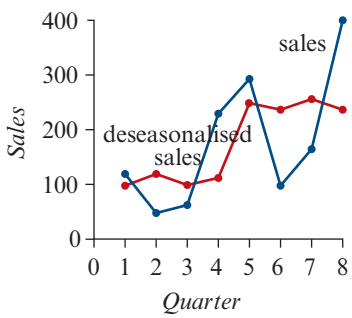
5 a deseasonalised number
$= 50.9 + 1.59 \times$ quarter number

b deseasonalised number = 76.34
reseasonalised (actual) number = 90
(to the nearest whole number)

6 a

Year	Q1	Q2	Q3	Q4
1	122	128	118	130
2	250	245	263	236

b



The deseasonalised sales appear to show an increasing trend over time.

c deseasonalised sales
$= 80.8 + 23.5 \times quarter$

d forecasted actual sales
$= 386.3 \times 1.13 = 437$

7 C **8** E

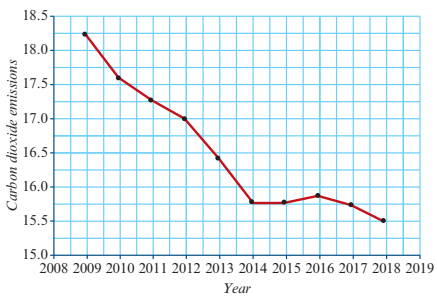
Chapter 5 review

Multiple-choice questions

1 E **2** E **3** A **4** B **5** E **6** C
7 D **8** D **9** C **10** D **11** B **12** C
13 B **14** A **15** E **16** D **17** D **18** A
19 B **20** E

Written-response questions

1 a



b Carbon dioxide emissions decreased between 2009 and 2014, then remained reasonable steady over the years 2014–2017, showing only irregular fluctuations. Between 2017 and 2018 there was a small decrease in carbon dioxide emissions.

c $CO_2\ emissions = 612.0 - 0.2958 \times year$

d 12.7

e Unreliable as we are extrapolating 8 years beyond the period in which the data were collected.

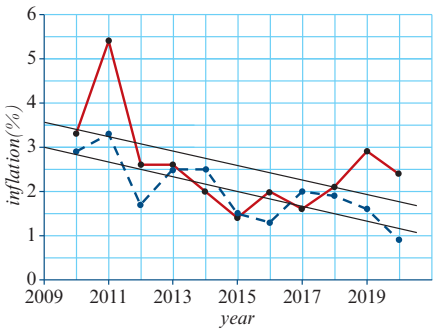
2 a i $inflation = 332 - 0.164 \times year$

b i $inflation = 339 - 0.167 \times year$

ISBN 978-1-009-11041-9

Cambridge University Press

a ii, b ii



c The trend lines are parallel. As such, they will never cross, so the inflation rate for China will remain higher than the inflation rate for Australia.

d 1.7

3 a

	Sum	Aut	Win	Spr
SI	0.29	0.36	1.37	1.98

b

	Sum	Aut	Win	Spr
Deseas	269	239	255	273

Chapter 6

Exercise 6A

1	A	**2**	B	**3**	B	**4**	B
5	D	**6**	B	**7**	D	**8**	E
9	E	**10**	E	**11**	D	**12**	A
13	D	**14**	A	**15**	A	**16**	E
17	C	**18**	B	**19**	B	**20**	B
21	C	**22**	E	**23**	D	**24**	C
25	A	**26**	D				

Exercise 6B

1	E	**2**	A	**3**	E	**4**	C
5	C	**6**	B	**7**	E	**8**	B
9	B	**10**	E	**11**	B	**12**	E
13	D	**14**	B	**15**	E	**16**	D
17	A	**18**	D				

Exercise 6C

1	C	**2**	C	**3**	A	**4**	D
5	B	**6**	B	**7**	E	**8**	B
9	D	**10**	A	**11**	E	**12**	E
13	B	**14**	A	**15**	E	**16**	C
17	D						

Exercise 6D

1	A	**2**	A	**3**	A	**4**	B
5	E	**6**	B	**7**	A	**8**	B
9	D	**10**	E	**11**	D	**12**	E
13	E	**14**	B	**15**	B	**16**	D
17	A						

Exercise 6E

1 a *age*, *distance*

b mean = 7.17 km, sd = 3.46 km

c $z = 1.7$

d

Study mode	*Gender* Female	Male
On campus	3	3
Online	4	2
Total	7	5

e i 60%

e ii Yes, there is an association between study mode preference and course. A higher percentage of students business chose to study online (60%), compared to only 36% for both students of Health and Social Science.

2 a The distribution of *distance* is positively skewed, with outliers at 17 km, 18 km, and 19 km.

b 30

c i Lower fence = -2, Upper fence = 14.

ii A distance of 1 km is within the fences.

d i 1 km **ii** 1.5 km

3 a On average, height increases by 0.815 cm for each additional 1 cm increase in arm span.

ISBN 978-1-009-11041-9

Cambridge University Press

b **i** Females: $r^2 = 64.6\%$

ii Males: $r^2 = 69.9\%$

iii Since the value of the coefficient of determination for males (69.9%) is higher than the value for females (64.6%), then we can say that arm span is a better predictor of height for males than for females.

c **i** The models predict that when both have arm span measurements of 160 cm, a male will be 1.8 cm taller than a female.

ii The models predict that when both have arm span measurements of 190 cm, a female will be 4.6 cm taller than a male.

iii The differences predicted are not reliable for a height of 160 cm as this value is outside the range of height data for males. The prediction is not reliable for a height of 190 cm as this value is outside the range of height data for females.

4 a There is a moderate strength, non-linear association between *expenditure* and *score*.

b y^2, $\log x$, $\frac{1}{x}$

c **i** The linearity assumption.

ii No, there is a clear structure in the residual plot. If the linearity assumption had been met the residuals would have been randomly scattered around a horizontal line at $y = 0$.

d **i** $score = 12.99 + 120.6 \times \log(expenditure)$

ii 495

5 a **i** \$12 000

ii \$11 000

b \$52 208.29

c **i** Slope = \$1525.80. On average the price of bitcoin is increasing by \$1525.80 each month.

ii \$92 837

iii Unreliable as we are extrapolating several years beyond the period in which the data were collected.

Chapter 7

Exercise 7A

1 a 2, 8, 14, 20, 26 **b** 5, 2, −1, −4, −7

c 1, 4, 16, 64, 256 **d** 64, 32, 16, 8, 4

2 a 6, 14, 30, 62, 126

b 24, 16, 12, 10, 9 **c** 1, 2, 5, 14, 41

d 124, 60, 28, 12, 4

3 a 4, 6, 8, 10, 12 **b** 24, 20, 16, 12, 8

c 2, 6, 18, 54, 162 **d** 50, 10, 2, 0.4, 0.08

e 5, 13, 29, 61, 125

f 18, 16.4, 15.12, 14.096, 13.2768

4 a 2, 5, 8, 11, 14 **b** 50, 45, 40, 35, 30

c 1, 3, 9, 27, 81 **d** 3, −6, 12, −24, 48

e 5, 9, 17, 33, 65 **f** 2, 7, 17, 37, 77

g −2, −1, 2, 11, 38

h −10, 35, −100, 305, −910

5 a 12, 57, 327, 1947, 11667

b 20, 85, 280, 865, 2620

c 2, 11, 47, 191, 767

d 64, 15, 2.75, −0.3125, −1.078125

e 48000, 45000, 42000, 39000, 36000

f 25000, 21950, 19205, 16734.50, 14511.05

6 a $A_2 = 6$ **b** $B_4 = -1$ **c** $C_3 = 27$

d $D_5 = 127$

7 a $V_0 = 4$, $V_{n+1} = V_n + 2$

b $V_0 = 24$, $V_{n+1} = V_n - 4$

c $V_0 = 2$, $V_{n+1} = 3V_n$

8 a $V_0 = 5$, $V_{n+1} = V_n + 5$

b $V_0 = 13$, $V_{n+1} = V_n - 4$

c $V_0 = 1$, $V_{n+1} = 4V_n$

d $V_0 = 64$, $V_{n+1} = 0.5V_n$

ISBN 978-1-009-11041-9

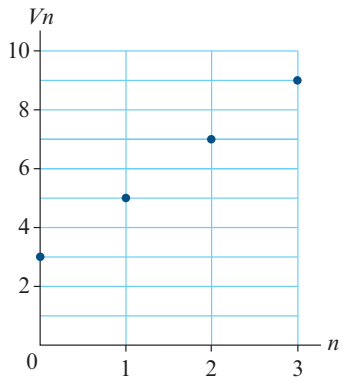
Cambridge University Press

9 6

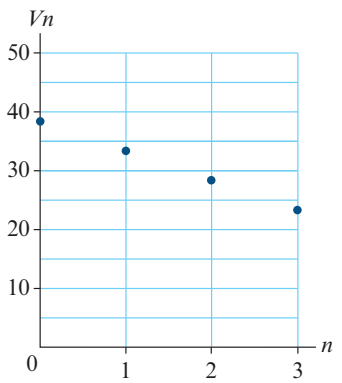
10 0 **11** E **12** D **13** E

Exercise 7B

1 a



b



2 a $V_0 = 8000$ **b** \$320

c $V_0 = 8000$, $V_{n+1} = V_n + 320$

3 a $H_0 = 41\,000$ **b** \$2542

c $H_0 = 41\,000$, $H_{n+1} = H_n + 2542$

4 a $V_0 = 2000$ **b** 14 years

$V_1 = 2000 + 76 = 2076$

$V_2 = 2076 + 76 = 2152$

$V_3 = 2152 + 76 = 2228$

5 a \$7518, \$8036, \$8554 **b** 6 years

6 a i \$15 000 **ii** \$525 **iii** 3.5%

b 29 years

7 a \$12 300

b $C_0 = 82\,000$, $C_{n+1} = C_n - 12\,300$

8 a $\frac{8}{100} \times 2800 = 224$ **b** 2800

c $W_0 = 2800$, $W_{n+1} = W_n - 224$

9 a \$2100, \$1700, \$1300 **b** 4

10 a \$22 195, \$21 390, \$20 585 **b** 17

11 a i \$1500 **ii** \$102 **iii** 6.8%

b \$684

c 14 years

12 a \$32 600

b $M_0 = 32\,600$, $M_{n+1} = M_n - 10$

13 a 450, 449.95, 449.90, 449.85, 449.80

b \$449

14 a \$47 800, \$47 600, \$47 400

b \$45 000 **c** 25 000 km

15 a 7200 **b** 72 000 **c** \$720 **d** 10%

16 C **17** C **18** B

Exercise 7C

1 a $A_n = 4 + 2n$, $A_{20} = 44$

b $A_n = 10 - 3n$, $A_{20} = -50$

c $A_n = 5 + 8n$, $A_{20} = 165$

d $A_n = 300 - 18n$, $A_{20} = -60$

2 a 5000 **b** \$270 **c** $V_n = 5000 + 270n$

d \$7430

3 a 12 000 **b** \$864

c $V_n = 12\,000 + 864n$ **d** \$19 776

4 a \$8000 **b** \$512

c i \$14 144 **ii** 16 years

5 a \$2000 **b** \$70

c i \$2420 **ii** 29 years

6 a \$5600 **b** \$1260 **c** $V_n = 5600 - 1260n$

d \$1820

7 a \$7000 **b** \$1225 **c** $V_n = 7000 - 1225n$

d 5 years

8 a \$1700 **b** \$212.50 **c** \$850

d \$212.50 **e** 8 years

9 a \$65 000 **b** \$3250 **c** 5%

d \$42 250 **e** 11 years

10 a \$29 000 **b** \$0.25 (25 cents)

c \$24 000 **d** 96 000 km

ISBN 978-1-009-11041-9

Cambridge University Press

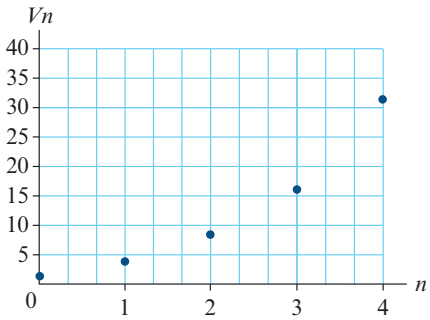
11 **a** \$9700 **b** \$0.388 per km
c $V_n = 35\,400 - 0.388n$
d 74 000 km

12 **a** **i** \$0.026875 **ii** \$69 687.50
iii \$20 156.25
b \$9218.75
c 1 486 400

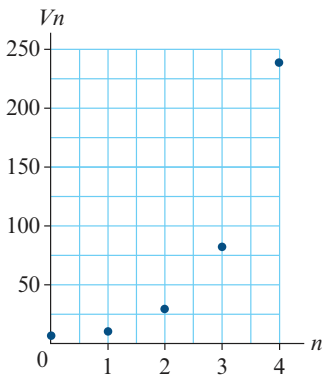
13 D **14** C

Exercise 7D

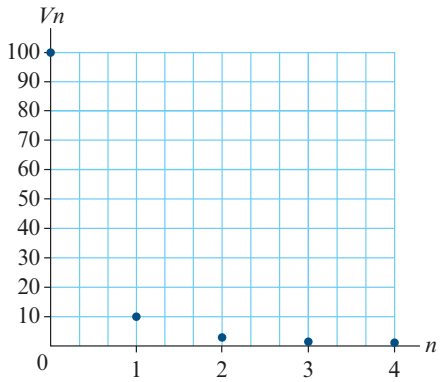
1 **a** 2, 4, 8, 16, 32



b 3, 9, 27, 81, 243



c 100, 10, 1, 0.1, 0.01



2 **a** $V_0 = 6000$
$V_1 = 1.042 \times 6000 = 6252$
$V_1 = 1.042 \times 6252 = 6514.58$
$V_2 = 1.042 \times 6514.58 = 6788.20$
b 7 years

3 **a** $V_0 = 20\,000$
$V_1 = 1.063 \times 20\,000 = 21\,260$
$V_1 = 1.063 \times 21\,260 = 22\,599.38$
$V_2 = 1.063 \times 22\,599.38 = 24\,023.14$
b 7 years

4 **a** \$5000 **b** 1.068
c $V_0 = 5000$, $V_{n+1} = 1.068 \times V_n$
d \$6947.46 **e** \$1947.46

5 **a** \$18 000 **b** 1.094
c $V_0 = 18\,000$, $V_{n+1} = 1.094V_n$
d \$25 783.50 **e** 4 years

6 $V_0 = 9800$, $V_{n+1} = 0.965V_n$

7 $M_0 = 28\,600$, $M_{n+1} = 0.926M_n$

8 **a** $V_0 = 18\,000$, $V_{n+1} = 0.955V_n$
b \$17 190, \$16 416.45, \$15 677.71, \$14 972.21, \$14 298.46
c \$15 677.71 **d** \$3701.54

9 **a** $W_0 = 4000$, $W_{n+1} = 0.959W_n$
b \$3527.90 **c** 755.46

10 **a** $S_0 = 13\,420$, $S_{n+1} = 0.888S_n$
b \$11 916.96, \$10 582.26, \$9397.05, \$8344.58, \$7409.99
c \$7409.99 **d** \$1185.21

11 C **12** E **13** E **14** C

Exercise 7E

1 **a** $V_n = 2^n \times 6$, $V_4 = 96$
b $V_n = 3^n \times 10$, $V_4 = 810$
c $V_n = 0.5^n \times 1$, $V_4 = 0.0625$
d $V_n = 0.25^n \times 80$, $V_4 = 0.3125$

2 **a** **i** 3000 **ii** 10%
b $V_n = 1.1^n \times 3000$
c \$4831.53

ISBN 978-1-009-11041-9

Cambridge University Press

3 **a** **i** \$2000 **ii** 6%
b $V_n = 1.06^n \times 2000$
c \$2524.95 **d** \$837.04
4 **a** $V_n = 1.125^n \times 8000$
b \$11 390.63 **c** \$3390.63
d \$1265.63
5 **a** **i** \$1200 **ii** 12%
b $V_n = 0.88^n \times 1200$ **c** \$490.41
6 **a** $V_n = 0.905^n \times 38\,500$
b \$23 372.42 **c** \$15 127.58
7 6 years **8** 100 years **9** 4
10 6% **11** \$9223.51 **12** \$32 397.17
13 C **14** E

Exercise 7F

1 **a** 0.40% **b** 2.08% **c** 0.40% **d** 0.14%
e 0.03%
2 **a** 6.48% **b** 5.8% **c** 14.82%
d 9.88% **e** 8.03%
3 **a** $V_0 = 8000, \quad V_{n+1} = 1.048\,V_n$
b $V_0 = 8000, \quad V_{n+1} = 1.012\,V_n$
c $V_0 = 8000, \quad V_{n+1} = 1.004\,V_n$
4 **a** $V_0 = 20\,000, \quad V_{n+1} = 1.005 \times V_n$
b $V_n = 1.005^n \times 20\,000$ **c** \$26 977
5 **a** $V_0 = 8000, \quad V_{n+1} = 1.012 \times V_n$
b $V_n = 1.012^n \times 8000$
c \$9231.16
6 **a** $V_0 = 7600, \quad V_{n+1} = 1.005 \times V_n$
b $V_n = 1.005^n V_n \times 7600$
c \$7791.91 **d** 139 months
7 **a** $V_0 = 3500, \quad V_{n+1} = 1.02 \times V_n$
b \$3788.51
8 **a** 4.68% **b** 4.70% **c** Monthly
9 **a** 8.25% **b** 8.24% **c** Monthly
10 **a** A – 8.62%, B – 8.11%
b A – \$3018.10, B – \$2837.08
c B – this loan will be charged less interest
11 **a** A – 5.43%, B – 5.61%
b A – \$7603, B – \$7860
c B – this investment will earn more interest
12 **a** 6.38% **b** 8.76% **c** 4.91% **d** 13.10%
13 6.78%
14 E **15** C **16** E **17** A

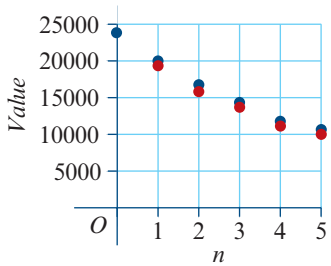
Chapter 7 review

Multiple-choice questions

1 C	**2** E	**3** D	**4** C	**5** A
6 A	**7** D	**8** A	**9** B	**10** A
11 D	**12** C	**13** C	**14** B	**15** A
16 B	**17** C	**18** D	**19** B	**20** C

Written-response questions

1 **a** $V_0 = 20\,000, \; V_{n+1} = V_n + 1880$
b \$29 400
c $W_0 = 20\,000, \; W_{n+1} = 1.094 W_n$
d $W_n = 1.094^n \times 20\,000$
e \$31 341.27
2 \$328.03
3 **a** **i** $V_0 = 22\,500, \; V_{n+1} = V_n - 2700$
ii \$9000
b **i** $V_0 = 22\,500, \; V_{n+1} = 0.84 V_n$
ii \$9409.77
c



4 **a** \$0.20
b Let V_n be the value of the vacuum cleaner after cleaning n offices.
$V_0 = 650, \; V_{n+1} = V_n - 0.20$
c \$250
5 **a** \$6575 **b** \$6722.75 **c** 6.9%
6 **a** $V_0 = 30\,000, \; V_{n+1} = 1.0075\,V_n$
b \$31 142.00
c \$32 814.21 **d** \$34 318.81

ISBN 978-1-009-11041-9

Cambridge University Press

7 $234.57

8 3.5

Chapter 8

Exercise 8A

1 a 2, 5, 11, 23, 47

b 50, 90, 170, 330, 650

c 128, 96, 80, 72, 68

2 a $500 **b** $100 **c** 1.03

d $V_0 = 500, V_{n+1} = 1.03V_n + 100$

3 a $300 000 **b** $50 000

c 1.052

d $V_0 = 300\,000, V_{n+1} = 1.052V_n + 50\,000$

4 a 1.003

b $V_0 = 3500, V_{n+1} = 1.003V_n + 150$

c $3821.48

5 a $V_0 = 1700, V_{n+1} = 1.008V_n + 100$

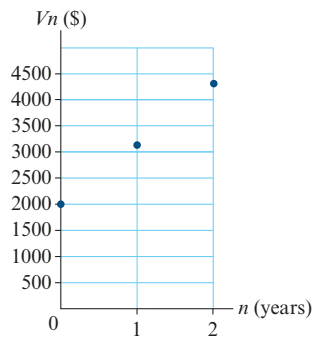
b $2395.38

6 $V_0 = 1500, V_{n+1} = 1.0002V_n + 4$

7 $V_0 = 24000, V_{n+1} = 1.005V_n + 500$, $27 766.81

8 a $2000 **b** $1000 **c** $4412.80

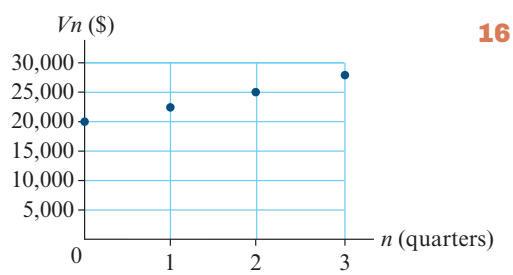
d



9 a $20 000 **b** $2000

c $27 689.06

d



10 8%

11 10%

12 C **13** B **14** C

Exercise 8B

1 $V_0 = 5000, V_{n+1} = 1.054V_n - 1400$

2 a $V_0 = 2000, D = 339$

b $R = 1.005$

c $V_0 = 2000, V_{n+1} = 1.005V_n - 339$

3 a $B_0 = 10\,000, B_{n+1} = 1.03B_n - 2600$

b $5331

4 $V_0 = 3500, V_{n+1} = 1.004V_n - 280$

5 $V_0 = 150\,000, V_{n+1} = 1.0014V_n - 650$

6 a $V_0 = 235\,000, V_{n+1} = 1.0001V_n - 150$

b $234 620.46

7 a $2500 **b** $626 **c** 8%

d $1117.03

8 a $5000 **b** $865 **c** $r = 12\%$

d $3361.85

9 a $V_0 = 20\,000, D = 3375$

b $R = 1.072$

c $V_0 = 20\,000, V_{n+1} = 1.072V_n - 3375$

10 a $V_0 = 750\,000, D = 4100$

b $R = 1.0045$

c $V_0 = 750\,000, V_{n+1} = 1.0045V_n - 4100$

11 a $V_0 = 40\,000, V_{n+1} = 1.015V_n - 10\,380$

b $10 217.70

12 a $5000 **b** $1030 **c** 12%

d $2030.50 **e** $3090

13 a $3052.65 **b** $6000

14 a $1 000 000 **b** $4000 **c** 2.88%

d $996 796.16

15 a $18 400 **b** 6.6%

c $9762.84

16 D **17** E **18** A

ISBN 978-1-009-11041-9

Cambridge University Press

Exercise 8C

1 a $14 000 **b** $1540 **c** $260

d $13 740

e

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	14 000.00
1	1800.00	1540.00	260.00	13 740

f

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	14 000.00
1	1800.00	1540.00	260.00	13 740.00
2	1800.00	1511.40	288.60	13 451.40
3	1800.00	1479.65	320.35	13 131.05

2 a $12 000 **b** 0.005

c $60 **d** $240 **e** $11 760

f

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	12 000.00
1	300.00	60.00	240.00	11 760.00

g

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	12 000.00
1	300.00	60.00	240.00	11 760.00
2	300.00	58.80	241.20	11 518.80
3	300.00	57.59	242.41	11 276.39

3 a $36 000 **b** 0.02

c

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	36 000.00
1	1000.00	720.00	280.00	35 720.00
2	1000	714.40	285.60	35 434.40
3	1000	708.69	291.31	35 143.09

4 a 1%

b $A = 16.75, B = 334.85, C = 342.17$

5 a $4000, $100 **b** 2.5%

c $A = 64.81, B = 505.16, C = 1076.65, D = 26.92, E = 530.73$

6 $A = 421.26, B = 458\,736.22$

7 a i $15.00 **ii** 0.25%

b $A = 12.53, B = 495.47, C = 4517.30$

8 a 1%

b $A = 53.02, B = 153.02, C = 5454.52$

9 D **10** D **11** D

ISBN 978-1-009-11041-9 Cambridge University Press

Exercise 8D

1 $554.16

2 $1262.60

3 $692.58

4 $771.27

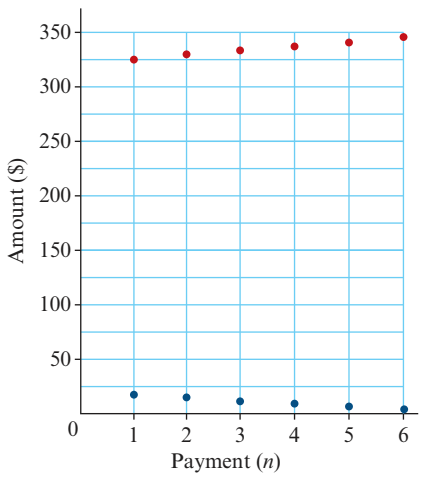
5 **a** 180.53 **b** 1380.53 **c** $624.67

6 **a** $A = 345.69, B = 1.72, C = 343.97$
b $4131.23 **c** $131.23

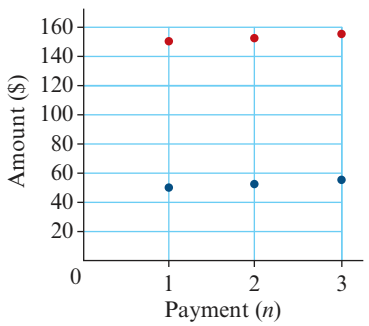
7 **a** $A = 3903.19, B = 34.82, C = 3868.37$
b $31 227.69 **c** $1227.69

8 $12 165.50, $165.50

9



10



11 E **12** C

Exercise 8E

1 **a** Negative **b** Negative
c $123 003.55 **d** $733 636.83

2 **a** $22 327.26 **b** $37 864.50

3 **a** Positive **b** Negative
c $6061.91

4 **a** $21 867.22 **b** $12 095.13

5 **a** $225 788.13 **b** $5452.89

6 **a** $34 093.96 **b** $344.64

7 **a** Negative **b** $28 674
c $6825.74

8 **a** $15 133.81 **b** $1732.49

9 **a** $7627.37 **b** $298.51

10 E **11** E **12** B

Exercise 8F

1 8.39%

2 **a** 2.7%
b **i** $741.19 **ii** 60 months

3 **a** $500
b **i** $6000 **ii** 6491.15
iii $491.15
c 23 months

4 55 payments

5 **a** $197 793.85
b **i** $2158.06 **ii** $774 237.82
iii $534 237.82

6 **a** $617.80 **b** $617.72
c $18 533.92 **d** $1533.92

7 **a** **i** $5253.40 **ii** $210 135.61
b **i** $4175.11 **ii** $4174.54
iii $250 506.03

8 $446 972.41

9 $2591.94

10 **a** 9.24% **b** $2348.50
c 57 months

11 96 quarters

12 **a** $4000 **b** $22 965.28

13 E

14 C

15 B

ISBN 978-1-009-11041-9

Cambridge University Press

Exercise 8G

1 $46 615.21
2 $178 558.60
3 **a** $81 939.67 **b** $67 141.09
4 $416.37
5 $338 807.90
6 **a** $5312.50 **b** $6500.67
c $6495.55 **d** $2 268 945.88
e $1 518 945.88
7 **a** 153 months **b** $229.96
8 **a** $V_0 = 125\,000$, $V_{n+1} = 1.0045V_n - 850$
b $11 966
9 **a** $S_0 = 150\,000$, $S_{n+1} = 1.005 \times S_n$
b $272 909.51 **c** 2.72%
10 **a** $29 697.05 **b** $34 378
11 **a** $3504.76 **b** $104.76
c $18929.68 **d** 58 payments
12 B
13 B
14 E

Exercise 8H

1 $600
2 $350
3 $462
4 $44 550
5 **a** $2775 **b** $32 775
6 **a** $2555 **b** $306 600
c $1 006 600
7 **a** $182 **b** $35 000 **c** $272.48
d $64 892.48
8 $40 000
9 $16 000
10 $28 000
11 2.9%
12 3.6%
13 4.8%
14 **a** $1312 **b** $78 720 **c** $86 400
d $1440 **e** 5.4%
15 C **16** B **17** D

Exercise 8I

1 **a** $9790.50 **b** $642 000
c $642 000
2 **a** $4800 **b** $57 600
3 **a** $2160 **b** It will increase
4 $100 000
5 $50 000
6 **a** $540 000 **b** $380 000
c $451 866.88
7 3%
8 6.25%
9 6.3%
10 **a** $17 850 **b** $350 000
c $M_0 = 350\,000$, $M_{n+1} = 1.00425M_n - 1487.50$
11 **a** $600 **b** $7600 **c** Option B
d $Z_0 = 200\,000$, $Z_{n+1} = 1.038Z_n - 7600$
12 B
13 D

Chapter 8 review

Multiple-choice questions

1 C	**2** D	**3** D	**4** B
5 D	**6** C	**7** D	**8** B
9 C	**10** C	**11** D	**12** E
13 D	**14** E	**15** C	**16** D
17 E	**18** C	**19** E	**20** D

Written-response questions

1 **a** $V_0 = 250\,000$, $V_{n+1} = 1.004V_n - 1800$
b $240 185.96 **c** 56 months

ISBN 978-1-009-11041-9

Cambridge University Press

d **i** \$1000 **ii** \$12 000
iii \$250 000

2 **a** \$781.25 **b** \$147 298.48
c 38 months
d **i** 41 payments **ii** \$3323.07

3 **a** \$656.65 **b** \$13 134
c \$3134

4 **a** 40 months **b** \$320.78
c \$4770.78

5 **a** \$247.04 **b** \$83 713.37

6 **a** \$1 175 244.58 **b** 290 months
c \$3300

Chapter 9

Exercise 9A

1 E **2** C **3** C **4** E **5** C **6** D
7 E **8** D **9** A **10** D **11** D **12** D
13 E **14** E **15** D

Exercise 9B

1 **a** \$8500 **b** \$222.44
c $A_0 = 8500, A_{n+1} = 1.013 \times A_n$
d 5.2% **e** 13 quarters

2 **a** $V_0 = 25\,000$,
$V_1 = 25\,000 - 936 = 24\,064$,
$V_2 = 24\,064 - 936 = 23\,128$,
$V_3 = 23\,128 - 936 = 22\,192$
b 36

3 **a** \$260 000
b $\frac{1170}{260\,000} \times 12 \times 100 = 5.4\%$
c $A = 1156.71$, $B = 993.29$, $C = 256\,053.46$

4 **a** 204 **b** \$29516.73 **c** The first

5 **a** \$4400 **b** $\frac{50}{20} = \$2.50$
c $H_n = 4450 - 2.50n$ **d** \$3950
e 581

6 **a** \$5520 **b** 22 months

Chapter 10

Exercise 10A

1 **a** 2×3 **b** 1×3 **c** 3×2 **d** 3×1
e 3×3

2 **a** 2×3 **b** 4×1 **c** 1×3

3 **a** 12 **b** 15 **c** 28

4 1×12, 12×1, 6×2, 2×6, 4×3, 3×4

5 **a** $\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ **b** $\begin{bmatrix} 3 & 5 \end{bmatrix}$ **c** $\begin{bmatrix} 9 & 8 \\ 1 & 9 \\ 0 & 1 \\ 7 & 5 \end{bmatrix}$

6 **a** square; 2×2; 4 **b** column 3×1; 3
c row; 1×4; 4

7 **a** $\begin{bmatrix} 1 & 3 & 5 \\ 0 & 4 & 7 \\ 0 & 0 & 2 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
c $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}$
d First 3 are symmetric.

8 **a** C, E **b** 3 **c** A **d** B
e 4, 2 **f** 3, 3 **g** 1, 5 **h** 3, 1
i 4, 2 **j** 9 **k** 5 **l** 0
m 1 **n** 0 **o** 4 **p** -1
q 3 **r** 3 **s** 1

9 $\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$ **10** $\begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$ **11** $\begin{bmatrix} -2 & -5 \\ -1 & -4 \\ 0 & -3 \end{bmatrix}$

12 $\begin{bmatrix} 2 & 5 & 10 \end{bmatrix}$ **13** $\begin{bmatrix} 4 & 9 \\ 9 & 16 \end{bmatrix}$

14 **a** $\begin{bmatrix} 1 & 2 \\ 0 & -2 \\ 3 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 4 & -2 \\ -4 & 6 \end{bmatrix}$ **c** $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$
d $\begin{bmatrix} 2 & 3 \end{bmatrix}$

ISBN 978-1-009-11041-9

Cambridge University Press

15 C **16** D **17** B **18** C
19 B

Exercise 10B

1 a $\begin{bmatrix} 4 & 2 & 1 \\ 6 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix}$, 3×3

b $\begin{bmatrix} 6 & 2 & 3 \end{bmatrix}$, 1×3

c $\begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$, 3×1; the total number of computers owned by the three households

2 a $\begin{bmatrix} 24 & 32 & 11 \\ 32 & 34 & 9 \end{bmatrix}$, 2×3;

b $\begin{bmatrix} 24 & 32 & 11 \end{bmatrix}$, 1×3

c $\begin{bmatrix} 24 \\ 32 \end{bmatrix}$, 2×1; the total number of small cars sold by the car dealers

3 a 4×4 **b** $\begin{bmatrix} 430 & 380 & 950 & 900 \end{bmatrix}$; 1×4;The total exports of B

c $\begin{bmatrix} 370 \\ 950 \\ 150 \\ 470 \end{bmatrix}$; 4×1

4 $\begin{bmatrix} 200 & 110 \\ 180 & 117 \\ 135 & 98 \\ 110 & 89 \\ 56 & 53 \\ 28 & 33 \end{bmatrix}$

5 $\begin{bmatrix} 3 & 5 & 8 & 7 & 0 & 2 & 3 & 6 \\ 4 & 2 & 2 & 9 & 0 & 0 & 0 & 9 \end{bmatrix}$

6 $\begin{bmatrix} 21 & 5 & 5 \\ 8 & 2 & 3 \\ 4 & 1 & 1 \\ 14 & 8 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

7 a $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

c $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

8 $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 3 \\ 1 & 3 & 0 \end{bmatrix}$

9 a Girls 3 and 4 are friends.
b Girls 2 and 5 are not friends.
c 3: girl 3 has three friends.
d girl 1, girl 3

10 a i Polar bears eat cod.
ii Nothing eats polar bears.

b
$$W = \begin{matrix} \\ P \\ S \\ C \\ Z \end{matrix} \begin{matrix} \begin{matrix} P & S & C & Z \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

11 C **12** B

Exercise 10C

1 $a = 9, b = 7$

2 a C, F

b A and B, C and F, D and E

c A and B, C and F, D and E

ISBN 978-1-009-11041-9

Cambridge University Press

d **i** $\begin{bmatrix} 4 & 4 \end{bmatrix}$ **ii** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

iii $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ **iv** $\begin{bmatrix} -2 & 2 \end{bmatrix}$

v $\begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix}$ **vi** $\begin{bmatrix} 9 & 3 \end{bmatrix}$

vii $\begin{bmatrix} 0 & 4 & 16 \\ 12 & 8 & 4 \end{bmatrix}$ **viii** $\begin{bmatrix} 0 & 4 & 16 \\ 12 & 8 & 4 \end{bmatrix}$

ix $\begin{bmatrix} -2 & 10 \end{bmatrix}$ **x** not defined

4 **a** $\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}$ **b** $\begin{bmatrix} -3 & -1 \\ 3 & 1 \end{bmatrix}$

c $\begin{bmatrix} 9 & 8 \\ 6 & 7 \end{bmatrix}$ **d** $\begin{bmatrix} 0 & 0 \end{bmatrix}$ **e** $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

f $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ **g** $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ **h** not defined

5 **a** $\begin{bmatrix} -2.2 & 1.1 \\ 7.7 & 4.4 \end{bmatrix}$ **b** $\begin{bmatrix} -0.2 & -13.8 \\ 1 & -3.7 \end{bmatrix}$

c $\begin{bmatrix} 5 & 8 & 7 \\ 16 & 0 & 3 \\ -1 & 5 & 4 \end{bmatrix}$ **d** $\begin{bmatrix} 0.6 & 2 & 1 & 3.2 \\ 1 & 0 & -0.6 & 2 \end{bmatrix}$

6 $x = 2, y = 4, z = 6, w = -4$

7 **a**

$$A = \begin{bmatrix} 2.4 \\ 3.5 \\ 1.6 \end{bmatrix} \quad B = \begin{bmatrix} 2.8 \\ 3.4 \\ 1.8 \end{bmatrix} \quad C = \begin{bmatrix} 2.5 \\ 2.6 \\ 1.7 \end{bmatrix} \quad D = \begin{bmatrix} 3.4 \\ 4.1 \\ 2.1 \end{bmatrix}$$

b $\begin{bmatrix} 11.1 \\ 13.6 \\ 7.2 \end{bmatrix}$; the total (yearly) DVD sales for each store

8 **a**

$$A = \begin{bmatrix} 16 & 104 & 86 \\ 75 & 34 & 94 \end{bmatrix} \quad B = \begin{bmatrix} 24 & 124 & 100 \\ 70 & 41 & 96 \end{bmatrix}$$

b $C = \begin{bmatrix} 40 & 228 & 186 \\ 145 & 75 & 190 \end{bmatrix}$; the total number of females and males enrolled in each of the three programs for the two years

c $D = \begin{bmatrix} 8 & 20 & 14 \\ -5 & 7 & 2 \end{bmatrix}$; the increase in the number of females and males enrolled in each of the three programs for the two years; a decrease in the number of men enrolled in weights classes

d $E = \begin{bmatrix} 48 & 248 & 200 \\ 140 & 82 & 192 \end{bmatrix}$

9 C **10** C **11** E

Exercise 10D

1 **a** i, ii, iv, v, vi, vii

b **i** [6] **ii** [2] **iii** $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ **iv** $\begin{bmatrix} -3 & 7 \end{bmatrix}$

c **i** [6] **ii** $\begin{bmatrix} 2 & 0 & -2 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ **iii** [0]

iv undefined

2 **a** [0] **b** [1] **c** [3] **d** $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

e $\begin{bmatrix} 5 & 5 \\ 1 & 2 \end{bmatrix}$ **f** $\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$

3 **a** $\begin{bmatrix} 1 & 2 & 3 \\ -3 & -6 & -9 \\ 5 & 10 & 15 \end{bmatrix}$ **b** $\begin{bmatrix} 6 & 7 \\ 19 & 18 \end{bmatrix}$

ISBN 978-1-009-11041-9

Cambridge University Press

c $\begin{bmatrix} 4 & 0 \\ 2 & -2 \\ 1 & 4 \end{bmatrix}$ **d** $\begin{bmatrix} -5 & 15 & 9 \\ -2 & 8 & 4 \\ -5 & 8 & 7 \end{bmatrix}$

4 a $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ post-multiply by this matrix.

b $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ pre-multiply by this matrix.

5 $\begin{bmatrix} 7 & 1 & 2 \\ 1 & 2 & 2 \\ 8 & 1 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 13 \end{bmatrix}$.

6 $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 9 & 0 & 2 \\ 1 & 7 & 3 \\ 8 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 18 & 10 & 9 \end{bmatrix}$

7 a $\begin{bmatrix} 22 \\ 12 \\ 18 \\ 12 \\ 30 \end{bmatrix}$ **b** $\begin{bmatrix} 8 & 10 & 6 & 14 & 11 \end{bmatrix}$

8 $\begin{bmatrix} 9 \\ 7 \\ 6 \\ 4 \\ 3 \\ 1 \end{bmatrix}$ **9** $\begin{bmatrix} 3000 \\ 2800 \\ 2600 \\ 2200 \end{bmatrix}$

10 a 2×3

b i $\begin{bmatrix} 184.50 \\ 236 \end{bmatrix}$

ii the total revenue from selling products A, B and C at Energy and Nourishing respectively

c number of columns in $P \neq$ number of rows in Q

11 $XY = \begin{bmatrix} 110\,000 \\ 116\,000 \\ 154\,000 \\ 58\,000 \end{bmatrix}$ It gives the total sales of each of the dealers.

12 a $\begin{bmatrix} 29 \\ 8.50 \end{bmatrix}$, John took 29 minutes to eat food costing \$8.50

b $\begin{bmatrix} 29 & 22 & 12 \\ 8.50 & 8.00 & 3.00 \end{bmatrix}$, John's friends took 22 and 12 minutes to eat food costing \$8.00 and \$3.00 respectively

13 a $\begin{bmatrix} 79 & 78 & 80 \\ 80 & 78 & 82 \end{bmatrix}$ **b** $\begin{bmatrix} 0.2 \\ 0.3 \\ 0.5 \end{bmatrix}$

c Semester 1: 79.2; Semester 2: 80.4

d Semester 1: 83.8; Semester 2: 75.2

e No, total score is 318.6

f 3 marks

14 $\begin{bmatrix} 5 & 5 \\ 5 & 10 \end{bmatrix}, \begin{bmatrix} 15 & 20 \\ 20 & 35 \end{bmatrix}, \begin{bmatrix} 50 & 75 \\ 75 & 125 \end{bmatrix}, \begin{bmatrix} 2250 & 3625 \\ 3625 & 5875 \end{bmatrix}$

15 $\begin{bmatrix} 5 & 7 \\ 7 & 26 \end{bmatrix}, \begin{bmatrix} 2 & 19 \\ 19 & 59 \end{bmatrix}, \begin{bmatrix} 17 & 40 \\ 40 & 137 \end{bmatrix}, \begin{bmatrix} 23 & 97 \\ 97 & 314 \end{bmatrix}$

16 $\begin{bmatrix} 24 & 30 & 36 \\ 38 & 59 & 64 \\ 33 & 54 & 51 \end{bmatrix}$

17 a $\begin{bmatrix} -1 & 5 \\ 5 & 2 \end{bmatrix}$ **b** $\begin{bmatrix} -3 & 8 \\ 6 & -3 \end{bmatrix}$

c $\begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix}$ **d** $\begin{bmatrix} 16 & 2 \\ 2 & 39 \end{bmatrix}$

e $\begin{bmatrix} 29 & -5 \\ -5 & 13 \end{bmatrix}$

18 A **19** C **20** D

ISBN 978-1-009-11041-9

Cambridge University Press

Exercise 10E

1 a i $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ **ii** $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

iii $\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$

b $AI = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$;

$IA = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$

$\therefore AI = IA = A$

c

$CI = \begin{bmatrix}1 & 2 & 0\\3 & 1 & 0\\0 & 1 & 2\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 2 & 0\\3 & 1 & 0\\0 & 1 & 2\end{bmatrix}$;

$IC = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\3 & 1 & 0\\0 & 1 & 2\end{bmatrix} = \begin{bmatrix}1 & 2 & 0\\3 & 1 & 0\\0 & 1 & 2\end{bmatrix}$

$CI = IC = C$

3 a 3 **b** −3 **c** 0 **d** −8

4 a $\begin{bmatrix}\frac{10}{11} & -\frac{2}{3}\\0 & \frac{1}{3}\end{bmatrix}$ **b** $\begin{bmatrix}\frac{20}{9} & \frac{1}{18}\\\frac{-50}{9} & \frac{1}{9}\end{bmatrix}$

c Matrix has no inverse, det $(D) = 0$

d $\begin{bmatrix}\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2}\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$

5 a $\begin{bmatrix}-9 & -3\\1 & 3\end{bmatrix}$ **b** $\begin{bmatrix}-28 & -15\\39 & 22\end{bmatrix}$

c $\begin{bmatrix}-14 & 9\\-9 & 8\end{bmatrix}$ **d** $\begin{bmatrix}1\\0\end{bmatrix}$ **e** $\begin{bmatrix}3.5\\2.5\\-1.5\end{bmatrix}$

f $\begin{bmatrix}-8\\10\end{bmatrix}$

6 $\begin{bmatrix}-1 & 2\\-3 & 5\end{bmatrix}$

7 a $X = A^{-1}C$

b $X = (AB)^{-1}C = B^{-1}A^{-1}C$

c $X = A^{-1}CB^{-1}$ **d** $X = A^{-1}C - B$

e $X = A^{-1}(C - B)$

f $X = (A - B)A^{-1} = I - BA^{-1}$

8 $x = -5000, y = 15\,000, z = 0$

9

Spray	P	Q	R
Barrels	$\frac{8}{13}$	$\frac{46}{39}$	$\frac{12}{13}$

10 a $\begin{bmatrix}0.1 & 0.25 & -0.4\\0.3 & -0.75 & 0.8\\-0.2 & 0.5 & -0.2\end{bmatrix}$

b

Product	P	Q	R
Number per day	13.5	0.5	13

11 Brad 20; Flynn 10; Lina 15

12 A **13** E **14** D

Exercise 10F

1 B only

2 a $X = \begin{bmatrix}\text{H} & \text{U} & \text{T} & \text{S}\end{bmatrix}$ **b** $n = 4$

3 $\begin{bmatrix}0 & 0 & 1 & 0\\0 & 0 & 0 & 1\\1 & 0 & 0 & 0\\0 & 1 & 0 & 0\end{bmatrix}$ **4** $\begin{bmatrix}X\\W\\Z\\Y\end{bmatrix}$

5 a $C = \begin{bmatrix}0 & 1 & 1\\1 & 0 & 1\\1 & 1 & 0\end{bmatrix}$ **b** $C^2 = \begin{bmatrix}2 & 1 & 1\\1 & 2 & 1\\1 & 1 & 2\end{bmatrix}$

c 2

6 a There is no direct communication link between the towers.

ISBN 978-1-009-11041-9

Cambridge University Press

b $T1$ and $T3$

c 1, 0

d There is a 2-step communication link between $T3$ and $T1$.

e 6

f
$$T = \begin{array}{c} \\ T1 \\ T2 \\ T3 \\ T4 \end{array} \begin{array}{c} \begin{array}{cccc} T1 & T2 & T3 & T4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

g $T1$ and $T4$

7
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

8 D **9** D

Exercise 10G

1 a
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

D, A, B, C

b
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

D, A, B, C

2 a

losers
$$\begin{array}{cc} & \\ & \\ \text{winners} & \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$A : 4,\ B : 1,\ C : 0,\ D : 2,\ E : 2$

A; D and E equal; B; C.

b
$$D^2 = \begin{bmatrix} 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \end{bmatrix}$$

c
$$T = \begin{bmatrix} 0 & 3 & 3 & 2 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 2 & 2 & 1 & 0 \end{bmatrix} \begin{array}{c} 9 \\ 1 \\ 0 \\ 3 \\ 5 \end{array}$$

The tie can be broken using two-step dominances to give the ranking A, E, D, B, C.

3 a
$$D = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \text{Score} \\ 2 \\ 3 \\ 1 \\ 1 \\ 3 \end{array}$$

b
$$D^2 = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 2 & 3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 & 0 \end{bmatrix}$$

ISBN 978-1-009-11041-9

Cambridge University Press

c

$$D + D^2 = \begin{matrix} \\ A \\ B \\ C \\ D \\ E \end{matrix} \begin{matrix} \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 2 & 0 \\ 2 & 0 & 3 & 3 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 2 & 3 & 0 \end{bmatrix} \end{matrix} \begin{matrix} \text{Score} \\ 4 \\ 9 \\ 2 \\ 4 \\ 7 \end{matrix}$$

The matrix $D + D^2$ gives the following ranking:

Rank	Player	Score
First	Bea	9
Second	Eve	7
Equal third	Ann and Deb	4
Fifth	Cat	2

4 a 10

b
- Ash defeats Carl and Dot
- Ben defeats Ash, Carl and Dot
- Carl defeats Dot
- Dot defeats Elle
- Elle defeats Ash, Ben and Carl

c Ben = Elle, Ash, Carl = Dot

5 a $\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$

b E, B, $A = C$, D

6 a $\begin{bmatrix} 0 & 2 & 2 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$ **b** A

7 a $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ **b** A, B, D, C

8 A, B, D, C

9 E, B, $A = C$, D

10 C **11** A **12** D

Chapter 10 review

Multiple-choice questions

1 C	**2** D	**3** B	**4** D
5 D	**6** A	**7** B	**8** C
9 D	**10** E	**11** A	**12** B
13 D	**14** E	**15** A	**16** D
17 D	**18** E	**19** D	**20** C
21 A	**22** C	**23** C	**24** C
25 A	**26** C	**27** C	**28** C
29 E			

Written-response questions

1 a $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 2 & 0 \\ 1 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

c $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

2 a 2×1 **b** 1×2

c Yes; number of columns in C equals the number of rows in J.

d $[162.41]$; $5 \times 30.45 + 4 \times 2.54 = 162.41$

e $\begin{bmatrix} 172.57 \\ 185.24 \end{bmatrix}$

3 a 456 **b** 2×2

c $B = \begin{bmatrix} 354 & 987 \\ 314 & 586 \end{bmatrix}$

d $C = \begin{bmatrix} 688 & 1863 \\ 527 & 1042 \end{bmatrix}$; the total number of books of each type in the two stores

e **i** 2×1

ii $\begin{bmatrix} 31\,236 \\ 18\,021 \end{bmatrix}$

iii total value of fiction and non-fiction books at bookshop 1

f $2A = \begin{bmatrix} 668 & 1752 \\ 426 & 912 \end{bmatrix}$

ISBN 978-1-009-11041-9

Cambridge University Press

4 a 1×5

b i $R = \begin{bmatrix} 90 & 135 & 165 & 150 & 60 \\ 48 & 72 & 63 & 88 & 32 \end{bmatrix}$

ii the number of students expected to get a C in Mathematics .

c i $F = \begin{matrix} \\ \end{matrix} \begin{matrix} M & P \end{matrix}$
$\begin{bmatrix} 220 & 197 \end{bmatrix}$

ii $FN = \begin{bmatrix} 220 & 197 \end{bmatrix} \begin{bmatrix} 600 \\ 320 \end{bmatrix} = [195\,040]$

The total fees paid are \$195 040.

5 a $N = \begin{bmatrix} 8 & 6 & 1 \end{bmatrix}$ **b** $NG = [575]$

c total number of points scored by Daniel

6 a 80 tonnes **b** 100 tonnes

c \$186, 000

d i 3×1

ii The price per tonne of each of the minerals

iii $\begin{bmatrix} 1000 \\ 700 \\ 300 \end{bmatrix}$

Chapter 11

Exercise 11A

1 a
$$\begin{matrix} \\ A \\ B \end{matrix} \begin{matrix} \begin{matrix} A & B \end{matrix} \\ \begin{bmatrix} 0.40 & 0.55 \\ 0.60 & 0.45 \end{bmatrix} \end{matrix}$$

b
$$\begin{matrix} \\ X \\ Y \end{matrix} \begin{matrix} \begin{matrix} X & Y \end{matrix} \\ \begin{bmatrix} 0.70 & 0.25 \\ 0.30 & 0.75 \end{bmatrix} \end{matrix}$$

c
$$\begin{matrix} \\ X \\ Y \\ Z \end{matrix} \begin{matrix} \begin{matrix} X & Y & Z \end{matrix} \\ \begin{bmatrix} 0.6 & 0.15 & 0.22 \\ 0.1 & 0.7 & 0.23 \\ 0.3 & 0.15 & 0.55 \end{bmatrix} \end{matrix}$$

d
$$\begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{matrix} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 0.45 & 0.35 & 0.15 \\ 0.25 & 0.45 & 0.20 \\ 0.30 & 0.20 & 0.65 \end{bmatrix} \end{matrix}$$

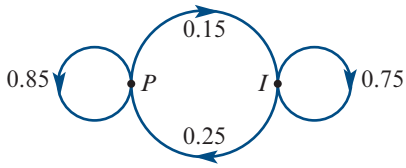
2
$$\begin{matrix} \\ O \\ B \end{matrix} \begin{matrix} \begin{matrix} O & B \end{matrix} \\ \begin{bmatrix} 0.96 & 0.98 \\ 0.04 & 0.02 \end{bmatrix} \end{matrix}$$

3
$$\begin{matrix} \\ F \\ P \end{matrix} \begin{matrix} \begin{matrix} F & P \end{matrix} \\ \begin{bmatrix} 0.80 & 0.14 \\ 0.20 & 0.86 \end{bmatrix} \end{matrix}$$

4 B

Exercise 11B

1 a $T = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix}$



b $0.85 \times 80 = 68$ **c** $0.25 \times 60 = 15$

d $0.15 \times 120 + 0.75 \times 40 = 48$

2 a i 10% **ii** 80% **iii** 10%

b i 680 **ii** 85

c i 1150 **ii** 0 **iii** 0

d All (100%) of the sea birds who nest at site A this year will nest at site A next year.

3 a i 76440 **ii** 7560

b i 5500 **ii** 1210 **iii** 266

4 a i 18 **ii** 6 **iii** 6

b i 84 **ii** 66 **iii** 30

c $\begin{bmatrix} 84 \\ 66 \\ 30 \end{bmatrix}$

ISBN 978-1-009-11041-9

Cambridge University Press

d i 21 ii 3 iii 6

e $\begin{bmatrix}180\end{bmatrix}$

5 B **6** E

Exercise 11C

1 a $S_1 = \begin{bmatrix}380\\220\end{bmatrix}$ **b** $S_2 = \begin{bmatrix}398\\202\end{bmatrix}$

c $S_3 = \begin{bmatrix}399.8\\200.2\end{bmatrix}$

2 a $S_5 = \begin{bmatrix}399.998\\200.02\end{bmatrix}$ **b** $S_7 = \begin{bmatrix}400\\200\end{bmatrix}$

c $S_{12} = \begin{bmatrix}400\\200\end{bmatrix}$

3 a $S_4 = \begin{bmatrix}5460\\4780\end{bmatrix}$ **b** $S_3 = \begin{bmatrix}5456\\4784\end{bmatrix}$

4 a i $S_1 = \begin{bmatrix}130\\170\end{bmatrix}$ ii $S_2 = \begin{bmatrix}151\\149\end{bmatrix}$

iii $S_3 = \begin{bmatrix}165.7\\134.3\end{bmatrix}$

b $T^5 = \begin{bmatrix}0.72269 & 0.55462\\0.27731 & 0.44538\end{bmatrix}$

c i $S_2 = \begin{bmatrix}151\\149\end{bmatrix}$ ii $S_3 = \begin{bmatrix}165.7\\134.3\end{bmatrix}$

iii $S_7 = \begin{bmatrix}191.8\\108.2\end{bmatrix}$

d See solutions

5 a i $S_1 = \begin{bmatrix}180\\130\\290\end{bmatrix}$ ii $S_2 = \begin{bmatrix}207\\136\\257\end{bmatrix}$

iii $S_3 \begin{bmatrix}225\\132.1\\242.9\end{bmatrix}$

b i $S_2 \begin{bmatrix}207\\136\\257\end{bmatrix}$ ii $S_3 = \begin{bmatrix}225\\132.1\\242.9\end{bmatrix}$

iii $S_7 = \begin{bmatrix}224.9\\129.7\\225.4\end{bmatrix}$

c See solutions

6 a $T = \begin{bmatrix}0.80 & 0.25\\0.20 & 0.75\end{bmatrix}$

b $S_0 = \begin{bmatrix}400\\400\end{bmatrix}$

c $S_1 = \begin{bmatrix}420\\380\end{bmatrix}$, 420 to Jill's and 380 to Pete's

d $S_5 = \begin{bmatrix}442.2\\357.8\end{bmatrix}$, 442 to Jill's and 358 to Pete's

e steady state solution: $S_s = \begin{bmatrix}444.4\\355.6\end{bmatrix}$, 444 to Jill's and 356 to Pete's

7 a $T = \begin{bmatrix}0.90 & 0.60\\0.10 & 0.40\end{bmatrix}$

b $S_0 = \begin{bmatrix}1500\\500\end{bmatrix}$

c $S_1 = \begin{bmatrix}1650\\350\end{bmatrix}$, 1650 are happy and 350 are unhappy

d $S_4 = \begin{bmatrix}1712.55\\287.45\end{bmatrix}$, 1713 are happy and 287 are unhappy

e steady state solution: $S = \begin{bmatrix}1714.3\\285.7\end{bmatrix}$, 1714 are happy and 286 are unhappy

8 a $S_0 = \begin{bmatrix}1200\\600\\200\end{bmatrix}$

ISBN 978-1-009-11041-9

Cambridge University Press

b $S_1 = \begin{bmatrix} 1270 \\ 440 \\ 290 \end{bmatrix}$, 1270 are happy

c $S_5 = \begin{bmatrix} 1310.33 \\ 429.82 \\ 259.85 \end{bmatrix}$, 1310 are happy

d steady state solution: $\begin{bmatrix} 1311.7 \\ 429.1 \\ 259.1 \end{bmatrix}$, 1312 are happy

9 A **10** E **11** A

Exercise 11D

1 a i $\begin{bmatrix} 80 \\ 120 \end{bmatrix}$ **ii** $\begin{bmatrix} 68.8 \\ 131.2 \end{bmatrix}$

b i
$$S_1 = TS_0 + R$$
$$= \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 100 \\ 100 \end{bmatrix} + \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$
$$= \begin{bmatrix} 80 \\ 120 \end{bmatrix} + \begin{bmatrix} 10 \\ 5 \end{bmatrix} = \begin{bmatrix} 90 \\ 125 \end{bmatrix}$$

ii
$$S_2 = TS_1 + R$$
$$= \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 90 \\ 125 \end{bmatrix} + \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$
$$= \begin{bmatrix} 79 \\ 136 \end{bmatrix} + \begin{bmatrix} 10 \\ 5 \end{bmatrix} = \begin{bmatrix} 89 \\ 141 \end{bmatrix}$$

c i
$$S_1 = TS_0 - B$$
$$= \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 100 \\ 100 \end{bmatrix} - \begin{bmatrix} -20 \\ 20 \end{bmatrix}$$
$$= \begin{bmatrix} 80 \\ 120 \end{bmatrix} - \begin{bmatrix} -20 \\ 20 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$$

ii
$$S_2 = TS_1 - B$$
$$= \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 100 \\ 100 \end{bmatrix} - \begin{bmatrix} -20 \\ 20 \end{bmatrix}$$
$$= \begin{bmatrix} 80 \\ 120 \end{bmatrix} - \begin{bmatrix} -20 \\ 20 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$$

2 a i $S_1 = \begin{bmatrix} 11500 \\ 8500 \\ 10000 \end{bmatrix}$ **ii** 7300

b *A: 30 000, B: 0, C: 0*; While the sea birds move between nesting sites each year, the '1' in the transition matrix indicates that, once a sea bird nests at site *A*, it continues to nest at this site. Meanwhile, some of the birds who nest at sites *B* and *C* each year will move to site *A* until, in the long term, all birds are nesting at site *A*.

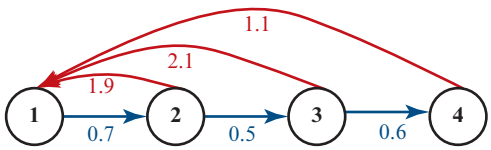
c i $\begin{bmatrix} 9500 \\ 9500 \\ 11000 \end{bmatrix}$ **ii** $\begin{bmatrix} 9000 \\ 9150 \\ 11850 \end{bmatrix}$ **iii** $\begin{bmatrix} 8507.5 \\ 8912.5 \\ 12\,580 \end{bmatrix}$

3 C **4** B

Exercise 11E

1 a i 1.9 **ii** 0.6

b



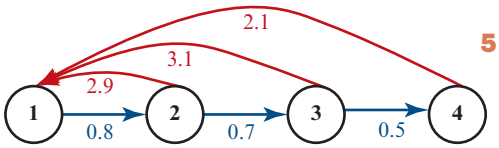
c i $\begin{bmatrix} 510 \\ 70 \\ 50 \\ 60 \end{bmatrix}$ **ii** $\begin{bmatrix} 784.8 \\ 212.8 \\ 178.5 \\ 21 \end{bmatrix}$ **iii** $\begin{bmatrix} 208\,276 \\ 103\,876 \\ 36\,984 \\ 15\,815.8 \end{bmatrix}$

ISBN 978-1-009-11041-9

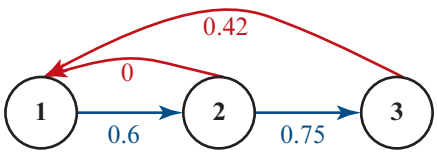
Cambridge University Press

d $\begin{bmatrix} 3483 \\ 1829 \\ 600 \\ 291 \end{bmatrix}$

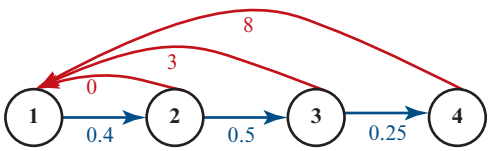
2 a



b



c

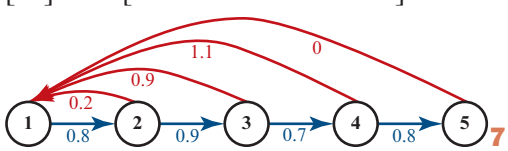


3 a $\begin{bmatrix} 0 & 1.3 & 2.4 \\ 0.7 & 0 & 0 \\ 0 & 0.6 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 2.3 & 3 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix}$

c $\begin{bmatrix} 0 & 1.4 & 2.6 & 0.6 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.05 & 0 \end{bmatrix}$

4 a $\begin{bmatrix} 15 \\ 20 \\ 30 \\ 15 \\ 10 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 0.2 & 0.9 & 1.1 & 0 \\ 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0 \end{bmatrix}$

c



d i $\begin{bmatrix} 48 \\ 12 \\ 18 \\ 21 \\ 12 \end{bmatrix}$ **ii** $\begin{bmatrix} 58 \\ 37 \\ 22 \\ 21 \\ 19 \end{bmatrix}$

e 37 **f** 90

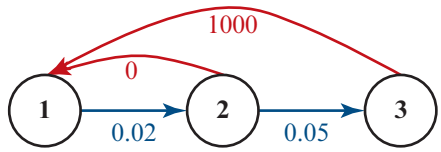
g i 111 **ii** 158 **iii** 235

h i 99 **ii** 145 **iii** 233

It appear that the population rate of increase approaches 10% per year. Further investigation confirms this.

5 a $\begin{bmatrix} 0 \\ 0 \\ 50 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 0 & 1000 \\ 0.02 & 0 & 0 \\ 0 & 0.05 & 0 \end{bmatrix}$

c



d i $\begin{bmatrix} 50\,000 \\ 0 \\ 0 \end{bmatrix}$ **ii** $\begin{bmatrix} 0 \\ 0 \\ 50 \end{bmatrix}$

iii $\begin{bmatrix} 50\,000 \\ 0 \\ 0 \end{bmatrix}$

e i $\begin{bmatrix} 50\,000 \\ 1 \\ 5 \end{bmatrix}$ **ii** $\begin{bmatrix} 50 \\ 100 \\ 50 \end{bmatrix}$

iii $\begin{bmatrix} 50\,000 \\ 1 \\ 5 \end{bmatrix}$

6 a i $\begin{bmatrix} 269 \\ 127 \\ 30 \end{bmatrix}$ **ii** $\begin{bmatrix} 356 \\ 168 \\ 40 \end{bmatrix}$ **iii** $\begin{bmatrix} 622 \\ 294 \\ 70 \end{bmatrix}$

i 427 **ii** 565 **iii** 986

b At this stage the rate of increase is approximately 5.7%

7 a i $\begin{bmatrix} 1400 \\ 240 \\ 100 \end{bmatrix}$ **ii** $\begin{bmatrix} 700 \\ 840 \\ 60 \end{bmatrix}$ **iii** $\begin{bmatrix} 2160 \\ 420 \\ 210 \end{bmatrix}$

ISBN 978-1-009-11041-9

Cambridge University Press

b **i** $\begin{bmatrix}976\\460\\91\end{bmatrix}$ **ii** $\begin{bmatrix}1241\\586\\115\end{bmatrix}$ **iii** $\begin{bmatrix}1579\\745\\146\end{bmatrix}$

c **i** $\begin{bmatrix}974\\460\\90\end{bmatrix}$ **ii** $\begin{bmatrix}1237\\584\\115\end{bmatrix}$ **iii** $\begin{bmatrix}1571\\741\\145\end{bmatrix}$

The population appears to be iincreasing at a rate of 27%

d

8 **a** **i** $\begin{bmatrix}0\\300\\0\end{bmatrix}$ **ii** $\begin{bmatrix}0\\0\\100\end{bmatrix}$ **iii** $\begin{bmatrix}200\\0\\0\end{bmatrix}$

b Population cycles through three states

c **i** Population decreases by 50% every three time periods

ii Population increases by 25% every three time periods

9 **a** $\begin{bmatrix}2800\\200\\200\\40\end{bmatrix}$ **b** $\begin{bmatrix}1080\\1400\\100\\20\end{bmatrix}$ **c** $\begin{bmatrix}4440\\540\\700\\10\end{bmatrix}$

10 **a** **i** Every 3 years, the population returns to 1000 newborns

ii Every 3 years, the population increases by 50% and returns to only newborns

iii Every 3 years, the population decreases by 40% and returns to only newborns

b Long-term growth rate 1.37; long-term ratio of age groups 818 : 130 : 52

11 D **12** D **13** A

Chapter 11 review

Multiple-choice questions

1 B **2** A **3** C **4** B **5** B
6 C **7** C **8** A **9** B **10** B
11 B **12** C **13** E

Written-response questions

1 **a** **i** $\begin{bmatrix}504\\244\end{bmatrix}$ **ii** 748

b $\begin{bmatrix}517\\257\end{bmatrix}$

c Diiscoo attendance is expected to increase to around 520 and stay at that level.

d Diiscoo attendance is expected to decrease to around 20 and stay at that level.

2

	Y	*M*	*O*
Y	0.1	0.2	0.4
M	0.9	0	0
O	0	0.8	0.6

3 **a**

$$\begin{bmatrix}0 & 0.1 & 0.9 & 0.2 & 0 & 0 & 0 & 0\\0.98 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\0 & 0.95 & 0 & 0 & 0 & 0 & 0 & 0\\0 & 0 & 0.95 & 0 & 0 & 0 & 0 & 0\\0 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0\\0 & 0 & 0 & 0 & 0.7 & 0 & 0 & 0\\0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0\\0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0\end{bmatrix}$$

b $S_2 = \begin{bmatrix}104.5\\107.8\\0\\90.25\\85.5\\31.5\\0\\0\end{bmatrix}$, $S_3 = \begin{bmatrix}28.83\\102.41\\102.41\\0\\81.225\\59.85\\15.75\\0\end{bmatrix}$

ISBN 978-1-009-11041-9

Cambridge University Press

c 1.035

4 a $\begin{bmatrix} 0.2 & 0.5 & 0.6 & 0.4 \\ 0.7 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.7 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} 980 \\ 692 \\ 488 \\ 344 \end{bmatrix}$

c $\begin{bmatrix} 972 \\ 680 \\ 484 \\ 342 \end{bmatrix}$, 0.992 **d** $\begin{bmatrix} 7467 \\ 4762 \\ 3038 \\ 1938 \end{bmatrix}$

e Day 56

Chapter 12

Exercise 12A

1 E **2** D **3** A **4** A **5** B **6** C
7 D **8** C **9** C **10** D **11** B **12** A
13 A **14** B **15** C **16** C **17** C **18** E
19 D **20** C **21** A **22** D **23** B **24** D
25 A

Exercise 12B

1 a 3×1 **b** 1×3

c HC; because the number of columns in H equals the number of rows in C

d i [696.72]

ii the number of Australian dollars ($696.72) that you would receive by converting your foreign currency into Australian dollars; $HC = 102 \times 1.316 + 262 \times 1.818 + 516 \times 0.167$

e $\begin{bmatrix} 566.21 \\ 137.46 \\ 647.21 \end{bmatrix}$

2 a $\begin{bmatrix} 0.67 & 0.28 \\ 0.33 & 0.72 \end{bmatrix}$

b $S_0 = \begin{bmatrix} 4000 \\ 6000 \end{bmatrix}$

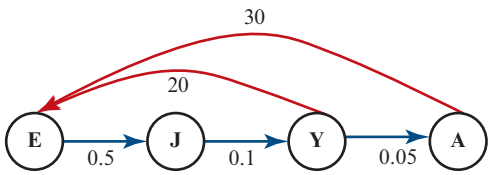
c $S_1 = \begin{bmatrix} 4360 \\ 5640 \end{bmatrix}$; 4360 fish in Lake Blue and 5640 fish in Lake Green

d $S_3 = \begin{bmatrix} 4555.156 \\ 5444.844 \end{bmatrix}$; 4555 fish in Lake Blue and 5445 fish in Lake Green

e $S_s = \begin{bmatrix} 4590.2 \\ 5409.8 \end{bmatrix}$; 4590 fish in Lake Blue and 5410 fish in Lake Green

3

a



b 50 **c** 920 **d** 5 weeks

e 5%

f i 1667 **ii** 1706

g 2%

4a i $\begin{bmatrix} 100 \\ 50 \end{bmatrix}$ **ii** 95

b $S_n = T^n S_1$ **c** 5 weeks

d 90

5a i 3×1 **ii** $k = 1.2$

b i A and B and A and C

ii $D \to B \to A \to C$

iii $C \to A \to B$ and $C \to D \to B$

c i $S_1 = \begin{bmatrix} 385 \\ 75 \\ 140 \end{bmatrix}$ **ii** 230

iii 70.56%

ISBN 978-1-009-11041-9

Cambridge University Press

d $B = \begin{bmatrix} 20 \\ 15 \\ 25 \end{bmatrix}$

6a 4×2

b **i** 435 **ii** 72.4%

c $L = \begin{bmatrix} 60 \\ 120 \\ 50 \\ 85 \end{bmatrix}$ $Q \times L = \begin{bmatrix} 5115 \end{bmatrix}$

7a $S_1 = \begin{bmatrix} 39 \\ 36 \\ 21 \\ 24 \end{bmatrix}$ $S_2 = \begin{bmatrix} 41.4 \\ 35.1 \\ 20.1 \\ 23.4 \end{bmatrix}$

b $S_{10} = \begin{bmatrix} 42.76 \\ 34.64 \\ 19.72 \\ 22.87 \end{bmatrix}$

(P) 43, (S) 35, (D) 20, (W) 23

c 3

d 36

e 3.9

f 47.4

8a 550

b The number of sandwiches sold in week 3.

c Hamburgers \$15, fish and chips \$14 sandwiches \$12

d $L = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$

Chapter 13

Exercise 13A

1 a **i** 3 **ii** 2 **iii** 1

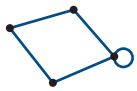
b 14

c town D and town H

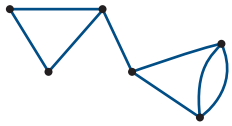
d

2 a

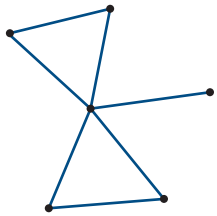
b



c

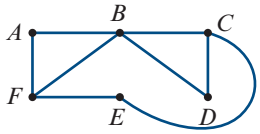


d

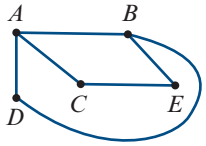


3 a i **b** ii **c** ii

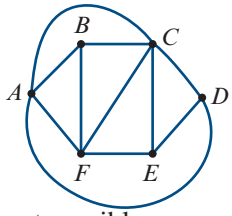
4 a



b



c



d not possible

5 a **i** $v = 8,\ f = 6,\ e = 12$

b **i** $v = 6,\ f = 8,\ e = 12$

c **i** $v = 7,\ f = 7,\ e = 12$

d **i** $v = 5,\ f = 3,\ e = 6$

e **i** $v = 5,\ f = 6,\ e = 9$

f **i** $v = 6,\ f = 4,\ e = 6$

6 a 4 **b** 12 **c** 19

7 7 **8** 14 **9** 15

10 C **11** E **12** C **13** B

14 E **15** C

ISBN 978-1-009-11041-9

Cambridge University Press

Exercise 13B

1 a

	A	B	C	D
A	0	1	1	0
B	1	0	1	1
C	1	1	0	0
D	0	1	0	0

b

	A	B	C	D
A	0	1	1	0
B	1	0	0	1
C	1	0	0	1
D	0	1	1	0

c

	A	B	C	D
A	0	1	0	0
B	1	0	0	0
C	0	0	0	1
D	0	0	1	0

d

	A	B	C	D
A	0	1	1	1
B	1	0	1	1
C	1	1	0	1
D	1	1	1	0

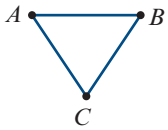
e

	A	B	C	D	E	F
A	0	1	1	0	0	0
B	1	0	0	1	0	0
C	1	0	0	1	0	0
D	0	1	1	0	0	0
E	0	0	0	0	0	1
F	0	0	0	0	1	0

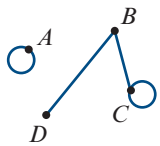
f

	A	B	C	D
A	0	0	0	0
B	0	0	0	1
C	0	0	0	2
D	0	1	2	0

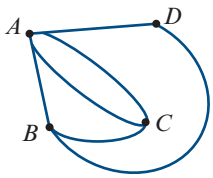
2 a



b



c



3 C is an isolated vertex.

4 Leading diagonals will all be '1'.

5

	A	B	C	D	E
A	0	1	1	1	1
B	1	0	1	1	1
C	1	1	0	1	1
D	1	1	1	0	1
E	1	1	1	1	0

6 E **7** A **8** E **9** A
10 C **11** B **12** E

Exercise 13C

1 a path **b** trail **c** path **d** walk
e trail **f** path

2 a walk **b** cycle **c** path **d** walk
e path **f** walk **g** **h**

3 a i Euler trail
ii $A–B–E–D–B–C–D–A–E$
b neither
c i Euler trail
ii $A–C–E–C–B–D–E–F$
d i Euler circuit
ii $A–B–C–E–D–C–A$
e i Euler circuit
ii $E–F–D–E–A–B–D–C–B–E$

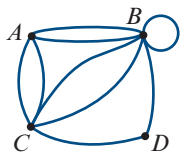
4 a $A–B–C–F–I–H–E–G–D–A$
b $A–B–C–D–E–F–A$
c $A–B-D–C–E–A$

5 $F–A–B–C–D–E–H–G$

ISBN 978-1-009-11041-9

Cambridge University Press

6 **a** 2 **b** 7 **c**



d Vertices are not all even.

7 **a** $v = 9, e = 12, f = 5$
$v - e + f = 2$

b **i** Hamiltonian cycle

ii Lake Bolac – Streatham – Nerrin Nerrin–Woorndoo–Mortlake –Hexham–Chatworth–Glenthompson–Wickliffe–Lake Bolac. and the reverse of this

c **i** Eulerian circuit

ii Not all vertices have an even degree

d **i** Lake Bolac - Wickliffe

ii LWMHCGWCWNSLWL

8 **a** Yes **b** Yes **c** No **d** Yes
e No **f** Yes **g** Yes

9 3

10 C **11** D **12** E **13** B
14 A

Exercise 13D

1 **a** *D–E* **b** 17 minutes
c 8 minutes **d** 36 minutes

2 11

3 **a** 34 **b** 56 **c** *E–B–A–E*, 22
d *A–E–F–G–I* or *A–C–F–G–I*

4 **a** *S–B–D–F*, 12
b $S - A - C - D - F$, 10
c $S - B - D - F$, 15
d $S - A - E - G - F$, 19

5 19 km

6 B **7** C **8** A **9** E

Exercise 13E

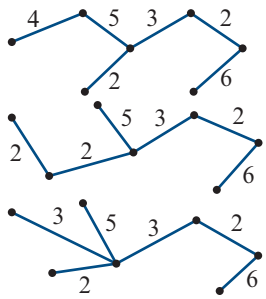
1 **a** $A - B - C - H$, 160
b $A - C - F - E - G - H$, 53
c $A - D - E - F - H$, 385
d $A - B - E - F - I - H$, 87

2 23 minutes

Exercise 13F

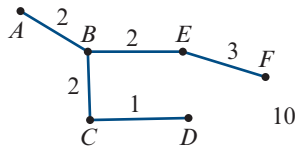
1 **a** 6

b

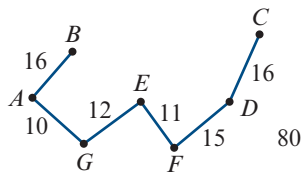


c 22, 20, 21 (Answers will vary)

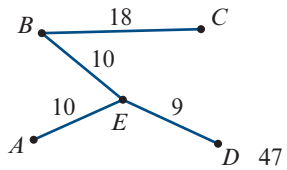
2 **a**



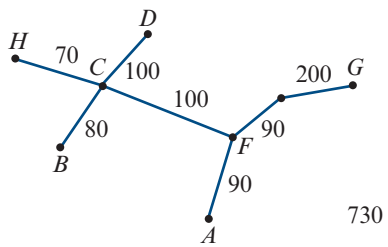
b



c



d



3 490 m

ISBN 978-1-009-11041-9

Cambridge University Press

4 A **5** B **6** C **7** E
8 B

Chapter 13 review

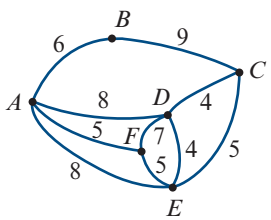
Multiple-choice questions

1 C **2** C **3** A **4** D
5 A **6** C **7** B **8** B
9 E **10** B **11** A **12** A
13 C **14** B **15** B **16** B

Written-response questions

1 a $A–B–C–F–G–E–X$, 11
b $A–C–E–I–H–X$, 127
c $A–D–E–G–H–J–M–X$, 55
d $A–B–D–E–G–I–L–M–X$, 49

2 a i



ii 24

iii

	A	B	C	D	E	F
A	0	1	0	1	1	1
B	1	0	1	0	0	0
C	0	1	0	1	1	0
D	1	0	1	0	1	1
E	1	0	1	1	0	1
F	1	0	0	1	1	0

b i 45 km
ii Some vertices are visited more than once.
iii $F–E–D–C–B–A–F$
iv 33 km (for route above; other answers possible)
c C and F

3 a 4 **b** 18
c $v = 6, e = 9, f = 5; v - e + f = 2$
d-l See solutions

Chapter 14

Exercise 14A

1 a 3 **b** 8
2 $C_1 = 14$, $C_2 = 12$, $C_3 = 21$
3 $C_1 = 12$, $C_2 = 16$, $C_3 = 16$
4 a 9 **b** 11 **c** 8 **d** 18
5 a A, 14; B, 23; C, 12; D, 16; E, not a cut that can be used to determine the maximum number of available seats
b It does not prevent flow from source (Arlie) to sink (Bowen).
c 12
6 a sink 1=10,sink 2=11
b sink 1 = 8, sink 2 = 18
7 a 9 **b** 18
8 a Cut passes through edges with weights 20,10,30,30
b 59 **c** 25
9 D **10** C **11** B

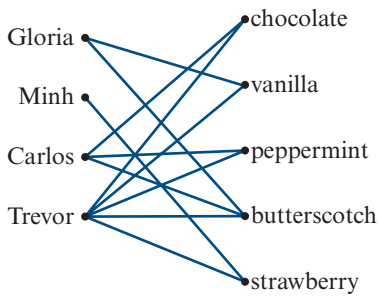
Exercise 14B

1 a Worker 1–Task 3; Worker 2–Task 1; Worker 3–Task 2
b Worker 1–Task 5; Worker 2–Task 6; Worker 3–Task 4
2 Niranjan – Cake; Nishara – Candles; Dinesh – Serviettes; Dhishani – Balloons
3 Two answers possible
Player 1 –Right Wing; Player 2 – Left wing; Player 3 –Centre; Player 4 – Right Defence; Player 5 – Left Defence
Or
Player 1 – Centre Player 2 – Right wing Player 3 – Left Defence Player 4 – Right Defence Player 5 – Left Wing
4 a two distinct groups of vertices (people and flavours)

ISBN 978-1-009-11041-9

Cambridge University Press

b



c 5

5 a *W*–*D*, *X*–*A*, *Y*–*B*, *Z*–*C*

b e.g., minimum cost is 11; *W*–*A*, *X*–*B*, *Y*–*D*, *Z*–*C*

6 Dimitri 800 m, John 400 m, Carol 100 m, Elizabeth 1500 m

7 Joe *C*, Meg *A*, Ali *B*

8 *A*–*Y*, *B*–*Z*, *C*–*X*, *D*–*W*

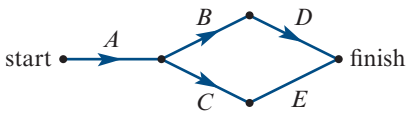
9 Champs Home, Stars Away, Wests Neutral; or Champs Neutral, Stars Away, Wests Home.
Cost = $20 000

10 *A* Mark, *B* Karla, *C* Raj, *D* Jess; or *A* Karla, *B* Raj, *C* Mark, *D* Jess; 55 km

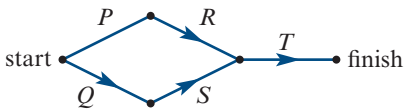
11 D **12** A **13** E **14** D

Exercise 14C

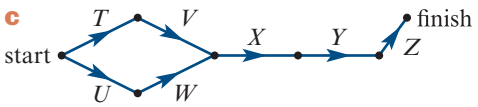
1 a



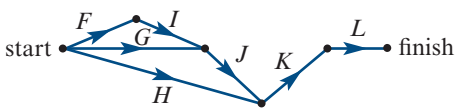
b



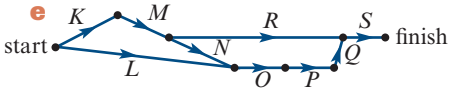
c



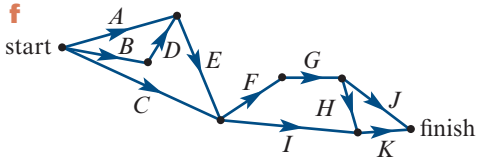
d



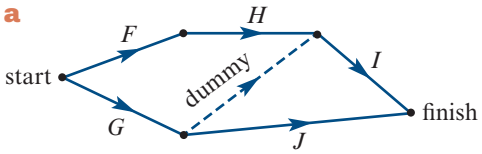
e



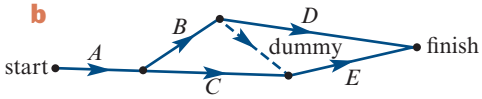
f



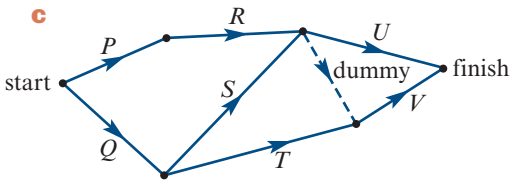
2 a



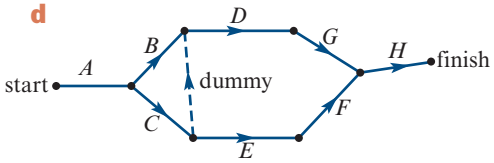
b



c



d



3

a

Activity	*Immediate predecessors*
A	–
B	–
C	*A*
D	*A*
E	*B*, *C*
F	*D*
G	*E*

b

Activity	*Immediate predecessors*
P	–
Q	*P*
R	*P*
S	*Q*
T	*Q*
U	*S*, *V*
V	*R*
W	*R*
X	*T*, *U*

ISBN 978-1-009-11041-9

Cambridge University Press

c

Activity	Immediate predecessors
J	–
K	–
L	J
M	N
N	K
O	K
P	N
Q	L, M
R	P
S	O, R
T	Q

d

Activity	Immediate predecessors
A	–
B	–
C	A
D	A
E	D, B
F	C, E
G	D, B
H	B

e

Activity	Immediate predecessors
P	–
Q	–
R	P
S	P
T	Q
U	R
V	S
W	S, T
X	U
Y	W
Z	V, X, Y

f

Activity	Immediate predecessors
A	–
B	A
C	A
D	A
E	B
F	C, D
G	D
H	E, F, G
I	G
J	I
K	H

4 a Remove panel.

b 'Order component' and 'Pound out dent'

5 a

Activity	Immediate predecessors
A	–
B	–
C	–
D	A
E	B, F
F	C
G	B, F
H	D, E
I	H
J	I, K
K	G
L	G
M	H
N	J, L
O	N

b $A - D - H - M$

$A - D - H - I - J - N - O$

c $B - E - H - M$

$B - E - H - I - J - N - O$

$B - G - K - J - N - O$

$B - G - L - N - O$

d $C - F - E - H - M$

ISBN 978-1-009-11041-9

Cambridge University Press

$C - F - E - H - I - J - N - O$
$C - F - G - K - J - N - O$
$C - F - G - L - N - O$

6 a D, F, H
b A, B, C, D, E, F, G, H

7 a G **b** K
c G is the immediate predecessor of both I and K, however activity K has other immediates predecesors not common to activity I

8 D **9** A **10** A

Exercise 14D

1 a $p = 12$ **b** $w = 10$
c $m = 8,\ n = 8$
d $a = 10,\ b = 18,\ c = 11$
e $f = 9,\ g = 12$
f $q = 8,\ r = 3,\ p = 5,\ n = 9$

2 a 3 **b** A–C **c** 5 **d** 13
e 2

3 a 12 **b** 10 **c** 9 **d** 1
e 3 **f** 9

4 a D–E–F **b** A: 1, B: 1, C: 15

5 a B–E–F–H–J
b A: 1, C: 14, D: 1, G: 1, I: 1

6 a

Activity	*Duration (weeks)*	*Immediate predecessors*
A	3	–
B	6	–
C	6	$A,\ B$
D	5	B
E	7	$C,\ D$
F	1	D
G	3	E
H	3	F
I	2	B

b B–C–E–G

7 a 8 **b** 11 **c** $C - F - G$
d B

8 a D, F, G **b** 13
c Activity H lies on the critical path and if delayed, the completion time of the project will be extended.
d 15 **e** F

9 a i 25 **ii** 29 **iii** 30
b 5 **c** 2

10 a B, C **b** A, B, C, D, E, G
c B, F, H, J **d** 4 **e** E, G, I

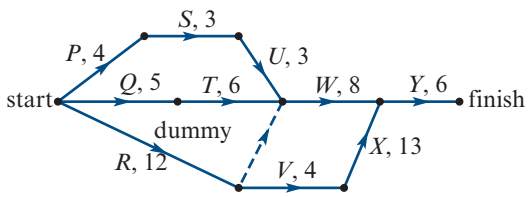
11

a

Activity	*Immediate predecessors*
A	–
B	–
C	–
D	B
E	B
F	A, D
G	C, E
H	F, G
I	F, G
J	G
K	H
L	I
M	J

b 3 **c** 9
d $B - E - G - H - K$
$B - E - G - I - L$ **e** 6

12 a



ISBN 978-1-009-11041-9

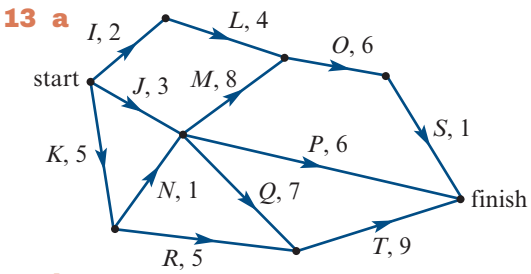
Cambridge University Press

b

Activity	*EST*	*LST*
P	0	11
Q	0	10
R	0	0
S	4	15
T	5	15
U	7	18
V	12	12
W	12	21
X	16	16
Y	29	29

c *R*–*V*–*X*–*Y* **d** 35 weeks

13 a



b

Activity	*EST*	*LST*
I	0	9
J	0	3
K	0	0
L	2	11
M	6	7
N	5	5
O	14	15
P	6	16
Q	6	6
R	5	8
S	20	21
T	13	13

c *K*–*N*–*Q*–*T* **d** 22 weeks

14 E **15** D **16** C **17** A
18 D

Exercise 14E

1 a *A* – *D*; *B* – *E* – *F*; *B* – *E* – *G* – *I*; *C* – *H* – *I*
b *B* – *E* – *G* – *I*, 21 hours
c 18 hours

2 a *A* – *B* – *F* – *H* **b** 21 days
c 20 days **d** $100

3 a *B*–*E*–*H*–*J* **b** 2 hours
c 6 hours **d** 14 hours

4 a 4 **b** 17 hours **c** $1200

5 a 22 days **b** $870

6 a *C*, *D*, *H*
b *B*, *E*, *H*, *I*, *J*
c **i** 21 days **ii** $450

7 a 21 days **b** 10 days **c** 5 days **d** *K*
e $6000

8 a 29 days **b** 6 **c** 4
d Two answers possible:
H, 2 *J*, 0 *K*, 2 *L*, 1 *M*, 1
H, 2 *J*, 0 *K*, 1 *L*, 1 *M*, 2

9 B **10** B **11** D

Chapter 14 review

Multiple-choice questions

1 D **2** D **3** A **4** A
5 C **6** B **7** E **8** D
9 D **10** E

Written-response questions

1 a Alvin should write the Body par 3
Billy should write the Body par 2
Chloe should write the Body par 1
Danielle should write the Conclusion
Elena should write the Introduction
b 43 minutes

2 a 26 **b** 15

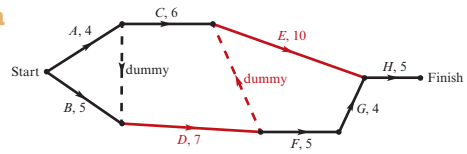
3 Rob – breaststroke, Joel – backstroke, Henk – freestyle, Sav – butterfly or Rob – breaststroke, Joel – butterfly, Henk – backstroke, Sav – freestyle. Time = 276

4 a 26 **b** 15

ISBN 978-1-009-11041-9

Cambridge University Press

5 a



b 12 **c** 1 hour **d** 4

e $B - D - E - H$ **f** 27 hours

g **i** $B - D - F - G - H$

ii 22 hours

h D, H must be in that order

Chapter 15

Exercise 15A

1 A	**2** E	**3** E	**4** B
5 C	**6** B	**7** A	**8** B
9 E	**10** C	**11** E	**12** D
13 D	**14** B	**15** E	**16** A
17 C	**18** C	**19** C	**20** E
21 B	**22** A	**23** D	**24** C
25 B	**26** C	**27** C	**28** E

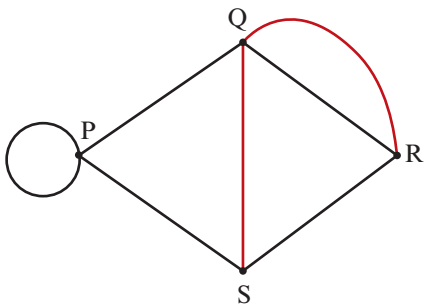
Exercise 15B

1 a 14 **b** 3 **c** 3

e $5 + 5 = 7 + 2, 9 = 9$

2

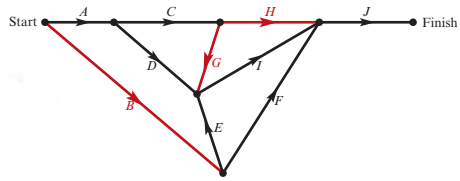
	P	Q	R	S
P	1	1	0	1
Q	1	0	2	1
R	0	2	0	1
S	1	1	1	0



3 a Cut 1 does not isolate the source from the sink.

b 26 **c** 22

4



5 a 9 **b** 7 **c** 1

d $B - D - E - G$ **e** 15

6 a $A \rightarrow 1, D \rightarrow 4, F \rightarrow 10, K \rightarrow 12,$

b $B–C–E–G–J–K$

7 a **i** 2.1 km

ii $PQRTSU$ or $PRQSTU$ or $PRQTSU$ or $PRTQSU$

b **i** $R–Q–P–R–T–Q–S–T–U–S$ or $R–Q–P–R–T–S–Q–T–U–S$

ii travel each road only once

8 a None of the edges overlap.

b $7 + 6 - 11 = 2$ **c** C **d** 297 km

e no **f** 79 km **g** 127 km

h 187 km

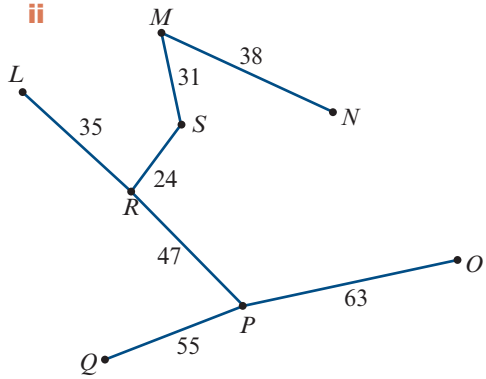
9 a 5 **b** 24 hours **c** 7 hours

10 11 megalitres/day

11 a 112 km

b **i** minimum spanning tree

ii



iii 293 km

c 306 km

12 a A, B, C

ISBN 978-1-009-11041-9

Cambridge University Press

b LST for B is 1, EST for E is 10, LST for I is 18

c **i** $A–D–F–I–J$ **ii** 27 months

d **i** $B–C–D–F–I–J$ **ii** 25 months

13 **a** $A–Z$, $B–W$, $C–X$, $D–Y$, or $A–Z$, $B–X$, $C–W$, $D–Y$

b \$130

14 **a** 15 weeks **b** \$8500 **c** 3

Chapter 16

Exercise 16A

Data analysis, probability and statistics

1 A	**2** C	**3** B	**4** D	**5** C
6 B	**7** B	**8** C	**9** A	**10** A
11 E	**12** E	**13** B	**14** B	**15** A
16 D	**17** B	**18** B	**19** A	**20** E

Recursion and financial modelling

21 C	**22** B	**23** D	**24** A	**25** B
26 C	**27** B	**28** D	**29** A	**30** C

Matrices

31 B	**32** B	**33** A	**34** E	**35** D
36 D	**37** E	**38** D	**39** E	**40** A
41 B				

Networks

42 C	**43** D	**44** B	**45** D	**46** E
47 B	**48** C	**49** C	**50** D	**51** C

Exercise 16B

Data analysis, probability and statistics

1 **a** mean = 54.042, stand dev = 2.717

b $z = -1.1$

c **i** 48.6 kg **ii** 2.5%

2 **a** EV: *number of distractions*, RV: *time*

b *time*

c **i** IQR = 6.2 seconds

ii Upper fence = $28.2 + 1.5 \times 6.2 = 37.5$

d 10 people

e From this information it can be concluded that the time taken to complete the task is associated with the number of distractions. The median time taken by the group who completed the task with no distractions was 25.0 seconds, faster than the group with a few distractions which has a median time of 26.2 seconds, which was in turn faster than the group with many distractions which took a median time of 29.2 seconds to complete the task.

3 **a** $r^2 = 84.8\%$

b 84.8% of the variation in *fuel consumption* can be explained by the variation in *speed*.

c 9.0 litres/100 km

d slope = 0.0218. On average, for each additional 1 km/hr increase in the *speed* of the car, the *fuel consumption* increases by 0.0218 litres/100 km.

e predicted value = 8.40, actual value= 8.30. Thus residual= −0.10.

4 **a** There is a strong, non-linear relationship between *efficiency* and *enthusiasm*.

b $\log y$, $\frac{1}{y}$, x^2

c $\log(efficiency) = 0.0205 + 0.0860 \times enthusiasm$

d 6.6

5 **a**

	Q1	Q2	Q3	Q4
SI	1.01	1.15	1.32	0.52

ISBN 978-1-009-11041-9

Cambridge University Press

b

Year	Q1	Q2	Q3	Q4
2022	62	60	61	63

6 a 130.6 cents/litre

b 141.1 cents/litre

c i Victoria, slope = 2.09. On average, the price of fuel in Victoria is increasing by 2.09 cents/litre each year.

ii NT, slope = 2.20. On average, the price of fuel in the NT is increasing by 2.20 cents/litre each year.

d i 156.7 cents/litre.

ii 169.7 cents/litre.

e The difference is predicted to increase over time. The cost of petrol in the NT is already higher than the cost of petrol in Victoria, and the cost is increasing at a higher rate in the NT (on average 2.20 cents/litre each year) than it is increasing in Victoria (on average 2.09 cents/litre each year).

Recursion and financial modelling

7 a \$6468.13 **b** 6.2%

c i \$5225 **ii** 9%

8 a \$39.23 **b** 6.42%

c $V_0 = 2600$, $V_{n+1} = 1.0025V_n + 140$

9 a \$79 **b** \$344.78

c \$375.48, \$375.60

Matrices

10 a 1×3

b $Q = \begin{bmatrix} 150 & 250 & 320 \\ 300 & 500 & 640 \\ 1050 & 1750 & 2240 \end{bmatrix}$

The number of shoppers who shopped from several sections at Radcliffs.

c $T \times A$

d i $S_1 = \begin{bmatrix} 2835 \\ 1940 \\ 2425 \end{bmatrix}$

ii $S_{50} = \begin{bmatrix} 2791.33 \\ 2577.88 \\ 1830.79 \end{bmatrix}$

iii The long term customer numbers are: HSL (2791), Radcliffs (2578) Cottonworths (1831)

11 a $L = \begin{bmatrix} 0 & 0.9 & 0.7 \\ 0.9 & 0 & 0 \\ 0 & 0.8 & 0 \end{bmatrix}$ **b** $S_0 = \begin{bmatrix} 0 \\ 100 \\ 0 \end{bmatrix}$

c $S_1 = \begin{bmatrix} 90 \\ 0 \\ 80 \end{bmatrix}$, $S_2 = \begin{bmatrix} 56 \\ 81 \\ 0 \end{bmatrix}$, $S_3 = \begin{bmatrix} 73 \\ 50 \\ 65 \end{bmatrix}$, $S_4 = \begin{bmatrix} 91 \\ 66 \\ 40 \end{bmatrix}$;

number of cases is increasing and spreading through the three stages

d $S_{40} = \begin{bmatrix} 5309 \\ 4258 \\ 3036 \end{bmatrix}$, $S_{41} = \begin{bmatrix} 5957 \\ 4778 \\ 3406 \end{bmatrix}$, 1.122

e Not sufficient to eradicate disease; growth rate after 40 weeks is approximately 1.027

f Sufficient to eradicate disease; growth rate after 40 weeks is approximately 0.94

12 a $S_1 = \begin{bmatrix} 29.6 \\ 14.4 \\ 14.4 \\ 21.6 \\ 6.4 \\ 0.0 \end{bmatrix}$, $S_{40} = \begin{bmatrix} 96.11 \\ 55.97 \\ 48.88 \\ 42.69 \\ 33.15 \\ 19.30 \end{bmatrix}$, $S_{41} = \begin{bmatrix} 99.04 \\ 57.67 \\ 50.37 \\ 43.99 \\ 34.16 \\ 19.89 \end{bmatrix}$

ISBN 978-1-009-11041-9

Cambridge University Press

c $S_0 = \begin{bmatrix} 715 \\ 416 \\ 363 \\ 317 \\ 247 \\ 144 \end{bmatrix}, S_1 = \begin{bmatrix} 735.9 \\ 429.0 \\ 374.4 \\ 326.7 \\ 253.6 \\ 148.2 \end{bmatrix}$

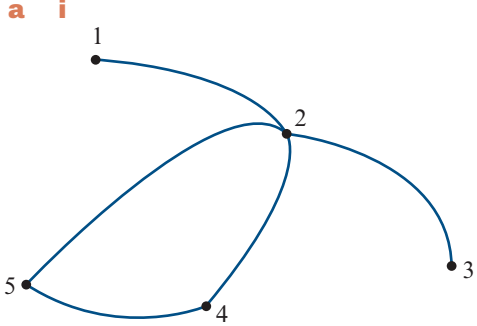
13 a 3×1

b $C \times A = [17900]$; the total cost of the seats.

c The product = [11600]; the total cost of the stalls and dress circle.

Networks

14 a i



ii $1 + 4 + 1 + 2 + 2 = 10$

b i Vertices D and E are odd.

ii E and F

iii $E - F - D - E - A - B - C - D$

c i Capacity = 20 + 25 + 30 = 75

ii Maximum flow = minimum cut = 15 + 15 + 30 = 60

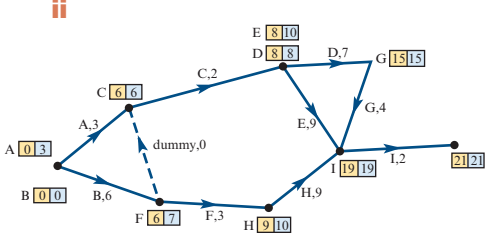
15 a i 2

ii C

b A on breastroke,
B on backstroke,
C on butterfly

c i 7 months

ii



iii A, E, F, H

iv 18 months

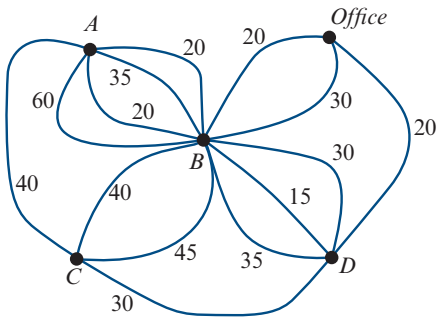
16 a $A - B - D - E - C - A$

b $B - C - E - D - E - C - A - B - D$; Must start and end at a vertex of odd degree.

c 9:54 am

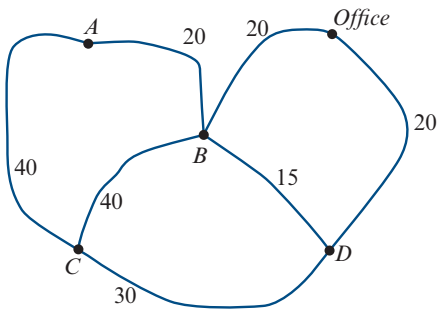
17 a i 4

ii



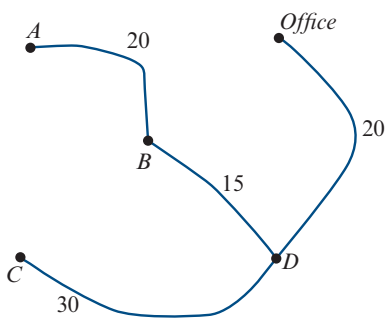
iii A, B, D

b i



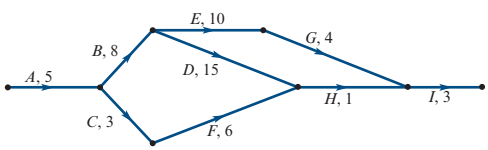
ii $44 400

c i



ii $20 400

18 a



b 32 weeks

ISBN 978-1-009-11041-9

Cambridge University Press

c **i**

Activity	Duration	EST	LST	Float
A	5	0	0	0
B	8	5	5	0
C	3	5	19	14
D	15	13	13	0
E	10	13	15	2
F	6	8	22	14
G	4	23	25	2
H	1	28	28	0
I	3	29	29	0

ii $A - B - D - H - I$

d **i** The project would be completed in a minimum of 30 weeks.

ii Nothing. This activity has a float of 14 and so it could be extended in duration by 14 weeks.

iii The project would be completed in a minimum of 37 weeks.

ISBN 978-1-009-11041-9

Cambridge University Press