Exercice 2

$$M = \left(\begin{array}{cc|cc} 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \hline \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{array}\right) \qquad \mu = 2$$

$$\Rightarrow M_{11} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & 0 \end{pmatrix} \qquad M_{12} = \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$M_{21} = \begin{pmatrix} \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix} \qquad M_{22} = \begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{pmatrix}$$

M_{11} :

Source	Degree	Dest.
A	3	B
B	2	A

M_{12} :

Source	Degree	Dest.
D	2	B

M_{21} :

Source	Degree	Dest.
A	3	C, D
B	2	D

M_{22} :

Source	Degree	Dest.
D	2	C

Exercise 3

Iterate : $v' = \beta M v + \frac{(1-\beta)}{|S|} e_S$

$$M = \begin{pmatrix} 0 & \frac{1}{2} & * & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{pmatrix}$$

$$\beta = 0.8 \quad \Rightarrow \quad \beta M = \begin{pmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{pmatrix}$$

a) $S = \{A\} \Rightarrow |S| = 1, \quad e_S = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$$\Rightarrow \frac{(1-\beta)}{|S|} e_S = \begin{pmatrix} \frac{1}{5} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow v' = \begin{pmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{pmatrix} v + \begin{pmatrix} \frac{1}{5} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

results:

$$\begin{pmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{pmatrix}, \begin{pmatrix} \frac{1}{2} \\ \frac{1}{6} \\ \frac{1}{6} \\ \frac{1}{6} \end{pmatrix}, \begin{pmatrix} \frac{2}{5} \\ \frac{2}{5} \\ \frac{1}{5} \\ \frac{1}{5} \end{pmatrix}, \dots, \sim \begin{pmatrix} 0.429 \\ 0.190 \\ 0.190 \\ 0.190 \end{pmatrix}$$

iteration: 0 1 2 ... 1000

b) $S = \{A, C\} \Rightarrow |S| = 2$

$$e_S = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \frac{(1-\beta)}{|S|} e_S = \begin{pmatrix} \frac{1}{10} \\ 0 \\ \frac{1}{10} \\ 0 \end{pmatrix}$$

$$\Rightarrow v' = \begin{pmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{pmatrix} v + \begin{pmatrix} \frac{1}{10} \\ 0 \\ \frac{1}{10} \\ 0 \end{pmatrix}$$

rezultati:

$$\begin{pmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{pmatrix}, \begin{pmatrix} \frac{2}{5} \\ \frac{1}{6} \\ \frac{4}{15} \\ \frac{1}{6} \end{pmatrix}, \begin{pmatrix} 0.38 \\ 0.173 \\ 0.273 \\ 0.173 \end{pmatrix}, \dots, \sim \begin{pmatrix} 0.386 \\ 0.171 \\ 0.271 \\ 0.171 \end{pmatrix}$$

iteracija: 0 1 2 ... 1000

Exercise 5

For TrustRank iteration: ($\beta = 0.8$)

$$v' = \beta M v + \frac{(1-\beta)}{|S|} e_S$$

, with $S = \{3\}$ and $e_S = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

For Page Rank iteration:

$$v' = \beta M v + \frac{(1-\beta)}{N} e$$

with $N = 4$ and $e = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

then for spam mass calculate

$$\frac{\text{Page Rank} - \text{TrustRank}}{\text{Page Rank}} = \text{spam mass}$$

for each page.

a)

$$M_1 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{pmatrix}$$ ← from figure 1.

from figure 2 ↓

Correct for dead end

$$\Rightarrow M_1 = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{4} & 0 \\ \frac{1}{3} & 0 & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{3} & 0 & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{4} & 0 \end{pmatrix} \qquad M_2 = \begin{pmatrix} 0 & \frac{1}{2} & 1 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \beta M_1 = \begin{pmatrix} 0 & \frac{2}{5} & \frac{1}{5} & 0 \\ \frac{4}{15} & 0 & \frac{1}{5} & \frac{2}{5} \\ \frac{4}{15} & 0 & \frac{1}{5} & \frac{2}{5} \\ \frac{4}{15} & \frac{2}{5} & \frac{1}{5} & 0 \end{pmatrix} \quad \langle L \rangle \quad \beta M_2 = \begin{pmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{pmatrix}$$

Results for TrustRank:

Figure 1:

$$\begin{pmatrix}1/4\\1/4\\1/4\\1/4\end{pmatrix}, \begin{pmatrix}0.2\\0.37\\0.17\\0.25\end{pmatrix}, \begin{pmatrix}0.182\\0.39\\0.19\\0.238\end{pmatrix}, \dots, \sim \begin{pmatrix}0.190\\0.384\\0.184\\0.241\end{pmatrix}$$

$\begin{pmatrix}0.15\\0.41\overline{6}\\0.21\overline{6}\\0.21\overline{6}\end{pmatrix}$ ← iteration 1

iteration: 0 1 2 3 ... 1000

Figure 2:

$$\begin{pmatrix}1/4\\1/4\\1/4\\1/4\end{pmatrix}, \begin{pmatrix}0.3\\0.3\overline{6}\\0.1\overline{6}\\0.1\overline{6}\end{pmatrix}, \begin{pmatrix}0.28\\0.34\overline{6}\\0.14\overline{6}\\0.22\overline{6}\end{pmatrix}, \dots, \sim \begin{pmatrix}0.269\\0.358\\0.158\\0.215\end{pmatrix}$$

b) first calculate PageRank as described:

Figure 1:

$$\begin{pmatrix}1/4\\1/4\\1/4\\1/4\end{pmatrix}, \begin{pmatrix}0.2\\0.2\overline{6}\\0.2\overline{6}\\0.2\overline{6}\end{pmatrix}, \dots, \sim \begin{pmatrix}0.208\\0.264\\0.264\\0.264\end{pmatrix}$$

iteration 0 1 ... 1000

Figure 2:

$$\begin{pmatrix}1/4\\1/4\\1/4\\1/4\end{pmatrix}, \begin{pmatrix}0.35\\0.2\overline{6}\\0.2\overline{6}\\0.2\overline{6}\end{pmatrix}, \dots, \sim \begin{pmatrix}0.321\\0.226\\0.226\\0.226\end{pmatrix}$$

Figure 2: $\sim \begin{pmatrix}0.161\\-0.582\\0.302\\0.0996\end{pmatrix}$

$\sim \begin{pmatrix}0.0857\\-0.456\\0.302\\0.0857\end{pmatrix}$

=> Spam mass calculated as described: Figure 1