SOEN 331: Introduction to Formal Methods for Software Engineering

Assignment 1

Propositional and Predicate Logic, Structures, Binary Relations, Functions and Relational Calculus

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1 Problem 1 (8 pts)

1.1 Problem:

You are shown a set of four cards placed on a table, each of which has a **number** on one side and a **symbol** on the other side. The visible faces of the cards show the numbers **2** and **7**, and the symbols $\square$, and $\bigcirc$.

Which card(s) must you turn over in order to test the truth of the proposition that *"If a card has an odd number on one side, then it has the symbol $\square$ on the other side"*? Explain your reasoning by deciding for <u>each</u> card whether it should be turned over and why.

1.2 Answer:

2 Problem 2 (8 pts)

2.1 Description

Consider the predicate *asks(a, b)* that is interpreted as *"a has asked b out on a date."*

1. Translate the following into English: $\forall\, a \exists\, b\ asks(a, b)$ and $\exists\, y \forall\, x\ asks(a, b)$.
2. Can we claim that $\forall\, a \exists\, b\ asks(a, b) \rightarrow \exists\, y \forall\, x\ asks(a, b)$? Discuss in detail.
3. Can we claim that $\exists\, y \forall\, x\ asks(a, b) \rightarrow \forall\, a \exists\, b\ asks(a, b)$? Discuss in detail.

2.2 Answer

3 Problem 3 (12 pts)

3.1 Formalize sentences and indicate formal type

Let's denote

P(x): scientist(x): "x is a scientist"

Q(x): honest(x): "x is honest"

We can formalize the statements as following

1. "No scientists are honest." $= \forall\, x, (P(x) \rightarrow \neg Q(x))$ = E form
2. "All scientists are crooked." $= \forall\, x, (P(x) \rightarrow \neg Q(x))$ = E form
3. "All scientists are honest." $= \forall\, x, (P(x) \rightarrow Q(x))$ = A form
4. "Some scientists are crooked." $= \exists\, x, (P(x) \wedge \neg Q(x))$ = O form
5. "Some scientists are honest." $= \exists\, x, (P(w) \wedge Q(x))$ = I form
6. "No scientist is crooked." $= \forall\, x, (P(x) \rightarrow Q(x))$ = A form
7. "Some scientists are not crooked." $= \exists\, x, (P(x) \wedge Q(x))$ = I form
8. "Some scientists are not honest." $= \exists\, x, (P(x) \wedge \neg Q(x))$ = O form

3.2 Identify pairs that are contradictories, contraries, subcontraries, and pairs that support subalteration

3.2.1 Pairs of contradictories

- (3) and (4)
- (6) and (4)
- (3) and (8)
- (6) and (8)
- (5) and (1)
- (5) and (2)
- (7) and (1)
- (7) and (2)

3.2.2 Pairs of contraries

- (3) and (1)
- (6) and (1)
- (3) and (2)
- (6) and (2)

3.2.3 Pairs of subcontraries

- (5) and (8)
- (7) and (8)
- (5) and (4)
- (7) and (4)

3.2.4 Pairs that support subalteration

- Subaltern: (5) - Superaltern: (3)
- Subaltern: (5) - Superaltern: (6)
- Subaltern: (7) - Superaltern: (3)
- Subaltern: (7) - Superaltern: (6)
- Subaltern: (4) - Superaltern: (1)
- Subaltern: (4) - Superaltern: (2)
- Subaltern: (8) - Superaltern: (1)
- Subaltern: (8) - Superaltern: (2)

4 Problem 4 (12 pts)

4.1 Description

Consider list $\Lambda = \langle w, x, y, z \rangle$, deployed to implement a Queue Abstract Data Type.

1. Let the head of Λ correspond to the front position of the Queue. Implement operations `enqueue(el, Λ)` and `dequeue(Λ)` using list construction operations. In both cases we can refer to Λ' as the state of the list upon successful termination of one of its operations.

2. Let us now reverse the way we manipulate our data structure and let the head of Λ correspond to the rear of the Queue.

   (a) What would be the result of $cons(el, \Lambda)$, and would it be a correct implementation for operation `enqueue(el, Λ)`?

   (b) What would be the result of $list(el, \Lambda)$, and would it be a correct implementation for operation `enqueue(el, Λ)`?

   (c) What would be the result of $concat(list(el), \Lambda)$, and would it be a correct implementation for operation `enqueue(el, Λ)`?

4.2 Answer

5 Problem 5 (12 pts)

5.1 Description

Let $A = \{0, 1, 2, 3, 4\}$ and relations R, S, T, and U on A defined as follows:

$$R = \{(0,0),(0,1),(0,3),(1,0),(1,1),(2,2),(3,0),(3,1),(3,3),(4,0),(4,1),(4,3),(4,4)\}$$

$$S = \{(0,1),(1,1),(2,3),(2,4),(3,0),(3,4),(4,0),(4,1),(4,4)\}$$

$$T = \{(0,3),(0,4),(2,1),(3,2),(4,2),(4,3)\}$$

$$U = \{(0,0),(0,1),(0,3),(1,0),(1,1),(1,3),(2,2),(3,0),(3,1),(3,3),(4,4)\}$$

Fill in the table below, using ✓, or ×.

		R	S	T	U
Reflexive					
Irreflexive					
Symmetric					
Asymmetric					
Antisymmetric					
Transitive					
Equivalence					
Partial order					

5.2 Answer

6 Problem 6 (8 pts)

6.1 Description

Consider the relation *"is a subtype of"* over the set {*rectangle*, *quadrilateral*, *square*, *parellelogram*, *rhombus*}.

1. Is this an *equivalence relation*?
2. Is this relation a *partial order*? If so, create a *Hasse diagram*, and identify *minimal* and *maximal* elements.

6.2 Answer

7 Problem 7 (8 pts)

7.1 Description

Consider the set $A = \{w, x, y, z\}$, and the relations

$$S = \{(w, x), (w, y), (x, w), (x, x), (z, x)\}$$

$$T = \{(w, w), (w, y), (x, w), (x, x), (x, z), (y, w), (y, y), (y, z)\}$$

Find the following compositions:

1. $S \circ T$
2. $T \circ S$
3. $T^{-1} \circ S^{-1}$

NOTE: Some authors (e.g. Rosen) adopt a different ordering of operands than the one we use in our lecture notes. Please follow the ordering (and the definition) of the lecture notes.

7.2 Answer

8 Problem 8 (12 pts)

8.1 Description

Consider sets $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{a, b, c, d, e, f\}$.

1. Determine the type of the correspondence in each of the following cases, or indicate if the correspondence is not a function.

   (a) $\{1 \mapsto b, 2 \mapsto c, 3 \mapsto e, 4 \mapsto d, 5 \mapsto f, 3 \mapsto a\}$

   (b) $\{1 \mapsto a, 2 \mapsto d, 3 \mapsto a, 4 \mapsto f, 5 \mapsto d, 6 \mapsto c\}$

   (c) $\{1 \mapsto c, 2 \mapsto b, 3 \mapsto d, 4 \mapsto e, 5 \mapsto e, 6 \mapsto f\}$

   (d) $\{1 \mapsto b, 2 \mapsto c, 3 \mapsto e, 4 \mapsto d, 5 \mapsto f, 6 \mapsto a\}$

   Fill in the table below, using ✓, or ×.

	Inective	Surjective	Bijective	Neither injective nor surjective	Not a function
(a)					
(b)					
(c)					
(d)					

2. Is it possible to construct a function $f : A \to B$ which is surjective and not injective? Discuss.

8.2 Answer

9 Problem 9 (20 pts)

9.1 Description

Consider the following relation:

$$laptops : Model \leftrightarrow Brand$$

where

$$\begin{aligned}
&laptops = \\
&\quad \{ \\
&\qquad legion5 \mapsto lenovo, \\
&\qquad macbookair \mapsto apple, \\
&\qquad xps15 \mapsto dell, \\
&\qquad spectre \mapsto hp, \\
&\qquad xps13 \mapsto dell, \\
&\qquad swift3 \mapsto acer, \\
&\qquad macbookpro \mapsto apple, \\
&\qquad dragonfly \mapsto hp, \\
&\qquad envyx360 \mapsto hp \\
&\quad \}
\end{aligned}$$

1. What is the domain and the range of the relation?

2. What is the result of the expression

   $$\{xps15, xps13, swift3, envyx360\} \lhd laptops$$

   What is the meaning of perator $\lhd$ and where would you deploy such operator in the context of a database management system?

3. What is the result of the expression

   $$laptops \rhd \{lenovo, hp\}$$

   What is the meaning of operator $\rhd$ and where would you deploy such operator in the context of a database management system?

4. What is the result of the expression

   $$\{legion5, xps15, xps13, dragonfly\} \ntriangleleft laptops$$

   What is the meaning of operator $\ntriangleleft$ and where would you deploy such operator in the context of a database management system?

5. What is the result of the expression

$$laptops \rhd \{apple, dell, hp\}$$

What is the meaning of operator $\rhd$ and where would you deploy such operator in the context of a database management system?

6. Consider the following expression

$$laptop \oplus \{ideapad \mapsto lenovo\}$$

(a) What is the result of the expression?

(b) What is the meaning of operator $\oplus$ and where would you deploy such operator in the context of a database management system?

(c) Does the result of the expression have a permanent effect on the database (relation)? If not, describe in detail how would you ensure a permanent effect.

9.2 Answer