#1 Chad Allen Zhao gets the hottest girl in the grade

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June 11, 2021

It was the instant after the last game of senior year, and Allen Zhao—the number one basketball prospect coming from the Class of 2022—had absolutely crushed it, the other team making a total of 2 points the entire game. Allen was running outside in celebration and the entire school was chasing him. Of course, Allen, despite all the glory and victory, was only looking for one person in the crowd.
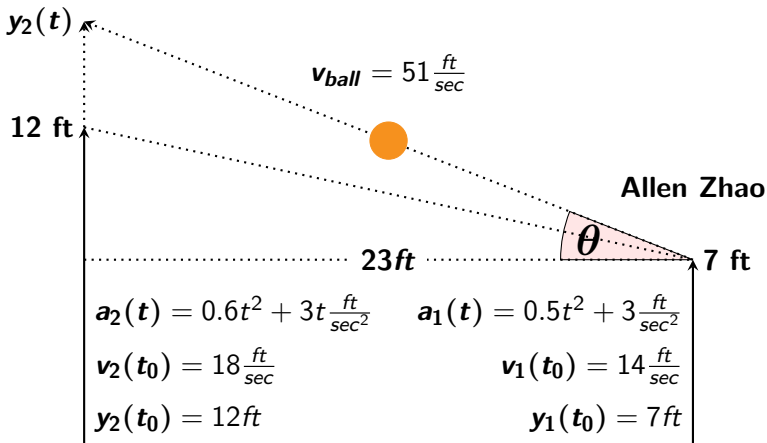
Allen Zhao and Tacko Fall go on a fast break with 20 seconds left on the game clock, in parallel motion $23ft$ apart. Allen must pass it directly to Fall given that Allen is accelerating at $(0.5t^2 + 3)\frac{ft}{sec^2}$ with an initial velocity of $14\frac{ft}{sec}$ and an initial position of 7 ft from the half-court and Fall is accelerating at $(0.6t^2 + 3t)\frac{ft}{sec^2}$ with an initial velocity of $18\frac{ft}{sec}$ and an initial position of $12ft$ from the half-court. Allen throws the ball at $51\frac{ft}{sec}$. Violet, watching the crowd has already calculated the answer to her challenge to Allen: to find $\frac{d\theta}{dt}$ when he throws the basketball at his initial position. Now, Allen must do the same.
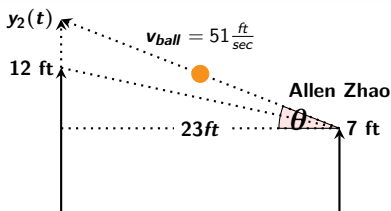
The Basketball Problem

Tacko Fall
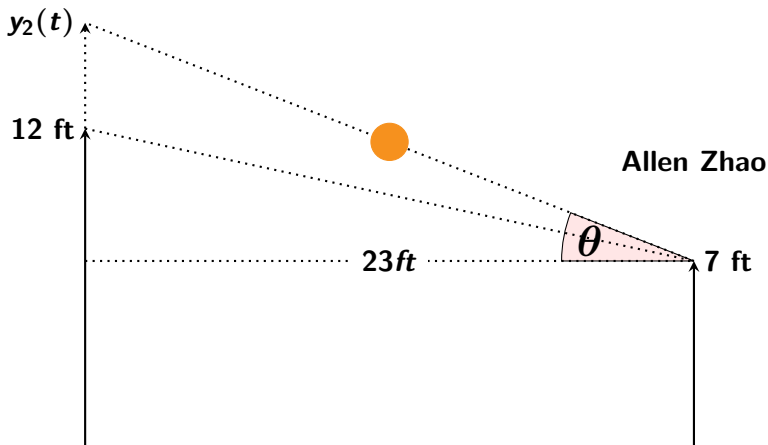
$y_2(t)$

$v_{ball} = 51\frac{ft}{sec}$

12 ft

Allen Zhao

θ

23ft

7 ft

$a_2(t) = 0.6t^2 + 3t\frac{ft}{sec^2}$ $a_1(t) = 0.5t^2 + 3\frac{ft}{sec^2}$

$v_2(t_0) = 18\frac{ft}{sec}$ $v_1(t_0) = 14\frac{ft}{sec}$

$y_2(t_0) = 12ft$ $y_1(t_0) = 7ft$

Tacko Fall

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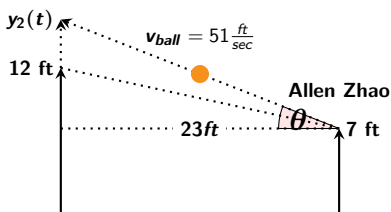
$y_2(t_0) = 12ft$ $y_1(t_0) = 7ft$

Before Allen uses any of his given information on the motion of Fall and himself, he should first look at the overarching equation for $\frac{d\theta}{dt}$.

$$tan(\theta) = \frac{(12ft - 7ft) + \Delta y_2 ft}{23ft}$$
$$\theta = tan^{-1}(\frac{5 + \Delta y_2}{23})$$
$$\frac{d\theta}{dt} = \frac{1}{23} \cdot \frac{1}{1 + (\frac{5+\Delta y_2}{23})^2} \cdot \frac{d(5 + \Delta y_2)}{dt}$$

Tacko Fall

$y_2(t)$

12 ft

$v_{ball} = 51\frac{ft}{sec}$

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$y_2(t_0) = 12ft$ $y_1(t_0) = 7ft$

$$\frac{d\theta}{dt} = \frac{1}{23} \cdot \frac{1}{1 + (\frac{5 + \Delta y_2}{23})^2} \cdot \frac{d(5 + \Delta y_2)}{dt}$$

Now, we need to solve for Δy_2. To do that, we need the time (t) it takes for the ball to travel to Fall's future position. By Pythagoras,

$$(51 \cdot t)^2 = 23^2 + (5 + \Delta y_2)^2$$

$$a_2(t) = 0.6t^2 + 3t \tfrac{ft}{sec^2} \qquad a_1(t) = 0.5t^2 + 3 \tfrac{ft}{sec^2}$$
$$v_2(t_0) = 18 \tfrac{ft}{sec} \qquad\qquad v_1(t_0) = 14 \tfrac{ft}{sec}$$
$$y_2(t_0) = 12ft \qquad\qquad y_1(t_0) = 7ft$$

Let's solve for Δy_2 and t. To do this, we will need to integrate $a_2(t)$, then $v_2(t)$ given the initial positions and velocities.

$$(51 \cdot t)^2 - 23^2 = (5 + \Delta y_2)^2$$
$$\sqrt{(51 \cdot t)^2 - 23^2} - 5 = \Delta y_2$$
$$\int_0^t (18 \frac{ft}{sec} + \int_0^t (0.6t^2 + 3t) \; dt) \; dt = \Delta y_2$$
$$\int_0^t (18 \frac{ft}{sec} + \int_0^t (0.6t^2 + 3t) \; dt) \; dt = \sqrt{(51 \cdot t)^2 - 23^2} - 5$$
$$t = 0.535sec, \Delta y_2 = 9.718ft$$

Now, we can finally solve for $\frac{d\theta}{dt} = \frac{1}{23} \cdot \frac{1}{1+(\frac{5+\Delta y_2}{23})^2} \cdot \frac{d(5+\Delta y_2)}{dt}$. To find $\frac{d(5+\Delta y_2)}{dt}$, we need to use relative motion. The rate of change of that segment of the triangle would be:

$$18\frac{ft}{sec} + \int_0^{0.535} (0.6t^2 + 3t)\ dt = 18.461\frac{ft}{sec}$$

With everything plugged in:

$$\frac{d\theta}{dt} = \frac{1}{23} \cdot \frac{1}{1+(\frac{14.718}{23})^2} \cdot 18.461 = 0.569\frac{rad}{sec}$$