1 Problem Statement

A delivery company in Oklahoma, USA, needs to pick the best spots for two main hubs from its 12 current locations (terminals). The main goal is to find the two hub locations that result in the minimum total transportation cost for the whole year.

The company must follow two simple rules:

- They must choose only two hubs.
- Every delivery spot must connect to only one of these two hubs.

The cost is based on distance, calculated at 0.74 USD per mile. Since drivers must go from the terminal to the hub and then come back to the terminal, the total distance used for cost calculation is always a full round trip. The distances (in miles) between all 12 locations are given in Table 1.

Table 1: Distance Matrix (Miles)

Location	**Altus**	**Ardmore**	**Bartlesville**	**Duncan**	**Edmond**	**Enid**	**Lawton**	**Muskogee**	**OKC**	**Ponca City**	**Stillwater**	**Tulsa**
Altus	0.0	169.8	291.8	88.2	153.9	208.2	54.2	274.2	141.1	245.0	209.2	248.0
Ardmore	169.8	0.0	248.6	75.9	112.5	199.0	115.8	230.4	100.5	202.2	162.6	204.6
Bartlesville	291.8	248.6	0.0	231.5	146.0	132.4	238.7	92.2	151.4	70.2	115.0	45.6
Duncan	88.2	75.9	231.5	0.0	93.5	137.5	34.1	213.5	80.9	184.8	145.3	187.8
Edmond	153.9	112.5	146.0	93.5	0.0	88.8	100.7	145.7	14.4	91.9	53.0	102.2
Enid	208.2	199.0	132.4	137.5	88.8	0.0	145.0	166.4	87.6	64.5	65.8	118.4
Lawton	54.2	115.8	238.7	34.1	100.7	145.0	0.0	220.6	88.0	191.9	152.5	194.9
Muskogee	274.2	230.4	92.2	213.5	145.7	166.4	220.6	0.0	140.4	142.5	119.2	48.1
OKC	141.1	100.5	151.4	80.9	14.4	87.6	88.0	140.4	0.0	104.7	66.6	107.6
Ponca City	245.0	202.2	70.2	184.8	91.9	64.5	191.9	142.5	104.7	0.0	41.9	96.5
Stillwater	209.2	162.6	115.0	145.3	53.0	65.8	152.5	119.2	66.6	41.9	0.0	71.2
Tulsa	248.0	204.6	45.6	187.8	102.2	118.4	194.9	48.1	107.6	96.5	71.2	0.0

Formulate the proposed problem in terms of Integer Linear Programming, implement it using the AMPL modelling language, and solve it in AMPL with appropriate solvers, by discussing the obtained computational results.

2 Mathematical model formulation

The objective of this problem is to minimize the maximum transportation cost between terminals and hubs, while satisfying all model constraints.

2.1 Input Data

- V_1: set of terminals.
- c_{ij}: transportation cost between terminal i and hub j.

We also define $V_2 = V_1$, representing the candidate locations for hub installation.

2.2 Decision Variables

- x_j, $\forall j \in V_2$: binary variable equal to 1 if a hub is located at site j, and 0 otherwise.
- y_{ij}, $\forall i \in V_1, j \in V_2$: binary variable equal to 1 if terminal i is assigned to hub j, and 0 otherwise.

2.3 Mathematical Model

In this case, we assumed w is the maximum transportation cost between terminal and the hub. It represented as the formular below:

$$w = \max_{i \in V_1} \sum_{j \in V_2} c_{ij}\, y_{ij}$$

The model can be formulated as follows:

$$\begin{aligned}
\min \quad & w \\
\text{s.t.} \quad & \sum_{j \in V_2} x_j = 2 && (1) \\
& \sum_{j \in V_2} y_{ij} = 1, \quad \forall i \in V_1 && (2) \\
& y_{ij} \leq x_j, \quad \forall i \in V_1, \forall j \in V_2 && (3) \\
& \sum_{j \in V_2} c_{ij} y_{ij} \leq w, \quad \forall i \in V_1 && (4) \\
& x_j \in \{0,1\}, \quad y_{ij} \in \{0,1\}
\end{aligned}$$

2.4 Model Interpretation

Constraint (1) guarantees that exactly two hubs are opened. Constraint (2) requires each terminal to be allocated to one hub only. Constraint (3) enforces the logical relationship between hub opening and assignment decisions. Constraint (4) defines w as an upper bound of the transportation cost for each terminal. Minimizing w results in minimizing the maximum cost.

We made the example Altus location, constraint (4) becomes:

$$\begin{aligned}
& 251.304\, y_{\text{Altus, Ardmore}} + 431.864\, y_{\text{Altus, Bartlesville}} + 130.536\, y_{\text{Altus, Duncan}} \\
& + 227.772\, y_{\text{Altus, Edmond}} + 308.136\, y_{\text{Altus, Enid}} + 80.216\, y_{\text{Altus, Lawton}} \\
& + 405.816\, y_{\text{Altus, Muskogee}} + 208.828\, y_{\text{Altus, OklahomaCity}} \\
& + 362.600\, y_{\text{Altus, PoncaCity}} + 309.616\, y_{\text{Altus, Stillwater}} + 367.040\, y_{\text{Altus, Tulsa}} \leq w
\end{aligned}$$

3 Implementation

The implementation of solving this mathematical model was showed in the code section. The problem was solved using the CPLEX solver, which returned a transportation cost w equal to 175.23 USD.

From the assignment matrix 2, it is observed that only the Duncan and Tulsa columns contain non-zero values, which confirms that the solver has selected these two sites as hub locations, in accordance with constraint 1. This allocation reflects the optimal configuration of the network: the demand nodes are divided into two service regions, each being efficiently served by one of the two selected hubs.

The results are as follows:

- Terminals Altus, Ardmore, Duncan, Edmond, Lawton and Oklahoma City are assigned to the hub located in Duncan.
- Terminals Bartlesville, Enid, Muskogee, Ponca City, Stillwater and Tulsa are assigned to the hub located in Tulsa.

Table 2: Terminal-to-Hub Assignment Matrix (y_{ij})

Terminal	Altus	Ardmore	Bartlesville	Duncan	Edmond	Enid	Lawton	Muskogee	OKC	Ponca City	Stillwater	Tulsa
Altus				x								
Ardmore				x								
Bartlesville												x
Duncan				x								
Edmond				x								
Enid												x
Lawton				x								
Muskogee												x
Oklahoma City				x								
Ponca City												x
Stillwater												x
Tulsa												x