University of Manitoba
Department of Statistics

STAT 2400

Introduction to Probability I

Sample Test #1 (A)

Question 1:

You are going to first roll a six-sided die, and then draw one card from a regular deck of 52 playing cards.

(A) What is the appropriate sample space Ω for this random experiment?

(B) What is the probability that the die and card show the same denomination? For this, consider the denomination of aces to be one.

Question 2:

Assume that 25% of the patients visiting a primary care physician are referred to a specialist. Assume also that 35% of the patients visiting that physician are referred for laboratory work. Finally, assume that 45% of the patients visiting that physician are not referred to a specialist or for lab work.

If we select a random a patient visiting that physician, what is the probability the patient was referred

(A) to a specialist and for lab work,

(B) to a specialist but not for lab work.

Question 3:

Three prizes are to be given to three students taking a Statistics class. In that class, there are 53 students, 23 of which are registered in the Faculty of Science, 12 of which are in Management, 10 of which are in Engineering and 8 that are not registered in any of these faculties. How many ways can the prizes be awarded if students cannot be awarded more than one prize and if

(A) the three prizes are identical,

(B) the three prizes are identical, but at least one prize goes to a student from Science,

(C) the three prizes are different,

(D) there is a first prize and two identical runner-up prizes,

(E) one prize is specifically for Science students, the other two are are identical and can be awarded to any student (including students from Science),

(F) one prize is specifically for Science students, one is for Management students and one is for Engineering students.

Question 4:

Assume that two events A and B are such that

$$\mathbb{P}(A) = 1/4 \qquad \text{and} \qquad \mathbb{P}(B) = 1/2.$$

Show that

(A) $1/2 \leq \mathbb{P}(A \cup B) \leq 3/4$,

(B) $\mathbb{P}(A \cap B) \leq 1/4$,

(C) $\mathbb{P}(A \triangle B) \geq 1/4$.

Hint: Recall that the symmetric difference of two sets A and B, denoted $A \triangle B$, is the set of all elements belonging to either A or B, but not both.

Question 5:

The second distributive law states that if A, B and C are all subsets of U, then

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C). \tag{1}$$

Using mathematical induction, prove the generalized version of this distributive law, that is, if A and $B_1, B_2, \ldots$ are all subsets of U, then

$$A \cup \left(\bigcap_{i=1}^{n} B_i \right) = \bigcap_{i=1}^{n} (A \cup B_i),$$

for any $n \geq 2$. For this, you can use (1) without proof.

Question 6:

Assume that $B_1, B_2, \ldots, B_n$ are events that form a partition of Ω.

(A) Clearly (and precisely) explain what the previous assumption means.

(B) Show that, for any event $A \subset \Omega$, we have that

$$\mathbb{P}(A^c) = 1 - \sum_{i=1}^{n} \mathbb{P}(A \cap B_i).$$

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