Project Euler #69: Totient maximum



This problem is a programming version of Problem 69 from projecteuler.net

Euler's Totient function, $\phi(n)$ [sometimes called the phi function], is used to determine the number of numbers less than n which are relatively prime to n. For example, as 1, 2, 4, 5, 7, and n, are all less than nine and relatively prime to nine, $\phi(9) = 6$.

\boldsymbol{n}	$Relatively\ Prime$	$\phi(n)$	$n/\phi(n)$
2	1	1	2
3	1, 2	2	1.5
4	1,3	2	2
5	1, 2, 3, 4	4	1.25
6	1,5	2	3
7	1, 2, 3, 4, 5, 6	6	$1.1666\dots$
8	1, 3, 5, 7	4	2
9	1, 2, 4, 5, 7, 8	6	1.5
10	1, 3, 7, 9	4	2.5

It can be seen that n=6 produces a maximum $n/\phi(n)$ for n<10. Find the value of n< N for which $n/\phi(n)$ is maximum. In case of multiple answers, print the minimum.

Input Format

First line contains $oldsymbol{T}$, denoting number of test cases. $oldsymbol{T}$ lines follow Each line contains $oldsymbol{N}$

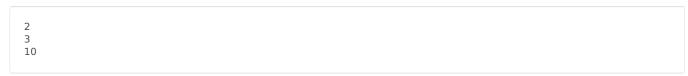
Constraints

 $\begin{aligned} &1 \leq T \leq 1000 \\ &3 \leq N \leq 10^{18} \end{aligned}$

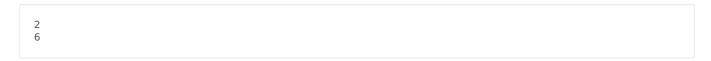
Output Format

Print the answer corresponding to each testcase on a new line.

Sample Input



Sample Output



Solution:

Project Euler # 69 (Hackerrank): Totient Maximum

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Euler Totient Function:

Number of Coprimes of
$$n = \psi(n) = n$$
. IT $(1 - \frac{1}{p})$

pln

• ratio =
$$\frac{n}{\varphi(n)} = \frac{1}{\prod(1-\frac{1}{p})}$$

$$\frac{n}{\varphi(n)} \max \Rightarrow T(1-\frac{1}{p}) \min \Rightarrow \frac{1}{p} \max_{n \in \mathbb{N}} \text{ and } K \max \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \text{ and } K \max_{n \in \mathbb{N}} \Rightarrow p \min_{n \in \mathbb{N}} \Rightarrow p$$

Algorithm:

How it norks:

• Take
$$p=2$$
 \Rightarrow p is min and $K=1$
• $n=p=2$. (n is still smaller than N \Rightarrow continue with $p=3$)

• Take
$$p=3$$
 = p_1 = 2. $3=6$ (n has prime factors 2 and 3)