University of Manitoba
Department of Statistics

STAT 2400

Introduction to Probability I

Sample Test #1 (B)

Question 1:

How many different license plates are there when valid license plates are made of

(A) any 3 letters (A to Z) followed by any 3 digits (0 to 9),

(B) any 3 letters and 3 digits in any order,

(C) any 6 letters and/or digits in any order, but repeated symbols are not allowed,

(D) any 6 letters and/or digits, but any letters necessarily come before any digits.

Question 2:

You first flip a coin 4 times and record the number of flips that result in *Heads*. Given this number, you then reflip the coin that many times and now record the number of flips that result in *Tails*.

(A) List the sample space Ω for this random experiment.

(B) Let E be the event that the observed number of *Tails* is odd. List the outcomes in E.

Question 3:

Assume that three events A, B and C are such that

$$\begin{array}{lll} \mathbb{P}(A) = 0.1, & \mathbb{P}(A \cup B) = 0.3, & \\ \mathbb{P}(B) = 0.3, & \mathbb{P}(B \cup C) = 0.9, & \mathbb{P}(A \cap B \cap C) = 0.1. \\ \mathbb{P}(C) = 0.7, & \mathbb{P}(A \cup C) = 0.7, & \end{array}$$

(A) Are A and B mutually exclusive? Justify your answer.

(B) Find $\mathbb{P}(A \cup B \cup C)$.

Question 4:

For integers $n \geq 1$, define the intervals

$$A_n = [1/n, 1 + 1/n].$$

Find $\bigcup_{n=1}^{\infty} A_n$ and $\bigcap_{n=1}^{\infty} A_n$.

Question 5:

Three couples (husband and wife) are paired at random on the dance floor, with each pair consisting of one man and one woman.

(A) Determine the probability that

 (i) each husband dances with his wife,

 (ii) at least one husband dances with his wife.

Now, assume a fourth woman enters the room and that each man is paired at random with one woman (thus leaving one woman without a partner).

(B) Find the probability that

 (i) the fourth woman is the one without a partner,

 (ii) each husband dances with his wife.

Question 6:

In the expansion of $(2x - y)^{47}$, what is the coefficient of x^2y^{45}?

Question 7:

Prove Boole's inequality: if $A_1, A_2, \ldots$ are all subsets of Ω, then

$$\mathbb{P}\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} \mathbb{P}(A_i)$$

for all $n \geq 1$.

Question 8:

Prove that, for all integers $n \geq 1$,

$$\sum_{k=1}^{n} \frac{k}{2^k}\binom{n}{k} = \frac{n}{2}\left(\frac{3}{2}\right)^{n-1}.$$

Hint: You can use the following identity without proof:

$$\sum_{k=0}^{n} \frac{1}{2^k}\binom{n}{k} = \left(\frac{3}{2}\right)^{n} \qquad \text{for all integers } n \geq 1.$$