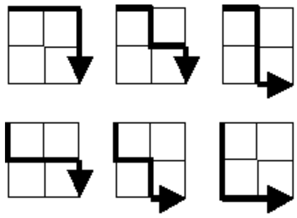
Project Euler #15: Lattice paths

This problem is a programming version of Problem 15 from projecteuler.net

Starting in the top left corner of a 2×2 grid, and only being able to move to the right and down, there are exactly 6 routes to the bottom right corner.



How many such routes are there through a $N \times M$ grid? As number of ways can be very large, print it modulo $(10^9 + 7)$.

Input Format

The first line contains an integer T , i.e., number of test cases.
Next T lines will contain integers N and M.

Constraints

- $1 \leqslant T \leqslant 10^3$
- $1 \leqslant N \leqslant 500$
- $1 \leqslant M \leqslant 500$

Output Format

Print the values corresponding to each test case.

Sample Input

```
2
2 2
3 2
```

Sample Output

```
6
10
```

Explanation

For 2×2 as shown in statement above.

The valid paths for a 2×2 grid shown in the example are the discreet (unique) permutations of {R, R, D, D}. We can list these as: {R, R, D, D}, {R, D, R, D}, {R, D, D, R}, {D, R, R, D}, {D, R, D, R} and {D, D, R, R}. You must have 2 Rs (rights) and 2 Ds (downs), the order not being important, and as long as you start from the top–left corner you will always end at the bottom–right corner.

A 4×4 grid example

To find the number of paths in a 4×4 grid you'll have to generate all the discreet permutations of {R, R, R, R, D, D, D, D}, where any combination of 4 Rs and 4 Ds will always be a valid path. If the Rs are first placed randomly in 4 of the 8 positions then they are considered independent. The Ds are considered dependent because they have no choice but to be placed into the remaining open slots.

For example, placing the 4 Rs randomly in any of the available 8 positions as {R, _, R, R, _, R, _, _} will dictate where the Ds get placed as {R, D, R, R, D, R, D, D}. You will have ${}_8C_4$ = 70 valid combinations. This could be interpreted as: how many *distinct* ways can you shuffle the characters in the string "RRRRDDDD".

Count the number of routes using the binomial coefficient

The number of contiguous routes for a square grid (n×n) is the central binomial coefficient or the center number in the $2n^{th}$ row of Pascal's triangle. The rows start their numbering at 0.

$$\binom{2n}{n} = \frac{(2n)!}{n! \times n!}$$

The formula in general for any rectangular grid (n×m) using the notation for the binomial coefficient is:

$$\binom{n+m}{n} = \frac{(n + m)!}{n! \times m!}$$