MATH 2080 2025F Assignment 1 (due Sept 25, in lab)

Name: ____________________ Student number: _______________

Use this as the cover page of your assignment. Use your own paper to write the solutions. Stable your pages.

To receive full marks, assignment solutions must be legible, presented in a linear order, and well-organized. Write on one side only.

1. Let $\sqrt{5}$ be the positive number that satisfies the equation $x^2 = 5$. Show that $\sqrt{5}$ is irrational. [3]

2. Prove the following De Morgan's law: $(A \cap B)^c = A^c \cup B^c$. [3]

3. Let $a, b \in \mathbb{R}$. Suppose that for every $\varepsilon > 0$ we have $a \leq b + \varepsilon$. Use proof by contradiction to show that $a \leq b$. [2]

MATH2080 Assignment 1 Solutions

1. Assume, to the contrary, that $\sqrt{5} \in \mathbb{Q}$. Then

$$\sqrt{5} = \frac{p}{q}, \quad p, q \in \mathbb{N} \text{ and are mutually prime.}$$

We have $p^2 = 5q^2$. $5 \mid p^2$ which implies $5 \mid p$ since 5 is prime. Then $5^2 \mid p^2 = 5q^2$ and so $5 \mid q^2$ which implies $5 \mid q$.

Thus, we have $5 \mid p$ and $5 \mid q$, contradicting to p and q being mutually prime.

Therefore, such p, q do not exist. So $\sqrt{5}$ must be irrational. $\square$

2. "$\subseteq$": let $x \in (A \cap B)^c$. Then $x \notin A \cap B$.

case 1. if $x \in A$, then $x \notin B \Rightarrow x \in B^c \subseteq A^c \cup B^c$.

case 2. if $x \notin A$, then $x \in A^c \subseteq A^c \cup B^c$

$\therefore$ In either case, $x \in A^c \cup B^c$. This shows

$$(A \cap B)^c \subseteq A^c \cup B^c. \qquad (2.1)$$

"$\supseteq$": Let $x \in A^c \cup B^c$. Then $x \in A^c$ or $x \in B^c$.

$\Rightarrow x \notin A$ or $x \notin B$. In either case, $x \notin A \cap B$. So $x \in (A \cap B)^c$. This shows

$$A^c \cup B^c \subseteq (A \cap B)^c. \qquad (2.2)$$

(2.1) and (2.2) combined yield

$$(A \cap B)^c = A^c \cup B^c \qquad \square$$

3. If $a \le b$ does not hold, then $a > b$. So $a - b = l > 0$.

Take $\varepsilon = \frac{l}{2}$. Then $\varepsilon > 0$. By the assumption of this question, $a \le b + \varepsilon \Rightarrow a - b \le \varepsilon$.

This means $l \le \frac{l}{2}$. Then $2l \le l$ which implies $l \le 0$, a contradiction.

Thus, $a \le b$ must hold. $\square$