

SOEN 331: Introduction to Formal Methods for
Software Engineering

Assignment 1

Propositional and Predicate Logic, Structures,
Binary Relations, Functions and Relational Calculus

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1 Problem 1 (8 pts)

1.1 Problem:

You are shown a set of four cards placed on a table, each of which has a **number** on one side and a **symbol** on the other side. The visible faces of the cards show the numbers **2** and **7**, and the symbols \square , and \bigcirc .

Which card(s) must you turn over in order to test the truth of the proposition that “*If a card has an odd number on one side, then it has the symbol \square on the other side*”? Explain your reasoning by deciding for each card whether it should be turned over and why.

1.2 Answer:

2 Problem 2 (8 pts)

2.1 Description

Consider the predicate $asks(a, b)$ that is interpreted as “ a has asked b out on a date.”

1. Translate the following into English: $\forall a \exists b asks(a, b)$ and $\exists y \forall x asks(a, b)$.
2. Can we claim that $\forall a \exists b asks(a, b) \rightarrow \exists y \forall x asks(a, b)$? Discuss in detail.
3. Can we claim that $\exists y \forall x asks(a, b) \rightarrow \forall a \exists b asks(a, b)$? Discuss in detail.

2.2 Answer

3 Problem 3 (12 pts)

3.1 Description

Let $scientist(x)$ denote the statement “x is a scientist”, and $honest(x)$ denote the statement “x is honest.” Formalize the following sentences and indicate their corresponding formal type.

1. “No scientists are honest.”
2. “All scientists are crooked.”
3. “All scientists are honest.”
4. “Some scientists are crooked.”
5. “Some scientists are honest.”
6. “No scientist is crooked.”
7. “Some scientists are not crooked.”
8. “Some scientists are not honest.”

Identify pairs that are contradictories, contraries, subcontraries, and pairs that support subalternation (clearly indicating superaltern and subaltern)

3.2 Answer

4 Problem 4 (12 pts)

4.1 Description

Consider list $\Lambda = \langle w, x, y, z \rangle$, deployed to implement a Queue Abstract Data Type.

1. Let the head of Λ correspond to the front position of the Queue. Implement operations `enqueue(e1, Λ)` and `dequeue(Λ)` using list construction operations. In both cases we can refer to Λ' as the state of the list upon successful termination of one of its operations.
2. Let us now reverse the way we manipulate our data structure and let the head of Λ correspond to the rear of the Queue.
 - (a) What would be the result of `cons(el, Λ)`, and would it be a correct implementation for operation `enqueue(e1, Λ)`?
 - (b) What would be the result of `list(el, Λ)`, and would it be a correct implementation for operation `enqueue(e1, Λ)`?
 - (c) What would be the result of `concat(list(el), Λ)`, and would it be a correct implementation for operation `enqueue(e1, Λ)`?

4.2 Answer

5 Problem 5 (12 pts)

5.1 Description

Let $A = \{0, 1, 2, 3, 4\}$ and relations R , S , T , and U on A defined as follows:

$$R = \{(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 1), (3, 3), (4, 0), (4, 1), (4, 3), (4, 4)\}$$

$$S = \{(0, 1), (1, 1), (2, 3), (2, 4), (3, 0), (3, 4), (4, 0), (4, 1), (4, 4)\}$$

$$T = \{(0, 3), (0, 4), (2, 1), (3, 2), (4, 2), (4, 3)\}$$

$$U = \{(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (1, 3), (2, 2), (3, 0), (3, 1), (3, 3), (4, 4)\}$$

Fill in the table below, using \checkmark , or \times .

	R	S	T	U
Reflexive				
Irreflexive				
Symmetric				
Asymmetric				
Antisymmetric				
Transitive				
Equivalence				
Partial order				

5.2 Answer

6 Problem 6 (8 pts)

6.1 Description

Consider the relation “*is a subtype of*” over the set $\{\text{rectangle, quadrilateral, square, parallelogram, rhombus}\}$.

1. Is this an *equivalence relation*?
2. Is this relation a *partial order*? If so, create a *Hasse diagram*, and identify *minimal* and *maximal* elements.

6.2 Answer

7 Problem 7 (8 pts)

7.1 Description

Consider the set $A = \{w, x, y, z\}$, and the relations

$$S = \{(w, x), (w, y), (x, w), (x, x), (z, x)\}$$

$$T = \{(w, w), (w, y), (x, w), (x, x), (x, z), (y, w), (y, y), (y, z)\}$$

Find the following compositions:

1. $S \circ T$
2. $T \circ S$
3. $T^{-1} \circ S^{-1}$

NOTE: Some authors (e.g. Rosen) adopt a different ordering of operands than the one we use in our lecture notes. Please follow the ordering (and the definition) of the lecture notes.

7.2 Answer

8 Problem 8 (12 pts)

8.1 Description

Consider sets $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{a, b, c, d, e, f\}$.

1. Determine the type of the correspondence in each of the following cases, or indicate if the correspondence is not a function.

(a) $\{1 \mapsto b, 2 \mapsto c, 3 \mapsto e, 4 \mapsto d, 5 \mapsto f, 3 \mapsto a\}$

(b) $\{1 \mapsto a, 2 \mapsto d, 3 \mapsto a, 4 \mapsto f, 5 \mapsto d, 6 \mapsto c\}$

(c) $\{1 \mapsto c, 2 \mapsto b, 3 \mapsto d, 4 \mapsto e, 5 \mapsto e, 6 \mapsto f\}$

(d) $\{1 \mapsto b, 2 \mapsto c, 3 \mapsto e, 4 \mapsto d, 5 \mapsto f, 6 \mapsto a\}$

Fill in the table below, using \checkmark , or \times .

	Injective	Surjective	Bijjective	Neither injective nor surjective	Not a function
(a)					
(b)					
(c)					
(d)					

2. Is it possible to construct a function $f: A \rightarrow B$ which is surjective and not injective? Discuss.

8.2 Answer

9 Problem 9 (20 pts)

9.1 Description

Consider the following relation:

$$laptops : Model \leftrightarrow Brand$$

where

$$laptops = \{ \begin{array}{l} legion5 \mapsto lenovo, \\ macbookair \mapsto apple, \\ xps15 \mapsto dell, \\ spectre \mapsto hp, \\ xps13 \mapsto dell, \\ swift3 \mapsto acer, \\ macbookpro \mapsto apple, \\ dragonfly \mapsto hp, \\ envyx360 \mapsto hp \end{array} \}$$

1. What is the domain and the range of the relation?
2. What is the result of the expression

$$\{xps15, xps13, swift3, envyx360\} \triangleleft laptops$$

What is the meaning of perator \triangleleft and where would you deploy such operator in the context of a database management system?

3. What is the result of the expression

$$laptops \triangleright \{lenovo, hp\}$$

What is the meaning of operator \triangleright and where would you deploy such operator in the context of a database management system?

4. What is the result of the expression

$$\{legion5, xps15, xps13, dragonfly\} \triangleleft laptops$$

What is the meaning of operator \triangleleft and where would you deploy such operator in the context of a database management system?

5. What is the result of the expression

$$laptops \triangleright \{apple, dell, hp\}$$

What is the meaning of operator \triangleright and where would you deploy such operator in the context of a database management system?

6. Consider the following expression

$$laptop \oplus \{ideapad \mapsto lenovo\}$$

- (a) What is the result of the expression?
- (b) What is the meaning of operator \oplus and where would you deploy such operator in the context of a database management system?
- (c) Does the result of the expression have a permanent effect on the database (relation)? If not, describe in detail how would you ensure a permanent effect.

9.2 Answer