

Q1: a) $y_5(x) = f(x_0) + (x-x_0)f[x_1, x_0] + (x-x_0)(x-x_1)f[x_2, x_1, x_0] + \dots$
 $+ (x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)f[x_4, x_3, x_2, x_1, x_0]$

	t	z	1st	2nd	3rd	4th
t_0	2	6				
t_1	2.1	767.752	17.52			
t_2	2.2	10.256	25.04	37.6		
t_3	2.7	36.576	52.64	46	12	
t_4	3	66	98.08	56.8	12	0
t_5	3.4	125.168	147.92	71.2	12	0

$$z_5(x) = z(t_0) + (t-t_0)z[t_1, t_0] + (t-t_0)(t-t_1)z[t_2, t_1, t_0] + (t-t_0)(t-t_1)(t-t_2)z[t_3, t_2, t_1, t_0] + (t-t_0)(t-t_1)(t-t_2)(t-t_3)z[t_4, t_3, t_2, t_1, t_0]$$

$$\Rightarrow z_5(x) = 6 + (t-2)(17.52) + (t-2)(t-2.1)(37.6) + (t-2)(t-2.1)(t-2.2)(12)$$

$$\Rightarrow z_5(2.5) = 23$$



$$b) f(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} 7.752 + \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} 10.256 + \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} 36.576 + \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} 66$$

$$\Rightarrow \cancel{f(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(2.1-2.2)(2.1-2.7)(2.1-3)} 7.752 + (x-x_1)(x-x_3)(x-x_4)} +$$

$$f(x) = \frac{(x-2.2)(x-2.7)(x-3)}{(2.1-2.2)(2.1-2.7)(2.1-3)} 7.752 + \frac{(x-2.1)(x-2.7)(x-3)}{(2.2-2.1)(2.2-2.7)(2.2-3)} 10.256 + \frac{(x-2.1)(x-2.2)(x-3)}{(2.7-2.1)(2.7-2.2)(2.7-3)} 36.576 + \frac{(x-2.1)(x-2.2)(x-2.7)}{(3-2.1)(3-2.2)(3-2.7)} 66$$

$$\Rightarrow \cancel{f(2.5) = z(2.5)} \Rightarrow z(2.5) = f(2.5) = 23$$

$\Rightarrow f(x_1(3.5)) = 3.125$

Q2: a) $y_0(x) = f(x_0) + (x-x_0)f[x_1, x_0] + (x-x_0)(x-x_1)f[x_2, x_1, x_0] + (x-x_0)(x-x_1)(x-x_2)f[x_3, x_2, x_1, x_0] + \dots$

	x	y	1 st	2nd	2 nd	3 rd
x_0	0	(26)	*			
x_1	1	15.5	(-10.5)			
x_2	2.5	5.375	-6.75	(1.5)		
x_3	3	3.5	-3.75	1.5	(0)	
x_4	4.5	2.375	-0.75	1.5	0	
x_5	5	3.5	2.25	1.5	0	
x_6	6	8	4.5	1.5	0	

$\Rightarrow y_0(x) = 26 - 10.5(x-0) + (x-0)(x-1)(1.5) + (x-0)(x-1)(x-2.5)(0)$

$\Rightarrow y_0(3.5) = 2.375$

Q2: b) $f(x) = \frac{(x-x_3)(x-x_4)(x-x_5) \cdot 5.375}{(x-x_3)(x-x_4)(x-x_5) \cdot 2.5} + \frac{(x-x_2)(x-x_4)(x-x_5) \cdot 3.5}{(x-x_2)(x-x_4)(x-x_5) \cdot 3.5} + \frac{(x-x_2)(x-x_3)(x-x_5) \cdot 2.375}{(x-x_2)(x-x_3)(x-x_5) \cdot 2.375} + \frac{(x-x_2)(x-x_3)(x-x_4) \cdot 3.5}{(x-x_2)(x-x_3)(x-x_4) \cdot 3.5}$

$\Rightarrow f(x) = \frac{(x-3)(x-4.5)(x-5) \cdot 5.375}{(2.5-3)(2.5-4.5)(2.5-5) \cdot 2.5} + \frac{(x-2.5)(x-4.5)(x-5) \cdot 3.5}{(3-2.5)(3-4.5)(3-5) \cdot 3.5} + \frac{(x-2.5)(x-3)(x-5) \cdot 2.375}{(4.5-2.5)(4.5-3)(4.5-5) \cdot 2.375} + \frac{(x-2.5)(x-3)(x-4.5) \cdot 3.5}{(5-2.5)(5-3)(5-4.5) \cdot 3.5}$

$\Rightarrow f(3.5) = 2.375$

Q3:

x	0	1	2.5	3	4.5	5	6
y	26	15.5	5.375	3.5	2.375	3.5	8

Using Newton Interpolating polynomial to determine y at $x=3.5$
 as in Q1, we have: $y(3.5) = 2.375$

Divided difference table

0th order $f[x_0] = f(x_0) = 26$

1st order $f[x_1, x_0] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{15.5 - 26}{1 - 0} = -10.5$

$x_1 - x_0 = 1 - 0$

2nd order $f[x_2, x_1, x_0] = f''$

	1 st	2 nd	3 rd	4 th	5 th	6 th
x_0	0	26				
x_1	1	15.5	-10.5			
x_2	2.5	5.375	-6.75	1.5		
x_3	3	3.5	-3.75	1.5	0	
x_4	4.5	2.375	-0.75	1.5	0	0
x_5	5	3.5	2.25	1.5	0	0
x_6	8	8	4.5	1.5	0	0