Homework week 1

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August 2022

1 Question 1:

a) Marginal distribution of P(x)

$$P_X(x) = \sum_j P(x_i; y_j)$$

X	X_1	X_2	X_3	X_4	X_5
$P_X(x)$	0.16	0.17	0.11	0.22	0.34

Marginal distribution of $P(y)$

$$P_Y(y) = \sum_i P(x_i; y_j)$$

Y	Y_1	Y_2	Y_3
$P_Y(y)$	0.26	0.47	0.27

b) Conditional distribution of $P(x_i|Y = y_1)$

$$P(x_i|Y = y_1) = \frac{P(x_i and Y=y_1)}{P(Y=y_1)}$$

X	X_1	X_2	X_3	X_4	X_5
$P(x\|Y=y_1)$	$\frac{1}{26}$	$\frac{2}{26}$	$\frac{3}{26}$	$\frac{10}{26}$	$\frac{10}{26}$

Conditional distribution of $P(x_i|Y = y_3)$

$$P(x_i|Y = y_3) = \frac{P(x_i and Y=y_3)}{P(Y=y_3)}$$

X	X_1	X_2	X_3	X_4	X_5
$P(x\|Y=y_3)$	$\frac{10}{27}$	$\frac{5}{27}$	$\frac{3}{27}$	$\frac{5}{27}$	$\frac{4}{27}$

2 Question 2:

Proof:

If X and Y are discrete variable:

$$\begin{aligned}
E_Y[E_X[x|y]] &= \sum_y p(y)E_X[x|y] \\
&= \sum_y p(y)\sum_x p(x|y)x \\
&= \sum_y \sum_x xp(y)p(x|y) \\
&= \sum_x x\sum_y p(x;y) \\
&= \sum_x xp(x) \\
&= E_X[X]
\end{aligned}$$

If X and Y are continuous variable:

$$\begin{aligned}
E_Y[E_X[x|y]] &= \int E_X[x|y]p(y)dy \\
&= \int (\int xp(x|y)dx)p(y)dy \\
&= \int\int xp(x|y)p(y)dxdy \\
&= \int\int xp(x,y)dxdy \\
&= \int xp(x)dx \\
&= E_X[X]
\end{aligned}$$

3 Question 3:

We denote:

X: "people use X"

Y: "people use Y"

and then we have:

$P(X) = 20.7\%$

$P(X) = 50\%$

$P(X|Y) = 36.5\%$

a) Probability of a randomly picked a person who use both X and Y:

$P(XandY) = P(X|Y) * P(Y) = 36.5\% * 50\% = 18.25\%$

b) Probability of a randomly picked person who use Y given that he/she not use X

$P(Y|\overline{X}) = \frac{P(Y and \overline{X})}{P(\overline{X})} = \frac{P(\overline{X}|Y)*P(Y)}{P(\overline{X})} = \frac{[1-P(X|Y)]*P(Y)}{1-P(X)} = \frac{[1-36.5\%]*50\%}{1-20.7\%}$
$\approx 40.037\%$

4 Question 4:

We have:

$$\begin{aligned} Var(X) &= E[(X-\mu)^2] \\ &= E[(X-E(X))^2] \\ &= E[X^2 - 2XE(X) + [E(X)]^2] \\ &= E[X^2] - 2E[XE(X)] + E[E(X)^2] \\ &= E[X^2] - 2E(X) * E(X) + E(X)^2 \\ &= E[X^2] - 2E(X)^2 + E(X)^2 \\ &= E[X^2] - E(X)^2 \end{aligned}$$

5 Question 5:

Assume that the player choose the 1^{st} door in the 1^{st} turn so the probability of the car in the 1^{st} door is $P(A) = \frac{1}{3}$ and the probability of the car in others doors is $P(\overline{A}) = \frac{2}{3}$. When Monty reveals the door which has the goat inside, the probability of the car in the leftover door is still remain $\frac{2}{3}$.
The player get the 2^{nd} turn to choose to stay on his/her 1^{st} choice or switch to the leftover door. Probability of the car in the 1^{st} door is $\frac{1}{3}$ and the probability of the car in the other door is $\frac{2}{3}$. So the best strategy is switch to the other door when Monty eliminate one of two door which has goat inside.