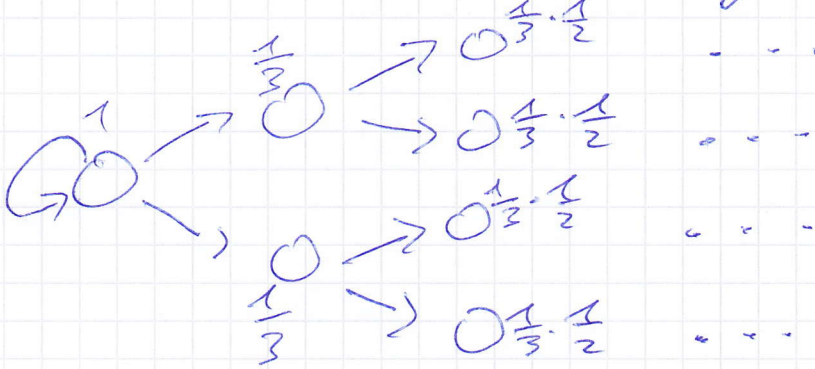
Exercise 4

a) After recursively removing the dead-end nodes we are left with only the root node that refers to itself:

which means that the only page rank is the page rank for the root node which is 1.

b) Using the first method in section 5.1.4 of the book, we reinsert the dead-ends and calculate their page rank in reverse order of removal:



That means

=> page rank of 0^{th}-level (root node) = 1

page rank of i^{th}-level, $i = 1, \dots, n$ $= \frac{1}{3}\cdot\left(\frac{1}{2}\right)^{i-1}$

If we sum them up we get:

$1 + 2\cdot\frac{1}{3} + 4\cdot\frac{1}{3}\cdot\frac{1}{2} + 8\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{2} + \dots$

$= 1 + \frac{2}{3}\left(1 + 2\cdot\frac{1}{2} + 4\cdot\left(\frac{1}{2}\right)^2 + \dots\right)$

$$= 1 + \frac{2}{3}\underbrace{(1 + 1 + 1 + \dots)}_{n}$$

$$= 1 + \frac{2}{3} \cdot n \gg 1$$

This is obviously bigger than 1 which means that the page ranks don't represent the distribution of a random surfer but we still get decent estimates of the relative importance of the pages.