

Lớp: Phân tích và thiết kế thuật toán – CS112.N22

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Bài học: Design and Analysis of Algorithms (Độ phức tạp thuật toán không đệ quy)

Nhóm 3:

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Ex1:

Câu 1:

```
//3
sum = 0;
for (i = 0; i < n; i++)
    for (j = i + 1; j <= n; j++)
        for (k = 1; k < 10; k++)
            sum = sum + i * j * k;
```

$$T_1 = O(1)$$

$$T_5 = O(1)$$

$$T_4 = O(1)$$

$$T_{45} = T_4 * T_5 = O(1) * O(1) = O(1)$$

$$T_3 = O(n)$$

$$T_{345} = T_3 * T_{45} = O(n) * O(1) = O(n)$$

$$T_2 = O(n)$$

$$T_{2345} = T_2 * T_{345} = O(n) * O(n) = O(n^2)$$

$$T_{12345} = T_1 + T_{2345} = O(1) + O(n^2) = O(n^2)$$

Câu 2:

```
//5
sum = 0;
thisSum = 0;
for (i = 0; i < n; i++) {
    thisSum += b[i];
    if (thisSum > sum)
        sum = thisSum;
    else
        thisSum = sum;
}
```

$$T_1 = O(1)$$

$$T_2 = O(1)$$

$$T_{12} = T_1 + T_2 = O(1) + O(1) = O(1)$$

$$T_6 = O(1)$$

$$T_8 = O(1)$$

$$T_5 = O(1)$$

$$T_{5678} = T_5 + \min(T_5, T_7) = O(1) + \min(O(1), O(1)) = O(1)$$

$$T_4 = O(1)$$

$$T_{45678} = T_4 + T_{5678} = O(1) + O(1) = O(1)$$

$$T_3 = O(n)$$

$$T_{3456789} = T_3 * T_{45678} = O(n) * O(1) = O(n)$$

$$T_{123456789} = T_{12} + T_{3456789} = O(1) + O(n) = O(n)$$

Ex2:

Câu 1:

```
//3
sum = 0;
i = 1;
while(i<=n) {
    j = n-i;
    while(j<=i) {
        sum = sum+j;
        j=j+1;
    }
    i=i+1;
}
```

1: 1 gán

2: 1 gán

3: $n + 1$ so sánh

4: n gán

5: $[\sum_{i=1}^n 0 \text{ if } n - i > i \text{ else } i - (n - i) + 1] + n$ so sánh

6: $\sum_{i=1}^n 0 \text{ if } n - i > i \text{ else } i - (n - i) + 1$ gán

7: $\sum_{i=1}^n 0 \text{ if } n - i > i \text{ else } i - (n - i) + 1$ gán

8: không tính('{}')

9: n gán

10: không tính('{}')

$$\begin{aligned}
& \sum_{i=1}^n 0 \text{ if } n - i > i \text{ else } i - (n - i) + 1 \\
&= \sum_{i=1}^n 2i - n + 1 \text{ if } i \geq \frac{n}{2} \\
&= \sum_{i=\frac{n}{2}}^n 2i - n + 1 \text{ if } n \% 2 == 0 \text{ else } \sum_{i=\frac{n+1}{2}}^n 2i - n + 1 \\
&= \frac{(\frac{n}{2}+1) * (2 * \frac{n}{2} + \frac{n}{2} * 2)}{2} - (\frac{n}{2} + 1) * n + (\frac{n}{2} + 1) \text{ if } n \% 2 == 0 \\
&\text{Else } \frac{(\frac{n+1}{2}) * [2 * (n+1) + (\frac{n-1}{2}) * 2]}{2} - (\frac{n+1}{2}) * n + (\frac{n+1}{2}) \\
&= \frac{n^2}{4} + n + 1 \text{ if } n \% 2 == 0 \text{ else } \frac{n^2}{4} + n + \frac{3}{4} \\
&\text{Gán}(n) = 2n + 2 + 2 * (\frac{n^2}{4} + n + 1 \text{ if } n \% 2 == 0 \text{ else } \frac{n^2}{4} + n + \frac{3}{4}) \\
&= \frac{n^2}{2} + 4n + 4 \text{ if } n \% 2 == 0 \text{ else } \frac{n^2}{2} + 4n + \frac{7}{2} \\
&\text{So sánh}(n) = n + 1 + (\frac{n^2}{4} + n + 1 \text{ if } n \% 2 == 0 \text{ else } \frac{n^2}{4} + n + \frac{3}{4}) + n \\
&= \frac{n^2}{4} + 3n + 2 \text{ if } n \% 2 == 0 \text{ else } \frac{n^2}{4} + 3n + \frac{7}{4} \\
&\text{T}(n) = \text{Gán}(n) + \text{So Sánh}(n) \\
&= (\frac{n^2}{2} + 4n + 4 \text{ if } n \% 2 == 0 \text{ else } \frac{n^2}{2} + 4n + \frac{7}{2}) \\
&\quad + (\frac{n^2}{4} + 3n + 2 \text{ if } n \% 2 == 0 \text{ else } \frac{n^2}{4} + 3n + \frac{7}{4}) \\
&= \frac{3n^2}{4} + 7n + 6 \text{ if } n \% 2 == 0 \text{ else } \frac{3n^2}{4} + 7n + \frac{21}{4} \\
&\Rightarrow \text{T}(n) = O(n^2)
\end{aligned}$$

Câu 2:

```

//4
s = 0;
i = 1;
while(i <= n){
    j = 1;
    while(j <= i*i) {
        s = s + 1;
        j = j + 1;
    }
    i = i + 1;
}

```

1: 1 gán

2: 1 gán

3: $n + 1$ so sánh

4: n gán

5: $(\sum_{i=1}^n i^2) + n$ so sánh

6: $\sum_{i=1}^n i^2$ gán

7: $\sum_{i=1}^n i^2$ gán

8: không tính('}')

9: n gán

10: không tính('}')

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\text{Gán}(n) = 2n + 2 + 2 * \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} \right) = \frac{2n^3}{3} + n^2 + \frac{7n}{3} + 2$$

$$\text{So sánh}(n) = n+1 + \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} \right) + n = \frac{n^3}{3} + \frac{n^2}{2} + \frac{13n}{6} + 1$$

$$T(n) = \text{Gán}(n) + \text{So sánh}(n)$$

$$= \left(\frac{2n^3}{3} + n^2 + \frac{7n}{3} + 2 \right) + \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{13n}{6} + 1 \right)$$

$$= n^3 + \frac{3n^2}{2} + \frac{9n}{2} + 3$$

$$\Rightarrow T(n) = O(n^3)$$

Câu 3:

```
//5
sum = 0;
i = 1;
while (i <= n) {
    j = n - i*i;
    while (j <= i*i) {
        sum = sum + i*j;
        j = j+1;
    }
    i = i+1;
}
```

1: 1 gán

2: 1 gán

3: $n + 1$ so sánh

4: n gán

5: $(\sum_{i=1}^n i^2 - (n - i^2) + 1 \text{ if } i^2 \geq n - i^2) + n$ so sánh

6: $\sum_{i=1}^n i^2 - (n - i^2) + 1 \text{ if } i^2 \geq n - i^2$ gán

7: $\sum_{i=1}^n i^2 - (n - i^2) + 1 \text{ if } i^2 \geq n - i^2$ gán

8: không tính('{}')

9: n gán

10: không tính('{}')

$$\sum_{i=1}^n i^2 - (n - i^2) + 1 \text{ if } i^2 \geq n - i^2$$

$$= \sum_{i=1}^n 2i^2 - n + 1 \text{ if } i \geq \sqrt[2]{\frac{n}{2}}$$

$$= \sum_{i=1}^n 2i^2 - n + 1 - \sum_{i=1}^{\sqrt[2]{n/2}} 2i^2 - n + 1$$

$$= 2 * \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} \right) - 2 * \left(\frac{\sqrt[2]{n/2}^3}{3} + \frac{\sqrt[2]{n/2}^2}{2} + \frac{\sqrt[2]{n/2}}{6} \right)$$

$$- (n - \sqrt[2]{n/2})(n - 1)$$

$$= \frac{2n^3}{3} + \frac{2n^{1,5}}{3\sqrt{2}} + \frac{5n}{6} - \frac{4n^{0,5}}{3\sqrt{2}}$$

$$\text{Gán}(n) = 2n + 2 + 2 * \left(\frac{2n^3}{3} + \frac{2n^{1,5}}{3\sqrt{2}} + \frac{5n}{6} - \frac{4n^{0,5}}{3\sqrt{2}} \right)$$

$$= \frac{4n^3}{3} + \frac{4n^{1,5}}{3\sqrt{2}} + \frac{11n}{3} - \frac{8n^{0,5}}{3\sqrt{2}} + 2$$

$$\text{So sánh}(n) = n + 1 + \left(\frac{2n^3}{3} + \frac{2n^{1,5}}{3\sqrt{2}} + \frac{5n}{6} - \frac{4n^{0,5}}{3\sqrt{2}} \right) + n$$

$$= \frac{2n^3}{3} + \frac{2n^{1,5}}{3\sqrt{2}} + \frac{17n}{6} - \frac{4n^{0,5}}{3\sqrt{2}} + 1$$

$$T(n) = \text{Gán}(n) - \text{So sánh}(n)$$

$$= \left(\frac{4n^3}{3} + \frac{4n^{1,5}}{3\sqrt{2}} + \frac{11n}{3} - \frac{8n^{0,5}}{3\sqrt{2}} + 2 \right) + \left(\frac{2n^3}{3} + \frac{2n^{1,5}}{3\sqrt{2}} + \frac{17n}{6} - \frac{4n^{0,5}}{3\sqrt{2}} + 1 \right)$$

$$= 2n^3 + \frac{2n^{1,5}}{\sqrt{2}} + \frac{13n}{2} - \frac{4n^{0,5}}{\sqrt{2}} + 3$$

$$\Rightarrow T(n) = O(n^3)$$