$$\sum_{n\geq 1} \underbrace{\frac{((2n)!)^3}{2^{6n}(n!)^6}}_{x_n}$$

Aplico criterio del cociente

$$\frac{x_{n+1}}{x_n} = \frac{((2n+2)!)^3}{2^{6n+6}((n+1)!)^6} \cdot \frac{2^{6n}(n!)^6}{((2n)!)^3} = \frac{1}{2^6}\frac{(2n+1)^3(2n+2)^3}{(n+1)^6} \to 1$$

$$\frac{(2n+1)^3 \cdot 2^3 \cancel{(n+1)^3}}{(n+1)^{\cancel{6}3}} \cdot \frac{1}{2^6} = \frac{(2n+1)^3 \cancel{2^3}}{(n+1)^3} \cdot \frac{1}{2^{\cancel{6}3}} = \frac{\cancel{2^3}(n+\frac{1}{2})^3}{(n+1)^3} \cdot \frac{1}{\cancel{2^3}}$$

JP No fue capaz de hacerlo y dice que lo intentemos nosotros XD

El criterio del cociente no da información, porque tiende a 1 por valores menores que 1

$a > 0$

$$\sum_{n\geq 1} \log\left(1+\frac{1}{n}\right) a^{\log n}$$

$$\log\left(1+\frac{1}{n}\right) a^{\log n} \sim \frac{1}{n} a^{\log n} = \frac{1}{n} e^{(\log a)(\log n)} = \frac{1}{n} n^{\log a} = \frac{1}{n^{1-\log a}}$$

$\alpha = a^{\log n}$

$\log \alpha = (\log n) \log a$

$\alpha = e^{(\log n)(\log a)} = n^{\log a}$

$1 - \log a > 1 \iff \log a < 0 \iff \underline{a < 1}$

Converge para valores de $a < 1$.

$$\sum_{n\geq 1} \underbrace{\frac{3^n n!}{\sqrt[3]{n}\; 5 \cdot 8 \cdot 11 \cdots (5+3n)}}_{x_n}$$

Aplicando criterio del cociente:

$$\frac{x_{n+1}}{x_n} = \frac{3(n+1)\sqrt[3]{n}}{\sqrt[3]{n+1}\,(8+3n)} = \sqrt[3]{\frac{n}{n+1}} \quad \frac{3n+3}{3n+8} \longrightarrow 1$$

Aplicando el criterio de Raabe:

$$n\left(1-\frac{x_{n+1}}{x_n}\right) = \underbrace{-n}_{v_n}\left(\frac{x_{n+1}}{x_n} - 1\right)$$

$$v_n(U_n - 1) \longrightarrow \underset{\overset{\shortparallel}{2}}{L} \iff U_n^{v_n} \to e^L$$

$$U_n^{v_n} = \left[\left(\frac{n+1}{n}\right)^n\right]^{\frac{1}{3}} \left(\frac{3n+8}{3n+3}\right)^n \longrightarrow e^{\frac{1}{3}} e^{\frac{5}{3}} = e^2$$

$$\sum_{n\geq 1} (-1)^{n+1} \Big(\underbrace{\frac{1\cdot 3\cdot 5\cdots (2n-1)}{2\cdot 4\cdot 6\cdots (2n)}}_{a_n}\Big)^{\alpha}$$

Si $\alpha \leq 0 \Rightarrow \frac{a_{n+1}}{a_n} \geq 1 \Rightarrow \{a_n\} \not\to 0$

$$\sum_{n\geq 1} a_n \qquad \frac{a_{n+1}}{a_n} = \left(\frac{2n+1}{2n+2}\right)^{\alpha}$$

Aplicando Raabe:

$$\left(\frac{a_n}{a_{n+1}}\right)^n = \left[\left(\frac{2n+2}{2n+1}\right)^n\right]^{\alpha} \longrightarrow e^{\frac{\alpha}{2}}$$

$\frac{\alpha}{2} > 1 \Rightarrow$ $\alpha > 2$ converge

$\alpha \leq 2$ no converge

$$R_n = n\left(1 - \frac{a_{n+1}}{a_n}\right) = n\left(1 - \frac{4n^2+4n+1}{4n^2+8n+4}\right) = n\left(\frac{4n+3}{4n^2+8n+4}\right)$$

Por lo que nuestra serie original converge absolutamente para $\alpha > 2$

$$\prod_{k=1}^{n}\left(1 - \frac{1}{2k}\right) = \prod_{k=1}^{n} \frac{2k-1}{2k} = \frac{1\cdot 3\cdot 5\cdots (2n-1)}{2\cdot 4\cdot 6\cdots (2n)} \longrightarrow 0$$

$$\log z_n = \sum_{k=1}^{n} \log\left(1 - \frac{1}{2k}\right) \to -\infty \qquad \sum_{n\geq 1} -\log\left(1 - \frac{1}{2n}\right)$$

$$-\log\left(1 - \frac{1}{2n}\right) \sim \frac{1}{2n}$$

$$\sum_{n\geq 1} (-1)^{n+1} \frac{\left(1+\frac{1}{n}\right)^n}{n}$$

$$\sum_{n\geq 1} \frac{1}{n} \underbrace{\left(1 + \frac{1}{n}\right)^n}$$

$$\frac{(n+1)^n}{n^{n+1}} = \frac{1}{n}\left(1+\frac{1}{n}\right)^n \longrightarrow 0$$

$$\frac{1}{n}\left(1+\frac{1}{n}\right)^n > \frac{1}{n}$$

$$\frac{(n+2)^{n+1}}{(n+1)^{n+2}} < \frac{(n+1)^n}{n^{n+1}}$$

$$(n+2)^{n+1} \cdot n^{n+1} < (n+1)^n \cdot (n+2)^{n+1}$$

$$(n(n+2))^{n+1} < [(n+1)^2]^{n+1} \iff n(n+2) < (n+1)^2 \iff 0 < 1$$

La sucesión converge

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d) $\displaystyle\sum_{n\geq 2} \underbrace{\frac{n^{\log n}}{(\log n)^n}}_{a_n}$

Por el criterio de la raíz:

$$\sqrt[n]{a_n} = \frac{n^{\frac{\log n}{n}}}{\log n} = \frac{e^{\frac{\log n}{n}\log n}}{\log n} = \frac{e^{\frac{(\log n)^2}{n}}}{\log n} \longrightarrow 0$$

h) $\displaystyle\sum_{n\geq 1} \left(e - \left(1 + \frac{1}{n^2}\right)^{n^2} \right)$

$$\left(1+\frac{1}{n^2}\right)^{n^2} < e < \left(1 + \frac{1}{n^2}\right)^{n^2+1}$$

$$0 < e - \left(1+\frac{1}{n^2}\right)^{n^2} < \left(1+\frac{1}{n^2}\right)^{n^2}\left(1 + \frac{1}{n^2} - 1\right) = \left(1+\frac{1}{n^2}\right)^{n^2} \cdot \frac{1}{n^2} < \frac{3}{n^2}$$

Es comparable a una serie de Riemann de razón 2.

El criterio de Raabe puede que dé información, el resto no.

f) $\displaystyle\sum_{n\geq 2} a^{\log(\log n)}$ $\quad a>0$

$$a^{\log(\log n)} = \alpha = e^{\log(\log n)\log a}$$

$$\log \alpha = \log(\log n)\log a$$

$$= \log(n)^{\log a} = \frac{1}{\log(n)^{-\log a}} \downarrow 0$$

Aplico el criterio de condensación de Cauchy:

$$\sum_{n\geq 1} 2^n a_{2^n} = \sum_{n\geq 1} 2^n \frac{1}{(\log(2^n))^{-\log a}} = \sum_{n\geq 1} 2^n \frac{1}{n^{-\log a}} \frac{1}{(\log 2)^{-\log a}}$$

Parece ser que la serie no converge nunca XD.

$$\sum_{n\geq 1} (-1)^{n+1} \frac{\log(n+2)}{n+2}$$

$$\frac{\log(n+2)}{n+2} > \frac{1}{n+2}$$

$\frac{\log(n+3)}{n+3} \leq \frac{\log(n+2)}{n+2} \Leftrightarrow$ Probamos que la función es decreciente

$\Leftrightarrow (n+2)\log(n+3) \leq (n+3)\log(n+2) \Leftrightarrow (n+3)^{n+2} \leq (n+2)^{n+3} \Leftrightarrow \left(\frac{n+3}{n+2}\right)^{n+2} \leq n+2 \Leftrightarrow$

$\Leftrightarrow \underbrace{\left(1+\frac{1}{n+2}\right)^{n+2}}_{\hat{e}} \leq n+2 \Leftrightarrow e < n+2 \quad \checkmark$

Por el criterio de Leibniz la serie converge a 0