Exercise 1

Since $\binom{N}{k}$ can be interpreted as the number of subsets of $\{1, \dots, N\}$ of size k, the sum $\sum_{k=0}^{N} \binom{N}{k}$ resembles the sum over all subset sizes which gives us the size of the powerset of $\{1, \dots, N\}$. The size of this powerset can be calculated by counting all combinations when excluding / including each individual element of the whole set $\{1, \dots, N\}$. ~~which~~ In other words you calculate

$$\underbrace{2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2}_{N \text{ times}} = 2^N.$$

Exercise 2

a) If $5 \cdot i \leq 100$ then $4 \cdot i, 3 \cdot i, \dots, i$ are of course ≤ 100 as well which gives us the items that are frequent with support threshold $= 5$:

$$i \in \{1, 2, \dots, 20\}$$

~~b)~~

c) $\left\lfloor \frac{100}{i} \right\rfloor$ gives us the number of baskets that contain i. So in order to obtain the sum of all basket sizes we can sum over all items using that term:

$$\sum_{i=1}^{100} \left\lfloor \frac{100}{i} \right\rfloor = \underline{482}$$

Exercise 4

a) there are $\binom{I}{2} = \frac{I(I-1)}{2}$ entries in the triangular matrix. Per count we need 4 Bytes which gives us:

$$\text{total space} = 4B \cdot \frac{I(I-1)}{2}$$

b) The largest possible number of pairs with a nonzero count can be obtained when all entries of the triangular matrix are filled with entries > 0.
So, ~~that~~ largest possible number $= \frac{I(I-1)}{2}$.

c) Since the triples method needs 12 Bytes per nonzero entry, the two methods need the same amount of space if $\frac{1}{3}$ of all possible pairs actually occur (nonzero entry).
$\Rightarrow$ So when less than $\frac{1}{3}$ of all possible entries occur, the triples method ~~[illegible]~~ uses less space.