



inputs Total Supporting Pages m
 Total Second Tier Pages k
 each second tier page links to m/k supporting pages
 each supporting page links to t
 t links to each of k second tier pages
 Probability for random teleport $\beta = 0.85$
 Page link contributed to t from outside is x
 Total Number of Pages in web n

Let y be PageRank of (t)

Equation to get sufficient values for

$$y = ax + b \frac{m}{n} + c \frac{k}{n}$$
 Find $a, b, \text{ and } c$

Solution Page Rank of each second tier page is.

$$y_k \Rightarrow \frac{\beta y}{k} + \frac{(1-\beta)}{n} \quad \text{--- (1)}$$

Page Rank of each of supporting pages connected to one second tier page

$$y_s \Rightarrow \frac{\beta y_k}{(m/k)} + \frac{(1-\beta)}{n} \Rightarrow \frac{k \beta y_k}{m} + \frac{(1-\beta)}{n}$$

- ① As we know each supporting Page is linked to t hence is contributing to its PageRank, ② also external Pages are contributing to PageRank of t . and ③ $(1-\beta)/n$ the share of fraction $(1-\beta)$ of PageRank that belongs to t .

Hence, external PageRank \downarrow x

β shared PageRank from supporting Pages \downarrow

$$y = x + \beta m \left(\frac{k\beta y_k}{m} + \frac{1-\beta}{n} \right) + \frac{(1-\beta)}{n}$$

$(1-\beta)/n$ share \downarrow

For big n can be ignored as ratio will be zero (given in question)

$$= x + \beta \cancel{m} \cdot \frac{k\beta y_k}{\cancel{m}} + \frac{m\beta(1-\beta)}{n}$$

$$= x + k\beta^2 y_k + \beta(1-\beta) \frac{m}{n}$$

Let βy_k in y_k from equation ①

$$y = x + k\beta^2 \left(\frac{\beta y}{k} + \frac{(1-\beta)}{n} \right) + \beta(1-\beta) \frac{m}{n}$$

$$= x + \frac{\cancel{k}\beta^2 \cdot \beta y}{\cancel{k}} + \beta^2(1-\beta) \frac{k}{n} + \beta(1-\beta) \frac{m}{n}$$

$$y = x + \beta^3 y + \beta^2(1-\beta) \frac{k}{n} + \beta(1-\beta) \frac{m}{n}$$

Solve for y .

$$y(1 - \beta^3) = x + \beta^2(1 - \beta) \frac{k}{n} + \beta(1 - \beta) \frac{m}{n}$$

$$y = \frac{1}{(1 - \beta^3)} \cdot x + \frac{\beta^2(1 - \beta)}{(1 - \beta^3)} \cdot \frac{k}{n} + \frac{\beta(1 - \beta)}{(1 - \beta^3)} \cdot \frac{m}{n}$$

Re-organize as per required order. of

$$y = ax + b \frac{m}{n} + c \frac{k}{n}$$

$$y = \frac{1}{(1 - \beta^3)} x + \frac{\beta(1 - \beta)}{(1 - \beta^3)} \frac{m}{n} + \frac{\beta^2(1 - \beta)}{(1 - \beta^3)} \frac{k}{n}$$

we have.

$$a = \frac{1}{1 - \beta^3} \quad , \quad b = \frac{\beta(1 - \beta)}{(1 - \beta^3)} \quad , \quad c = \frac{\beta^2(1 - \beta)}{(1 - \beta^3)}$$

$$\text{given } \beta = 0.85$$

$$a = 2.59151, \quad b = \frac{0.1275}{0.385875} \approx 0.33042, \quad c = \frac{0.108375}{0.385875} \approx 0.280855$$
