Homework week 5

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1 Question 1

We have

$$X = \begin{bmatrix} -x_1^T- \\ -x_2^T- \\ ... \\ -x_n^T- \end{bmatrix} \in R^{n*D}$$

If we want to reduce the dimension of X by choosing M most important features in X (M < D), doing that by create B matrix

$$B = \begin{bmatrix} | & | & & | \\ b_1 & b_2 & ... & b_m \\ | & | & & | \end{bmatrix} \in R^{D*M}$$

$$R^D \to R^M : \begin{cases} x_1 \to z_1 \\ ... \\ x_n \to z_n \end{cases}$$

$$Z = X*B = \begin{bmatrix} -x_1^T- \\ -x_2^T- \\ ... \\ -x_n^T- \end{bmatrix} \begin{bmatrix} | & | & & | \\ b_1 & b_2 & ... & b_m \\ | & | & & | \end{bmatrix} = \begin{bmatrix} x_1^T b_1 & x_1^T b_2 & & x_1^T b_m \\ x_2^T b_1 & x_2^T b_2 & & x_2^T b_m \\ ... & ... & ... & ... \\ x_n^T b_1 & x_n^T b_2 & & x_n^T b_m \end{bmatrix} = \begin{bmatrix} -z_1^T- \\ -z_2^T- \\ ... \\ -x_n^T- \end{bmatrix}$$

We want the new coordinate have the highest variance and the mean = 0

$$\mu_x = \frac{x_1 + x_2 + ... + x_n}{N}$$

$$x^{'} = x - \mu_x \to \mu_{x'} = 0$$

$$\mu_z = \frac{x_1^{'T} * b_1 + x_2^{'T} * b_1 + ... + x_n^{'T} * b_1}{N} = \frac{b_1^T \sum_{i=1}^N x_i^{'}}{N} = b_1^T \mu_x^{'} = 0$$

$$\begin{aligned} Var_z &= \frac{\sum_{i=1}^N (x_i^{'T} b_1 - \mu_z)^2}{N} = \frac{\sum_{i=1}^N (x_i^{'T} b_1)^2}{N} = \frac{\sum_{i=1}^N b_1^T x_i^{'} x_i^{'T} b_1}{N} \\ &= b_1^T \frac{\sum_{i=1}^N x_i^{'} x_i^{'T}}{N} b_1 = b_1^T \frac{\sum_{i=1}^N (x - \mu_x)(x - \mu_x)}{N} b_1 = b_1^T cov(X, X) b_1 \end{aligned}$$

$$\begin{aligned} \max \quad & b_1^T S b_1 \\ \text{s.t.} \quad & ||b_1||_2^2 = 1 \end{aligned}$$

Langrae function

$$\begin{aligned} L(\lambda) &= b_1^T S b_1 + \lambda(1 - ||b_1||_2^2) \\ \frac{\partial L(\lambda)}{\partial b_1} &= 2b_1^T S - \lambda 2 b_1 = 0 \\ \rightarrow S b_1 &= \lambda b_1 \rightarrow \begin{cases} b_1 \text{ is eigenvector} \\ \lambda \text{ is eigenvalue} \end{cases} \end{aligned}$$

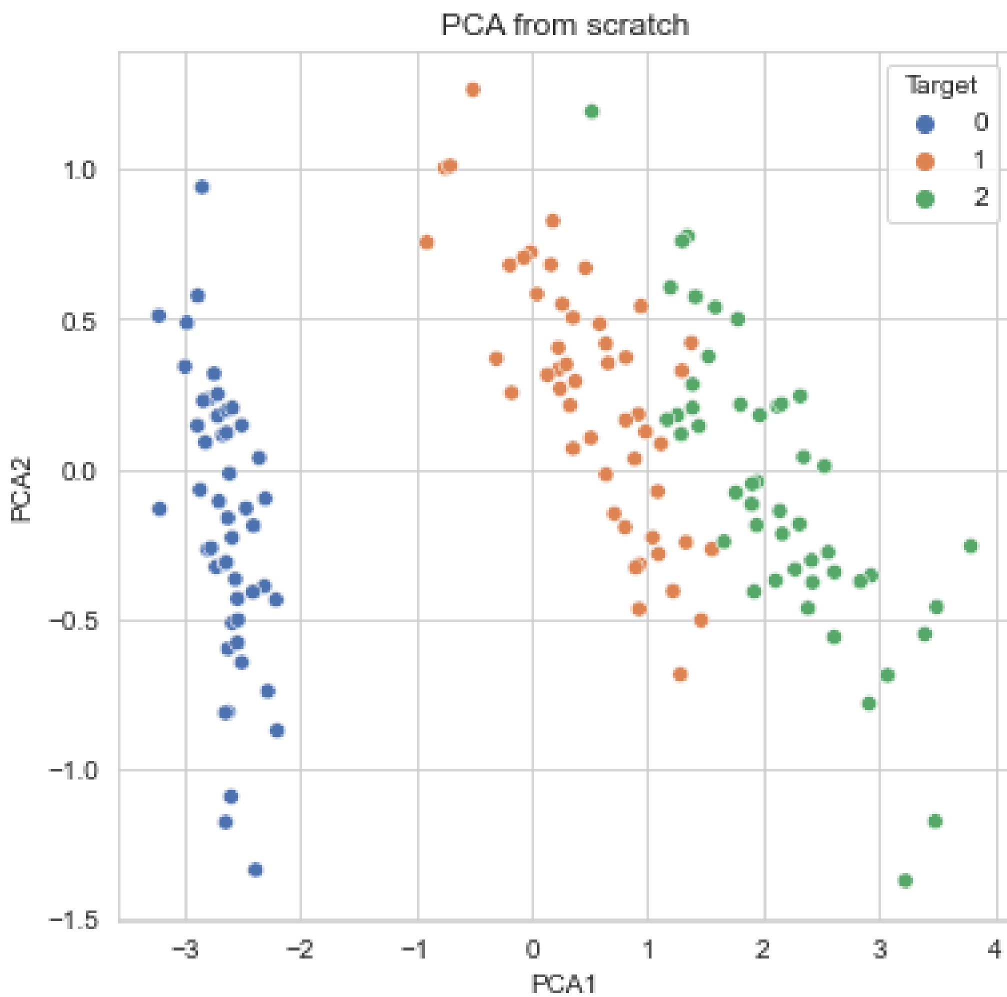
$$\begin{aligned} \frac{\partial L(\lambda)}{\partial \lambda} &= 1 - ||b_1||_2^2 = 0 \\ b_1^T b_1 = 1 & \end{aligned}$$

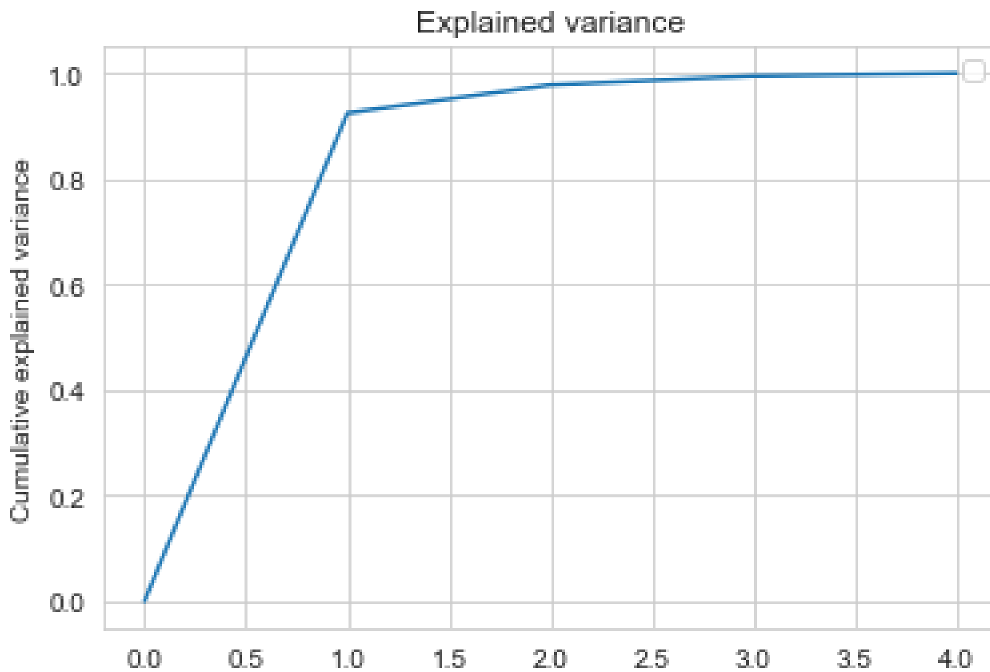
$$Var = b_1^T S b_1 = b_1^T \lambda b_1 = \lambda b_1^T b_1 = \lambda$$

we sort the eigenvalue in decending order and choose D fisrt largest eigenvector to create B matrix $\rightarrow Z = Bx^{'}$

2 Question 2

3 Question 3





4 Question 4

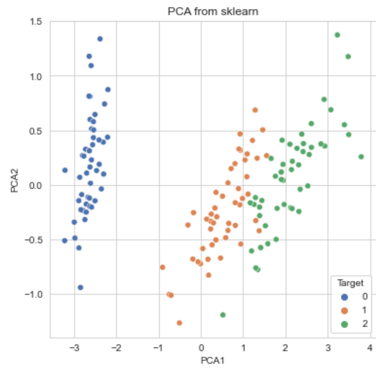
The output of PCA algorithm in Sklearn package is little bit different from the PCA algorithm implemented from scratch. The PCA from sklearn lists the entries of eigenvectors rowwise, while the eigenvectors calculated from np.linalg.eig() function is lists columnwise. This different will lead to the different in eigenvalues and the final outputs. This problem really does not affect the classification model (model might have different weight but the prediction will the same)

PCA from scratch(numpy)

PCA.get_components()

$$\begin{bmatrix} 0.361387 & -0.656589 \\ -0.084523 & -0.730161 \\ 0.856671 & 0.173373 \\ 0.358289 & 0.075481 \end{bmatrix}$$

PCA from sklearn



pca.components_

$$\begin{bmatrix} 0.361387 & -0.084523 & 0.856671 & 0.358289 \\ 0.656589 & 0.730161 & -0.173373 & -0.075481 \end{bmatrix}$$