Project Euler #28: Number spiral diagonals

This problem is a programming version of Problem 28 from projecteuler.net

Starting with the number 1 and moving to the right in a clockwise direction a 5 by 5 spiral is formed as follows:

$$\begin{array}{ccccc} \mathbf{21} & 22 & 23 & 24 & \mathbf{25} \\ 20 & \mathbf{7} & 8 & \mathbf{9} & 10 \\ 19 & 6 & \mathbf{1} & 2 & 11 \\ 18 & \mathbf{5} & 4 & \mathbf{3} & 12 \\ \mathbf{17} & 16 & 15 & 14 & \mathbf{13} \end{array}$$

It can be verified that the sum of the numbers on the diagonals is 101.
What is the sum of the numbers on the diagonals in a $N \times N$, (N is odd) spiral formed in the same way?
As the sum will be huge you have to print the result mod $(10^9 + 7)$

Input Format

The first line contains an integer T , i.e., number of test cases.
Next T lines will contain an integer N.

Constraints

$1 \leq T \leq 10^5$
$1 \leq N < 10^{18}, \text{N is odd}$

Output Format

Print the values corresponding to each test case.

Sample Input

```
2
3
5
```

Sample Output

```
25
101
```

Solution:

	$f_1(n)$	$f_2(n)$	$f_3(n)$	$f_4(n)$
$n = 1$	1	(1)	(1)	(1)
$n = 3$	3	5	7	9
$n = 5$	13	17	21	25
$n = 7$	31	37	43	49
sum	48	60-1	72-1	84-1
	sum_1	sum_2	sum_3	sum_4

(1) is only used for building polynomial model

Step 1: Build Polynomial Model

Take $f_1(n)$ as example:

n	1	3	5	7
$f_1(n)$	1	3	13	31

$$f_1(n) = an^3 + bn^2 + cn + d$$

$n=1$: $a + b + c + d = 1 = f_1(1)$

$n=3$: $27a + 9b + 3c + d = 3 = f_1(3)$

$n=5$: $125a + 25b + 5c + d = 13 = f_1(5)$

$n=7$: $343a + 49b + 7c + d = 31 = f_1(7)$

$\Rightarrow a = 0,\ b = 1,\ c = -3,\ d = 3 \Rightarrow \boxed{f_1(n) = n^2 - 3n + 3}$

→ Do the same with f_2, f_3 and $f4$, we have:

$$\boxed{\begin{aligned} f_2(n) &= n^2 - 2n + 2 \\ f_3(n) &= n^2 - n + 1 \\ f_4(n) &= n^2 \end{aligned}}$$

$N = \frac{n+1}{2}$: Number of numbers from 1 to n

Step 2: Calculate sum of f(n)

- $sum_1 = \sum f_1(n) = (1^2 + 3^2 + \ldots + n^2) - 3.(1 + 3 + \ldots + n) + 3.N$
- Sum of odd squares:
  $$1^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(n+2)}{6}$$
- Sum of arithmetic progression:
  $$1 + 3 + \ldots + n = \frac{N.(a_1 + a_n)}{2} = \frac{N.(1+n)}{2}$$

$$\Rightarrow sum_1 = \frac{n(n+1)(n+2)}{6} - 3.\frac{N(1+n)}{2} + 3N$$

→ Do the same with ~~&~~ sum_2, sum_3 and sum_4, we have:

$$sum_2 = \frac{n(n+1)(n+2)}{6} - 2 . \frac{N(1+n)}{2} + 2N$$

$$sum_3 = \frac{n(n+1)(n+2)}{6} - 1 . \frac{N(1+n)}{2} + 1.N$$

$$sum_4 = \frac{n(n+1)(n+2)}{6}$$

$$\Rightarrow sum = sum_1 + sum_2 + sum_3 + sum_4 - 3.1$$