3.3 The menu at a restaurant has five choices of beverage, three salads, six entrees and four desserts. How many complete meals are available with beverage, salad, entree, and dessert?

3.4 An advertisement for *i*Dolls™ states: "Choose from 69 billion combinations to create a one-of-a-kind doll." The ad goes on to say that there are 39 choices for hairstyle, 19 for eye color, 8 for hair color, 6 for face shape, 24 for lip color, 5 for freckle pattern, 5 for line of clothing, 6 for blush color, and 5 for skin tone. Exactly how many possibilities are there for these options?

3.5 An identification number consists of an ordered arrangement of eight decimal digits. How many identification numbers can be formed if

a) there are no restrictions? **b)** no digit can occur twice?
c) no digit can agree with its predecessor?
d) no digit can agree with either of its two immediate predecessors?

3.6 How many batting orders are possible for the nine starting players on a baseball team?

3.7 The author of this book spoke with a representative of the United States Postal Service and obtained the following information about zip codes. A five-digit zip code consists of five digits of which the first three give the sectional center and the last two the post office or delivery area. In addition to the five-digit zip code, there is a trailing *plus four zip code.* The first two digits of the plus four zip code give the sector or several blocks and the last two the segment or side of the street. For the five-digit zip code, the first four digits can be any of the digits 0–9 and the fifth any of the digits 1–8. For the plus four zip code, the first three digits can be any of the digits 0–9 and the fourth any of the digits 1–9.

a) How many possible five-digit zip codes are there?
b) How many possible plus four zip codes are there?
c) How many possibilities are there including both the five-digit zip code and the plus four zip code?

3.8 Telephone numbers in the United States consist of a three-digit area code followed by a seven-digit local number. Suppose that neither the first digit of an area code nor the first digit of a local number can be a zero but that all other choices are acceptable.

a) How many different area codes are possible?
b) For a given area code, how many local telephone numbers are possible?
c) How many telephone numbers are possible?

3.9 In Example 2.3 on page 27, we considered the random experiment of rolling two dice. Use the BCR to determine the number of possible outcomes

a) for this random experiment. **b)** in which the sum of the dice is 5.
c) in which doubles are rolled. **d)** in which the sum of the dice is even.

3.10 In Example 2.4 on page 27, we considered the random experiment of observing a mechanical or electrical unit consisting of five components and determining which components are working and which have failed. Use the BCR to find the number of possible outcomes

a) for this random experiment.
b) in which exactly one of the five components is not working.
c) in which at least one of the five components is not working.
d) in which at most one of the five components is not working.

3.11 An alphabet has six letters a, b, c, d, e, and f. How many four-letter words (i.e., ordered arrangements of four of the six letters) can be formed if

a) there are no restrictions? **b)** no letter can occur twice?
c) the first letter must be a or b? **d)** the letter c must occur at least once?

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3.12 Let A and B be two finite sets with the same number of elements—say, n.
a) How many functions are there from A to B?
b) How many of these functions are one-to-one?

3.13 Five people—say, a, b, c, d, and e—are arranged in a line. How many arrangements are there in which
a) c is before d? **b)** d is not first?

3.14 Suppose that you and your best friend are among n people to be arranged in a line. How many arrangements are possible in which exactly k people are between you and your friend?

3.15 Four married couples attend a banquet.
a) How many ways can they be seated on one side of a straight (i.e., noncircular) table in such a way that each husband sits next to his wife?
b) Repeat part (a) for a circular table.

3.16 Consider a domino that consists of two subrectangles, each marked with a number from 1 to n. How many such dominos are possible? *Note:* A number pair is not ordered.

3.17 If n people attend a party and each pair of people shake hands, how many handshakes will there be?

3.18 A poker hand consists of 5 cards dealt from an ordinary deck of 52 playing cards.
a) In five-card stud, the order in which the cards are dealt matters. How many five-card stud hands are possible?
b) In five-card draw, the order in which the cards are dealt doesn't matter. How many five-card draw hands are possible?

3.19 In how many ways can n distinguishable balls be arranged in n distinguishable boxes so that
a) no box is empty? **b)** exactly one box is empty? **c)** at least one box is empty?

Theory Exercises

3.20 Use mathematical induction to complete the proof of the BCR, as given in Proposition 3.1 on page 87.

Advanced Exercises

3.21 In this exercise, you are to obtain the number of subsets of a finite set, Ω.
a) Suppose that Ω consists of three elements—say, $\{a, b, c\}$. List the possible subsets of Ω and, from your list, determine the number of subsets.
b) In part (a), determine the number of subsets by using the BCR. *Hint:* For a given subset, each element of Ω either is a member of that subset or it isn't.
c) Suppose that Ω consists of n elements. Determine the number of subsets of Ω.

3.22 This exercise provides an alternative derivation of the result in Exercise 3.21(c). The eight subsets of $\{a, b, c\}$ are $\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, and $\{a, b, c\}$. Although listing all 16 subsets of $\{a, b, c, d\}$ is relatively easy, you are to answer the following questions by other methods, which can be applied in more complicated problems where you may not be able to write a complete list easily.
a) To create a subset of the set $\{a, b, c, d\}$, first choose a subset of $\{a, b, c\}$, and then decide whether to add d to that subset. Use that idea and the BCR to find the number of subsets of $\{a, b, c, d\}$.

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Solution We can think of the possible results as partitioning the eight finishing places into three groups, one of size 3 for the U.S. athletes, one of size 2 for the Canadian athletes, and one of size 3 for the Mexican athletes. For instance, the three groups of finishing places $\{2, 5, 7\}$, $\{3, 4\}$, and $\{1, 6, 8\}$ would correspond to the three U.S. athletes finishing in second, fifth, and seventh places; the two Canadian athletes finishing in third and fourth places; and the three Mexican athletes finishing in first, sixth, and eighth places. Hence, by the partitions rule,

$$\binom{8}{3,\ 2,\ 3} = \frac{8!}{3!\,2!\,3!} = 560$$

results are possible. ■

Note that multinomial coefficients reduce to binomial coefficients in the case of two groups ($k = 2$). Indeed, if m_1 and m_2 are nonnegative integers whose sum is m, then

$$\binom{m}{m_1,\ m_2} = \frac{m!}{m_1!\,m_2!} = \frac{m!}{m_1!\,(m-m_1)!} = \binom{m}{m_1}.$$

This result makes sense from a conceptual point of view: The number of possible ordered partitions of m objects into two distinct groups of sizes m_1 and m_2 equals the number of ways that we can choose m_1 objects from the m objects to constitute the first group, which is $\binom{m}{m_1}$. The remaining $m - m_1 = m_2$ objects must constitute the second group.

EXERCISES 3.2 Basic Exercises

3.23 Show for $k, j \in \mathcal{N}$, with $j \leq k$, that $k! = k(k-1)\cdots(k-j+1)(k-j)!$.

3.24 Determine the value of

a) $(7)_3$. **b)** $(5)_2$. **c)** $(8)_4$. **d)** $(6)_0$. **e)** $(9)_9$.

3.25 At a movie festival, a team of judges is to pick the first, second, and third place winners from the 18 films entered. Use permutation notation to express the number of possibilities and then evaluate that expression.

3.26 Investment firms usually have a large selection of mutual funds from which an investor can choose. One such firm has 30 mutual funds. Suppose that you plan to invest in 4 of these mutual funds, 1 during each quarter of next year. Use permutation notation to express the number of possibilities and then evaluate that expression.

3.27 The sales manager of a clothing company needs to assign seven salespeople to seven different territories. How many possibilities are there for the assignments?

3.28 Determine the value of

a) $\binom{7}{3}$. **b)** $\binom{5}{2}$. **c)** $\binom{8}{4}$. **d)** $\binom{6}{0}$. **e)** $\binom{9}{9}$.

3.29 The Internal Revenue Service (IRS) decides that it will audit the returns of 3 people from a group of 18. Use combination notation to express the number of possibilities and then evaluate that expression.

3.30 At a lottery, 100 tickets were sold and three prizes are to be given. How many possible outcomes are there if

a) the prizes are equivalent? **b)** there is a first, second, and third prize?

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3.31 An economics professor is using a new method to teach a junior-level course with an enrollment of 42 students. The professor wants to conduct in-depth interviews with the students to get feedback on the new teaching method but doesn't want to interview all 42 of them. She decides to interview a sample of 5 students from the class. How many different samples are possible?

3.32 The Powerball® is a multistate lottery that was introduced in April 1992 and is now sold in 24 states, the District of Columbia, and the Virgin Islands. To play the game, a player first selects five numbers from the numbers 1–53 and then chooses a Powerball number, which can be any number between 1 and 42, inclusive. How many possibilities are there?

3.33 In the game of *keno*, there are 80 balls, numbered 1–80. From these 80 balls, 10 are selected at random.

a) How many different outcomes are possible?

b) If a player specifies 20 numbers, in how many ways can he get all 10 numbers selected?

3.34 A club has 14 members.

a) How many ways can a governing committee of size 3 be chosen?

b) How many ways can a president, vice president, and treasurer be chosen?

c) How many ways can a president, vice president, and treasurer be chosen if two specified club members refuse to serve together?

3.35 How many license plates are there consisting of three digits and three letters if there is no restriction on where the digits and letters are placed?

3.36 A five-card draw poker hand consists of 5 cards dealt from an ordinary deck of 52 playing cards. The order in which the cards are received is unimportant. Note that, in sequence, an ace can play as either the lowest or highest card. In other words, the hierarchy of card denominations, from lowest to highest, is ace, 2, 3, ..., 10, jack, queen, king, ace. Determine the number of possible hands of the specified type.

a) Straight flush: five cards of the same suit in sequence

b) Four of a kind: $\{w, w, w, w, x\}$, where w and x are distinct denominations

c) Full house: $\{w, w, w, x, x\}$, where w and x are distinct denominations

d) Flush: five cards of the same suit, not all in sequence

e) Straight: five cards in sequence, not all of the same suit

f) Three of a kind: $\{w, w, w, x, y\}$, where w, x, and y are distinct denominations

g) Two pair: $\{w, w, x, x, y\}$, where w, x, and y are distinct denominations

h) One pair: $\{w, w, x, y, z\}$, where w, x, y, and z are distinct denominations

3.37 Repeat Exercise 3.36 for the game of five-card stud, where the order in which the cards are received matters.

3.38 Refer to the inclusion–exclusion principle, Proposition 2.10 on page 73. Determine the number of summands in each sum.

3.39 The U.S. Senate consists of 100 senators, 2 from each state. A committee consisting of 5 senators is to be formed.

a) How many different committees are possible?

b) How many are possible if no state can have more than 1 senator on the committee?

3.40 Refer to Example 3.17 on page 105. How many possible results are there in which the United States has exactly two finishers in the top three and one in the bottom three?

3.41 Without doing any calculations, explain why

a) $\binom{n}{k} = \binom{n}{n-k}$. **b)** $\binom{n}{k} = \binom{n}{k,\, n-k}$.

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3.42 A teacher plans to construct a 25-question exam from previous exams, in which there are 50 true–false questions and 80 multiple-choice questions. Ignoring the order of the questions,

a) how many exams can be constructed?

b) how many exams can be constructed that consist of exactly 13 true–false questions?

c) how many exams can be constructed that consist of all true–false questions?

3.43 How many distinct arrangements are there of the letters in the word PERSEVERE?

3.44 Consider a group of 30 football players from which 8 are to be selected as quarterbacks, 7 as fullbacks, and 4 as centers.

a) In how many ways can this selection be made?

b) Now suppose that the group of 30 football players consists of 15 quarterbacks, 9 fullbacks, and 6 centers. Further suppose that the 8 quarterbacks to be selected must be chosen from among the 15 quarterbacks and likewise for the fullbacks and centers. How many different selections are possible?

3.45 From a list of 20 candidates, you must choose a committee of 8 and, from among those 8, you must choose a president, a secretary, and a treasurer. In parts (a)–(c), imagine (but don't draw) a tree diagram representing the process whose first and second steps are as specified. Then use the BCR in a way suggested by the structure of the tree diagram, together with permutation and combination rules, to write an expression for the number of ways to make the necessary choices. Finally, evaluate that expression.

a) The first step is the choice of the 8 committee members; the second step is the choice of the 3 officers from among those 8.

b) The first step is the choice of the 5 nonofficer committee members; the second step is the choice of the 3 officers from among the 15 remaining candidates.

c) The first step is the choice of the 3 officers; the second step is the choice of the 5 nonofficer committee members from among the 17 remaining candidates.

d) Rather than 20 candidates, suppose that there are n candidates; rather than 3 officers, j officers; and rather than 8 committee members, including the 3 officers, k committee members, including the j officers. Without mentioning factorials, explain how parts (a), (b), and (c) suggest the truth of the identities

$$\binom{n}{k}(k)_j = \binom{n}{k-j}\big(n-(k-j)\big)_j = (n)_j\binom{n-j}{k-j}.$$

e) Now establish the identities in part (d) by a different method—namely, by using the permutations and combinations rules and some algebra.

Theory Exercises

3.46 Binomial theorem: Use combinatorial analysis to prove the *binomial theorem;* that is,

$$(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k},$$

for all numbers a and b and positive integers n.

3.47 Pascal's triangle: In this exercise, you are asked to examine an interesting pattern that occurs when the coefficients of the expansions of $(a+b)^n$, for $n = 0, 1, 2, \ldots,$ are written in the form of a triangle, called *Pascal's triangle.* Subsequently, you are to prove an identity involving binomial coefficients that establishes the pattern mathematically.

a) Consider the expressions $(a+b)^n$, for $n = 0, 1, 2, \ldots.$ Use the binomial theorem (Exercise 3.46) to expand each expression up to and including $n = 6$.

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EXERCISES 3.3 Basic Exercises

3.56 Provide the details for the solution of Example 3.18(b) on page 111.

3.57 Four cards are dealt from an ordinary deck of 52 playing cards. What is the probability that the denominations (face values) of the cards are

a) all the same? **b)** all different?

3.58 In a small lottery, 10 tickets—numbered $1, 2, \ldots, 10$—are sold. Two numbers are drawn at random for prizes. You hold tickets numbered 1 and 2. What is the probability that you win at least one prize?

3.59 An ordinary deck of 52 playing cards is shuffled and dealt. What is the probability that

a) the seventh card dealt is an ace? **b)** the first ace occurs on the seventh card dealt?

3.60 From an urn containing M red balls and $N - M$ black balls, a random sample of size n is taken without replacement. Find the probability that exactly j black balls are in the sample.

3.61 The birthday problem: A probability class has 38 students.

a) Find the probability that at least 2 students in the class have the same birthday. For simplicity, assume that there are always 365 days in a year and that birth rates are constant throughout the year. *Hint:* Use the complementation rule.

b) Repeat part (a) if the class has N students.

c) Evaluate the probability obtained in part (b) for $N = 1, 2, \ldots, 70$. Use a computer or calculator to do the number crunching.

d) What is the smallest class size for which the probability that at least 2 students in the class have the same birthday exceeds 0.5?

3.62 An urn contains four red balls and six black balls. Balls are drawn one at a time at random until three red balls have been drawn. Determine the probability that a total of seven balls is drawn if the sampling is

a) without replacement. **b)** with replacement.

3.63 Four mathematicians, three chemists, and five physicists are seated randomly in a row. Find the probability that all the members of each discipline sit together.

3.64 Suppose that a random sample of size n without replacement is taken from a population of size N. For $k = 1, 2, \ldots, n$, determine the probability that k specified members of the population will be included in the sample.

3.65 Suppose that a random sample of size n with replacement is taken from a population of size N.

a) Determine the probability that no member of the population is selected more than once.

b) Show that the probability in part (a) approaches 1 as $N \to \infty$. Interpret this result.

3.66 Refer to Example 3.19 on page 111. Determine the probability that the number of defective TVs selected is

a) exactly one. **b)** at most one. **c)** at least one.

3.67 Refer to Example 3.20(b) on page 113. In this exercise, you are to obtain $P(E)$—the probability that a specified member of the population will be included in the sample—in two additional ways.

a) Compute $N(E)$ directly and then apply Equation (3.4) on page 110 to determine $P(E)$.

b) For $k = 1, 2, \ldots, n$, let A_k denote the event that the kth member selected is the specified member. Without doing any computations, explain why $P(A_k) = 1/N$, for $k = 1, 2, \ldots, n$. Conclude that $P(E) = n/N$. Explain your reasoning.

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3.68 Refer to Example 3.21 on page 114.

a) For $N = 1, 2, \ldots, 6$, determine the probability that at least one woman gets her own key.

b) Compare your answers in part (a) to the approximate probability of $1 - e^{-1}$ given at the end of Example 3.21.

c) Comment on the accuracy of using $1 - e^{-1}$ to approximate the probability that at least one woman gets her own key.

3.69 In an extrasensory-perception (ESP) experiment, a psychologist takes 10 cards, numbered 1–10, and shuffles them. Then, as she looks at each card, the subject writes the number he thinks is on the card.

a) How many possibilities exist for the order in which the subject writes the numbers?

b) If the subject doesn't have ESP, what is the probability that he writes the numbers in the correct order—that is, in the order that the cards are actually arranged?

3.70 Suppose that you have a key ring with N keys, exactly one of which is your house key. Further suppose that you get home after dark and can't see the keys on the key ring. You randomly try one key at a time until you get the correct key, being careful not to mix the keys you have already tried with the ones you haven't.

a) Use counting techniques to determine the probability that you get the correct key on the nth try, where n is an integer between 1 and N, inclusive.

b) Solve part (a) without doing any computations.

c) Determine the probability that you get the correct key on or before the nth try by using (i) a direct counting and (ii) your result from part (a) and the additivity property of a probability measure.

3.71 Refer to Exercise 3.70, but now suppose that you mix the keys you have already tried with the ones you haven't.

a) Use counting techniques to determine the probability that you get the correct key for the first time on the nth try, where n is any positive integer.

b) Determine the probability that you get the correct key on or before the nth try by using (i) a direct counting and (ii) your result from part (a) and the additivity property of a probability measure. Here n is any positive integer. *Hint:* Use the complementation rule.

3.72 A student takes a true–false test consisting of 15 questions. Assuming that the student guesses at each question, find the probability that the student gets

a) at least 1 question correct. **b)** a 60% or better on the exam.

3.73 Refer to Exercise 3.36 on page 107.

a) Determine the probability of each of the five-card draw poker hands considered there.

b) For the hands considered in part (a), are the five-card stud poker probabilities (where the order in which the cards are received matters) different from the five-card draw poker probabilities? Explain your answer.

3.74 A gene consists of 10 subunits, each of which is normal or mutant. For a particular cell, that gene consists of 3 mutant subunits and 7 normal subunits. Before the cell divides into two daughter cells—say, cell 1 and cell 2—the gene duplicates. The corresponding gene of cell 1 consists of 10 subunits chosen at random from the 6 mutant subunits and 14 normal subunits; cell 2 gets the remaining subunits. Determine the probability that one of the daughter cells consists of all normal subunits.

3.75 If n balls are distributed randomly into n boxes, what is the probability that exactly one box is empty?

3.76 Suppose that you and your best friend are among n people to be arranged randomly in a line. Determine the probability that exactly k people are between you and your friend.

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3.88 From a group of 50 basketball players, 5 are to be selected as centers, 10 as forwards, and 10 as guards. In how many ways can this selection be made?

3.89 From the 100 members of the U.S. Senate, four committees are to be formed consisting of 10, 7, 15, and 8 members. If no senator is permitted to serve on more than one committee, in how many ways can these committees be formed?

3.90 In Example 3.7 on page 96, we considered exacta wagering in horse racing. Two similar wagers are the quinella and the trifecta. In a quinella wager, the bettor picks the two horses that she believes will finish first and second, but not in a specified order. In a trifecta wager, the bettor picks the three horses she thinks will finish first, second, and third in a specified order. For a 12-horse race, determine the number of possible

a) quinella wagers. **b)** trifecta wagers.

3.91 If a die is rolled eight times, in how many ways can it come up 4 twice, 5 three times, 6 once, and 1 twice?

3.92 A company employs 10 stenographers.

a) If 3 are to be assigned to the executive suite, 3 to the marketing department, and 4 to a general stenographic pool, in how many ways can the assignments be made?

b) If the 3 stenographers in the executive suite are to be assigned to the president, executive vice president, and financial vice president, with the remaining 7 stenographers assigned as in part (a), in how many different ways can they now be assigned?

c) If the 3 stenographers in the executive suite are to be assigned as in part (b), the 3 in the marketing department are to be assigned to the general manager, manager for domestic marketing, and manager for foreign marketing, and the remaining 4 as in part (a), in how many different ways can they now be assigned?

3.93 Determine the number of distinguishable arrangements of the letters in the word

a) MISSISSIPPI. **b)** MASSACHUSETTS.

3.94 In five-card draw poker, what is the probability that you get at least one ace on the deal?

3.95 A bridge hand consists of 13 cards dealt from an ordinary deck of 52 playing cards. Determine the probability that a bridge hand

a) contains exactly two of the four aces.

b) has an 8-4-1 distribution—eight cards of one suit, four of another, and one of another.

c) has a 5-5-2-1 distribution.

d) is void in a specified suit. **e)** is void in at least one suit.

3.96 Refer to Exercise 3.95. In the actual game of bridge, there are four players—designated North, South, East, and West. The 52 cards are dealt one by one, alternating from one player to the next, until all 52 cards are distributed. Determine the probability that

a) each player gets 1 ace.

b) North and South together have exactly 3 aces.

3.97 Balls are drawn at random without replacement from an urn containing 10 red balls and 16 black balls. Find the probability that it takes at least three draws to get the first red ball.

3.98 All dice considered in this problem are balanced.

a) If two dice are rolled—one red and one green—find the probability that the green die shows a larger number than the red die.

b) If three dice are rolled, find the probability that at least two show the same number.

3.99 A bowl contains 10 chips, numbered 1–10. Two chips are randomly selected from the bowl. Determine the probability that the sum of the numbers on the chips obtained is 10 if the sampling is

a) without replacement. **b)** with replacement.

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3.100 An urn contains 10 red, 20 white, and 30 blue balls. If 6 balls are selected at random from the urn, determine the probability that 1 red, 2 white, and 3 blue balls are obtained if the sampling is
a) without replacement. **b)** with replacement.

3.101 In a lot containing N items, exactly M are defective. Items are inspected one at a time without replacement. Determine the probability that the kth item inspected ($k \geq M$) will be the last defective one in the lot.

3.102 An Arizona state lottery, called *Lotto,* is played as follows: The player selects six numbers from the numbers 1–42 and buys a ticket for \$1. There are six winning numbers, which are selected at random from the numbers 1–42. To win a prize, a *Lotto* ticket must contain three or more of the winning numbers. A ticket with exactly three winning numbers is paid \$2. The prize for a ticket with exactly four, five, or six winning numbers depends on sales and on how many other tickets were sold that have exactly four, five, or six winning numbers, respectively. If you buy one *Lotto* ticket, determine the probability that
a) you win the jackpot; that is, your six numbers are the same as the six winning numbers.
b) your ticket contains exactly four winning numbers.
c) you don't win a prize.
d) Repeat part (a) if you buy N tickets.

3.103 Four married couples are randomly paired at a dinner table—the four women on one side of the table and the four men on the other side. What is the probability that no man sits across from his wife?

3.104 An elevator starts with six passengers and stops at eight floors. Find the probability that no two passengers get off on the same floor.

3.105 A random sample of size n is taken without replacement from a population of size N. Find the probability that k specified members of the population are included in the sample.

3.106 A random sample of size n is taken without replacement from a population of size N. The n members obtained are noted and then returned to the population. Subsequently, a random sample of size m is taken without replacement from the population, and the m members obtained are noted.
a) Determine the probability that the two samples have exactly k members in common by considering the first sample as "marking" the members of the population.
b) Repeat part (a) by considering the second sample as "marking" the members of the population.
c) Use parts (a) and (b) to establish the following combinatorial identity without doing any calculations.

$$\frac{\binom{n}{k}\binom{N-n}{m-k}}{\binom{N}{m}} = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}}$$

d) Establish the combinatorial identity in part (c) by using the combinations rule and elementary algebra.

3.107 Suppose that a die is tossed n times.
a) How many possible outcomes are there?
b) How many outcomes are there for which none of the first $n-1$ tosses are 6 and the nth toss is a 6?
c) If the die is balanced, determine the probability that none of the first $n-1$ tosses are 6 and the nth toss is a 6.

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