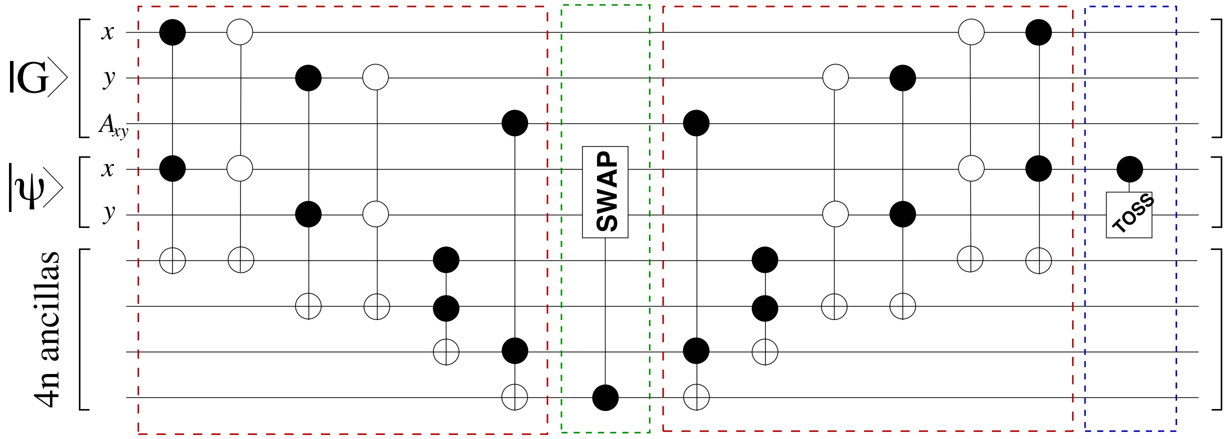

FIG. 1: Schematic circuit for one step of a discrete quantum walk. The graph is encoded in the state $|G\rangle$ and the walk position and coin state are recorded in $|\psi\rangle$. The three qubit gates are Toffoli gates conditioned on 1 (filled circles) or 0 (open circles) flipping the qubit under the cross. The conditional SWAP gate is a set of Fredkin gates on each pair of qubits, and the TOSS operation can be chosen to suit the particular application.

Given $|G\rangle$, a single step of the quantum walk algorithm can be done efficiently, i.e., in $O(\text{poly}(n))$ gates per step. Each of the three qubit gates can be performed using a small number (typically 5 to 10) one and two qubit gates, see e.g., Ref. [9] for more details on construction of gates from more elementary quantum operations. The required number of three-qubit gates per step is thus $6n$ for the comparison of the values of (x, y, A_{xy}) down to one qubit, plus n for the conditional swap, plus another $6n$ to uncompute the ancillas, plus however many operations are required to toss the value of the y register. The total number of gates is thus $\gtrsim 14n$, which is linear in n, comfortably within the original prescription that "efficient" means $O(\text{poly}(\log N))$. The state $|G\rangle$ is unchanged by each step of the quantum walk, but the state $|\psi\rangle$ is being entangled with it, so at the end of the walk when the position $|\psi(x)\rangle$ is measured, this destroys $|G\rangle$ as well.

Several points are notable about this implementation. First, this shows that both discrete and continuous formulations of quantum walks can treat any undirected graph (previously the discrete, coined quantum walks only applied to fixed degree graphs). Second, even before discussing whether the quantum walk itself gives advantages over classical algorithms, the storage involved to contain the description of the graph, $2n+1$ qubits, is an exponential improvement over $N(N-1)/2$ classical bits. Third, it is interesting that it turns out to depend on the graph being undirected for both the discrete and continuous time methods: there are parallels with Abelian and non-Abelian groups and the hidden subgroup problem that deserve further investigation. (See [10, 11] for some discussion of possibilities on directed graphs.)

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