## Assignment 2

## 1 Tensorflow Softmax

- (a) See file q1\_softmax.py
- (b) See file q1\_softmax.py
- (c) The purpose of placeholder is to hold our input data, and the purpose of feed dictionaries is to feed input values to placeholders. See implementation in file q1\_classifier.py
  - (d) See file q1\_classifier.py
- (e) See file q1\_classifier.py. After the model's  $train\_op$  is called, the prediction  $y\_hat$  is computed in the forward propagation and the gradient of loss with respect to W and b is computed during the back propagation. The variable W and b will be changed.

## 2 Neural Transition-Based Dependency Parsing

(a) The parsing procedure is as follows:

stack	buffer	new dependency	transition
[ROOT]	[I, parsed, this, sentence, correctly]		Initial Configuration
[ROOT, I]	[parsed, this, sentence, correctly]		SHIFT
[ROOT, I, parsed]	[this, sentence, correctly]		SHIFT
[ROOT, parsed]	[this, sentence, correctly]	$parsed \rightarrow I$	LEFT-ARC
[ROOT, parsed, this]	[sentence, correctly]		SHIFT
[ROOT, parsed, this, sentence]	[correctly]		SHIFT
[ROOT, parsed, sentence]	[correctly]	sentence→this	LEFT-ARC
[ROOT, parsed]	[correctly]	parsed→sentence	RIGHT-ARC
[ROOT, parsed, correctly]			SHIFT
[ROOT, parsed]		$parsed \rightarrow correctly$	RIGHT-ARC
[ROOT]		ROOT→parsed	RIGHT-ARC

Table. 1: Parsing Procedure

- (b) A sentence containing n words will be parsed in  $2 \times n$  steps. This is true because every word will be shifted into the stack once, and will be removed from the stack once, therefore, the total number of steps is  $2 \times n$ .
  - (c) See file q2\_parser\_transitions.py
  - (d) See file q2\_parser\_transitions.py
  - (e) See file q2 initialization.py
  - (f) The constant  $\gamma$  can be expressed as:

$$\gamma = \frac{1}{1 - p_{drop}}$$

This is true because the expected value of  $h_{drop}$  is  $(1 - p_{drop})h$ .

(g)

- (i) By using m, the amount of steps we take at each update will now become the running average of gradients of all times. And the current gradient calculated only contribute a little to the m, therefore it is more stable and can stop the updates from varying too much.
- (ii) The parameters that have smaller gradient will get larger updates. This helps the learning because it will smooth the updates we make and try to update all parameters equally.
  - (h) See file q2\_parser\_model.py and predictions in file q2\_test.predicted.pkl

The best UAS my model achieves on the dev set is 88.70, and the UAS achieves on the test set is 88.87.

## 3 Recurrent Neural Networks: Language Modeling

(a)

(i) Assuming k is the right word at position t+1, then we have:

$$\begin{split} \frac{1}{\exp\left(-J^{(t)}(\theta)\right)} &= \frac{1}{\exp\left(\sum_{j=1}^{|V|} y_j^{(t)} \log \hat{y}_j^{(t)}\right)} \\ &= \frac{1}{\hat{y}_k^t} \\ &= \frac{1}{\bar{P}(\boldsymbol{x}_{pred}^{(t+1)} = \boldsymbol{x}^{(t+1)} | \boldsymbol{x}^{(t)}, \dots, \boldsymbol{x}^{(1)})} \\ &= \operatorname{PP}\left(y^{(t)}, \hat{y}^{(t)}\right) \end{split}$$

(ii) By using the result above, we have:

$$\log \left( \prod_{t=1}^{T} PP(y^{(t)}, \hat{y}^{(t)}) \right)^{1/T} = \log \left( \frac{1}{\exp\left(-\sum_{t=1}^{T} J^{(t)}(\theta)\right)} \right)^{1/T}$$
$$= \frac{1}{T} \sum_{t=1}^{T} J^{(t)}(\theta)$$

For any positive function, minimizing its logarithm is equivalent to minimizing f itself, therefore minimizing the geometric mean perplexity is the same as minimizing the function above, which is the arithmetic mean cross-entropy loss.

- (iii) For a single word, the perplexity is |V|. When |V|=10000, the cross-entropy loss is  $\log 10000\approx 9.21$ 
  - (b) Let's define:

$$z^{(t)} = W_h h^{(t-1)} + W_e e^{(t)} + b_1$$

$$\theta^{(t)} = U h^{(t)} + b_2$$

$$\delta_1^{(t)} = \frac{\partial J}{\partial \theta^{(t)}}$$

$$\delta_2^{(t)} = \frac{\partial J}{\partial z^{(t)}} = \delta_1^{(t)} \frac{\partial \theta^{(t)}}{\partial h^{(t)}} \frac{\partial h^{(t)}}{\partial z^{(t)}}$$

We then have:

$$\begin{split} & \delta_{1}^{(t)} = \hat{y}^{(t)} - y^{(t)} \\ & \frac{\partial J^{(t)}}{\partial U} = \delta_{1}^{(t)} h^{(t)^{T}} \\ & \frac{\partial J^{(t)}}{\partial h^{(t)}} = U^{T} \delta_{1}^{(t)} \\ & \delta_{2}^{(t)} = (U^{T} \delta_{1}^{(t)}) \odot \left( z^{(t)} \odot (1 - z^{(t)}) \right) \\ & \frac{\partial J^{(t)}}{\partial W_{e}} |_{(t)} = \delta_{2}^{(t)} e^{(t)^{T}} \\ & \frac{\partial J^{(t)}}{\partial W_{h}} |_{(t)} = \delta_{2}^{(t)} h^{(t-1)^{T}} \\ & \frac{\partial J^{(t)}}{\partial h^{(t-1)}} = W_{h}^{T} \delta_{2}^{(t)} \end{split}$$

(c)

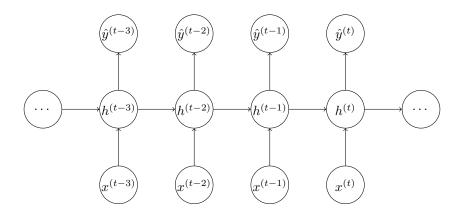


Figure. 1: The "Unrolled" Network for 3 Timesteps

The gradients can be calculated as follows:

$$\begin{split} &\frac{\partial J^{(t)}}{\partial e^{(t-1)}} = W_e{}^T \bigg( \gamma^{(t-1)} \odot \big( z^{(t-1)} \odot (1-z^{(t-1)}) \big) \bigg) \\ &\frac{\partial J^{(t)}}{\partial W_e}|_{(t-1)} = \delta_2^{(t)} e^{(t)^T} + \bigg( \gamma^{(t-1)} \odot \big( z^{(t-1)} \odot (1-z^{(t-1)}) \big) \bigg) e^{t-1^T} \\ &\frac{\partial J^{(t)}}{\partial W_h}|_{(t-1)} = \delta_2^{(t)} h^{t-1^T} + \bigg( \gamma^{(t-1)} \odot \big( z^{(t-1)} \odot (1-z^{(t-1)}) \big) \bigg) h^{t-2^T} \end{split}$$

- (d) Given  $h^{(t-1)}$ , first we need  $O(d \times |V| + D_h \times D_h + D_h \times d + |V| \times D_h)$  to do forward propagation. Then we need  $O(|V| \times D_h + |V| \times D_h + D_h \times d + D_h \times D_h + D_h \times D_h)$  to do back propagation.
- (e) The total number of operations we need to do forward propagation is simply  $O(T \times (d \times |V| + D_h \times D_h + D_h \times d + |V| \times D_h))$ , and the total number of operations we need to do back propagation is:

$$O(3 \times T \times (|V| \times D_h + D_h \times d + D_h \times D_h))$$

(f) The term  $|V| \times D_h$  is likely to be the largest if we use a large corpus, and it's from the output layer (the one that calculates the output according to current hidden state) of the RNN.