NT219- Cryptography

Week 4: Modern Symmetric Ciphers

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What is cryptograph?

- Cryptology= Cryptography + Cryptanalysis

What?

Goals

- Confidentiality
- Privacy

Cipher systems

- Sysmmetric (AES)
- Asymmetric (RSA, ECC, CRYSTALS-KYBER)

- Integrity
- Authentication

Hash functions
Message authentication code (MAC)
Digital signature (digital certificate)

- Non-repudiation (Accountability)

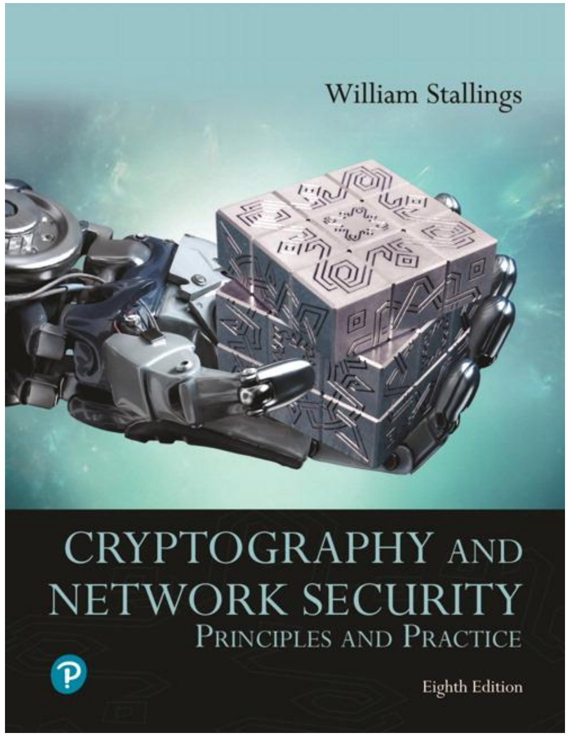
- Availability

Outline

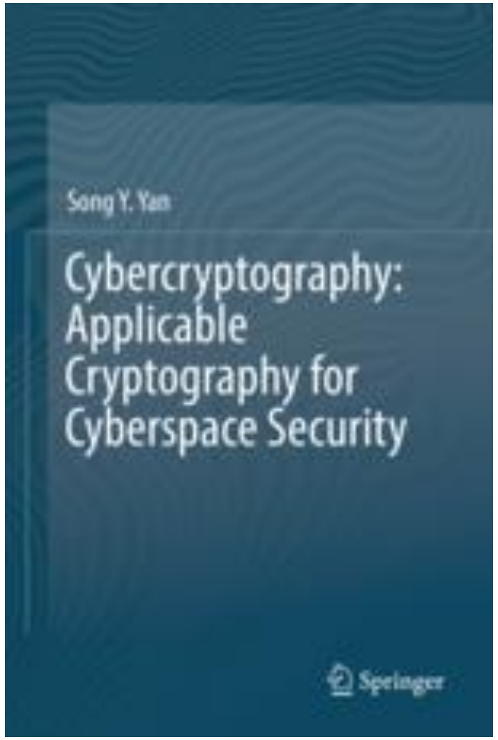
- Cryptanalysis Stream Cipher
- Block cipher
  - Data Encryption Standard (DES)
  - Advanced Encryption Standard (AES)
  - Some other ciphers
    - Searchable encryption

Textbooks and References

- Text books

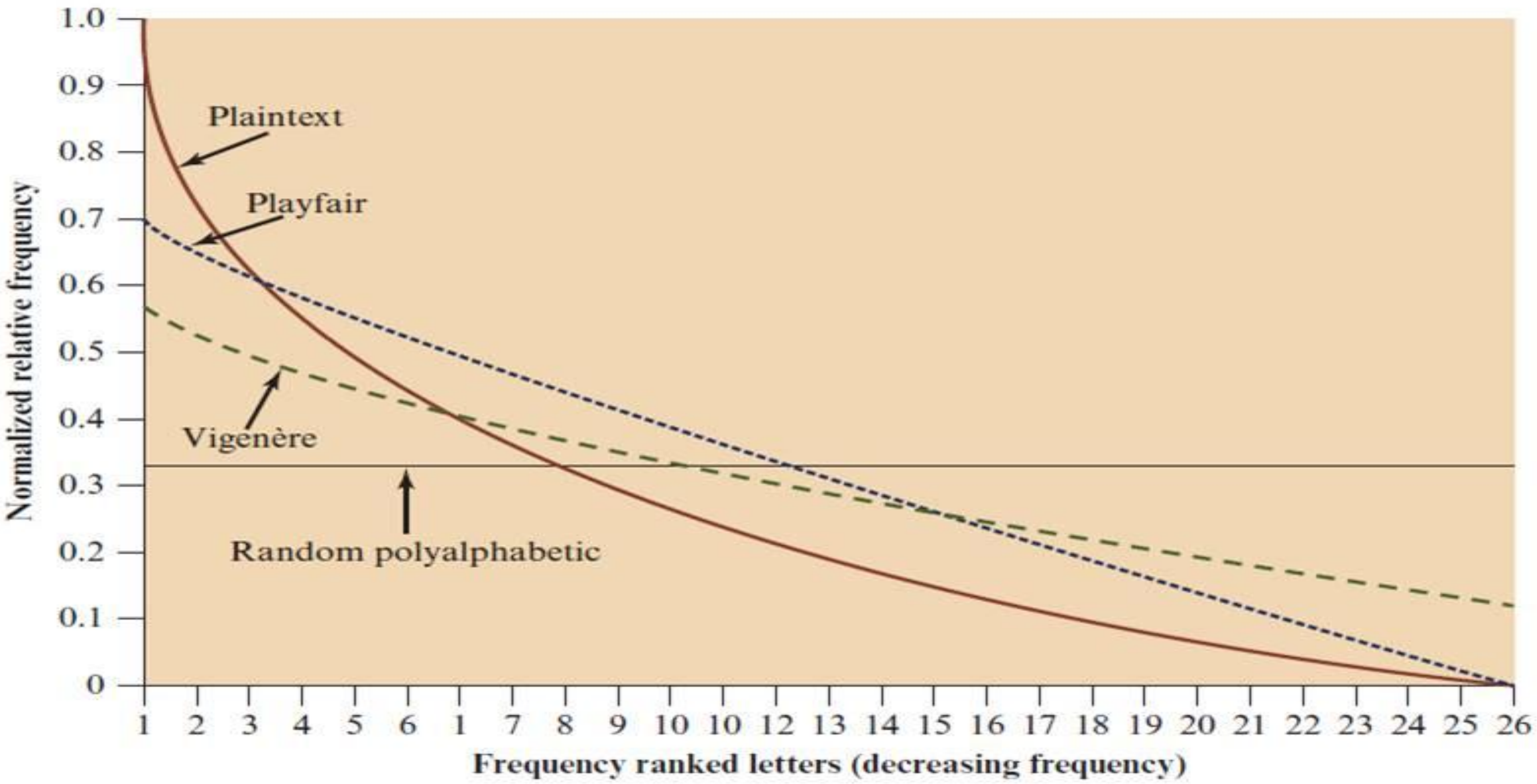


[1] Chapter 4,6



[2] Chapter 5

Classical symmetric cipher cryptanalysis



Transposition ciphers

Goals: scrambles the positions of characters

(1) Rail fence cipher

(2) Columnar Transposition Cipher

https://en.wikipedia.org/wiki/Transposition_cipher

Columnar Transposition Cipher

- Is a more complex transposition
- Write the message in a rectangle, row by row, and read the message off, column by column, but permute the order of the columns
  - The order of the columns then becomes the key to the algorithm

Key:	4	3	1	2	5	6	7
Plaintext	a	t	t	a	c	k	p
	o	s	t	p	o	n	e
	d	u	n	t	i	l	t
	w	o	a	m	x	y	z

Ciphertext

Ciphertext: TTNAAPTMTSUOAODWCOIXKNLYPETZ

Stream Cipher (1 of 8)

Vigenère cipher

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

- **Plaintext stream**

M	=	A	T	T	A	C	K	A	T	D	A	W	N
		0	19	19	0	2	10	0	19	3	0	22	13

- **Secret key (Keystream)**

K'	=	L	E	M	O	N	L	E	M	O	N	L	E
		11	4	12	14	13	11	4	12	14	13	11	4

➢ **Ciphertext**

C	=	L	X	F	O	P	V	E	F	R	N	H	R
		11	23	5	14	15	21	4	5	17	13	7	17

$C = c_1 c_2 \cdots c_i \cdots$ where $c_i = m_i + k_i \bmod 26$

Stream Cipher

- **Secret key (Keystream)**

$$K = k_1 k_2 \cdots k_i \cdots$$

- **Plaintext stream**

$$M = m_1 m_2 \cdots m_i \cdots$$

m_i : bit or byte

- **Ciphertext**

$$C = c_1 c_2 \cdots c_i \cdots$$

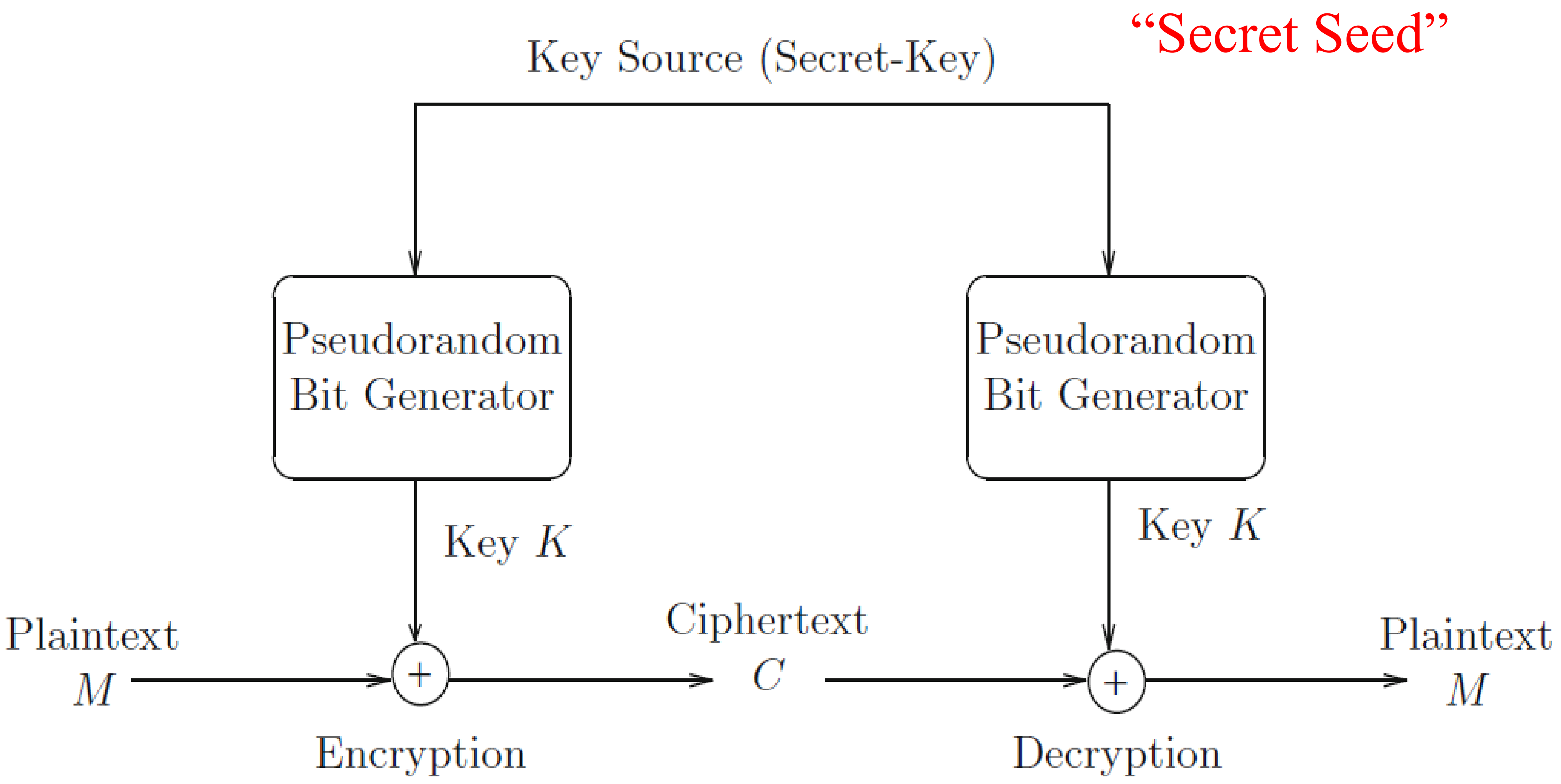
where $c_i = m_i \overline{\oplus} k_i$

$k1 \quad k2 \quad k3 \quad \ldots \quad k_n$

$m1 \quad m2 \quad m3 \quad \ldots \quad m_n$

$k1 \oplus m1 \quad k2 \oplus m2 \quad \ldots \quad k_n \oplus m_n$

Cryptanalysis Stream Cipher



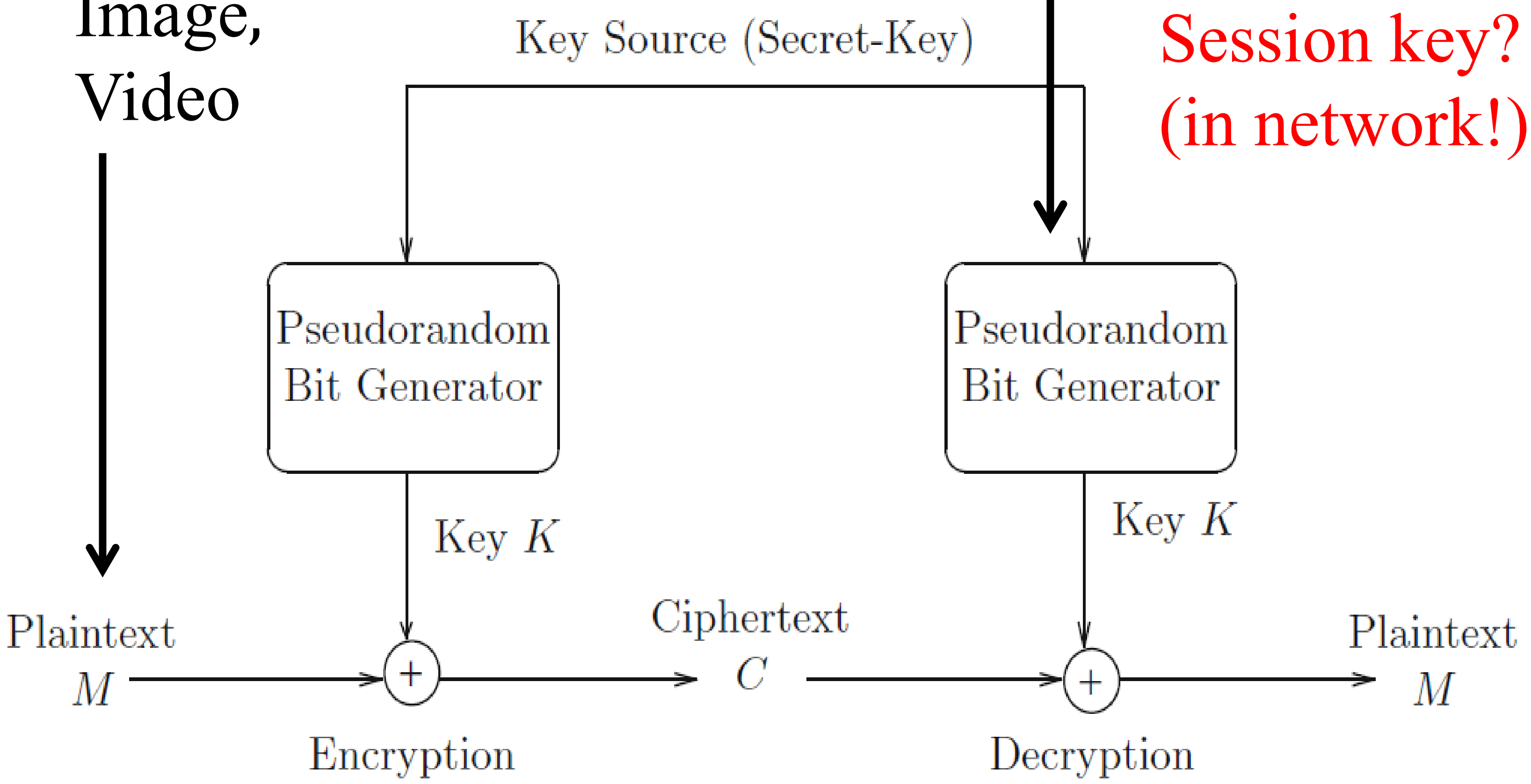
Cryptanalysis Stream Cipher

$C_1 = K \oplus P_1$

Chosen plaintext attack!

- Known: (P_1, C_1)
- K?
- Attack other cipher C_n

Image, Video



One-time key!
Session key?
(in network!)

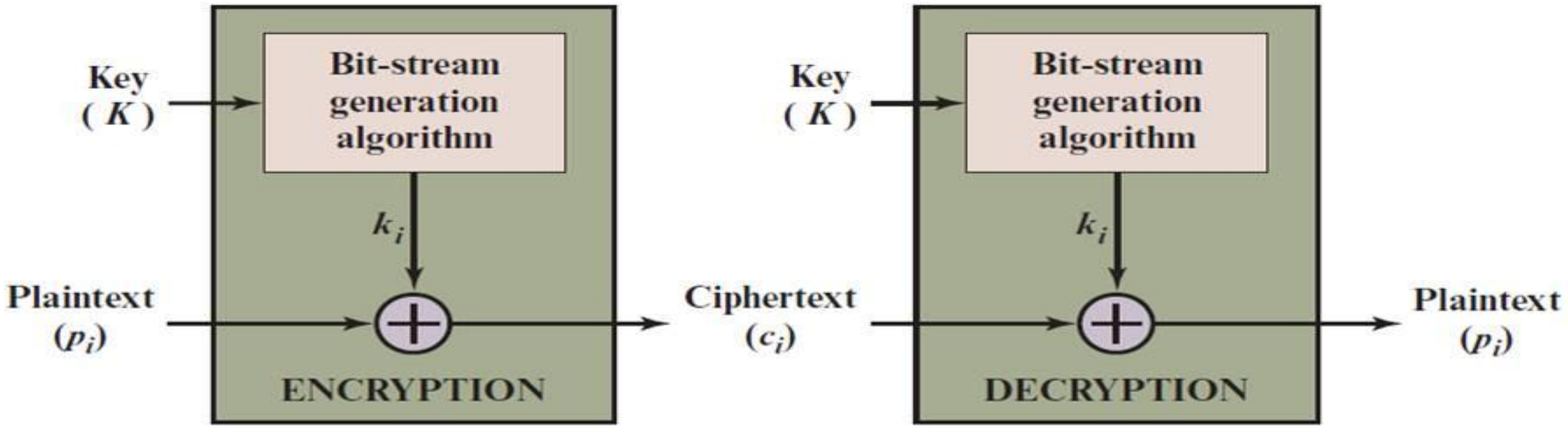
Outline

- Stream Cipher
- Block cipher
  - Data Encryption Standard (DES)
  - Advanced Encryption Standard (AES)
  - Some other ciphers
    - Searchable encryption

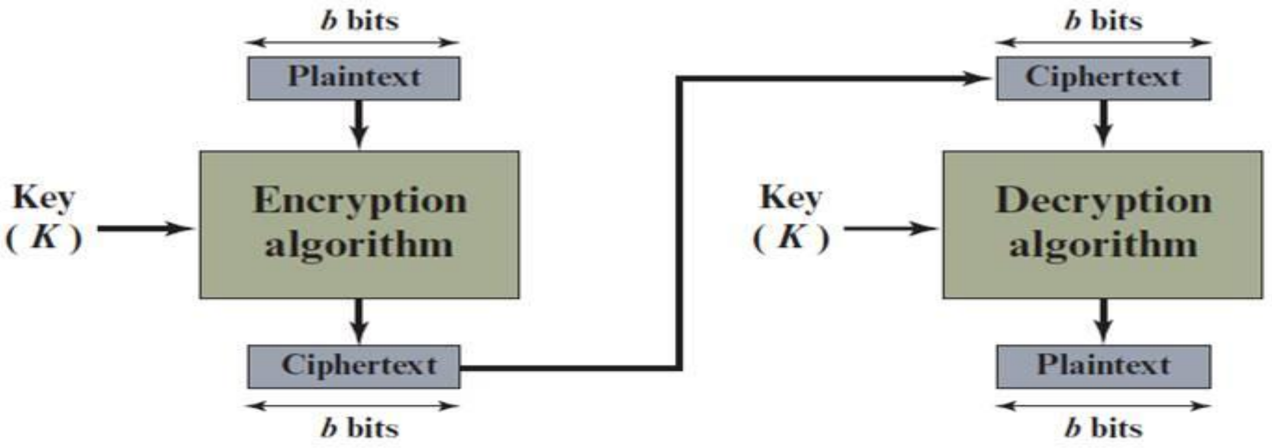
Block Cipher

- **A *block of plaintext* is treated as a whole** and used to produce a *Ciphertext block of equal length*
- Typically, a block size of 64 or 128 bits is used
- As with a stream cipher, the two users share a symmetric encryption key
- **The majority of network-based symmetric cryptographic applications make use of block ciphers**

Stream Cipher Vs. Block Cipher



(a) Stream cipher using algorithmic bit-stream generator



(b) Block cipher

Encryption and Decryption Tables for Substitution Cipher

Mono Alphabetic Substitution

```
A   B   C   D   E   F   G   H   I   J   K   L   M   N   O   P   Q   R   S   T   U   V   W   X   Y
--+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+---+-
U   G   E   T   J   M   A   P   R   V   X   S   K   I   O   L   W   B   D   Z   C   Y   H   Q   F
```

BLock?

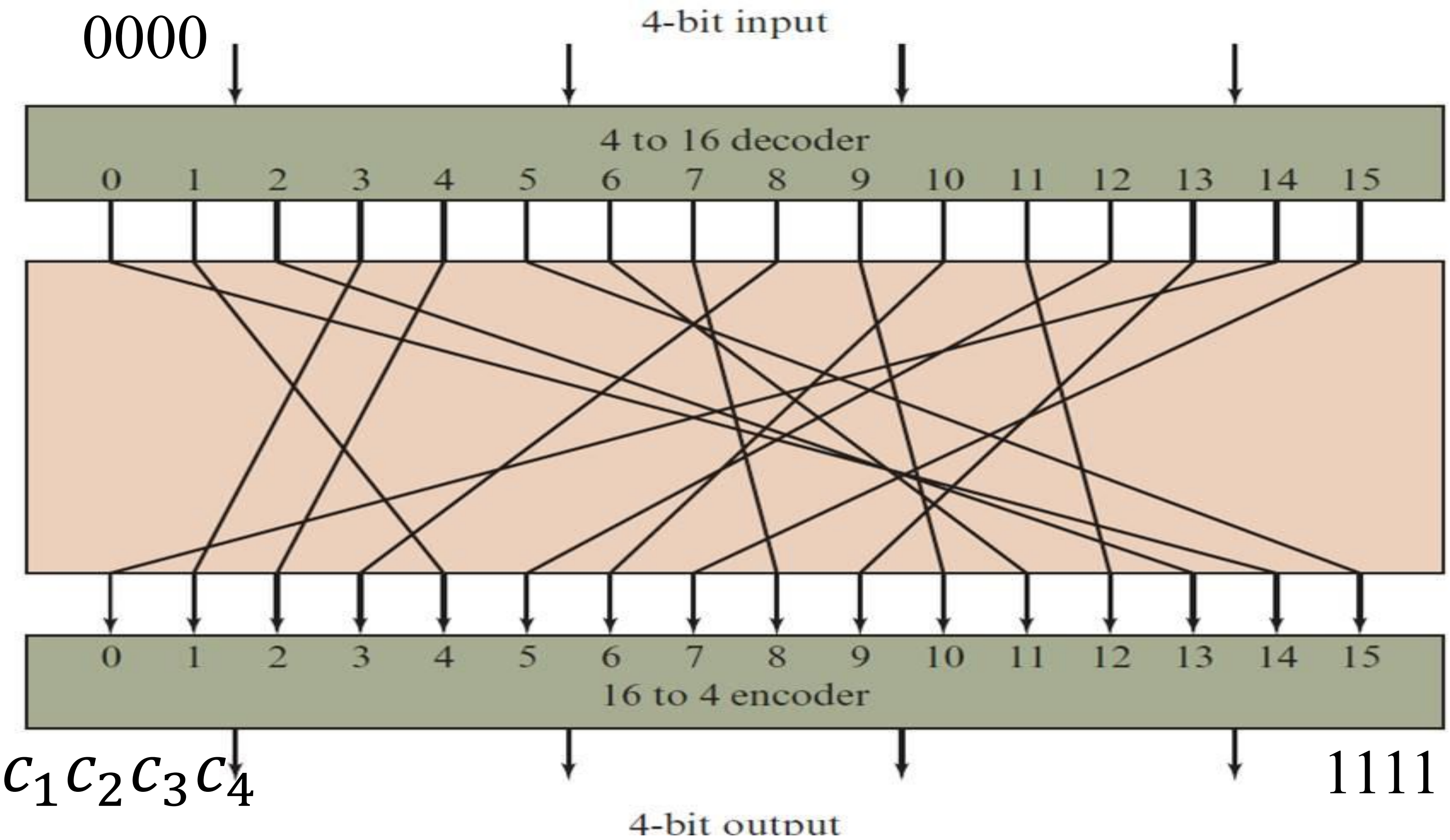
Plaintext	Ciphertext
0000	1110
0001	0100
0010	1101
0011	0001
0100	0010
0101	1111
0110	1011
0111	1000
1000	0011
1001	1010
1010	0110
1011	1100
1100	0101
1101	1001
1110	0000

Block Substitution

$b_1 b_2 b_3 b_4$

4-bits block substitution

0000



$Keys: 2^4!$

$c_1 c_2 c_3 c_4$

1111

Block Substitution

How many possible substitutions for a block n-bit?

$b_1 b_2 b_3 \ldots . b_n$

$c_1 c_2 c_3 \ldots . c_n$

$Keys\colon\ 2^n!$

Feistel Cipher

- Feistel proposed the use of a cipher that alternates substitutions and permutations
- **Substitutions**
  - Each plaintext element or group of elements is uniquely replaced by a corresponding ciphertext element or group of elements
- **Permutation**
  - No elements are added or deleted or replaced in the sequence, rather the order in which the elements appear in the sequence is changed
- Is a practical application of a proposal by Claude Shannon to develop a product cipher that alternates confusion and diffusion functions
- Is the structure used by many significant symmetric block ciphers currently in use

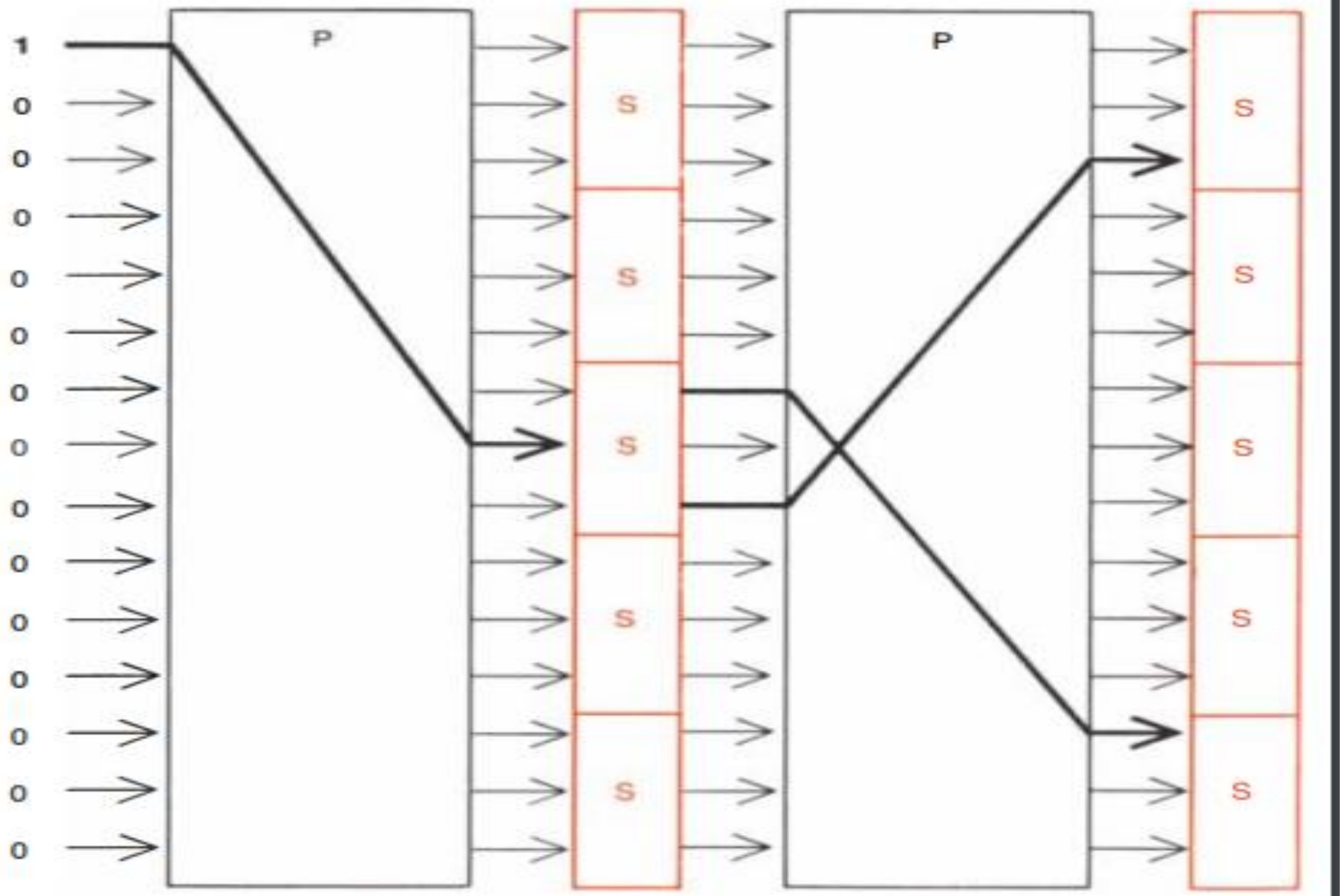
Feistel, H. (1973). Cryptography and computer privacy. Scientific american, 228(5), 15-23.

Diffusion and Confusion

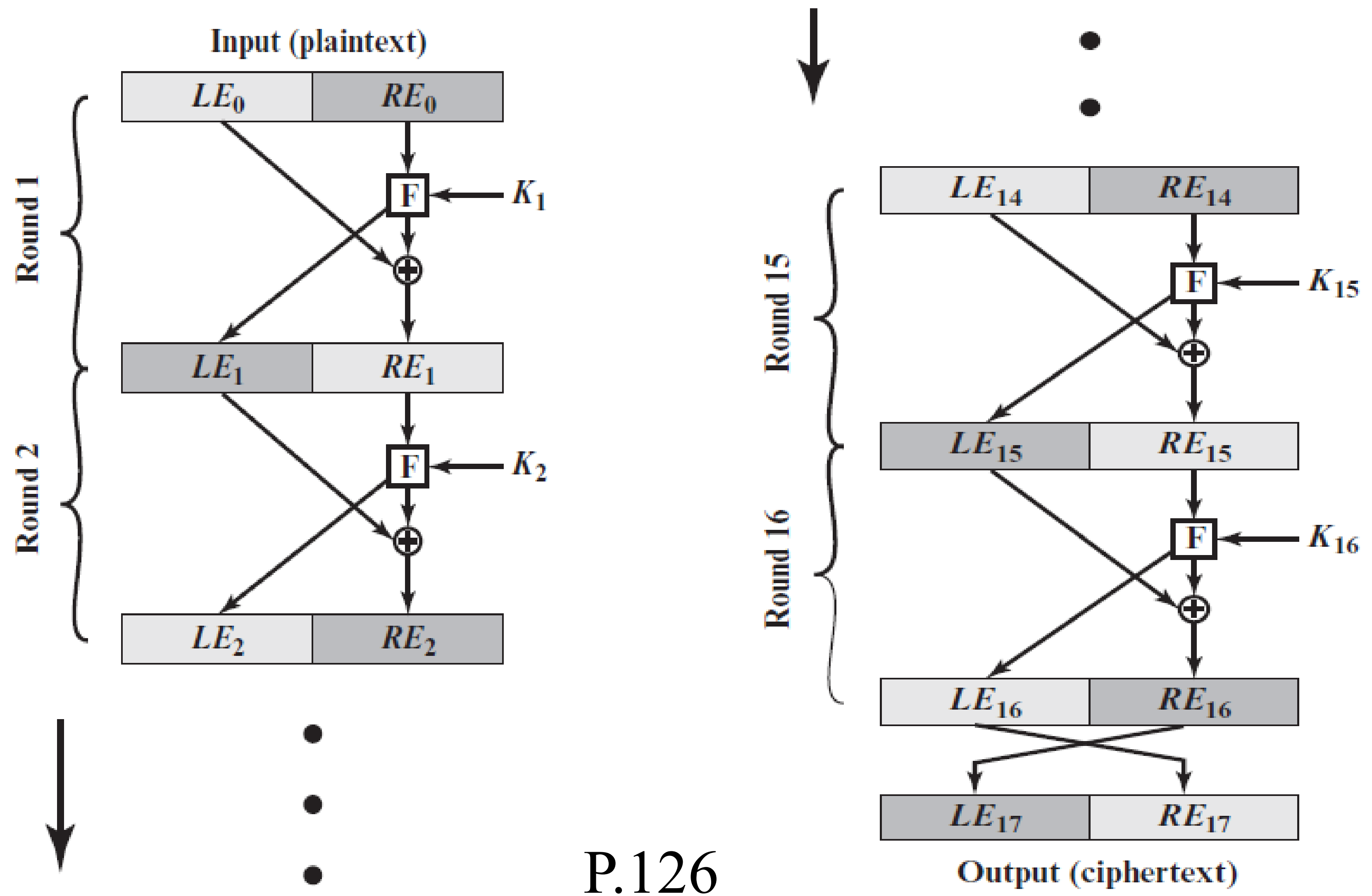
- Terms introduced by Claude Shannon to capture the two basic building blocks for any cryptographic system
  - Shannon's concern was to thwart cryptanalysis based on statistical analysis

- **Diffusion**
  - The statistical structure of the **plaintext** is dissipated into long-range statistics of the **ciphertext**
  - This is achieved by having each plaintext digit affect the value of many ciphertext digits
- **Confusion**
  - Seeks to make the relationship between the statistics of the **ciphertext** and the value of the **encryption key** as complex as possible
  - Even if the attacker can get some handle on the statistics of the ciphertext, the way in which the key was used to produce that ciphertext is so complex as to make it difficult to deduce the key

Feistel Cipher



Feistel Encryption and Decryption (16 rounds)



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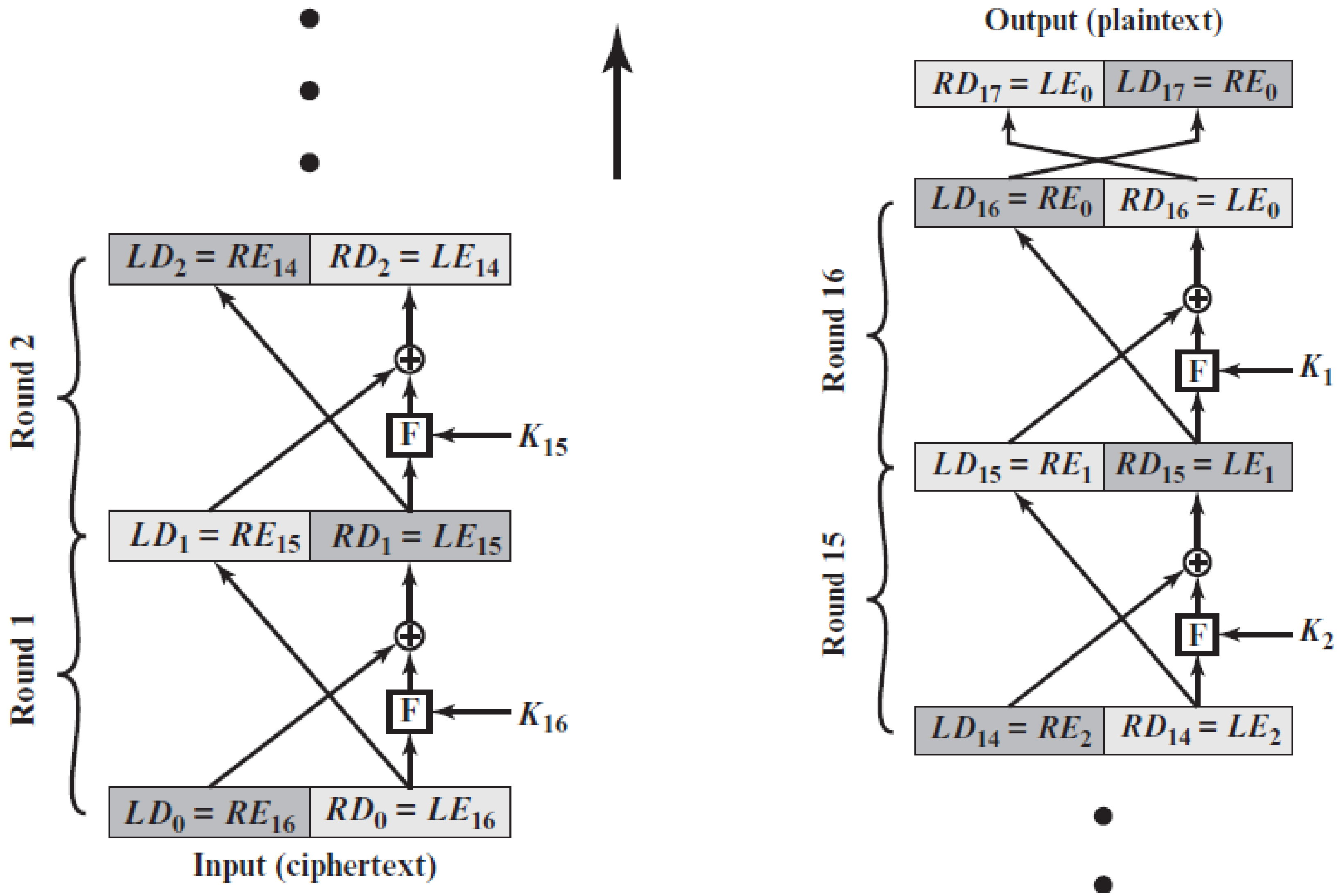
Feistel Encryption and Decryption (16 rounds)



The Feistel Cipher Scheme (FCS)

- Divide M into blocks of $2l$-bits long (pad the last block if needed)
- Use only the XOR and Substitution operations
- Generate *n* sub-keys of a fixed length from the encryption key K: $K_1,\ldots,K_n$
- Divide a $2l$-bit block input into two parts: L_0 and R_0, both of size l (the suffix and prefix of the block, respectively)
- Perform a substitution function F on an l-bit input string and a sub-key to produce an l-bit output
- Encryption and decryption each executes n rounds of the same sequence of operations

FCS Encryption and Decryption

FCS Encryption

- Let $M = L_0R_0$; execute the following operations in round i, $i = 1, \ldots, n$:

  $L_i = R_{i-1}$

  $R_i = L_{i-1} \oplus F(R_{i-1}, K_i)$

- Let $L_{n+1} = R_n$, $R_{n+1} = L_n$ and $C = L_{n+1}R_{n+1}$

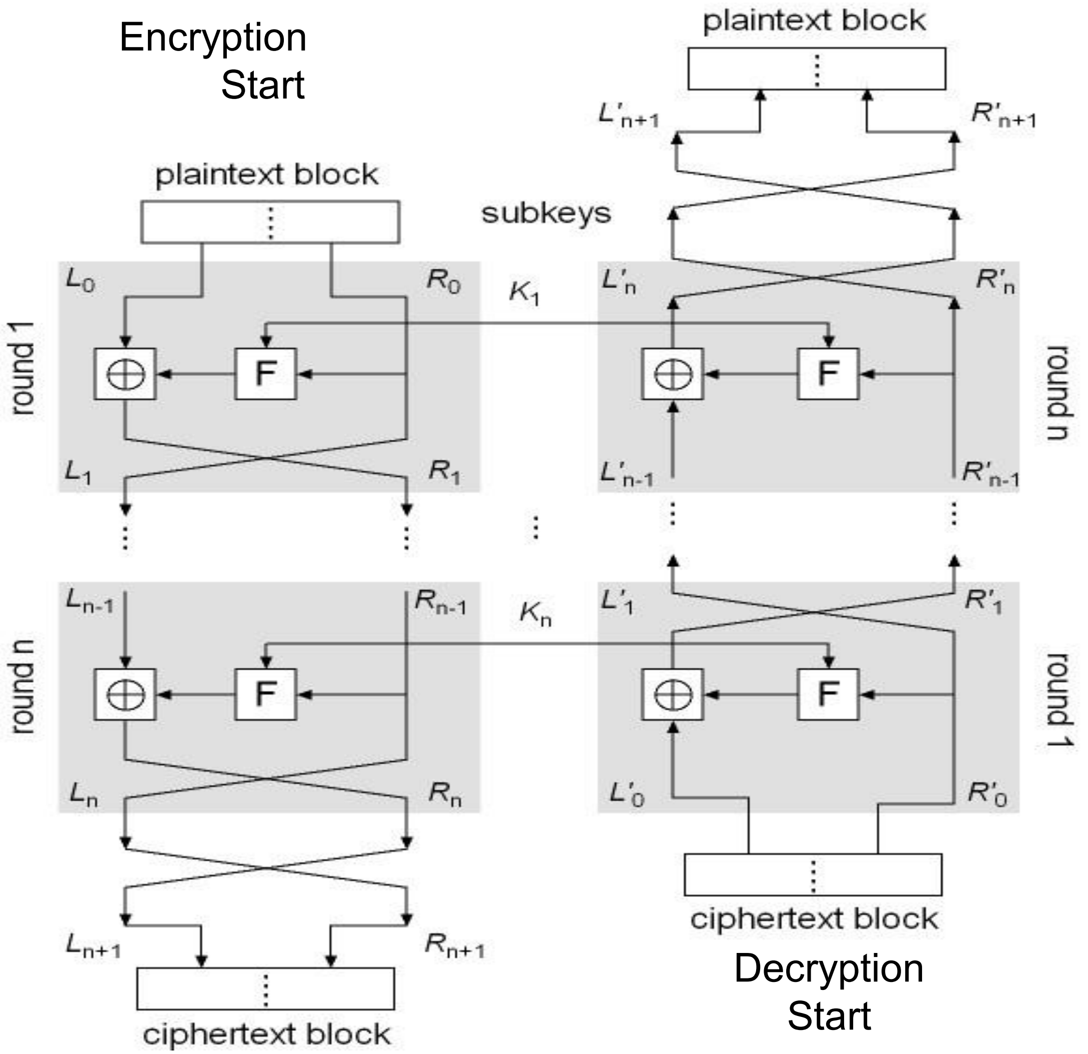
FCS Decryption

- Symmetrical to encryption, with sub-keys in reverse order
- Rewrite C as $C = L'_0R'_0$
- Execute the following in round i ($i = 1, \ldots, n$):

  $L'_i = R'_{i-1}$

  $R'_i = L'_{i-1} \oplus F(R'_{i-1}, K'_{n-i+1})$

- Let $L'_{n+1} = R'_n$, $R'_{n+1} = L'_n$
- We will show that $M = L'_{n+1}R'_{n+1}$



Proof of FCS decryption

- Will show that $C = L_{n+1}R_{n+1} = L'_0R'_0$ is transformed back to $M = L_0R_0$ by the FCS Decryption algorithm
- Prove by induction the following equalities:

  (1) $L'_i = R_{n-i}$ (2) $R'_i = L_{n-i}$

- **Basis**: $L_0' = L_{n+1} = R_n$, $R_0' = R_{n+1} = L_n$; (1) and (2) hold
- **Hypothesis**: Assume when $i \leq n$:

  $L_{i-1}' = R_{n-(i-1)}$ $R_{i-1}' = L_{n-(i-1)}$

- **Induction step**:
  $L'_i = R'_{i-1}$ (by decrypt. alg.) $= L_{n-i+1}$ (by hypothesis) $= R_{n-i}$ (by encrypt. alg.)
  Hence (1) is true

- $R'_i = L'_{i-1} \oplus F(R'_{i-1}, K_{n-i+1})$
  $= R_{n-(i+1)} \oplus F(L_{n-(i+1)}, K_{n-i+1})$
  $= [L_{n-i} \oplus F(R_{n-i}, K_{n-i+1})] \oplus F(R_{n-i}, K_{n-i+1})$
  $= L_{n-i}$
  Hence (2) true

Feistel Cipher Design Features (1 of 2)

- Block size
  - Larger block sizes mean greater security but reduced encryption/decryption speed for a given algorithm
- Key size
  - Larger key size means greater security but may decrease encryption/decryption speeds
- Number of rounds
  - The essence of the Feistel cipher is that a single round offers inadequate security but that multiple rounds offer increasing security
- Subkey generation algorithm
  - Greater complexity in this algorithm should lead to greater difficulty of cryptanalysis

Feistel Cipher Design Features (2 of 2)

- Round function F
  - Greater complexity generally means greater resistance to cryptanalysis
- Fast software encryption/decryption
  - In many cases, encrypting is embedded in applications or utility functions in such a way as to preclude a hardware implementation; accordingly, the speed of execution of the algorithm becomes a concern
- Ease of analysis
  - If the algorithm can be concisely and clearly explained, it is easier to analyze that algorithm for cryptanalytic vulnerabilities and therefore develop a higher level of assurance as to its strength

Outline

- Stream Cipher
- Block cipher
  - Data Encryption Standard (DES)
  - Advanced Encryption Standard (AES)
  - Some other ciphers
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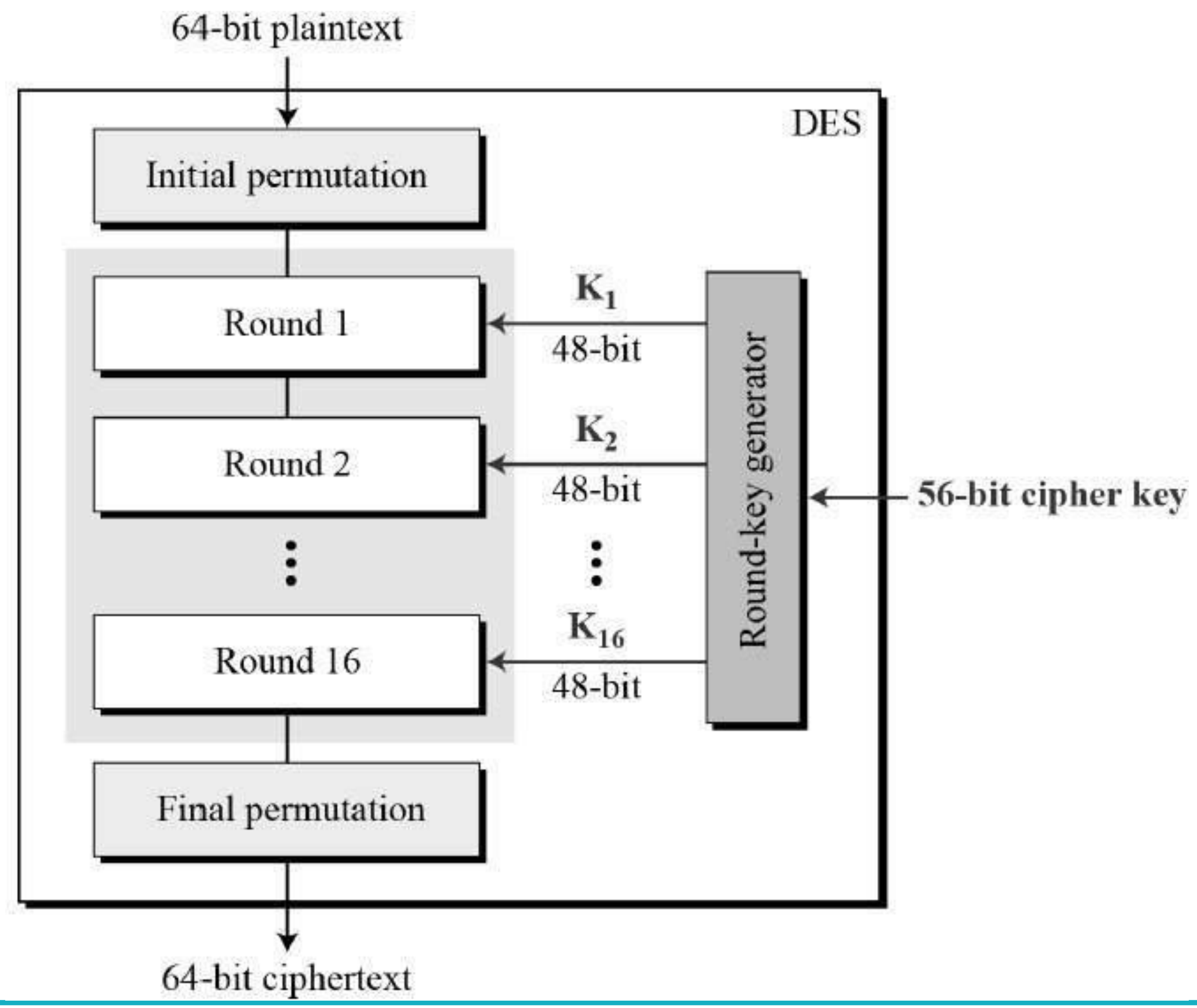
Data Encryption Standard (DES)

- Issued in 1977 by the National Bureau of Standards (now NIST) as Federal Information Processing Standard 46
- Was the most widely used encryption scheme until the introduction of the Advanced Encryption Standard (AES) in 2001
- Algorithm itself is referred to as the Data Encryption Algorithm (DEA)
  - Data are encrypted in 64-bit blocks using a 56-bit key
  - The algorithm transforms 64-bit input in a series of steps into a 64-bit output
  - The same steps, with the same key, are used to reverse the encryption

DES Encryption Algorithm



DES encryption steps

- Rewrite $IP(M) = L_0R_0$,

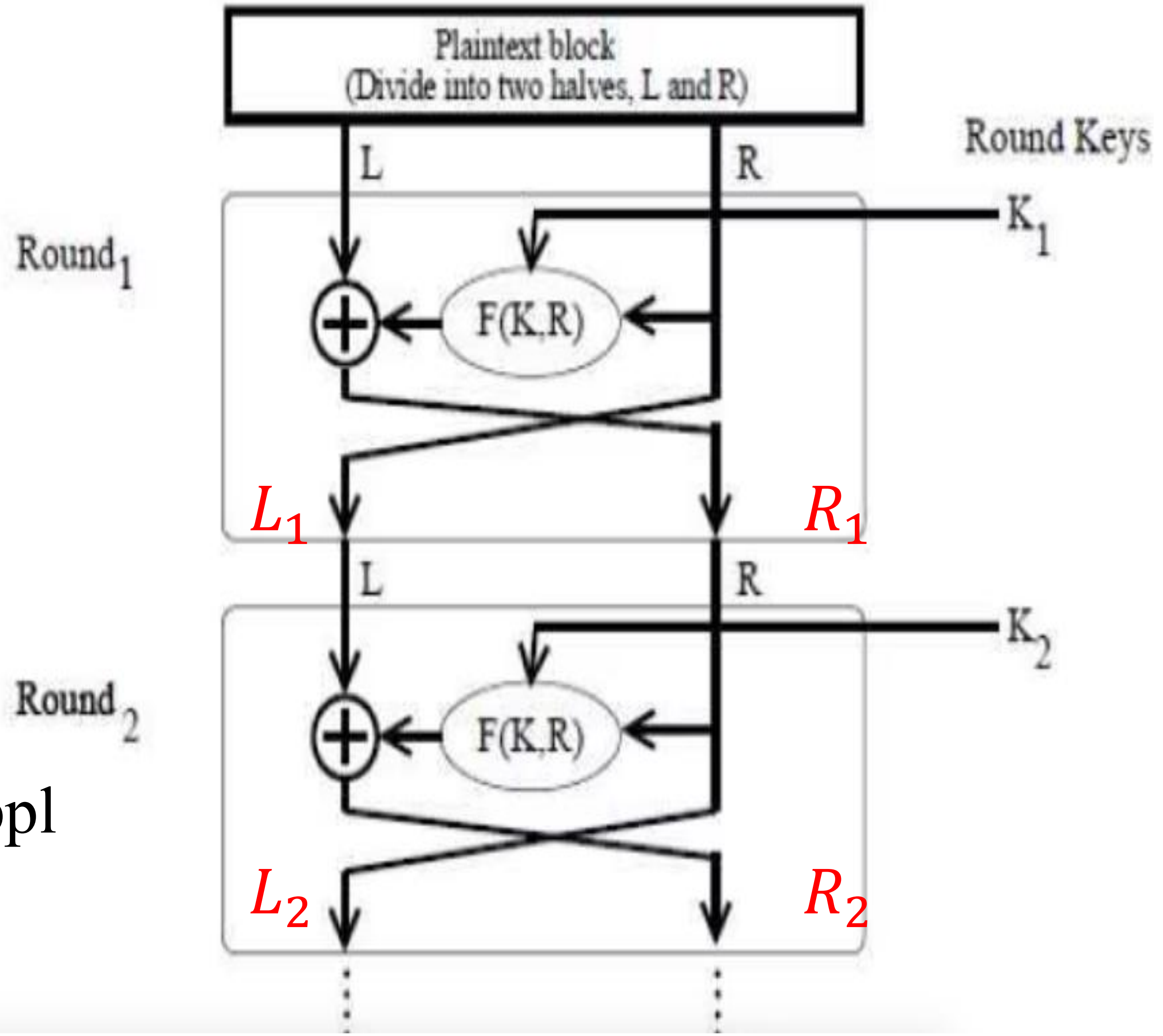
where $|L_0| = |R_0| = 32$

- For i = 1, 2, …, 16, execute the following operations in order:

$$L_i = R_{i-1}$$
$$R_i = L_{i-1} \oplus F(R_{i-1}, K_i)$$

- Let $C = IP^{-1}(R_{16}L_{16})$.

https://en.wikipedia.org/wiki/DES_supplementary_material



DES Sub-Key Generation

- The block size of DES is 64 bits and the encryption key is 56 bits, which is represented as a 64-bit string $K = k_1\ k_2 \ldots k_{64}$
- DES uses 16 rounds of iterations with 16 sub-keys
- Sub-key generation:
  1. Remove the $8i$-th bit (i = 1, 2, …, 8) from K
  2. Perform an ***initial permutation*** on the remaining 56 bits of K, denoted by $IP_{key}(K)$
  3. Split this 56-bit key into two pieces: U_0V_0, both with 28 bits
  4. Perform Left Circular Shift on U_0 and V_0 a defined number of times, producing U_iV_i:

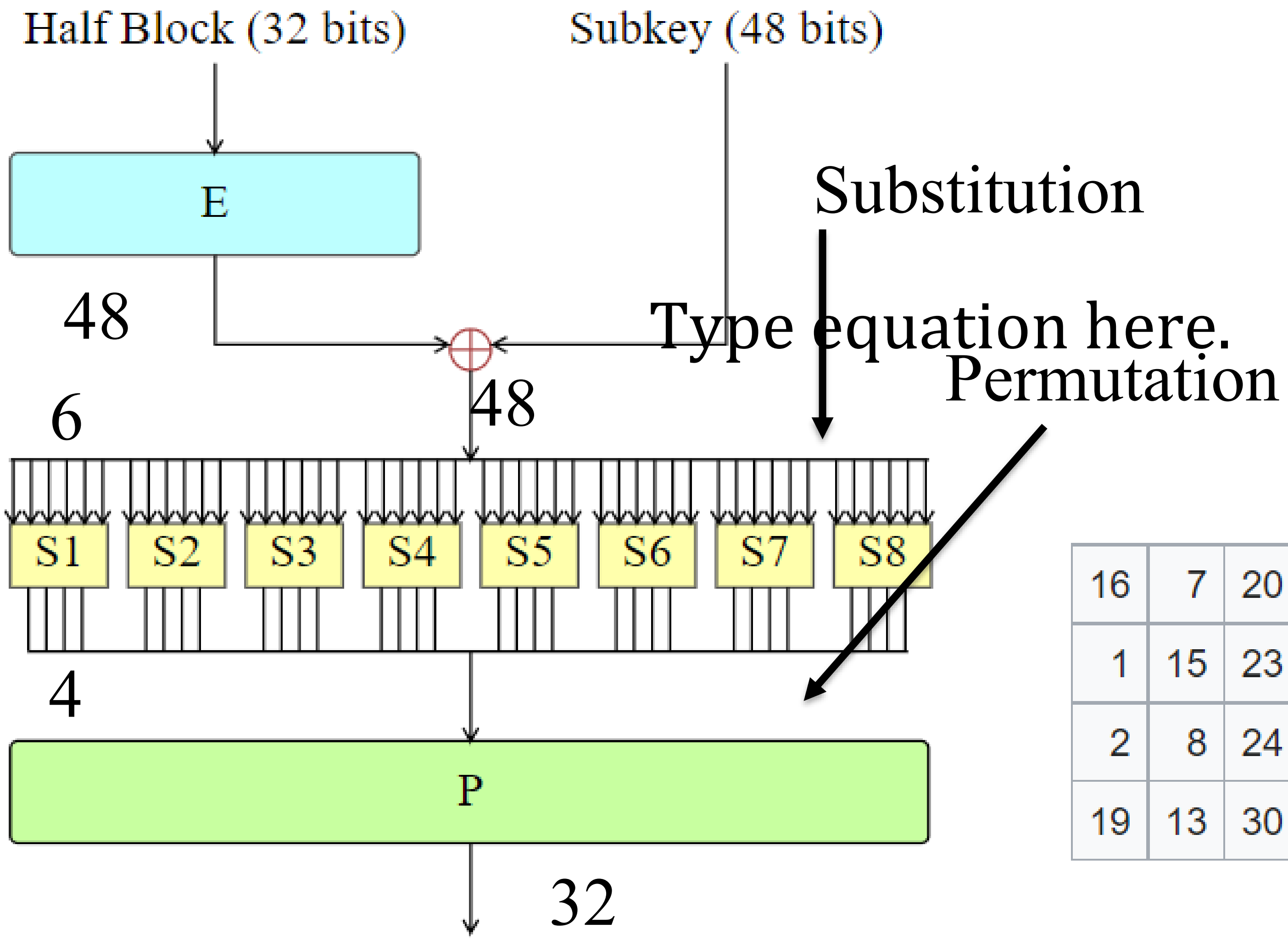
     $$U_i = LS_{z(i)}(U_{i-1}), \qquad V_i = LS_{z(i)}(V_{i-1})$$
  5. Permute the resulting U_iV_i using a defined compress permutation, resulting in a 48-bit string as a sub-key, denoted by K_i

     $$K_i = P_{key}(U_i\ V_i)$$

https://www.geeksforgeeks.org/data-encryption-standard-des-set-1/

DES function $F(R_{i-1}, K_i)$

$$F(R_{i-1}, K_i) = P(S(EP(R_{i-1}) \oplus K_i)), i = 1,\ldots,16$$



P

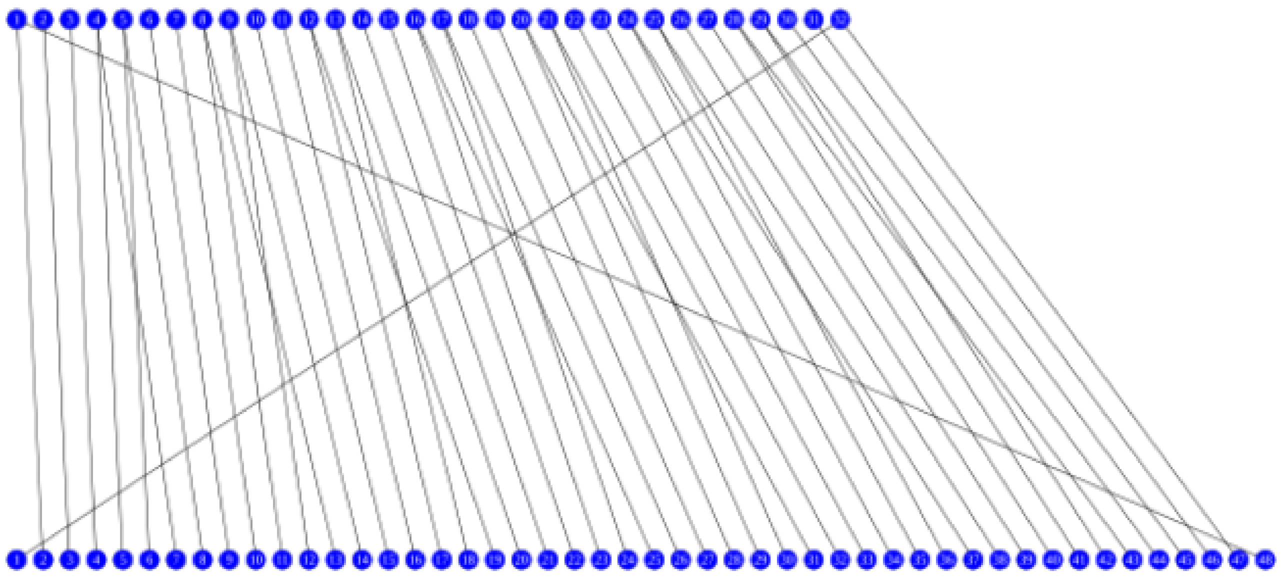
16	7	20	21	29	12	28	17
1	15	23	26	5	18	31	10
2	8	24	14	32	27	3	9
19	13	30	6	22	11	4	25

DES function $F(R_{i-1}, K_i)$

Expansion function (E) [edit]

E

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1



DES Substitution Boxes

S_5		Middle 4 bits of input															
		0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Outer bits	00	0010	1100	0100	0001	0111	1010	1011	0110	1000	0101	0011	1111	1101	0000	1110	1001
	01	1110	1011	0010	1100	0100	0111	1101	0001	0101	0000	1111	1010	0011	1001	1000	0110
	10	0100	0010	0001	1011	1010	1101	0111	1000	1111	1001	1100	0101	0110	0011	0000	1110
	11	1011	1000	1100	0111	0001	1110	0010	1101	0110	1111	0000	1001	1010	0100	0101	0011

Input: “**0**1101**1**”
Output: “1001”

DES Substitution Boxes

- The DES substitution function F is defined below:

  $$F(R_{i-1}, K_i) = P(S(EP(R_{i-1}) \oplus K_i)),\ i = 1,\ldots,16$$

- First, permute R_i using $EP(R_i)$ to produce a 48-bit string x
- Next, XOR x with the 48-bit sub key K_i to produce a 48-bit string y
- Function S turns y into a 32-bits string z, using eight 4x16 special matrices, called S-boxes
  - Each entry in an S-box is a 4-bit string
  - Break y into 8 blocks, each with 6-bits
  - Use the i^{th} matrix on the i^{th} block $b_1b_2b_3b_4b_5b_6$
  - Let b_1b_6 be the row number, and $b_2b_3b_4b_5$ the column number, and return the corresponding entry
  - Each 6-bit block is turned to a 4-bit string, resulting in a 32-bit string z
- Finally, permute z using P to produce the result of DES's F function
- This result, XOR'd with L_{i-1}, is R_i

https://en.wikipedia.org/wiki/DES_supplementary_material

Is DES good enough?

- Security strength of DES
  - Number of rounds
  - Length of encryption key
  - Construction of the substitute function
- DES was used up to the 1990’s.
- People began to take on the DES Challenges to crack DES
- Only uses 56-bit keys = $2^{56} \sim 7.2 \times 10^{16}$ keys
- Brute-force will work with current technology
  - In 1997 on Internet in a few months
  - In 1998 on dedicated h/w (EFF) in a few days
  - In 1999 above combined in 22 hours

What to Do Next?

- Start over
- New standards begin to be looked into
- On the other hand, can we extend the use of DES?

Block Cipher Design Principles

- The greater the number of rounds, the more difficult it is to perform cryptanalysis
- In general, the criterion should be that the number of rounds is chosen so that known cryptanalytic efforts require greater effort than a simple brute-force key search attack
- If DES had 15 or fewer rounds, differential cryptanalysis would require less effort than a brute-force key search

Block Cipher Design Principles

- The heart of a Feistel block cipher is the function F
- The more nonlinear F, the more difficult any type of cryptanalysis will be
- The SAC and BIC criteria appear to strengthen the effectiveness of the confusion function

The algorithm should have good avalanche properties

- Strict avalanche criterion (SAC)
  - States that any output bit j of an S-box should change with probability 1/2 when any single input bit i is inverted for all i , j
- Bit independence criterion (BIC)
  - States that output bits j and k should change independently when any single input bit i is inverted for all i , j , and k

Block Cipher Design Principles

- With any Feistel block cipher, the key is used to generate one subkey for each round
- In general, we would like to select subkeys to maximize the difficulty of deducing individual subkeys and the difficulty of working back to the main key
- It is suggested that, at a minimum, the key schedule should guarantee key/ciphertext Strict Avalanche Criterion and Bit Independence Criterion