Homework week 3

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Octorber 2022

1 Question 1:

We have:

$$t_n = y(x, w) + \epsilon$$
$$p(t_n) = \mathcal{N}(t_n|y(x, w), \epsilon^2)$$
$$p(t|x, w, \beta) = \prod_{n=1}^{N} \mathcal{N}(t_n|y(x_n, w); \sigma^2)$$
$$\mathcal{N}(t_n|y(x_n, w), \sigma^2) = \frac{1}{\sqrt{2\pi\beta^{-1}}} e^{(t-y(x_n,w))^2 \frac{\beta}{2}}$$

$$\begin{aligned} log \prod_{i=1}^{N} \mathcal{N}(t_n|y(x_n, w); \beta^{-1} &= \sum_{i=1}^{N} log(\frac{1}{\sqrt{2\pi\beta^{-1}}} e^{(t_n-y(x_n,w))^2 \frac{\beta}{2}} \\ &= \sum_{i=1}^{N} (\frac{-1}{2} log(2\pi\beta^{-1}) - (t_n - y(x_n, w))^2 \\ &\approx -\sum_{i=1}^{N} (t_n - y(x_n - w))^2 \end{aligned}$$

Now we have to :

$$\sum_{i=1}^{N} (t_n - y(x_n - w))^2$$

X = $\begin{bmatrix} 1 & X_1 \\ 1 & X_2 \\ ... & ... \\ 1 & X_n \end{bmatrix}$; t = $\begin{bmatrix} t_1 \\ t_2 \\ ... \\ t_n \end{bmatrix}$; w = $\begin{bmatrix} w_0 \\ w_1 \\ ... \\ w_d \end{bmatrix}$; y = $\begin{bmatrix} y_1 \\ y_2 \\ ... \\ y_n \end{bmatrix} = \begin{bmatrix} w_0 + w_1 x_{11} + ... + w_d x_{1d} \\ w_0 + w_1 x_{21} + ... + w_d x_{2d} \\ ... \\ w_0 + w_1 x_{n1} + ... + w_d x_{nd} \end{bmatrix}$

t - y = $\begin{bmatrix} t_1 - y_1 \\ t_2 - y_2 \\ ... \\ t_n - y_n \end{bmatrix} \Longrightarrow ||t - y||_2^2 = \sum_{i=1}^{n} (t_i - y_i)^2 = L$

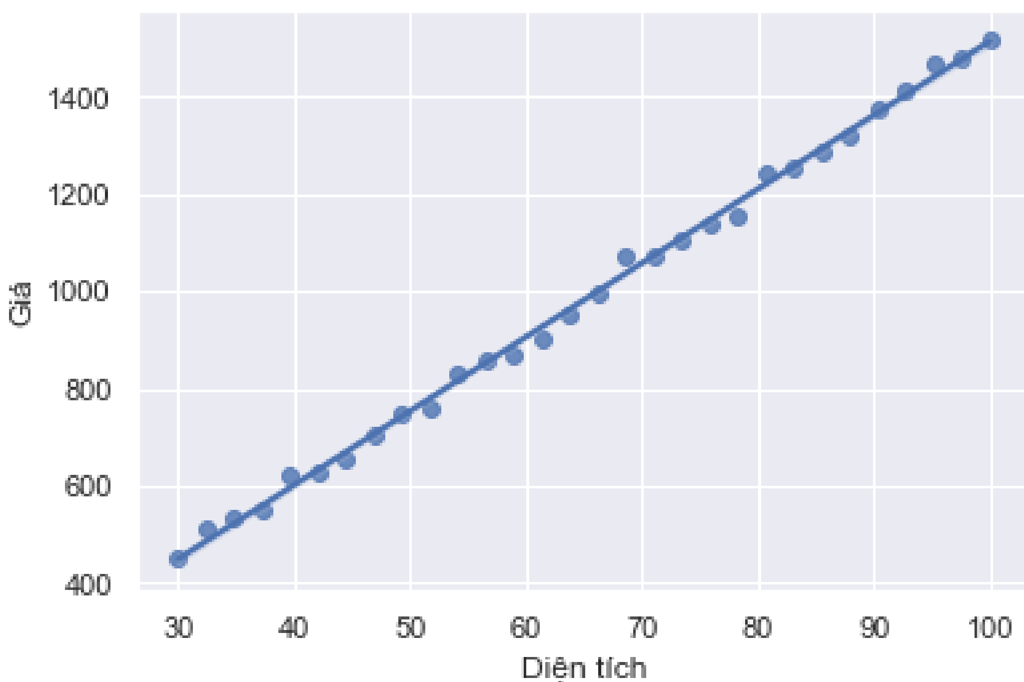
$$\implies L = ||t - y||_2^2 = ||t - XW||_2^2$$

$$\begin{aligned} \frac{\partial L}{\partial W} = 2X^T(t - XW) = 0 & \\ \iff X^T t &= X^T X W \\ \iff W &= (X^t X)^{-1} X^T t \end{aligned}$$

2 Question 2:

a) Linear Regression model for house price prediction:

$$\text{HousePrice} = -7.064 + 15.211 * \text{HouseArea}$$



b) Prediction:

Area	Price
50	753.49
100	1514.05
150	2274.59

3 Question 3:

Linear Regression model for house price prediction in Boston:

$$\begin{aligned}\text{Price} = 36.459 &- 0.108011 * \text{CRIM} + 0.046420 * \text{ZN} + 0.020559 * \text{INDUS} \\ &+ 2.686734 * \text{CHAS} - 17.766611 * \text{NOX} + 3.809865 * \text{RM} + 0.000692 * \text{AGE} \\ &- 1.475567 * \text{DIS} + 0.306049 * \text{RAD} - 0.012335 * \text{TAX} \\ &- 0.952747 * \text{PTRATIO} + 0.009312 * \text{B} - 0.524758 * \text{LSTAT}\end{aligned}$$

4 Question 4:

X is a m*n matrix X^TX is a n*n matrix

If X in linearly indepedence when $X\vec{v} = 0$ have only trivial solution $\vec{v} = \vec{0}$, and $\vec{v} \in N(X)$. We have:

$$\begin{aligned} X\vec{v} &= 0 \\ \implies X^TX\vec{v} &= 0 \\ \implies \vec{v} &\in N(X^TX)\end{aligned}$$

Hence $N(X) \subseteq N(X^TX)$

If X^TX in linearly indepedence when $X^TX\vec{v} = 0$ have only trivial solution $\vec{v} = \vec{0}$, and $\vec{v} \in N(X^TX)$. We have:

$$\begin{aligned} X^TX\vec{v} &= 0 \\ \to \vec{v}^TX^TX\vec{v} &= \vec{v}^T0 \\ \to (X\vec{v})^T(X\vec{v}) &= 0 \\ \to ||X\vec{v}||_2^2 &= 0 \\ \to X\vec{v} &= 0 \\ \to \vec{v} &\in N(X)\end{aligned}$$

Hence $N(X^TX) \subseteq N(X)$

Therefore $N(X) = N(X^TX)$, so X^TX invertible $\leftrightarrow X^TX$ linearly indepedence $\leftrightarrow X$ linearly indepedence $\leftrightarrow X$ full rank