University of Manitoba
Department of Statistics

STAT 2400

Introduction to Probability I

Sample Test #1 (C)

Question 1:

Every morning, in the schoolyard, Ms. Smith's 20 students are lined up before they enter the school hallway. How many different student lineups are there if

(A) the students are lined up completely at random;

(B) the smallest student and tallest student of the class necessarily come first and last, respectively;

(C) Andrew, Carrie and Kathy will not be separated in the line;

(D) Andrew, Carrie and Kathy will not be separated in the line, but Andrew is also the tallest student in the class and has to be last in line;

(E) Andrew and Carrie have been fighting and so, won't be next to each other in line;

(F) boys and girls should alternate in the line (assuming the class has 10 boys and 10 girls).

Question 2:

Let A and B be events of Ω such that

$$\mathbb{P}(A) = 0.2, \qquad \mathbb{P}(A \cup B) = 0.6 \qquad \text{and} \qquad \mathbb{P}(A \cap B) = 0.1.$$

(A) Determine $\mathbb{P}(B)$.

(B) Determine $\mathbb{P}(A \cup B^c)$.

Question 3:

The letters A, A, A, A, A, B, B, C, D, R and R are written on 11 balls that are otherwise identical. These balls are put into a box and drawn one by one, without replacement, until the box is empty, thus forming a word.

What is the probability the word ABRACADABRA is formed?

Question 4:

An urn contains 4 red balls and 6 black balls. Balls are drawn one at a time, without replacement until all red balls have been removed. Let

$$A_i : \text{the } 4^{\text{th}} \text{ red ball is drawn on the } i^{\text{th}} \text{ draw.}$$

Find $\mathbb{P}(A_i)$ for $i = 4, 5, \ldots, 10$.

Question 5:

Two fair dice are rolled n times in succession.

What is the probability that a double six is obtained at least once?

Question 6:

A company has a position open and has identified three highly qualified applicants: John, Barbara and Martin. However, because the company has only a few female employees, Barbara estimates her chances of being hired to be 50% higher than John's and Martin's.

According to this, what is the probability that Barbara will be hired.

Question 7:

State Kolmogorov's axioms of probability.

Question 8:

Assume that A and B are two events of Ω.

(A) Prove Bonferroni's inequality:

$$\mathbb{P}(A \cap B) \geq \mathbb{P}(A) + \mathbb{P}(B) - 1.$$

(B) Assume that $\mathbb{P}(A) = \mathbb{P}(B) = 1$. Show that

$$\mathbb{P}(A \cap B) = \mathbb{P}(A \cup B) = 1.$$

Question 9:

Prove that for $0 \leq i \leq k \leq m \leq n$,

$$\binom{n}{m}\binom{m}{k}\binom{k}{i} = \binom{n}{i}\binom{n-i}{k-i}\binom{n-k}{m-k}.$$

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